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Unemployment and Business Cycles*

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Abstract

We develop and estimate a general equilibrium model that accounts for key business cycle properties of macroeconomic aggregates, including labor market variables. In sharp contrast to leading New Keynesian models, wages are not subject to exogenous nominal rigidities. Instead we derive wage inertia from our specification of how firms and workers interact when negotiating wages. Our model outperforms the standard Diamond-Mortensen-Pissarides model both statistically and in terms of the plausibility of the estimated structural parameter values. Our model also outperforms an estimated sticky wage model.

Keywords: unemployment, business cycles, wage inertia, Bayesian estimation

JEL: E2, E24, E32

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1. Introduction

Employment and unemployment fluctuate a great deal over the business cycle. Macroeconomic models have difficulty accounting for this fact. See for example the classic real business cycle models of Kydland and Prescott (1982) and Hansen (1985). Models that build on the search-theoretic framework of Diamond (1982), Mortensen (1985) and Pissarides (1985) (DMP) also have difficulty accounting for the volatility of labor markets, see Shimer (2005a). In both classes of models, the problem is that real wages rise sharply in business cycle expansions, thereby limiting firms' incentives to expand employment. The proposed solutions depend on controversial assumptions, such as high labor supply elasticities or high replacement ratios.¹

Empirical New Keynesian models have been relatively successful in accounting for the cyclical properties of employment. However, they do so by assuming that wage-setting is subject to nominal rigidities and that employment is demand-determined.² These assumptions prevent the sharp rise in wages that limits the employment responses in standard models. Empirical New Keynesian models have been criticized on at least four grounds. First, these models do not explain wage inertia, they just assume it. Second, agents in the model would not choose the wage arrangements that are imposed upon them by the modeler.³ Third, empirical New Keynesian models are inconsistent with the fact that many wages are constant for extended periods of time. In practice, these models assume that agents who do not reoptimize their wage simply index it to technology growth and inflation.⁴ So, these models predict that all wages are always changing. Fourth, these models cannot be used to examine some key policy issues such as the effects of an extension of unemployment benefits.

In this paper we develop and estimate a model that accounts for the response of key macro aggregates, including labor market variables like wages, employment, job vacancies and unemployment to identified monetary policy shocks, neutral technology shocks and investment-specific technology shocks. In contrast to leading empirical New Keynesian models, we do not assume that wages are subject to exogenous nominal rigidities. Instead, we derive wage inertia as an equilibrium outcome. Like empirical New Keynesian models, we

¹For discussions of high labor supply elasticities in real business cycle models, see for example, Rogerson and Wallenius (2009) and Chetty, Guren, Manoli and Weber (2012). For discussions of the role of high replacement ratios in DMP models see for example, Hagedorn and Manovskii (2008) and Hornstein, Krusell and Violante (2010).

²For example, Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) and Gali, Smets and Wouters (2012) assume that nominal wages are subject to Calvo-style rigidities.

³This criticism does not necessarily apply to a class of models initially developed by Hall (2005). We discuss these models in the conclusion.

⁴See, for example, Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), Justiniano, Primiceri and Tambalotti (2010), Christiano, Trabandt and Walentin (2011), and Gali, Smets and Wouters (2012).

assume that price setting is subject to exogenous nominal Calvo-style rigidities. Guided by the micro evidence on prices, we assume that firms which do not reoptimize their price must keep it unchanged, i.e. no price indexation.

We take it as given that a successful model must have the property that wages are relatively insensitive to the aggregate state of the economy. Our model of the labor market builds on Hall and Milgrom (2008), henceforth HM.⁵ In practice, by the time workers and firms sit down to bargain, they know there is a surplus to be shared if they can come to terms. So, rather than just going their separate ways in the wake of a disagreement, workers and firms continue to negotiate.⁶ This process introduces a delay in the time required to make a deal. This delay is costly for both workers and firms. HM's key insight is that if the associated costs are relatively insensitive to the aggregate state of the economy, then negotiated wages will inherit that insensitivity.

This paper investigates whether a dynamic general equilibrium model which embeds this source of wage inertia can account for key business cycle properties of labor markets. We show that it does. In the wake of an expansionary shock, wages rise by a relatively small amount, so that firms receive a substantial fraction of the rents associated with employment. Consequently, firms have a strong incentive to expand their labor force. In addition, the muted response of wages to aggregate shocks makes firms' marginal costs relatively acyclical. This acyclicity of marginal costs enables our model to account for the inertial response of inflation even with modest exogenous nominal rigidities in prices.

In our benchmark model, we assume that workers and firms bargain over the current wage rate in each period, taking as given the outcome of future wage negotiations (*period-by-period bargaining*). We also consider an alternative approach in which firms and workers bargain just once, when they first meet. At that time, they bargain over the present discounted value of the wages that prevail throughout the duration of their match (*present discounted value bargaining*). In this approach, firms and workers are indifferent about how wages are paid out over dates and states of nature, as long as the present discounted value of wage payments is consistent with the outcome of their negotiation. We show that the two approaches to bargaining lead to identical equilibrium allocations, though to possibly different spot wages. For example, present discounted value bargaining is consistent with the nominal wage of an individual worker being constant for extended periods of time and wages of job changers being more volatile than those of incumbent workers.

We adopt period-by-period bargaining as our benchmark specification. First, it allows us to incorporate wage data into our empirical analysis. Second, the present discounted value

⁵For a paper that uses a reduced form version of HM in a calibrated real business cycle model, see Hertweck (2006).

⁶This perspective on bargaining has been stressed in Rubinstein (1982), Binmore (1985) and Binmore, Rubinstein and Wolinsky (1986).

bargaining makes strong assumptions about agents' ability to commit to streams of wage payments. We defer analyzing the time consistency of these commitments to future work.

We estimate our model using a Bayesian variant of the strategy in Christiano, Eichenbaum and Evans (2005), henceforth CEE. That strategy involves minimizing the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified vector autoregression (VAR) for 12 post-war, quarterly U.S. times series that includes key labor market variables. We contrast the empirical properties of our model with estimated versions of leading alternatives. The first alternative is a variant of our model in which the labor market corresponds closely to the standard DMP model. The second alternative is a version of the standard New Keynesian sticky wage model of the labor market proposed in Erceg, Henderson and Levin (2000), henceforth EHL. The version that we emphasize does not allow for wage indexation because the resulting implications are strongly at variance with micro data on nominal wages of incumbent workers. For comparability we also report results for a version of the sticky wage model with wage indexation.

We show that our model outperforms the DMP model in terms of both model fit and the plausibility of the estimated structural parameter values. For example, in the estimated DMP model, the replacement ratio of income for unemployed workers is substantially higher than the upper bound suggested by existing microeconomic evidence. Our models also outperforms the DMP model based on the metrics adopted in the labor market search literature. Authors like Shimer (2005a) emphasize that the standard deviation of labor market tightness (vacancies divided by unemployment) is orders of magnitude higher than the standard deviation of labor productivity. Our model has no difficulty in accounting for the statistics that Shimer (2005a) emphasizes.

Finally, we show that our model outperforms the sticky wage New Keynesian model with no wage indexation in terms of statistical fit. The statistical fit of the model with indexation is marginally worse than our model. We conclude that given the limitations of the sticky wage models, there is simply no need to work with them. The alternating offer bargaining model has stronger micro foundations, fits the data better and can be used to analyze a broader set of labor market variables, e.g. job vacancies and job finding rates.

Our paper is organized as follows. Section 2 describes the labor market of our model in isolation. Section 3 integrates the labor market model into a simple New Keynesian model without capital. We use this model to discuss the intuition about how our model of the labor market works in a general equilibrium setting with sticky prices. Section 4 describes our empirical model. Section 5 describes our econometric methodology. Section 6 presents our empirical results. Section 7 contains concluding remarks.⁷

⁷A technical appendix is available at: sites.google.com/site/mathiastrabandt/home/downloads/CETtechapp.pdf.

2. The Labor Market

In this section we discuss our model of the labor market. We assume there is a large number of identical, competitive firms that produce a homogeneous good using labor. Let ϑ_t denote the marginal revenue from hiring an additional worker. Here, we treat ϑ_t as an exogenous stochastic process. In the next section, we embed the labor market in a general equilibrium model and determine the equilibrium process for ϑ_t .

At the beginning of period t , a firm pays a fixed cost, κ , to meet a worker with probability one. We refer to this specification as the hiring cost specification. Once a worker and a firm meet, they engage in bilateral bargaining. If bargaining results in agreement (as it always does in equilibrium) the worker begins production immediately.

We denote the number of workers employed in period t by l_t . The size of the labor force is normalized to one. At the end of the period, a fraction $1 - \rho$ of randomly selected employed workers is separated from their firm. These workers join the ranks of the unemployed and search for work. So, at the end of the period, there are $1 - \rho l_t$ workers searching for a job. In period $t + 1$ a fraction, f_{t+1} , of searching workers meet a firm and the complementary fraction becomes unemployed. With probability ρ , a worker who is employed at time t remains with the same firm in period $t + 1$. With probability $(1 - \rho) f_{t+1}$ this worker moves to another firm in period $t + 1$. Finally, with probability $(1 - \rho)(1 - f_{t+1})$ this worker is unemployed in period $t + 1$. Our measure of unemployment in period t is $1 - l_t$. We think of workers that change jobs between t and $t + 1$ as job-to-job movements in employment. There are $(1 - \rho) f_{t+1} l_t$ workers of this type. With our specification, the job-to-job transition rate is substantial and procyclical, consistent with the data (see Shimer, 2005b). While controversial, the standard assumption that the job separation rate is acyclical has been defended on empirical grounds (see Shimer, 2005b).⁸ Finally, we think of the time period as one quarter.

Let w_t^p denote the expected present discounted value of the wage payments by a firm to a worker that it is matched with:

$$w_t^p = w_t + \rho E_t m_{t+1} w_{t+1}^p. \quad (2.1)$$

Here w_t denotes the time t wage rate. The discount factor m_{t+1} is an exogenous stochastic process. In our general equilibrium model (see sections 3 and 4), we determine the endogenous equilibrium process for m_t . Let J_t denote the value to a firm of employing a worker in period t :

$$J_t = \vartheta_t^p - w_t^p, \quad (2.2)$$

⁸For a different view, see Fujita and Ramey (2009) who argue that the job separation rate is counter-cyclical.

where ϑ_t^p denotes the expected present discounted value of ϑ_t ,

$$\vartheta_t^p = \vartheta_t + \rho E_t m_{t+1} \vartheta_{t+1}^p. \quad (2.3)$$

Because there is free entry into the labor market, firm profits must be zero. It follows that,

$$\kappa = J_t. \quad (2.4)$$

We denote by V_t the value to a worker of being matched with a firm that pays w_t in period t :

$$V_t = w_t + E_t m_{t+1} [\rho V_{t+1} + (1 - \rho) (f_{t+1} \bar{V}_{t+1} + (1 - f_{t+1}) U_{t+1})]. \quad (2.5)$$

Here, \bar{V}_{t+1} denotes the value of working for another firm in period $t + 1$. In equilibrium, $\bar{V}_{t+1} = V_{t+1}$. Finally, U_{t+1} in (2.5) is the value of being an unemployed worker in period $t + 1$. It is convenient to rewrite (2.5) as follows:

$$V_t = w_t^p + A_t, \quad (2.6)$$

where

$$A_t = (1 - \rho) E_t m_{t+1} [f_{t+1} \bar{V}_{t+1} + (1 - f_{t+1}) U_{t+1}] + \rho E_t m_{t+1} A_{t+1}. \quad (2.7)$$

Note that V_t consists of two components. The first is the expected present value of the wages received by a worker from a firm that he is matched with at time t . The second corresponds to the expected present value of the payments that a worker receives in all dates and states when he is separated from that firm.

The value of unemployment, U_t , is given by,

$$U_t = D + \tilde{U}_t, \quad (2.8)$$

where \tilde{U}_t denotes the continuation value of unemployment:

$$\tilde{U}_t \equiv E_t m_{t+1} [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}]. \quad (2.9)$$

In (2.8), D denotes goods received by an unemployed worker from the government.

The number of employed workers evolves as follows:

$$l_t = (\rho + x_t) l_{t-1}. \quad (2.10)$$

Here x_t denotes the hiring rate so that the number of new hires in period t is equal to $x_t l_{t-1}$. The job finding rate is given by,

$$f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}}. \quad (2.11)$$

The numerator is the number of newly hired workers at the beginning of time t . The denominator is the number of workers who are searching for work at the end of time $t - 1$.

2.1. Wage Determination: Alternating Offer Bargaining

Our baseline specification assumes period-by-period bargaining. That is, we assume that workers and firms bargain in period t over the period t wage rate, taking as given the outcome of future bargains that will occur as long as they remain matched. Future wage agreements matter for current negotiations via their present discounted value, \tilde{w}_t^p :

$$\tilde{w}_t^p \equiv E_t \sum_{l=1}^{\infty} \rho^l [m_{t+1} \dots m_{t+l}] w_{t+l} = \rho E_t m_{t+1} w_{t+1}^p. \quad (2.12)$$

The bargaining problem of all workers is the same, regardless of how long they have been matched with a firm. This result follows from our assumptions that hiring costs are sunk at the time of bargaining and the expected duration of a match is independent of how long a match has already been in place.

Consistent with Hall and Milgrom (2008), wages are determined according to the alternating offer bargaining protocol proposed in Rubinstein (1982) and Binmore, Rubinstein and Wolinsky (1986). Each time period (a quarter) is subdivided into M periods of equal length, where M is even. Firms make a wage offer at the start of the first subperiod. They also make offers at the start of every subsequent odd subperiod in the event that all previous offers have been rejected. Similarly, workers make a wage offer at the start of all even subperiods in case all previous offers have been rejected. Because M is even, the last offer is made, on a take-it-or-leave-it basis, by the worker. In subperiod $j = 1, \dots, M - 1$, the recipient of an offer can either accept or reject it. If the offer is rejected the recipient may declare an end to the negotiations or he may plan to make a counteroffer at the start of the next subperiod. In the latter case there is a probability, δ , that bargaining breaks down.

Consider a firm that makes a wage offer, $w_{j,t}$, in subperiod for $j < M$, j odd. The firm sets $w_{j,t}$ as low as possible subject to the condition that the worker does not reject it. Other things equal, the firm would like to make an offer that worker accepts because a lack of agreement delays the onset of production. So, it is optimal for the firm to offer the lowest wage subject to the worker not rejecting it. The resulting wage offer, $w_{j,t}$ satisfies the following indifference condition on the part of the worker:

$$V_{j,t} = \delta U_{j,t} + (1 - \delta) \left[\frac{1}{M} D + V_{j+1,t} \right]. \quad (2.13)$$

We assume that when an agent is indifferent between accepting and rejecting an offer, he accepts it. The left hand side of (2.13), $V_{j,t}$, denotes the value to a worker of accepting the wage offer $w_{j,t}$,

$$V_{j,t} = w_{j,t} + \tilde{w}_t^p + A_t, \quad (2.14)$$

where \tilde{w}_t^p and A_t are taken as given by a worker-firm bargaining pair. The right hand side of (2.13) represents the worker's disagreement payoff, i.e. the value to a worker of rejecting the

wage offer, with the intention of making a counteroffer. The first term on the right hand side of (2.13) reflects the possibility that negotiations exogenously break down and the worker becomes unemployed. The value of becoming unemployed at time t , in subperiod j is given by $U_{j,t}$,

$$U_{j,t} = \frac{M - j + 1}{M} D + \tilde{U}_t,$$

where the term involving D reflects our assumption that the worker receives unemployment benefits in period t in proportion to the number of subperiods spent in unemployment.

The second term on the right hand side of (2.13) reflects the fact that with probability $1 - \delta$ the worker will receive unemployment benefits for a period $1/M$ and make a counteroffer $w_{j+1,t}$ to the firm which he expects to be accepted. Equation (2.13) represents the relevant indifference condition assuming that the worker's disagreement payoff exceeds the value of his outside option, $U_{j,t}$. In practice one must verify that this condition holds.

Next, consider the problem of a worker who makes an offer in subperiod, j , where $j < M$ and j is even. Other things equal, the worker would like to make an offer that the firm accepts because a lack of agreement delays the payment of wages whose value exceeds unemployment benefits. So, it is optimal for the worker to offer the highest wage subject to the firm not rejecting it. The resulting wage offer, $w_{j,t}$, satisfies the following indifference condition on the part of the firm:

$$J_{j,t} = \delta \times 0 + (1 - \delta) [-\gamma + J_{j+1,t}]. \quad (2.15)$$

The left hand side of (2.15) denotes the value to a firm of accepting the wage offer $w_{j,t}$,

$$J_{j,t} = \frac{M - j + 1}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{j,t} + \tilde{w}_t^p), \quad (2.16)$$

where

$$\tilde{\vartheta}_t^p \equiv E_t \sum_{l=1}^{\infty} \rho^l [m_{t+1} \dots m_{t+l}] \vartheta_{t+l} = \rho E_t m_{t+1} \vartheta_{t+1}^p. \quad (2.17)$$

The first term on the right hand side of (2.16) is the value to the firm of accepting the worker's offer and beginning production immediately. The term $(M - j + 1) / M$ in (2.16) reflects our assumption that one worker produces $1/M$ goods in each subperiod during which production occurs. The term $\tilde{\vartheta}_t^p$ represents the expected present value of future marginal revenues associated with a worker, while $(w_{j,t} + \tilde{w}_t^p)$ represents the expected present value of wage payments to the worker if the firm accepts the wage offer $w_{j,t}$.

The expression on the right side of (2.15) is the firm's disagreement payoff. If the firm rejects the worker's offer with the intention of making a counteroffer there is a probability, δ , that negotiations break down and the firm goes to its outside option whose value is zero. With probability $1 - \delta$ the firm makes a counteroffer, $w_{j+1,t}$, in the next subperiod. To make a counteroffer, the firm incurs a cost, γ . The second expression in the square bracketed

term in (2.15) reflects that the value associated with a successful firm counteroffer, $w_{j+1,t}$. Equation (2.15) represents the relevant indifference condition governing the worker's wage offer assuming that the firm's disagreement payoff exceeds the value of its outside option, i.e., zero. In practice one must verify that this condition holds.

Finally, consider subperiod M in which the worker makes a take-it-or-leave-it offer. The worker chooses the highest possible wage subject to the condition that the firm does not reject it and go to the outside option. Since the latter has zero value, we can write the firm's indifference condition as:⁹

$$J_{M,t} = 0, \quad (2.18)$$

where

$$J_{M,t} = \frac{1}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{M,t} + \tilde{w}_t^p). \quad (2.19)$$

We now summarize how to compute the equilibrium wage rate. Note that $w_{j,t}$ and \tilde{w}_t^p always appear as a sum in the equilibrium conditions, (2.13), (2.15) and (2.18),

$$w_{j,t}^p \equiv w_{j,t} + \tilde{w}_t^p, \quad (2.20)$$

for $j = 1, \dots, M$. We can solve for w_M^p given the variables that are exogenous to the worker-firm pair. Then, (2.13) for $j = M - 1$ can be solved for w_{M-1}^p and (2.15) can be solved for w_{M-2}^p . In this way, the equilibrium conditions can be used to solve uniquely for a set of values

$$w_{1,t}^p, w_{2,t}^p, w_{3,t}^p, \dots, w_{M,t}^p, \quad (2.21)$$

conditional on variables that are exogenous to the worker-firm bargaining pair. The equilibrium present discounted value of the wage, w_t^p , is just $w_{1,t}^p$. Because (2.13), (2.15) and (2.18) are simple linear equations, they can be solved analytically for w_t^p :

$$w_t^p = \frac{1}{\alpha_1 + \alpha_2} [\alpha_1 \vartheta_t^p + \alpha_2 (U_t - A_t) + \alpha_3 \gamma - \alpha_4 (\vartheta_t - D)], \quad (2.22)$$

where

$$\begin{aligned} \alpha_1 &= 1 - \delta + (1 - \delta)^M \\ \alpha_2 &= 1 - (1 - \delta)^M \\ \alpha_3 &= \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1 \\ \alpha_4 &= \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2. \end{aligned}$$

⁹Note that in this expression (and elsewhere) the firm's outside option does not have to take into account that it is costly for the firm to meet another worker at the start of period $t + 1$. This reflects that in our environment the firm's decision to undertake an expense to meet a worker is unrelated to its labor market experience in previous periods.

It can be shown that α_i , for $i = 1, 2, 3, 4$ are strictly positive. Appendix A contains a detailed derivation of the previous equation.

Finally, we use (2.1) and (2.12) to compute the period t wage rate w_t , conditional on w_t^p and a given set of beliefs about future wages as summarized by \tilde{w}_t^p . Our analysis indicates that w_t^p is uniquely determined conditional on the variables that are exogenous to the worker-firm pair, $\vartheta_t, \vartheta_t^p, U_t, A_t$. In principle the additive decomposition of w_t^p into the current wage rate, w_t , and future payments summarized by \tilde{w}_t^p is not uniquely determined. We resolve this potential non-uniqueness in the timing of wage payments by our assumption that wages in each date are the same time-invariant function of a small set of state variables.

Given laws of motion for ϑ_t and m_{t+1} , a period-by-period bargaining equilibrium is a stochastic process for the ten variables,

$$J_t, w_t^p, V_t, U_t, l_t, f_t, x_t, A_t, \tilde{w}_t^p, w_t, \quad (2.23)$$

which satisfies the following equilibrium conditions, (i) the first eight variables in (2.23) satisfy the eight equations, (2.2), (2.4), (2.6), (2.7), (2.8), (2.10), (2.11) and (2.22), (ii) the stochastic process, \tilde{w}_t^p in (2.23), satisfies (2.12), and (iii) w_t satisfies (2.1),

$$w_t = w_t^p - \tilde{w}_t^p.$$

In the following subsection, we exploit the *block recursive structure* of this equilibrium, i.e. the fact that the first eight variables in (2.23) can be computed without reference to w_t and \tilde{w}_t^p .

We conclude this subsection by noting that in the standard DMP setup, w_t^p is determined by the following *Nash sharing rule*:

$$J_t = \frac{1 - \eta}{\eta} (V_t - U_t) \quad (2.24)$$

where $0 \leq \eta \leq 1$ is the share of the total surplus given to workers. It is straightforward to show that (2.22) can be re-written as an *Alternating Offer Bargaining sharing rule*:

$$J_t = \beta_1 (V_t - U_t) - \beta_2 \gamma + \beta_3 (\vartheta_t - D), \quad (2.25)$$

where $\beta_i = \alpha_{i+1}/\alpha_1$, for $i = 1, 2, 3$. So, as in the Nash sharing rule, w_t^p depends on J_t and $(V_t - U_t)$, with weights determined by the parameters describing the environment of the model economy. However, there are two constant terms involving γ and D that are, by assumption, not a function of the state of the economy, as well as a separate term in ϑ_t . In section 3 we provide intuition for how the parameters of agents' environment affect the sensitivity of wages to different shocks to the economy.

2.2. Implications for Wages

In our baseline model bargaining occurs on a period-by-period basis. We now consider the present discounted value bargaining arrangement and show that it leads to the identical quantity allocation as period-by-period bargaining. Moreover the resulting equilibrium is consistent with the following empirical observations: (i) incumbent workers and firms do not renegotiate wages every period; (ii) the nominal wage of incumbent workers is often constant for extended periods of time; and (iii) the volatility of wages paid to new hires is higher than the volatility of wages paid to incumbent workers.

Under present discounted value bargaining, workers and firms only bargain once, namely when they first meet. The negotiations pertain to the wage that the firm will pay to the worker in each date and state of nature where they remain matched. Since workers and firms only care about the present discounted value of wages, we suppose that they begin by bargaining over the scalar, w_t^p . The structure of bargaining parallels the period-by-period bargaining framework discussed in the previous subsection, with obvious modifications.

As before, we divide the quarter in which the firm and worker first meet into M subperiods. At the start of the first subperiod, the firm makes an offer, which we denote by $w_{1,t}^p$. The firm makes the lowest possible offer subject to the constraint that the offer is not rejected by the worker. This constraint implies that $w_{1,t}^p$ satisfies a version of (2.13) in which $w_{j,t} + \tilde{w}_t^p$ in (2.14) is replaced by $w_{j,t}^p$, for $j = 1$. To evaluate the right side of (2.13) the firm must know the worker's counteroffer, $w_{2,t}^p$, in case the worker rejects the firm's offer. The worker's counteroffer satisfies a version of (2.15) with $w_{j,t} + \tilde{w}_t^p$ in (2.16) replaced by $w_{j,t}^p$, for $j = 2$. But, to evaluate the right side of (2.15) with $j = 2$ the worker must know the firm's counteroffer, $w_{3,t}^p$, in case the firm rejects $w_{2,t}^p$, and so on. It follows that to make its initial offer $w_{1,t}^p$, the firm must solve for $w_{j,t}^p$, for all $j > 1$. If all offers $w_{j,t}^p$, $j = 1, \dots, M - 1$ are rejected, then the worker makes a final take-it-or-leave-it offer, $w_{M,t}^p$. This offer has the property that a version of (2.18) holds, the modification being that $w_{M,t} + \tilde{w}_t^p$ in (2.19) is replaced with $w_{M,t}^p$.

The solution to this sequence of equations is a unique set, (2.21), which can be computed by iterating through the subperiods beginning with $j = M$ and working backwards. By construction, the offer $w_{1,t}^p$ is accepted and corresponds to the equilibrium present discounted value of the wage, w_t^p . The M equations whose solution yields w_t^p are exactly the same as the equation used to solve the period-by-period bargaining problem. So, the solution has the same characterization, (2.22).

An agreement between a worker and firm constitutes a sequence of wage rates indexed by each of the dates and states of nature over the duration of their match, subject to the constraint that the sequence is consistent with the agreed-upon value of w_t^p . We denote by

\mathcal{W}_t the sequence of date and state contingent wage rates that the firm and the worker agree on. We think of a given specification of \mathcal{W}_t as a particular wage payment scheme. Given laws of motion for ϑ_t and m_{t+1} a present discounted value bargaining equilibrium is a set of nine stochastic processes,

$$J_t, w_t^p, V_t, U_t, l_t, f_t, x_t, A_t, \mathcal{W}_t. \quad (2.26)$$

The first eight of these stochastic processes satisfy the same eight conditions described after (2.23) and the wage payment scheme \mathcal{W}_t must satisfy the restriction that the expected present value of its elements is equal to w_t^p .

Suppose that we have a set of the ten objects in (2.23) that satisfy the relevant equilibrium conditions. The first eight of these objects satisfy the same conditions required of the first eight objects in (2.26). Also let \mathcal{W}_t consist of the sequence of wage rates in the period-by-period bargaining equilibrium. The expected present value of that sequence of wage rates is w_t^p . We conclude:

Proposition 2.1. *A period-by-period bargaining equilibrium is a present discounted value bargaining equilibrium.*

An implication of this proposition is that the equilibrium in our baseline model is consistent with empirical observation (i), namely incumbent workers and firms do not renegotiate wages every period. This consistency follows because we can interpret the period-by-period bargaining equilibrium as a particular present discounted value bargaining equilibrium in which the wage rates in each date and state of nature correspond to the relevant wage rates in our period-by-period bargaining equilibrium.

The equilibrium in our baseline model is also in principle consistent with empirical observations (ii) and (iii), namely that the nominal wage of incumbent workers often does not change for long periods of time and the wages of newly hired workers are more volatile than those of incumbent workers. The reason is that under present discounted value bargaining, the following payment scheme is an equilibrium: a worker that bargains in period t receives a fixed nominal wage payment, W_t , for each date and state in which the match continues. This equilibrium is clearly consistent with observations (ii) and (iii).

A potential difficulty with the way that we account for observations (ii) and (iii) is that the equilibrium may not be time consistent. By time consistency of a present value bargaining equilibrium we mean that if present value bargaining was re-started at some time in the future, the firm and worker would again agree on the same state and date contingent wage rates they agreed on when they first met. Clearly not all present discounted value bargaining equilibria are time consistent. For example, suppose that \mathcal{W}_t involves the firm paying w_t^p to the worker in period t and zero thereafter. If bargaining were re-opened at a later date,

the worker would not accept a zero wage payment. So, in general, present discounted value bargaining requires that we make strong assumptions about agents' ability to commit.

3. Incorporating the Labor Market Model into a Simple Macroeconomic Framework

In this section we incorporate the labor market model of the previous section into the benchmark New Keynesian macroeconomic model using a structure that is very similar to Ravenna and Walsh (2008). We use this framework to explore the intuition for how the alternating offer bargaining model of the labor market helps to account for the cyclical behavior of key macroeconomic variables.

3.1. Simple Framework

As in Andolfatto (1995) and Merz (1996), we assume that each household has a unit measure of workers. Because workers experience no disutility from working, they supply their labor inelastically. An employed worker brings home the real wage, w_t . An unemployed worker receives D goods in government-provided unemployment compensation. Unemployment benefits are financed by lump-sum taxes paid by the household. Workers maximize their expected income, subject to the labor market arrangements described in the previous section. By the law of large numbers, this strategy maximizes the total income of the household. Workers maximize expected income in exchange for perfect consumption insurance from the household. All workers have the same concave preferences over consumption. So, the optimal insurance arrangement involves allocating the same level of consumption, C_t , to each worker.

The household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln C_t$$

subject to the budget constraint:

$$P_t C_t + B_{t+1} \leq W_t l_t + (1 - l_t) P_t D + R_{t-1} B_t - T_t.$$

Here $0 \leq l_t \leq 1$ denotes the fraction of household members who are employed. In addition, T_t denotes lump-sum taxes net of lump-sum transfers and profits. Also B_{t+1} denotes purchases of bonds in period t and R_{t-1} denotes the gross nominal interest rate on bonds purchased in the previous period. Finally, the variables W_t and P_t denote the nominal wage rate and price of the final good.

A final homogeneous good, Y_t , is produced by competitive and identical firms using the following technology:

$$Y_t = \left[\int_0^1 (Y_{j,t})^{\frac{1}{\lambda}} dj \right]^\lambda, \quad (3.1)$$

where $\lambda > 1$. The representative firm chooses specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to the production function (3.1). The firm's first order condition for the j^{th} input is:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\frac{\lambda}{\lambda-1}} Y_t. \quad (3.2)$$

As in Ravenna and Walsh (2008), the j^{th} input good is produced by a monopolist *retailer*, with production function,

$$Y_{j,t} = \exp(a_t) h_{j,t},$$

where $h_{j,t}$ is the quantity of the intermediate good purchased by the j^{th} producer. This intermediate good is purchased in competitive markets at the after-tax price $(1 - \nu) P_t^h$ from a *wholesaler*. Here, ν represents a subsidy (financed by a lump-sum tax on households) which has the effect of eliminating the monopoly distortion in the steady state. That is, $1 - \nu = 1/\lambda$ where λ denotes the steady state markup. In the retailer production function, a_t denotes a technology shock that has the law of motion:

$$a_t = \tau a_{t-1} + \varepsilon_t,$$

where ε_t is the *i.i.d.* shock to technology and $|\tau| < 1$.

The monopoly producer of $Y_{j,t}$ sets $P_{j,t}$ subject to Calvo sticky price frictions. In particular,

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1 - \xi \end{cases}. \quad (3.3)$$

Here, \tilde{P}_t denotes the optimal price set by the fraction $1 - \xi$ of producers who have the opportunity to reoptimize. Note that we do not allow for price indexation. So, the model is consistent with the observation that many prices remain unchanged for extended periods of time (see Eichenbaum, Jaimovich and Rebelo, 2011, and Klenow and Malin, 2011).

Let,

$$s_t \equiv \frac{\vartheta_t}{\exp(a_t)}, \quad (3.4)$$

where $\vartheta_t = P_t^h / P_t$ so that $(1 - \nu) s_t$ denotes the retail firm's real marginal cost. Also, let

$$h_t = \int_0^1 h_{j,t} dj.$$

The wholesalers who produce h_t correspond to the perfectly competitive firms modeled in the previous section. Recall that they produce h_t using labor only and that labor has a fixed marginal productivity of unity. The total supply of the intermediate good is given by l_t which equals the total quantity of labor used by the wholesalers. So, clearing in the market for intermediate goods requires

$$h_t = l_t. \quad (3.5)$$

We adopt the following monetary policy rule:

$$\ln(R_t/R) = \rho_R \ln(R_{t-1}/R) + (1 - \rho_R) [r_\pi \ln(\pi_t/\pi) + r_y \ln(l_t/l)] + \varepsilon_{R,t} \quad (3.6)$$

where $\pi_t = P_t/P_{t-1}$ denotes the gross inflation rate and $\varepsilon_{R,t}$ is a monetary policy shock. In addition, a time series variable without a time subscript refers to its value in nonstochastic steady state.

3.2. Integrating the Labor Market into the Simple Framework

There are four points of contact between the model in this section and the one in the previous section. The first point of contact is the labor market in the wholesale sector where the real wage is determined as in section 2. The second point of contact is via ϑ_t in (3.4), which corresponds to the real price that appears in the previous section (see (2.3)). The third point of contact occurs via the asset pricing kernel, m_{t+1} , which is given by:

$$m_{t+1} = \beta \frac{C_t}{C_{t+1}}. \quad (3.7)$$

The fourth point of contact is the resource constraint which specifies how the homogeneous good, Y_t , is allocated among its possible uses:

$$C_t + \kappa x_t l_{t-1} = Y_t, \quad (3.8)$$

where

$$Y_t = \exp(a_t) l_t. \quad (3.9)$$

Here, $\kappa x_t l_{t-1}$ denotes the cost of generating new hires in period t . The expression on the right side of (3.9) is the production function for the final good. The absence of price distortions in this expression reflects Yun's (1996) result that these distortions can be ignored in (3.9) when linearizing around a nonstochastic steady state without price distortions.

From the perspective of the model in this section, the prices in the previous section correspond to real prices. So, w_t corresponds to the real wage rate, where conversion to real is accomplished using P_t . That is, workers and firms bargain over real wages according to the alternating wage offer arrangement described in section 2. See the technical appendix for a list of the equilibrium equations.

3.3. Quantitative Results in the Simple Model

This subsection displays the dynamic response of our simple model to monetary policy and technology shocks. In addition, we discuss the sensitivity of these responses to the wage bargaining parameters, δ, γ, D and M . The first subsection below reports a set of baseline parameter values for the model. The second subsection presents and discusses impulse responses.

3.3.1. Baseline Parameterization

Table 1 lists the baseline parameter values. The values for parameters that are common to the simple macro model and the medium-sized DSGE model are equal to the prior means that we use when we estimate the parameters of the latter model. We set the parameters of the monetary policy rule, (3.6), r_π, r_y, ρ_R equal to 1.7, 0.1 and 0.7, respectively. We set the discount factor β to $1.03^{-0.25}$ so that the implied steady state real interest rate is the same as in the medium-sized DSGE model. We assume the steady state gross markup, λ , to be 1.2 and set the degree of price stickiness, ξ , to our prior mean of 0.66. In addition, we set δ to 0.005, which is the value used by HM. The parameter M is equal to 60 which roughly corresponds to the number of business days in a quarter. We assume $\rho = 0.9$, which implies a match survival rate that is consistent with both HM and Shimer (2012a).¹⁰

We calibrate three model parameters, D, κ and γ to hit three steady state targets. In particular, we require (i) a steady state unemployment rate, $1 - l$, of 5.5%; (ii) a steady state value of 1 percent for the ratio of hiring costs to gross output, i.e., $\kappa xl/Y = 0.01$; and (iii) a steady state value of 0.4 for the replacement ratio, D/w . The resulting values for D, κ and γ are reported in Table 2 which also summarizes other steady state properties of the model. The calibrated value for γ implies that the firm must pay 0.61 of a day's worth of the revenue to generate a counteroffer.

We assume that the AR(1) parameter for the law of motion for technology, τ , is equal to 0.95. Finally, for simplicity, we assume that the gross steady state inflation rate, π , is equal to unity.

3.3.2. Impulse Responses

Figures 1 and 2 display the dynamic responses of the model economy to monetary policy and technology shocks, respectively. We report results for the baseline parameterization.

¹⁰Denote the probability that a worker separates from a job at a monthly rate by $1 - \tilde{\rho}$. The probability that a person employed at the end of a quarter separates in the next three months is $(1 - \tilde{\rho}) + \tilde{\rho}(1 - \tilde{\rho}) + \tilde{\rho}^2(1 - \tilde{\rho}) = (1 - \tilde{\rho})(1 + \tilde{\rho} + \tilde{\rho}^2)$. Shimer (2012a) reports that $\tilde{\rho} = 1 - 0.034$, implying a quarterly separation rate of 0.0986. HM assume a similar value of 0.03 for the monthly separation rate. This value is also consistent with Walsh's (2003) summary of the empirical literature.

In addition, we display results for four other parameterizations, each of which changes the value of one parameter relative to the baseline case. In particular we raise δ from 0.005 in the baseline parameterization to 0.0075. Also, we lower γ from 0.01 to 0.009, we decrease D from 0.396 to 0.376 and we lower M from 60 to 50.

Figure 1 displays the dynamic response of the model economy to a negative 25 annualized basis point monetary policy shock, $\varepsilon_{R,t}$. In the baseline model, real wages respond by a relatively small amount with the peak rise equal to 0.03 percent. Inflation also responds by a relatively small amount, with a peak rise of 0.06 percent (on an annual basis). At the same time, there is a substantial increase in consumption, which initially jumps by about 0.13 percent. Finally, the unemployment rate drops by 0.14 percentage points in the impact period of the shock.

The basic intuition for how a monetary policy shock affects the economy in our model is as follows. As in standard New Keynesian sticky price models, an expansionary monetary policy shock drives the real interest rate down, inducing an increase in the demand for final goods. This rise induces an increase in the demand for the output of sticky price retailers. Since they must satisfy demand, the retailers purchase more of the wholesale good. Therefore, the relative price of the wholesale good increases and the marginal revenue product (ϑ_t) associated with a worker rises. Other things equal, this motivates wholesalers to hire more workers and increases probability that an unemployed worker finds a job. The latter effect induces a rise in workers' disagreement payoffs. The resulting increase in workers' bargaining power generates a rise in the real wage. Given our assumptions about parameter values, alternating offer bargaining mutes the increase in real wages, thus allowing for a large rise in employment, a substantial decline in unemployment, and a small rise in inflation. If the rise in the real wage was large, the incentive of employers to hire more workers would be weaker and a monetary policy shock would have less of an expansionary effect.

To provide intuition for the quantitative role of alternating offer bargaining, Figure 1 displays the economy's response to a monetary policy shock for different values of δ, γ, D and M . To understand how the model economy responds to shocks, it is useful to use the value of unemployment, U_t , as an indicator of general economic conditions. Shocks that expand economic activity tend to simultaneously raise U_t . In what follows, we provide intuition about how the parameters governing the alternating offer sharing rule, δ, γ, D , and M influence the responsiveness of the wage w_t to U_t . To do so, we consider a bargaining session between a single worker and a single firm. We consider the response of the wage negotiated by this firm-worker pair to a rise in U_t experienced idiosyncratically by that pair. For convenience we assume the experiment occurs when the economy is in nonstochastic steady state. By this we mean a situation in which all aggregate shocks are fixed at their unconditional means, aggregate variables are constant and there is ongoing idiosyncratic

uncertainty at the worker-firm level.

Let i denote the particular worker-firm pair under consideration. Let U^i denote the value of unemployment to the worker in the i^{th} worker-firm pair. The variable, w^i denotes the wage negotiated by the i^{th} worker-firm pair. The object of interest is w_U^i , the elasticity of w^i with respect to U^i , where

$$w_U^i \equiv \frac{d \log w^i}{d \log U^i} = \frac{U}{w} W_U^i, \quad W_U^i \equiv \frac{dw^i}{dU^i}. \quad (3.10)$$

In what follows, we assume that firm and worker disagreement payoffs exceed the value of their outside options. In (3.10), w and U denote the economy-wide average value of the wage rate and of the value of unemployment, respectively, in nonstochastic steady state.

Consider the impact of reducing γ . A decrease in γ raises the disagreement payoff of the firm, putting the worker in a weaker bargaining position. So, other things equal, a fall in γ leads to a decrease in w^i . As it turns out, this decrease is the same, regardless of the value of U^i , so that W_U^i is independent of γ . It follows that γ affects w_U^i entirely through its effect on U/w . The zero profit condition for firms implies that the steady state value of the real wage is independent of the bargaining parameters. So, γ affects w_U^i only through its impact on U . A decrease in γ places downward pressure on *all* worker-firm pair wages and therefore on w . Since w does not respond to γ , the value of U must rise to neutralize the downward pressure on w . But the rise in U leads to a rise in w_U^i . Consistent with this intuition, Figure 1 shows that a decrease in γ does not affect the steady state real wages but does increase the steady value of employment and consumption. At the same time it reduces steady state unemployment relative to the baseline case. Also consistent with the previous intuition, the fall in γ increases the response of real wages and inflation to a monetary policy shock while it leads to a smaller change in employment and consumption.

The intuition for the effect of a decline in D is very similar to the intuition for the effect of a fall in γ . A decline in D leaves the steady real wage unaffected but leads to a fall in steady state unemployment. Other things equal, a fall in D puts the worker in a worse bargaining position and leads to a fall in w^i . As it turns out, the decrease in w^i is the same for all values of U^i , so that W_U^i is independent of D . Consequently, D affects w_U^i only through its impact on U . A decrease in D places downward pressure on *all* worker-firm pair wages and therefore on w . Since w does not respond to a change in D , the value of U must rise to neutralize the downward pressure on w . A rise in U increases the worker's disagreement payoff and his bargaining power, thereby exerting countervailing upward pressure on w . This reasoning underlies the intuition for why a decrease in D leads to a rise in U and w_U^i . Consistent with this intuition, Figure 1 shows that a decrease in D does not affect the steady state real wage, increases the steady value of employment and consumption, and reduces steady state unemployment. Also, as the previous intuition suggests, the fall in D increases the response

of real wages and inflation to a monetary policy shock while it decreases the responses of employment and consumption to that shock.

Consider next the impact of increasing δ . In terms of the steady state, consumption rises, unemployment falls, while inflation and the real wage are unaffected. Figure 1 shows that the dynamic responses of the real wage and inflation to an expansionary monetary policy shock are stronger than in the baseline case. At the same time, consumption and unemployment respond by less than in the baseline case. To understand the basic intuition for the dynamic responses, note that shocks which increase the value of workers' outside option, as summarized by U_t , tend to increase the real wage rate, and dampen the effects of an expansionary shock. In the extreme case of $\delta = 0$, there is no chance that workers and firms are thrown to their outside options during negotiations. So, the value of unemployment, U_t , simply does not directly enter into the indifference conditions (2.13), governing workers' and firms' offers. As a result, the real wage should not depend much on cyclical shocks that affect U_t . By continuity, a rise in δ increases the importance of U_t in worker's disagreement payoff which make the real wage more sensitive to shocks. The stronger response of the real wage reduces the incentive of firms to hire workers, thereby limiting the expansionary effects of the shock. Finally, the larger rise in the real wage places upward pressure on the marginal costs of retailers, leading to higher inflation than in the baseline parameterization.

Now consider the impact of a lower value of M . Figure 1 indicates a fall in M leads to stronger dynamic responses of the real wage and inflation to an expansionary monetary policy shock. To understand this result consider the extreme case where M is very large. Equations (2.16) and (2.18) imply

$$J_{M,t} = \frac{1}{M}\vartheta_t + \tilde{\vartheta}_t^p - (w_{M,t} + \tilde{w}_t^p) = 0.$$

Then (2.2), (2.4), (2.12) and (2.17) imply that

$$w_{M,t} = \frac{1}{M}\vartheta_t + \rho\kappa E_t m_{t+1}. \quad (3.11)$$

Suppose that m_{t+1} and therefore the interest rate is constant. Then as M goes to infinity, $w_{M,t}$ would become constant and does not depend on cyclical shocks to the economy. In general, we would expect that this cyclical insensitivity is inherited by $w_{1,t} (\equiv w_t)$, $w_{2,t}$, ..., $w_{M-1,t}$, a conjecture that is consistent with our numerical results. By continuity, we expect the real wage to be more sensitive to shocks when M is smaller. This intuition is consistent with the results reported in Figure 1.

Finally, note that in subperiod M , the worker makes a take-it-or-leave-it offer. The cost to the firm of rejecting the offer is zero, which is obviously not a function of the state of the economy. While firms and workers never actually negotiate in the last subperiod of

a quarter, that out-of-equilibrium possibility still affects the actual real wage rate. Our intuition suggests that the non state-contingent value to the firm of rejecting the final take-it-or-leave-it offer made by a worker, should make the actual real wage less sensitive to a shock. To pursue this intuition we solved a version of model in which the firm makes a take-or-leave-it offer in subperiod M . The cost to the worker of rejecting such an offer does, to some extent, depend on the state of the economy. Consistent with our intuition, in this version of the model, the real wage is more sensitive to a policy shock than the baseline model. That said, the other features of the alternating offer bargaining offer model still act to mute the response of real wages to the shock and allow for a substantial expansion in aggregate economic activity.

Figure 2 displays the dynamic responses of our baseline model and the four alternatives to a 0.1 percent innovation in technology. In the baseline model, real wages rise but by a relatively modest amount. Inflation also falls by a modest amount, with a peak decline of about 0.1 percent (on an annual basis). Notice that unemployment falls by a substantial amount with a peak decline of about 0.07 percent. The effect of either lowering γ , D and M or raising δ is to make inflation more responsive to the technology shock while the decline in unemployment is muted relative to the baseline parameterization.

In sum, in this section we have shown that our labor market model can potentially account for the cyclical properties of key labor market variables. In the next section we analyze whether it actually provides an empirically convincing account of those properties. To that end we embed it in a medium-sized DSGE model which we estimate and evaluate.

4. An Estimated Medium-sized DSGE Model

In this section, we describe a medium-sized DSGE model similar to one in CEE, modified to include our labor market framework. The first subsection describes the problems faced by households and goods producing firms. We discuss the labor market in second subsection. The third subsection specifies the law of motion of the three shocks to agents' environment. These include a monetary policy shock, a neutral technology shock and an investment-specific technology shock. The last subsection briefly presents a version of the model embodying the standard DMP specification of the labor market, i.e. wages are determined by a Nash sharing rule and firms face vacancy posting costs. In addition, we also present a version of the model with sticky wages as proposed in EHL. These alternative models represent important benchmarks for comparison.

4.1. Households and Goods Production

The basic structure of the representative household's problem is the same as in section 3.1. Here, we allow for habit persistence in preferences, time-varying unemployment benefits, and the accumulation of physical capital, K_t .

The preferences of the representative household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln (C_t - bC_{t-1}).$$

The parameter b controls the degree of habit formation in household preferences. We assume $0 \leq b < 1$. The household's budget constraint is:

$$P_t C_t + P_{I,t} I_t + B_{t+1} \leq (R_{K,t} u_t^K - a(u_t^K) P_{I,t}) K_t + (1 - l_t) P_t D_t + l_t W_t + R_{t-1} B_t - T_t. \quad (4.1)$$

As above, T_t denotes lump-sum taxes net of transfers and firm profits and D_t denotes the unemployment compensation of an unemployed worker. In contrast to (2.8), D_t is exogenously time-varying to ensure balanced growth. In (4.1), B_{t+1} denotes beginning-of-period t purchases of a nominal bond which pays rate of return, R_t at the start of period $t + 1$, and $R_{K,t}$ denotes the nominal rental rate of capital services. The variable u_t^K denotes the utilization rate of capital. As in CEE, we assume that the household sells capital services in a perfectly competitive market, so that $R_{K,t} u_t^K K_t$ represents the household's earnings from supplying capital services. The increasing convex function $a(u_t^K)$ denotes the cost, in units of investment goods, of setting the utilization rate to u_t^K . The variable $P_{I,t}$ denotes the nominal price of an investment good and I_t denotes household purchases of investment goods.

The household owns the stock of capital which evolves according to,

$$K_{t+1} = (1 - \delta_K) K_t + [1 - S(I_t/I_{t-1})] I_t.$$

The function $S(\cdot)$ is an increasing and convex function capturing adjustment costs in investment. We assume that $S(\cdot)$ and its first derivative are both zero along a steady state growth path.

As in our simple macroeconomic model, we assume that a final good is produced by a perfectly competitive representative firm using the technology, (3.1). The final good producer buys the j^{th} specialized input, $Y_{j,t}$, from a retailer who uses the following technology:

$$Y_{j,t} = k_{j,t}^\alpha (z_t h_{j,t})^{1-\alpha} - \phi_t. \quad (4.2)$$

The retailer is a monopolist in the product market and is competitive in the factor markets. Here $k_{j,t}$ denotes the total amount of capital services purchased by firm j . Also, ϕ_t represents an exogenous fixed cost of production which grows in a way that ensures balanced growth.

The fixed cost is calibrated so that profits are zero along the balanced growth path. In (4.2), z_t is a technology shock whose properties are discussed below. Finally, $h_{j,t}$ is the quantity of an intermediate good purchased by the j^{th} retailer. This good is purchased in competitive markets at the price P_t^h from a *wholesaler*, whose problem is discussed in the next subsection. Analogous to CEE, we assume that to produce in period t , the retailer must borrow $P_t^h h_{j,t}$ at the start of the period at the interest rate R_t . The retailer repays the loan at the end of period t when he receives his sales revenues. The j^{th} retailer sets its price, $P_{j,t}$, subject to its demand curve, (3.2), and the Calvo sticky price friction (3.3). Recall that we do not allow for automatic indexation of prices to either steady state or lagged inflation.

4.2. Wholesalers and the Labor Market

The structure of the labor market is the same as in section 2. Each wholesaler employs a measure of workers. Let l_{t-1} denote the representative wholesaler's labor force at the end of $t-1$. A fraction $1-\rho$ of these workers separates exogenously. So, the wholesaler has a labor force of ρl_{t-1} at the start of period t . At the beginning of period t the wholesaler selects its hiring rate, x_t , which determines the number of new workers that it meets at time t . As in the small macro model, we assume that the wholesaler's cost of hiring is a linear function of the hiring rate and is denominated in units of the final consumption good. The value of a worker to the wholesaler, J_t , is also denominated in units of the final consumption good. To ensure balanced growth we replace κ by an exogenous stochastic process that is uncorrelated with the state of the economy. We denote the time t value of this process by κ_t .

To hire $x_t l_{t-1}$ workers, the wholesaler must post $x_t l_{t-1}/Q_t$ vacancies. Here Q_t denotes the aggregate vacancy filling rate which firms take as given and is further described below. We assume that posting vacancies is costless.

The job finding rate is given by (2.11) where x_t and l_{t-1} denote the economy-wide value of the corresponding wholesaler-specific variables. Individual workers view x_t and l_{t-1} as being exogenous and beyond their control. The values of employment and unemployment, U_t and V_t are denoted in units of the final good. In order to ensure balanced growth, we replace D by an exogenous stochastic process that is uncorrelated with the state of the economy. We denote the time t value of this process by D_t .

After setting x_t , the firm has access to l_t workers (see equation (2.10)). Each of these workers engages in bilateral bargaining with a representative of the firm, taking the outcome of all other negotiations as given. The equilibrium wage rate, w_t , i.e. W_t/P_t is the outcome of the alternating offer bargaining process described in section 2. To ensure balanced growth we replace γ by an exogenous stochastic process that is uncorrelated with the state of the economy. We denote the time t value of this process by γ_t . As before, we verify numerically that all bargaining sessions conclude successfully with the firm and worker agreeing to an

employment contract. Thus, in equilibrium the representative wholesaler employs all l_t workers with which it has met, at wage rate w_t . Production begins immediately after wage negotiations are concluded and the wholesaler sells the intermediate good at the real price, $\vartheta_t \equiv P_t^h/P_t$.

To summarize, the labor market equilibrium conditions coincide with the ones derived in section 2 except that κ , D and γ are replaced by κ_t , D_t and γ_t .

4.3. Market Clearing, Monetary Policy and Functional Forms

The total amount of intermediate goods purchased by retailers from wholesalers is:

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Recall that the output of intermediate goods produced by wholesalers is equal to the number of workers they employ. So, the supply of intermediate goods is l_t . As in the simple model, market clearing for intermediate goods requires $h_t = l_t$. The capital services market clearing condition is:

$$u_t^K K_t = \int_0^1 k_{j,t} dj.$$

Market clearing for final goods requires:

$$C_t + (I_t + a(u_t^K)K_t)/\Psi_t + \kappa_t x_t l_{t-1} + G_t = Y_t. \quad (4.3)$$

The right hand side of the previous expression denotes the quantity of final goods. The left hand side represents the various ways that final goods are used. Homogeneous output, Y_t , can be converted one-for-one into either consumption goods, goods used to hire workers, or government purchases, G_t . In addition, some of Y_t is absorbed by capital utilization costs. Finally, Y_t can be used to produce investment goods using a linear technology in which one unit of the final good is transformed into Ψ_t units of I_t . Perfect competition in the production of investment goods implies,

$$P_{I,t} = \frac{P_t}{\Psi_t}.$$

The asset pricing kernel, m_{t+1} , is constructed using the marginal utility of consumption, which we denote by $u_{c,t}$:

$$u_{c,t} = (C_t - bC_{t-1})^{-1} - \beta b E_t (C_{t+1} - bC_t)^{-1}.$$

Then,

$$m_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}.$$

We adopt the following specification of monetary policy:

$$\ln(R_t/R) = \rho_R \ln(R_{t-1}/R) + (1 - \rho_R) [r_\pi \ln(\pi_t/\pi) + r_y \ln(\mathcal{Y}_t/\mathcal{Y})] + \sigma_R \varepsilon_{R,t}.$$

Here, π denotes the monetary authority's target inflation rate. The steady state inflation rate in our model is equal to π . The shock, $\varepsilon_{R,t}$, is a unit variance, zero mean disturbance to monetary policy. Also, R and \mathcal{Y} denote the steady values of R_t and \mathcal{Y}_t . The variable, \mathcal{Y}_t , denotes Gross Domestic Product (GDP):

$$\mathcal{Y}_t = C_t + I_t/\Psi_t + G_t.$$

We assume that G_t grows exogenously in a way that is consistent with balanced growth. Working with the data from Fernald (2012) we find that the growth rate of total factor productivity is well described by an *i.i.d.* process. Accordingly, we assume that $\ln \mu_{z,t} \equiv \ln(z_t/z_{t-1})$ is *i.i.d.* We also assume that $\ln \mu_{\Psi,t} \equiv \ln(\Psi_t/\Psi_{t-1})$ follows an AR(1) process. The parameters that control the standard deviations of both processes are denoted by (σ_z, σ_Ψ) . The autocorrelation of $\ln \mu_{\Psi,t}$ is denoted by ρ_Ψ .

Recall that our model exhibits growth stemming from neutral and investment-specific technological progress. The variables $Y_t/\Phi_t, C_t/\Phi_t, w_t/\Phi_t$ and $I_t/(\Psi_t\Phi_t)$ converge to constants in nonstochastic steady state, where

$$\Phi_t = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t$$

is a weighted average of the sources of technological progress. If objects like the fixed cost of production, the cost of hiring, the cost to a firm of preparing a counteroffer, government purchases, and unemployment transfer payments were constant, they would become irrelevant over time. To avoid this implication, it is standard in the literature to suppose that such objects grow at the same rate as output, which in our case is given by Φ_t . An unfortunate implication of this assumption is that technology shocks of both types immediately affect the vector of objects $[\phi_t, \kappa_t, \gamma_t, G_t, D_t]'$. It seems hard to justify such an assumption. To avoid this problem, we proceed as in Christiano, Trabandt and Walentin (2012) and Schmitt-Grohé and Uribe (2012) who assume that government purchases, G_t , are a distributed lag of unit root technology shocks, i.e. G_t is cointegrated with Y_t but has a smoother stochastic trend. In particular, we assume that

$$[\phi_t, \kappa_t, \gamma_t, G_t, D_t]' = [\phi, \kappa, \gamma, G, D]' \Omega_t.$$

where Ω_t is $diag(\Omega_t^i)$ with $i \in \{\phi, \kappa, \gamma, G, D\}$ and Ω_t^i denotes a distributed lag of past values of Φ_t defined by,

$$\Omega_t^i = \Phi_{t-1}^{\theta_i} (\Omega_{t-1}^i)^{1-\theta_i}. \quad (4.4)$$

Here $0 < \theta_i \leq 1$ are parameters to be estimated. Note that Ω_t^i grows at the same rate as Φ_t in the long-run. When θ_i is very close to zero, Ω_t^i is virtually unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find very attractive on a priori grounds. In practice, we constrain the first four diagonal elements of Ω_t to be the same. For reasons discussed below, we found that it is useful to allow θ_D to take on a separate value.

We assume that the cost of adjusting investment takes the form:

$$S(I_t/I_{t-1}) = 0.5 \exp \left[\sqrt{S''} (I_t/I_{t-1} - \mu \cdot \mu_\Psi) \right] + 0.5 \exp \left[-\sqrt{S''} (I_t/I_{t-1} - \mu \cdot \mu_\Psi) \right] - 1.$$

Here, μ and μ_Ψ denote the unconditional growth rates of Φ_t and Ψ_t . The value of I_t/I_{t-1} in nonstochastic steady state is $(\mu \cdot \mu_\Psi)$. In addition, S'' represents a model parameter that coincides with the second derivative of $S(\cdot)$, evaluated in steady state. It is straightforward to verify that $S(\mu \cdot \mu_\Psi) = S'(\mu \cdot \mu_\Psi) = 0$.

We assume that the cost associated with setting capacity utilization is given by,

$$a(u_t^K) = 0.5\sigma_a\sigma_b(u_t^K)^2 + \sigma_b(1 - \sigma_a)u_t^K + \sigma_b(\sigma_a/2 - 1)$$

where σ_a and σ_b are positive scalars. We normalize the steady state value of u_t^K to one. This pins down the value of σ_b given an estimate of σ_a .

Finally, we discuss how vacancies are determined. We posit a standard matching function:

$$x_t l_{t-1} = \sigma_m (1 - \rho l_{t-1})^\sigma (l_{t-1} v_t)^{1-\sigma}, \quad (4.5)$$

where $l_{t-1} v_t$ denotes the total number of vacancies and v_t denotes the vacancy rate. Given x_t and l_{t-1} , we use (4.5) to solve for v_t . Recall that we defined the total number of vacancies by $x_t l_{t-1}/Q_t$. We can solve for the aggregate vacancy filling rate Q_t using

$$Q_t = \frac{x_t}{v_t}. \quad (4.6)$$

The equilibrium of our model has a particular recursive structure. We can first solve all model variables, apart from v_t and Q_t . These two variables can then be solved for using (4.5) and (4.6).

4.4. Alternative Labor Market Models

In this subsection we consider alternative labor market models that we include in our DSGE framework. First, we describe our version of the DMP model, which is characterized by search costs and a Nash sharing rule. Second, we describe the sticky nominal wage model of EHL.

4.4.1. The DMP Model

In this subsection, we describe the version of the medium-sized DSGE model which we refer to as the ‘Nash Sharing, Search’ specification. We replace our alternating offer sharing rule, (2.22) by the *Nash sharing rule* (2.24) which we repeat for convenience,

$$J_t = \frac{1 - \eta}{\eta}(V_t - U_t).$$

We incorporate DMP-style search costs into our DSGE model as follows. We assume that vacancies are costly and that posting vacancies is the only action the firm takes to meet a worker. With probability Q_t a vacancy results in a meeting with a worker. The aggregate rate at which workers are hired, x_t , depends on the aggregate vacancy rate, v_t , according to (4.6).

The cost of setting the vacancy rate to v_t is given by:

$$\kappa_t v_t l_{t-1}. \tag{4.7}$$

The probability Q_t is determined by the matching function, (4.5).

Compared to our baseline hiring cost setup, three changes are required to incorporate the search cost specification into the medium-sized DSGE model.

First, the free entry/zero profit condition, (2.4) is replaced by:

$$\kappa_t = Q_t J_t. \tag{4.8}$$

Free entry in the search cost specification implies that the marginal cost of posting a vacancy is equal to the expected return. Second, we add Q_t and v_t to the list of variables that must be simultaneously solved for using (4.5) and (2.4). The third change involves replacing the hiring cost term in (4.3) with the vacancy cost term (4.7) in the resource constraint. Doing so we obtain:

$$C_t + (I_t + a(u_t^K)K_t)/\Psi_t + \kappa_t v_t l_{t-1} + G_t = Y_t. \tag{4.9}$$

We conclude by discussing an important feature of the search cost specification. Define labor market tightness as:

$$\Gamma_t = \frac{v_t l_{t-1}}{1 - \rho l_{t-1}}. \tag{4.10}$$

Relations (4.5) and (4.6) imply that Q_t takes the following form,

$$Q_t = \sigma_m \Gamma_t^{-\sigma}.$$

It follows that the probability of filling a vacancy is decreasing in labor market tightness.

4.4.2. The Sticky Wage Model

We now describe a modification of the medium-sized DSGE model which incorporate the sticky nominal wage framework of EHL. We replace the wholesale production sector with the following environment. The final homogeneous good, Y_t , is produced by competitive and identical firms using technology (3.1). The specialized inputs used in the production of Y_t are produced by retailers using capital services and a homogeneous labor input. The final good producer buys the j^{th} specialized input, $Y_{j,t}$, from a retailer who produces the input using technology (4.2). Capital services are purchased in competitive rental markets. In (4.2), $h_{j,t}$ refers to the quantity of a homogeneous labor input that firm j purchases from ‘labor contractors’. These contractors produce the homogeneous labor input by combining a range of differentiated labor inputs, $h_{i,t}$, using the following technology:

$$h_t = \left[\int_0^1 (h_{i,t})^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \quad \lambda_w > 1. \quad (4.11)$$

Labor contractors are perfectly competitive and take the wage rate, W_t , of h_t as given. They also take the wage rate, $W_{i,t}$, of the i^{th} labor type as given. Profit maximization on the part of contractors yields to the labor demand curve:

$$h_{i,t} = \left(\frac{W_t}{W_{i,t}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t. \quad (4.12)$$

Substituting (4.11) into (4.12) and rearranging, we obtain:

$$W_t = \left[\int_0^1 W_{i,t}^{\frac{1}{1-\lambda_w}} di \right]^{1-\lambda_w}. \quad (4.13)$$

Specialized labor inputs are supplied by a large number of identical households. The representative household has many members corresponding to each type i of labor and provides complete insurance to all of its members in return for their wage income. The household’s budget constraint is given by (4.1) except that D_t is equal to zero. This constraint reflects our assumption that the household owns the capital stock, sets the utilization rate and makes investment decisions.

It is optimal for the household to assign an equal amount of consumption to each of its members. The household’s utility function is given by:

$$\ln(C_t - bC_{t-1}) - A \int_0^1 \frac{h_{i,t}^{1+\psi}}{1+\psi} di. \quad (4.14)$$

Here, A is a positive constant and $h_{i,t}$ denotes hours worked by the i^{th} member of the household. The wage rate of the i^{th} type of labor, $W_{i,t}$, is determined outside the representative household by a monopoly union that represents all i -type workers across all households.

The monopoly union faces Calvo-type nominal rigidities when setting the wage. With probability $1 - \xi_w$ the union can optimize the wage $W_{t,i}$ and with probability ξ_w it cannot. There is no wage indexation so that in the latter case, the nominal wage rate is given by:

$$W_{i,t} = W_{i,t-1}. \tag{4.15}$$

The union maximizes the welfare of its members. For a more detailed exposition of the model and its solution, see CEE.

5. Econometric Methodology

We estimate our model using a Bayesian variant of the strategy in CEE that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified VAR for post-war quarterly U.S. times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt and Walentin (2011), henceforth CTW.

To facilitate comparisons, our analysis is based on the same VAR as used in CTW who estimate a 14 variable VAR using quarterly data that are seasonally adjusted and cover the period 1951Q1 to 2008Q4. As in CTW, we identify the dynamic responses to a monetary policy shock by assuming that the monetary authority sees the contemporaneous values of all the variables in the VAR and a monetary policy shock affects only the Federal Funds Rate contemporaneously. As in Altig, Christiano, Eichenbaum and Linde (2011), Fisher (2006) and CTW, we make two assumptions to identify the dynamic responses to the technology shocks: (i) the only shocks that affect labor productivity in the long-run are the innovations to the neutral technology shock, z_t , and the innovation to the investment-specific technology shock, Ψ_t and (ii) the only shock that affects the price of investment relative to consumption in the long-run is the innovation to Ψ_t . These identification assumptions are satisfied in our model. Standard lag-length selection criteria lead CTW to work with a VAR with 2 lags.¹¹

There is an ongoing debate over whether or not there is a break in the sample period that we use. Implicitly, our analysis sides with those authors who argue that the evidence of parameter breaks in the middle of our sample period is not strong. See for example Sims and Zha (2006) and Christiano, Eichenbaum and Evans (1999).

¹¹See CTW for a sensitivity analysis with respect to the lag length of the VAR.

We include the following variables in the VAR:¹²

$$\left(\begin{array}{c} \Delta \ln(\text{relative price of investment}_t) \\ \Delta \ln(\text{real } GDP_t/\text{hours}_t) \\ \Delta \ln(\text{GDP deflator}_t) \\ \text{unemployment rate}_t \\ \ln(\text{capacity utilization}_t) \\ \ln(\text{hours}_t) \\ \ln(\text{real } GDP_t/\text{hours}_t) - \ln(\text{real wage}_t) \\ \ln(\text{nominal } C_t/\text{nominal } GDP_t) \\ \ln(\text{nominal } I_t/\text{nominal } GDP_t) \\ \ln(\text{vacancies}_t) \\ \text{job separation rate}_t \\ \text{job finding rate}_t \\ \ln(\text{hours}_t/\text{labor force}_t) \\ \text{Federal Funds rate}_t \end{array} \right). \quad (5.1)$$

Given an estimate of the VAR we can compute the implied impulse response functions to the three structural shocks. We stack the contemporaneous and 14 lagged values of each of these impulse response functions for 12 of the VAR variables in a vector, $\hat{\psi}$. We do not include the job separation rate and the size of the labor force because our model assumes those variables are constant. We include these variables in the VAR to ensure the VAR results are not driven by an omitted variable bias.

The logic underlying our model estimation procedure is as follows. Suppose that our structural model is true. Denote the true values of the model parameters by θ_0 . Let $\psi(\theta)$ denote the model-implied mapping from a set of values for the model parameters to the analog impulse responses in $\hat{\psi}$. Thus, $\psi(\theta_0)$ denotes the true value of the impulse responses whose estimates appear in $\hat{\psi}$. According to standard classical asymptotic sampling theory, when the number of observations, T , is large, we have

$$\sqrt{T} \left(\hat{\psi} - \psi(\theta_0) \right) \overset{a}{\sim} N(0, W(\theta_0, \zeta_0)).$$

Here, ζ_0 denotes the true values of the parameters of the shocks in the model that we do not formally include in the analysis. Because we solve the model using a log-linearization procedure, $\psi(\theta_0)$ is not a function of ζ_0 . However, the sampling distribution of $\hat{\psi}$ is a function of ζ_0 . We find it convenient to express the asymptotic distribution of $\hat{\psi}$ in the following form:

$$\hat{\psi} \overset{a}{\sim} N(\psi(\theta_0), V), \quad (5.2)$$

where

$$V \equiv \frac{W(\theta_0, \zeta_0)}{T}.$$

¹²See section A of the technical appendix in CTW for details about the data.

For simplicity our notation does not make the dependence of V on θ_0, ζ_0 and T explicit. We use a consistent estimator of V . Motivated by small sample considerations, that estimator has only diagonal elements (see CTW). The elements in $\hat{\psi}$ are graphed in Figures 3 – 5 (see the solid lines). The gray areas are centered, 95 percent probability intervals computed using our estimate of V .

In our analysis, we treat $\hat{\psi}$ as the observed data. We specify priors for θ and then compute the posterior distribution for θ given $\hat{\psi}$ using Bayes' rule. This computation requires the likelihood of $\hat{\psi}$ given θ . Our asymptotically valid approximation of this likelihood is motivated by (5.2):

$$f(\hat{\psi}|\theta, V) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\hat{\psi} - \psi(\theta))' V^{-1}(\hat{\psi} - \psi(\theta))\right]. \quad (5.3)$$

The value of θ that maximizes the above function represents an approximate maximum likelihood estimator of θ . It is approximate for three reasons: (i) the central limit theorem underlying (5.2) only holds exactly as $T \rightarrow \infty$, (ii) our proxy for V is guaranteed to be correct only for $T \rightarrow \infty$, and (iii) $\psi(\theta)$ is calculated using a linear approximation.

Treating the function, f , as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of θ conditional on $\hat{\psi}$ and V is:

$$f(\theta|\hat{\psi}, V) = \frac{f(\hat{\psi}|\theta, V) p(\theta)}{f(\hat{\psi}|V)}. \quad (5.4)$$

Here, $p(\theta)$ denotes the priors on θ and $f(\hat{\psi}|V)$ denotes the marginal density of $\hat{\psi}$:

$$f(\hat{\psi}|V) = \int f(\hat{\psi}|\theta, V) p(\theta) d\theta.$$

The mode of the posterior distribution of θ can be computed by maximizing the value of the numerator in (5.4), since the denominator is not a function of θ . The marginal density of $\hat{\psi}$ is required for an overall measure of the fit of our model. To compute the marginal likelihood, we use the standard Laplace approximation. In our analysis, we also find it convenient to compute the marginal likelihood based on a subset of the elements in $\hat{\psi}$ (see Appendix B for details).

6. Results

In this section we present the empirical results for our model, the ‘Alternating Offer, Hiring’ model. In addition, we report results for a version of our model with the search cost specification, the ‘Alternating Offer, Search’ model and the Nash sharing model with search

or hiring costs, the ‘Nash Sharing, Search’ and ‘Nash Sharing, Hiring’ models. The ‘Nash Sharing, Search’ model is our version of the DMP model. Finally, we report results for the ‘Sticky Wage’ model.

In the first three subsections we discuss results for the different models. In the final subsection we assess the models’ ability to account for the statistics that Shimer (2005a) uses to evaluate the standard DMP model.

We set the values for a subset of the model parameters a priori. These values are reported in Panel A of Table 3. We also set the steady state values of five model variables, listed in Panel B of Table 3. We specify β so that the steady state annual real rate of interest is three percent. The depreciation rate on capital, δ_K , is set to imply an annual depreciation rate of 10 percent. The values of μ and μ_Ψ are determined by the sample average of real per capita GDP and real investment growth in our sample. We assume the monetary authority’s inflation target is 2.5 percent and that profits of intermediate good producers are zero in steady state. We set the rate at which vacancies create job-worker meetings, Q , to 0.7, as in den Haan, Ramey and Watson (2000) and Ravenna and Walsh (2008). We set the steady state unemployment rate to the average unemployment rate in our sample, implying a steady state value of u equal to 0.055. As in the simple macro model, we set M equal to 60 and ρ equal to 0.9. Finally, we assume that the steady state value of the ratio of government consumption to gross output is 0.20.

All remaining model parameters are estimated subject to the restrictions summarized in Table 3. Table 4 presents prior and posterior distributions for all of the estimated objects in the models.

6.1. The Estimated ‘Alternating Offer, Hiring’ Model

A number of features of the posterior mode of the estimated parameters in the ‘Alternating Offers, Hiring’ model are worth noting. First, the posterior mode of ξ implies a moderate degree of price stickiness, with prices changing on average once every 2.4 quarters. This value lies within the range reported in the literature. For example, according to Nakamura and Steinsson (2012), the recent micro-data based literature finds that the price of the median product changes roughly every 1.5 quarters when sales are included, and every 3 quarters when sales are excluded. Second, the posterior mode of δ implies that there is a roughly 0.3% chance of an exogenous break-up in negotiations when a wage offer is rejected. Third, the posterior modes of our model parameters, along with the assumption that the steady state unemployment rate equals 5.5%, implies that it costs firms 0.27 days of marginal revenue to prepare a counteroffer during wage negotiations (see Table 5). Fourth, the posterior mode of steady state hiring costs as a percent of total wages of newly hired workers is equal to

6.7%.¹³ Silva and Toledo (2009) report that, depending on the exact costs included, the value of this statistic is between 4 and 14 percent, a range that encompasses the corresponding statistic in our model. Fifth, the posterior mode for the replacement ratio is 0.67. Based on a summary of the literature, Gertler, Sala and Trigari (2008) argue that a plausible range for the replacement ratio is 0.4 to 0.7. The lower bound is based on studies of unemployment insurance benefits, while the upper bound takes into account informal sources of insurance.¹⁴ Sixth, the posterior mode of θ_i , $i \in \{\phi, \kappa, \gamma, G\}$, governing the responsiveness of $[\phi_t, \kappa_t, \gamma_t, G_t]$ to technology shocks, is close to zero (0.014). So, these variables are very unresponsive in the short-run to technology shocks. Interestingly, the posterior mode for θ_D is relatively large compared to the posterior of the other θ_i parameters. If we set θ_D to 0.014, then unemployment benefits initially move by very little after a technology shock. Other things equal, the model then generates a rise in employment and falls in unemployment and inflation that are too large relative to the responses emerging from the VAR. This result is one way to see that our model has no difficulty in resolving the Shimer puzzle. We return to this important point below. Seventh, the posterior modes of the parameters governing monetary policy are similar to those reported in the literature (see for example Justiniano, Primiceri, and Tambalotti, 2010).

The solid black lines in Figures 3-5 present the impulse response functions to monetary policy shock, a neutral-technology shock and an investment-specific technology shock, implied by the estimated VAR. The grey areas represent 95 percent probability intervals. The solid lines with the circles correspond to the impulse response functions of our model evaluated at the posterior mode of the structural parameters. Figure 3 shows that the model does very well at reproducing the estimated effects of an expansionary monetary policy shock, including the hump-shaped rise of real GDP and hours worked and the muted response of inflation. Notice that real wages respond by much less than hours to the monetary policy shock. Even though the maximal rise in hours worked is roughly 0.14%, the maximal rise in real wages is only 0.06%. Significantly, the model accounts for the hump-shaped fall in the unemployment rate as well as the rise in the job finding rate and vacancies that occur after an expansionary monetary policy shock. The model does understate the rise in the capacity utilization rate. The sharp rise of capacity utilization in the estimated VAR may reflect that our data on the capacity utilization rate pertains to the manufacturing sector, which may overstate the average response across all sectors in the economy.

¹³Table 4 reports the hiring cost to gross output ratio in steady state which is defined as: $s_l = 100\kappa n_\kappa x l / y$. Here n_κ is equal to Ω_t^κ / Φ_t evaluated at steady state. Given s_l and the real wage, w , it is straightforward to compute hiring costs as a share of the wage of newly hired workers: $100n_\kappa \kappa / w$.

¹⁴Aguiar, Hurst and Karabarbounis (2012) find that workers who become unemployed increase the amount of time that they spend on home production by roughly 30 percent. This increase in home production could potentially rationalize a replacement ratio that is higher than 0.7.

From Figure 4 we see that the model does a good job of accounting for the estimated effects of a neutral technology shock. Of particular note is that the model reproduces the estimated sharp fall in the inflation rate that occurs after a positive neutral technology shock, a feature of the data stressed in Altig, Christiano, Eichenbaum and Linde (2011) and Paciello (2009). Also, the model generates a sharp fall in the unemployment rate along with a large rise in job vacancies and the job finding rate. Finally, Figure 5 shows that the model does a good job of accounting for the estimated response of the economy to an investment-specific technology shock.

6.2. The Estimated Sticky Wage Model

In this subsection we discuss the empirical properties of the sticky wage model and compare its performance to the ‘Alternating Offer, Hiring’ model. Recall that our sticky wage model rules out indexation of wages to technology and inflation. We comment on a version of the model that allows for such indexation at the end of the subsection.

Table 3 reports parameter values of the ‘Sticky Wage’ model that we set a priori. Note in particular that we fix ξ_w to 0.75 so that wages change on average once a year.¹⁵ Table 4 reports the posterior modes of the estimated sticky wage model parameters. Several results emerge from the table. First, the posterior mode of the coefficient on inflation in the Taylor rule, r_π , is substantially higher than the corresponding posterior mode in the ‘Alternating Offer, Hiring’ model (2.09 versus 1.36). Second, the degree of price stickiness is higher in the ‘Sticky Wage’ model than in the ‘Alternating Offer, Hiring’ model. In the sticky wage model, prices are estimated to change on average roughly once a year compared to 2.4 quarters in the ‘Alternating Offer, Hiring’ model .

Figures 3-5 show that with two important exceptions, the sticky wage model does reasonably well at accounting for the estimated impulse response functions. These exceptions are that the model understates the responses of inflation to a neutral technology shock and a monetary policy shock.

We want to compare the fit of our baseline model with that of the sticky wage model. The marginal likelihood is a standard measure of fit. However, using it here is complicated by the fact that the two models do not address the same data. For example, the sticky wage model has no implications for vacancies and the job finding rate.¹⁶ To obtain a measure of fit based

¹⁵We encountered numerical problems in calculating the posterior mode of model parameters when we did not place a dogmatic prior on ξ_w . We suspect that this problem stems from indeterminacy of the equilibrium for various configurations of the parameter values. As Ascari, Benzoni and Castelnuovo (2011) stress, the range of parameter values for which the indeterminacy problem arises is substantially larger in sticky wage models without indexation relative to models with indexation.

¹⁶Galí (2011) has shown how to derive implications for the unemployment rate from the sticky wage model. For a discussion of this approach see Galí, Smets and Wouters (2012), Christiano (2012) and Christiano, Trabandt and Walentin (2012).

on a common data set, we integrate out unemployment, the job finding rate and vacancies from the marginal likelihood associated with our baseline model.¹⁷ The marginal likelihoods based on the impulse response functions of the nine remaining variables are reported in Table 4 (see ‘Laplace, 9 Variables’). The marginal likelihood for our baseline model is 67 log points higher than it is for the sticky wage model. We conclude that, subject to the approximations that we used to compute the marginal likelihood function, there is substantial statistical evidence in favor of the ‘Alternating Offer, Hiring’ model relative to the sticky wage model.

Finally, we estimated a version of the sticky wage model where we allow for wage indexation. In particular, we assume that if a labor supplier cannot re-optimize his wage, it changes by the steady state growth rate of output, μ , times the lagged inflation rate. Table 4 reports that, relative to the no indexation sticky wage model, the posterior modes of many of the model parameters move towards those reported for the ‘Alternating Offer, Hiring’ model. See for example the habit persistence parameter b , the Taylor rule parameter r_π and the markup parameter λ . The impulse response functions of our model and the sticky wage model with indexation are qualitatively very similar, Table 4 indicates that the marginal likelihood of the latter model is about 4 log points lower than our baseline model. Overall, we conclude that the performance of the two models is similar. But the performance of the sticky wage model depends very much on the troubling wage indexation assumption.

6.3. The Estimated DMP Model

In this subsection, we compare the performance of our version of the DMP model with the ‘Alternating Offer, Hiring’ model. Recall that there are two differences between these models: the assumption of hiring versus search costs and the way that wages are determined. To assess the importance of each difference, we proceed as follows. First, we modify the baseline model by replacing the alternating offer bargaining specification with the Nash sharing rule of the DMP model (see subsection 4.4.1). We consider two cases: one with search costs (the ‘Nash Sharing, Search’ model) and one with hiring costs (the ‘Nash Sharing, Hiring’ model). Second, we compare the empirical performance of these models. Finally, we isolate the role of hiring versus search costs in the ‘Alternating Offer, Hiring’ model by considering a version of this model in which hiring costs are replaced by search costs, the ‘Alternating Offer, Search’ model. This version of the model is closest in spirit to HM who assume that there are search costs rather than hiring costs.

The solid black lines in Figures 6-8 are the impulse responses, implied by the estimated VAR, to a monetary policy shock, a neutral technology shock and an investment-specific technology shock. The grey areas represent 95 percent probability intervals. The solid

¹⁷See Appendix B for a detailed derivation of how we integrate out these variables.

lines with the circles, the dashed lines, the dashed lines broken by dots and the thick solid line correspond to the impulse response functions of the ‘Alternating Offer, Hiring’, ‘Nash Sharing, Hiring’, ‘Nash Sharing, Search’, and ‘Alternating Offer, Search’ models, respectively. All impulse response functions are constructed using the posterior mode of the structural parameters estimated for the ‘Alternating Offer, Hiring’ model. Conditional on the values of the other structural parameters, we calibrate the value of η in the Nash models to obtain a steady state rate of unemployment equal to 5.5%. The values of η in the ‘Nash Sharing, Search’ and ‘Nash Sharing, Hiring’ models are 0.48 and 0.66, respectively.

From Figure 6 we see that the responses of output, hours worked, job finding rates, unemployment, vacancies, consumption and investment to a monetary policy shock are weakest in the ‘Nash Sharing, Search’ model. That model also gives rise to the strongest real wage and inflation responses. These findings are closely related to the Shimer (2005a) critique of the DMP model.

Comparing the ‘Nash Sharing, Search’ and ‘Nash Sharing, Hiring’ models we see that switching from the search cost to the hiring cost specification improves the performance of the model. In particular, output, job finding rates, unemployment, vacancies, consumption and investment exhibit stronger responses to a monetary policy shock, while real wages and inflation exhibit weaker responses. The basic intuition is that the search cost specification implies that yields on posting vacancies are countercyclical. This force mutes the effects of an expansionary monetary policy shock. A similar result emerges comparing the ‘Alternating Offer, Hiring’ and ‘Alternating Offer, Search’ models.

From Figure 7 we see that the weakest responses of output, hours worked, job finding rates, unemployment, vacancies, consumption and investment to a neutral technology shock arise in the ‘Nash Sharing, Search’ model. Also, consistent with Shimer (2005a), vacancies, job finding rates and unemployment are essentially unresponsive to the shock. As in Figure 6, moving from a search cost to a hiring cost specification improves the performance of the model. Finally, Figure 8 shows that similar but less dramatic conclusions emerge from considering an investment-specific technology shock.

We now consider the results of estimating the ‘Nash Sharing, Search’ and ‘Nash Sharing, Hiring’ models. Consider first the posterior mode of the estimated structural parameters (see Table 4). The key result here is that for the ‘Nash Sharing, Search’ and the ‘Nash Sharing, Hiring’ models, the posterior modes of the replacement ratio are 0.96 and 0.90, respectively. The high value of the replacement ratio enables the Nash sharing models to account for the response of unemployment to the three structural shocks that we consider. Indeed the impulse response functions of the ‘Alternating Offer, Hiring’ and the two Nash Sharing models are visually similar (see Figures A1 - A3 in the technical appendix). This finding is reminiscent of Hagedorn and Manovskii’s (2008) argument that a high replacement

ratio has the potential to boost the volatility of unemployment and vacancies in search and matching models.

The ‘Alternating Offer, Hiring’ model outperforms all Nash models, based on the marginal likelihood. Table 4 reports that the marginal likelihood for that model is 38 and 27 log points higher than it is for the ‘Nash Sharing, Search’ and ‘Nash Sharing, Hiring’ models, respectively.¹⁸ We infer that, subject to the approximations that we have made in calculating the marginal likelihood function, there is substantial statistical evidence in favor of the ‘Alternating Offer, Hiring’ model.

Finally, we investigate the relative importance of hiring versus search costs in our preferred model. To this end, we estimated the ‘Alternating Offer, Search’ model. From Table 4, we see that there are two significant changes in the posterior mode of the structural parameters relative to those of the ‘Alternating Offer, Hiring’ model. First, the probability of a bargaining-breakup falls from 0.3% percent to 0.06%. So, the outside option, U_t , has a more limited effect on bargaining. Second, search costs are driven to a relatively low value as a percent of gross output, 0.17%. The latter value corresponds to a share of search costs relative to the wages of new hires wages of about 2%. This value lies below the lower bound of the range suggested in Silva and Toledo (2009). In effect, the search part of the ‘Alternating Offer, Search’ model is driven out of the model. From Table 4 we see that the marginal likelihood for our baseline model is 9 log points higher than it is for the ‘Alternating Offer, Search’ model. These results imply that moving from search to hiring costs improves the empirical performance of the model. An additional reason to favor the hiring cost specification comes from the micro evidence in Yashiv (2000), Cheremukhin and Restrepo-Echavarria (2010) and Carlsson, Eriksson and Gottfries (2013). Third, and most importantly, the improvement from moving from ‘Nash Sharing’ models to ‘Alternating Offer’ models is larger than the improvement due to moving from search to hiring costs in the ‘Alternating Offer’ models (see for example the marginal likelihood values in Table 4).

6.4. The Cyclical Behavior of Unemployment and Vacancies

We have argued that our model can account for the estimated response of unemployment and vacancies to monetary policy shocks, neutral technology shocks and investment-specific technology shocks. Our methodology is quite different than the one used in much of the relevant labor market search literature. In this subsection we show that our model also does well when we assess its performance using the procedures adopted in that literature. Shimer (2005a) considers a real version of the standard DMP model in which labor productivity

¹⁸Using different models estimated on macro data of various countries, Christiano, Trabandt and Walentin (2011b), Furlanetto and Goshenny (2012a,b) and Justiniano and Michelacci (2011) also conclude that a hiring cost specification is preferred to a search cost specification.

shocks and the job separation rate are exogenous stationary stochastic processes. He argues that shocks to the job separation rate cannot be very important because they lead to a positively sloped Beveridge curve.

Shimer (2005a) deduces the model’s implications for HP-filtered moments which he compares to the analog moments in U.S. data. He focuses on the volatility of vacancies divided by unemployment relative to the volatility of labor productivity. Shimer also looks at the persistence and the correlation among these variables.¹⁹ Shimer (2005a) emphasizes that the standard DMP model fails to account of the volatility of vacancies divided by unemployment, $\sigma(v/u)$, relative to the volatility of labor productivity, $\sigma(Y/l)$. We refer to the ratio $\sigma(v/u)/\sigma(Y/l)$ as the ‘volatility ratio’.²⁰ Shimer (2005a) reports that the volatility ratio is about 20 in U.S. data. But in the standard DMP model analyzed by Shimer (2005a), the volatility ratio is only roughly 2.

In the spirit of Shimer’s (2005a) analysis, we consider a version of our model in which the only source of uncertainty is a neutral technology shock, z_t . As in the estimated DSGE model we assume that the growth rate of the neutral technology shock is *i.i.d.*. We set the standard deviation of the innovation to that shock, σ_{μ_z} , to 0.7%, a value that implies that the standard deviation of HP-filtered real GDP in the model and the data are the same.²¹

Table 6 reports estimates for various moments of the data and the implications of the ‘Alternating Offer, Hiring’ and ‘Nash Sharing, Search’ models for these moments. Consider first the results of calculating model moments using parameter values equal to the estimated posterior mode of the ‘Alternating Offer, Hiring’ model. The key finding is that the volatility ratio implied by the ‘Alternating Offer, Hiring’ model is 27.8 which effectively reproduces the analog statistic in our data, i.e. 27.6. In this sense our model is *not* subject to the Shimer critique. Notice that our model also accounts very well for the standard deviations and, with one exception, the first-order autocorrelations of vacancies, unemployment and labor productivity. The exception is that the model somewhat understates the first-order autocorrelation of vacancies. Finally, the model does quite well in reproducing the unconditional correlations between vacancies, unemployment and labor productivity. Consider next the implications of the ‘Nash Sharing, Search’ model. Consistent with Shimer (2005a), this model generates a much smaller value of the volatility ratio, namely 13.2. In addition, the model has problems reproducing volatilities, as well as some of the first-order autocorrelations and correlation statistics. For example, the model generates the wrong sign for the

¹⁹See Shimer (2005a), Table 1, page 28. Hagedorn, and Manovskii (2008) consider the same statistics.

²⁰Here, $\sigma(\cdot)$ denotes the standard deviation of a time series variable after it has been HP-filtered.

²¹This value of σ_{μ_z} is much larger than the mode of the posterior in the estimated medium-sized DSGE model. This fact reflects that in this subsection we are attributing all movements in real GDP to neutral technology shocks. We do not maintain such an assumption when we estimate the model. With such a large value of σ_{μ_z} there are occasions in a long simulated time series where variables like the unemployment rate rise to high levels not observed in the data.

correlation between unemployment and labor productivity (0.2 in the model and -0.3 in the data).

Interestingly, the volatility ratio implied by the ‘Nash Sharing, Search’ is higher than the one reported in Shimer (2005a) for the DMP model. The difference in results reflects that our medium-sized DSGE model is considerably more complex than the model used by Shimer (2005a). We have examined the case when we eliminate habit formation, the working capital channel and physical capital from our model. Further, we also suppose that firms change prices roughly once a quarter ($\xi = 0.15$). Under these assumptions - which brings our model as close as possible to the one studied by Shimer (2005a) - it turns out that the ‘volatility ratio’ is equal to 24.4 in the ‘Alternating Offer, Hiring’ and only 2.6 in the ‘Nash Sharing, Search’ model.

Finally, we evaluate the implications of the ‘Nash Sharing, Search’ model using the posterior mode of the parameter estimates for that model. Among other things, the replacement ratio for this model is 0.96. Consistent with Hagedorn and Manovskii (2008), we find that the ‘Nash Sharing, Search’ model is able to roughly account for the ‘volatility ratio’. That ratio is 27.6 in our data and 24.5 in the ‘Nash Sharing, Search’ model.

Hagedorn and Manovskii (2008) and Shimer (2005a) work with stationary representations of the neutral technology shock. To assess the robustness of our results we redid our calculations assuming that z_t is an AR(2) process with roots equal to 1.35 and -0.45 and a standard deviation of the shock equal to 0.3. With these parameter values the estimated ‘Alternating Offer, Hiring’ model roughly matches the point estimates of the four volatility moments, the four first-order autocorrelation moments and the six correlation moments reported in Table 6. Importantly, the ‘volatility ratio’ in the model is roughly equal to 30, while it is 27.6 in the data. In contrast, with this parameterization of the technology process, the ‘volatility ratio’ in the ‘Nash Sharing, Search’ model is only equal to 10. If we work with the mode of the estimated version of the latter model, we are able to account for the ‘volatility ratio’. So, the results that we obtain with the stationary technology process are very similar to those reported above.

Viewed as a whole, the results of this section corroborate our argument that the ‘Alternating Offer, Hiring’ model does well at accounting for the cyclical properties of key labor market variables and outperforms the competing models that we consider. The result holds whether we assess the model using our impulse response methodology or use the statistics stressed in the relevant literature.

7. Conclusion

This paper constructs and estimates an equilibrium business cycle model which can account for the response of the U.S. economy to neutral and investment-specific technology shocks as well as monetary policy shocks. The focus of our analysis is how labor markets respond to these shocks. Significantly, our model does not assume that wages are sticky. Instead, we derive inertial wages from our specification of how firms and workers interact when negotiating wages. This inertia can be interpreted as applying to the period-by-period wage, or to the present value of the wage package negotiated at the time that a worker and firm first meet. It remains an open question which implications for optimal policy of existing DSGE models are sensitive to abandoning the sticky wage assumption. We leave the answer to this question to future research.

We have been critical of standard sticky wage models in this paper. Still, Hall (2005) describes one interesting line of defense for sticky wages. He introduces sticky wages into the DMP framework in a way that satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement (Hall, 2005, p. 50). A sketch of Hall's logic is as follows: in a model with labor market frictions, there is a gap between the reservation wage required by a worker to accept employment and the highest wage a firm is willing to pay an employee. This gap, or bargaining set, fluctuates with the shocks that affect the surplus enjoyed by the worker and the employer. When calibrated based on aggregate data, the fluctuations in the bargaining set are sufficiently small and the width of the set is sufficiently wide, that an exogenously sticky wage rate can remain inside the set for an extended period of time. Gertler and Trigari (2009) and Shimer (2012b) pursue this idea in a calibrated model while Gertler, Sala and Trigari (2008) do so in an estimated, medium-sized DSGE model.²² A concern about this strategy for justifying sticky wages is that the microeconomic shocks which move actual firms' bargaining sets are far more volatile than what the aggregate data suggest. As a result, it may be harder to use the preceding approach to rationalize sticky wages than had initially been recognized. An important task is to discriminate between the approach taken in this paper and the approach proposed in Hall (2005).

We wish to emphasize that our approach follows HM in assuming that the cost of disagreement in wage negotiations is relatively insensitive to the state of the business cycle. This assumption played a key role in the empirical success of our model. Assessing the empirical plausibility of this assumption using microeconomic data is a task that we leave to future research.

²²See also Krause and Lubik (2007), Christiano, Ilut, Motto and Rostagno (2008) and Christiano, Trabandt and Walentin (2011b).

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Appendix

A. The Equilibrium Wage Rate

We develop an analytic expression relating the equilibrium wage rate to economy-wide variables taken as given by firms and workers when bargaining.

It is useful to re-state the indifference conditions for the worker and the firm given in the main text:

$$\begin{aligned}
w_{j,t} + \tilde{w}_t^p + A_t &= \delta \left[\frac{M-j+1}{M} D + \tilde{U}_t \right] + (1-\delta) \left[\frac{D}{M} + w_{j+1,t} + \tilde{w}_t^p + A_t \right] && \text{for } j = 1, 3, \dots, M-1 \\
\frac{M-j+1}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{j,t} + \tilde{w}_t^p) &= (1-\delta) \left[-\gamma + \frac{M-j}{M} \vartheta_t + \tilde{\vartheta}_t^p - (w_{j+1,t} + \tilde{w}_t^p) \right] && \text{for } j = 2, 4, \dots, M-2 \\
\frac{\vartheta_t}{M} + \tilde{\vartheta}_t^p - (w_{j,t} + \tilde{w}_t^p) &= 0 && \text{for } j = M
\end{aligned}$$

Rewrite the previous expressions and abbreviate variables taken as given during the wage bargaining:

$$\begin{aligned}
w_{j,t} + \tilde{w}_t^p &= \underbrace{\frac{D}{M} + \delta(U_t - A_t)}_a - \underbrace{\frac{\delta D}{M} j}_{c_j} + (1-\delta)(w_{j+1,t} + \tilde{w}_t^p) && \text{for } j = 1, 3, 5, \dots, M-1 \\
w_{j,t} + \tilde{w}_t^p &= \underbrace{\frac{\vartheta_t}{M} + \delta \vartheta_t^p + (1-\delta)\gamma}_b - \underbrace{\frac{\delta \vartheta_t}{M} j}_{d_j} + (1-\delta)(w_{j+1,t} + \tilde{w}_t^p) && \text{for } j = 2, 4, 5, \dots, M-2 \\
w_{j,t} + \tilde{w}_t^p &= \left(\frac{1-M}{M} \right) \vartheta_t + \vartheta_t^p && \text{for } j = M
\end{aligned}$$

Or, in short:

$$\begin{aligned}
w_{j,t} + \tilde{w}_t^p &= a - c_j + (1-\delta)(w_{j+1,t} + \tilde{w}_t^p) && \text{for } j = 1, 3, 5, \dots, M-1 \\
w_{j,t} + \tilde{w}_t^p &= b - d_j + (1-\delta)(w_{j+1,t} + \tilde{w}_t^p) && \text{for } j = 2, 4, 5, \dots, M-2
\end{aligned}$$

Write out:

$$\begin{aligned}
w_t^p = w_{1,t} + \tilde{w}_t^p &= a - c_1 + (1-\delta)(w_{2,t} + \tilde{w}_t^p) \\
w_{2,t} + \tilde{w}_t^p &= b - d_2 + (1-\delta)(w_{3,t} + \tilde{w}_t^p) \\
w_{3,t} + \tilde{w}_t^p &= a - c_3 + (1-\delta)(w_{4,t} + \tilde{w}_t^p) \\
w_{4,t} + \tilde{w}_t^p &= b - d_4 + (1-\delta)(w_{5,t} + \tilde{w}_t^p) \\
&\dots \\
w_{M-1,t} + \tilde{w}_t^p &= a - c_{M-1} + (1-\delta)(w_{M,t} + \tilde{w}_t^p)
\end{aligned}$$

Substituting several times results in the following pattern:

$$\begin{aligned}
w_t^p &= a + (1-\delta)^2 a + (1-\delta)^4 a + (1-\delta)^6 a \\
&\quad + (1-\delta) b + (1-\delta)^3 b + (1-\delta)^5 b \\
&\quad - c_1 - (1-\delta)^2 c_3 - (1-\delta)^4 c_5 - (1-\delta)^6 c_7 \\
&\quad - (1-\delta) d_2 - (1-\delta)^3 d_4 - (1-\delta)^5 d_6 \\
&\quad + (1-\delta)^7 (w_{8,t} + \tilde{w}_t^p)
\end{aligned}$$

Rearrange:

$$\begin{aligned}
w_t^p &= a + (1 - \delta)^2 a + (1 - \delta)^4 a + (1 - \delta)^6 a + \dots + (1 - \delta)^{M-2} a \\
&\quad + (1 - \delta) b + (1 - \delta)^3 b + (1 - \delta)^5 b + \dots + (1 - \delta)^{M-3} b \\
&\quad - c_1 - (1 - \delta)^2 c_3 - (1 - \delta)^4 c_5 - (1 - \delta)^6 c_7 - \dots - (1 - \delta)^{M-2} c_{M-1} \\
&\quad - (1 - \delta) d_2 - (1 - \delta)^3 d_4 - (1 - \delta)^5 d_6 - \dots - (1 - \delta)^{M-3} d_{M-2} \\
&\quad + (1 - \delta)^{M-1} (w_{M,t} + \tilde{w}_t^p)
\end{aligned}$$

Or, equivalently:

$$\begin{aligned}
w_t^p &= a \left[1 + (1 - \delta)^2 + (1 - \delta)^4 + (1 - \delta)^6 + \dots + (1 - \delta)^{M-2} \right] \\
&\quad b (1 - \delta) \left[1 + (1 - \delta)^2 + (1 - \delta)^4 + (1 - \delta)^6 + \dots + (1 - \delta)^{M-4} \right] \\
&\quad - c_1 - (1 - \delta)^2 c_3 - (1 - \delta)^4 c_5 - (1 - \delta)^6 c_7 - \dots - (1 - \delta)^{M-2} c_{M-1} \\
&\quad - (1 - \delta) d_2 - (1 - \delta)^3 d_4 - (1 - \delta)^5 d_6 - \dots - (1 - \delta)^{M-3} d_{M-2} \\
&\quad + (1 - \delta)^{M-1} \left[\left(\frac{1 - M}{M} \right) \vartheta_t + \vartheta_t^p \right]
\end{aligned} \tag{A.1}$$

Note that:

$$\begin{aligned}
S &= 1 + x + x^2 + x^3 + \dots + x^n \\
xS &= x + x^2 + x^3 + \dots + x^n + x^{n+1}
\end{aligned}$$

Subtract and rearrange:

$$S = \frac{1 - x^{n+1}}{1 - x}$$

So that:

$$1 + x + x^2 + x^3 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x} \tag{A.2}$$

Using (A.2), we can write the square brackets multiplying a and b in (A.1) as:

$$\left[1 + \underbrace{(1 - \delta)^2}_x + \underbrace{(1 - \delta)^4}_{x^2} + \underbrace{(1 - \delta)^6}_{x^3} + \dots + \underbrace{(1 - \delta)^{M-2}}_{x^{(M-2)/2}} \right] = \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2}$$

and

$$\left[1 + \underbrace{(1 - \delta)^2}_x + \underbrace{(1 - \delta)^4}_{x^2} + \underbrace{(1 - \delta)^6}_{x^3} + \dots + \underbrace{(1 - \delta)^{M-4}}_{x^{(M-4)/2}} \right] = \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2}$$

Hence,

$$\begin{aligned}
w_t^p &= \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} a + b(1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} \\
&\quad - \left[c_1 + (1 - \delta)^2 c_3 + (1 - \delta)^4 c_5 + \dots + (1 - \delta)^{M-2} c_{M-1} \right] \\
&\quad - (1 - \delta) \left[d_2 + (1 - \delta)^2 d_4 + (1 - \delta)^4 d_6 + \dots + (1 - \delta)^{M-4} d_{M-2} \right] \\
&\quad + (1 - \delta)^{M-1} \left[\left(\frac{1 - M}{M} \right) \vartheta_t + \vartheta_t^p \right]
\end{aligned} \tag{A.3}$$

The square bracket in the last line in (A.3) can be written as,

$$\begin{aligned}
&\left[d_2 + (1 - \delta)^2 d_4 + (1 - \delta)^4 d_6 + \dots + (1 - \delta)^{M-4} d_{M-2} \right] \\
&= 2 \frac{\delta \vartheta_t}{M} \left[1 + (1 - \delta)^2 2 + (1 - \delta)^4 3 + \dots + (1 - \delta)^{M-4} \frac{(M - 2)}{2} \right]
\end{aligned}$$

Note that differentiating both sides of:

$$1 + x + x^2 + x^3 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

yields

$$1 + 2x + 3x^2 + \dots + nx^{n-1} = \frac{-(n+1)x^n(1-x) + (1-x^{n+1})}{(1-x)^2}$$

Hence, the square bracket of the last line in (A.3) can be expressed more compactly as:

$$\begin{aligned}
&\left[1 + \underbrace{(1 - \delta)^2 2}_x + \underbrace{(1 - \delta)^4 3}_{x^2} + \underbrace{(1 - \delta)^6 4}_{x^3} + \dots + \underbrace{(1 - \delta)^{M-4} \frac{(M - 2)}{2}}_{x^{(M-4)/2} \frac{2}{n}} \right] \\
&= \frac{\left(1 - (1 - \delta)^M \right) - \frac{M}{2} (1 - \delta)^{(M-2)} (1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2}
\end{aligned}$$

Finally, the terms involving c in (A.3) can be rewritten as:

$$\begin{aligned}
&\left[c_1 + (1 - \delta)^2 c_3 + (1 - \delta)^4 c_5 + (1 - \delta)^6 c_7 + \dots + (1 - \delta)^{M-2} c_{M-1} \right] \\
&= \frac{\delta D}{M} \left[1 + (1 - \delta)^2 3 + (1 - \delta)^4 5 + (1 - \delta)^6 7 + \dots + (1 - \delta)^{M-2} (M - 1) \right] \\
&= \frac{\delta D}{M} 2 \left[1/2 + (1 - \delta)^2 2 + (1 - \delta)^4 3 + (1 - \delta)^6 4 + \dots + (1 - \delta)^{M-2} M/2 \right] \\
&\quad - \frac{\delta D}{M} \left[1 + (1 - \delta)^2 + (1 - \delta)^4 + (1 - \delta)^6 + \dots + (1 - \delta)^{M-2} \right] + \frac{\delta D}{M} \\
&= 2 \frac{\delta D}{M} \frac{\left(1 - (1 - \delta)^{M+2} \right) - \left(1 + \frac{M}{2} \right) (1 - \delta)^M (1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2} \\
&\quad - \frac{\delta D}{M} \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2}
\end{aligned}$$

Pulling everything together, we can write (A.3) as:

$$\begin{aligned}
w_t^p &= \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \left[\frac{D}{M} + \delta (U_t - A_t) \right] \\
&+ (1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} \left[\frac{\vartheta_t}{M} + \delta \vartheta_t^p + (1 - \delta) \gamma \right] \\
&- 2 \frac{\delta D \left(1 - (1 - \delta)^{M+2} \right) - (1 + \frac{M}{2}) (1 - \delta)^M (1 - (1 - \delta)^2)}{M (1 - (1 - \delta)^2)^2} \\
&+ \frac{\delta D}{M} \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \\
&- (1 - \delta) 2 \frac{\delta \vartheta_t \left(1 - (1 - \delta)^M \right) - \frac{M}{2} (1 - \delta)^{(M-2)} (1 - (1 - \delta)^2)}{M (1 - (1 - \delta)^2)^2} \\
&+ (1 - \delta)^{M-1} \left[\left(\frac{1 - M}{M} \right) \vartheta_t + \vartheta_t^p \right]
\end{aligned}$$

Collecting terms gives:

$$\begin{aligned}
w_t^p &= \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2} \delta (U_t - A_t) + \left[(1 - \delta) \delta \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} + (1 - \delta)^{M-1} \right] \vartheta_t^p \\
&+ (1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} (1 - \delta) \gamma \\
&+ \left[\frac{(1 + \delta) \frac{1 - (1 - \delta)^M}{1 - (1 - \delta)^2}}{-2 \delta \frac{(1 - (1 - \delta)^{M+2}) - (1 + \frac{M}{2}) (1 - \delta)^M (1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2}} \right] \frac{D}{M} \\
&+ \left[\frac{(1 - \delta) \frac{1 - (1 - \delta)^{M-2}}{1 - (1 - \delta)^2} + (1 - \delta)^{M-1} (1 - M)}{- (1 - \delta) 2 \delta \frac{(1 - (1 - \delta)^M) - \frac{M}{2} (1 - \delta)^{(M-2)} (1 - (1 - \delta)^2)}{(1 - (1 - \delta)^2)^2}} \right] \frac{\vartheta_t}{M}
\end{aligned}$$

Simplifying, using straightforward algebra yields:

$$\begin{aligned}
(2 - \delta) w_t^p &= \left(1 - (1 - \delta)^M \right) (U_t - A_t) + \left(1 - \delta + (1 - \delta)^M \right) \vartheta_t^p \\
&+ \frac{1}{\delta} \left((1 - \delta)^2 - (1 - \delta)^M \right) \gamma \\
&+ \frac{(1 - \delta)^M (1 - \delta - (2 - \delta) M) - (1 - \delta)}{2 - \delta} \left[\frac{\vartheta_t}{M} - \frac{D}{M} \right]
\end{aligned}$$

After some further rewriting, we can express the previous expression as the following alternating offer bargaining sharing rule:

$$(\alpha_1 + \alpha_2) w_t^p = \alpha_1 \vartheta_t^p + \alpha_2 (U_t - A_t) + \alpha_3 \gamma - \alpha_4 (\vartheta_t - D)$$

where

$$\begin{aligned}\alpha_1 &= 1 - \delta + (1 - \delta)^M \\ \alpha_2 &= 1 - (1 - \delta)^M \\ \alpha_3 &= \alpha_2 \frac{1 - \delta}{\delta} - \alpha_1 \\ \alpha_4 &= \frac{1 - \delta}{2 - \delta} \frac{\alpha_2}{M} + 1 - \alpha_2.\end{aligned}$$

Note that $\alpha_1, \dots, \alpha_4 > 0$. Alternatively, we can write the alternating offer bargaining sharing rule in terms of the following variables:

$$\alpha_1 J_t = \alpha_2 (V_t - U_t) - \alpha_3 \gamma + \alpha_4 (\vartheta_t - D).$$

Finally, notice that for $M \rightarrow \infty$, the sharing rule becomes:

$$J_t = \frac{1}{1 - \delta} \left[V_t - U_t - \frac{(1 - \delta)^2}{\delta} \gamma \right].$$

B. Marginal Likelihood for a Subset of Data

We derive the marginal likelihood function for a common subset of data. We denote the full set of data by the $N \times 1$ vector, $\hat{\psi}$. We decompose $\hat{\psi}$ into two parts:

$$\hat{\psi} = \begin{bmatrix} \hat{\psi}_1 \\ \hat{\psi}_2 \end{bmatrix},$$

where $\hat{\psi}_i$ is $N_i \times 1$, $i = 1, 2$ and $N_1 + N_2 = N$. We have a marginal likelihood for $\hat{\psi}$:

$$f(\hat{\psi}) = \int f(\hat{\psi}|\theta) p(\theta) d\theta,$$

where $f(\hat{\psi}|\theta)$ denotes the likelihood of $\hat{\psi}$ conditional on the model parameters, θ . Also, $p(\theta)$ denotes the priors. We seek the marginal likelihood for the subset of data $\hat{\psi}_1$, which is defined as:

$$f(\hat{\psi}_1) = \int f(\hat{\psi}) d\hat{\psi}_2.$$

For this, we rely heavily on the Laplace approximation to $f(\hat{\psi})$:

$$\frac{f(\hat{\psi}|\theta^*) p(\theta^*)}{(2\pi)^{-\frac{M}{2}} |g_{\theta^*\theta^*}|^{\frac{1}{2}}}, \tag{B.1}$$

where $g_{\theta^*\theta^*}$ denotes the second derivative of $\log f(\hat{\psi}|\theta) p(\theta)$ with respect to θ , evaluated at the mode, θ^* . Also, M denotes the number of elements in θ . Note that we can write the matrix V as follows:

$$V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix},$$

Where V_{11} is the upper $N_1 \times N_1$ block of V and V_{22} is the lower $N_2 \times N_2$ block. The zeros on the off-diagonal of V reflect our assumption that V is diagonal. Using this notation, we write our approximation to the likelihood (5.3) as follows:

$$\begin{aligned} f(\hat{\psi}|\theta^*) &= (2\pi)^{-\frac{N_1}{2}} |V_{11}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_1 - \psi_1(\theta^*))' V_{11}^{-1} (\hat{\psi}_1 - \psi_1(\theta^*)) \right] \\ &\quad \times (2\pi)^{-\frac{N_2}{2}} |V_{22}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_2 - \psi_2(\theta^*))' V_{22}^{-1} (\hat{\psi}_2 - \psi_2(\theta^*)) \right]. \end{aligned}$$

Substituting this expression into (B.1), we obtain the following representation of the marginal likelihood of $\hat{\psi}$:

$$\begin{aligned} f(\hat{\psi}) &= (2\pi)^{\frac{M-N_1}{2}} |g_{\theta^*\theta^*}|^{-\frac{1}{2}} |V_{11}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_1 - \psi_1(\theta^*))' V_{11}^{-1} (\hat{\psi}_1 - \psi_1(\theta^*)) \right] \\ &\quad \times (2\pi)^{-\frac{N_2}{2}} |V_{22}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_2 - \psi_2(\theta^*))' V_{22}^{-1} (\hat{\psi}_2 - \psi_2(\theta^*)) \right] p(\theta^*) \end{aligned}$$

Now it is straightforward to compute our approximation to $f(\hat{\psi}_1)$:

$$\begin{aligned} f(\hat{\psi}_1) &= \int_{\hat{\psi}_2} f(\hat{\psi}) d\hat{\psi}_2 \\ &= (2\pi)^{\frac{M-N_1}{2}} |g_{\theta^*\theta^*}|^{-\frac{1}{2}} |V_{11}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_1 - \psi_1(\theta^*))' V_{11}^{-1} (\hat{\psi}_1 - \psi_1(\theta^*)) \right] p(\theta^*). \end{aligned}$$

Here, we have used

$$\int_{\hat{\psi}_2} (2\pi)^{-\frac{N_2}{2}} |V_{22}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\psi}_2 - \psi_2(\theta^*))' V_{22}^{-1} (\hat{\psi}_2 - \psi_2(\theta^*)) \right] d\hat{\psi}_2 = 1,$$

which follows from the fact that the integrand is a density function.

Table 1: Parameters and Steady State Values in the Small Macro Model

Parameter	Value	Description
<i>Panel A: Parameters</i>		
β	$1.03^{-0.25}$	Discount factor
ξ	2/3	Calvo price stickiness
λ	1.2	Price markup parameter
ρ_R	0.7	Taylor rule: interest rate smoothing
r_π	1.7	Taylor rule: inflation coefficient
r_y	0.1	Taylor rule: employment coefficient
ρ	0.9	Job survival probability
M	60	Max. bargaining rounds per quarter
δ	0.005	Probability of bargaining session break-up
τ	0.95	AR(1) technology
<i>Panel B: Steady State Values</i>		
$400(\pi - 1)$	0	Annual net inflation rate
l	0.945	Employment
D/w	0.4	Replacement ratio
$\kappa xl/Y$	0.01	Hiring cost to output ratio

^a Sticky wages as in Erceg, Henderson and Levin (2000).

Table 2: Small Model Steady States and Implied Parameters for ‘Alternating Offer, Hiring’ Model

Variable	Model	Description
C	0.94	Consumption
Y	0.95	Gross output
e^a	1	Steady state technology
ϑ	1	Marginal revenue of wholesalers
w	0.99	Real wage
U	129.1	Value of unemployment
V	130.0	Value of work
J	0.1	Firm value
u	0.055	Steady state unemployment rate
f	0.63	Job finding rate
D	0.396	Unemployment benefits
κ	0.1	Hiring cost parameter
$\gamma/(\vartheta/M)$	0.61	Counteroffer cost as share of daily revenue

Table 3: Non-Estimated Parameters and Calibrated Variables in the Medium-sized Model

Parameter	Value	Description
<i>Panel A: Parameters</i>		
δ_K	0.025	Depreciation rate of physical capital
β	0.9968	Discount factor
ρ	0.9	Job survival probability
M	60	Max. bargaining rounds per quarter (alternating offers model)
λ_w	1.2	Wage markup parameter (sticky wage model)
ξ_w	0.75	Wage stickiness (sticky wage model)
$400\log(\mu)$	1.7	Annual output per capita growth rate
$400\log(\mu \cdot \mu_\Psi)$	2.9	Annual investment per capita growth rate
<i>Panel B: Steady State Values</i>		
$400(\pi - 1)$	2.5	Annual net inflation rate
<i>profits</i>	0	Intermediate goods producers profits
Q	0.7	Vacancy filling rate
u	0.055	Unemployment rate
G/Y	0.2	Government consumption to gross output ratio

Table 4: Priors and Posteriors of Parameters for the Medium-sized Model

Model #		Prior		Posterior					
				Alternating Offer Bargaining		Nash Sharing		Sticky Wage Wage Indexation:	
				Hiring \mathcal{M}_1	Search \mathcal{M}_2	Hiring \mathcal{M}_3	Search \mathcal{M}_4	No \mathcal{M}_5	Yes \mathcal{M}_6
		D,Mean,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std
<i>Price Setting Parameters</i>									
Price Stickiness	ξ	B,0.66,0.15	0.58 ,0.03	0.64 ,0.04	0.70 ,0.02	0.74 ,0.02	0.74 ,0.02	0.65 ,0.03	
Price Markup Parameter	λ	G,1.20,0.05	1.43 ,0.04	1.43 ,0.04	1.42 ,0.04	1.43 ,0.04	1.25 ,0.05	1.36 ,0.04	
<i>Monetary Authority Parameters</i>									
Taylor Rule: Smoothing	ρ_R	B,0.70,0.15	0.86 ,0.01	0.86 ,0.01	0.84 ,0.01	0.84 ,0.01	0.78 ,0.01	0.86 ,0.01	
Taylor Rule: Inflation	r_π	G,1.70,0.15	1.36 ,0.11	1.39 ,0.12	1.37 ,0.12	1.39 ,0.12	2.09 ,0.15	1.48 ,0.13	
Taylor Rule: GDP	r_y	G,0.10,0.05	0.04 ,0.01	0.04 ,0.01	0.04 ,0.01	0.04 ,0.01	0.01 ,0.01	0.09 ,0.03	
<i>Preferences and Technology</i>									
Consumption Habit	b	B,0.50,0.15	0.83 ,0.01	0.83 ,0.01	0.82 ,0.01	0.82 ,0.01	0.70 ,0.02	0.76 ,0.02	
Capacity Util. Adj. Cost	σ_a	G,0.50,0.30	0.08 ,0.04	0.06 ,0.03	0.06 ,0.04	0.05 ,0.03	0.04 ,0.02	0.04 ,0.03	
Investment Adj. Cost	S''	G,8.00,2.00	13.67 ,1.8	13.80 ,1.9	13.41 ,1.9	13.50 ,1.9	5.31 ,0.82	7.94 ,1.10	
Capital Share	α	B,0.33,0.03	0.24 ,0.02	0.24 ,0.02	0.25 ,0.02	0.25 ,0.02	0.32 ,0.02	0.29 ,0.02	
Techn. Diffusion g, ϕ, κ, γ	θ_i	B,0.50,0.20	0.01 ,0.01	0.02 ,0.01	0.01 ,0.01	0.01 ,0.01	0.04 ,0.02	0.02 ,0.01	
Technology Diffusion D	θ_D	B,0.50,0.20	0.74 ,0.15	0.66 ,0.19	0.12 ,0.03	0.10 ,0.02	-	-	
<i>Labor Market Parameters</i>									
Prob. of Barg. Breakup	100δ	G,0.50,0.40	0.30 ,0.06	0.06 ,0.06	-	-	-	-	
Replacement Ratio	D/w	B,0.40,0.10	0.67 ,0.06	0.69 ,0.07	0.90 ,0.01	0.96 ,0.01	-	-	
Hiring-Search Cost/Y	s_l	G,1.00,0.30	0.50 ,0.16	0.17 ,0.04	0.55 ,0.17	0.47 ,0.14	-	-	
Match. Function Param.	σ	B,0.50,0.10	0.56 ,0.03	0.52 ,0.04	0.56 ,0.03	0.50 ,0.04	-	-	
Inv. Labor Supply Elast.	ψ	G,1.00,0.25	-	-	-	-	0.89 ,0.20	2.19 ,0.33	
<i>Shocks</i>									
Std. Monetary Policy	σ_R	G,0.65,0.05	0.60 ,0.03	0.62 ,0.03	0.62 ,0.03	0.62 ,0.03	0.64 ,0.04	0.62 ,0.03	
Std. Neutral Technology	σ_{μ_z}	G,0.10,0.05	0.14 ,0.01	0.14 ,0.02	0.17 ,0.01	0.17 ,0.02	0.31 ,0.02	0.25 ,0.02	
Std. Invest. Technology	σ_Ψ	G,0.10,0.05	0.11 ,0.02	0.11 ,0.02	0.11 ,0.02	0.11 ,0.02	0.15 ,0.02	0.14 ,0.02	
AR(1) Invest. Technology	ρ_Ψ	B,0.75,0.10	0.73 ,0.06	0.73 ,0.06	0.74 ,0.06	0.75 ,0.05	0.58 ,0.06	0.63 ,0.06	
<i>Memo Items</i>									
Log Marg. Likelihood (Laplace, 12 Variables):			302.0	291.5	279.3	263.5	-	-	
Log Marg. Likelihood (Laplace, 9 Variables ^b):			328.2	319.1	301.1	290.3	260.9	324.5	
Post. Odds - $\mathcal{M}_1 : \mathcal{M}_i, i = 1, \dots, 6$ (9 Variab.):			1:1	9e3:1	6e11:1	3e16:1	2e29:1	1:40.4	

Notes: s_l denotes the steady state hiring or search cost to gross output ratio (in percent). For model specifications where particular parameter values are not relevant, the entries in this table are blank.

^a Sticky wage model as in Erceg, Henderson and Levin (2000).

^b Common dataset across all models, i.e. when unemployment, vacancies and job finding rates are excluded.

Table 5: Medium-sized Model Steady States and Implied Parameters at Posterior Mode in ‘Alternating Offer, Hiring’ Model

Variable	Value	Description
K/Y	6.71	Capital to gross output ratio (quarterly)
C/Y	0.58	Consumption to gross output ratio
I/Y	0.22	Investment to gross output ratio
l	0.945	Steady state labor input
R	1.014	Gross nominal interest rate (quarterly)
R^{real}	1.0075	Gross real interest rate (quarterly)
mc	0.701	Marginal cost (inverse markup)
σ_b	0.036	Capacity utilization cost parameter
Y	1.07	Gross output
ϕ/Y	0.43	Fixed cost to gross output ratio
σ_m	0.66	Level parameter in matching function
f	0.63	Job finding rate
ϑ	0.85	Marginal revenue of wholesaler
x	0.1	Hiring rate
J	0.06	Value of firm
V	258.1	Value of work
U	257.7	Value of unemployment
v	0.14	Vacancy rate
w	0.84	Real wage
$\gamma/(\vartheta/M)$	0.27	Counteroffer costs as share of daily revenue

Table 6: Data vs. Medium-Sized Model With Unit-Root Neutral Technology Shock

<i>Volatility Statistics</i>					
	$\sigma(u)$	$\sigma(v)$	$\sigma(v/u)$	$\sigma(Y/l)$	$\sigma(v/u)/\sigma(Y/l)$
Data	0.13	0.14	0.26	0.009	27.6
‘Alternating Offer, Hiring’ Model	0.17	0.13	0.29	0.010	27.8
‘Nash Sharing, Search’ Model (DMP)	0.06	0.07	0.12	0.009	13.2
<i>First Order Autocorrelations</i>					
	u	v	v/u	Y/l	
Data	0.86	0.90	0.89	0.70	
‘Alternating Offer, Hiring’ Model	0.84	0.46	0.71	0.78	
‘Nash Sharing, Search’ Model (DMP)	0.60	0.13	0.36	0.83	
<i>Correlations</i>					
	u, v	$u, v/u$	$v, v/u$	$u, Y/l$	
Data	-0.91	-0.98	0.98	-0.28	
‘Alternating Offer, Hiring’ Model	-0.85	-0.97	0.95	-0.19	
‘Nash Sharing, Search’ Model (DMP)	-0.81	-0.95	0.96	0.21	
	$v, Y/l$	$v/u, Y/l$			
Data	0.37	0.33			
‘Alternating Offer, Hiring’ Model	0.26	0.23			
‘Nash Sharing, Search’ Model (DMP)	-0.04	-0.13			

Notes: u , v and Y/l denote the unemployment rate, vacancies and labor productivity; $\sigma(\cdot)$ is the standard deviation of these variables. All data are in log levels and hp-filtered with smoothing parameter 1600. The sample period is 1951Q1 to 2008Q4. Data sources are the same as those used for the estimation of the medium-sized model. Similar to Shimer (2005), we simulate the model using a unit-root neutral technology shock. See the main text for details.

Figure 1: Small Model Impulse Responses to a 25 ABP Monetary Policy Shock

—●— Baseline - - - Higher δ - · - Lower γ - · - Lower D ··· Lower M

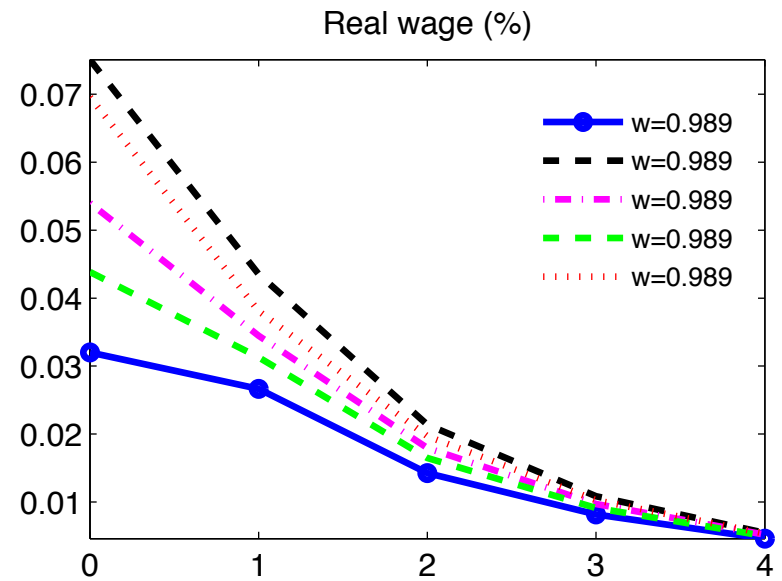
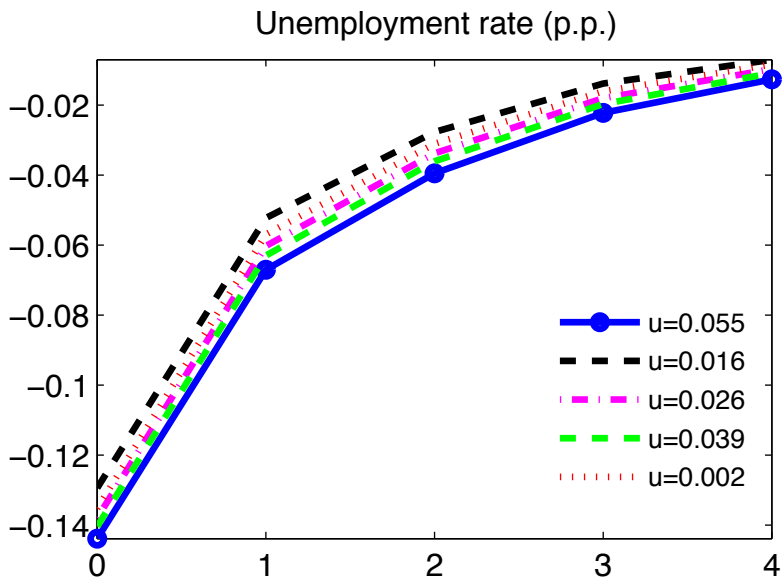
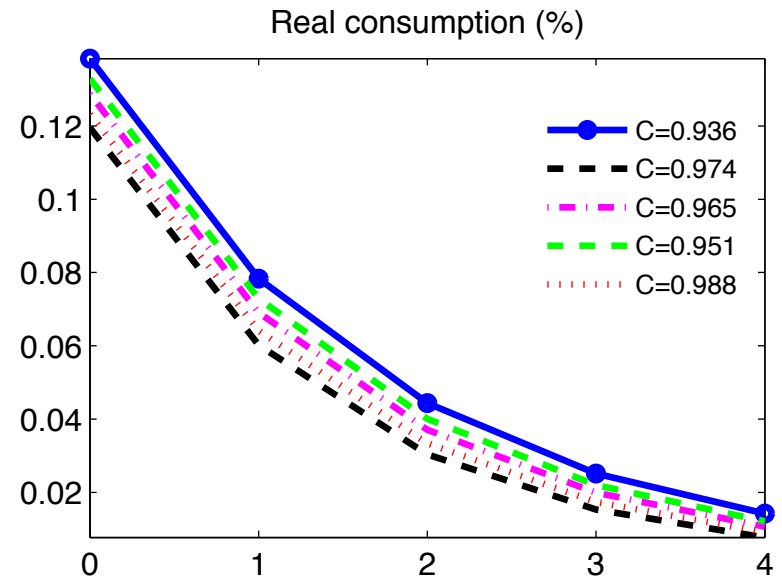
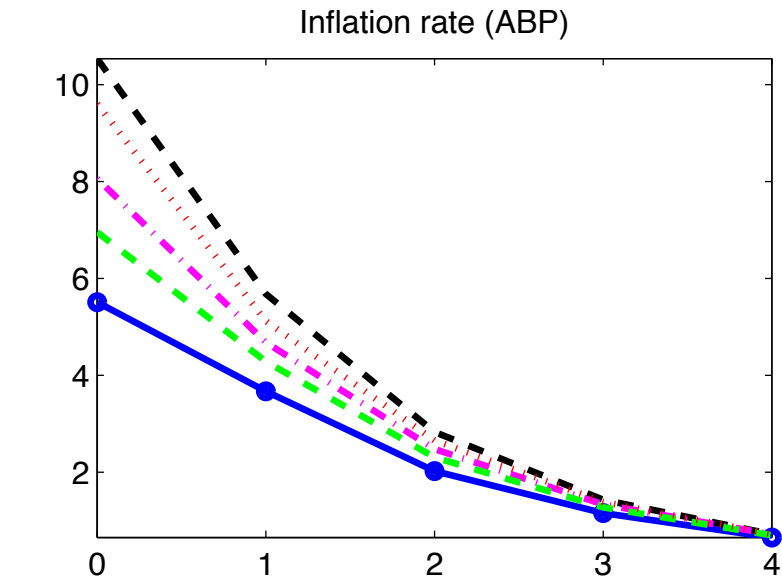


Figure 2: Small Model Impulse Responses to a 0.1 Percent Technology Shock

● Baseline
 - - Higher δ
 - · - Lower γ
 - - - Lower D
 · · · Lower M

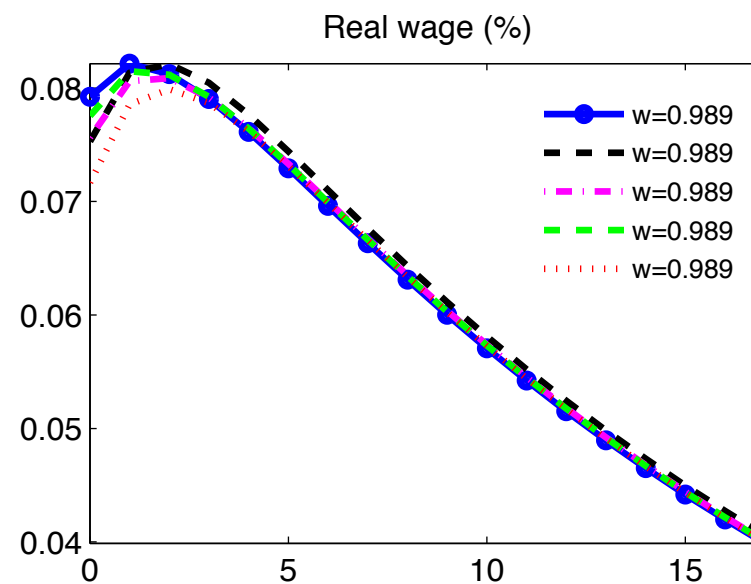
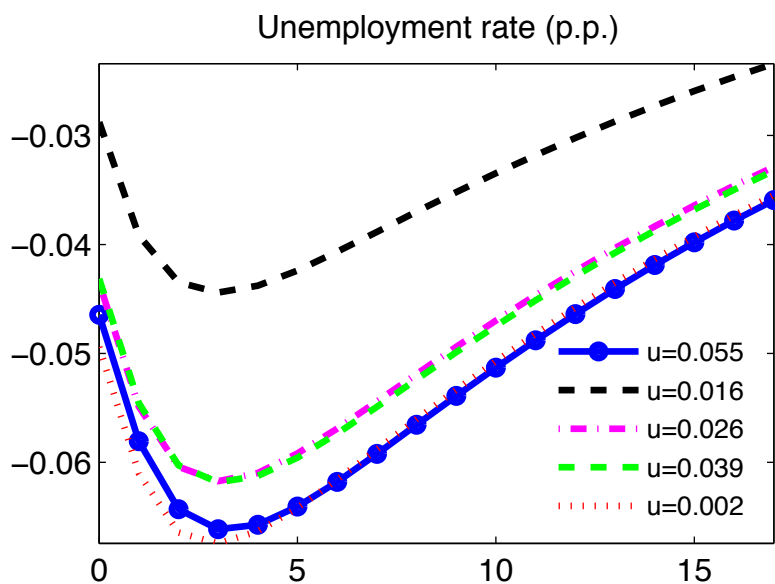
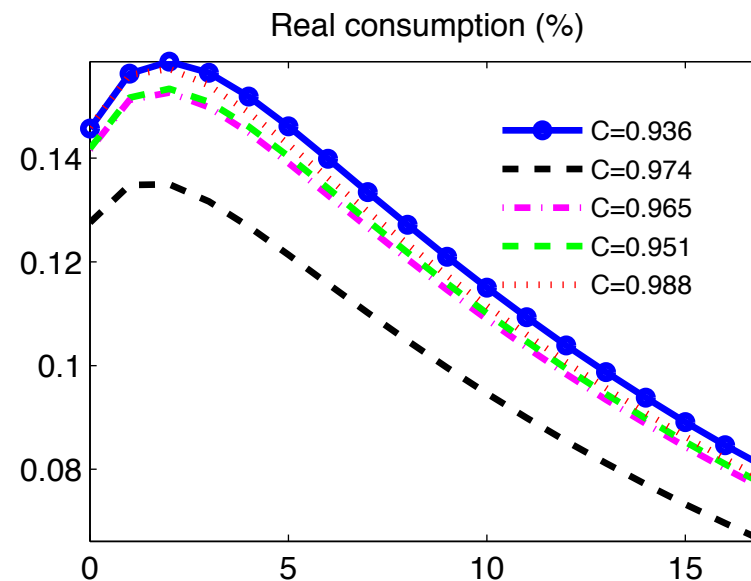
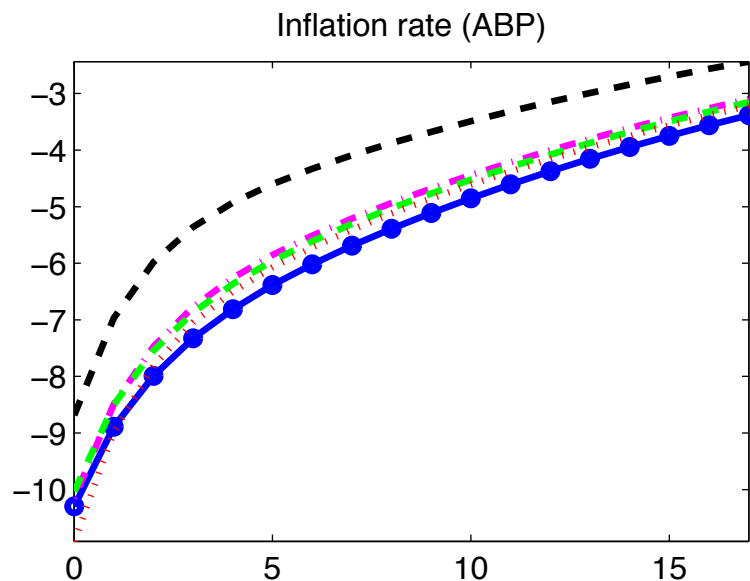
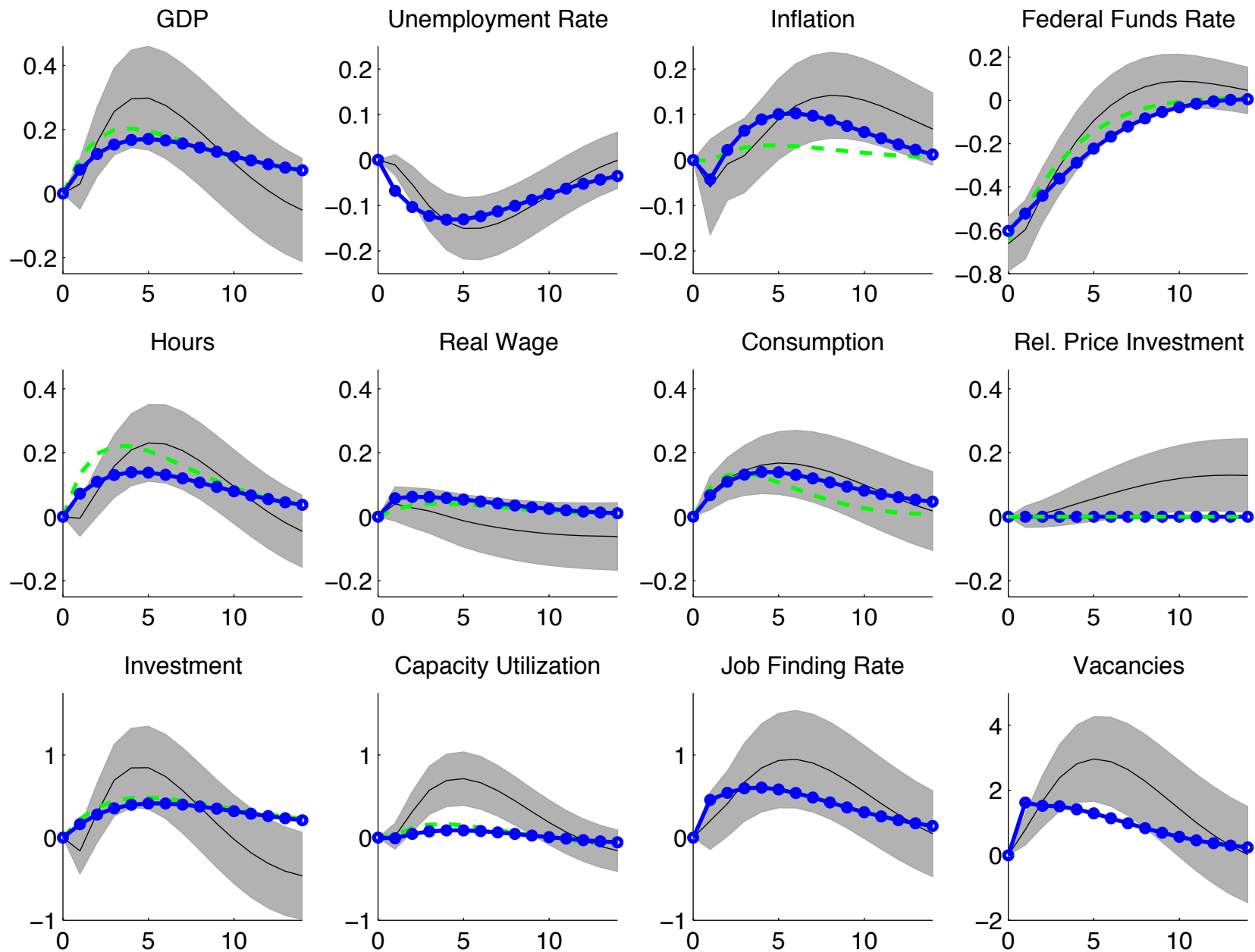


Figure 3: Medium-Sized Model Impulse Responses to a Monetary Policy Shock

VAR 95%
 VAR Mean
 Sticky Wages (No Indexation)
 Alternating Offer Bargaining



Notes: x-axis: quarters, y-axis: percent

Figure 4: Medium-Sized Model Impulse Responses to a Neutral Tech. Shock

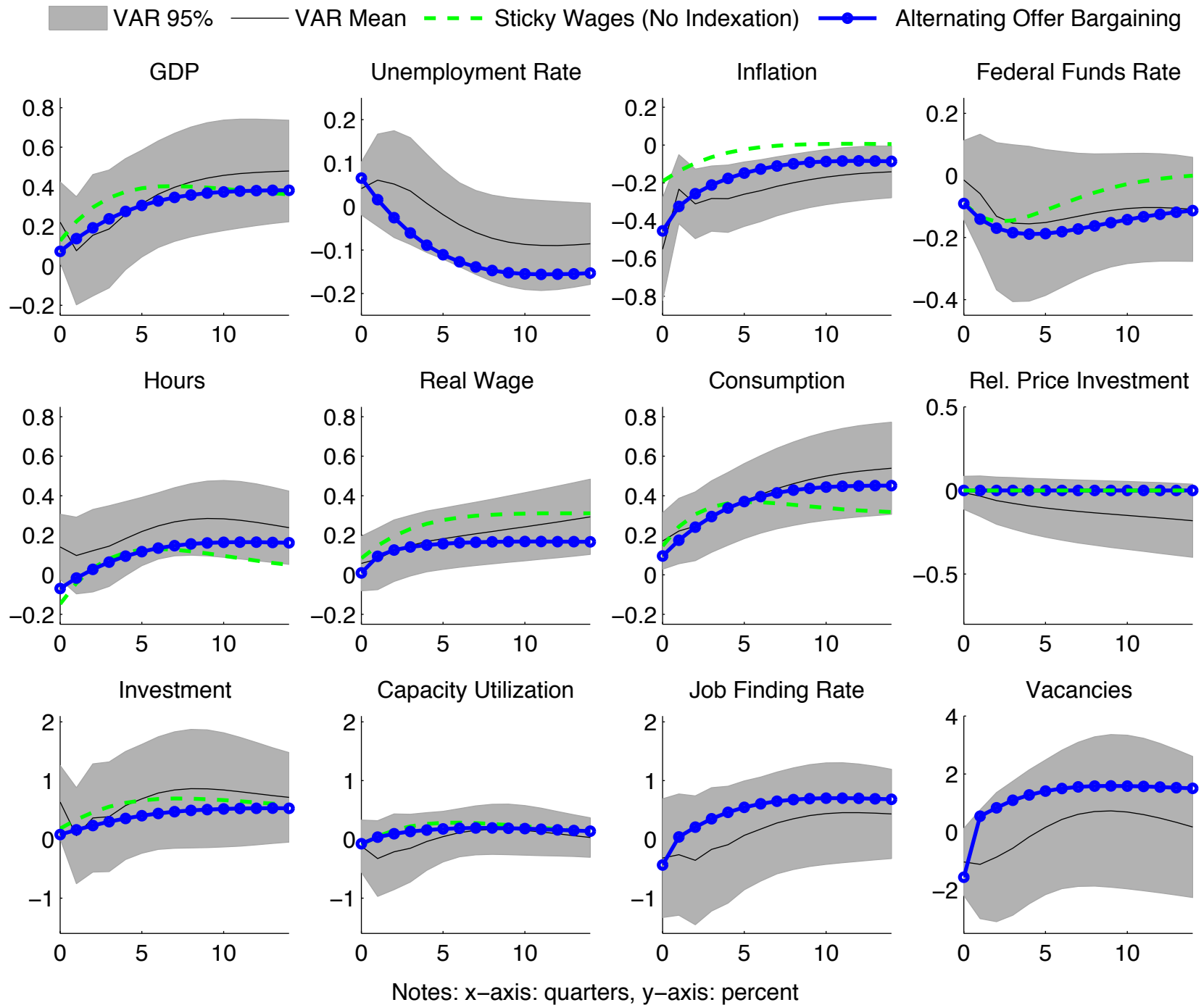
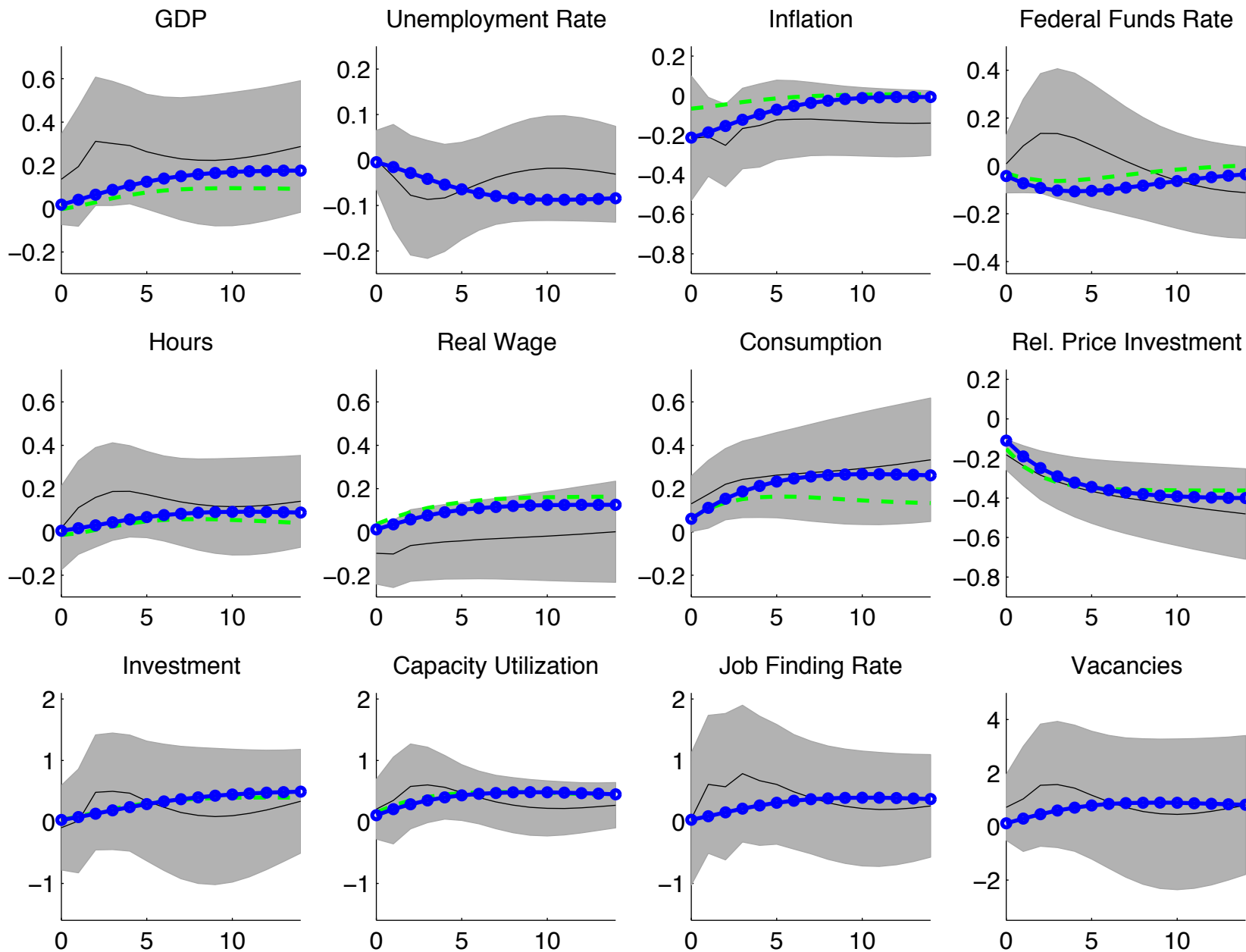


Figure 5: Medium-Sized Model Responses to an Investment Specific Tech. Shock

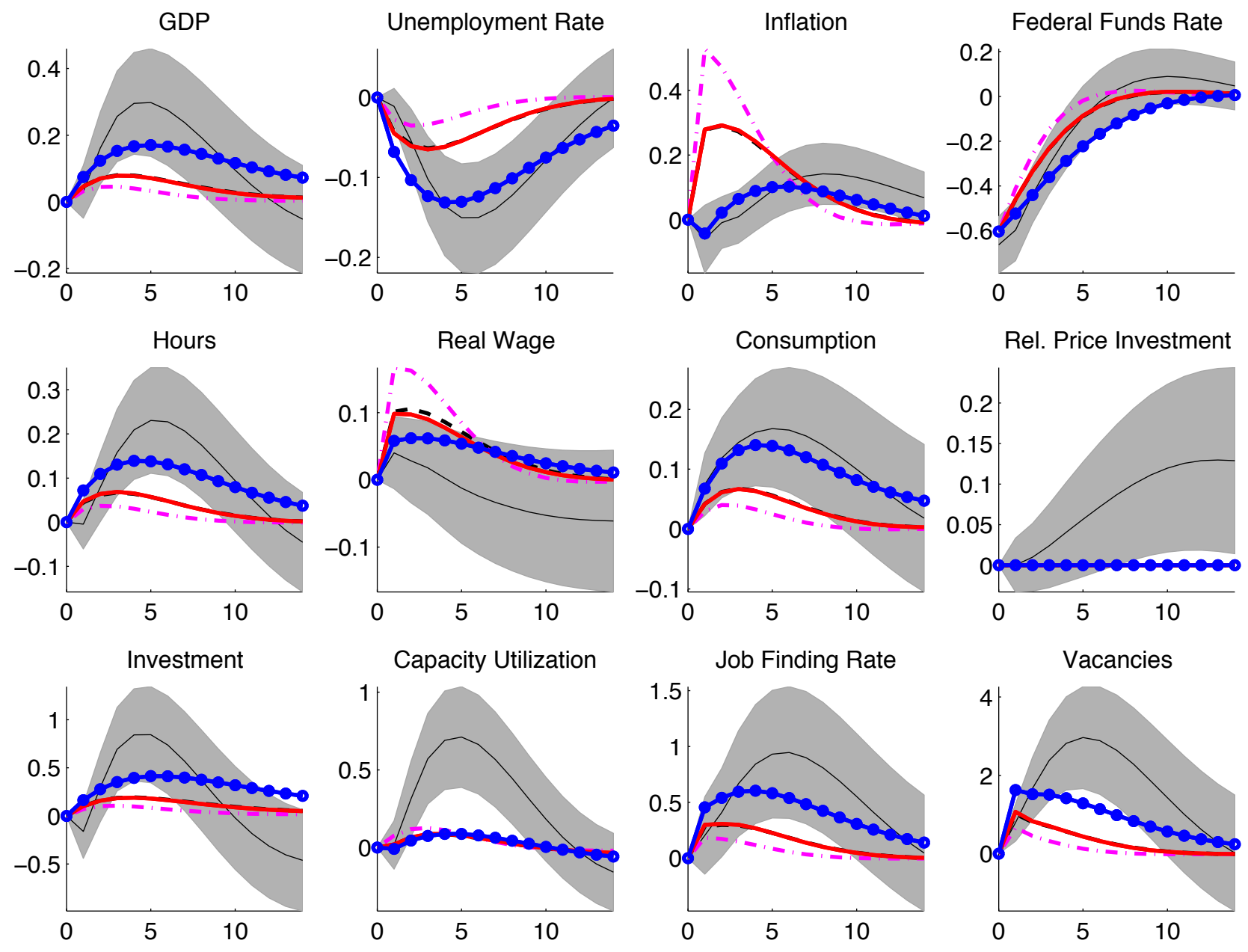
VAR 95%
 VAR Mean
 Sticky Wages (No Indexation)
 Alternating Offer Bargaining



Notes: x-axis: quarters, y-axis: percent

Figure 6: Medium-Sized Model Impulse Responses to a Monetary Policy Shock

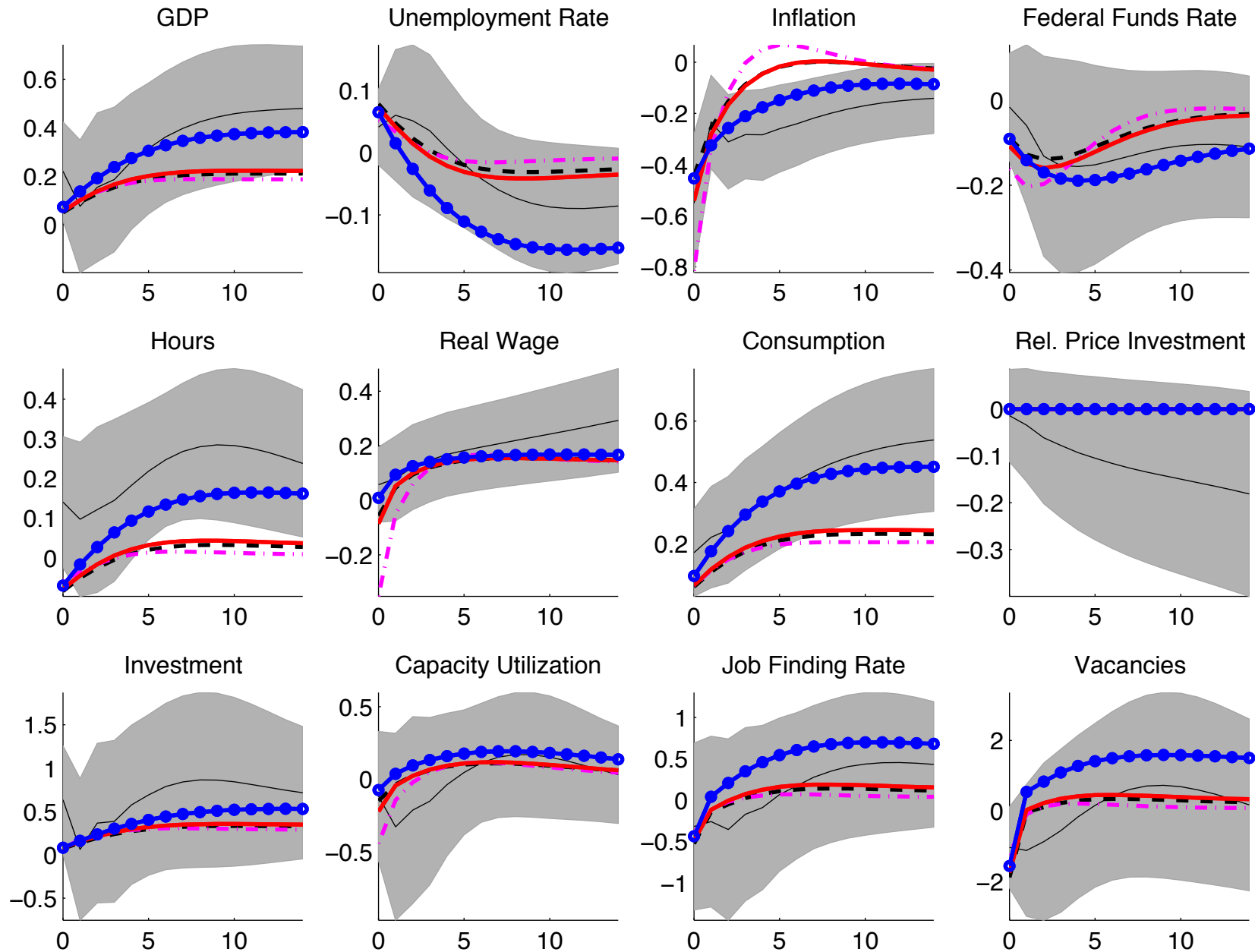
VAR 95%
 VAR Mean
 AltOfferHiring
 AltOfferSearch
 NashHiring
 NashSearch



Notes: x-axis: quarters, y-axis: percent

Figure 7: Medium-Sized Model Impulse Responses to a Neutral Tech. Shock

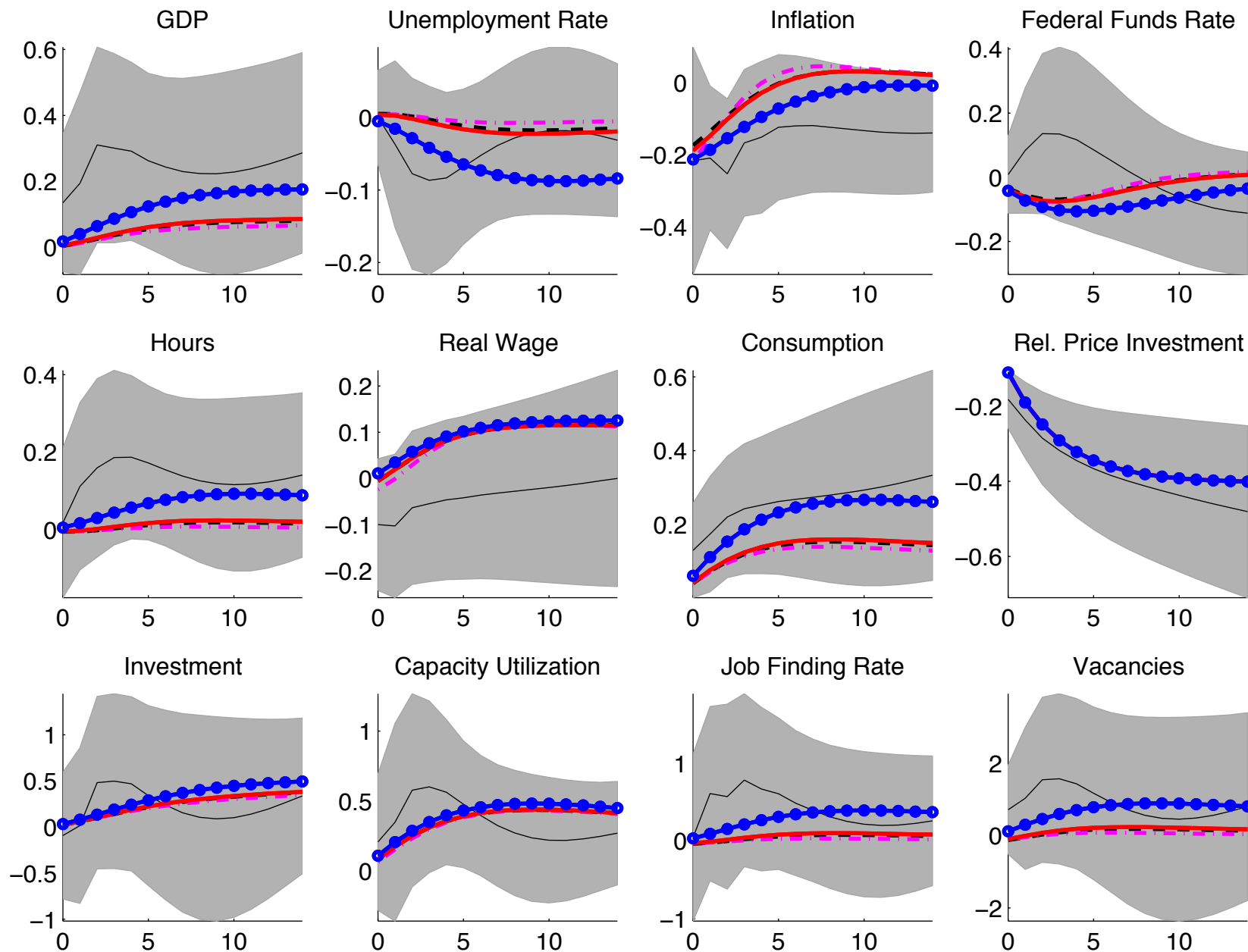
VAR 95%
 VAR Mean
 AltOfferHiring
 AltOfferSearch
 NashHiring
 NashSearch



Notes: x-axis: quarters, y-axis: percent

Figure 8: Medium-Sized Model Responses to an Investment Specific Tech. Shock

VAR 95%
 VAR Mean
 AltOfferHiring
 AltOfferSearch
 NashHiring
 NashSearch



Notes: x-axis: quarters, y-axis: percent