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A Collateral Theory of Endogenous Debt Maturity*

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Abstract

This paper studies optimal debt maturity when firms cannot issue state contingent claims and must back promises with collateral. We establish a trade-off between long-term borrowing costs and short-term rollover costs. Issuing both long- and short-term debt balances financing costs because different debt maturities allow firms to cater risky promises across time to investors most willing to hold risk. Contrary to existing theories predicated on information frictions or liquidity risk, we show that collateral is sufficient to explain the joint issuance of different types of debt: safe "money-like" debt, risky short- and long-term debt. The model predicts that borrowing costs are lowest, leading to more leverage and production, when firms issue multiple debt maturities. Lastly, we show that "hard" secured debt covenants are redundant when collateral is scarce because they act as perfect substitutes for short-term debt.

Keywords: Collateral, Debt Maturity, Investment, Cost of Capital, Debt covenants

JEL Codes: D92, G11, G12, G31, G32, E22

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1 Introduction

In this paper, we study how a firm should optimally structure its debt maturity when assets must be pledged as collateral to back promises. There are no information frictions between the firm and its potential creditors, so the model can be thought to apply to large corporations for whom there is abundant public information with access to debt instruments that span the maturity spectrum. The novelty of this approach is that we endogenize the joint determination of debt maturity, financing costs, and investment scale in a general equilibrium setting without asymmetric or private information or liquidity risky.

A firm issues non-contingent promises (debt) that are backed by collateral. Long-term debt has positive credit spreads (yield spread of a bond over the risk-free rate) when collateralized promises are risky. Short-term debt issued today is safe and "money-like" when the firm rolls over expiring claims tomorrow. The short-term promises issued tomorrow may also have positive credit spreads, similar to long-term debt. Specifically, short-term credit spreads are high in bad states and potentially expose equity holders to rollover losses, while short-term debt can be rolled over at the risk-free rate in good states. The total cost of short-term debt is the expected face-value of the promises needed to honor expiring claims tomorrow. Our main result is that the optimal debt structure is the joint issuance of safe "money-like" short-term debt and risky long-term debt today with safe (risky) short-term in good (bad) states tomorrow; hence, a well defined interior solution. A debt structure with multiple debt maturities is the least costly and allows firms to increase leverage, investment, production, and output.

The intuition is the following. Risky interest rates rise the more debt a firm issues at any point time because debt is issued to investors with lower valuations for risky assets. Borrowing costs are low if relatively few risky promises are made to investors period-by-period, or if the firm's collateral capacity is high. For a given collateral capacity, safe short-term debt substitutes for expensive long-term debt, but must be rolled over tomorrow. Substituting away from long-term debt lowers long-term borrowing costs to the point where they are just equal to expected short-term rollover costs tomorrow. Issuing all long-term debt today is expensive and prohibits the firm from selling risky debt to investors tomorrow who have higher marginal valuations for risky assets. Alternatively, issuing all safe short-term debt raises expected rollover costs tomorrow. Issuing some risky long-term debt today also acts as a substitute for potentially expensive rollover costs tomorrow. Short-term borrowing costs tomorrow fall as the firm issues risky long-term debt today. In equilibrium, the firm balances maturity specific debt costs and substitutes between debt maturities to the point where expected costs across all maturities are equalized. Even though the firm issues fewer risky promises in each time period, the price of each promise issued is highest (borrowing costs are lowest) when the firm issues multiple maturities, which allows for more leverage, investment and output.

After establishing that a variety of debt maturities is generally optimal, we explore the effects of "hard" protective debt covenants on equilibrium debt maturity. Specifically, we allow long-term debt to be secured by one of the most common covenants found in long-term corporate debt indentures–the negative pledge covenant. A negative pledge covenant is a distinct way to guarantee assets are available to the creditors at hand if a debtor defaults. In particular, the negative pledge stipulates that a debtor cannot use any of its assets as security for subsequent debt obligations without securing the current issuance. Without the covenant, raising the face value of short-term debt to rollover claims in bad states, dilutes the value of existing long-term claims. The covenant protects long-term debt holders from debt dilution. Though almost ubiquitous in corporate indentures, the impact of negative pledge covenants, to our knowledge, has not been rigorously studied by economists.[1,2]

To model the negative pledge, we assume long-term debt holders are entitled to receive a *pro rata* share of firm assets that are stipulated in their indenture, irrespective of what happens with its short-term obligations. We show that the delivery on long-term debt with the covenant is equivalent to the delivery in an economy in which there is no maturity mis-match between assets and liabilities, and hence no dilution. The price any investor is willing to pay for a protected long-term bond rises when the effects of short-term debt dilution are eliminated. In equilibrium, the firm responds by re-optimizing its maturity structure away from short-term debt toward long-term debt. Consistent with our model's predictions, Billet, King, and Mauer (2007) find that debt covenants and short-term debt act as substitutes; firms

[1]By contrast, corporate legal scholars view negative pledge covenants are highly important. Legal scholar Philip Wood (2007) states that the negative pledge clause is "one of the most fundamentally important covenants in an unsecured term loan agreement."

[2]The curious reader can flip to section 4, table 1 to see a coarse breakdown of negative pledge data in primary public debt indentures obtained from Mergent-FISD. In essence, these covenants are more likely to appear in medium-to-long-term debt contracts and in debt contracts issued by non-financial sector firms.

with more covenants in their public debt indentures tend to issue relatively more long-term debt than firms with few or no covenants. More interesting, the covenant does not affect real economic outcomes because it only serves to reallocate the value of scarce collateral from short-term to protected long-term creditors. Equilibrium risky interest rates adjust back to their values before the covenant was introduce as the firm issues more risky long-term promises. Hard protective covenants are redundant with scarce collateral because they do not affect the fundamental collateral capacity of the risky asset.

Our model is collateral economy, "C-model", that features an optimizing agent endowed only with a risky inter-period production technology (firm). [3] The production technology is a long-term, two-period project. Keeping with the spirit of Fostel and Geanakoplos (FG) (2008, 2012, 2015 and 2016) C-models, for a given amount of collateral, firms can only borrow more by paying a higher interest rate, and will pay a lower interest rate if they borrow less. The need to post collateral leads to an upward sloping capital supply curve rather than the perfectly elastic supply curve *assumed* in corporate finance models. Debt financing is least costly when the firm issues promises to investors who, for a given interest rate, require the least amount of collateral. Issuing multiple debt maturities allows risky promises to be made throughout the duration of an investment project, which concentrates debt to investors who value risky promises most. Concentrating risky promises to investors with the highest willingness to hold risk allows the firm to leverage its assets, invest, and produce more. In sum, the need for collateral to back promises affects the debt maturity choice, the cost of capital, and investment demand and production without asymmetric or private information or liquidity risk.

The model predicts that multiple debt maturities comprise an optimal debt liability structure, which is born out by the data. For example, firms typically issue debt into distinct maturity bins rather than issuing a single debt maturity (Choi, Hackbarth, and Zechner (2016)).[4] The model predicts that borrowing costs will be lower and firms can borrow more against any fundamental asset value when it issues multiple maturities. These prediction are exactly borne out empirically by Norden, Rooenboom, and Wang (2016) who find that firms with granular maturity structures have better access to credit and borrow at lower rate spreads. Our model

[3] See Geanakoplos (2003 and 2009) and Fostel and Geanakoplos (FG) (2008, 2012, 2015 and 2016) for the workhorse C-model.

[4] IBM as an extreme, had 12 different bond maturities outstanding, ranging from one year to 89 years.

also rationalizes why the largest corporations, who are the least likely to be subject to information asymmetries and liquidation risk, have active commercial paper programs (Kahl, Shivdasani, and Yihui Wang (2015). Issuing commercial paper acts as a *substitute* for more expensive long-term debt because it reduces the amount of long-term promises made for a fixed amount of collateral. Rolling over the short-term claims by using collateral tomorrow is marginally less expensive than issuing additional risky long-term claims today. We show that the substitution effect holds even when rolling over short-term debt causes equity holders to absorb rollover losses (when collateral capacity is low). Furthermore, as the collateral capacity of the firm increases, short-term debt leads to rollover gains further strengthening its use as part of the firm's debt structure. Lastly, the collateral capacity of firm assets jointly determine maturity, leverage, and the "growth option" of investment. Our model suggests that empirical studies must treat these three variables as endogenous and jointly estimate them. In the cross section, maturity, leverage, and growth will differ based on firm or industry specific collateral capacities, pledgability, or on differences in the recovery rate of the securities issued.

The organization of the paper is as follows: related literature is below. Section 2 introduces the model, agents, the different debt contracts considered. Section 3 characterizes the equilibrium debt liability structure and comparative static results. Section 4 introduces the covenant and provides a numerical example of the model. Section 5 concludes. All proofs that are not obvious from the text are contained in the appendix.

Related literature

Debt maturity matters in traditional models due to improved investment incentives arising from debt overhang (Meyers (1977), He and Diamond (2014)), optimal default timing (He and Milbradt (2016)), information asymmetries (Flannery (1986), Kale and Noe (1990)), control rights (Diamond (1991, 1993), and inefficient continuation policies (Houston and Venkataraman (1994)). We add to these theories by proposing collateral frictions as an alternative explanation for why firms issue multiple debt maturities.[5] A recent exception to this literature is Hugonnier, Malamud,

[5]Heterogeneity is at the heart of Jung and Subramanian (2014), but an agency problem gives maturity a role in their model. Specifically, heterogeneous beliefs between managers and equity holders leads to a tradeoff between manager optimism and long-term debt issuance. Our model also has a flavor of this effect, but heterogeneity between the firm and investors is only material for determining what portion long-term debt constitutes of total debt issuance.

and Morellec (2015). The authors show how capital supply frictions in a dynamic capital structure model lead to optimal leverage distributions rather than target leverage ratios. Another distinction in our model are non-exclusive banking relationships between debtors and creditors. Non-exclusive banking is quite natural considering how large corporations typically raise capital in both public and private debt markets. For example, Dass and Massa (2014), using Lipper eMAXX data, highlight that the average corporation has 17 institutional investors acting as creditors at any point in time (see also Detragiache, Garella, and Guiso (2000)).[6]

The joint issuance of debt maturities also arises in Diamond (1991 and 1993) and Houston and Venkataraman (1994). These papers argue that using both short- and long-term debt helps balance inefficient liquidation incentives arising from private information.[7] However, the empirical evidence for the liquidation and information friction story is not overly supportive. Graham and Harvey (2001) find that firms are typically *not* concerned with information asymmetries and agency problems. Billet, King, and Mauer (2007) do not find evidence that liquidity risk drives short-term debt use for rated firms. Johnson (2003) finds that the liquidity risk effects of short-term debt matter almost exclusively for unrated firms, and virtually non-existent for rated firms. In this light, we do not model agency problems, and show that safe short-term debt, resembling commercial paper, is actually sufficient for firms to use multiple maturities, providing a direct contrast to existing theories.

Diamond and He (2014) highlight the subtle effects of debt maturity on debt overhang and investment incentives. The optimal debt maturity balances the symptoms of short- and long-term debt overhang; earlier default versus reduced investment incentives. However, they consider different debt maturities with equivalent market values and a fixed asset. We do not consider overhang effects because how debt maturities are structure in our model effects the *ex ante* value of the asset/project

[6]Large firms typically raise capital from a syndicate of creditors rather than a single creditor even when considering private loan markets. Using supervisory data on bank holding companies, Caglio, Darst, and Parolin (2016) show that larger corporates borrow from, on average, 6 banks compared to small firms that tend to borrow from one.

[7]Proposition 2 of Diamond (1991) shows that short-term debt is the unique funding outcome in the model absent liquidation risk or loss of control rents. The agency problem is therefore necessary in his model to obtain an equilibrium with multiple maturities. Firm borrowing is fixed in Diamond (1991) and firms can borrow up to the fixed amount at the same interest rate. The need for collateral in our model implies that firms must pay higher borrowing costs if they wish to make additional promises. That is why even without liquidation risk, short-term funding is not the unique funding outcome.

the firm undertakes.

Most dynamic debt maturity models in continuous time tend to focus on refinancing policies, optimal leverage ratios, and target average debt maturity (see Leland (1994, and 1998)). He and Milbradt (2016) bring debt maturity to the forefront of these models. They emphasize the joint determination of default and maturity by showing that a firm actively manages maturing debt depending on the firm's distance to default. The firm issues short- or long-term debt and commits to a constant book leverage policy, *i.e.* maintains a constant aggregate face value of outstanding debt. Rollover losses arise as equity holders must absorb any cash flow shortfall when maturing bonds are refinanced when credit conditions deteriorate. The rollover losses feed back to the default decision by equity holders, leading to earlier default. Though we do not focus on the timing of default[8], our model has a similar feedback mechanism in which the anticipated short-term debt rollover losses induce equity holders to substitute toward more initial long-term debt issuance. Our model characterizes equity holders' optimal up-front debt financing strategy using collateral rather than a firm's existing or on-going debt management strategy. We view our paper as complementary.

Brunnermeier and Oehmke (2013) show that financial firms' inability to commit to a maturity structure leads short-term debt to dilute long-term debt. In their model with a fixed supply of assets, the firm increases aggregate debt liabilities when new debt is issued to repay expiring claims, which dilutes the per-claim value of existing debt. The equilibrium debt maturity structure unravels to inefficient short-term debt with costly liquidation. Equity holders cannot absorb losses in their model as they can in our model and He and Milbradt (2016). He and Xiong (2012b), with a fixed maturity structure, show how short-term debt can amplify default risks when liquidity risk is present because equity holders will default at earlier valuation thresholds. Default timing is fixed in our model, but maturity is allowed to adjust.

Lastly, Geanakoplos (2009) and He and Xiong (2012a) study debt financing in incomplete asset markets with heterogeneous agents. In their models, short-term debt is the unique equilibrium debt maturity because a sequence of short-term claims allows agents to take maximum leverage. The difference in their models is that all

[8]As explained later, while our model does allow us focus on the timing of default, we abstract away from default on short-term debt in the intermediate period to highlight that our mechanism is unrelated to existing hypothesis on agency problems or rent seeking behavior.

agents own risky assets, some of whom have higher valuations than others, and agents can borrow against the assets by issuing safe promises to obtain leverage. Issuing consecutive short-term claims allows optimists to borrow against the lowest value of the asset one period in the future. Though the price of the asset falls after one period of bad news (it increases after good news), it is still higher than the lowest possible asset payout in the final period. The asset's intermediate price and the lowest payout in the final period represent the sequence of promises optimists make when leveraging the asset with short-term claims. Issuing a long-term claim can only be levered using the lowest payout of the asset in the final period, which is always less than the price of the asset at an intermediate state.

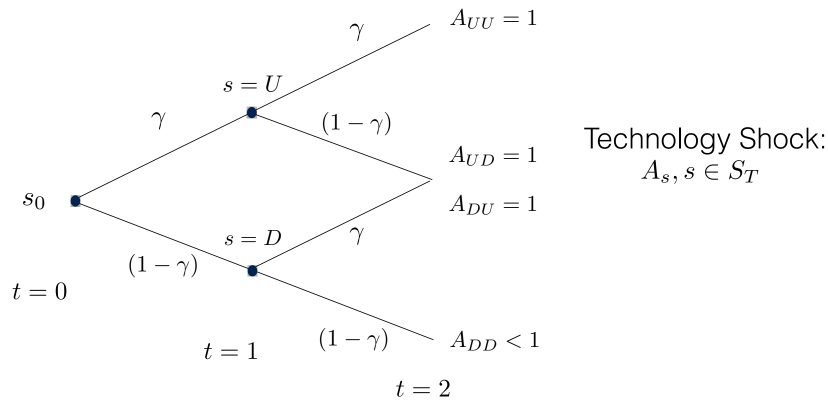
2 Model

2.1 Time and uncertainty

The model is a dynamic three-period version of FG (2016) with time $t = \{0, 1, 2\}$. Uncertainty is denoted by a tree of state events $s \in S$ with root s_0, intermediate states $s \in S$ that take values $\{U, D\}$, and a set of terminal nodes denoted $S_T = \{UU, UD, DU, UU\} \subset S$. Thus, the complete state-tree has elements $S = \{U, D, UU, UD, DU, DD\}$. Furthermore, each state $s \neq s_0$ has a unique predecessor denoted s^*. The timing of intermediate state realization U is denoted $t(U) = 1$ and terminal state UU as $t(UU) = 2$. Let state realization U be up or a "good" state and D be down or a "bad" state.

The economy receives a technology shock at $t = 2$, $A_s, s \in S_T$. The value of the technology shock is conditional on the information revealed at $t = 1$. We assume that good news at $t = 1$ always results in a good technology shock, whose value is normalized to 1: $A_s = 1, s \neq DD$. Bad news at $t = 1$ raises the uncertainty of the technology shock at $t = 2$, akin to "scary bad news" in Geanakoplos (2009). Specifically, the technology shock is good at terminal node $s = DU$, and bad at terminal node $s = DD$, $A_{DD} < 1$. Note that this uncertainty structure is also the same as in He and Xiong (2012) and Diamond and He (2014). Figure 1 depicts the economy's state tree.

Figure 1: Economy State Tree



2.2 Debt contracts

There is a single durable consumption good available in the economy at $t = 0$, which is the numeraire and treated as cash. There are two types of promises that can be made, each with different maturity. A promise that matures after one period is called short-term and a promise that matures after two periods is called long-term. All promises are non-contingent, and pay zero-coupons. Promises can be interpreted as debt contracts. Let the quantity of debt issued at any state and time be $q_s, s \in S/s_T$. The quantity of long-term debt issued at $t = 0$ is denoted q_0^{ℓ}. Short-term debt may be issued at $t = 0, 1$. The quantity of short-term debt issued at $t = 0$ is given by q_0^{ς} and the quantity of short-term debt issued at $t = 1$ is given by q_s^{ς}, $s = U, D$. Following much of the literature, we assume equal seniority between short- and long-term debt. Our key assumption is that all debt must be back by collateral. Collateral serves as the payment enforcement mechanism in the economy. Creditors have the right to confiscate debtor collateral up to the value of the promise but nothing more. The collateral rate of a debt contract is given by a delivery function, $d_{S_T}(\cdot)$. We return to the debt delivery functions in section 2.4.

2.3 Agents

Firm

Firms represent our major departure from the C-models of Geanakoplos (2009), He and Xiong (2012), and FG (2008, 2010, 2012). We introduce an outside firm with an inter-period production technology. Specifically, we assume a single firm is owned and operated by a manager (equity claimant) with access to two-period decreasing returns to scale production technology.[9] The production function is denoted by $f(I_0; \alpha, A_s) = A_s I_0^{\alpha}$, $\alpha < 1$, $s \in S_T$, where I_0 is the amount of capital the manager raises and puts into production. We will see that the parameters A_{S_T} and α determine the fundamental collateral rate. We assume the firm owner has no cash endowment and does not generate cash flow at $t = 1$, and that new promises do not scale the project's original size.[10] As in Brunnermeier and Oehmke (2012) and He and Milbradt (2016), we assume the firm cannot commit to a maturity structure at $t = 0$. The firm chooses between the maturity profiles, q_0^{ℓ} and/or q_0^{s}, at $t = 0$.[11]

Investors

There exists at $t = 0$ a continuum of uniformly distributed investors with unit mass, $h \in H \sim U[0, 1]$, each of whom is endowed with a unit of the durable consumption good in all non-terminal states, $e^h, e_s^h, s \neq S_T$. The uniform distribution allows one to rank investors according to the likelihood each places on the subsequent state being good, denoted by h. Investors are risk-neutral, expected utility maximizers that consume at $t = 2$, and do not discount the future. Without loss of generality, we assume investors have different priors (see Fostel and Geanakoplos (2015)). The specific reason for heterogeneity is not relevant. One could equally assume investors differ in a measure of risk aversion; have different endowments across states, which produces different marginal utilities across states; or have different degrees of "patience." The critical assumption is the heterogeneity of marginal utilities across investors. We choose to think about beliefs because it is most familiar in these models.

Investors also have access to riskless a storage technology. Investors form portfolios consisting of cash and promises purchased from firms. Let $\overline{h_s}, s \in S_T$ denote the product of all h along the path from 0 to s_T. For example, investor $\mathring{h}$ expects that

[9] The firm owner also being the firm manager immediately removes the agency problem from our setup.

[10] Alternatively, one could assume that there is complete limited commitment at the interim in which no cash flows can be verified at a reasonable cost so debt repayments cannot come from cash flow.

[11] We restrict the analysis to debt issuance and do not allow for equity financing. This allows us to focus the analysis entirely on the endogenous composition of debt issuance in terms of the debt liability structure. Incorporating equity is a natural extension to the model.

all promises default at $s = DD$ with probability $\left(1-\mathring{h}\right)^2$ and are repaid in all other states, $s \in S_{T/DD}$, with probability $1-\left(1-\mathring{h}\right)^2$. The von-Neumann-Morgenstern preferences are given by:

$$U^h\left(x_{UU}, x_{UD}, x_{DU}, x_{DD}\right) \;\; = \sum_{s\in S_T} \overline{h_s} x_s \tag{1}$$

A natural interpretation of investors in our model of corporate debt financing would be insurance companies or pension funds who are buy-and-hold investors with long-dated liabilities. Moreover, these institutional investors receive funds via premiums and 401k contributions over time, rather than receive all their capital at once, which is captured by the state-contingent endowment process.[12]

2.4 Debt repayment

In this section we describe the maturity specific debt repayment functions that all agents rationally anticipate to price risky promises. We assume that agents cannot be coerced into honoring promise except for having collateral seized. All debts are collateralized by future output. We assume all agents know how the collateral cash flows depend on future states, and that all collateral value is pledgeable. Hence, we are assuming there are no collateral "cash-flow problems" (see FG (2016).[13] For simplicity, we normalize the face value of all debt contracts to 1.

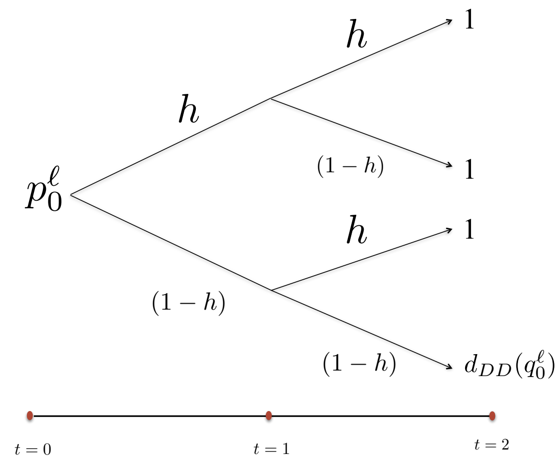
Short-term Debt

Short-term debt repayment is conditional on whether the firm rolls over debt at $t = 1$. Specifically, short-term debt will be "safe" and provide money-like claims if it is always rolled over; otherwise, short-term debt will be risky due to liquidation at $t = 1$. We shut down the risky short-term debt channel for two reasons: 1) to explicitly differentiate our mechanism from the control rent and inefficient liquidation

[12]This interpretation is quite natural since insurance companies and pension funds currently hold about \$4.4 trillion in outstanding U.S. corporate debt (37% of all outstanding). Moreover, mutual funds currently hold about \$1.86 trillion in outstanding corporate debt (15% of all outstanding). In sum, these three classes of investors hold over 50% of all outstanding U.S. corporate debt, which makes them the most likely marginal buyers in the primary corporate debt market. The remainder of debt is spread across 14 other broad classes of bond holders (Source: Flow-of-Funds L.213).

[13]Traditional macro/finance models such as Kiyotaki and Moore (1993) assume that creditors can confiscate land, but not the fruit produced by the land. Corporate finance models following Holmstrom and Tirole (1997) assume an information asymmetry between borrowers and lenders. Borrowing too much in these models reduces cash-flow and reduces incentives to work hard to produce good cash flows.

Figure 2: Long Term Financing



channels of Diamond (1991) and Houston and Venkataraman (1994), and 2) the empirical evidence of liquidity risk on maturity choice for rated corporates in general, and highly rated corporates in particular, is at best weak.

Let $d_s\left(q_0^{\varsigma}\right)$ describe debt delivery as a function of short-term debt issued at $t=0$. Short-term debt delivery must be $d_s\left(q_0^{\varsigma}\right)=1$ when debts are rolled over and short-term prices $p_0^{\varsigma}=1$.[14] The firm must issue q_s^{ς} one-period debt contracts to rollover expiring claims in order to ensure their "safety." Failure to repay short-term debt tomorrow results in interrupted production and subsequent forfeiture and liquidation of all firm assets. Short-term debt issued at $t=0$ must be risky if there is a non-zero chance that assets are liquidated at $t=1$. We make distinct our "long-term" collateral story by assuming that the liquidation value of the firm, L, is sufficiently small that creditors will not issue risky short-term debt at $t=0$, though not necessary for multiple maturities to arise.[15] Safe short-term debt issued today derives from

[14]This implies that absent liquidation, all $t=0$ short-term bonds are financed at the risk-free rate, similar to Flannery (1986) and Diamond (1991). Moreover, FG (2012) show that the types of leverage contracts endogenously chosen by agents will be those characterized by increased uncertainty following bad news, which we take as given in the model.

[15]Note that the production process is two-period, which means that interrupting it one period early is assumed to be quite costly to creditors in terms of finding a second best use or replacing management to continue production. Multiple debt maturities will arise even with defaultable short-term debt issued at $t=0$ if the claims are priced sufficiently high. In such cases, the firm issues risky short- and long-term debt at $t=0$, and safe short-term debt at $t=1$.

the fact that investors and firms know that failure to repay tomorrow results in an equity claim equal to 0.[16] The only way the firm retains an equity claim in a terminal state is by repaying creditors. The firm uses its asset as collateral to issue the promises risky tomorrow that ensure full repayment of short-term claims issued today. Conditional on good news, short-term debt is rolled over at the risk free rate because the firm has sufficient collateral to repay debt at $S_T = \{UU, UD\}$. In this case, $d_s\left(q_U^{\varsigma}\right) = 1, s = UU, UD, DU$. Conversely, debt delivery is uncertain conditional on bad news. Specifically, we assume that all debts can be repaid at $s = DU$, and $d_{DU}\left(q_D^{\varsigma}\right) = 1$, but collateral is insufficient to honor obligations at $s = DD$ and $d_{DD}\left(q_D^{\varsigma}\right) = \frac{A_{DD} I_0^{\alpha}}{q_D^{\varsigma} + q_o^{\ell}} < 1$.[17] In sum, debt delivery is given by

$$d_s\left(q_{s^*}^{\varsigma}\right) = \begin{cases} 1, & s \in S_{T/DD} \\ \frac{A_{DD} I_0^{\alpha}}{q_D^{\varsigma} + q_o^{\ell}}, & s = DD \end{cases}. \tag{2}$$

The sequence of short-term debt contract payouts is depicted in figure 3.

Long-term Debt

Because of equal seniority, long-term and short-term debt deliver the same units of consumption in all states. Equilibrium debt prices, p_0^{ℓ} and p_D^{ς}, will differentiate the expected returns on different maturities. Let $d_s\left(q_0^{\ell}\right)$ denote the long-term debt deliver function, which depends on the final-period state, $s \in S_T$. When short-term debt is issued and successfully rolled over, both long- and short-term deliveries are given by (2), $d_s\left(q_0^{\ell}\right) = d_s\left(q_{s^*}^{\varsigma}\right)$, or generically $d_s(\cdot), s \in S_T$.

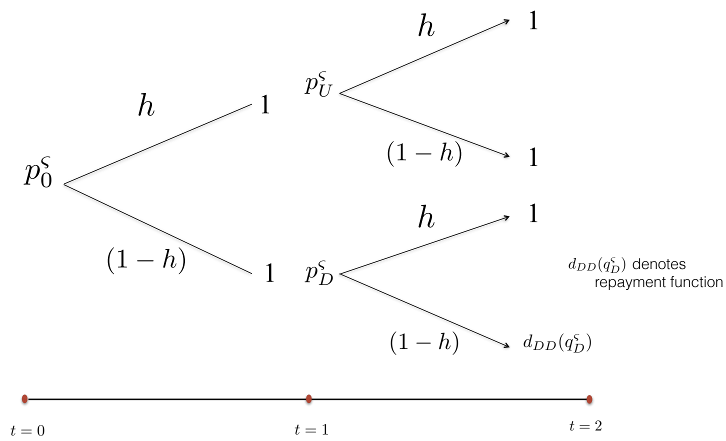
2.5 Firm maximization problem

The firm chooses an initial investment amount I_0 and debt maturity structure ρ to maximize expected profits (equity) denoted $E_0[\pi] = \Pi$, where ρ denotes the portion of investment capital that is raised by issuing long-term debt. The firm weighs the relative expected borrowing costs between the two debt maturities against the marginal product of capital to determine its debt liability structure. Let γ denote

[16] Implicitly, we are assuming absolute priority holds and that all debts are accelerated, similar to Brunnermeier and Oehmke (2012).

[17] This is simply restating absolute priority ala Merton (1974) via a collateral constraint. Equity receives nothing when debt holders are not repaid *ex post*, but collateral delivery is required to obtain debt *ex ante*. We will show that $A_{DD} < \alpha$ is sufficient for $d_{DD}(\cdot) < 1$, and consider this parametrization throughout the paper to focus on risky debt.

Figure 3: Short-Term Financing



the probability of good news in the following period.[18] Formally, the firm maximizes the following problem[19]

$$\begin{cases} \max\limits_{I_0,\rho} \prod = & \left\{ \sum_{s\in S_T} \overline{\gamma}_s \max\left[A_s I_0^{\alpha} - q_0^{\ell} d_s\left(q_0^{\ell}\right) - q_{s^*}^{\varsigma} d_s\left(q_{s^*}^{\varsigma}\right), 0\right] \right\} \\ \text{s.t.} & I_0 = p_0^{\ell} q_0^{\ell} + p_0^{\varsigma} q_0^{\varsigma} \\ & p_s^{\varsigma} q_s^{\varsigma} = q_0^{\varsigma},\ s = U, D \\ & 0 \le \rho = \frac{p_0^{\ell} q_0^{\ell}}{I_0} \le 1 \end{cases} \tag{3}$$

where $\overline{\gamma}$ denotes the product of all γ along the path from 0 to $s \in S_T$. At $t = 1$, in either state $s = \{U, D\}$, the firm must decide whether it is beneficial to roll-over the short-term component of its debt portfolio. The firm repays short-term debt holders by raising $p_s^{\varsigma} q_s^{\varsigma}$, $s = \{U, D\}$, for which it pays $q_s d_s\left(q_{s^*}^{\varsigma}\right), s \in S_T$ on maturity.

[18]We will show that γ does not determine then general existence of multiple maturities as an equilibrium outcome. The fact that γ is known to the firm and may not be equivalent to the marginal investor's expectation of good news is not completely without loss of generality. γ will determine the relative amount of long- versus short-term claims, $0 < \rho < 1$, that makeup the optimal debt liability structure. However, one can solve the model by restricting γ to almost surely equal the marginal buyer's expectation so that there is a "true" state probability. This approach will pin down a unique ρ for all A_{DD} rather than have a state-space consisting of (A_{DD}, γ)-pairs.

[19]The probability the firm believes that $s_2 = UU$ is given by γ^2 , $s_2 = UD$ is given by $\gamma(1-\gamma)$, $s_2 = DU$ is given by $(1-\gamma)\gamma$, and finally $s_2 = DD$ by $(1-\gamma)^2$. Also, here we are assuming the firm knows the true state probabilities and investors simply agree to disagree. We will show in Section 3.2 that this assumption is not crucial in determining an interior optimum. It will however, determine the tilt between long- and short-term debt.

The firm ultimately owes $\left(q_0^\ell + q_{s^*}^\varsigma\right) d_s(\cdot)$ at $t = 2$ when all short-term debt is rolled over, keeps the residual equity claim, but walks away with 0 at $t = 1$ if it does not repay short-term debt.

The maximization problem is subject to the following constraints: the amount of capital the firm can use for production has to be raised by issuing bonds at $t = 0$. Conditional on rolling over short-term debt at $t = 1$, the firm issues new short-term debt, $q_0^\varsigma = p_s^\varsigma q_s^\varsigma$, $s = \{U, D\}$. Lastly, the portion of the firm's investment that is raised through long-term debt, ρ, is naturally bound between 0 and 1. All debt deliveries are backed collateral output.

2.6 Investor maximization problem

We can now characterize the investors' budget sets. Given $t = 0$ debt prices, $\left(p_0^\ell, p_0^\varsigma, p_{s^*}^\varsigma\right)$, each investor, $h \in H$, chooses cash holdings, $\left\{x_0^h, x_{s^*}\right\}$, debt holdings, $\left\{q_0^{\ell,h}, q_0^{\varsigma,h}, q_{s^*}^\varsigma\right\}$, and final period consumption decisions, $\left\{x_s^h\right\}$, to maximize utility given by (1) subject to the budget set defined by:

$$
\begin{aligned}
B^h\left(p_0^\ell, p_0^\varsigma, p_{s^*}^\varsigma\right) \quad = \quad & \Big\{ \left(x_0, x_{s^*}, q_0^\ell, q_0^\varsigma, q_{s^*}^\varsigma, x_s\right)_{h\in H} \in R_+ \times R_+ \times R_+ \times R_+ \times R_+ \times R_+ : \\
& x_0^h + p_0^\ell q_0^{\ell,h} + p_0^\varsigma q_0^{\varsigma,h} = e_0^h, \\
& x_{s^*}^h + p_{s^*}^\varsigma q_{s^*}^\varsigma = e_1^h \\
& x_s^h = x_0^h + x_{s^*}^{h_1} + d_s\left(q_0^\ell\right) + d_{s^*}\left(q_0^\varsigma\right) + d_s\left(q_{s^*}^\varsigma\right), s \in S_T \Big\}.
\end{aligned}
$$

Each investor may use their initial cash endowment to purchase either type of debt at $t = 0$, and the endowment received at $t = 1$ to purchase short-term debt at $t = 1$. All cash that is not used to purchase debt is carried forward to consume at $t = 2$. All final period consumption comes from debt purchases and cash holdings.

2.7 Equilibrium

Equilibrium is a collection of debt prices, firm investment decision, investor cash holdings, debt holdings, and final consumption decisions

$$
\begin{gathered}
\left(\left(p_0^\ell, p_0^\varsigma, p_{s^*}^\varsigma\right), I_0, \left(x_0, q_0^\ell, q_0^\varsigma, x_s\right)_{h\in H} \left(x_1, q_{s^*}^\varsigma, x_s\right)_{h\in H}\right) \in \\
\left(R_+ \times R_+ \times R_+\right) \times R_+ \times \left(R_+ \times R_+ \times R_+ \times R_+\right) \times \left(R_+ \times R_+ \times R_+\right)
\end{gathered}
$$

such that the following are satisfied:

1. $\int_0^1 x_0^h dh_0 + p_0^\ell \int_0^1 q_0^{\ell,h} dh + p_0^\varsigma \int_0^1 q_0^{\varsigma,h} = \int_0^1 e_0^h dh$
2. $\int_0^1 x_{s^*}^h dh + p_{s^*}^\varsigma \int_0^1 q_{s^*}^{\varsigma,h} dh = \int_0^1 e_1^h dh$
3. $f_s(I_0) = \pi_s + q_{s^*} d_s\left(q_0^\ell\right) + q_{s^*} d_s\left(q_{s^*}^\varsigma\right)$
4. $I_0 = p_0^\ell \int q_0^{\ell,h} dh + p_0^\varsigma \int q_0^{\varsigma,h} dh$
5. $p_{s^*}^\varsigma \int_0^1 q_{s^*}^{\varsigma,h} dh = q_0^\varsigma$
6. $\pi(I_0, \rho) \geq \pi(\hat{I_0}, \rho)$, $\forall\, \hat{I}_0 \geq 0$ and $0 \leq \hat{\rho} \leq 1$
7. $\left(x_0^h, q_0^{\ell,h}, q_0^{\varsigma,h}, q_{s^*}^\varsigma, x_{s^*}, x_s^h\right) \in B^h\left(p_0^\ell, p_0^\varsigma, p_{s^*}^\varsigma\right) \Rightarrow U^h(x) \leq U^h\left(x^h\right),\ \forall h$

   $s \in S_T$

Conditions (1) and (2) state that all investor cash endowment at $t = 0, 1$ respectively, is used for consumption or debt purchases. The $t = 2$ goods market clearing condition states that all firm output goes toward repaying debt and firm profits as shown by condition (3). Condition (4) states that all of the capital raised by issuing both short- and long-term debt at $t = 0$ is used as input in final goods production. Condition (5) says all capital raised at $t = 1$ is used to fully repay $t = 0$ short-term creditors. Condition (6) states that the firm chooses investment to maximize profits, while condition (7) states that investors choose portfolios of debt and cash holdings to maximize their respective utilities given their budget sets.

3 Optimal debt maturity

We conjecture a candidate maturity structure and check to see if it is optimal. This entails determining whether or not the firm's debt liabilities consists of both long- and short-term debt, only long-term debt, or only short-term debt. We begin by establishing that a well-defined tradeoff between long- and short-term debt exists in this model. The main result is to show that issuing safe short-term debt today is in fact always credible under the model's assumptions, and is sufficient for multiple debt maturities to comprise the endogenous debt liability structure.

3.1 The tradeoff between long- and short-term debt

As in Brunnermeier and Oehmke (2012), we assume that all debts are accelerated if short-term debt is not rolled over at $t = 1$, and the investment project is fully liquidated to repay outstanding creditors, resulting in the expected value of equity being zero.[20] Short-term debt is rolled over if the firm can realizes profits at $t = 2$ after repaying both long- and short-term debt obligations with non-zero probability. We can focus our analysis conditional that $s = D$ because uncertainty is fully resolved and all debts are repaid at $s = U$.

The downside of short-term debt is the exposure to uncertain rollover costs at $s = D$. Specifically, short-term debt is subject to repricing at $s_1 = D$ where the firm must increase the outstanding face-value of short-term claims to rollover existing claims, $q_D^{\varsigma} = \frac{q_0^{\varsigma}}{p_D^{\varsigma}} > q_0^{\varsigma}$.[21] The benefit of short-term debt is that it will be fully collateralized if $s = U$, hence the firm borrows at the risk free rate for the duration of the project. Turning to long-term debt, the cost of long-term debt is that it is always *ex ante* risky because of insufficient collateral at $s = DD$, meaning the firm will always pay a positive credit spread in states it retains equity at $t = 2$. The benefit of long-term debt is that it insulates the firm from potentially higher rollover costs at $s = D$. Long-term debt borrowing cost and short-term rollover cost define the trade-off the firm faces when structuring its debt liabilities.

Equilibrium characterized by both long- and short-term debt contracts must have the expected cost of raising an additional unit of capital equalized across debt maturities. This intuition is given by the firm's first order conditions for an interior maximum:

$$\frac{\left[1-(1-\gamma)^2\right]}{p_0^{\ell}} = \frac{1}{p_0^{\varsigma}}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right] \tag{4}$$

$$\alpha I_0^{\alpha-1}\left[1-(1-\gamma)^2\right] = \frac{\rho\left[1-(1-\gamma)^2\right]}{p_0^{\ell}} + \frac{(1-\rho)}{p_0^{\varsigma}}\left[\gamma + \frac{\gamma(1-\gamma)}{p_D^{\varsigma}}\right]. \tag{5}$$

Equation (4) says the expected borrowing costs must be the same if debt maturity is interior. The corner solution for all short-term debt is given by $\rho = 0$, while $\rho = 1$ is the corner solution for all long-term debt. Equation (5) says that the expected

[20] Debt acceleration clauses are the most common covenant found in public debt indentures.

[21] This represents one key difference between our model and those of He and Milbrandt (2016) and the Leland (1994, 1998) models; The face-value of outstanding debt is not fixed in our model, $p_D^{\varsigma} < 1 \Rightarrow q_0^{\varsigma} \neq q_D^{\varsigma}$. Both debt prices and quantities are endogenous.

marginal product of capital in states where firm's retain equity, $\forall s \in S_T/S_{DD}$ with probability $1-(1-\gamma)^2$, must be equal to the maturity-weighted expected marginal cost of issuing debt. Simple inspection of (4) shows that $p_0^\ell > p_D^\varsigma$ is necessary for both long- and short-term debt to co-exist.[22]

Moving forward, it will be useful to define the endogenous collateral rate in (2). Using equations (4), (5), and the funding constraints and definition of ρ from (3), one obtains

$$d_{DD}(\cdot) = \frac{A_{DD}}{\alpha}\left(\frac{p_D^\varsigma}{(1-\rho)\,p_0^\ell + \rho p_D^\varsigma}\right) \tag{6}$$

and $d_s(\cdot) = 1, s \in S_{T/DD}$. First, notice that $A_{DD} < \alpha \implies d_{DD}(\cdot) < 1$. In other words, sufficient down-side risk, $A_{DD} < \alpha$, ensures that promises are made at an interest rate above the risk-free rate, $p_0^\ell, p_D^\varsigma < 1$. Second, note that $d_{DD}\left(q_0^\ell\right) = \frac{A_{DD}}{\alpha}$ if and only if long-term debt is the only risky promise made, $\rho = 1$, and there is no maturity mismatch. The collateral rate is determined purely by production fundamentals, A_{DD}, α, which we refer to as the asset's fundamental collateral capacity or collateral rate. Lastly, $d_{DD}(\cdot) < d_{DD}\left(q_0^\ell\right)$ when $0 < \rho < 1$. The collateral rate that investors anticipate, conditional on two periods of bad news, is lower if the firm issues both risky long- and short-term debt. The reason why investors recover less when short-term debt is risky is because short-term debt dilutes the per claim value investors receive when the risky interest rate rises at $s = D$. If prices fall, then the firm must issue more promises tomorrow than is issues today in order to keep its original promises safe. And for a fixed fundamental collateral capacity, issuing additional new promises means less recovery for existing claimants.

Proposition 1 $p_0^\ell, p_D^\varsigma < 1$ *if and only if* $A_{DD} < \alpha$, *long-term and short-term debt at* $s = D$ *are risky. If* $\left(q_0^\ell, q_0^\varsigma, q_s^\varsigma\right) > 0$, *then* $d_{DD}^\mu(\cdot) < d_{DD}^\ell\left(q_0^\ell\right)$ *and short-term debt dilutes the per claim value of existing debt claims at* $s = D$.

We can also define the firm's asset leverage. The loan-to-value, LTV, equals $\frac{I_0}{I_0^\alpha} = I_0^{(1-\alpha)}$. The margin for the loan, m, equals $1 - LTV = 1 - I_0^{(1-\alpha)}$. Leverage, l, equals $\frac{1}{m}$. Using (4) and (5) one obtains[23]

$$l = \frac{1}{1-\alpha p_0^{\ell *}(\cdot)} \tag{7}$$

[22]The firm would always choose short-term debt if it borrows risk-free at $t = 0$ *and* on better terms at $t = 1$ relative to issuing long-term debt.

[23]The expression can be written in terms of equilibrium risky short-term debt as well because in expectation the two risky prices must the same. But doing so is slightly more messy.

where $p_0^{\ell^*}(\cdot)$ is the equilibrium price of risky long-term debt as a function of parameters (A_{DD}, γ, α). The following relationship between the cost of debt financing and leverage is immediate.

Proposition 2 *In a collateral equilibrium, the least costly debt financing strategy corresponds to maximal leverge.*

Note that leverage is endogenous and pro-cyclical in the price of risky short-term debt at $t = 1$ because p_D^s is proportional to p_0^ℓ via (4).Equation (7) foreshadows how the firm should structure its debt liabilities; mainly, obtain risky debt financing at the lowest possible cost.

3.2 Equilibrium debt maturity

Full characterization of equilibrium requires pinning down marginal buyers. At initial node s_0 there will be a *marginal buyer*, h_0. Every agent $h > h_0$ will buy long-term debt and every agent $h < h_0$ will hold a portfolio of money-like securities. There will be an additional *marginal buyer* at $s = D$, h_D. All investors $h > h_D$ will purchase risky short-term debt at $t = 1$, and all agents $h < h_D$ will remain in cash. Figure 4 shows, for a given investment demand and collateral capacity, marginal buyer regimes for all long-term debt (in red), all short-term debt (in green), and a combination of long- and short-term debt (in black). The blue shaded area at $t = 0$ represents the portion of risk long-term debt that is substituted into safe short-term debt when the firm issues both debt maturities rather than only long-term debt. Similarly, the blue shaded region at $t = 1$ represents the risky short-term debt portion that is substituted into long-term debt at $t = 0$ when the firm issues both debt maturities rather than all short-term debt.

Marginal buyers price risky assets in expectation. For example, at $t = 0$, the expected return to holding a risky long-term bond to maturity must equivalent to holding cash, either because cash is the only other asset available in the economy, $(\rho = 1)$, or because safe short-term debt is also issued, $0 < \rho < 1$.

$$\frac{1 - (1 - h_0)^2 + (1 - h_0)^2 \, d_s(\cdot)}{p_0^\ell} = 1. \tag{8}$$

It is clear that with a fixed collateral capacity, defined within $d_s(\cdot)$, issuing more debt requires paying a higher interest rate as the equilibrium marginal buyer puts more weight on the worst case delivery. Likewise, increasing the collateral capacity of the firm's assets raises risky debt prices because all investors receive more delivery in the worst state. Similarly, at $t = 1$, a marginal buyer prices risky short-term debt by equating the expected return to holding cash:

$$\frac{h_1 + (1 - h_1)\, d_s(\cdot)}{p_s^{\varsigma}} = 1. \tag{9}$$

Issuing both debt maturities allows the firm to cater its risky debt claims to buyers across time with the highest marginal valuations to hold risk rather than issuing all risky claims at a single point in time and paying relatively higher borrowing cost.

Short-term debt will be "safe" at $t = 0$ if and only if it is unconditionally rolled over at $t = 1$. The condition for rollover is that the firm's equity value must be greater than or equal to zero after repaying both long- and short-term debts:

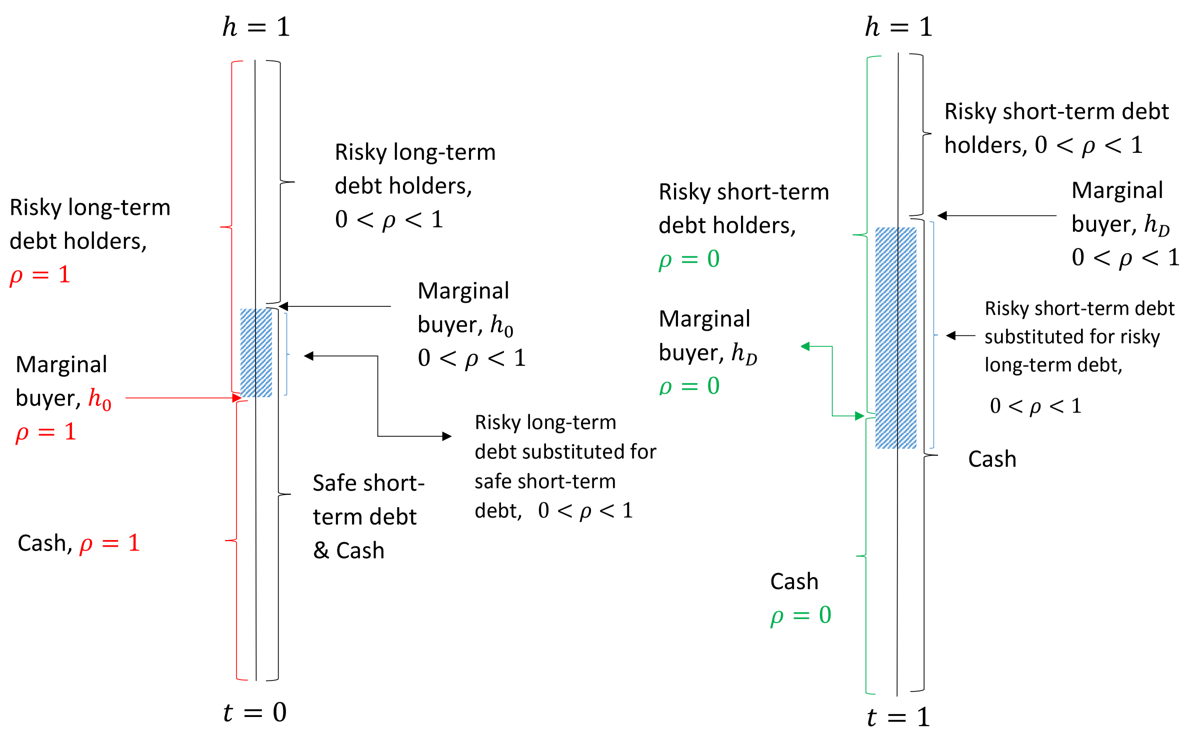
$$I_0^{\alpha} \geq q_0^{\ell} + q_0^{\varsigma}. \tag{10}$$

Note that state probabilities, γ, do not factor in this decision because the firm only retains equity when it fully repays all debts in coinciding states. It is clear that the price of short-term debt issued at $t = 0$ must be $p_0^{\varsigma} = 1$ if the firm continues production into $t = 2$.

Proposition 3 *Optimal debt financing is characterized by issuing safe short- and risky long-term debt at $t = 0$ and risky (safe) short-term debt in future bad (good) states. Leverage, investment, and production are all highest when multiple debt maturities comprise the debt liability structure.*

The intuition is that for any given investment amount and fundamental collateral capacity, $\left(I_0, \frac{A_{DD}}{\alpha}\right)$, safe short-term debt substitutes for risky long-term debt at $t = 0$, similar to what figure 4 shows. The firm issues fewer total long-term promises, which concentrates the remaining risky promises to investors most willing to hold risk, resulting in lower interest rates. Additionally, the promises needed to ensure short-term debt is rolled over at $t = 1$ are also concentrated to investors with high willingness to hold risk. The firm effectively reallocates its promises away from investors today, who require more collateral to borrow at a give interest rate, to investors tomorrow who require less collateral. For a fixed collateral rate, issuing risky debt with multiple maturities effectively places the risky promises to investors across time who require less collateral to finance debt at a given interest rate.

Figure 4: Marginal buyer regimes



A debt financing strategy of all short-term debt today will not be optimal because the expected rollover costs it must incur tomorrow to maintain *ex ante* short-term debt safety are too high. All of the firm's risky promises are issued at $s = D$. Given the fixed collateral rate, the marginal buyer requires a higher interest rate to finance debt. Substituting a portion of the promises needed tomorrow to long-term creditors today will save on short-term rollover costs because today's promises can be made to investors more willing to finance debt.

Proposition 4 *Short-term debt at $t = 0$ is safe if and only if*

$$\epsilon^* \equiv \alpha \frac{1 - \rho^*}{(1 - \alpha\rho^*)} \leq \frac{p_D^{\varsigma^*}}{p_0^{\ell^*}} < 1 \tag{11}$$

where $*s$ denote optimal allocations.

The gist of proposition 2 is that issuing safe short-term debt is possible as long as there is a balance between the price of risky promises issued today versus tomorrow, $\frac{p_D^{\varsigma^*}}{p_0^{\ell^*}}$. Equations (8) and (9) show that the ratio of risky debt prices, $\frac{p_D^{\varsigma^*}}{p_0^{\ell^*}}$, the firm balances over time is fundamentally related to how different investors price risky debt securities through marginal buyer priors, $h_0$and h_D. In equilibrium, risky debt prices are pinned down through market clearing; the supply of risky claims the firm issues must equal investor demand to hold risky debt: $\frac{\left(1-h_0^*\right)}{p_0^{\ell^*}} = q_0^{\ell^*}$ for long-term debt at $t = 0$ and $\frac{\left(1-h_D^*\right)}{p_D^{\varsigma^*}} = q_D^{\varsigma^*}$ for risky short-term debt issued at $t = 1$. In short, the more risky claims the firm issues at a single point in time, the lower the price it fetches for its risky promises. If it becomes too expensive to rollover short-term debt (issue long-term debt,)–$p_D^{\varsigma^*}$ is low (high) relative to $p_0^{\ell^*}$–then the firm should switch to all long-term (short-term) funding.

Before continuing, we draw the key distinction between our model and the models of Geanakoplos (2003 and 2009), Fostel and Geanakoplos (2008, 2010), and He and Xiong (2012a). In these models, all agents are endowed with both a risk-less and risky asset. Some agents (optimists) want to hold more risky assets than others due to different marginal utilities. Leveraged optimists buy all the risky assets by issuing riskless promises equal to the risky asset's value in the worst state. Issuing short-term claims allows agents to borrow against the asset's intermediate-state and terminal-state value, but long-term claims only allow agents to borrow against the terminal-state value. Thus, for optimists who price the asset in equilibrium, short-term debt always dominates long-term debt. These models are best suited to describe debt financing of financial assets for which the use of leverage is paramount. Banks, hedge funds, and institutional investors typically use leverage to make their asset purchases.

By contrast, the "firm" in our model is endowed with a risky production technology and issues debt claims using its technology as collateral to produce and consume, while investors have riskless assets that they use to purchase firm debt. Optimists use their riskless asset to buy risky debt claims. The firm maximizes its equity value by concentrating its risky claims to optimists. Using multiple debt maturities allows the firm to smooth debt financing cost across time by issuing risky claims to optimists rather than consolidating costs into a single maturity bucket that places debt into the hands of less optimistic investors. We view our model of collateralized production as one that fits with how large corporations fund their real investment and growth projects.

With this distinction, proposition 3 implies that safe short-term debt should be used *in conjunction* with risky-long term debt because it will help lower aggregate risky financing costs. This intuition rationalizes the existence of corporate commercial paper (CP) programs. In fact, through the lens of our model, short-term CP are the safe money-like claims issued at $t = 0$. The CP issuance is refinanced by a potentially risky debt issuance at $t = 1$. This interpretation is consistent with the "bridge financing" story of Kahl, Shivdasani, and Wang (2015), but is fundamentally based on the allocation of promises across time backed fixed collateral. Moreover, the fact that issuing safe claims is sufficient for an interior debt maturity structure contrasts the liquidation risk story underpinning Diamond (1991) and Houston and Venkataraman (1994). By extension, firms obtain more leverage against their assets when issuing multiple debt maturities. For a given collateral capacity, more leverage leads to more borrowing, investment, and production.

Underlying our collateral mechanism is a substitution effect between short- and long-term debt. One can see the substitution effect by examining short-term debt's contribution to the overall value of the firm. Specifically, the output attributable to rolling over short-term debt at $t = 1$ is given by $I_0^\alpha - (\rho I_0)^\alpha = I_0^\alpha (1 - \rho^\alpha)$. This represents the "hypothetical" amount of output that is generated by rolling-over short-term debt.[24] Whether or not the additional output covers the rollover costs is

[24]We say this is hypothetical because if the firm did not rollover its short-term debt then it would

given by $I_0^\alpha(1-\rho^\alpha) \geq q_0^s$. When this condition is satisfied, it means that short-term financing increases the expected value of equity–rollover gains rather than losses–in addition to reducing risky long-term financing costs. When the condition does not hold it means the gains to equity derive exclusively from reducing risky long-term borrowing costs, which outweigh the rollover losses. We show in the appendix, in general, the condition does not hold for low values of A_{DD} but always holds for high values of A_{DD}. The intuition is that there is more down-side risk when A_{DD} is low, which makes the expected cost of rolling over short-term debt more expensive than the production benefits, which are determined by α. The only "benefit" of using short-term financing with high down side risk is entirely attributable to the *substitution effect* of reducing the amount risky long-term promises issued and the associated financing costs. Alternatively, less down-side risk lowers short-term expected rollover costs at $t = 1$. Not only does using short-term debt reduce the amount of long-term debt issued at $t = 0$, but the attractive expected refinancing terms available at $t = 1$ serve to increase the expected value of equity at $t = 2$. For intermediate values of A_{DD}, the condition depends on the value of γ. We return to effects of γ after the comparative statics discussion in section 3.3.

Proposition 5 *In a collateral economy with multiple debt maturities, the substitution effect of safe short-term debt always serves to reduce long-term borrowing costs.*

Figure 5 shows how γ and A_{DD} determine when short-term debt leads to rollover losses–the substitution effect is the only benefit of short-term debt–versus when issuing short-term debt leads to rollover gains.[25] The white region represents the parameter values for which there are short-term rollover losses and the gray region are the parameters for which there are short-term rollover gains. There are threshold values of $A_{DD} = \left\{\underline{A}_{DD}, \overline{A}_{DD}\right\}$ for which $A_{DD} < \underline{A}_{DD}$ always leads to rollover losses and $\overline{A}_{DD} < A_{DD} < \alpha$ always leads to rollover gains.

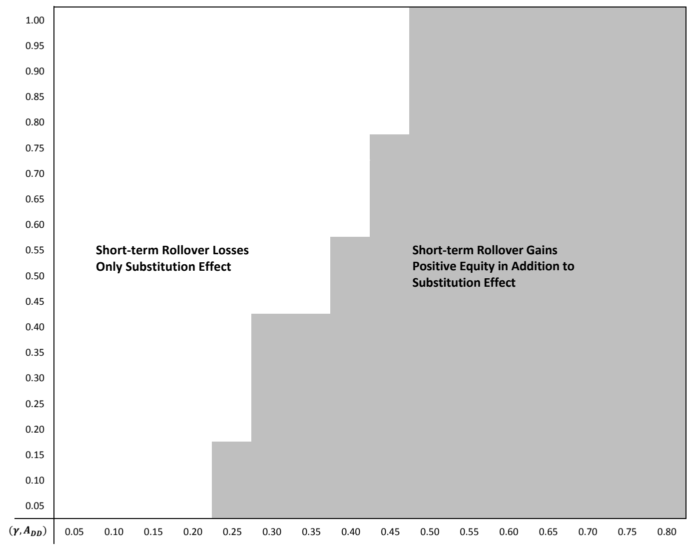
3.3 Debt maturity optimization and comparative statics

This section briefly discusses the model's comparative static results related to how the maturity profile is optimized toward long- or short-term debt depending on model

default altogether and not produce anything at $t = 2$. Thus, we think of this as what the firm adds to its output by issuing risky short-term debt at $t = 1$ to rollover existing claims.

[25]We arbitrarily choose $\alpha = 0.8$ for the exposition and numerical exercise section 4.3. The qualitative results do not depend on α.

Figure 5: Economy State-Space



parameters. The predictions of our model are broadly consistent with existing empirical studies, but are based on using collateral to make promises across time rather than asymmetric or private information.

Let $0 < \rho^* (\alpha, \gamma, A_{DD}) < 1$ denote the equilibrium amount of long-term debt issued for any given set of state parameters. Specifically, γ is the likelihood that good news arrives in the following period, from the firm's perspective. A_{DD} determines the amount of collateral the firm can pledge at $s = DD$ and is a measure of down-side risk, while α is the returns to scale parameter.

More short-term debt is issued the more likely good news arrives in $t = 1$, $\frac{\partial \rho}{\partial \gamma} < 0$. The reason is that the likelihood of rolling over short-term debt at the risk-free rate increases, which lowers expected rollover costs relative to long-term funding. We can interpret γ as a measure of management "optimism" which is consistent with the empirical findings of Landier and Thesmar (2008) and Graham et. al (2013) that management optimism leads to more short-term debt issuance, controlling for firm risky factors and leverage.

More short-term debt is issued the more collateral the firm can pledge at $s = DD$, $\frac{\partial \rho}{\partial A_{DD}} < 0$. The reason is that risky short-term debt prices at $t = 1$ are more responsive to movements in A_{DD} than risky long-term debt prices at $t = 0$. To see why, consider any given investor, h. This investor puts more weight on $s = DD$ at time $t = 1$ than she does at $t = 0$ due to scary bad news, $(1 - h) > (1 - h)^2$. The value of an investor's claim at $s = DD$, irrespective of maturity, is the delivery rate given by (6). Investor h therefore values the recoverable claim more at $t = 1$ than at $t = 0$. The interpretation is that more short-term debt is issued the higher are expected cash-flows, or the lower is down-side risk, because collateral capacity is expanded. More collateral capacity reduces the expected rollover costs of short-term financing.

Lastly, there are two interpretations for α: 1) a measure of firm productivity (higher curvature), and 2) the firm's "growth option." The more productive the firm, the more long-term debt the firm issues. The reason is related to the observations in Diamond and He (2014). Mainly, the value of long-term debt responds more to firm fundamentals than short-term debt when bad news increases uncertainty relative to good news ($p^{\varsigma}_U = 1, p^{\varsigma}_D < 1$). All uncertainty is completely resolved at $s = U$ in our model. By extension, there is more uncertainty at $s = D$. Changes in fundamentals affect short-term debt values only at $t = 1$ when short-term debt issued at $t = 0$ is risk free. Conversely, changes in fundamentals always affect risky long-term debt. Thus, $\left(\frac{\partial \rho}{\partial \alpha}\right) < 0$ because the firm is more productive as $\alpha \downarrow$.

The returns to scale parameter α is also a measure of the firm's "growth option." Empirical studies measure growth options as the market-to-book value of assets. In the model, the market value of the firm's assets is simply the amount it produces because there is only one asset whose price is normalized to 1. The book value of the firm's asset is the amount of capital it raises to produce, or the book value of its liabilities. The market-to-book value of the firm is given by[26]

$$\text{market-to-book} = \frac{I_0^{\alpha}}{I_0} = I_0^{(\alpha-1)} = \frac{1}{\alpha p_0^{\ell}\left(\alpha, A_{DD}, \gamma\right)}. \tag{12}$$

Notice that the growth option of the firm is inextricably linked to the exogenous parameters of the model through the market price of debt. Therefore, growth options are endogenously determined along with the firm's maturity choice and leverage through an asset's fundamental collateral capacity. Empirical treatment of the growth options as exogenous to leverage and maturity choices is not justified within

[26] We use the first order conditions (4) and (5) to derive the market-to-book in terms of the long-term bond price, p_0^{ℓ}. It can also be expressed in terms of short-term bond prices since the expected costs across maturities must be the same in an interior maturity equilibrium.

the framework of our model. The joint endogeneity of growth options and maturity choice may help explain why the empirical literature reports mixed results (Barclay and Smith (1995) and Guedes and Opler (1996) find that growth options and maturity are negatively related. Stohs and Mauer (1996) and Johnson (2003) find a positive relationship, while Billet et. al. (2007) find no relationship when controlling for covenants).

To sum up,

Proposition 6 *Debt maturity is optimized more long-term:*

- *the lower the likelihood of good states, low γ,*
- *the lower are expected cash-flows or the higher is down-side risk, low A_{DD}.*

We now return to the substitution effect of short-term debt for intermediate values of A_{DD} as a function of γ. Return to figure (5), which shows that the threshold value of A_{DD} above (below) which there are rollover gains (losses) from issuing short-term debt is weakly increasing in γ. Low γ implies the down-side risk of using short-term debt is more likely to materialize tomorrow. From the comparative statics above, the firm issues less short-term debt (reduces its short-term promises) at $t = 0$ to reduce its expected rollover costs. The fall in expected rollover costs means that the range of collateral values (the set of A_{DD} for a fixed α) for which rollover gains materialize expands. The main principle is that reducing the amount of promises made tomorrow in bad states reduces the equity dilution effect of short-term debt because rollover costs will be relatively low. By contrast, more promises made tomorrow lead to higher rollover costs and stronger debt dilution.

4 Protected debt with endogenous maturity

In this section we show that protective debt covenants are redundant when collateral is required to issue debt. To anticipate the result, protecting long-term debt raises any individual investor's expected per claim value. The firm responds by re-optimizing its maturity more toward the cheaper protected debt and away from the more expensive unprotected claim. The re-optimized firm changes the relative supply of different debt maturities it offers, which brings relative equilibrium prices back to pre-protected levels. No *real* outcomes are affected when the firm simply substitutes one maturity for another, because the covenant does not affect the fundamental collateral capacity of the firm's asset.

Our treatment of protected long-term debt can be thought either as an explicit collateral pledge or earmark, or the inclusion of a negative pledge covenant that explicitly spells out how long-term debt is secured from short-term debt dilution. The benefit of thinking about negative pledge covenants, as detailed below, is two fold: 1) negative pledges are among the most common covenants found in public debt indentures, 2) given their prominence, surprisingly little is known in the academic literature of their impact. We thus attempt to fill this void with the support of strong practical relevance.

4.1 Protected debt and the negative pledge covenant

We begin be describing the negative pledge covenant. Negative pledges are among the most common covenants found in public debt indentures and widely recognized by the law and economics profession (see Bjerre (1999), Wood (2007, 2008)). The covenant stipulates that the firm cannot issue secured debt in the future without securing the current debt issue. For example, Billet et. al. (2007) classify negative pledge covenants as "Secured Debt Restrictions" because they restrict the security of future debt issues. Table III in their paper shows that negative pledges are typically the 3rd or 4th most common covenant, behind cross default or acceleration, asset sale, and merger clauses. Negative pledges are more common than leverage, dividend, and share repurchase restrictions. Table 1 gives a general sense for the basic statistics on types of bonds that contain a negative pledge covenant. They are more prone in medium-to-long-term non-financial corporate indentures.

We motivate the negative pledge as follows. In our model, as in Brunnermeier and Oehmke (2013) and He and Milbradt (2016), short-term debt dilutes the value of long-term debt, even when all debts have equal seniority *ex ante.*[27] Short-term debt issued at $t = 1$ is collateralized at $s = DD$ by output at $t = 2$ but so is long-term debt issued at $t = 0$. The effect of short-term debt on long-term debt is that each long-term investor receives less per unit debt owned in default the more short-term

[27]See equation (6) in section 3.2.

Table 1: Negative pledge covenant

	Negative pledge covenant	
	Yes	No
Non-financial	14,783	11,424
Financial	3,117	4,825
< 5yr	2,244	2,376
5yr - 30 yr	15,284	13,401
Total	17,900	16,249

debt the firm needs to rollover (the same pie is split between more pieces).[28]

A portion of long-term debt buyers at $t = 0$ will not provide short-term rollover financing at $t = 1$, i.e. $h_0 < h_D$. Therefore, long-term investors $h_0 < h < h_D$ cannot be assured that the firm will be able to secure funding and proceed to $t = 2$. Long-term creditors can demand that their debts are secured via a negative pledge. As we show below, the negative pledge thus stipulates *ex ante* exactly what pieces (assets) long-term creditors receive, which effectively makes the collateral value of long-term debt the same as if the firm were funded only with long-term debt.

We assume that the negative pledge ensures that ρ portion of the firm assets are used as collateral exclusively for long-term funding, irrespective of short-term debt financing at $t = 1$. These assets cannot be used as collateral for short-term debt without violating the pledge and opening the firm up to costly litigation. The remainder of the assets, $(1 - \rho)$, are used as collateral to secure short-term rollover financing.

With the covenant, the recovery values given by (2) now become

$$\begin{cases} d_{DD}\left(q_0^\ell\right) = & \frac{\rho A_{DD} I_0^\alpha}{q_0^\ell}, \text{ long-term recovery} \\ d_{DD}\left(q_D^\varsigma\right) = & \frac{(1-\rho) A_{DD} I_0^\alpha}{q_D^\varsigma}, \text{ short-term recovery} \end{cases}.$$

The collateral, or future output, is split between long-term creditors protected by the pledge and short-term creditors who fund the short-term debt rollover at $t = 1$.

[28]The necessary condition for an interior optimum maturity structure, $p_0^\ell > p_D^\varsigma$ is sufficient for short-term debt to dilute the per claim value of long-term bonds.

Using the first order conditions for an interior maximum, (4) and (5), along with the funding constraints in program (3) that relate q_0^ℓ and q_1^ς to I_0 and ρ, the debt delivery functions can be written as:

$$\begin{cases} d_{DD}\left(\hat{q}_0^\ell\right) = & \frac{A_{DD}}{\alpha} \\ d_{DD}\left(\hat{q}_D^\varsigma\right) = & \frac{A_{DD}}{\alpha}\left(\frac{p_D^\varsigma}{p_0^\ell}\right) \end{cases}. \tag{13}$$

With the negative pledge, the maturity specific collateral rates in (13) behave as if $\rho = 1$ for long-term debt and $\rho = 0$ for short-term debt in (6), even though $0 < \rho < 1$.

Proposition 7 *Secured long-term debt mitigates short-term debt dilution. Any given investor is willing to pay more for secured long-term debt than unprotected debt subject to dilution.*

An immediate implication of proposition 6 is that the firm's debt maturity will be optimized more toward long-term financing when long-term creditors are protected from dilution.

Corollary 1 *The debt maturity mix is optimized more long-term when long-term creditors are protected with secured covenants.*

The result that the firm substitutes away from short-term debt toward more protected long-term debt is consistent with the empirical findings of Billet et. al. (2007). Interestingly, substitution effects arise with collateral purely through relative prices and not through reduced agency conflicts. Instead, it is the ability of the firm to split its collateral to back different debt maturities that generates the substitution effect. Covenants that reallocate scarce collateral lower protected debt prices and incentivize shifting risky debt issuance more heavily toward the protected maturity.

The general equilibrium effects are more subtle. Specifically, the firm increases the supply of risky long-term bonds it issues but reduces the supply of risky short-term bonds. In equilibrium, the relative prices of the two debt maturities must be equivalent in expectation (see equation (4)). The firm substitutes between maturities to the point where the relative price difference between risky long- and short-term debt are the same. There is no real effect on investment, output, or production, because the fundamental collateral capacity of firm assets is unchanged; collateral is simply is reallocated across the suppliers of credit.

Proposition 8 *In a collateral equilibrium with risky debt and no other frictions, secured debt covenants do not affect real outcomes.*

4.2 Numerical example

This section provides a numerical illustration of the economy with and without the negative pledge covenant. We choose the following parameters $\{\alpha = 0.8, 0 < A_{DD} \leq \alpha, 0 < \gamma \leq 1\}$. Figure (6) shows the equilibrium marginal investor regime for $A_{DD} = 0.5$ and $\gamma = 0.8$. What is important to note is that all risky debt financing is done by relative optimists. At $t = 0$ optimists use their cash to buy long-term bonds while relative pessimists buy safe short-term bonds. At $t = 1$, all risky short-term debt is also purchased by subset of the optimists. The more long-term debt the firm issues, the more investors it must seek to finance its risky long-term debt issuance pushing the marginal buyer at $t = 0$ down further. Issuing safe promises to relative pessimists at $t = 0$ enables the firm to issue its risky long-term promises to more optimists at $t = 0$ and roll those claims through relative optimists at $t = 1$.

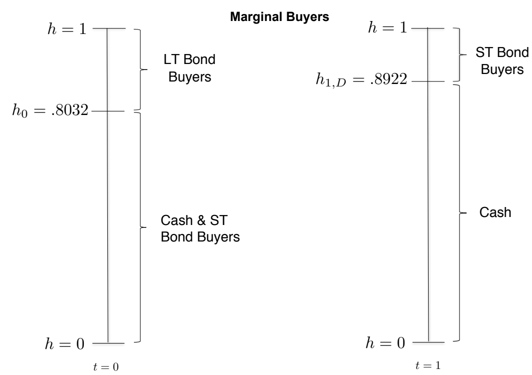
Table 2 highlights the major effects of the secured debt covenant for various $(A_{DD,}\gamma)$-pairs. The top (bottom) panel contains the endogenous variables for the economy with (without) the covenant. The numbers in red highlight the key changes. First, note that all debt prices, investment levels and profits are unchanged across the two panels. More (Less) long-term (short-term) debt is issued in the economy with the covenant. The covenant simply tilts the maturity through the relative number of risky long-term promises made, $\rho \uparrow$. The firm substitutes away from short-term debt.

The last set of results are the comparative statics for changes in down-side risk, A_{DD}, and good state probability, γ. The first two rows of either panel show how variables change as A_{DD} decreases for a fixed γ, while the third row shows change as γ falls for the same A_{DD} as the first row. First consider risky debt prices, p^{ς}_{D} and p^{ℓ}_{0}. More down-side risk at $t = 2$ lowers all risky debt prices, resulting in lower investment and profits. The firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$. Second consider a decrease in good state probability for the same A_{DD} as in the first row. The bottom row shows that the firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$, resulting in lower long-term debt prices, but higher

Table 2: Endogenous Variables

Covenant	p_0^ς	p_D^ς	p_0^ℓ	q_0^ς	q_D^ς	q_0^ℓ	I_0	ρ	Π
(A_{DD},γ)=(.5, .8)	1	.941	.989	.145	.154	.167	.311	.533	.075
(A_{DD},γ)=(.2, .8)	1	.894	.980	.136	.153	.163	.297	.539	.072
(A_{DD},γ)=(.5, .5)	1	.957	.985	.107	.112	.199	.304	.646	.057
No Covenant	p_0^ς	p_D^ς	p_0^ℓ	q_0^ς	q_D^ς	q_0^ℓ	I_0	ρ	Π
(A_{DD},γ)=(.5, .8)	1	.941	.989	.149	.158	.163	.311	.519	.075
(A_{DD},γ)=(.2, .8)	1	.894	.980	.138	.154	.162	.297	.534	.072
(A_{DD},γ)=(.5, .5)	1	.957	.985	.113	.115	.197	.304	.638	.057

Figure 6: Regime: Portfolio - Rollover



risky short-term debt prices. The firm also invests less and is less profitable. The comparative statics confirm the major predictions of the model.

5 Conclusion

This paper characterizes optimal debt financing in an economy where borrowers must use collateral to borrow and investors are heterogeneous. For a fixed collateral rate, issuing both long- and short-term debt is generally the least costly debt financing strategy. Issuing risky debt at various points in time smooths financing cost relative to issuing risky securities all in one time period. The model rationalizes why large corporates use commercial paper as bridge financing to finance long-term investment projects because it reduces the amount of risky long-term promises made. Further, the model predicts that firms will use more short-term debt when managers operating in shareholders best interest are optimistic about investment returns, or when expected cash flows are high, or down-side risk is low. We also show how growth options and leverage are endogenous to the firm's debt maturity choice because the price of the securities issued are affected by maturity, which in turn affects investment returns. Finally, we show how protective debt covenants found in long-term debt contracts can prevent dilution and increase the value of long-term claims, and lead to more long-term financing. However, protective covenants are redundant in general equilibrium with collateral requirements because they doe not change the collateral capacity of the asset backing risky promises.

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A Appendix Omitted Proofs

Proof of Proposition 1: Suppose $A_{DD} = \alpha$. Issuing only long-term debt permits financing at the risky free rate, $p_0^\ell = 1$. Short-term debt co-exists iff $p_D^\varsigma = 1$ as well because otherwise the l.h.s of (4) fails to equal the r.h.s. Maturity becomes irrelevant if there is no downside risk to collateral. Thus, $A_{DD} < \alpha$ being sufficient for $p_0^\ell < 1$ is also sufficient for (6) to be less than 1 because $p_D^\varsigma < p_0^\ell < 1\ \forall 0 < \rho < 1$. Thus, it is clear from (6) that $d_{DD}\left(\cdot\right)\big|_{0<\rho<1} < d_{DD}\left(q_0^\ell\right)\big|_{\rho=1}$ for all (A_{DD}, α)-pairs and risky short-term debt dilutes existing long-term debt. *Q.E.D.*

Proof of Proposition 3: We show for any investment amount I_0, issuing $q_0^\ell > 0$ and $q_D^\varsigma > 0$ is cost reducing relative to either $q_0^\ell = 0$ or . First, Note that (4) and (5) can be combined to express the firm's marginal product equal to either only the marginal cost of long-term debt or the marginal cost of short-term debt reflecting the fact that an interior maximum must be characterized by maturity cost equivalence at the margin. Suppose all short-term debt is rolled over so that $p_0^\varsigma = 1$ always. Suppose maturity is irrelevant, and the firm can obtain the same terms of financing all long-term or via interior solution. Let I_0^* be the optimal investment amount for some parameter set $\Gamma\left(\alpha, A_{DD}, \gamma\right)$. The firm is indifferent to raising I_0^* by issuing all long-term debt, $Q = \tilde{q}_0^\ell$, at price $\tilde{p}_0^\ell$ or to issuing both long- and short-term debt, $Q = \hat{q}_0^\ell + \hat{q}_0^\varsigma$, at prices $\hat{p}_0^\ell$ and $\hat{p}_0^\varsigma = 1$. Clearly it must be the case that $\tilde{q}_0^\ell > \hat{q}_0^\ell$, $\forall \hat{q}_0^\varsigma > 0$, and since the firm takes prices as given, it must be the case that $\tilde{p}_0^\ell > \hat{p}_0^\ell$. Market clearing–the supply of financing equals the firm's demand for financing–for all long-term debt is given by $\left(1 - \tilde{h}_0\right) = \tilde{p}_0^\ell \tilde{q}_0^\ell = I_0^*$ and for both long- and short-term debt by $\left(1 - \hat{h}_0\right) + \left(1 - \hat{h}_D\right) = \hat{p}_0^\ell \hat{q}_0^\ell + \hat{p}_D^\varsigma \hat{q}_D^\varsigma = I_0^*$. Equating the market clearing conditions for the same I_0^* gives $\left(1 - \tilde{h}_0\right) = \left(1 - \hat{h}_0\right) + \left(1 - \hat{h}_D\right)$. This can only hold if $\hat{h}_D = 1$ meaning that $\hat{q}_D = 0$–no short-term debt is issued–or if $\tilde{h}_0 < \hat{h}_0$–the marginal long-term bond buyer in an interior solution is more optimistic than the marginal bond buyer in the corner solution. The more optimistic the investor, the higher the price she is willing to pay $\implies \tilde{p}_0^\ell < \hat{p}_0^\ell$, which contradicts $\tilde{q}_0^\ell > \hat{q}_0^\ell$, $\forall \hat{q}_0^\varsigma > 0$. The same logic will also show that the firm can never be indifferent between all short-term financing and a combination of short- and long-term. *Q.E.D.*

Proof of Proposition 4: Combining (4) and (5) and plugging into (10) immediately gives (11). Note that $\epsilon\left(\rho; \alpha\right) \in \left(\alpha, 0\right), 0 < \rho < 1$ and clearly decreases in the arguments that increase ρ. Proposition 5 shows that $\frac{\partial \rho}{\partial \gamma} < 0$ and $\frac{\partial \rho}{\partial A_{DD}} < 0$, meaning that $\left.\frac{\partial\epsilon(\rho;A_{DD},\gamma)}{\partial A_{DD}}\right|_{\alpha} < 0$ and $\left.\frac{\partial\epsilon(\rho;A_{DD},\gamma)}{\partial\gamma}\right|_{\alpha} < 0$. Therefore, $A_{DD}\downarrow 0$ and $\gamma\downarrow 0 \implies \epsilon \overset{\lim}{\to} 0$. Any risky bond price ratio $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} > 0$ will satisfy (11) for small values of A_{DD} and γ because $\rho \overset{\lim}{\to} 1$. Its less obvious that (11) is always satisfied when $\rho \overset{\lim}{\to} 0$ because $\epsilon \overset{\lim}{\to} \alpha$. The reason is that moving from all short- to an interior solution involves reducing the safe short-term debt issued at $t = 0$ in favor or risky long-term debt which is always costly at $t = 0$. By contrast, moving from all long to an interior involves issuing less risky long-term for safe short-term at $t = 0$, for which the cost benefits are always clear. $\epsilon \overset{\lim}{\to} \alpha$ as $\gamma \uparrow 1$ because $\rho \to 0$. As long as $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \geq \alpha$ as $\gamma \uparrow 1$, condition (11) will hold for all γ because $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \uparrow$ as $\gamma \downarrow 0$ and $\epsilon \uparrow$ .Similarly, if $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \geq \alpha$ holds for $A_{DD} \to 0$, then it will hold for all $A_{DD} \to \alpha$ because $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \uparrow$ as $A_{DD}\downarrow$. For the numerical example in Table 1 of appendix B, $\frac{p_D^{\varsigma}}{p_{0,}^{\ell}} \approxeq 0.95$, with $\alpha = 0.7$, $\gamma = 0.8$, and $A_{DD} = 0.5$. We can show numerically that (11) does indeed hold $\forall\,(A_{DD},\gamma)\,-$pairs. *Q.E.D.*

Proof of Proposition 5: Using (4) and (5) and plugging into $I_0^{\alpha}\,(1-\rho^{\alpha})$, one obtains

$$\hat{\epsilon} \equiv \alpha\frac{(1-\rho)}{(1-\rho^{\alpha})} \leq \frac{p_D^{\varsigma}}{p_0^{\ell}} < 1 \tag{14}$$

Notice that (14) and (11) are similar but for how α enters into the denominator. It is easy to verify that $\hat{\epsilon} > \epsilon \iff \alpha < \frac{1}{\rho^{(1-\alpha)}}$. The condition $\alpha < \frac{1}{\rho^{(1-\alpha)}}$ holds $\forall 0 < \alpha < 1$, $0 < \rho < 1$. Therefore, any interior $\rho\,(\Gamma)$ that pins down $\frac{p_D^{\varsigma^*}}{p_0^{\ell^*}}$ for which (11) holds necessarily satisfies (14). From Propositions 2 and 3, we know that an interior solution always arises because it's cost reducing relative to issuing all long- or short-term debt. Thus, even for the parameters space for which (14) does not hold, it is still beneficial for the firm to issue both long- and short-term debt rather than issue all of either debt maturity precisely because substituting across risky debt maturities lowers the cost of moving away from one maturity even as moving into the other maturity increases its cost. *Q.E.D.*

Proof of Proposition 6: From (4) and a given set of risky debt prices $\left(p_0^{\ell^*}, p_D^{\varsigma^*}\right)$, $\uparrow\gamma$ increases the l.h.s more than the right. If the firm issues more long-term debt, $\rho\uparrow$, long-term debt prices fall and short-term debt prices rise, causing further deviation from the necessary equality. Thus, the firm must issue more short-term debt, $\frac{\partial\rho}{\partial\gamma} < 0$, raising long-term debt prices and lowering short-term debt prices to a new set of equilibrium prices $\left(\tilde{p}_0^{\ell}, \tilde{p}_D^{\varsigma}\right)$. From (6) we know that long-term debt holders and risky short-term debt holders expect the same delivery at $s = DD$. For a given set of initial prices, $\left(p_0^{\ell*}, p_D^{\varsigma*}\right)$, and corresponding marginal buyers, (h_0^*, h_D^*), raising A_{DD} increases deliveries. However, long-term debt holders at $t = 0$ place $(1 - h_0)^2$ weight on $s = DD$ while short-term debt holders at $t = 1$ place $(1 - h_D^{\varsigma})$ weight on $s = DD$. Unless $h_0 \ll h_D$, in which case the firm is issuing almost all long-term debt and $\rho \overset{\lim}{\to} 1$, short-term debt buyers at $t = 1$ place more weight on deliveries that long-term debt holders at $t = 0$. But we know from Propositions 2 and 3 that if $\rho \overset{\lim}{\to} 1$ the firm will find it beneficial to issue more short-term debt to take advantage of safe debt financing at $t = 0$ and optimistic capital at $t = 1$. Thus almost surely, risky short-term debt holders at $t = 1$ are more responsive to changes in A_{DD} than long-term debt holders at $t = 0$. *Q.E.D.*

Proof of Proposition 6: The first part is given in the text by (13). It is clear from (8) and (9) that any given buyer must pay a higher price for securities with higher deliveries. *Q.E.D.*

Proof of Corollary 1: From Proposition 6 and (13) we know that $d_{DD}\left(q_0^{\ell}\right) > d_{DD}\left(q_D^{\varsigma}\right)$ when long-term indentures include the covenant for a given (I_0^*, ρ^*). Suppose the firm does not alter its debt structure and $\rho^*$is unchanged. Then, long-term debt prices must rise to a new level reflecting greater marginal valuations, $p_0^{\ell c} > p_0^{\ell*}$, where the superscript c denotes prices with the covenant. But if long-term debt is now cheaper in equilibrium, then the maturity structure for a given (I_0^*, ρ^*)cannot be optimizing and the firm must adjust. Thus the firm issue more long-term debt and reduces its short-term debt, leaving I_0^*unchanged and $\rho^c > \rho^*$so lowering $\downarrow p_0^{\ell c} = p_0^{\ell*}$. *Q.E.D.*

Proof of Proposition 7: Follows immediately from the proof of Corollary 1 and investment optimality in (4) and (5). *Q.E.D.*

B Appendix

B.1 Multiple debt maturity funding

The ten endogenous variables are $\left(p_0^\varsigma, p_0^\ell, p_D^\varsigma, q_0^\varsigma, q_0^\ell, q_D^\varsigma, I_0, \rho, h_0, h_D\right)$. The system of equations, along with (4) and (5) is:

$$p_0^\varsigma = 1 \tag{15}$$

$$1 = \frac{1-(1-h_0)^2+(1-h_0)^2 d_{DD}\left(q_0^\ell\right)}{p_0^\ell} \tag{16}$$

$$1 = \frac{h_D+(1-h_D)\, d_{DD}\left(q_D^\varsigma\right)}{p_D^\varsigma} \tag{17}$$

$$I_0 = p_0^\ell q_0^\ell + p_0^\varsigma q_0^\varsigma \tag{18}$$

$$\rho = \frac{p_0^\ell q_0^\ell}{I_0} \tag{19}$$

$$q_0^\varsigma = p_s^\varsigma q_s^\varsigma \tag{20}$$

$$1-h_0 = p_0^\ell q_0^\ell \tag{21}$$

$$1-h_D = p_D^\varsigma q_D^\varsigma \tag{22}$$

The first three equations are bond pricing equations. Equation (15) shows that short-term bonds issued at time 0 are risk free because all short-term debt is rolled over at time 1. Equation (16) states that long-term bonds are priced based on the time 0 marginal investor's expectations because he is indifferent between buying the bond and holding a cash equivalent asset. Similarly, equation (17) states that time 1 short-term bonds are priced based on the time 1 marginal investor's expectations because cash is the only other alternative asset. Equation (18) says that the amount of capital the firm raises in the bond market is equal to the investment it puts into its production technology. Equations (4) and (5) are the first order conditions *w.r.t.* the portfolio allocation ρ and investment level I_0, respectively. The necessary condition for the firm to issue a portfolio of both long and short-term bonds in (4) says that on the margin the expected cost of issuing either type of bond must be the same. The left hand side of (5) is the expected marginal product of capital irrespective of whether or not it is issued via long-term or short-term bonds. The right hand side is the expected-weighted marginal cost of capital. Equation (19) sets ρ equal to the portion of the firm's investment that is raised via long-term debt. Equation (20) shows that the firm will issue as many short-term bonds at time 1 as it takes to fully repay its time 0 short-term creditors. Equations (21) and (22) are, respectively, the long-term and time 1 short-term bond market clearing conditions.