Technical Appendix to ‘Optimal Monetary Policy in a Model with Distinct Core and Headline Inflation Rates’

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1 Overview

This section shows how to derive the first order conditions for the model presented in the main text of the paper. This section also shows how to take a log-linear approximation to the model's first order conditions, and then manipulate them into a particularly simple form that is akin to that of the model of Erceg, Henderson, and Levin (2000). Section 2 derives a quadratic approximation to the welfare function following the approach of Rotemberg and Woodford (1997).

1.1 Households' Problem

Recall that there is a continuum of households of measure 1, indexed by h. Each household supplies a differentiated labor service $N_t(h)$ to a goods-producing sector at a wage rate $W_t(h)$. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose to produce an aggregate L_t. The labor aggregator solves the following problem:

$$\min_{L_t(h)\forall h} \int_0^1 W_t(h)N_t(h)dh + W_t \left[L_t - \left(\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} dh \right)^{1+\theta_w} \right]. \tag{1}$$

The first-order conditions from this problem and the zero-profit condition for the aggregator imply that:

$$N_t\left(h\right) = \left[\frac{W_t\left(h\right)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t, \tag{2}$$

$$L_t = \left[\int_0^1 \left(N_t\left(h\right)\right)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w}, \tag{3}$$

$$W_t = \left[\int_0^1 W_t\left(h\right)^{\frac{-1}{\theta_w}} dh \right]^{-\theta_w}. \tag{4}$$

It is natural to interpret W_t as the aggregate wage index. The utility functional of household h is:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \mathbb{W}_{t+j}\left(h\right), \tag{5}$$

where the period utility function $\mathbb{W}_t(h)$ has the time separable form:

$$\mathbb{W}_t(h) = \mathbb{U}(C_t(h)) - \mathbb{V}(N_t(h)) = \frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{\chi_0}{1+\chi} N_{t+j}(h)^{1+\chi}. \tag{6}$$

Here $C_t(h)$ and $N_t(h)$ denote each household's total consumption and hours of labor in period t, respectively. The intertemporal elasticity of consumption, $\frac{1}{\sigma}$, satisfies $\frac{1}{\sigma} > 0$, and we assume that $0 < \beta < 1$, $\chi > 0$, and $\chi_0 > 0$.

The households' utility maximization problem is given by

$$\max_{C_t(h), W_t(h), B_{t+1}(h) \forall s, \lambda_t(h)} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{\chi_0}{1+\chi} N_{t+j}(h)^{1+\chi} \right) \tag{7}$$

$$+\lambda_{t+j}(h) \left[-P_{ct} C_t(h) - \int_s \xi_{t,t+1} B_{t+1}(h) + B_t(h) \right. \tag{8}$$

$$\left. +(1+\tau_w) W_t(h) N_t(h) + P_{ot} Y_{ot} + \Gamma_t(h) + T_t(h) \right], \tag{9}$$

where $N_t(h)$ is understood to follow the labor demand schedule in equation (2) above. Households' wage setting is subject to Calvo contracts renewed with probability $(1 - \xi_w)$. When wages are not reset, they are updated based on the steady-state wage inflation. The first-order conditions of this utility maximization problem yield:

$$C_t(h)^{-\sigma} = \lambda_t(h) P_{ct}, \tag{10}$$

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\xi_w \beta)^j \left\{ -\chi_0 N_{t+j}(h)^{\chi} \frac{\partial N_{t+j}(h)}{\partial W_t(h)} \right.$$

$$\left. +\lambda_{t+j}(h) \left[(1+\tau_w) N_{t+j}(h) + (1+\tau_w) W_t(h) \frac{\partial N_{t+j}(h)}{\partial W_t(h)} \right] \right\} = 0, \tag{11}$$

$$\frac{1}{1+i_t} = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t}, \tag{12}$$

$$P_{ct} C_t(h) + \int_s \xi_{t,t+1} B_{t+1}(h) - B_t(h) =$$

$$+(1+\tau_w) W_t(h) N_t(h) + P_{ot} Y_{ot} + \Gamma_t(h) + T_t(h), \tag{13}$$

where we defined i_t to be the economy's risk-free rate, i.e. $\frac{1}{1+i_t} = \int_s \xi_{t,t+1} ds$. Invoking our complete markets assumption, $C_t(h) = C_t$ and $\lambda_t(h) = \lambda_t$ for all h.

A household's total consumption in each period depends, in turn, on its purchases both of a composite nonenergy consumption good $C_{nt}(h)$, and of energy $O_{Ct}(h)$, according to the aggregator:

$$C_t(h) = C_{nt}(h)^{1-\omega_{oc}} O_{ct}(h)^{\omega_{oc}}. \tag{14}$$

The household's minimization problem associated with producing $C_t(h)$ is:

$$\min_{C_{nt}(h), O_{ct}(h), P_{ct}} P_{nt} C_{nt}(h) + P_{ot} O_{ct}(h) + P_{ct}\left[C_t(h) - C_{nt}(h)^{1-\omega_{oc}} O_{ct}(h)^{\omega_{oc}}\right]. \tag{15}$$

The first-order conditions are:

$$P_{nt} = P_{ct}(1-\omega_{oc}) C_{nt}(h)^{-\omega_{oc}} O_{ct}(h)^{\omega_{oc}}, \tag{16}$$

$$P_{ot} = P_{ct}\omega_{oc} C_{nt}(h)^{1-\omega_{oc}} O_{ct}(h)^{\omega_{oc}-1}, \tag{17}$$

plus the technology constraint in equation (14). Dividing equation (16) by (17), and aggregating over households, yields:

$$O_{ct} = \frac{\omega_{oc}}{1-\omega_{oc}} \frac{P_{nt}}{P_{ot}} C_{nt}, \tag{18}$$

which we interpret as the oil demand equation for households.

1.2 Wholesalers' Problem

The wholesale good is produced by a representative firm. This firm minimizes the cost of producing the wholesale good taking prices as given, subject to its technology of production. It is convenient to split the budgeting problem into two stages. In the top stage, the wholesaler purchases a composite capital-labor input V_t at a price P_{Vt} and an oil input O_{pt} at a price P_{ot}. The wholesaler combines these inputs to produce the wholesale good, Y_{wt}, whose price is P_{wt}.

The minimization problem is the following:

$$\min_{V_t, O_{pt}, P_{wt}} P_{vt} V_t + P_{ot} O_{pt} \tag{19}$$

$$+P_{wt}\left(Y_{wt} - V_t^{1-\omega_{op}} O_{pt}^{\omega_{op}}\right). \tag{20}$$

The first-order conditions for the minimization problem in (20) are:

$$P_{vt} = P_{wt}(1-\omega_{op}) \frac{Y_{wt}}{V_t}, \tag{21}$$

$$P_{ot} = P_{wt}\omega_{op} \frac{Y_{wt}}{O_{pt}}, \tag{22}$$

$$Y_{wt} = V_t^{1-\omega_{op}} O_{pt}^{\omega_{op}}. \tag{23}$$

Dividing equation (21) by (22) yields:

$$\frac{P_{vt}}{P_{ot}} = \frac{1-\omega_{op}}{\omega_{op}} \left(\frac{O_{pt}}{V_t} \right), \tag{24}$$

which we interpret as the oil demand for production. From the above equation we can see that the ratio $\frac{O_{pt}(f)}{V_t(f)}$ is equalized across wholesalers.

At the first stage of production, the wholesaler also rents capital K_t and labor L_t from households at factor prices R_{kt} and W_t, respectively. The wholesaler chooses factor inputs so as to solve the following cost minimization problem:

$$\min_{K_t, L_t, P_{vt}} R_{kt}K_t(f) + W_tL_t + P_{vt}\left[V_t - K_t^{\alpha}(Z_tL_t)^{1-\alpha}\right], \tag{25}$$

where Z_t is a technology shock common across wholesalers. The first-order conditions of the above problem are:

$$R_{kt} = P_{vt}\alpha K_t^{\alpha-1}(Z_tL_t)^{1-\alpha}, \tag{26}$$

$$W_t = P_{vt}(1-\alpha)K_t^{\alpha}(Z_tL_t)^{-\alpha}Z_t, \tag{27}$$

$$V_t = K_t^{\alpha}\left(Z_tL_t\right)^{1-\alpha}. \tag{28}$$

From equation (27), we get that:

$$\frac{P_{vt}}{P_{nt}} = \frac{\frac{W_t}{P_{nt}}}{(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} Z_t^{1-\alpha}}. \tag{29}$$

1.3 Bundlers and Retailers

The composite good Y_{Nt} is produced by a representative firm (or "bundler") according to the technology:

$$C_{nt} = Y_{nt} = \left[\int_0^1 \left(Y_{nt}\left(f\right)\right)^{\frac{1}{1+\theta_p}} df\right]^{1+\theta_p}, \tag{30}$$

The representative firm purchases the underlying retail goods at prices $P_{nt}(f)$, and sells the composite good to households at a price of P_{nt}.

The bundler's problem is the following:

$$\min_{Y_{nt}(f)\forall f, P_{nt}} \int_0^1 P_{nt}(f)Y_{nt}(f)df + P_{nt}\left[Y_{nt} - \left(\int_0^1 Y_{nt}(f)^{\frac{1}{1+\theta_p}}\right)^{1+\theta_p}\right]. \tag{31}$$

The first-order conditions and the zero-profit conditions from the above minimization problem can be manipulated to yield:

$$Y_{nt}(f) = \left[\frac{P_{nt}(f)}{P_{nt}}\right]^{-\frac{1+\theta_p}{\theta_p}} Y_{nt}, \tag{32}$$

$$P_{nt} = \left[\int_0^1 P_{nt}(f)^{-\frac{1}{\theta_p}}\right]^{-\theta_p}, \tag{33}$$

$$Y_{nt} = \left(\int_0^1 Y_{nt}(f)^{\frac{1}{1+\theta_p}}\right)^{1+\theta_p}. \tag{34}$$

Retail goods are produced by monopolistically competitive firms. Each retailer f purchases a homogenous "wholesale" good Y_{wt}, and transforms it into a particular type of retail good according to a simple linear production function:

$$Y_{nt}(f) = Y_{wt}(f), \tag{35}$$

where $Y_{wt}(f)$ denotes purchases of the wholesale good by producer f. Retailers set the price of their respective output goods in Calvo-style staggered contracts with a probability $1-\xi_p$ of receiving a signal to re-optimize its contract price, $P_{nt}(f)$. Those firms not receiving a signal to re-optimize adjust their price by the steady state inflation rate. A firm f that receives a signal to adjust solves the following maximization problem:

$$\max_{P_{nt}(f)} \mathbb{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[(1+\tau_p)\pi^j P_{nt}(f) Y_{nt+j}(f) - P_{wt+j} Y_{nt+j}(f)\right], \tag{36}$$

taking its demand schedule (32) and the price of wholesale goods P_{wt} as given. The first-order condition from this problem is:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[(1+\tau_p)\pi^j Y_{n,t+j}(f) + (1+\tau_p)\pi^j \frac{\partial Y_{nt+j}(f)}{\partial P_{nt}(f)} - P_{wt+j}\frac{\partial Y_{nt+j}(f)}{\partial P_{nt}(f)}\right] = 0. \tag{37}$$

1.4 Resource Constraints

In addition to the constraint in equation (30) that all output of the nonenergy good is consumed by households, clearing of the wholesale market requires that the cumulative demand of retailers equals the available supply, thus:

$$\int_0^1 Y_{wt}(f)df = Y_{wt} = C_{nt} = \int_0^1 C_{nt}(h)dh. \tag{38}$$

Moreover, energy market clearing implies that the energy demand of households and wholesale firms equal the exogenous flow endowment:

$$\int_0^1 O_{ct}(h)dh + O_{pt} = Y_{ot}. \tag{39}$$

The factor input markets need to clear; labor and capital demand from the wholesaler need to equal households' supply, thus:

$$L_t = \int_0^1 L_t(h)dh, \tag{40}$$

$$K_t = \bar{K}. \tag{41}$$

Finally, the exogenous flow endowment, Y_{Ot}, is itself the sum of a (nearly) permanent and temporary component:

$$Y_{ot} = Y_{ot}^P + Y_{ot}^T. \tag{42}$$

The permanent and temporary components, represented as log-deviations from their steady-state values, evolve according to:

$$y_{ot}^P = \rho^P y_{ot-1}^P + \epsilon_t^P, \tag{43}$$

$$y_{Ot}^T = \rho^T y_{ot-1}^T + \epsilon_t^T. \tag{44}$$

1.5 Flexible Wage/Price Economy

When wages are reset every period, the utility maximization problem is modified to yield the following first-order condition, instead of equation (11) above:

$$-\chi_0 N_t(h)^\chi \frac{\partial N_t(h)}{\partial W_t(h)} + \lambda_t(h)\left[(1+\tau_w)N_t(h) + (1+\tau_w)W_t(h)\frac{\partial N_t(h)}{\partial W_t(h)}\right] = 0. \tag{45}$$

Dividing by $(1+\tau_w)N_t(h)$ yields:

$$\chi_0 N_t(h)^\chi \frac{1}{1+\tau_w}\frac{\frac{\partial N_t(h)}{\partial W_t(h)}}{N_t(h)} = \lambda_t(h)\left[1 + \frac{\frac{\partial N_t(h)}{\partial W_t(h)}}{\frac{N_t(h)}{W_t(h)}}\right]. \tag{46}$$

Invoking our complete markets assumption and from equation (10), $\lambda_t(h) = \frac{C_t}{P_{ct}}$. Furthermore, note that, from equation (2):

$$\frac{\frac{\partial N_t(h)}{\partial W_t(h)}}{\frac{N_t(h)}{W_t(h)}} = -\frac{1+\theta}{\theta}.$$

Using the above elasticity in the previous equation and rearranging:

$$\frac{W_t(h)}{P_{ct}} = \left(\frac{1+\theta_w}{1+\tau_w}\right) \chi_0 N_t(h)^{\chi} C_t^{\sigma}. \tag{47}$$

Analogously, when intermediate prices are reset every period, the following first-order condition replaces equation (37) above.

$$(1+\tau_p)Y_{nt}(f) + [(1+\tau_p)P_{nt}(f) - P_{wt}] \frac{\partial Y_{nt}(f)}{\partial P_{nt}(f)} = 0. \tag{48}$$

Dividing by $(1+\tau_p)Y_{nt}(f)$:

$$1 + \left[P_{nt} - \left(\frac{1}{1+\tau_p} P_{wt}\right)\right] \frac{\frac{\partial Y_{nt}(f)}{\partial P_{nt}(f)}}{Y_{nt}(f)} = 0. \tag{49}$$

Multiplying by P_{nt} and noticing that $\frac{\partial Y_{nt}(f)}{\partial P_{nt}(f)} \frac{P_{nt}(f)}{Y_{nt}(f)} = -\frac{1+\theta_P}{\theta_p}$, the above equation becomes:

$$P_{nt}(f) + \left[P_{nt}(f) - \frac{1}{1+\tau_p} P_{wt}\right] \left(\frac{1+\theta_p}{\theta_p}\right) = 0. \tag{50}$$

Invoking symmetry, $P_{nt}(f) = P_{nt}$ for all f. Thus, rearranging the above equation yields:

$$P_{nt} = \frac{1+\theta_p}{1+\tau_p} P_{wt}. \tag{51}$$

1.6 Steady States

We focus on a steady state in which all prices, P_n, P_v, P_o, P_c, are equal to 1 and and the stock of capital, $\bar{K}$, is fixed at 1. Note that the omission of the time subscript t denotes a variable's steady state value. We show that our choice of prices implies a restriction on the steady-state value of Z when we take the value of Y_o, the supply of oil, as given. Furthermore, we show that for any $L > 0$, given our choice of preferences, there is a value of χ that supports that value of L in steady state.

We also choose τ_p and θ_p so that $\tau_p = \theta_p$. Thus, from equation (51):

$$P_w = P_n = 1. \tag{52}$$

From the wholesalers' cost minimization problem

$$V = (1 - \omega_{op})Y_w, \tag{53}$$

$$O_p = \omega_{op} Y_w. \tag{54}$$

Similarly, from the households' cost minimization problem

$$C_n = (1 - \omega_{oc})C, \tag{55}$$

$$O_c = \omega_{oc} C. \tag{56}$$

From the market clearing condition for oil $Y_o = O_c + O_p$. From equation (38), we know that $Y_w = C_n$. Thus, combining equations (54), (55), and (56) yields:

$$\frac{O_c}{Y_o} = \frac{\omega_{oc}}{\omega_{oc} + \omega_o(1 - \omega_{oc})}, \tag{57}$$

$$\frac{O_p}{Y_o} = 1 - \frac{O_c}{Y_o}. \tag{58}$$

Equivalently, one can also see that

$$\left(\frac{\omega_{oc}}{1 - \omega_{oc}}\right)\left(\frac{O_p}{Y_o}\right) = \omega_{op}\left(\frac{O_c}{Y_o}\right) \tag{59}$$

.

From equation (27)

$$W = (1 - \alpha)N^{-\alpha}\bar{K}^{\alpha}Z^{1-\alpha}, \tag{60}$$

and from equation (45)

$$W = \chi_0 L^{\chi} C^{\sigma}. \tag{61}$$

From equation (2), in a symmetric steady state, N(h)=N=L. Thus, combining the preceding 2 equations:

$$(1 - \alpha)L^{-\alpha}\bar{K}^{\alpha}Z^{1-\alpha} = \chi_0 L^{\chi} C^{\sigma}. \tag{62}$$

Next, we express C in terms of L and Z. From equation (14)

$$C = C_n^{1-\omega_{oc}} O_c^{\omega_{oc}}. \tag{63}$$

Combining the above equation with equation (56) yields:

$$C = \omega_{oc}^{\frac{\omega_{oc}}{1+\omega_{oc}}} C_n. \tag{64}$$

In turn, from equations (38) and (23), we can express C_n as:

$$C_n = \omega_{op}^{\frac{\omega_{op}}{1-\omega_{op}}} V. \tag{65}$$

Since $V = \bar{K}^{\alpha}(ZL)^{1-\alpha}$, then from the above two equations:

$$C = \omega_{oc}^{\frac{\omega_{oc}}{1+\omega_{oc}}} \omega_{op}^{\frac{\omega_{op}}{1-\omega_{op}}} \bar{K}^{\alpha}(ZL)^{1-\alpha}. \tag{66}$$

Thus, combining the equation above with equation (62), it becomes clear that for any choice of L and Z, there is a unique value of χ_0 that supports that choice. Finally, for any choice of L and given our choice of unitary prices, the oil market clearing condition, equation (42), implies a restriction on the steady state value of Z given by:

$$\omega_{oc}\left[\omega_{oc}^{\frac{\omega_{oc}}{1+\omega_{oc}}} \omega_{op}^{\frac{\omega_{op}}{1-\omega_{op}}} \bar{K}^{\alpha}(ZL)^{1-\alpha}\right] + \omega_{op}\left[\omega_{op}^{\frac{\omega_{op}}{1-\omega_{op}}} \bar{K}^{\alpha}(ZL)^{1-\alpha}\right] = Y_o, \tag{67}$$

where Y_o is an exogenous quantity.

The steady-state relationships given below will prove useful in the derivation of the second-order approximation to the welfare loss function.

Combining equations (10) and (11) in steady state yields

$$\mathbb{U}_C = \mathbb{V}_N \frac{1}{W}. \tag{68}$$

Noticing that $W = (1-\alpha)\frac{V}{N}$ and $V = (1-\omega_{op})(1-\omega_{oc})C$, we obtain

$$\mathbb{U}_C \bar{C}(1-\omega_{oc}) = \mathbb{V}_N \bar{N} \frac{1}{(1-\alpha)(1-\omega_{op})}. \tag{69}$$

Note that the first and second derivative of the subutility functional for leisure $\mathbb{V}(N_t(h)) = \frac{\chi_0}{1+\chi} N_{t+j}(h)^{1+\chi}$ satisfy:

$$\left(\mathbb{V}_N \bar{N} + \mathbb{V}_{NN}(\bar{N})^2\right) = \mathbb{V}_N \bar{N}(1+\chi), \tag{70}$$

and

$$\left(\mathbb{V}_N \bar{N} \frac{\theta_w}{1+\theta_w} + \mathbb{V}_{NN}(\bar{N})^2\right) = \mathbb{V}_N \bar{N}\left(\frac{\theta_w}{1+\theta_w} + \chi\right). \tag{71}$$

1.7 Linearized Conditions for an Equilibrium

Below we give a set of log-linear equations that, together with the shock processes, equations (42) to (44), and a characterization of monetary policy describe the model's equilibrium.

1. *Production of wholesale good*

   Combining equations (23) and (28), and remembering that aggregate capital is fixed, yields:

$$y_{wt} = (1 - \omega_{op})(1 - \alpha)\left[z_t + l_t\right] + \omega_{op}\; o_{pt}. \tag{72}$$

2. *Oil demand by firms* From equation (24), we have that

$$o_{pt} = -\left(\frac{p_{ot}}{p_{vt}}\right) + v_t. \tag{73}$$

   But from equations (28) and(29),

$$v_t = (1 - \alpha)\left(l_t + z_t\right). \tag{74}$$

   Furthermore, $\frac{p_{ot}}{p_{vt}} = \frac{p_{ot}}{p_{nt}} - \frac{p_{vt}}{p_{nt}}$, and from equation (29) we get that:

$$\frac{p_{vt}}{p_{nt}} = \frac{w_t}{p_{nt}} + \alpha l_t - (1 - \alpha)z_t. \tag{75}$$

   Substituting (74) and (75) into (73) we get:

$$o_{pt} = -\frac{p_{ot}}{p_{nt}} + \frac{w_t}{p_{nt}} + \alpha\, l_t - (1 - \alpha)z_t + (1 - \alpha)\left(l_t + z_t\right), \tag{76}$$

   which simplifies to:

$$o_{pt} = -\frac{p_{ot}}{p_{nt}} + \frac{w_t}{p_{nt}} + l_t. \tag{77}$$

   Define $\psi_{ot} = \frac{p_{ot}}{p_{nt}}$ and $\eta_t = \frac{w_t}{p_{nt}}$. Then, we rewrite the above equation as:

$$o_{pt} = -\psi_{ot} + \eta_t + l_t. \tag{78}$$

3. *Real marginal cost*

   From equations 21 to 23, we can see that:

$$\frac{p_{wt}}{p_{nt}} = (1 - \omega_{op})\frac{p_{vt}}{p_{nt}} + \omega_{op}\frac{p_{ot}}{p_{nt}}. \tag{79}$$

   Combining the above equation with (73) we get that:

$$\frac{p_{wt}}{p_{nt}} = (1 - \omega_{op})\left[\frac{w_t}{p_{nt}} + \alpha\, l_t - (1-\alpha) z_t\right] + \omega_{op}\,\frac{p_{ot}}{p_{nt}}. \tag{80}$$

   Define $\phi_{nt} = \frac{p_{wt}}{p_{nt}}$. Then, also using the definition for ψ_{ot} and η_t, the above equation becomes:

$$\phi_{nt} = (1 - \omega_{op})\left[\eta_t + \alpha\, l_t - (1-\alpha) z_t\right] + \omega_{op}\,\psi_{ot}. \tag{81}$$

4. *Production of household consumption good*

   From equation (14)

$$c_t = (1 - \omega_{oc})\; c_{nt} + \omega_{oc}\; o_{ct}. \tag{82}$$

5. *Households' oil demand*

   From equation (18) we get that:

$$o_{ct} = -\frac{p_{ot}}{p_{nt}} + c_{nt}. \tag{83}$$

6. *Relative price for consumer goods* From equations (14), (16), and (17), we get that:

$$\frac{p_{ct}}{p_{nt}} = \omega_{oc}\psi_{ot}. \tag{84}$$

7. *Resource constraint for nonoil good*

   From equation (38):

$$y_{wt} = c_{nt}. \tag{85}$$

8. *Resource constraint for oil*

   From equation (39)

$$y_{ot} = \frac{O_c}{Y_o}\, o_{ct} + \left(1 - \frac{O_c}{Y_o}\, o_{ct}\right) o_{pt}, \tag{86}$$

   where $\frac{O_c}{Y_o} = \frac{\omega_{oc}}{\omega_{oc}+\omega_o(1-\omega_{oc})}$, as derived above.

9. *Price setting*

   With Calvo-style contracts, linearizing equation (37) yields:

$$\pi_{nt} = \beta\, \mathbb{E}_t \pi_{Nt+1} + \kappa_p \phi_{nt}, \tag{87}$$

   where $\pi_{nt} = p_{nt} - p_{nt-1}$ and $\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$.

   With flexible prices, from equation (51)

$$\phi_{nt} = 0. \tag{88}$$

10. *Wage setting*

    With Calvo-style contracts, linearizing equation (11) yields:

$$\omega_t = \beta\, \mathbb{E}_t \omega_{t+1} + \kappa_w \left(\frac{p_{ct}}{p_{nt}} + \sigma c_t + \chi l_t - \frac{w_t}{p_{nt}} \right), \tag{89}$$

    where $\omega_t = w_t - w_{t-1}$ and $\kappa_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w}$.

    With flexible wages, from equation 47:

$$\eta_t = \frac{p_{ct}}{p_{nt}} + \chi\ l_t + \sigma\ c_t. \tag{90}$$

11. *Consumption Euler*

    Combining equations (10) and (12):

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}\,(i_t - \mathbb{E}_t \pi_{ct+1}). \tag{91}$$

12. *Product real wage identity*

$$\omega_t - \pi_{nt} = \eta_t - \eta_{t-1}. \tag{92}$$

13. *Consumer price inflation identity*

$$\pi_{ct} = \pi_{nt} + \omega_{oc} \left(\psi_{ot} - \psi_{ot-1} \right). \tag{93}$$

1.8 Simplifying the linear conditions for an equilibrium

In this section we show how to rewrite the log-linear conditions that characterize the aggregate supply block of the model into a form that depends only on price inflation (for retail goods), wage inflation, the product real wage, the real wage gap (the deviation between the real wage and the real wage that would prevail under price and wage flexibility), and the employment gap. As noted in the text, this representation is very useful insofar as the second- order approximation to welfare depends only on these variables. To simplify the notation, in this and following sections we have at times dropped the expectation operator accompanying variables dated $t+1$.

We begin by using the oil market equilibrium conditions to express the deviation of the relative price energy from its flexible price value, i.e., $\psi_{ot} - \psi_{ot}^*$, solely in terms of the employment gap $l_t - l_t^*$, and the real wage gap $\eta_t - \eta_t^*$. Substituting the wholesaler's energy demand function into the oil resource constraint yields:

$$y_{ot} = o_{pt} + \frac{O_c}{Y_o} \left(o_{ct} - o_{pt} \right) = l_t + \eta_t - \psi_{ot} + \frac{O_c}{Y_o} \left(o_{ct} - o_{pt} \right). \tag{94}$$

Thus, oil supply equals demand, where the latter may be expressed as wholesale producers' demand plus an adjustment factor that depends on the ratio of the energy demand of households relative to that of wholesale producers.

Solving equation (81) for η_t and substituting into equation (78), then using (72), we obtain that:

$$o_{pt} = y_{wt} - \psi_{ot} + \frac{p_{wt}}{p_{nt}}. \tag{95}$$

From equation (86)

$$y_{ot} = o_{pt} + \frac{O_c}{Y_o}\left(o_{ct} - o_{pt}\right). \tag{96}$$

Subtracting equation (95) from (83) yields:

$$o_{ct} - o_{pt} = -\phi_{nt}, \tag{97}$$

where $\phi_{nt} = log\left(\frac{P_{wt}}{P_{nt}}\right)$. Intuitively, household energy demand depends on the price of energy relative to that of retail goods $\left(\frac{P_{ot}}{P_{nt}}\right)$, while wholesalers' demand depends on the price of energy relative to that of wholesale goods $\left(\frac{P_{ot}}{P_{wt}}\right)$. Thus, a fall in the markup of retail over wholesale goods (i.e., a rise in marginal cost ψ_{nt}) should reduce the energy demand of households relative to that of wholesale producers. Substituting equation (97) into (94), rearranging terms, and then substituting for real marginal cost using (81) yields:

$$\begin{aligned}\psi_{ot} &= -y_{ot} + l_t + \eta_t - \frac{O_c}{Y_o}\psi_{nt} \\ &= -y_{ot} + l_t + \eta_t - \frac{O_c}{Y_o}\left((1-\omega_{op})(\eta_t + \alpha l_t - (1-\alpha)z_t) + \omega_{op}\psi_{ot}\right).\end{aligned} \tag{98}$$

Solving for the relative price of oil gives:

$$\begin{aligned}\left(1 + \omega_{op}\frac{O_c}{Y_o}\right)\psi_{ot} &= -y_{ot} + \left(1 - \frac{O_c}{Y_o}(1-\omega_{op})\alpha\right) l_t \\ &+ \left(1 - \frac{O_c}{Y_o}(1-\omega_{op})\right)\eta_t + (1-\alpha)\frac{O_c}{Y_o}(1-\omega_{op})z_t.\end{aligned} \tag{99}$$

Recalling that $\frac{O_c}{Y_o} = \frac{\omega_{oc}}{\omega_{op}(1-\omega_{oc})+\omega_{oc}}$, the coefficients in equation (99) may be rewritten:

$$\left(1 + \omega_{op}\frac{O_c}{Y_o}\right) = 1 + \frac{\omega_{op}\omega_{oc}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}} = \frac{\omega_{op} + \omega_{oc}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}, \tag{100}$$

and

$$1 - \left(\frac{O_c}{Y_o}\right)(1-\omega_{op}) = 1 + \omega_{op}\left(\frac{O_c}{Y_o}\right) - \left(\frac{O_c}{Y_o}\right) = \frac{\omega_{op} + \omega_{oc} - \omega_{oc}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}} = \frac{\omega_{op}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}. \tag{101}$$

Substituting into (99) gives:

$$\begin{aligned}\left(\frac{\omega_{op} + \omega_{oc}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}\right)\psi_{ot} &= -y_{ot} + \left(\frac{\omega_{op}}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}\right)\eta_t + \\ &\left(\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc} - \alpha\omega_{oc}(1-\omega_{op})}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}\right) l_t \\ &+(1-\alpha)\left(\frac{\omega_{oc}(1-\omega_{op})}{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}\right) z_t.\end{aligned} \tag{102}$$

Solving for the relative price of oil yields:

$$\begin{aligned}\psi_{ot} \quad &= -\left(\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}}{\omega_{op}+\omega_{oc}}\right) y_{ot} + \left(\frac{\omega_{op}}{\omega_{op}+\omega_{oc}}\right) \eta_t + \\ &\left(\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}-\alpha\omega_{oc}(1-\omega_{op})}{\omega_{op}+\omega_{oc}}\right) l_t \\ &+(1-\alpha)\left(\frac{\omega_{oc}(1-\omega_{op})}{\omega_{op}+\omega_{oc}}\right) z_t.\end{aligned} \tag{103}$$

Noting that the numerator of the coefficient on l_t can be expressed:

$$\begin{aligned}\omega_{op}(1-\omega_{oc})+\omega_{oc}-\alpha\omega_{oc}(1-\omega_{op}) \quad &= \omega_{op} + (1-\alpha)\omega_{oc}(1-\omega_{op}) \qquad (104) \\ &= \alpha\omega_{op} + (1-\alpha)\left(\omega_{op}+\omega_{oc}(1-\omega_{op})\right) \\ &= \alpha\omega_{op} + (1-\alpha)\left(\omega_{oc}+\omega_{op}(1-\omega_{oc})\right),\end{aligned}$$

the relative price of oil can be written alternatively as:

$$\begin{aligned}\psi_{ot} \quad &= -\left(\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}}{\omega_{op}+\omega_{oc}}\right) y_{ot} + \left(\frac{\omega_{op}}{\omega_{op}+\omega_{oc}}\right) \eta_t \\ &+\left(\frac{\alpha\omega_{op}+(1-\alpha)\left(\omega_{oc}+\omega_{op}(1-\omega_{oc})\right)}{\omega_{op}+\omega_{oc}}\right) l_t \\ &+(1-\alpha)\left(\frac{\omega_{oc}(1-\omega_{op})}{\omega_{op}+\omega_{oc}}\right) z_t.\end{aligned} \tag{105}$$

Because this relationship also obtains in the flexible price equilibrium, it is convenient to express the percentage deviation of the oil price from its flexible price value, or oil price gap, as:

$$\begin{aligned}\psi_{ot}-\psi_{ot}^* \quad &= \left(\frac{\omega_{op}}{\omega_{op}+\omega_{oc}}\right)(\eta_t-\eta_t^*) \\ &+\left(\frac{\alpha\omega_{op}+(1-\alpha)\left(\omega_{oc}+\omega_{op}(1-\omega_{oc})\right)}{\omega_{op}+\omega_{oc}}\right)(l_t-l_t^*).\end{aligned} \tag{106}$$

We can now use this result to reformulate the expression for marginal cost. Using equation (81) above, recalling that it also holds in the flexible price equilibrium, one gets:

$$\phi_{nt}-\phi_{nt}^* = (1-\omega_{op})\left(\eta_t-\eta_t^*+\alpha(l_t-l_t^*)\right) + \omega_{op}\left(\psi_{ot}-\psi_{ot}^*\right). \tag{107}$$

Substituting for the "oil price gap" yields:

$$\begin{aligned}\phi_{nt}-\phi_{nt}^* \quad &= \quad (1-\omega_{op})\left(\eta_t-\eta_t^*+\alpha(l_t-l_t^*)\right) + \omega_{op}\left(\frac{\omega_{op}}{\omega_{op}+\omega_{oc}}\right)(\eta_t-\eta_t^*) \\ &+\omega_{op}\left(\frac{\alpha\omega_{op}+(1-\alpha)\left(\omega_{oc}+\omega_{op}(1-\omega_{oc})\right)}{\omega_{op}+\omega_{oc}}\right)(l_t-l_t^*).\end{aligned} \tag{108}$$

Grouping terms, we have:

$$\begin{aligned}\phi_{nt} - \phi_{nt}^* \quad &= \left(1 - \omega_{op} + \frac{\omega_{op}^2}{\omega_{op} + \omega_{oc}}\right)(\eta_t - \eta_t^*) \\ &+ \left[\alpha\left(1 - \omega_{op} + \frac{\omega_{op}^2}{\omega_{op} + \omega_{oc}}\right) + (1-\alpha)\omega_{op}\left(\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}}\right)\right](l_t - l_t^*). \end{aligned} \tag{109}$$

The coefficient on $(\eta_t - \eta_t^*)$ may be rewritten as:

$$1 - \omega_{op} + \frac{\omega_{op}^2}{\omega_{op} + \omega_{oc}} = \frac{\omega_{op} + \omega_{oc} - \omega_{op}^2 - \omega_{op}\omega_{oc} + \omega_{op}^2}{\omega_{op} + \omega_{oc}} = \frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}}, \tag{110}$$

we can express real marginal cost exclusively in terms of the real wage and employment gaps:

$$\phi_{nt} - \phi_{nt}^* = \left[\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}}\right](\eta_t - \eta_t^*) + [\alpha + (1-\alpha)\omega_{op}](l_t - l_t^*). \tag{111}$$

Equivalently, defining:

$$\lambda_{MPL} = \alpha + (1-\alpha)\omega_{op}, \tag{112}$$

real marginal cost may be expressed:

$$\phi_{nt} - \phi_{nt}^* = \left[\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}}\right](\eta_t - \eta_t^*) + \lambda_{MPL}(l_t - l_t^*). \tag{113}$$

It is also straightforward to use the condition for the relative price of oil to derive an expression for the "MRS gap" that depends only on employment and real wage gaps. Defining the MRS as the cost of leisure in terms of the non-oil consumption good, i.e., $MRS_{nt} = \omega_{oc}\psi_{ot} + \sigma c_t + \chi l_t$. The term $MRS_{nt} - \eta_t$ can be written as:

$$MRS_{nt} - \eta_t = (MRS_{nt} - \eta_t^*) - (\eta_t - \eta_t^*). \tag{114}$$

Thus $MRS_{nt} - \eta_t$ equals the MRS gap minus the real wage gap. In turn, the latter can be rewritten as:

$$(MRS_{nt} - \eta_t^*) + (\eta_t^* - \eta_t) = \omega_{oc}(\psi_{ot} - \psi_{ot}^*) + \sigma(c_t - c_t^*) + \chi(l_t - l_t^*) + \eta_t^* - \eta_t.$$

Using equations (82), (83), and (74), one can get:

$$c_t = (1 - \omega_{op})(1-\alpha)(l_t + z_t) + \omega_{op} o_{pt} - \omega_{oc}\psi_{ot}. \tag{115}$$

Substitute the firms' oil demand equation (78) in the above expression. The consumption gap, can then be written as:

$$c_t - c_t^* = (1-\alpha)\left[1 - \omega_{oc} - \omega_{op}(1-\omega_{oc})\right](l_t - l_t^*), \tag{116}$$

where we have eliminated the oil price gap using equation (106). Hence,

$$\begin{aligned} MRS_{nt} - \eta_t \quad &= (MRS_{nt} - \eta_t^*) - (\eta_t - \eta_t^*) \\ &= \omega_{oc}\left(\frac{\omega_{op}}{\omega_{op} + \omega_{oc}}\right)(\eta_t - \eta_t^*) \\ &+ \omega_{oc}\left(\frac{\alpha\omega_{op} + (1-\alpha)\left(\omega_{oc} + \omega_{op}(1-\omega_{oc})\right)}{\omega_{op} + \omega_{oc}}\right)(l_t - l_t^*) \\ &+ \sigma(c_t - c_t^*) + \chi(l_t - l_t^*) - (\eta_t - \eta_t^*). \end{aligned} \tag{117}$$

Grouping terms yields:

$$MRS_{nt} - \eta_t = \left(\frac{\omega_{oc}\omega_{op}}{\omega_{op} + \omega_{oc}} - 1\right)(\eta_t - \eta_t^*) + \lambda_{MRS}(l_t - l_t^*), \tag{118}$$

where we have defined:

$$\begin{aligned} \lambda_{MRS} \quad &= \quad \omega_{oc}\left[\alpha\frac{\omega_{op}}{\omega_{op} + \omega_{oc}} + (1-\alpha)\left(\frac{\omega_{oc} + \omega_{op}(1-\omega_{oc})}{\omega_{op} + \omega_{oc}}\right)\right] \\ &+ \quad \sigma(1-\alpha)(1 - \omega_{oc} - \omega_{op}(1-\omega_{oc})) + \chi. \end{aligned} \tag{119}$$

Noting that:

$$\frac{\omega_{oc}\omega_{op}}{\omega_{op} + \omega_{oc}} - 1 = \frac{\omega_{op}(\omega_{oc} - 1) - \omega_{oc}}{\omega_{op} + \omega_{oc}}, \tag{120}$$

we may express the MRS gap as:

$$MRS_{nt} - \eta_t = \left[\frac{\omega_{op}(\omega_{oc} - 1) - \omega_{oc}}{\omega_{op} + \omega_{oc}}\right](\eta_t - \eta_t^*) + \lambda_{MRS}(l_t - l_t^*). \tag{121}$$

Accordingly, upon substituting equations (111) and (122) into the price and wage-setting equations, respectively, the aggregate supply block of the model may be expressed in the simple form:

1. *Price-Setting*

$$\pi_{nt} = \beta\pi_{n,t+1} + \kappa_P\left(\left[\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}}\right](\eta_t - \eta_t^*) + \left[\alpha + (1-\alpha)\omega_{op}\right](l_t - l_t^*)\right) \tag{122}$$

2. *Wage-Setting*

$$\omega_t = \beta\omega_{t+1} + \kappa_w \left(\left[\frac{\omega_{op}(1-\omega_{oc}) + \omega_{oc}}{\omega_{op} + \omega_{oc}} \right] (\eta_t - \eta_t^*) + \lambda_{MRS}(l_t - l_t^*) \right) \tag{123}$$

3. *Product Real Wage Evolution*

$$\eta_t = \eta_{t-1} + \omega_t - \pi_{nt} \tag{124}$$

The demand block closes the model and is given below for completeness:

4. *IS Equation*

$$(l_t - l_t^*) = (l_{t+1} - l_{t+1}^*) - \frac{1}{\sigma(1-\alpha)(1-\omega_{oc})(1-\omega_{op})} (i_t - \pi_{ct+1} - r_{ct}^*) \tag{125}$$

5. *Relating headline and core inflation*

$$\begin{aligned}
&\pi_{ct} = \pi_{nt} + \omega_{oc} \left(\psi_{ot}^* - \psi_{ot-1}^* \right) \\
&+\omega_{oc} \left(\frac{\omega_{op}}{\omega_{op} + \omega_{oc}} \right) \left[\eta_t - \eta_t^* - (\eta_{t-1} - \eta_{t-1}^*) \right] \\
&+\omega_{oc} \left[\frac{\alpha\omega_{op}}{\omega_{op} + \omega_{oc}} + (1-\alpha) \left(\frac{\omega_{oc} + \omega_{op}(1-\omega_{oc})}{\omega_{op} + \omega_{oc}} \right) \right] \left[(l_t - l_t^*) - (l_{t-1} - l_{t-1}^*) \right]
\end{aligned} \tag{126}$$

6. *Monetary policy rule*

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i) \left[\gamma_{\pi n}\pi_{nt} + \gamma_{\pi c}\pi_{ct} + \gamma_{\pi nt}\pi_{nt+1} + \gamma_{\pi ct}\pi_{ct+1} + \gamma_l (l_t - l_t^*) \right] \tag{127}$$

7. *The flex price/wage model*

$$\begin{aligned}
&r_{ct}^* = \sigma(1-\alpha)(1-\omega_{op})(1-\omega_{oc}) \left[z_{t+1} - z_t + l_{t+1}^* - l_t^* \right] \\
&+\sigma \left[(1-\omega_{oc})\omega_{op} + \omega_{oc} \right] (y_{ot+1} - y_{ot})
\end{aligned} \tag{128}$$

$$\eta_t^* = (1-\alpha)(1-\omega_{op})z_t + \omega_{op} y_{ot} - \lambda_{MPL} l_t^* \tag{129}$$

$$\begin{aligned} l_t^* = & \frac{1}{\lambda_{MRS}^f + \lambda_{MPL}} \left[\omega_{op}\left(1-\sigma\right) + \omega_{oc}(1-\omega_{op})(1-\sigma)\right] y_{ot} \\ & + \left[(1-\alpha)(1-\omega_{op}) - \sigma(1-\omega_{op})(1-\alpha) - \omega_{oc}(1-\omega_{op})(1-\alpha)(1-\sigma)\right] z_t, \end{aligned} \tag{130}$$

where

$$\lambda_{MRS}^f = \chi + \sigma(1-\omega_{op})(1-\alpha) + \omega_{oc}(1-\omega_{op})(1-\alpha)(1-\sigma)$$
$$\lambda_{MPL} = \alpha + (1-\alpha)\omega_{op}$$

2 The Welfare Loss Function

In the derivation of the welfare loss function we make recurrent use of two facts given below. For a variable A_t,

$$\frac{A_t - A}{A} \approx a_t + \frac{1}{2} a_t^2, \tag{R1}$$

where $a_t = \log A_t - \log A$. And furthermore, if A_t is defined as:

$$A_t = \left(\int_0^1 A_t(z)^\rho dz \right)^{\frac{1}{\rho}},$$

then

$$a_t = \int_0^1 \left(\log A_t(z) - \log A\right) dz + \frac{1}{2} \rho \; var_z \, a_t(z),$$

which yields:

$$\begin{aligned} a_t &= \int_0^1 a_t(z) dz + \frac{1}{2} \rho \; var_z \, a_t(z) \\ &= E_z a_t(z) + \frac{1}{2} \rho \; var_z \, a_t(z). \end{aligned} \tag{R2}$$

From now on, we impose the additional parametric restriction $\sigma = 1$.

As discussed in the main text, we measure social welfare as the conditional expectation of average household lifetime utility:

$$SW_0 = E_0 \int_0^1 \left[\sum_{t=0}^{\infty} \beta^t \mathbb{W}_t\left(h\right) \right] dh = E_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \mathbb{W}_t\left(h\right) dh \right] = E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{W}_t, \tag{131}$$

defining period social welfare $\mathbb{W}_t$ is simply the average period utility level of households $\int_0^1 \mathbb{W}_t\left(h\right) dh$. Given the separable form of household period utility, the period (social) welfare function may be written as:

$$\mathbb{W} = \mathbb{U}(C) - \int_0^1 \mathbb{V}\left(N(h)\right) dh = \mathbb{U}(C) - E_h \mathbb{V}\left(N(h)\right), \tag{132}$$

recalling that complete markets for consumption and separable preferences over consumption and labor implies that consumption is equalized across households (and hence does not depend on h).

Following Rotemberg and Woodford (1999), we take a second order approximation to the social welfare function around the Pareto efficient steady state (the steady state in which both wage and price inflation are constant (at π), monopolistic distortions are eliminated through appropriate subsidies, and all exogenous shocks are set equal to their unconditional means).

The challenging part of deriving of the approximation to the conditional welfare function consists in obtaining the second order approximation to the *period* welfare function W_t (given the period loss function and our assumption of Calvo-style contracts, the discounted conditional loss function follows immediately using results in e.g., Woodford (2003)). Hence, we begin by deriving an approximation to the period welfare function, and because all variables enter contemporaneously, find it convenient to omit time subscripts in this part of the derivation.

We approximate the two parts of the period welfare function given in equation (132) separately and combine the results at the end. First consider a second order arithmetic approximation to $E_h \mathbb{V}\left(N(h)\right)$:

$$E_h \mathbb{V}\left(N(h)\right) \approx \overline{\mathbb{V}} + \int_0^1 \mathbb{V}_N \bar{N} \frac{dN(h)}{\bar{N}} dh + \frac{1}{2} \int_0^1 \mathbb{V}_{NN} (\bar{N})^2 (\frac{dN(h)}{\bar{N}})^2 dh. \tag{133}$$

Using result R1, we obtain a second order logarithmic approximation to $E_h\mathbb{V}(N(h))$:

$$E_h\mathbb{V}(N(h)) \approx \overline{\mathbb{V}} + \mathbb{V}_N\bar{N}(E_hn(h) + \frac{1}{2}E_hn(h)^2) + \frac{1}{2}\mathbb{V}_{NN}(\bar{N})^2(E_hn(h)^2). \tag{134}$$

Given that the aggregate labor index L is given by:

$$L = \left[\int_0^1 (N(h))^{\frac{1}{1+\theta_w}} dh\right]^{1+\theta_w}, \tag{135}$$

result R2 implies that a second order logarithmic approximation can be expressed as:

$$l = ln(L) - ln(L) \approx E_hn(h) + \frac{1}{2}\left(\frac{1}{1+\theta_w}\right) var_hn(h). \tag{136}$$

We now turn to the production side of the economy to solve for effective hours l. Given our assumption that the production function of the wholesale goods producers is Cobb-Douglas, it may be expressed in log percentage deviations form as:

$$y_w = (1-\omega_{op})(1-\alpha)(z+l) + \omega_{op}o_p. \tag{137}$$

Solving for effective labor yields:

$$l = \frac{1}{(1-\omega_{op})(1-\alpha)}(y_w - \omega_{op}o_p - (1-\omega_{op})(1-\alpha)z). \tag{138}$$

Recalling the resource constraint for the uses of the wholesale good from the text:

$$Y_w = \int_0^1 Y_w(f)df, \tag{139}$$

result R2 implies the second order approximation:

$$y_w = E_fy_w(f) + \frac{1}{2}var_fy_w(f) = E_fy_n(f) + \frac{1}{2}var_fy_n(f), \tag{140}$$

where the second equality reflects the simple linear production function for retailers $y_n(f) = y_w(f)$. Given that the index of aggregate non-oil output Y_n is:

$$Y_n = \left[\int_0^1 (Y_n(f))^{\frac{1}{1+\theta_p}} df\right]^{1+\theta_p}, \tag{141}$$

result R2 implies a second order approximation of the form:

$$y_n \approx E_fy_n(f) + \frac{1}{2}\left(\frac{1}{1+\theta_p}\right) var_fy_n(f). \tag{142}$$

Substituting for $E_f y_n(f)$ using (139) yields:

$$y_n \approx y_w - \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f). \tag{143}$$

We can now use equation (142) to solve for y_w, and then substitute the result into (138) to obtain an expression for effective labor of the form:

$$\begin{aligned} l \quad &\approx \frac{1}{(1-\omega_{op})(1-\alpha)}\left[y_n - \omega_{op} o_p - (1-\omega_{op})(1-\alpha)z\right] \\ &+\frac{1}{2}\frac{1}{(1-\omega_{op})(1-\alpha)}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f). \end{aligned} \tag{144}$$

Because the production functions for firms and households are Cobb-Douglas, the condition determining the relative use of oil by firms and households is:

$$\frac{O_p}{O_c} = \omega_{op}\left(\frac{1-\omega_{oc}}{\omega_{oc}}\right)\left(\frac{Y_w}{Y_n}\right)(\Phi_n) = \omega_m\left(\frac{Y_w}{Y_n}\right)(\Phi_n), \tag{145}$$

where ω_m is simply a composite parameter defined as $\omega_m = \omega_{op}\left(\frac{1-\omega_{oc}}{\omega_{oc}}\right)$, and where we have defined $\Phi_n = \frac{P_w}{P_n}$ (thus, Φ_n is the real marginal cost of producing retail goods, or the inverse of the price markup in the retail sector). Using the resource constraint for oil,

$$\frac{Y_o}{O_c} = 1 + \frac{O_p}{O_c} = \Psi, \tag{146}$$

where Ψ is simply a definition. From the identity:

$$\frac{O_p}{Y_o} = \frac{O_p}{O_c}\frac{O_c}{Y_o} = \frac{O_p}{O_c}\frac{1}{\Psi}. \tag{147}$$

We may take logs to obtain:

$$o_p = y_o + (o_p - o_c) - \psi = y_o + (y_w - y_n) + \psi_n - \psi, \tag{148}$$

where the second equality uses equation (145) to solve for $o_p - o_c$. Using equation (143), equation (148) may be written as:

$$o_p \approx y_o + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f) + \phi_n - \psi. \tag{149}$$

We next must take a second order log approximation of ψ. Noting first from (145) and (146) that:

$$\psi = ln\left(1 + \omega_m e^{ln(Y_w) - ln(Y_n)} e^{ln(\Phi_n)}\right) - ln\left(1+\omega_m\right), \tag{150}$$

the second order approximation to ψ is easily seen to be:

$$\psi \approx \left(\frac{\omega_m}{1+\omega_m}\right)(\phi_n + y_w - y_n) + \frac{1}{2}\left(\frac{\omega_m}{(1+\omega_m)^2}\right){\phi_n}^2. \tag{151}$$

Since in the steady state $\frac{O_c}{Y_o} = \frac{1}{1+\omega_m}$ and $\frac{O_p}{Y_o} = \frac{\omega_m}{1+\omega_m}$ – with the former immediate from equations (145) and (146), and the latter from the resource constraint – the approximation to ψ may be expressed equivalently as:

$$\psi \approx \left(\frac{O_p}{Y_o}\right)\left(\phi_n + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f)\right) + \frac{1}{2}\left(\frac{O_p}{Y_o}\right)\left(\frac{O_c}{Y_o}\right){\phi_n}^2. \tag{152}$$

Substituting this into equation (149) gives the desired second order approximation to o_p:

$$o_p \approx y_o + \frac{O_c}{Y_o}\phi_n + \frac{1}{2}\left(\frac{O_c}{Y_o}\right)\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f) - \left(\frac{O_c}{Y_o}\right)\left(\frac{O_p}{Y_o}\right)\phi_n^2. \tag{153}$$

Substituting for o_p in equation (144) yields an expression for effective labor of the form:

$$\begin{aligned} l \quad &\approx \frac{1}{(1-\omega_{op})(1-\alpha)}\left[(y_n - \omega_{op}y_o - (1-\omega_{op})(1-\alpha)z) - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right] \\ &-\frac{1}{2}\frac{\omega_{op}}{(1-\omega_{op})(1-\alpha)}\left(\frac{O_c}{Y_o}\right)\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f) \\ &+\frac{1}{2}\frac{1}{(1-\omega_{op})(1-\alpha)}\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f) \\ &+\frac{1}{2}\frac{\omega_{op}}{(1-\omega_{op})(1-\alpha)}\left(\frac{O_c}{Y_o}\right)\left(\frac{O_p}{Y_o}\right)\phi_n^2. \end{aligned} \tag{154}$$

Using equations (136) and (154), we may now solve for $E_h n(h)$ as follows:

$$\begin{aligned} E_h n(h) \quad &\approx l - \frac{1}{2}\frac{1}{1+\theta_w}var_h n(h) \\ &\approx \frac{1}{(1-\omega_{op})(1-\alpha)}\left[(y_n - \omega_{op}y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right] \\ &-\frac{1}{2}\frac{\omega_{op}}{(1-\omega_{op})(1-\alpha)}\left(\frac{O_c}{Y_o}\right)\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f) \\ &+\frac{1}{2}\frac{1}{(1-\omega_{op})(1-\alpha)}\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f) \\ &+\frac{1}{2}\frac{\omega_{op}}{(1-\omega_{op})(1-\alpha)}\left(\frac{O_c}{Y_o}\right)\left(\frac{O_p}{Y_o}\right)\phi_n^2 - \frac{1}{2}\frac{1}{1+\theta_w}var_h n(h). \end{aligned} \tag{155}$$

The approximation to the component of the welfare function (134) also depends on the second moment $E_h n(h)^2$. Note that:

$$E_h n(h)^2 = var_h n(h) + (E_h n(h))^2, \tag{156}$$

and also note that (155) implies a second-order approximation to $(E_h n(h))^2$ given by:

$$(E_h n(h))^2 \approx \left[\frac{1}{(1-\omega_{op})(1-\alpha)}\right]^2 \left[y_n - \omega_{op} y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right]^2. \tag{157}$$

It follows that the second moment $E_h n(h)^2$ can be expressed:

$$\begin{aligned} E_h n(h)^2 \quad & \approx var_h n(h) + \\ & \left[\frac{1}{(1-\omega_{op})(1-\alpha)}\right]^2 \left[y_n - \omega_{op} y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right]^2. \end{aligned} \tag{158}$$

Using (155) and (158), we now can substitute for $E_h n(h)$ and $E_h n(h)^2$ in (134) to obtain:

$$\begin{aligned} E_h \mathbb{V}\left(N(h)\right) \quad & \approx \overline{\mathbb{V}} + \mathbb{V}_N \bar{N} E_h n(h) + \frac{1}{2}\left(\mathbb{V}_N \bar{N} + \mathbb{V}_{NN}(\bar{N})^2\right) E_h n(h)^2 \\ & \approx \overline{\mathbb{V}} + \mathbb{V}_N \bar{N} \frac{1}{(1-\omega_{op})(1-\alpha)} \left[(y_n - \omega_{op} y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right] \\ & + \frac{1}{2}\mathbb{V}_N \bar{N} \frac{1}{(1-\omega_{op})(1-\alpha)} \left[1 - \omega_{op}\left(\frac{O_c}{Y_o}\right)\right]\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f) \\ & + \frac{1}{2}\mathbb{V}_N \bar{N} \frac{1}{(1-\omega_{op})(1-\alpha)} \omega_{op}\left(\frac{O_c}{Y_o}\right)\left(\frac{O_p}{Y_o}\right)\phi_n^2 \\ & + \frac{1}{2}\left(\mathbb{V}_N \bar{N} \frac{\theta_w}{1+\theta_w} + \mathbb{V}_{NN}(\bar{N})^2\right) var_h n(h) \\ & + \frac{1}{2}\left(\mathbb{V}_N \bar{N} + \mathbb{V}_{NN}(\bar{N})^2\right)\left[\frac{1}{(1-\omega_{op})(1-\alpha)}\right]^2 \\ & \left[(y_n - \omega_{op} y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right]^2. \end{aligned} \tag{159}$$

We next consider approximating the component of the period welfare function associated with consumption. A second order arithmetic approximation to $\mathbb{U}(C)$ yields:

$$\mathbb{U}(C) \approx \overline{\mathbb{U}} + \mathbb{U}_C \bar{C} \frac{dC}{\bar{C}} + \frac{1}{2}\mathbb{U}_{CC}(\bar{C})^2 \frac{dC}{\bar{C}}^2. \tag{160}$$

Using R2, we obtain a second order logarithmic approximation to $\mathbb{U}(C)$:

$$\mathbb{U}(C) \approx \overline{\mathbb{U}} + \mathbb{U}_C \bar{C} c + \frac{1}{2}(\mathbb{U}_C \bar{C} + \mathbb{U}_{CC} \bar{C})^2 c^2. \tag{161}$$

Under the assumption that the technology for producing the final consumption good is Cobb-Douglas, i.e.,

$$C = C_n^{1-\omega_{oc}} O_c^{\omega_{oc}}, \tag{162}$$

and substituting for O_c using equation (146), it follows that the logarithmic percentage deviation of aggregate consumption from its steady state value may be expressed:

$$c = (1-\omega_{oc})c_n + \omega_{oc}y_o - \omega_{oc}\psi. \tag{163}$$

Using the second order approximation for ψ given by equation (152), the second order approximation to c is given by:

$$c \approx (1-\omega_{oc})y_n + \omega_{oc}y_o - \omega_{oc}\left(\frac{O_p}{Y_o}\right)\left[\left(\phi_n + \frac{1}{2}\omega_{oc}\left(\frac{O_c}{Y_o}\right){\phi_n}^2 + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f)\right)\right]. \tag{164}$$

Substituting (164) into the component of the welfare function associated with consumption (161) yields:

$$\begin{aligned}
\mathbb{U}(C) \quad &\approx \overline{\mathbb{U}} + \mathbb{U}_C\bar{C}c \\
&\approx \overline{\mathbb{U}} + \mathbb{U}_C\bar{C}\left[(1-\omega_{oc})y_n + \omega_{oc}y_o\right] \\
&-\mathbb{U}_C\bar{C}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\left[\left(\phi_n + \frac{1}{2}\omega_{oc}\left(\frac{O_c}{Y_o}\right){\phi_n}^2 + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f)\right)\right],
\end{aligned} \tag{165}$$

where the term $\frac{1}{2}(\mathbb{U}_C\bar{C} + \mathbb{U}_{CC}\bar{C})^2c^2$ drops given our assumption that the subutility function over consumption is logarithmic.

Using equations (159) and (165), we derive the period welfare loss, which is defined as the deviation of the period social welfare from the level that would prevail under price and wage flexibility:

$$\begin{aligned}
W_{loss} \quad &\approx \mathbb{W} - \mathbb{W}^* = \mathbb{U}(C) - \mathbb{U}(C^*) - \left[E_h\mathbb{V}\left(N(h)\right) - \mathbb{V}\left(N^*\right)\right] \\
= \quad & \mathbb{U}_C\bar{C}(1-\omega_{oc})\left[y_n - y_n^*\right] \\
- \quad & \mathbb{U}_C\bar{C}(1-\omega_{oc})\left(\frac{\omega_{oc}}{1-\omega_{oc}}\right)\left(\frac{O_p}{Y_o}\right)\left[\phi_n + \frac{1}{2}\left(\frac{O_c}{Y_o}\right)\phi_n^2 + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f)\right] \\
- \quad & \mathbb{V}_N\bar{N}\frac{1}{(1-\alpha)(1-\omega_{op})}\left[y_n - y_n^*\right]
\end{aligned}$$

$$
\begin{aligned}
+ \quad & \mathbb{V}_N\bar{N}\frac{1}{(1-\alpha)(1-\omega_{op})}\left(\frac{O_c}{Y_o}\right)\omega_{op}\left[\phi_n - \frac{1}{2}\left(\frac{O_p}{Y_o}\right)\phi_n^2 + \frac{1}{2}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f)\right] \\
- \quad & \frac{1}{2}\left(\mathbb{V}_N\bar{N} + \mathbb{V}_{NN}(\bar{N})^2\right)\left[\frac{1}{(1-\omega_{op})(1-\alpha)}\right]^2 \\
& \left(\left[(y_n - \omega_{op}y_o - (1-\omega_{op})(1-\alpha)z - \omega_{op}\left(\frac{O_c}{Y_o}\right)\psi_n\right]^2 - \left[y_n^* - \omega_{op}y_o - (1-\omega_{op})(1-\alpha)z\right]^2\right) \\
- \quad & \frac{1}{2}\mathbb{V}_N\bar{N}\frac{1}{(1-\omega_{op})(1-\alpha)}\left(\frac{\theta_p}{1+\theta_p}\right) var_f y_n(f) \\
+ \quad & \frac{1}{2}\left(\mathbb{V}_N\bar{N}\frac{\theta_w}{1+\theta_w} + \mathbb{V}_{NN}(\bar{N})^2\right) var_h n(h). \qquad (166)
\end{aligned}
$$

In deriving the above, we exploited the fact that $\phi_{nt}^* = 0$. For convenience, we repeat below equations (69), (59), (70), and (71), derived in Section 1.6:

$$
\mathbb{U}_C\bar{C}(1-\omega_{oc}) = \mathbb{V}_N\bar{N}\frac{1}{(1-\alpha)(1-\omega_{op})},
$$

$$
\left(\frac{\omega_{oc}}{1-\omega_{oc}}\right)\left(\frac{O_p}{Y_o}\right) = \omega_{op}\left(\frac{O_c}{Y_o}\right),
$$

$$
\left(\mathbb{V}_N\bar{N} + \mathbb{V}_{NN}(\bar{N})^2\right) = \mathbb{V}_N\bar{N}\left(1+\chi\right),
$$

$$
\left(\mathbb{V}_N\bar{N}\frac{\theta_w}{1+\theta_w} + \mathbb{V}_{NN}(\bar{N})^2\right) = \mathbb{V}_N\bar{N}\left(\frac{\theta_w}{1+\theta_w} + \chi\right).
$$

Using equations (69) and (59), it is evident that the first four lines of equation (166) can be reduced to:

$$
\frac{1}{2}\mathbb{U}_C\bar{C}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\left(-\left(\frac{O_c}{Y_o}\right) - \left(\frac{O_p}{Y_o}\right)\right)\phi_n^2 = -\frac{1}{2}\mathbb{U}_C\bar{C}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\phi_n^2. \qquad (167)
$$

Thus, all linear terms drop from the period welfare loss function.

Next, from our analysis of the flexible price and wage equilibrium it is apparent that employment is invariant to oil shocks with log utility in consumption, i.e., $l^* = 0$, implying that the production function of wholesale producers can be written as:

$$
y_w^* = (1-\omega_{op})(1-\alpha)z + \omega_{op}\ y_o. \qquad (168)
$$

Given that $y_w^* = y_n^*$ under price flexibility, it follows that:

$$
y_n^* = (1-\omega_{op})(1-\alpha)z + \omega_{op}\ y_o. \qquad (169)
$$

Using the latter, it is apparent that the term in the loss function premultiplied by $\left(\mathbb{V}_N\bar{N}+\mathbb{V}_{NN}(\bar{N})^2\right)$ (i.e., on lines 5 and 6) can be written as:

$$\frac{1}{2}\left(\mathbb{V}_N\bar{N}+\mathbb{V}_{NN}(\bar{N})^2\right)\left[\frac{1}{(1-\omega_{op})(1-\alpha)}\right]^2\left[y_n-y_n^*-\omega_{op}\left(\frac{O_c}{Y_o}\right)\psi_n\right]^2. \tag{170}$$

With these reductions, the period welfare loss function may be written in the simple form:

$$\begin{aligned}\frac{W_{loss}}{\mathbb{U}_C C} &= \frac{\mathbb{W}-\mathbb{W}^*}{\mathbb{U}_C C} \qquad (171)\\ &= -\frac{1}{2}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\phi_n^2-\frac{1}{2}\frac{(1-\omega_{oc})(1+\chi)}{(1-\alpha)(1-\omega_{oc})}\left[y_n-y_n^*-\omega_{op}\left(\frac{O_c}{Y_o}\right)\psi_n\right]^2\\ &\quad -\frac{1}{2}(1-\omega_{oc})\left(\frac{\theta_p}{1+\theta_p}\right)var_f y_n(f)-\frac{1}{2}(\chi+\frac{\theta_w}{1+\theta_w})(1-\alpha)(1-\omega_{oc})(1-\omega_{op})var_h n(h).\end{aligned}$$

We now use very standard results (Rotemberg and Woodford, 1999) to express cross sectional dispersion in output and wages in terms of time series variation in prices and wages. Accordingly, the demand curve facing the monopolistic retailers may be expressed in logarithmic form as:

$$y_n(f)=-\left(\frac{1+\theta_p}{\theta_p}\right)(p_n(f)-p_n)+y_n. \tag{172}$$

Thus, the cross-sectional output dispersion term may be expressed in terms of cross-sectional price dispersion as:

$$var_f y_n(f)=\left[\frac{1+\theta_p}{\theta_p}\right]^2 var_f p_n(f). \tag{173}$$

Similarly, given that the labor demand curve facing the monopolistically households may be expressed in logarithmic form as:

$$n(h)=-\left(\frac{1+\theta_w}{\theta_w}\right)(w(h)-w)+l, \tag{174}$$

the cross-sectional employment dispersion term may be expressed in terms of cross-sectional wage dispersion as:

$$var_h n(h)=\left[\frac{1+\theta_w}{\theta_w}\right]^2 var_h w(h). \tag{175}$$

Substituting (173) and (175) into the period welfare loss function yields:

$$\begin{aligned}
\frac{\mathbb{W}_t - \mathbb{W}_t^*}{\mathbb{U}_c C} = \quad & -\frac{1}{2}\frac{(1+\chi)(1-\omega_{oc})}{(1-\omega_{op})(1-\alpha)}\left[y_{nt} - y_{nt}^* - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_n\right]^2 \\
& -\frac{1}{2}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\phi_{nt}^2 \\
& -\frac{1}{2}(1-\omega_{oc})\left(\frac{1+\theta_p}{\theta_p}\right) var_f p_{nt}(f) \\
& -\frac{1}{2}(1-\omega_{oc})(1-\omega_{op})(1-\alpha)\left(\frac{1+\theta_w}{\theta_w}\right)\left(1+\frac{1+\theta_w}{\theta_w}\chi\right) var_h w_t(h), \qquad (176)
\end{aligned}$$

Under Calvo-style pricing, the time series evolution of $var_f p_{nt}(f)$ is given by:

$$var_f p_{nt}(f) = \frac{\xi_p}{1-\xi_p}\pi_{nt}^2 + \xi_p var_f p_{n,t-1}(f), \qquad (177)$$

where we must now use time subscripts to indicate the time dimension. Similarly, the time series evolution of $var_h w_t(h)$ is given by:

$$var_h w(h) = \frac{\xi_w}{1-\xi_w}\omega_t^2 + \xi_w var_h w_{t-1}(h). \qquad (178)$$

Using (177) and (178) and the period welfare function, it follows that the (time zero) conditional discounted social welfare loss may be expressed:

$$\begin{aligned}
& E_0 \sum_{t=0}^{\infty}\beta^t\left[\frac{\mathbb{W}_t - \mathbb{W}_t^*}{\mathbb{U}_c C}\right] = -\frac{1}{2}\frac{(1+\chi)(1-\omega_{oc})}{(1-\omega_{op})(1-\alpha)}\sum_{t=0}^{\infty}\beta^t E_0\left[y_{nt} - y_{nt}^* - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_{nt}\right]^2 \qquad (179) \\
& -\frac{1}{2}\omega_{oc}\left(\frac{O_p}{Y_o}\right)\sum_{t=0}^{\infty}\beta^t E_0\phi_{nt}^2 \\
& -\frac{1}{2}(1-\omega_{oc})\left(\frac{1+\theta_p}{\theta_p}\right)\left(\frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}\right)\sum_{t=0}^{\infty}\beta^t E_0\pi_{nt}^2 \\
& -\frac{1}{2}(1-\omega_{oc})(1-\omega_{op})(1-\alpha)\left(\frac{1+\theta_w}{\theta_w}\right)\left(1+\frac{1+\theta_w}{\theta_w}\chi\right)\left(\frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\right)\sum_{t=0}^{\infty}\beta^t E_0\omega_t^2,
\end{aligned}$$

where $\frac{O_p}{Y_o} = 1 - \frac{\omega_{oc}}{\omega_{oc}+\omega_o(1-\omega_{oc})}$ and $\frac{O_c}{Y_o} = \frac{\omega_{oc}}{\omega_{oc}+\omega_o(1-\omega_{oc})}$. The above equation corresponds to equation (23) in the main text, and completes our derivation of the welfare function.

Notice that we can rewrite the loss function above in terms of the labor gap, instead of the output gap by noticing that: $\frac{1}{(1-\omega_{op})^2(1-\alpha)^2}\left[y_{nt} - y_{nt}^* - \omega_{op}\left(\frac{O_c}{Y_o}\right)\phi_{nt}\right]^2 = (l_t - l_t^*)^2$, from equation (154).

3 Optimal Monetary Policy

This section shows how to set up the optimal monetary policy under commitment using our quadratic approximation of the welfare loss function. For this purpose, we consider the problem of choosing paths for l_t, η_t, π_{nt}, ω_t, and ϕ_{nt} to minimize $E_0\left(\sum_{t=0}^{\infty}\beta^t L_t\right)$, where

$$L_t = -\frac{1}{2}\left[\lambda_1\phi_{nt}^2 + \lambda_2\left(l_t - l_t^*\right)^2 + \lambda_3\pi_{nt}^2 + \lambda_4\omega_t^2\right]$$

and

$$\begin{aligned}
\lambda_1 &= \omega_{oc}\left(\frac{O_p}{Y_o}\right), \\
\lambda_2 &= (1+\chi)\,(1-\omega_{oc})(1-\omega_{op})\,(1-\alpha), \\
\lambda_3 &= (1-\omega_{oc})\left(\frac{1+\theta_p}{\theta_p}\right)\frac{\xi_p}{(1-\beta\xi_p)\,(1-\xi_p)}, \\
\lambda_4 &= (1-\omega_{oc})\,(1-\omega_{op})\,(1-\alpha)\left(\frac{1+\theta_w}{\theta_w}\right)\left(1+\frac{1+\theta_w}{\theta_w}\chi\right)\frac{\xi_w}{(1-\beta\xi_w)\,(1-\xi_w)},
\end{aligned}$$

subject to the constraints that the sequences must satisfy equations (82), (83), (84) and (*real* marginal cost). Note that we have replaced the output gap by the employment gap.

The Lagrangian of this problem can be written as:

$$\begin{aligned}
&\min_{l_t,\eta_t,\pi_{nt},\omega_t,\phi_{nt}} E_0\sum_{t=0}^{\infty}\beta^t L_t \\
&+\beta^t\mu_{1,t}\left[\pi_{nt} - \beta\pi_{n,t+1} - \kappa_P\left(\left[\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}}{\omega_{op}+\omega_{oc}}\right](\eta_t-\eta_t^*) + \left[\alpha+(1-\alpha)\omega_{op}\right](l_t-l_t^*)\right)\right] \\
&+\beta^t\mu_{2,t}\left[\omega_t - \beta\omega_{t+1} - \kappa_w\left(\left[\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}}{\omega_{op}+\omega_{oc}}\right](\eta_t-\eta_t^*) + \lambda_{MRS}(l_t-l_t^*)\right)\right] \\
&+\beta^t\mu_{3,t}\left[\eta_t - \eta_{t-1} - \omega_t + \pi_{nt}\right] \\
&+\beta^t\mu_{4,t}\left[\phi_{nt} - \left[\frac{\omega_{op}(1-\omega_{oc})+\omega_{oc}}{\omega_{op}+\omega_{oc}}\right](\eta_t-\eta_t^*) - \left[\alpha+(1-\alpha)\omega_{op}\right](l_t-l_t^*)\right].
\end{aligned}$$