Appendix: Derivation of Equation (24)

Growth-Led Exports: Is Variety the Spice of Trade?

Take the logarithm of equation (23) and totally differentiate. Make the following notational simplifications: $P^X/R=PX$, $P^E{*}=PE{*}$, $P^D{*}=PD{*}$.

$$\text{dlog}(NX) = \underbrace{-\sigma\ \text{dlog}(PX/PE{*})}_{(1)} + \underbrace{\text{dlog}(E{*}/PE{*})}_{(2)} + \underbrace{\text{dlog}[Y/(Y+Y{*})]}_{(3)}$$

$$+ \underbrace{(\sigma-1)\ \text{dlog}(A{*})}_{(4)} + \underbrace{\text{dlog}[(Y+Y{*})/Y{*}]}_{(5)} + \underbrace{\text{dlog}(Z/Z{*})}_{(6)}$$

$$+ \underbrace{(1-\sigma)\ \text{dlog}(PE{*}/PD{*})}_{(7)} - \underbrace{\text{dlog}\{1 + Z\ Y\ [PX/(PD{*}\ A{*})]^{1-\sigma}/(Z{*}\ Y{*})\}}_{(8)}$$

Terms (1)-(3) above are the same as in equation (24) except that "d" is replaced by "Δ". Making use of $\text{dlog}(X)=dX/X$, term (8) can be written:

<u>Term 8</u>

$$-\{Z\ Y\ (1-\sigma)\ PX\ [PD{*}\ A{*}\ dpX - PX\ (PD{*}\ dA{*} + A{*}\ dPD{*})]/[Z{*}\ Y{*}\ (PD{*}\ A{*})^3] + [Z{*}\ Y{*}\ (Y\ dZ + Z\ dY) - Z\ Y\ (Y{*}\ dZ{*} + Z{*}\ dY{*})]/(Z{*}\ Y{*})^2\}/\{1 + Z\ Y\ [PX/(PD{*}\ A{*})]^{1-\sigma}/(Z{*}\ Y{*})\}$$

Use of initial conditions -- $A{*}=1$, $PX=PD{*}$, $Z=Z{*}$ -- allows simplification to

$$-\{Y\ (1-\sigma)\ (dPX - PD{*}\ dA{*} - dPD{*})/(Y{*}\ PD{*}) + [Y{*}\ Y\ (dZ - dZ{*}) + Z(Y{*}\ dY - Y\ dY{*})]/(Z\ Y{*2})\}/(1 + Y/Y{*})$$

dividing both numerator and denominator by $(1+Y/Y{*})$

$$[Y/(Y{*}+Y)](1-\sigma)dA{*} - [Y/(Y{*}+Y)](1-\sigma)(dPX - dPD{*})/PD{*} - [Y/(Y{*}+Y)](dZ - dZ{*})/Z - (Y{*}\ dY - Y\ dY{*})/[(Y{*}+Y)Y{*}]$$

Term 4

$(\sigma\text{-}1)dA^*/A^*$ $(A^*=1)$

Combine with first term of simplified term 8 to yield

$(\sigma\text{-}1)[Y^*/(Y+Y^*)]dA^*/A^*$ which is the fourth term in equation (24).

Term 5

$[Y^*/(Y+Y^*)][Y^*(dY+dY^*)-(Y+Y^*)dY^*]/Y^{*2}$

$= [Y^*/(Y+Y^*)][Y^*\ dY - Y\ dY^*]/Y^{*2}$

$= [Y^*\ dY - Y\ dY^*]/[(Y+Y^*)Y^*]$

which cancels out the fourth term of simplified term 8.

Term 6

$(Z^*/Z)(Z^*\ dZ - Z\ dZ^*)/Z^{*2}$ $(Z=Z^*)$

$= (dZ - dZ^*)/Z$

Combine with third term of simplified term 8 to yield

$[Y^*/(Y+Y^*)](dZ - dZ^*)/Z$ which is the fifth term in equation (24).

Term 7

Use the definition of PE* in equation (22), defining
$w = PD^*\ D^*/E^*$ and $(1-w) = PX\ X/E^*$.

$(1-\sigma)\ d\log\{[w\ PD^* + (1-w)\ PX]/PD^*\}$

$= (1-\sigma)[PD^*(w\ dPD^* + PD^*\ dW + dPX - w\ dPX - PX\ dW) - w\ PD^*\ dPD^*$
$\quad - PX\ dPD^* + w\ PX\ dPD^*]/PD^{*2}$

Substituting the initial condition: PX = PD*.

$(1-\sigma)(dPX - w\ dPX - dPD^* + w\ dPD^*)/PD^*$

$= (1-\sigma)(1-w)(dPX - dPD^*)/PD^*$

Under the initial condition of no home bias ($A^*=1$) the share of imports in expenditures ($1-w$) equals exporter's share of world output $[Y/(Y+Y^*)]$.

$(1-\sigma)[Y/(Y+Y^*)](dPX - dPD^*)/PD^*$
which cancels out the second term of simplified term 8.