Board of Governors of the Federal Reserve System
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FOMC Secretariat

Date: October 25, 2011

To: Research Directors

From: Deborah J. Danker

Subject: Supporting Document for DSGE Models Update

The attached document supports the update on the projections of the DSGE models that was distributed in yesterday's memo, "DSGE Models Update."

System DSGE Project: Research Directors Drafts

October 24, 2011

Projections from EDO: Current Outlook November FOMC Meeting

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October 18, 2011

1 The Outlook for 2011 to 2014

The EDO model projects economic growth a touch above trend and low inflation while the policy rate is pegged to its effective lower bound until the second half of 2013, in line with the latest FOMC statement.

The normalization of the model's risk premia from their elevated levels immediately following the crisis has thus far been unusually slow and households and firms now anticipate that this pattern of slow normalization will persist for the near-term as well. Consequently, the current sizeable gap between actual production and its long-term trend closely only modestly over the projection. Inflation remains low as wage pressures are weak relative to labor productivity, reflecting the declines in household wealth over the past several years, low level of hours worked anticipated over the next few years, and the rapid increases in productivity seen in 2009.

This model forecast takes as data market expectations as of 2011:Q3 that the policy rate will remain at its effective lower bound until 2013Q3, followed by a gradual rise thereafter. Conditional on these expectations and its usual observables, EDO projects that real GDP will advance at a pace modestly above trend going forward– about 2 1/2 percent, on average, over 2012-2014, as shown in figure 1. The above-trend pace

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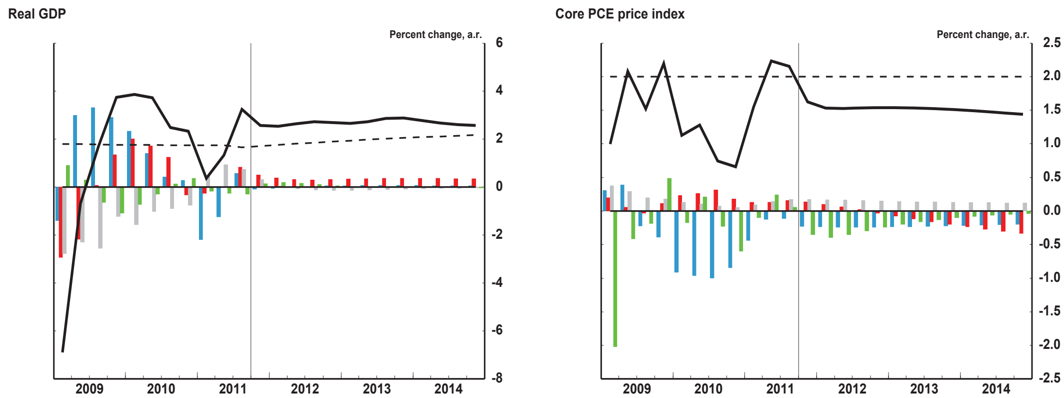
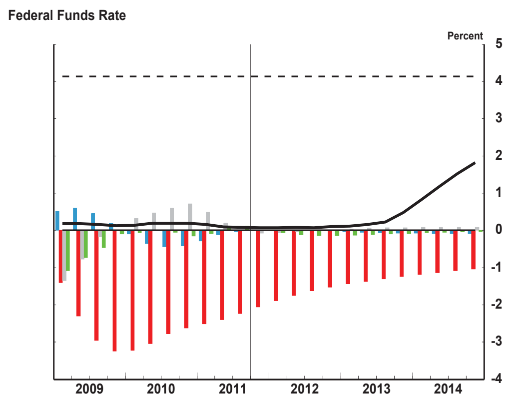
of growth is accompanied by inflation around 1.5 percent per year, noticeably below the target of 2 percent, as a consequence of labor market slack.[1]

Figure 1: Recent History and Forecasts

EDO Projection Summary





	2011 Q4/Q4	2012 Q4/Q4	2013 Q4/Q4	2014 Q4/Q4
Real GDP (a)	1.9	2.7	2.8	2.7
Credible set (c)	.1-3.4	-.1-3.9	.9-4.5	1.3-4.8
Core PCE Price index (a)	1.9	1.5	1.5	1.5
Credible set (c)	1.7-2.1	.8-2.1	.7-2.1	.7-2.1
Federal Funds Rate (b)	0.1	0.1	0.5	1.8
Credible set (c)	.0-.7	.0-1.9	.0-2.3	.6-3.6

(a) Q4/Q4 percent change, (b) Q4 level, (c) 68 percent

Black, solid line -- Data (through 2010Q4) and projections; Black, dashed line -- Steady-state or trend values
Contributions (bars): Red -- Financial; Blue -- Technology; Silver -- Monetary policy; Green -- Other

The decomposition of the projections for these variables shown in figure 1 highlights the important role that the adverse shocks to financial conditions in 2008 and early 2009 play in shaping the recession in that period and the projected recovery,

[1]The EDO model has been shown to forecast as well as, or better than, alternatives in a number of papers (e.g., Edge, Kiley, and Laforte (2010) and Wieland and Wolters (2010); however, forecasting is very challenging, and models generally perform similar to, but not better than, simple time series alternatives, or consensus forecasts.

especially in the later years. Specifically, the figures decompose the movements in real GDP, the federal funds rate, and core inflation into the contributions from financial (risk premium) shocks, monetary policy shocks, productivity movements, and other disturbances (largely markup, or Phillips-curve, shocks); the first two are traditional "demand" shocks, and the latter two are traditional "supply" shocks.[2] As shown in the federal funds rate chart, the need to accommodate the adverse impact of the strain in the financial conditions (the red bars) is the most largest factor holding the federal funds rate at its affective lower bound throughout most of the projection. The recovery in real GDP projected for 2012-14 is essentially entirely the result of the projected gradual step-up in demand that should accompany lower risk premia, again illustrated by the contribution of the red bars in the GDP chart. The easing provided by forward guidance boosts real GDP growth in the early quarters of the projection.

As mentioned previously, the forecast conditions on (rational) private-sector expectations of a policy rate path consistent with the latest statement of the FOMC. This anticipated path is rationalized by augmenting the model's usual exogenous shock processes to include eight quarters of anticipated shocks to both the monetary policy reaction function and the household's Euler equation for nominal risk-free assets. Interestingly, these additional observables do little to change the EDO forecasts for output and inflation estimated from the model's usual information set, since the two classes of anticipated shocks are estimated to have largely offsetting effects on most variables other than the federal funds rate. On the one hand, the model explains the expected policy rate path as a response to continued strains from financial conditions going forward. Indeed, the economy-wide risk premium remains at its current level for the first two years of the projection instead of gradually falling as its pre-crisis dynamics would have implied. On the other hand, however, even given this additional weakness in the forecast, the stance of monetary policy appears very accommodative, essentially offsetting the impact of higher risk-premia.

[2]The contributions of the demand shocks now incorporate the effects of their anticipated counterparts.

2 The Dynamics of the Model to Anticipated Shocks

This section explores the response of key economic aggregates to the aforementioned anticipated monetary policy and economy-wide risk premium shocks to better understand their contribution to the projection. Figure 2 displays the dynamics of real GDP growth in response to the anticipated shocks for eight different horizons. As expected, the anticipation of future contractionary policy shock or higher risk premium depress real activity. The later positive contributions to real GDP growth results from output eventually returning to the steady-state from its lower levels. We note the broad symmetry of the responses across the two types of shock. This symmetry reflects the fact that both shocks essentially affect the economy through the willingness of households to hold the risk-free nominal asset. As far as this arbitrage condition is concerned, however, a one-off anticipated increase in the level of the federal funds rate can be perfectly offset by corresponding one-off anticipated decrease in the premium on holding that asset.

Figure 2: Impulse Responses – Real GDP Growth

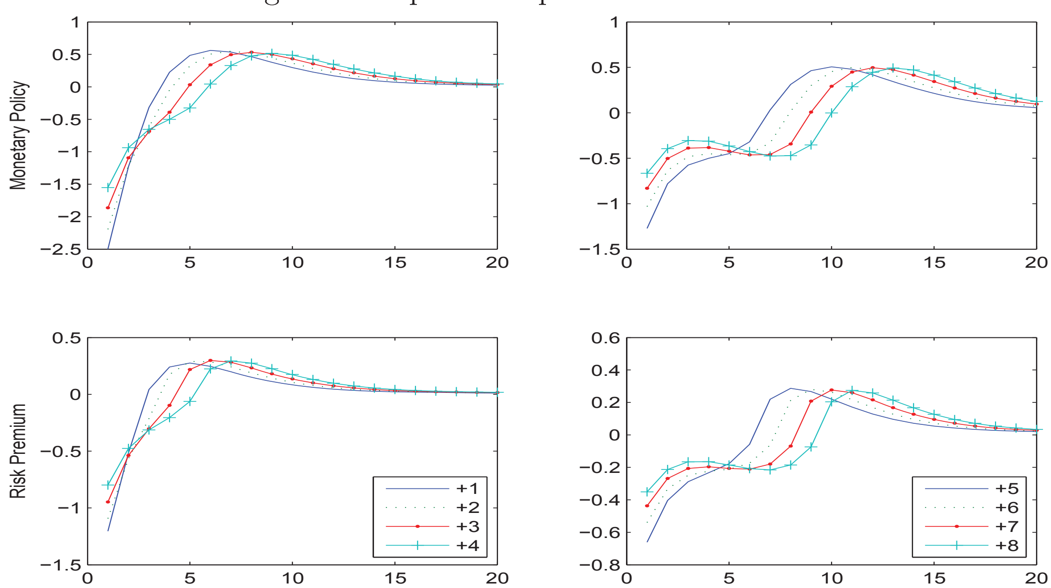
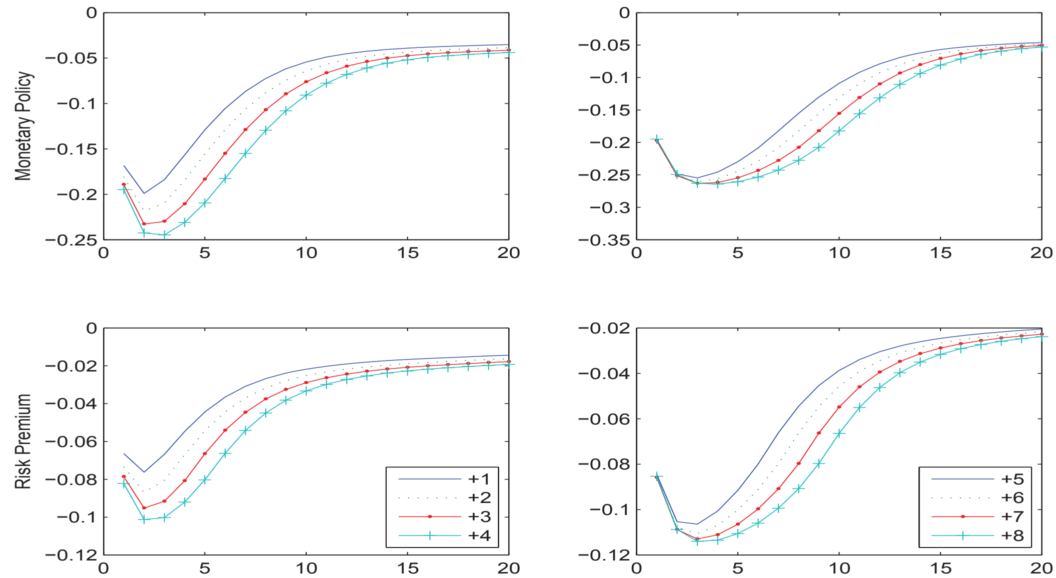
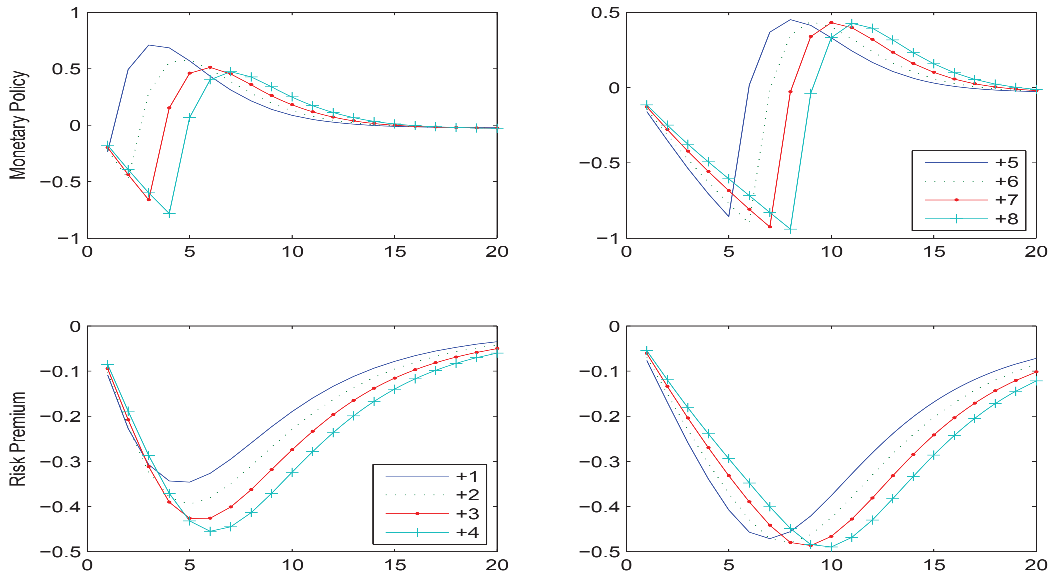


Figure 3: Impulse Responses – Core PCE Inflation



An anticipated positive shock to the monetary policy rule gives private agents an incentive to tilt their portfolios in favor of the nominal risk-free asset. Because that asset is in fixed nominal supply, inflation falls in advance of the realization of the shock, as shown in figure 3. Production falls, due to the presence of nominal rigidities. As seen in figure 4, the fed funds rate actually declines as well in anticipation of the future realization of a positive (hence contractionary) monetary policy shock as the interest rate endogeneously adjusts to the declines in output and prices. It is only upon or after (for the most distant horizons) its realization that the interest rate becomes positive – albeit not by as much as the size of the shock. This dynamic explains the positive contributions observed (see the gray bars in the lower left panel of figure 1) early on in the projection from (anticipated) monetary policy shocks to the fed funds rate. They correspond to the anticipated movements in the fed funds rate that lead the realization of the furthest negative shocks used to impose the ZLB in anticipation until 2013Q2.

Figure 4: Impulse Responses – Fed Funds Rate



3 An Overview of Key Model Features

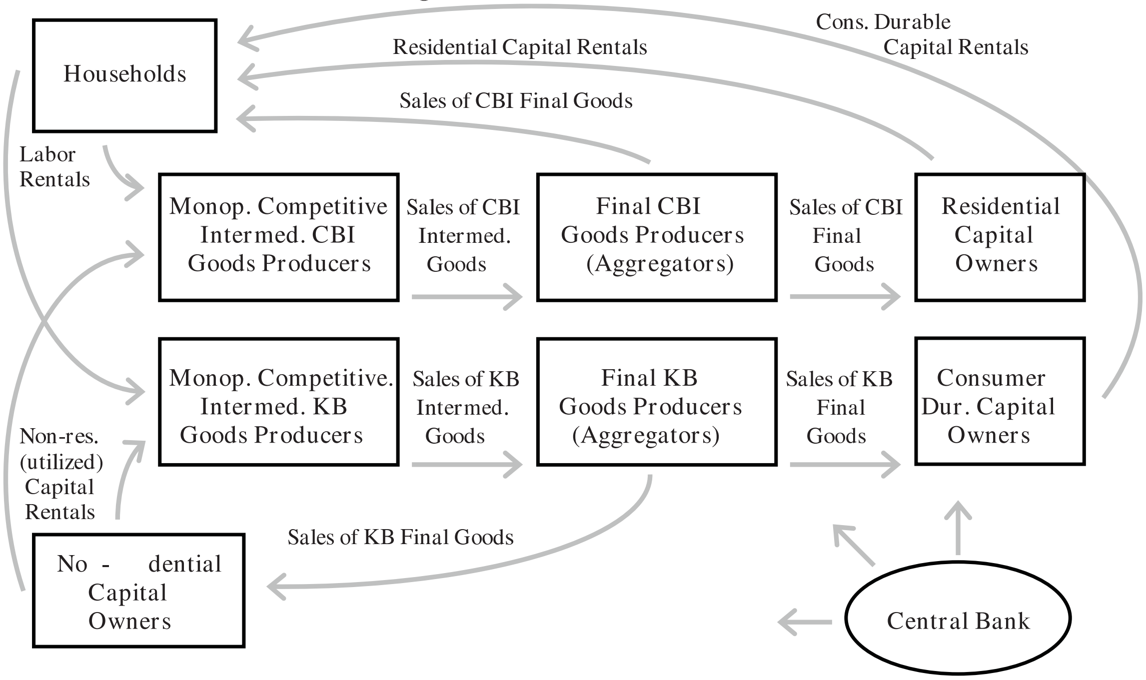
Figure 5 provides a graphical overview of the model. While similar to most related models, EDO has a more detailed description of production and expenditure than most other models.[3]

Specifically, the model possesses two final good sectors in order to capture key long-run growth facts and to differentiate between the cyclical properties of different categories of durable expenditure (e.g., housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics).

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer non-durable goods and non-housing services, consumer durable goods, residential investment, and

[3]Chung, Kiley, and Laforte (2011) provide much more detail regarding the model specification, estimated parameters, and model propeties.

Figure 5: Model Overview



non-residential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer non-durable goods and services are sold directly to households; consumer durable goods, residential capital goods, and non-residential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer non-durable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy's two final goods producing sectors, while consumer durable goods and non-residential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the non-durable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

This remainder of this section provides an overview of the key properties of the model. In particular, the model has five key features:

- Production of goods and services occurs in two sectors, with differential rates of technological progress across sectors.

- A disaggregated specification of household preferences and firm production processes that leads to separate modeling of nondurables and services consumption, durables consumption, residential investment, and business investment.
- Risk premia associated with different investment decisions play a central role in the model. These include A) an aggregate risk-premium, or natural rate of interest, shock driving a wedge between the short-term policy rate and the interest rate facing private decisionmakers (as in Smets and Wouters (2007)) and B) fluctuations in the discount factor/risk premia facing the intermediaries financing household (residential and consumer durable) and business investment.
- A new-Keynesian structure for price and wage dynamics.
- A monetary policy that reacts to inflation and a measure of resource utilization.

3.1 Two-sector production structure

It is well known (e.g., Edge, Kiley, and Laforte (2010)) that real outlays for business investment and consumer durables have substantially outpaced those on other goods and services, while the prices of these goods (relative to others) has fallen. For example, real outlays on consumer durables have far outpaced those on other consumption, while prices for consumer durables have been flat and those for other consumption have risen substantially; as a result, the ratio of nominal outlays in the two categories has been much more stable, although consumer durable outlays plummeted in the Great Recession. Many models fail to account for this fact.

EDO accounts for this development by assuming that business investment and consumer durables are produced in one sector and other goods and services in another sector. Specifically, production by firm j in each sector s (where s equals kb for the sector producing business investment and consumer durables sector and cbi for the sector producing other goods and services) is governed by a Cobb-Douglas production function with sector-specific technologies:

$$X_t^s(j) \ = \left(Z_t^m Z_t^s L_t^s(j)\right)^{1-\alpha} \left(K_t^{u,nr,s}(j)\right)^{\alpha}, \text{ for } s = cbi, kb. \tag{1}$$

In 1, Z^m represents (labor-augmenting) aggregate technology, while Z^s represents (labor-augmenting) sector-specific technology; we assume that sector-specific technological change affects the business investment and consumer durables sector only; L^s is labor input and $K^{u,nr,s}$ is capital input (that is, utilized *non-residential business* capital (and hence the nr and u terms in the superscript). Growth in this sector-specific technology accounts for the long-run trends, while high-frequency fluctuations allow the possibility that investment-specific technological change is an important source of business cycle fluctuations.

3.2 The structure of demand

EDO differentiates between several categories of expenditure. Specifically, business investment spending determines non-residential capital used in production, and households value consumer nondurables goods and services, consumer durable goods, and residential capital (e.g., housing). Differentiation across these categories is important, as fluctuations in these categories of expenditure can differ notably, with the cycles in housing and business investment, for example, occurring at different points over the last three decades.

Valuations of these goods and services, in terms of household utility, is given by the following utility function:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \varsigma^{cnn} \ln(E_t^{cnn}(i) - h E_{t-1}^{cnn}(i)) + \varsigma^{cd} \ln(K_t^{cd}(i)) + \varsigma^{r} \ln(K_t^{r}(i)) - \varsigma^{l} \frac{(L_t^{cbi}(i) + L_t^{kb}(i))^{1+\nu}}{1+\nu} \Big\}, \tag{2}$$

where E^{cnn} represents expenditures on consumption of nondurable goods and services, K^{cd} and K^r represent the stocks of consumer durables and residential capital (housing), $L^{cbi} + L^{kb}$ represents the sum of labor supplied to each productive sector (with hours worked causing disutility), and the remaining terms represent parameters (such as the discount factor, relative value in utility of each service flow, and the elasticity of labor supply).

By modeling preferences over these disaggregated categories of expenditure, EDO attempts to account for the disparate forces driving consumption of nondurables and durables, residential investment, and business investment – thereby speaking to issues such as the surge in business investment in the second half of the 1990s or the housing cycle the early 2000s recession and the most recent downturn. Many other models do not distinguish between developments across these categories of spending.

3.3 Risk premia, financial shocks, and economic fluctuations

The structure of the EDO model implies that households value durable stocks according to their expected returns, including any expected service flows, and according to their risk characteristics, with a premium on assets which have high expected returns in adverse states of the world. However, the behaviour of models such as EDO is conventionally characterized under the assumption that this second component is negligible. In the absence of risk adjustment, the model would then imply that households adjust their portfolios until expected returns on all assets are equal.

Empirically, however, this risk adjustment may not be negligible and, moreover, there may be a variety of factors, not explicitly modelled in EDO, which limit the ability of households to arbitrage away expected return differentials across different assets. To account for this possibility, EDO features several exogenous shocks to the rates of return required by the household to hold the assets in question. Following such a shock – an increase in the premium on a given asset, for example– households will wish to alter their portfolio composition to favor the affected asset, leading to changes in the prices of all assets and, ultimately, to changes in the expected path of production underlying these claims.

The "sector-specific" risk shocks affect the composition of spending more than the path of GDP itself. This occurs because a shock to these premia leads to sizable substitution across residential, consumer durable, and business investment; for example, an increase in the risk premia on business investment leads households to shift away from business investment and towards residential investment and consumer durables. Consequently, it is intuitive that a large fraction of the non-cyclical, or idiosyncratic, component of investment flows to physical stocks will be accounted for by movements in the associated premia.

Shocks to the required rate of return on the nominal risk-free asset play an especially large role in EDO. Following an increase in the premium, in the absence of nominal rigidities, the households' desire for higher real holdings of the risk-free asset would be satisfied entirely by a fall in prices, i.e., the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be off-set, in part, through a fall in real income, a decline which is distributed across all spending components. Because this response is capable of generating comovement across spending categories, the model naturally exploits such shocks to explain the business cycle. Reflecting this role, we denote this shock as the "aggregate risk-premium".

3.4 New-Keynesian Price and Wage Phillips Curves

As in most of the related literature, nominal prices and wages are both "sticky" in EDO. This friction implies that nominal disturbances – that is, changes in monetary policy – have effects on real economic activity. In addition, the presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

Given the widespread use of the New-Keynesian Phillips curve, it is perhaps easiest to consider the form of the price and wage Phillips curves in EDO at the estimated parameters. The price Phillips curve (governing price adjustment in both productive sectors) has the form:

$$\pi_t^{p,s} = 0.22\pi_{t-1}^{p,s} + 0.76E_t\pi_{t+1}^{p,s} + .017mc_t^s + \theta_t^s \tag{3}$$

where mc is marginal cost and θ is a markup shock. As the parameters indicate, inflation is primarily forward-looking in EDO.

The wage (w) Phillips curve for each sector has the form:

$$w_t^s = 0.01 \quad w_{t-1}^s + 0.95E_t \quad w_{t+1}^s + .012\left(mrs_t^{c,l} - w_t^s\right) + \theta_t^w + adj.\,costs. \tag{4}$$

where mrs represents the marginal rate of substitution between consumption and leisure. Wages are primarily forward looking and relatively insensitive to the gap between households' valuation of time spent working and the wage.

3.5 The Monetary Policy Rule

The estimated monetary policy rule has standard features – the policy interest rate responds inertially to inflation and a deviation of output from a trend level:

$$r_t = 0.76 r_{t-1} + (1 - 0.76) \;\; 1.50 \quad P_t^{PCE} + 1.20\,(y_t - trend) \;\; + \;\; {}_t^{Rshock}. \tag{5}$$

$${}_t^{Rshock} = \rho^{Rshock} \delta_{t-1}^{Rshock} + \;\; {}_t^{R} \tag{6}$$

The long-run responses to the output gap and inflation are very similar to those in the literature. The measure of trend output is based on a production-function concept – that is, trend output is the level of output consistent with labor input and the utilization of capital at long-run levels, given the current level of productive capital; this output concept is a Divisia aggregate of production in the two sectors discussed earlier.

3.6 Summary of Model Specification

To summarize, fluctuations in all economic variables are driven by eleven structural shocks. It is most convenient to summarize these shocks into four broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.
- Financial, or intertemporal, shocks: This category consists of shocks to risk premia. In EDO, variation in risk premia – both the premium households' receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries – are purely exogenous. Nonetheless, the specification captures aspects of related models with more explicit financial sectors (e.g., Bernanke, Gertler, and Gilchrist (1999)).
- Monetary policy shocks.
- Other shocks: This category is dominated by shocks to price and wage markups, or Phillips curve shock; it als includes the shock to autonomous demand, which is quantitatively not important in EDO.

4 Estimation: Data and Properties

4.1 Data

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy's equilibrium, casts this resulting system in its state-space representation for the set of (in our case 12) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

Because of the detailed modeling of demand, EDO can consider more data on expenditure than other related models to inform its parameter estimates and projections. The model is estimated using 12 data series over the sample period from 1984:Q4 to 2011:Q1. The series are:

1. The growth rate of real gross domestic product (ΔGDP);
2. The growth rate of real consumption expenditure on non-durables and services (ΔC);
3. The growth rate of real consumption expenditure on durables (ΔCD);
4. The growth rate of real residential investment expenditure (ΔRes);
5. The growth rate of real business investment expenditure (ΔI);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,total}$);
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,core}$);
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
9. Hours, which equals hours of all persons in the non-farm business sector from the Bureau of Labor Statistics (H);[4]

[4]We remove a low-frequency trend from hours via the Hodrick-Prescott filter with a smoothing parameter of 128000; our model is not designed to capture low frequency trends in population growth or labor force participation.

10. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index (ΔRW);
11. The federal funds rate (R).
12. The yield on the 2-yr. U.S. Treasury security (RL).

Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

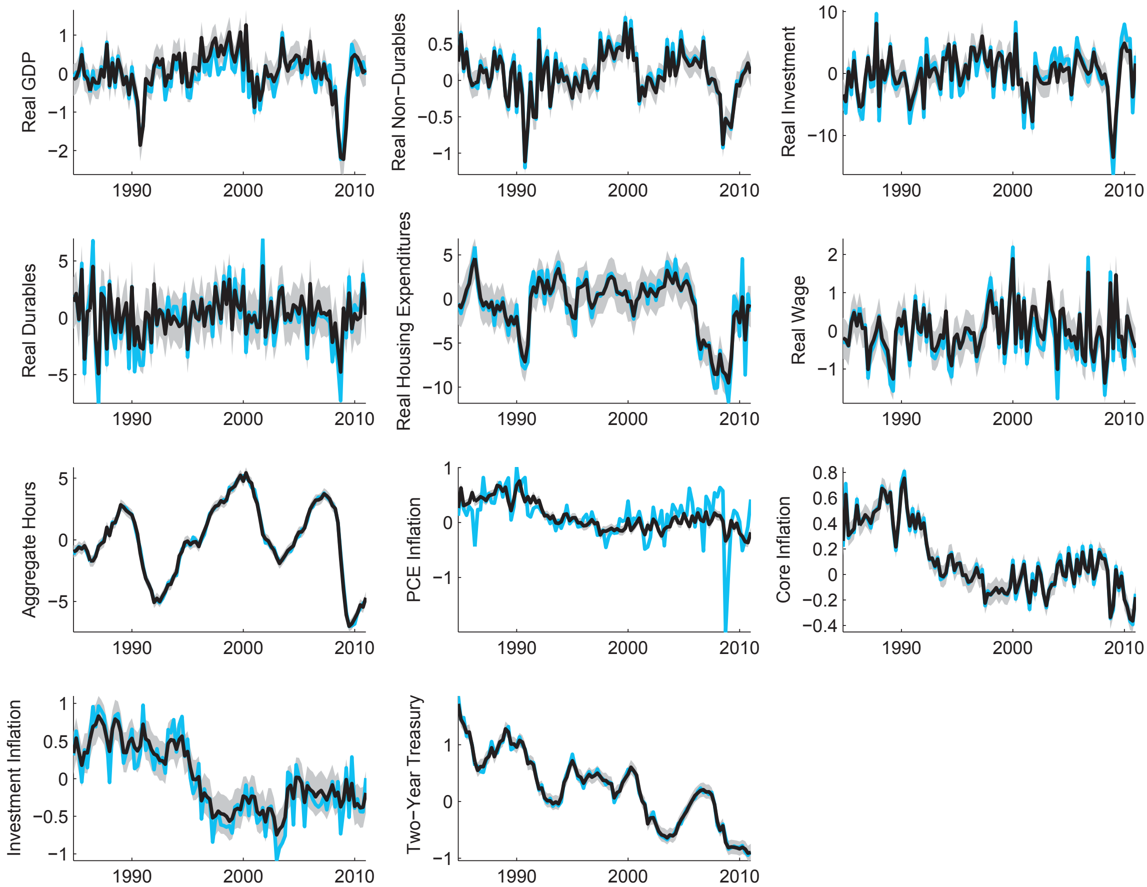
Figure 6 presents the observed data (in blue) and the observable data net of the model's estimated measurement error (in black), along 95 percent confidence intervals. For series other than overall PCE price inflation, measurement error is a moderate portion of movements in the series. The larger role for measurement error in accounting for the path of PCE price inflation reflects the absence of separate sectors for food and energy in the model.

4.2 Estimates of shocks and exogenous fundamentals

Figures 7 and 8 report modal estimates of the model's structural shocks and the persistent exogenous fundamentals (i.e., risk premia and autonomous demand). These series have recognizable patterns for those familiar with U.S. economic fluctuations. For example, the risk premia jump at the end of the sample, reflecting the financial crisis and the model's identification of risk premia, both economy-wide and for housing, as key drivers.

Of course, these stories from a glance at the exogenous drivers yield applications for alternative versions of the EDO model and future model enhancements. For example, the exogenous risk premia can easily be made to have an endogenous component following the approach of Bernanke, Gertler, and Gilchrist (1999) (and indeed we have considered models of that type). At this point we view incorporation of such mechanisms in our baseline approach as premature, pending ongoing research on financial frictions, banking, and intermediation in dynamic general equilibrium models. Nonetheless, the EDO model captured the key financial disturbances during the last several years in its current specification, and examining the endogenous factors that explain these developments will be a topic of further study.

Figure 6: Smoothed Observables and Data

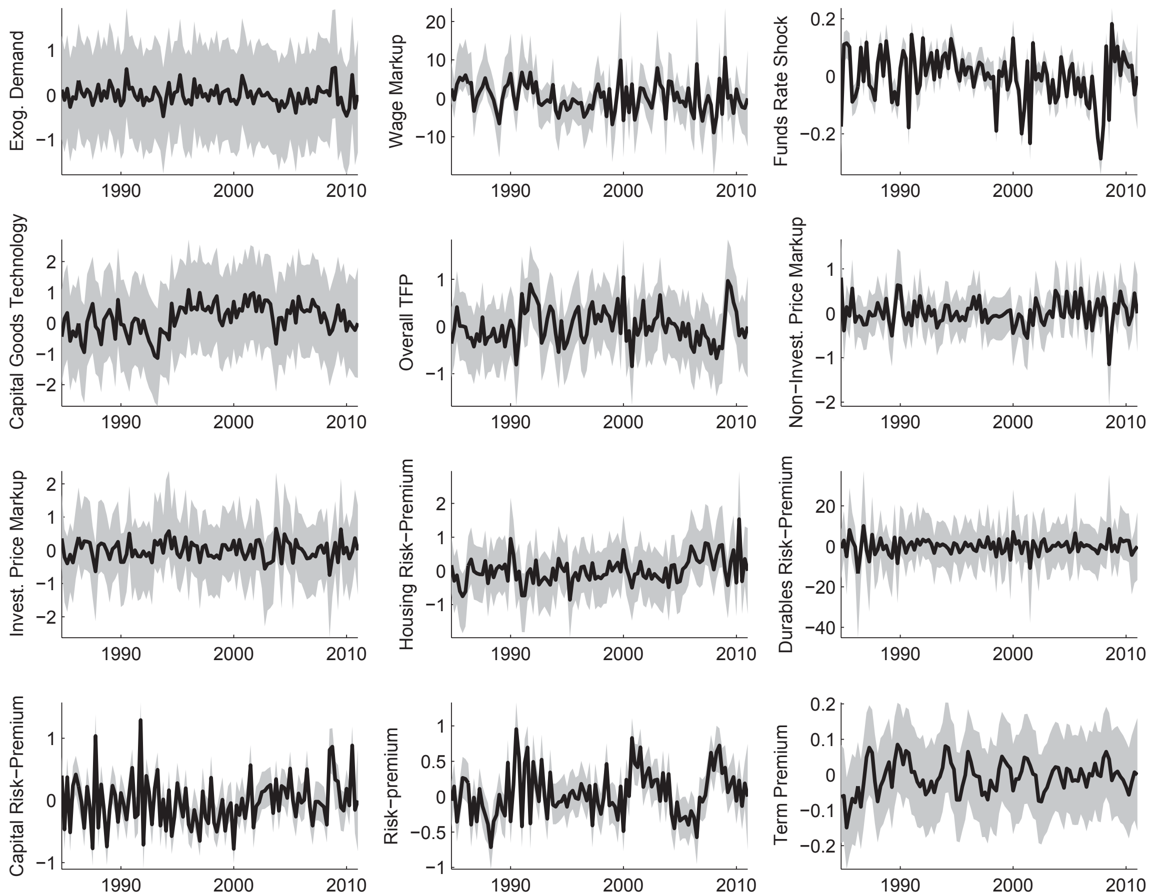


4.3 Variance Decompositions and impulse responses

We provide detailed variance decompositions and impulse response in Chung, Kiley, and Laforte (2011), and only highlight the key results here.

Volatility in hours per capita is accounted for primarily by the economy-wide risk premium and business investment risk premium shocks at horizons between one and sixteen quarters. The large role for risk premia shocks in the forecast error decomposition at business cycle horizons illustrates the importance of this type of "demand" shock for volatility in the labor market. This result is notable, as hours per capita is the series most like a "gap" variable in the model – that is, house per

Figure 7: Innovations to Exogenous Processes



capita shows persistent cyclical fluctuations about its trend value.

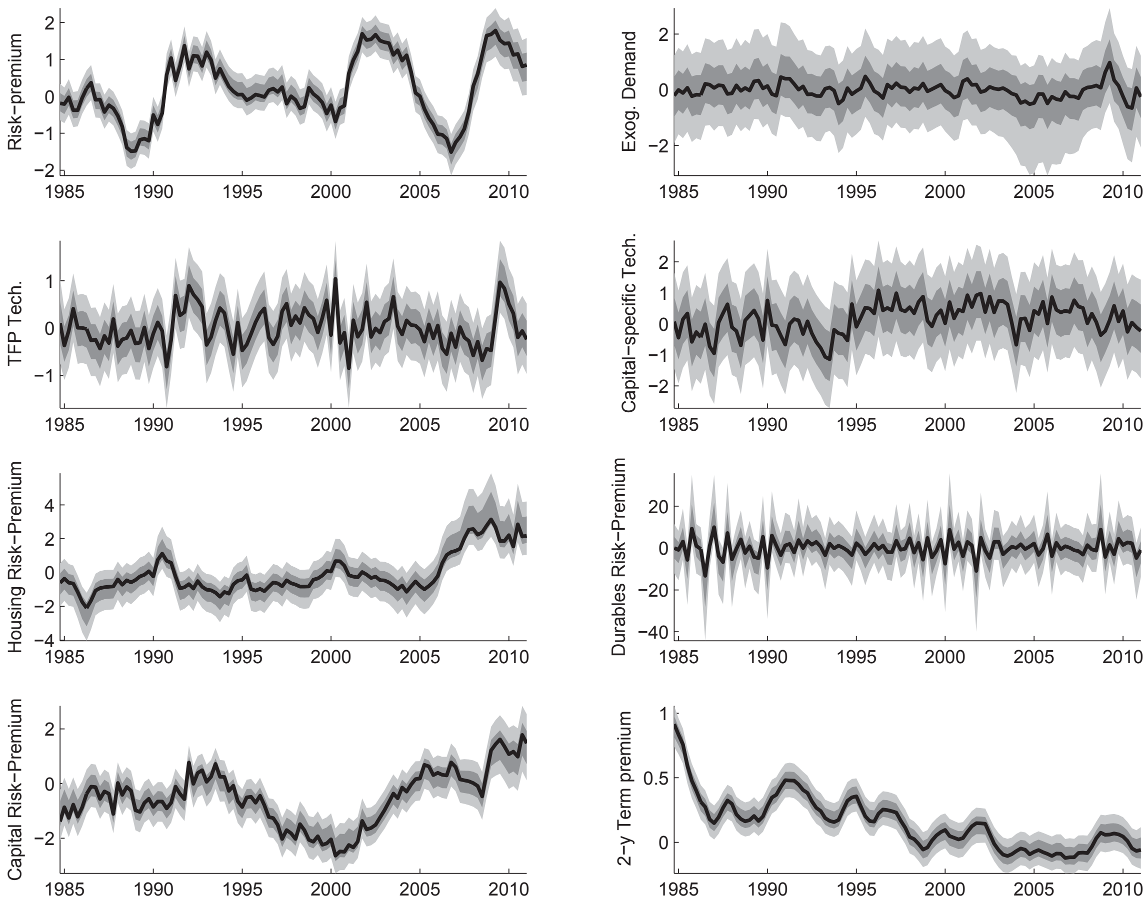
Volatility in aggregate GDP growth is accounted for primarily by the technology shocks in each sector, although the economy-wide risk premium shock contributes non-negligibly to the unconditional variance of GDP growth.

Volatility in core inflation is accounted for primarily by the markup shocks in the short run and technology shocks in the long run.

Volatility in the federal funds rate is accounted for primarily by the economywide risk premium.

Volatility in expenditures on consumer non-durables and non-housing services is, in the near horizon, accounted for predominantly by economy-wide and

Figure 8: Exogenous Drivers



non-residential investment specific risk-premia shocks.

Volatilities in expenditures on consumer durables, residential investment, and non-residential investment are, in the near horizon, accounted for predominantly by their own sector specific risk-premium shocks.

With regard to impulse responses, we previously highlight the responses to the most important shock, the aggregate risk premium, in figure **??**. As we noted, this shock looks like a traditional demand shock, with an increase in the risk premium lowering real GDP, hours worked, and inflation; monetary policy offsets these negative effects somewhat by becoming more accommodative. As for responses to other disturbances, the impulse responses to a monetary policy innovation captures the conventional wisdom regarding the effects of such shocks. In particular, both household and business expenditures on durables (consumer durables, residential investment, and nonresidential investment) respond strongly (and with a hump-shape) to a contractionary policy shock, with more muted responses by nondurables and services consumption; each measure of inflation responds gradually, albeit more quickly than in some analyses based on vector autoregressions (VARs).[5]

Shocks to sectoral risk premia principally depress spending in the associated category of expenditure (e.g., an increase in the residential risk premium lowers residential investment), with offsetting positive effects on other spending (which is "crowded in").

Following an economy-wide technology shock, output rises gradually to its long-run level; hours respond relatively little to the shock (in comparison to, for example, output), reflecting both the influence of stick prices and wages and the offsetting income and substitution effects of such a shock on households willingness to supply labor.

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[5]This difference between VAR-based and DSGE-model based impulse responses has been highlighted elsewhere – for example, in the survey of Boivin, Kiley, and Mishkin (2010).

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FRBNY DSGE Model: Research Directors Draft

October 19, 2011

Overview

The FRBNY DSGE model forecast is obtained using data released through 2011Q2 augmented, for 2011Q3, with observations on the federal funds rate and the Baa corporate bond spread, as well as the NY Fed staff forecast for real GDP growth, Core PCE inflation and hours. The projections are conditional on the federal funds rate being 25bp through 2013Q2, in line with the current FOMC statement.

The model projects weak growth in economic activity, as it did in June. Real growth is 1.8% in 2011 (Q4/Q4), picks up in 2012 (Q4/Q4) to 2.6%, and returns to slightly below 2% in 2013-2014. Core inflation is 1.8% in 2011 (Q4/Q4), notably higher than in the June projections, due to recent strong readings. In spite of this, inflation forecasts from 2012 onwards are actually lower than in June, due to the projected weakness in economic activity. There is significant uncertainty around the real GDP forecasts, with a non negligible risk of recession.

The main drivers of the subdued real GDP and inflation outlook are the same forces behind the Great Recession, namely the two shocks associated with frictions in the financial system: spread and MEI (marginal efficiency of investment) shocks, whose impact is long-lasting.

General Features of the Model

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by Bernanke, Gertler, and Gilchrist (1999). The actual implementation of the credit frictions closely follows Christiano, Motto, and Rostagno (2009).

In this section, we briefly describe the microfoundations of the model, including the optimization problem of the economic agents and the nature of the exogenous processes. The innovations to these processes, which we refer to as "shocks," are the drivers of macroeconomic fluctuations. The model identifies these shocks by matching the model dynamics with six quarterly data series: real GDP growth, core PCE inflation, the labor share, aggregate hours worked, the effective federal funds rate (FFR), and the spread between Baa corporate bonds and 10-year Treasury yields. Model parameters are estimated from 1984Q1 to the present using Bayesian methods. Details on the structure of the model, data sources, and results of the estimation procedure can be found in the accompanying "FRBNY DSGE Model Documentation" note.

The economic units in the model are households, firms, banks, entrepreneurs, and the government. (Figure 1 describes the interactions among the various agents, the frictions and the shocks that affect the dynamics of this economy.)

Households supply labor services to firms. The utility they derive from leisure is subject to a random disturbance, which we call "labor supply" shocks (this shock is sometimes also referred to as a "leisure" shock). Labor supply shocks capture exogenous movements in labor supply due to such factors as demographics and labor market imperfections. The labor market is also subject to frictions because of nominal wage rigidities. These frictions play an important role in the extent to which various shocks affect hours worked. Households also have to choose the amount to consume and save. Their savings take the form of deposits to banks and purchases of government bills. Household preferences take into account habit persistence, a characteristic that affects their consumption smoothing decisions.

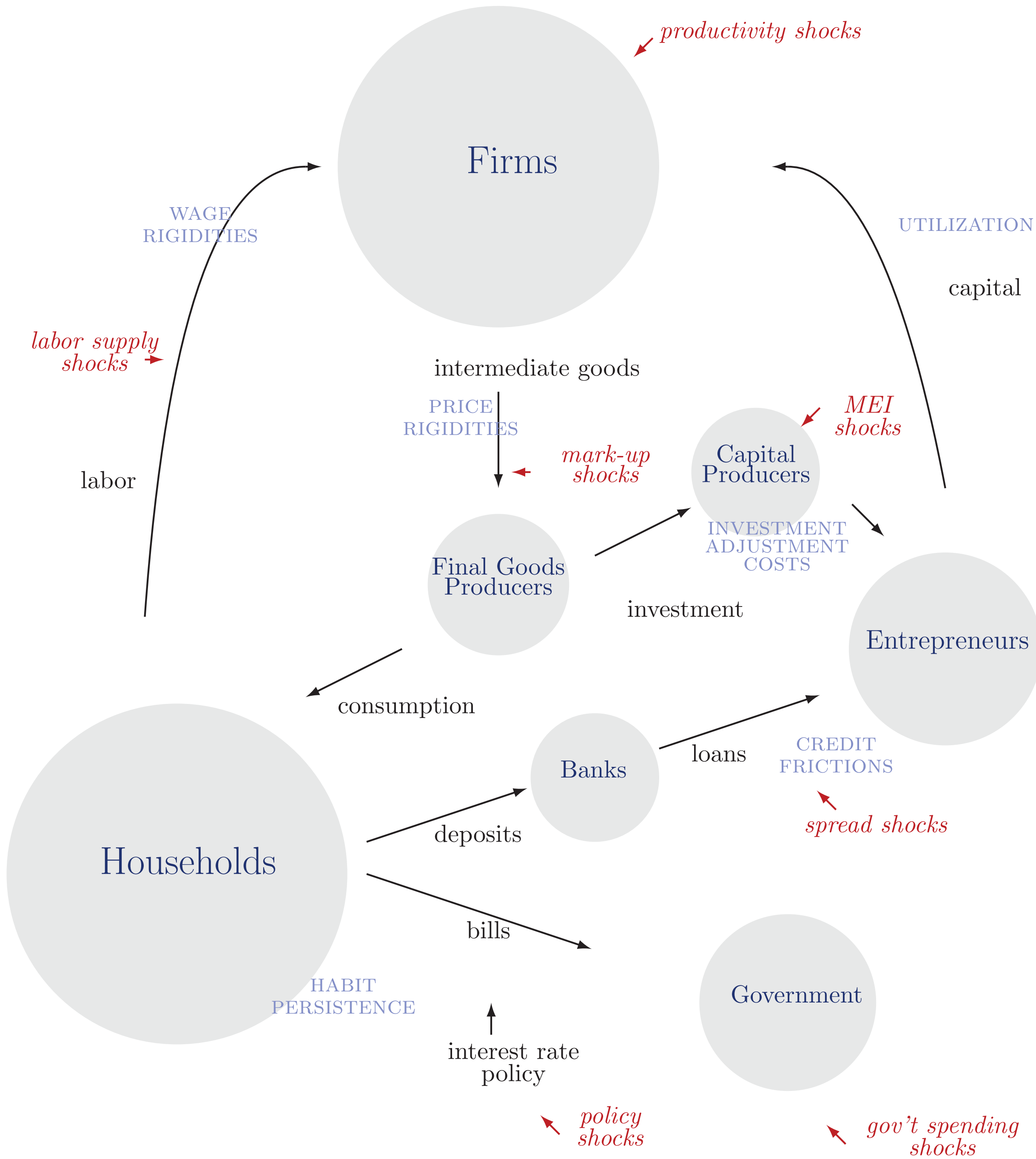
Monopolistically competitive firms produce intermediate goods, which a competitive firm aggregates into the single final good that is used for both consumption and investment. The production function of intermediate producers is subject to "total factor productivity" (TFP) shocks. Intermediate goods markets are subject to price rigidities. Together with wage rigidities, this friction is quite important in allowing demand shocks to be a source of business cycle fluctuations, as countercyclical mark-ups induce firms to produce less when demand is low. Inflation evolves in the model according to a standard, forward-looking New Keynesian Phillips curve, which determines inflation as a function of marginal costs, expected future inflation, and "mark-up" shocks. Mark-up shocks capture exogenous changes in the degree of competitiveness in the intermediate goods market. In practice, these shocks capture unmodeled inflation pressures, such as those arising from fluctuations in commodity prices.

Financial intermediation involves two actors, *banks* and *entrepreneurs*, whose interaction captures imperfections in financial markets. These actors should not be interpreted in a literal sense, but rather as a device for modeling credit frictions. Banks take deposits from households and lend them to entrepreneurs. Entrepreneurs use their own wealth and the loans from banks to acquire capital. They then choose the utilization level of capital and rent the capital to intermediate good producers. Entrepreneurs are subject to idiosyncratic disturbances in their ability to manage the capital. Consequently, entrepreneurs' revenue may not be enough to repay their loans, in which case they default. Banks protect against default risk by pooling loans to all entrepreneurs and charging a spread over the deposit rate. Such spreads vary endogenously as a function of the entrepreneurs' leverage, but also exogenously depending on the entrepreneurs' riskiness. Specifically, mean-preserving changes in the volatility of entrepreneurs' idiosyncratic shocks lead to variations in the spread (to compensate banks for changes in expected losses from individual defaults). We refer to these exogenous movements as "spread" shocks. Spread shocks capture financial intermediation disturbances that affect entrepreneurs' borrowing costs. Faced with higher borrowing costs, entrepreneurs reduce their demand for capital, and investment drops. With lower aggregate demand, there is a contraction in hours worked and real wages. Wage rigidities imply that hours worked fall even more (because nominal wages do not fall enough). Price rigidities mitigate price contraction, further depressing aggregate demand.

Capital producers transform general output into capital goods, which they sell to the entrepreneurs. Their production function is subject to investment adjustment costs: producing capital goods is more costly in periods of rapid investment growth. It is also subject to exogenous changes in the "marginal efficiency of investment" (MEI). These MEI shocks capture exogenous movements in the productivity of new investments in generating new capital. A positive MEI shock implies that fewer resources are needed to build new capital, leading to higher real activity and inflation, with an effect that persists over time. Such MEI shocks reflect both changes in the relative price of investment versus that of consumption goods (although the literature has shown the effect of these relative price changes to be small), and most importantly financial market imperfections that are not reflected in movements of the spread.

Finally, the *government* sector comprises a monetary authority that sets short-term interest rates according to a Taylor-type rule and a fiscal authority that sets public spending and collects lump-sum taxes to balance the budget. Exogenous changes in government spending are called "government" shocks (more generally, these shocks capture exogenous movements in aggregate demand). All exogenous processes are assumed to follow independent AR(1) processes with different degrees of persistence, except for i.i.d. "policy" shocks, which are exogenous disturbances to the monetary policy rule.

Figure 1: Model Structure



The Model's Transmission Mechanism

In this section, we illustrate some of the key economic mechanisms at work in the model's equilibrium. We do so with the aid of the impulse reponse functions to the main shocks hitting the economy, which we report in figures 7 to 13.

We start with the shock most closely associated with the Great Recession and the severe financial crisis that characterized it: the spread shock. As discussed above, this shock stems from an increase in the perceived riskiness of borrowers, which induces banks to charge higher interest rates for loans, thereby widening credit spreads. As a result of this increase in the expected cost of capital, entrepreneurs' borrowing falls, hindering their ability to channel resources to the productive sector via capital accumulation. The model identifies this shock by matching the behavior of the Baa corporate bond rate over 10-year Treasuries, and the spread's comovement with output growth, inflation, and the other observables. Figure 7 shows the impulse responses of the variables used in the estimation to a one-standard-deviation innovation in the spread shock. An innovation of this size increases the observed spread by roughly 35 basis points (bottom right panel). This leads to a reduction in investment and consequently to a reduction in output growth (top left panel) and hours worked (top right panel). The fall in the level of hours is fairly sharp in the first year and persists for many quarters afterwards, leaving the labor input not much higher than at the trough five years after the impulse. Of course, the effects of this same shock on GDP growth, which roughly mirrors the change in the level of hours, are much more short-lived. Output growth returns to its steady state level about two years after the shock hits, but it barely moves above it after that, implying no catch up of the level of GDP towards its previous trend. The persistent drop in the level of economic activity due to the spread shock also leads to a prolonged decline in real marginal costs - which in this model map one-to-one into the labor share (middle left panel)- and, via the New Keynesian Phillips curve, in inflation (middle right panel). Finally, policymakers endogenously respond to the change in the inflation and real activity outlook by cutting the federal funds rate (bottom left panel).

Very similar considerations hold for the MEI shock, which represents a direct hit to the "technological" ability of entrepreneurs to transform investment goods into productive capital, rather than an increase in their funding cost. Although the origins of these two shocks are different, the fact that they both affect the creation of new capital implies very similar effects on the observable variables, as shown by the impulse responses in figure 8. In particular, a positive MEI shock also implies a very persistent increase in investment, output and hours worked, as well as in the labor share and hence inflation. The key difference between the two impulses, which is also what allows us to tell them apart empirically, is that the MEI shock leaves spreads virtually unchanged (bottom right panel).

Another shock that plays an important role in the model, and whose estimated contribution to the Great Recession and its aftermath increased in light of the latest data revisions, is the TFP shock. As shown in figure 9, a positive TFP shock has a large and persistent effect on output growth, even if the reponse of hours is muted in the first few quarters (and slightly negative on impact). This muted response of hours is due to the presence of nominal rigidities, which prevent an expansion of aggregate demand sufficient to absorb the increased ability of the economy to supply output. With higher productivity, marginal costs and thus the labor share fall, leading to lower inflation. The policy rule specification implies that this negative correlation between inflation and real activity, which is typical of supply shocks, produces countervailing forces on the interest rate, which as a result moves little. These dynamics make the TFP shock particularly suitable to account for the first phase of the recovery, in which GDP growth was above trend, but hours and inflation remained weak. With the recent softening of the expansion, though, the role of TFP shocks is fading.

The last shock that plays a relevant role in the current economic environment is the mark-up shock, whose impulse response is depicted in figure 10. This shock is an exogenous source of inflationary pressures, stemming from changes in the market power of intermediate goods producers. As such, it leads to higher inflation and lower real activity, as producers reduce supply to increase their desired markup. Compared to those of the other prominent supply shock in the model, the TFP shock, the effects of markup-shocks feature significantly less persistence. GDP growth falls on impact after mark-ups increase, but returns above average after about one year. Inflation is sharply higher, but only for a couple of quarters, leading to a temporary spike in the nominal interest rate, as monetary policy tries to limit the pass-through of the shock to inflation. Unlike in the case of TFP shocks, however, hours fall immediately, mirroring the behavior of output.

Forecasts

Unconditional Forecast	2011 (Q4/Q4)		2012 (Q4/Q4)		2013 (Q4/Q4)		2014 (Q4/Q4)	
	Oct	June	Oct	June	Oct	June	Oct	June
Core PCE Inflation	1.5 (1.3,1.7)	0.8 (0.2,1.4)	1.0 (0.2,1.6)	1.2 (0.3,1.9)	1.3 (0.4,2.0)	1.5 (0.6,2.4)	1.6 (0.6,2.4)	1.8 (0.8,2.6)
Real GDP Growth	0.9 (0.2,1.5)	2.7 (-0.2,4.8)	1.7 (-1.6,4.2)	2.1 (-1.3,5.0)	1.6 (-2.1,4.5)	2.0 (-1.2,5.4)	1.8 (-1.6,5.1)	2.2 (-0.9,5.7)

Conditional Forecast*	2011 (Q4/Q4)		2012 (Q4/Q4)		2013 (Q4/Q4)		2014 (Q4/Q4)	
	Oct	June	Oct	June	Oct	June	Oct	June
Core PCE Inflation	1.8 (1.6,2.0)	1.4 (1.0,1.7)	1.0 (0.3,1.7)	1.1 (0.3,1.8)	1.3 (0.4,2.0)	1.5 (0.6,2.3)	1.6 (0.7,2.4)	1.7 (0.8,2.6)
Real GDP Growth	1.8 (1.1,2.4)	2.1 (0.7,3.2)	2.6 (-0.8,5.0)	2.0 (-1.3,4.6)	1.8 (-1.8,4.8)	1.9 (-1.2,5.2)	1.8 (-1.5,5.1)	2.2 (-1.0,5.6)

*The unconditional forecasts use data up to 2011Q2, the quarter for which we have the most recent GDP release, as well as the federal funds rate and spreads data for 2011Q3. In the conditional forecasts, we further include the 2011Q3 FRBNY staff projections for GDP growth, core PCE inflation, and hours worked as additional data points. Numbers in parentheses indicate 68 percent probability intervals.

We detail the forecast of three main variables over the horizon 2011-2014: real GDP growth, core PCE inflation and the federal funds rate. The federal funds rate expectations are constrained to be 25bp through 2013Q2. We capture policy anticipation by adding anticipated monetary policy shocks to the central bank's reaction function, following Laseen and Svensson (2009).

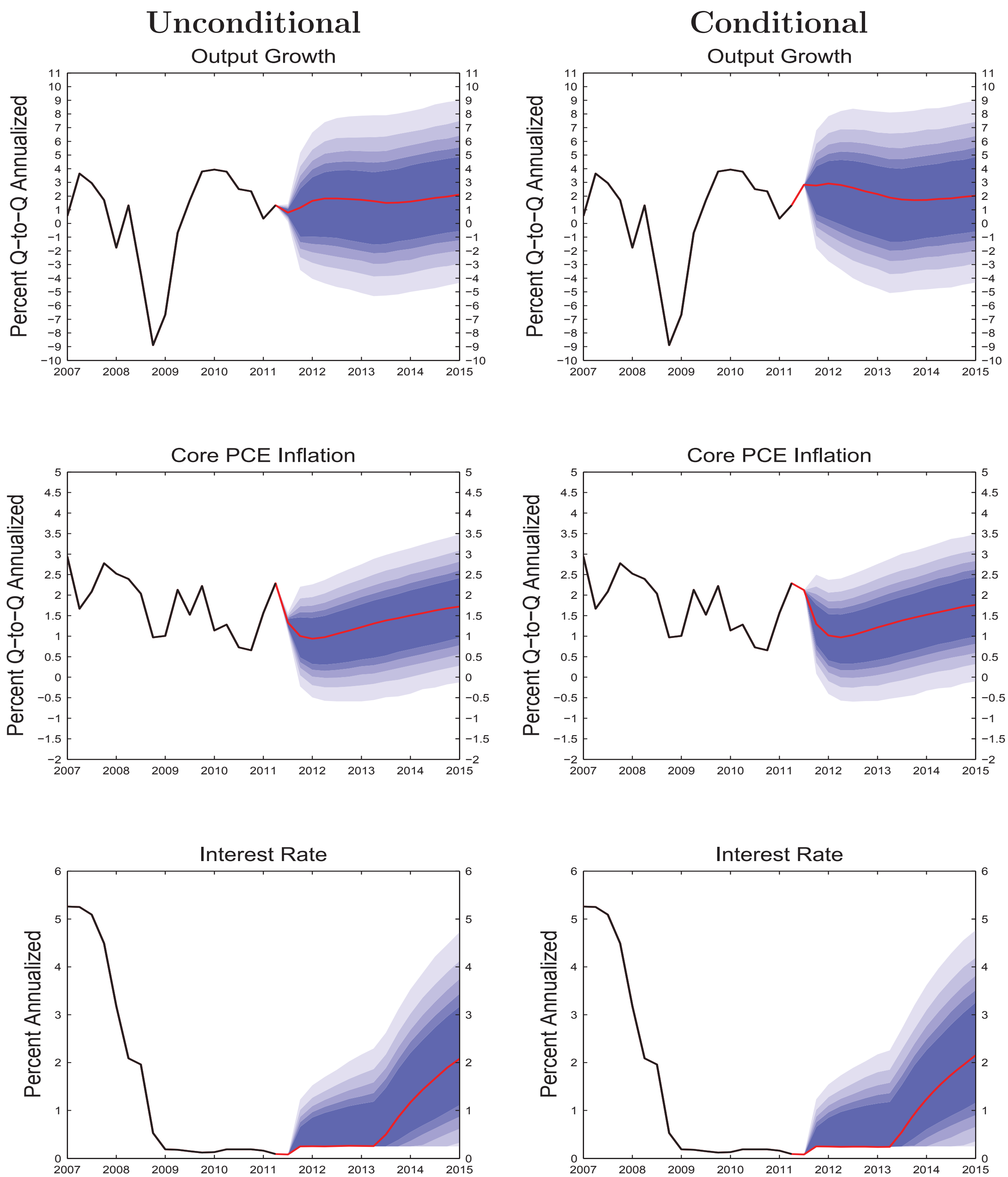
The table above presents Q4/Q4 forecasts for real GDP growth and inflation for 2011-2014, with 68 percent probability intervals. We include two sets of forecasts. The *unconditional* forecasts use data up to 2011Q2, the quarter for which we have the most recent GDP release, as well as the federal funds rate and spreads data for 2011Q3, which are currently available. In the *conditional* forecasts, we further include the 2011Q3 FRBNY staff projections for GDP growth, core PCE inflation, and hours worked as additional data points (as of October 19, the staff projections for 2011Q3 are 2.8% for output growth, 2.1% for core PCE inflation, and 0.1% growth for hours worked). Treating the staff forecasts as data allows us to incorporate into the DSGE forecasts information about the current quarter that is not yet available in the data. In addition to providing the current forecasts, for comparison we report the forecasts included in the memo discussed at the June FOMC meeting (we did not report forecasts for 2014 in that memo).

Figure 2 presents quarterly forecasts, both unconditional (left panels) and conditional (right panels). In the graphs, the black line represents data, the red line indicates the mean forecast, and the shaded areas mark the uncertainty associated with our forecast as 50, 60, 70, 80 and 90 percent probability intervals. Output growth and inflation are expressed in terms of percent annualized rates, quarter to quarter. The interest rate is the annualized quarterly average. The bands reflect both parameter uncertainty and shock uncertainty. Figure 3 compares the current forecasts with those produced for the June FOMC meeting.

Our inflation forecasts are notably higher for 2011 than they were in June, as the FRBNY DSGE model was surprised by the strong readings for core PCE inflation in 2011. In spite of this, inflation forecasts from 2012 onwards are actually lower than in June. This is due to the weaker forecasts for economic activity, as discussed below. For both the 2012 and 2013 forecast horizons the 68% confidence bands for Q4/Q4 inflation are within the 0-2% interval, implying that the model places great probability on the event that inflation is below the implicit FOMC target at least through 2013. In 2014 the expected inflation (Q4/Q4) is 1.6%, and the 68% bands are between 0.7 and 2.4%.

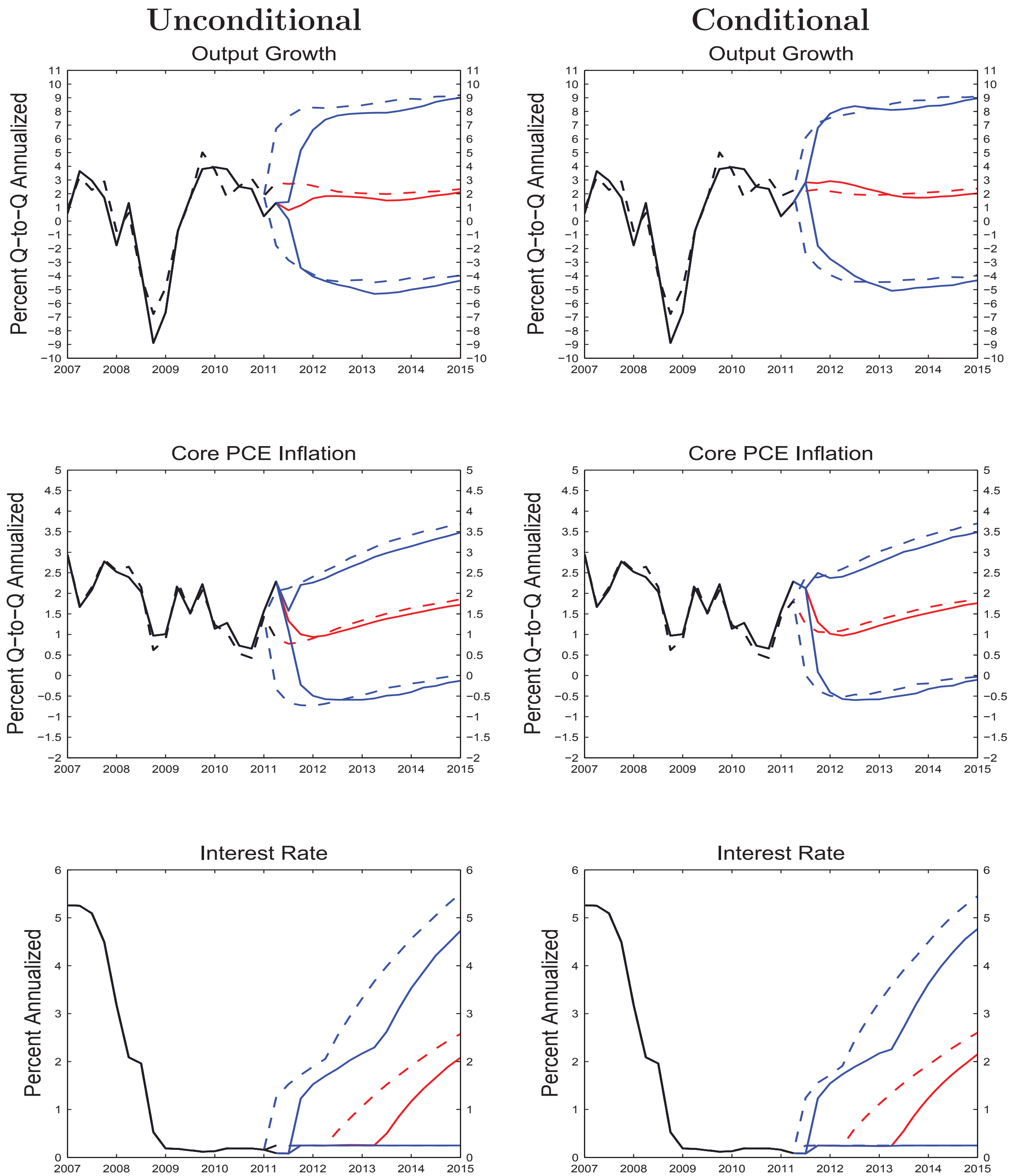
The FRBNY DSGE forecasts for real output growth in June were already fairly weak, as they projected growth between 2 and 3% for the remainder of 2011 and around 2% in 2012 and 2013. Current forecasts are even weaker in 2011 as a result of NIPA revisions and weak new data. The mean projections for output growth in 2011 (Q4/Q4) are 0.9% and 1.8% for the unconditional and conditional forecasts, respectively. Note from Figure 2 that the FRBNY staff projection for 2011Q3 is about 2% higher than the model's unconditional forecast. This relatively more upbeat assessment of current conditions carries over to 2011Q4 and to the end of 2012, explaining the difference in Q4/Q4 forecasts for 2012 (2.6 versus 1.7% for the conditional and unconditional forecast, respectively). In 2013 and 2014 the two forecasts are quite similar, and envision growth slightly below 2% in both years. There is significant uncertainty around the forecasts: the 25th percentile of the output growth forecast distribution is below zero throughout the forecast distribution, implying that the chance of negative readings for growth is larger than 1/4 in any given quarter. The 75th percentile is between 3.5 and 5.0%.

Figure 2: Forecasts



Black lines indicate data, red lines indicate mean forecasts, and shaded areas mark the uncertainty associated with our forecast as 50, 60, 70, 80, and 90 percent probability intervals.

Figure 3: Change in Forecasts



Solid and dashed red lines represent the mean for current and June's forecast, respectively. Solid and dashed blue lines represent 90 percent probability intervals.

Interpreting the Forecasts

To understand the forecasts, we use the shock decomposition shown in Figure 4. This figure quantifies the importance of each shock for output growth, core PCE inflation, and the federal funds rate (FFR) from 2007 on, by showing the extent to which each of the disturbances contributes to keeping the variables from reaching their long-run values. Specifically, in each of the three panels the solid line (black for realized data, red for mean forecast) shows the variable in deviation from its steady state (for output, the numbers are per capita, as the model takes population growth as exogenous; for both output and inflation, the numbers are quarter-to-quarter annualized). The bars represent the contribution of each shock to the deviation of the variable from steady state, that is, the counterfactual values of output growth, inflation, and the federal funds rate (in deviations from the mean) obtained by setting all other shocks to zero. By construction, for each observation the bars sum to the value on the solid line.

The figure shows that all three variables of interest are currently below their steady-state values, and are forecast to stay so through the end of the forecast horizon. Two of the shocks most responsible for the Great Recession, the so-called "financial" shocks (Spread and MEI), are still the main drivers of the outlook a few years after the end of the recession. This is quite evident for inflation and interest rates, where it is clear that MEI and Spread shocks (azure and purple bars, respectively) play a key role in keeping these two variables below steady state.

This feature of the DSGE forecast is less evident for real output growth, as the contribution of MEI shocks seems small, particularly toward the end of the forecast horizon, and the contribution of Spread shocks is negligible (and positive). However, recall that a small, but still negative, effect on output *growth* implies that the effect of the MEI shocks on the *level* of output is getting *larger*, even several quarters after the occurrence of the shock. Similarly, the fact that the growth impact of Spread shock is positive but very small implies that the level of output is very slowly returning to trend. This is evident in the protracted effect of Spreads and MEI shocks on aggregate hours, shown in the impulse responses of Figures 7 and 8, respectively, and discussed above. In turn, the fact that economic activity is well below trend pushes inflation and consequently interest rates (given the Fed's reaction function) below steady state.

Some more insight about the interpretation of the "financial" shocks – MEI and Spread shocks – can be obtained from Figure 5. This figure shows the recent history of the shocks, expressed in standard deviation units. The panel labeled "Spread" shows that during the Great Recession there were two large Spread shocks, one in 2007 and one in concurrence with the Lehman Brothers default (recall from Figure 7 that positive Spread shocks raise spreads and have negative impact on economic activity). The panel labeled "MEI" shows that MEI shocks were mostly negative from 2009 onwards, that is, *after* the end of the recession (recall from Figure 8 that negative MEI shocks have negative impact on economic activity). These shocks therefore seem to capture the headwinds from the financial crisis. (The Spread shocks capture headwinds associated with a large spread while the MEI shocks capture financial headwinds that are not reflected necessarily in the spread.)

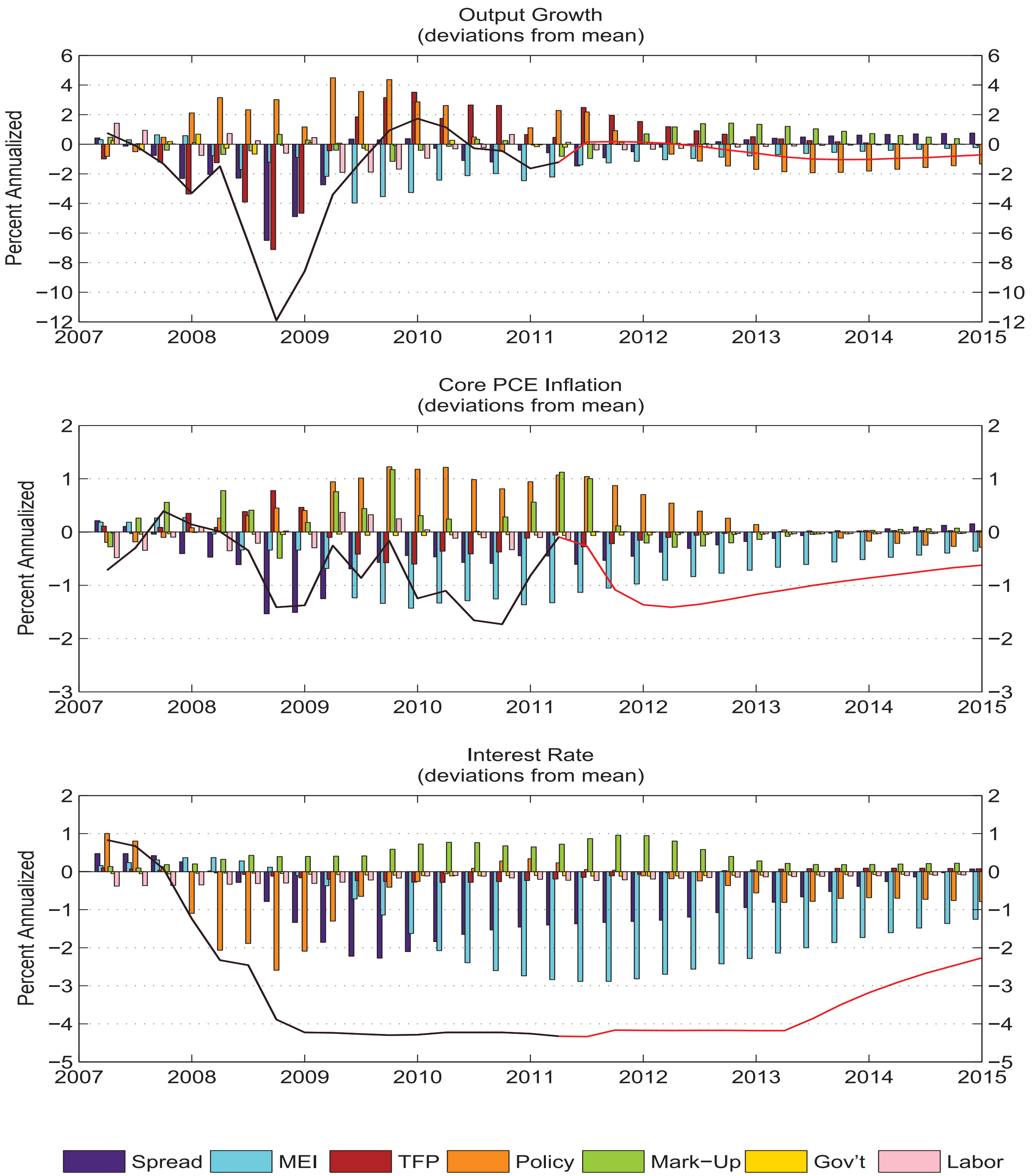
In discussing the weak outlook for real activity we emphasized so far the impact of shocks on deviations of the level of output from trend. But the trend may also shift, as pointed out by the literature on unit roots. The FRBNY DSGE model allows for a stochastic trend in total factor productivity, hence in output, and shifts in the trend can obviously have a significant impact on the outlook. Is there evidence of any such shifts? Figure 6 addresses this question. The figure shows the stochastic trend in TFP (solid lines) together with the deterministic component of that trend (dashed blue lines), both expressed in logarithms and normalized to zero at the beginning of 2007Q1. Deviations of the solid from the dashed line represent shifts in the trend. Because of the decline in productivity during the recession, by 2009 the trend had shifted down by about 6% relative to the deterministic drift. The pick-up in productivity in the second half of 2009, however, almost completely erased that gap, which now stands at about 0.5%. That shows that there is little evidence that shifts in the trend are responsible for the bleak outlook.

Other shocks, beside the "financial" shocks, play an important role in explaining the forecasts. For instance, the model attributes much of the rise in core inflation in 2011 to price mark-up shocks. By assumption these shocks push inflation above marginal costs. Figure 10 shows that mark-up shocks capture large but transitory movements in inflation, such as those due to oil price fluctuations. As a result, the large positive mark-up shock behind the up-tick in inflation in recent quarters has almost no effect on the inflation forecasts. Mark-up shocks also appear to sustain output growth in 2012 and 2013. This is due to the fact that the effect of mark-up shocks on the *level* of output, while negative, vanishes over time, producing a positive effect in terms of growth rates.

Finally, according to the model, monetary policy shocks were largely expansionary in recent history, and especially in 2008. These shocks include both contemporaneous and anticipated deviations from the feedback rule, which we use to implement the lower bound through 2013Q2. Notably, the impact of policy shocks on the interest rate is currently very small, implying that the level of the interest rate is not too far from that implied by the estimated policy rule. In 2013 and 2014 the impact of these shocks becomes larger: the impact of the forward guidance, combined with the interest rates smoothing component of the policy which limits quarter-to-quarter adjustments, implies that the renormalization path is lower than that implied by the estimated rule. (Note however that this path is associated with a rather dim outlook for output and inflation, indicating that the estimated interest rate rule may be far from optimal under current circumstances.)

Policy shocks play an important role in pushing inflation upward in the aftermath of the recession. Interestingly, while policy shocks have a positive impact on current output growth, they have a negative impact on growth from 2012 onward. As much as this result may seem counter-intuitive at first, it is actually the natural consequence of the fact that the impact of expansionary monetary policy on the *level* of output, while still positive, is fading, implying that the effect on the *growth rate* is currently negative (as the level of output returns to its trend from below). This is partly because the stimulative effect of the "extended period" language is front-loaded, and hence had most impact in the current quarter.

Figure 4: Shock Decomposition



The shock decomposition is presented for the conditional forecast. The solid lines (black for realized data, red for mean forecast) show each variable in deviation from its steady state. The bars represent the shock contributions; specifically, the bars for each shock represent the counterfactual values for the observables (in deviations from the mean) obtained by setting all other shocks to zero.

Figure 5: Shock Histories

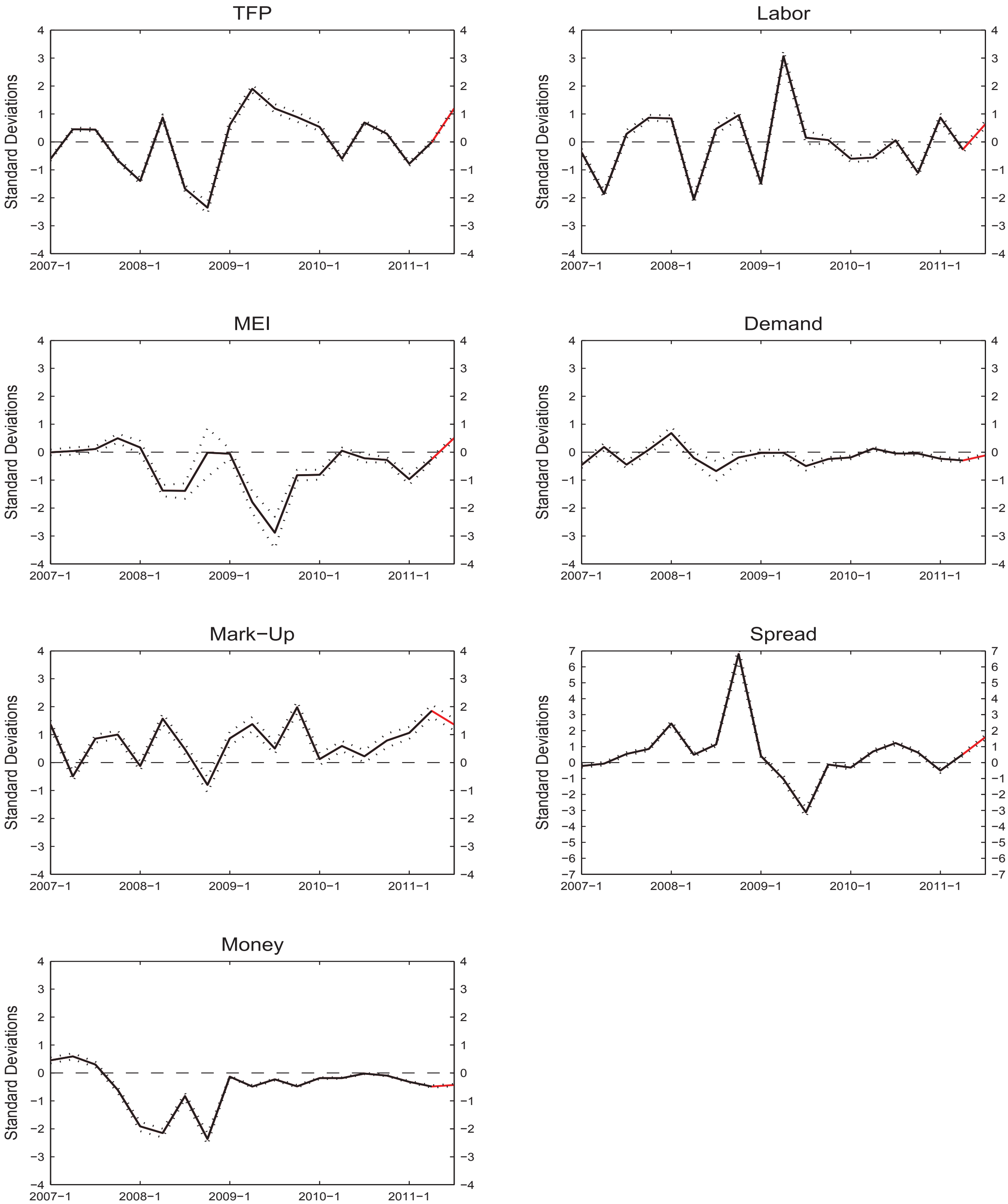
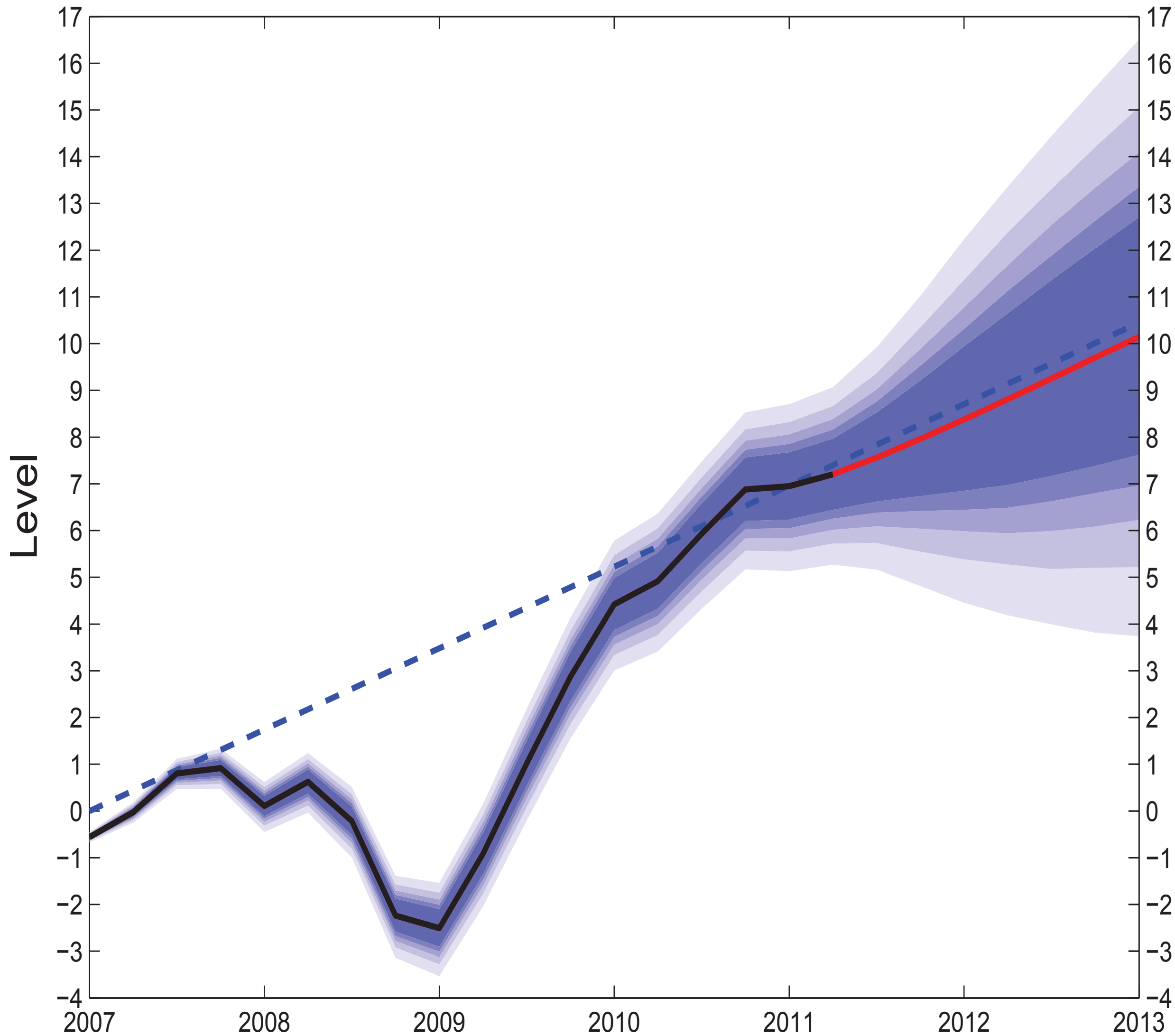


Figure 6: Trend Components of Productivity



The trend in productivity is decomposed as the sum of a deterministic component (dotted blue line) and a stochastic component (solid line; black for realized data and red for forecast). Shaded areas mark the uncertainty associated with our estimate as 50, 60, 70, 80, and 90 percent probability intervals.

Figure 7: Responses to a Spread Shock

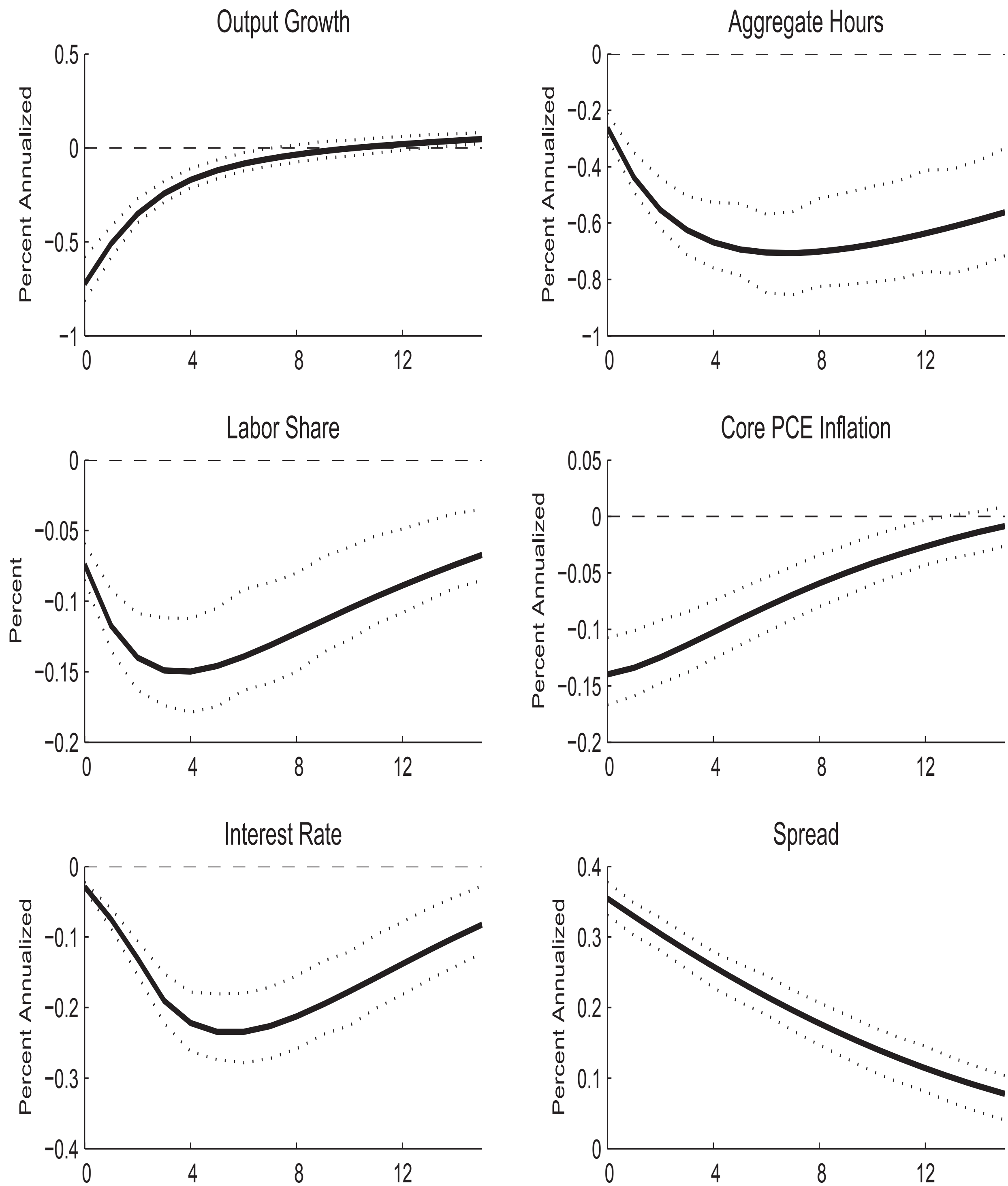


Figure 8: Responses to an MEI Shock

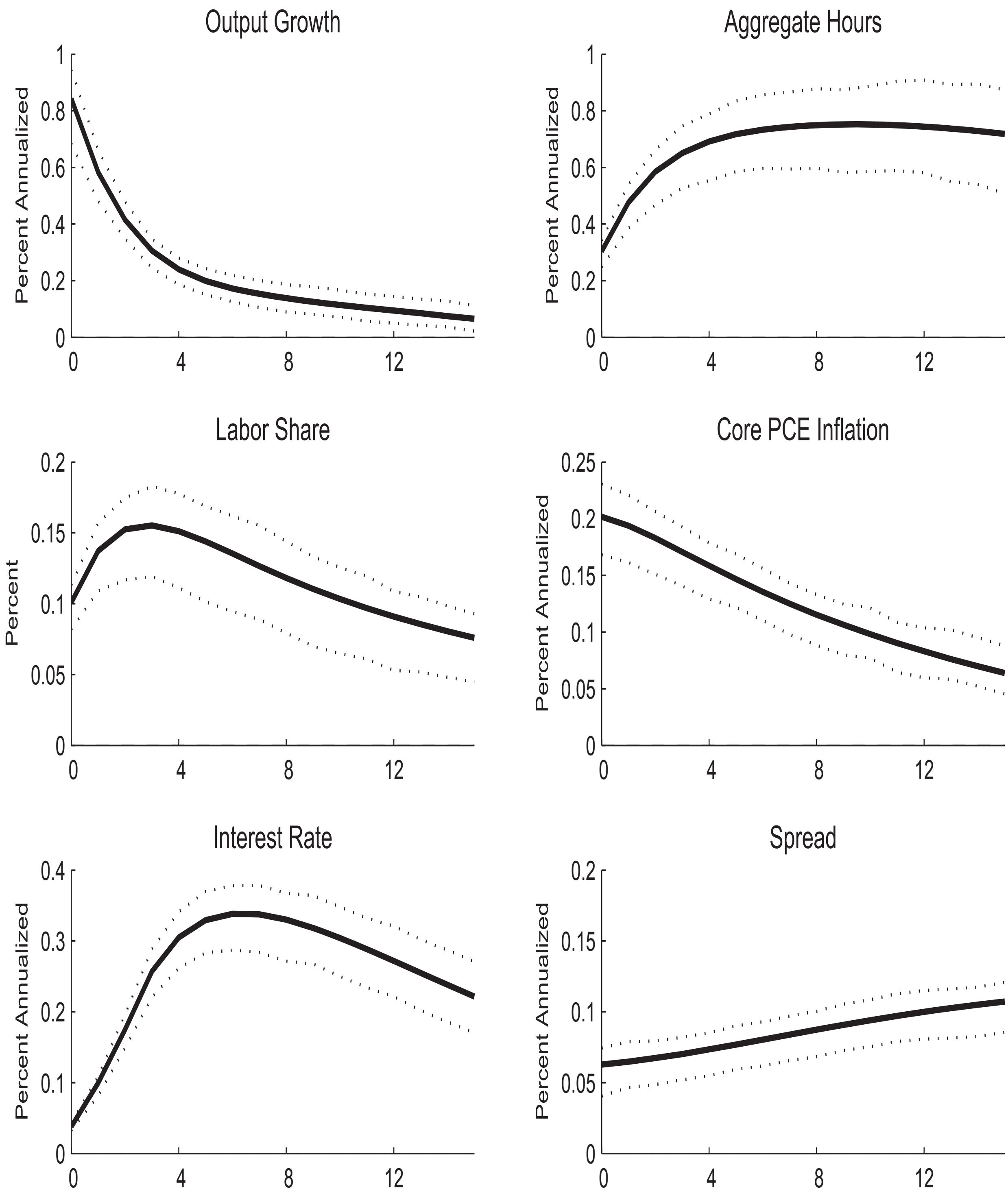


Figure 9: Responses to a TFP Shock

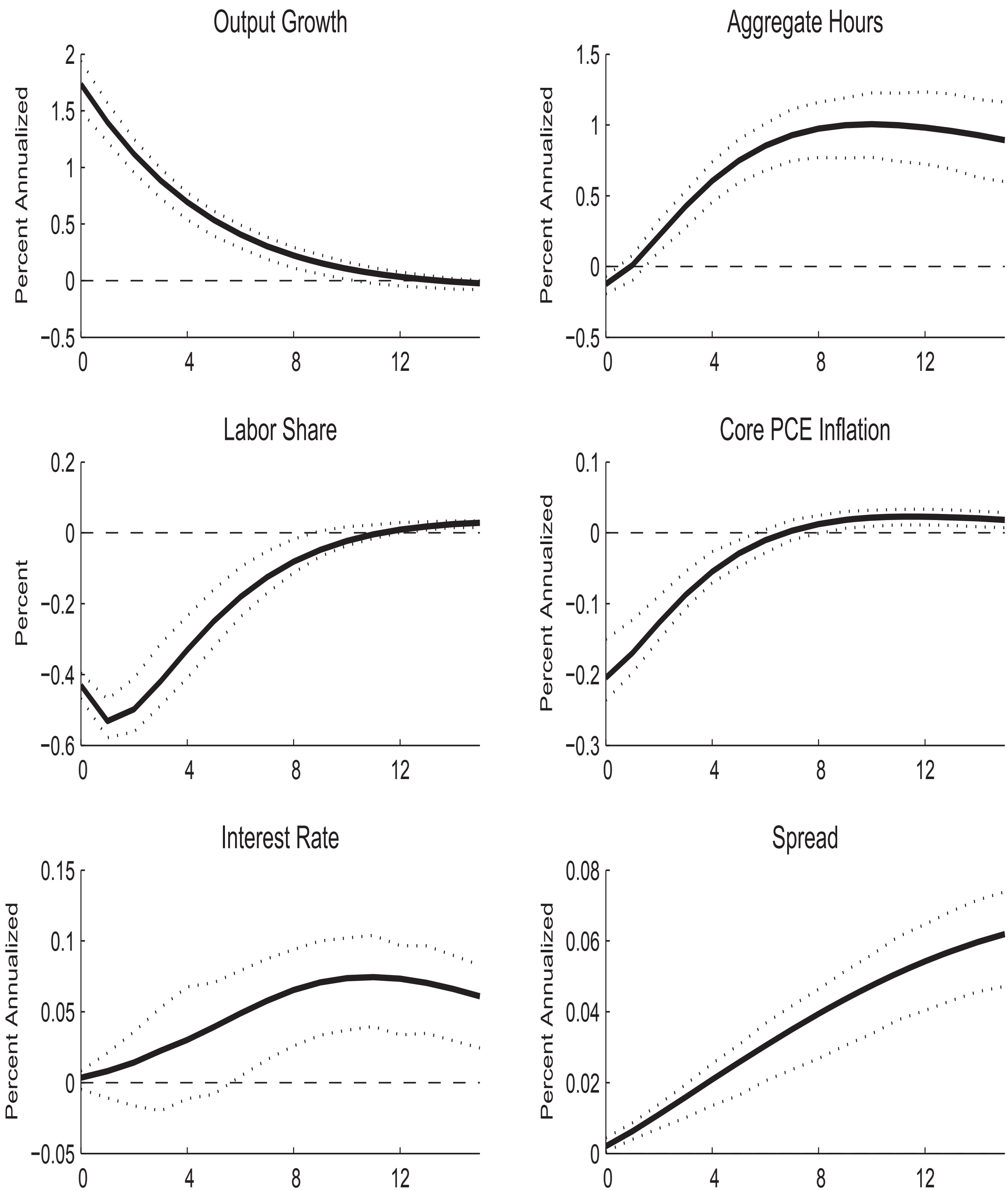


Figure 10: Responses to a Mark-up Shock

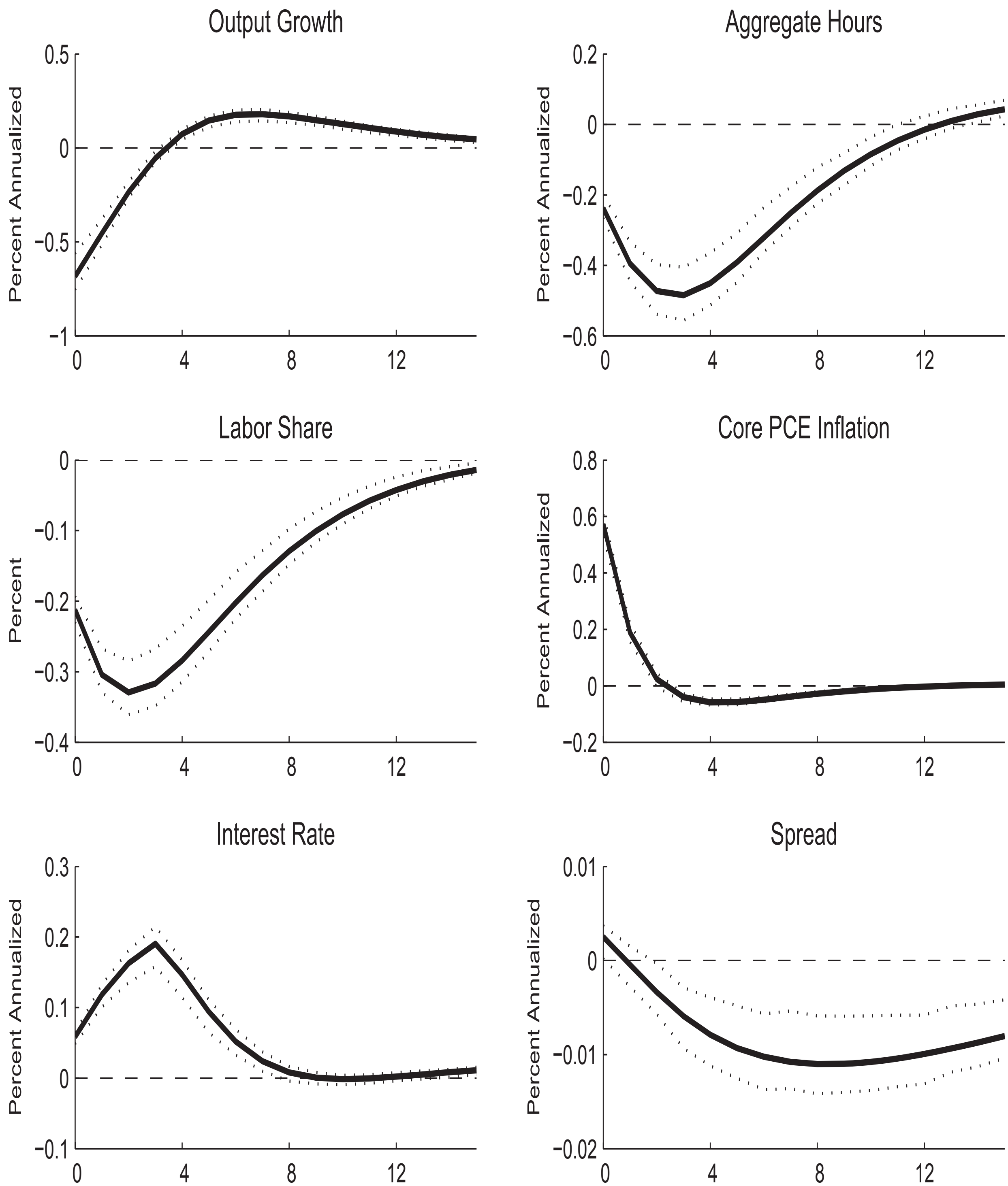


Figure 11: Responses to a Monetary Policy Shock

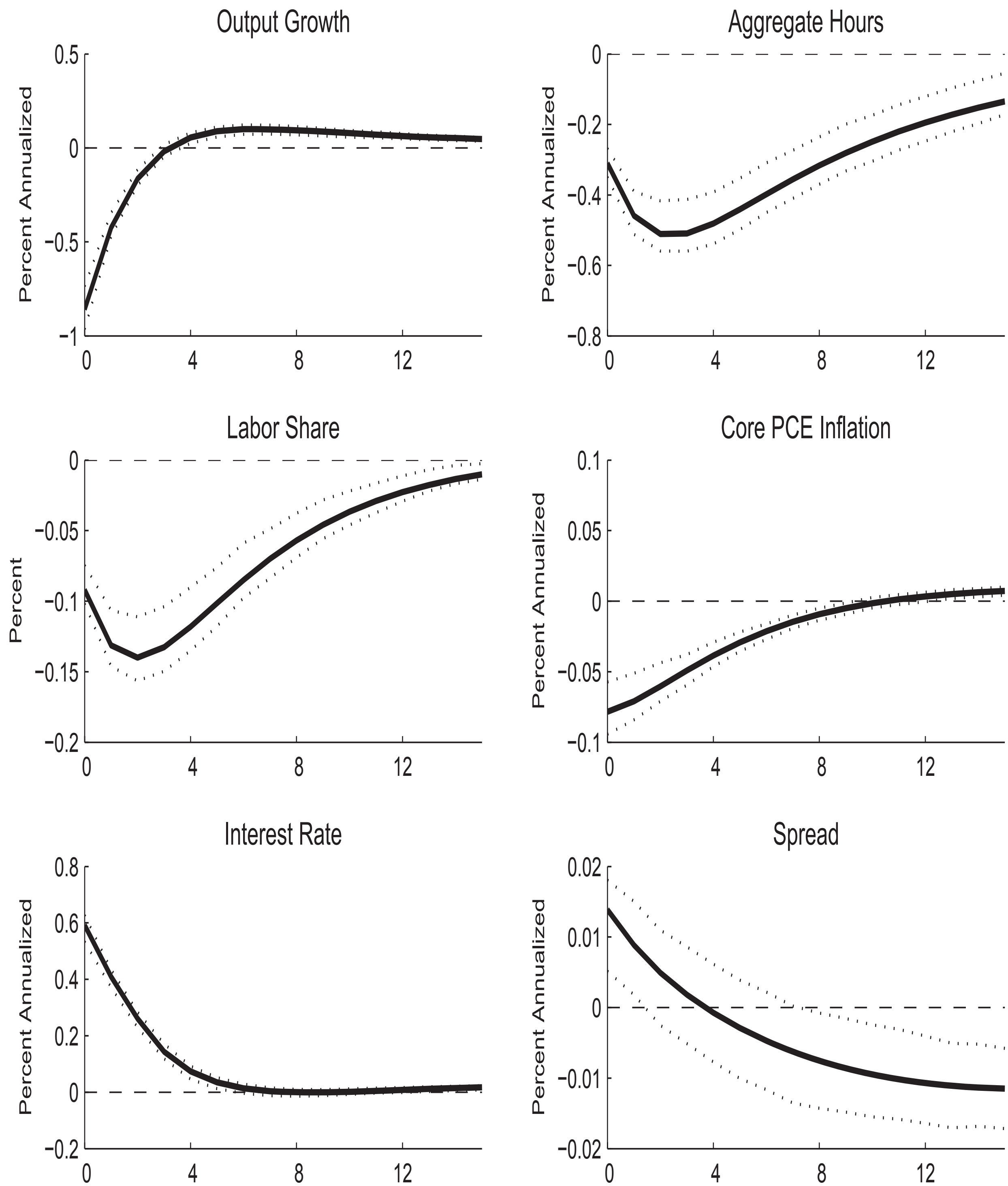


Figure 12: Responses to a Labor Supply Shock

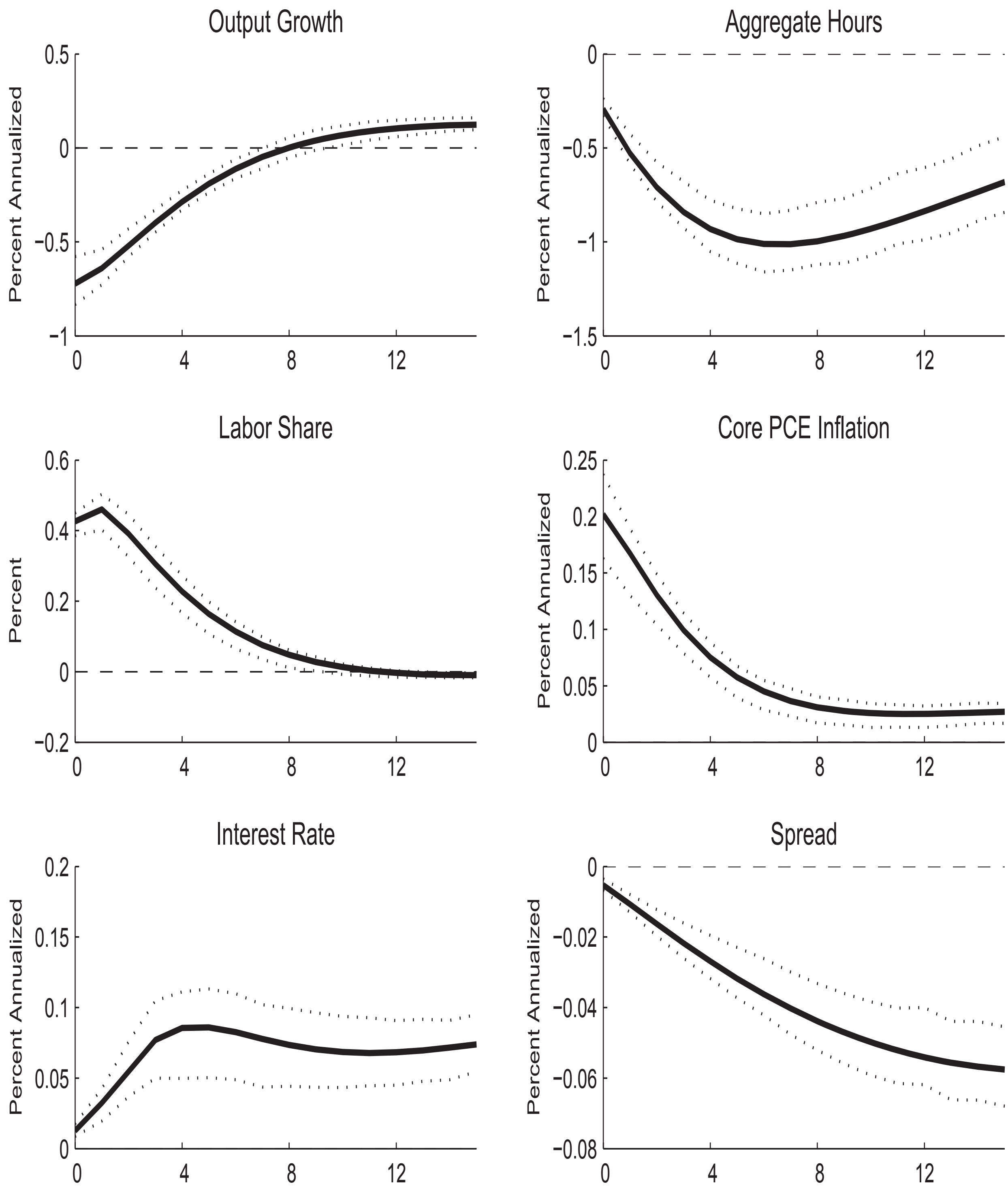
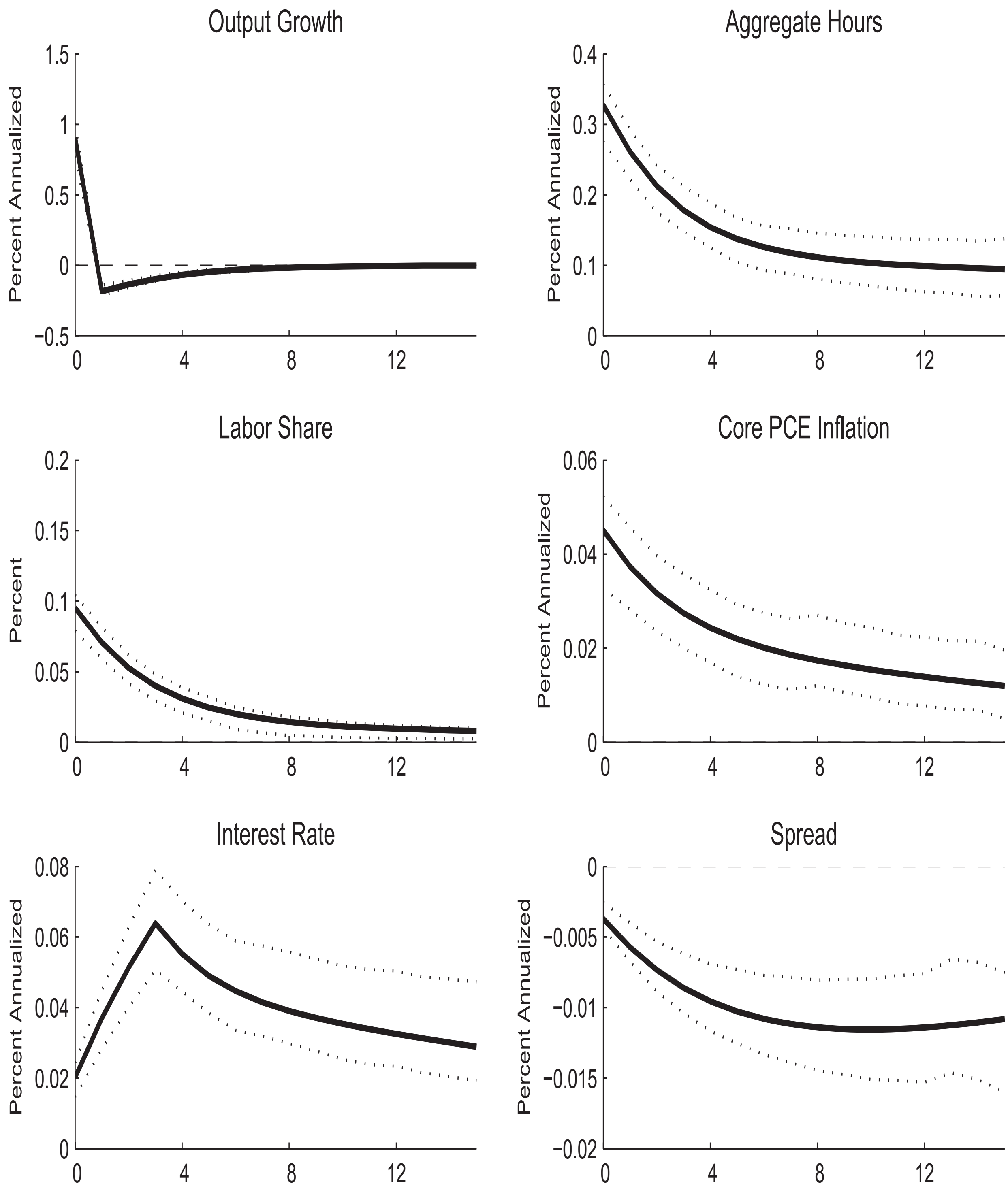


Figure 13: Responses to a Government Spending Shock



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Detailed Philadelphia (PRISM) Forecast Overview

October 2011

Keith Sill

Forecast Summary

The FRB Philadelphia DSGE model denoted PRISM, projects that real GDP growth will rebound fairly strongly over the forecast horizon with real output growth approaching 5 percent by mid 2012. Inflation is projected to be well contained at 1.5 percent through 2011, even with significantly above-trend output growth. For this forecast round, we have implemented the assumption that the federal funds rate remains in a range of 0 to 0.25 percent through mid-2013. Monetary policy begins to tighten by the third quarter of 2013 in accord with the estimated monetary policy rule, and reaches a bit over 2 percent in 2014Q4. Currently, many of the model's state variables are well below their steady-state values. In particular, consumption, investment, and the capital stock are low relative to steady state, and absent any shocks, the model would predict a rapid recovery. These state variables have been below steady state since the end of the recession. The relatively slow recovery to date and the low inflation that has recently characterized U.S. economic activity require the presence of shocks to offset the strength of the model's internal propagation channels.

The Current Forecast and Shock Identification

The PRISM model is an estimated New Keynesian DSGE model with sticky wages, sticky prices, investment adjustment costs, and habit persistence. The model is similar to the Smets & Wouters 2007 model and is described more fully in Schorfheide, Sill, and Kryshko 2010. Unlike in that paper though, we estimate PRISM directly on core PCE inflation rather than projecting core inflation as a non-modeled variable. Details on the model and its estimation are available in a Technical Appendix that was distributed for the June FOMC meeting or is available on request.

The current forecasts for real GDP growth, core PCE inflation, and the federal funds rate are shown in Figures 1a-1c along with the 68 percent probability coverage intervals. The forecast uses data through 2011Q2 supplemented by observations on 2011Q3 from the most recent Macroadvisers forecast. The model takes third quarter output growth of 2.4 percent as given and the projection begins with 2011Q4. PRISM sees a strong rebound in real GDP growth, which rises to about 5 percent by mid 2012. Output growth begins to taper off a bit in 2013, and falls to a 4 percent pace by 2014Q4. While output growth is fairly robust, core PCE inflation stays moderate, dropping from a bit over 2 percent in mid 2011to 1.5 percent through most of the forecast horizon. Based on the 68 percent coverage interval, the model sees little chance of deflation over the next 3 years and almost no chance of recession. The federal funds rate is constrained at the zero bound through mid-2013. Thereafter, the model dynamics take over and the funds rate rises to a bit over 2 percent in 2014Q4.

The key factors driving the projection are shown in the forecast shock decompositions (shown in Figures 2a-2c) and the smoothed estimates of the model's primary shocks (shown in Figure 3, where they are normalized by standard deviation). The primary shocks driving above-trend real output growth over the next 3 years are financial shocks in the form of discount factor shocks (labeled Fin) and labor supply shocks (labeled Labor). PRISM estimates a long series of largely negative shocks to labor supply since 2008. These shocks have a persistent negative effect on hours worked and so account for a large share of the decline in output in 2009 and 2010. These shocks have pushed hours worked well below steady state, and as they unwind over the projection period the labor market recovers and output growth is pushed above trend.

The model also estimates a sequence of largely negative discount factor shocks since 2008. All else equal, these shocks push down current consumption and push up investment, with the effect being very persistent. Consequently, consumption (nondurables + services) is well below the model's estimated steady state at this point. As these shocks wane over the projection period, consumption growth picks up to an above-4 percent pace over most of the next three years. The negative discount factor shocks worked to push investment growth above steady state since 2009, but the effects were to some extent offset by negative shocks to the marginal efficiency of investment. As these MEI shocks unwind, they give a strong upward impetus to investment growth over 2012-2014. But this effect is offset in part from the waning of discount factor shocks that are putting downward pressure on investment growth. All told though, investment growth runs at about a 7 percent pace in 2012, falling back to about 3 percent growth by the end of the forecast horizon.

The forecast for core PCE inflation is largely a story of upward pressure from the unwinding of labor supply shocks being offset by downward pressure from the waning of discount factor shocks. Negative discount factor shocks have a strong and persistent negative effect on marginal cost and inflation in the estimated model. Compared, for example, to a negative MEI shock that lowers real output growth by 1 percent, a negative discount factor shock that lower real output growth by 1 percent leads to a 3 times larger drop in inflation that is more persistent (see the impulse responses in Figures 6a and 6b). The negative discount factor shock leads to capital deepening and higher labor productivity. Consequently, marginal cost and inflation fall. The negative effect of discount factor shocks on inflation is estimated to have been quite significant since the end of 2008. As these shocks unwind over the projection period there is a decreasing, but still substantial, downward effect on inflation over the next three years. Shocks to price markups also help explain the recent strength of core PCE inflation, but their effects are not very persistent so that inflation is projected to decline in the near-term.

Partly offsetting the downward pressure on inflation from discount factor shocks is the upward pressure coming from labor supply shocks. Labor supply shocks that push down aggregate hours also serve to put upward pressure on the real wage and hence marginal cost. The effect is persistent, so as the labor supply shocks unwind over the forecast horizon they exert a strong, but waning, negative pull on inflation. On balance the effect of these opposing forces is to keep inflation below 2 percent through the forecast horizon.

The Unconditional Forecast

Pinning down the federal funds rate at the zero lower bound through mid-2013 (using fully anticipated monetary policy shocks) has a significant impact on the PRISM forecast. Figures 4a-c show the forecast and shock decompositions for the unconditional forecast (ie, a forecast that does not constrain the funds rate path). The forecasted path for real GDP growth is about 1 percentage point higher over the next 3 years. The projection for core PCE inflation is above 2 percent over the forecast horizon and the federal funds rate begins to rise immediately, reaching nearly 4 percent by 2012Q4. Thus, the forecast is considerably stronger if the funds rate is not constrained at the ZLB through mid-2013.

The shock decompositions provide guidance on the difference between the two forecasts. When the federal funds rate is constrained at the ZLB through mid-2013, it has a strong effect on expected inflation through the monetary policy reaction function (which puts little estimated weight on the output gap). Agents in the model fully anticipate that the funds rate will be low and with policy completely credible, expect that future inflation will be low given the policy reaction function relationship between interest rates and inflation. This feedback has an effect on the estimated current state of the economy and on how historical shocks get allocated by the Kalman filter. In particular, absent constraining the federal funds rate, discount factor shocks play a smaller role in the shock decompositions for inflation and output growth and MEI shocks play a larger role in order for history to be consistent with the higher inflation projection (see Figure 5). Indeed, the smoothed shock estimates show smaller estimated shocks for the discount factor and larger estimated shocks for the marginal efficiency of investment in the case where the funds rate is unconstrained. Consequently, there is less upward impetus on consumption over the forecast horizon and more upward pressure on investment growth. Since discount factor shocks play a smaller role in the historical shock decomposition, there is less downward pressure on inflation over the forecast horizon as the shocks unwind. On the other hand, the larger role for MEI shocks in the historical decomposition implies upward pressure on inflation in 2013 and 2014 as the shocks unwind.

On balance, constraining the funds rate in the forecast has the effect of altering the composition of shocks away from the discount factor shock and toward the MEI shock. The result is higher inflation and output growth over the forecast horizon. With inflation running above target and strong output growth, PRISM forecasts that the funds rate should begin rising immediately, reaching near 3 percent by mid-2013 -- roughly 300 basis points above the constrained path federal funds rate at that point. By 2014Q4, the unconstrained funds rate is near 4 percent, roughly 200 basis point above the constrained funds rate path forecast.

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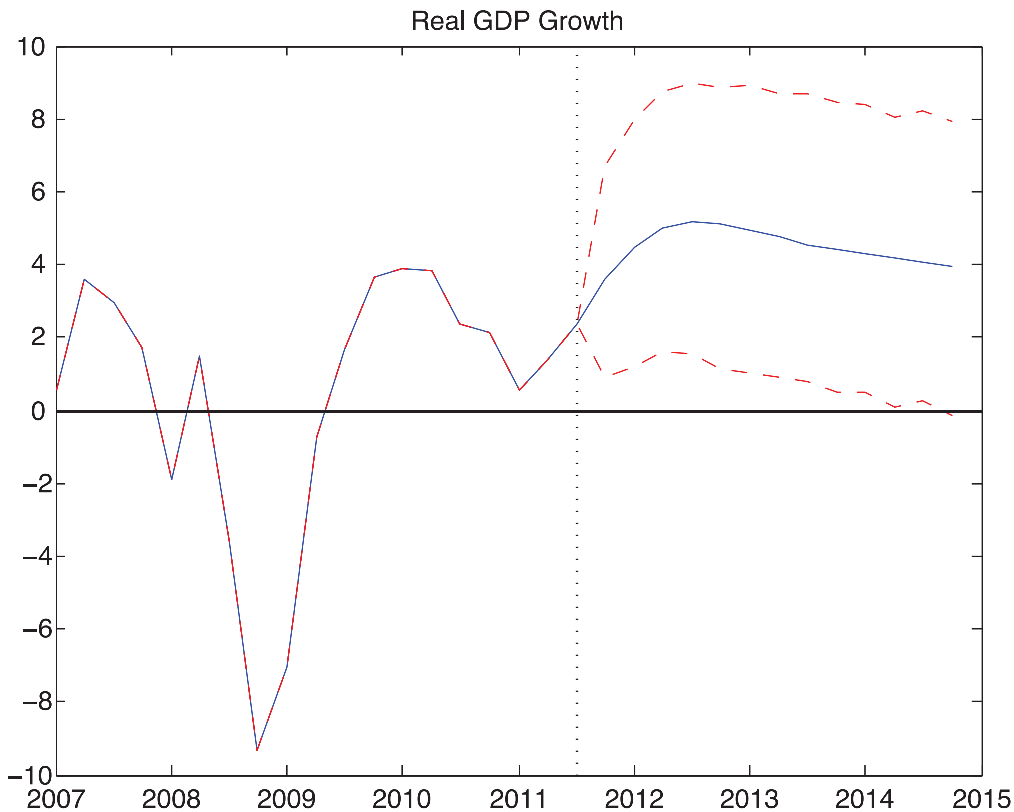
Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586-606.

Figure 1a



Real GDP Growth
10
8
6
4
2
0
−2
−4
−6
−8
−10
2007
2008
2009
2010
2011
2012
2013
2014
2015

Figure 1b

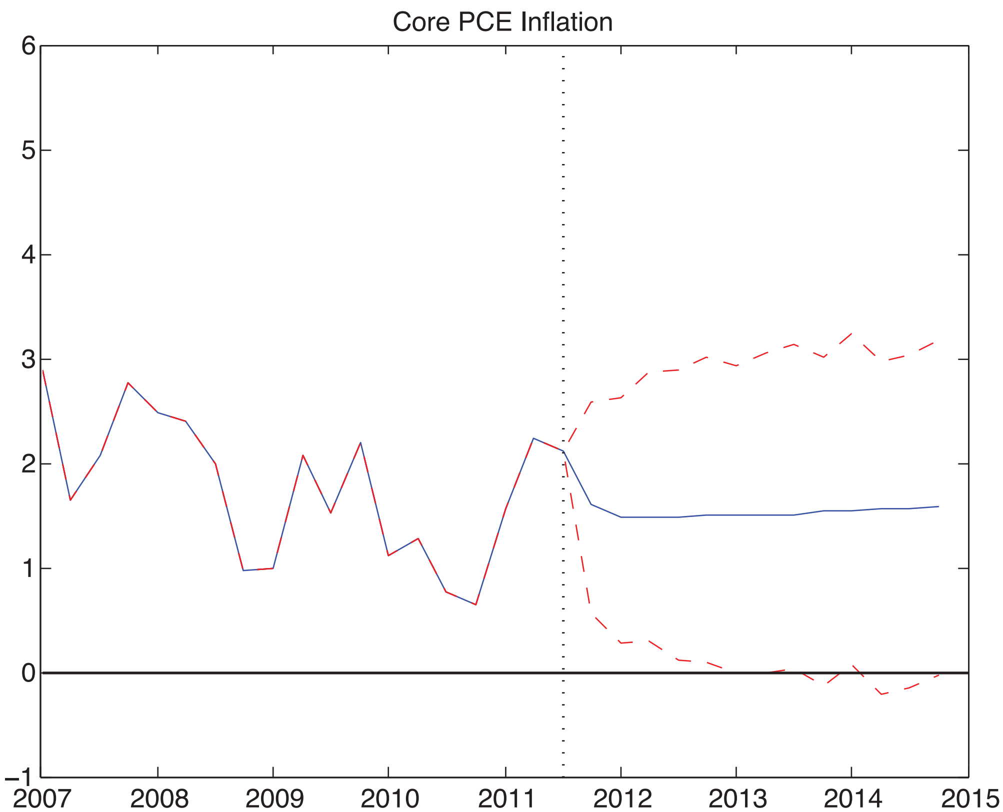

Core PCE Inflation
6
5
4
3
2
1
0
−1
2007
2008
2009
2010
2011
2012
2013
2014
2015

Figure 1c

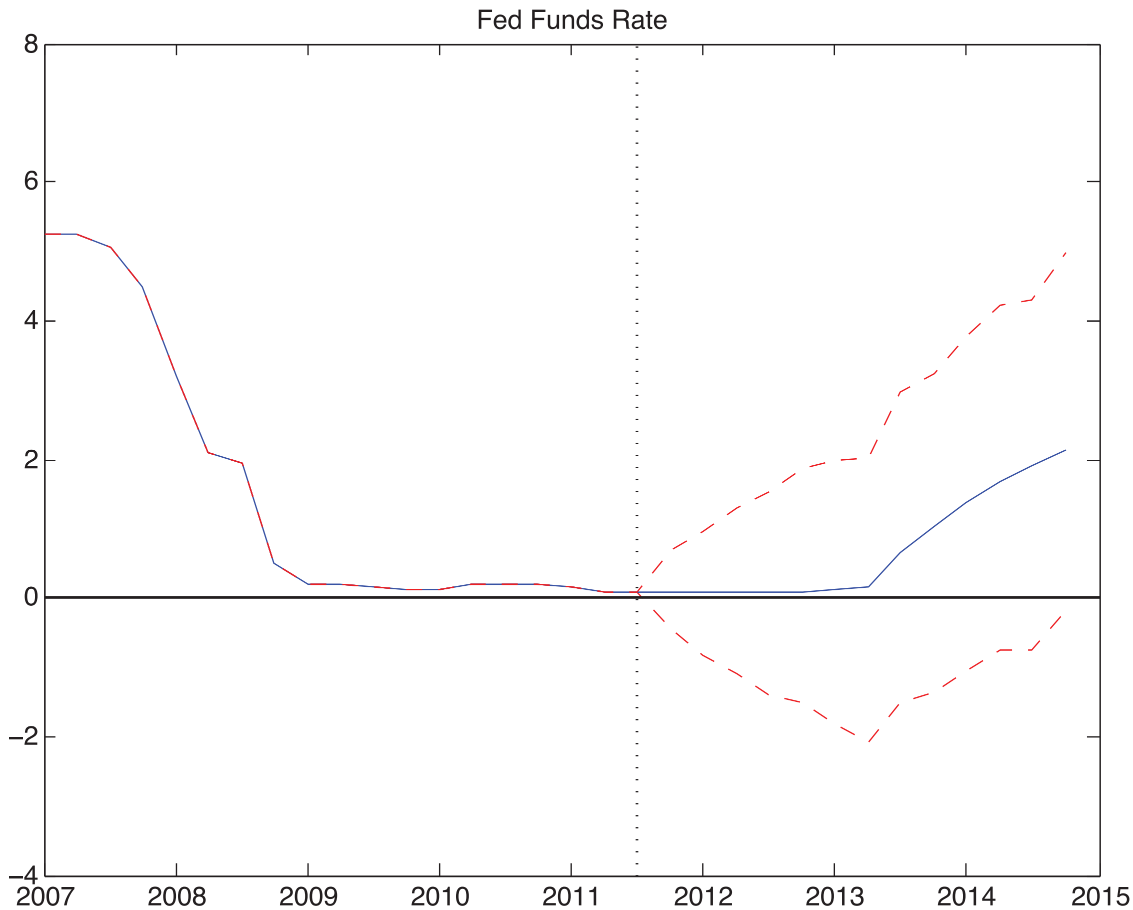
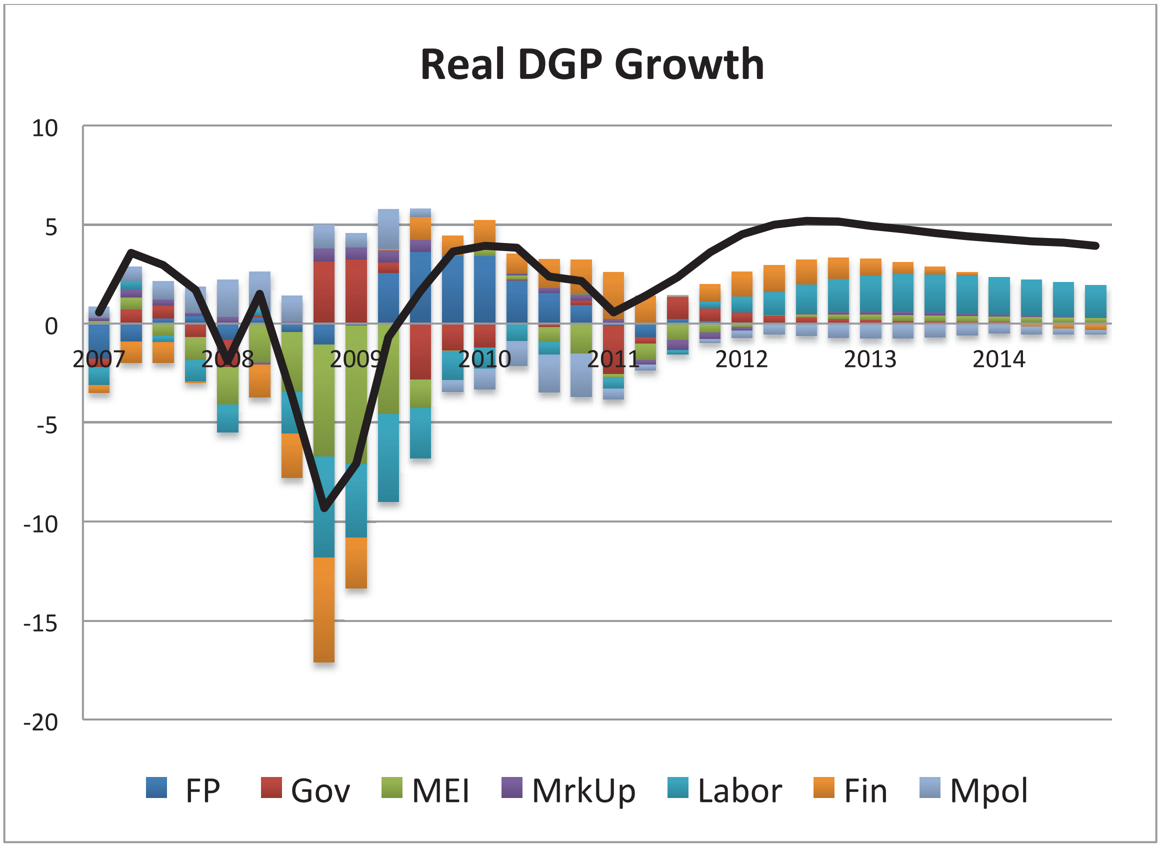

Fed Funds Rate
8
6
4
2
0
−2
−4
2007
2008
2009
2010
2011
2012
2013
2014
2015

Figure 2a
Conditional Forecast



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

Figure 2b
Conditional Forecast

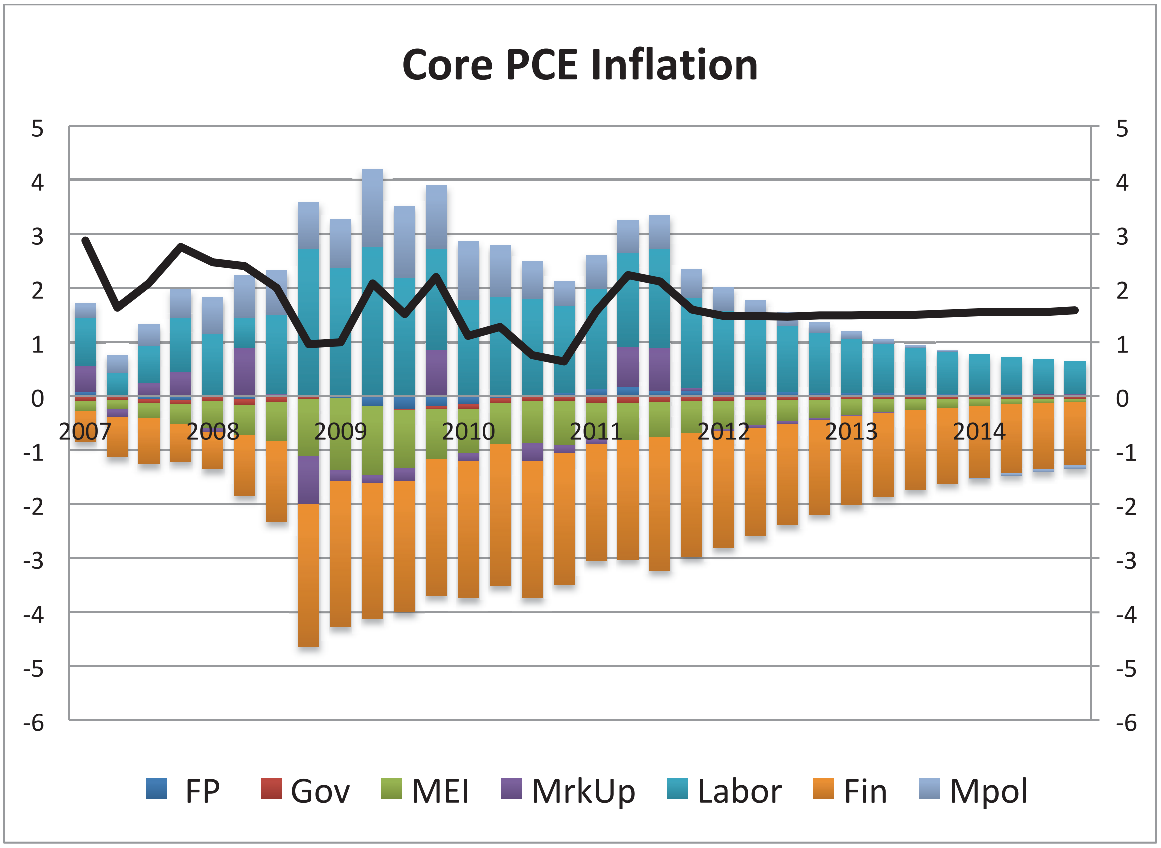


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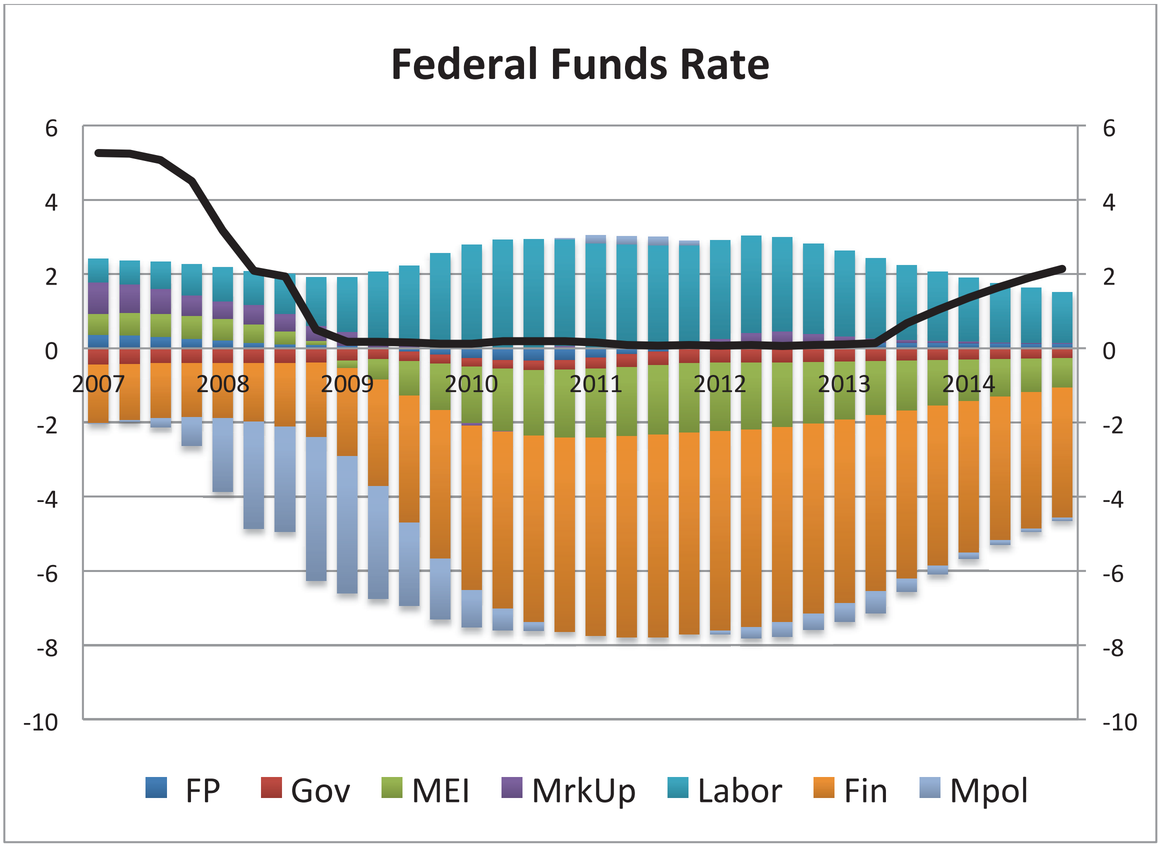
Figure 2c
Conditional Forecast



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MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

Figure 3
Smoothed Shock Estimates For Conditional Forecast Model
(normalized by standard deviation)

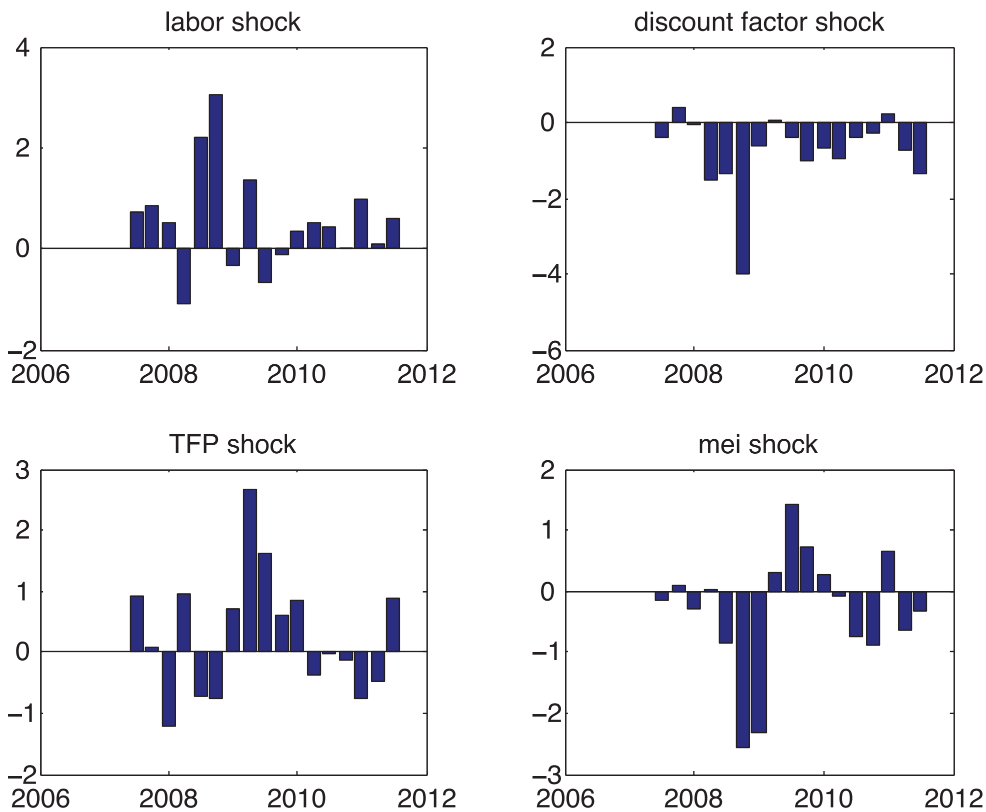
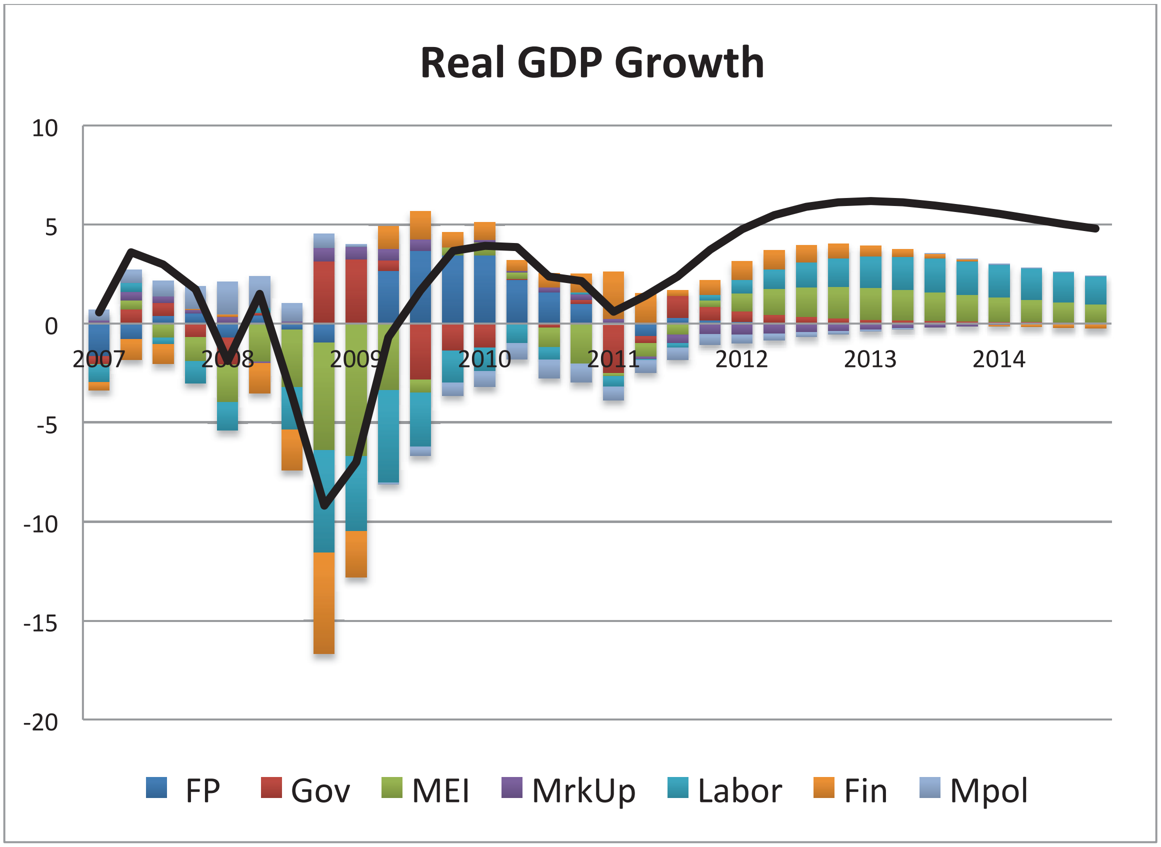


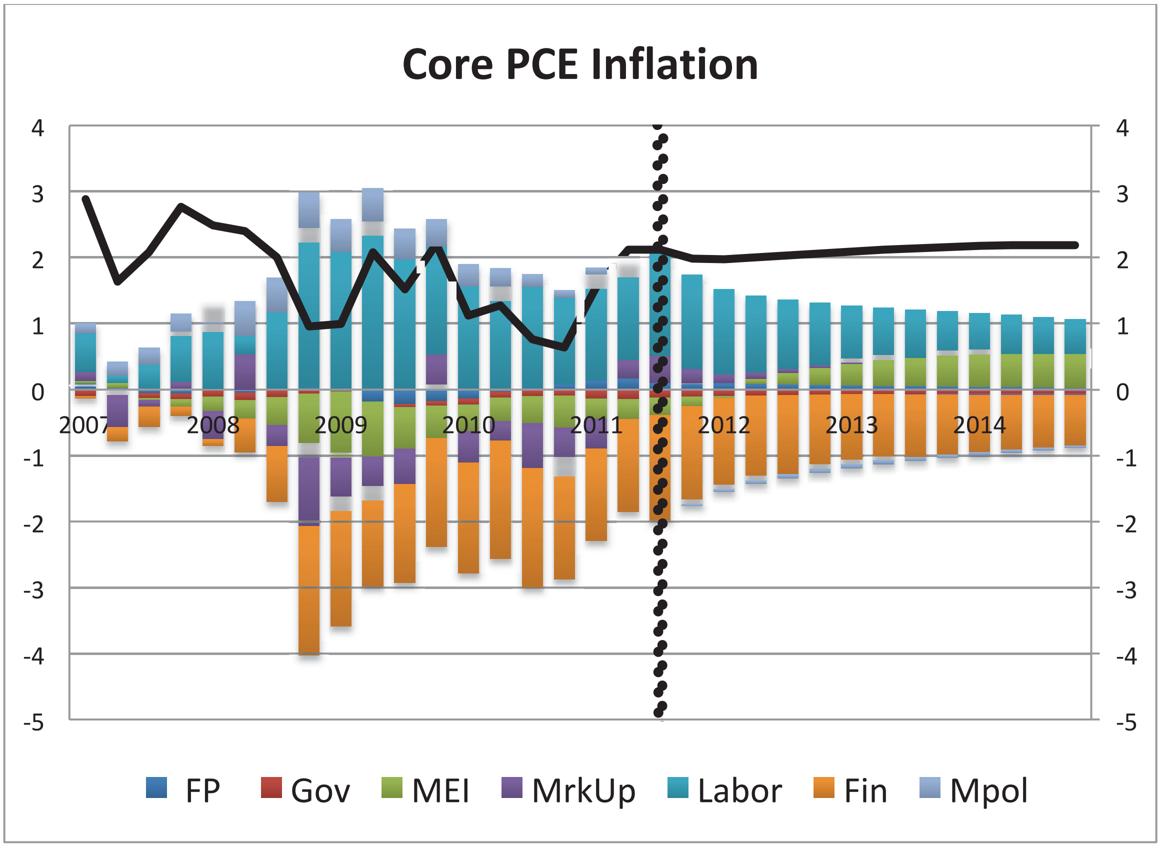
Figure 4a
Unconditional Forecast



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

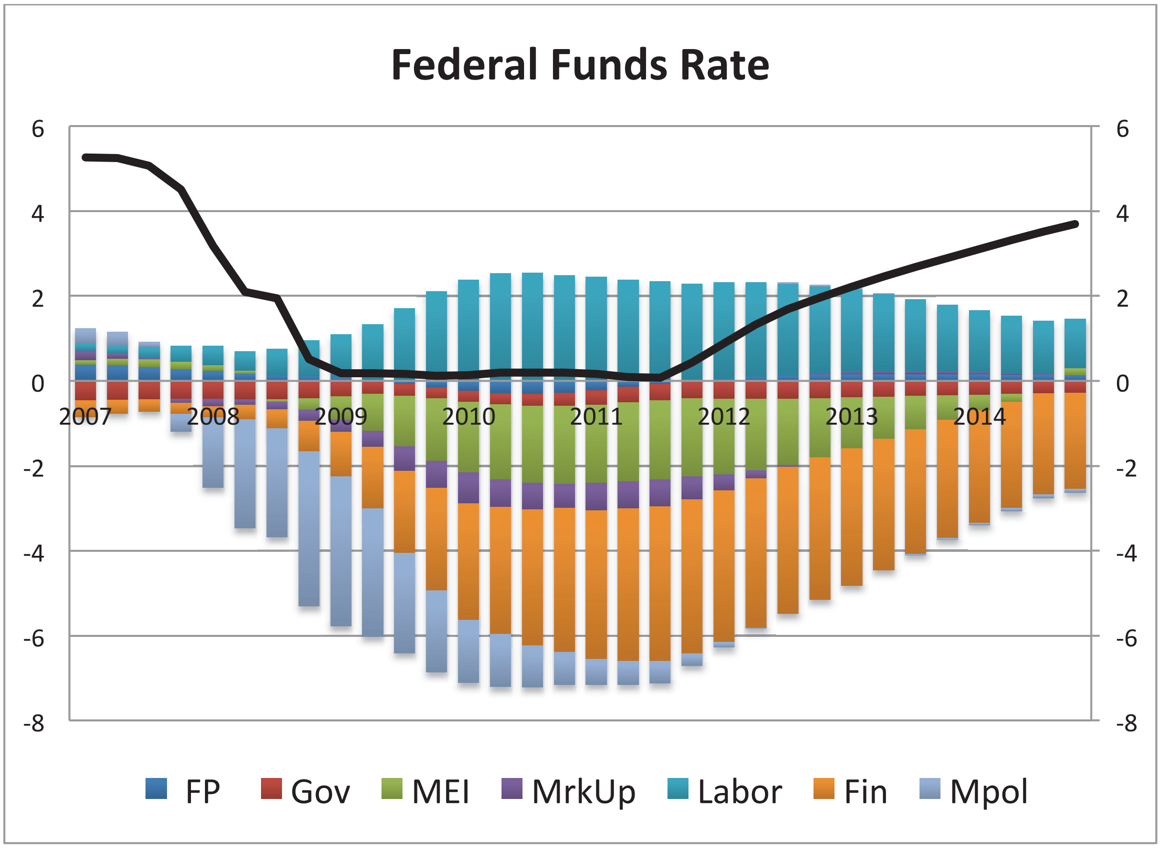
Figure 4b
Unconditional Forecast



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

Figure 4c
Unconditional Forecast



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

Figure 5
Smoothed Shock Estimates from Unconstrained Forecast Model
(normalized by standard deviation)

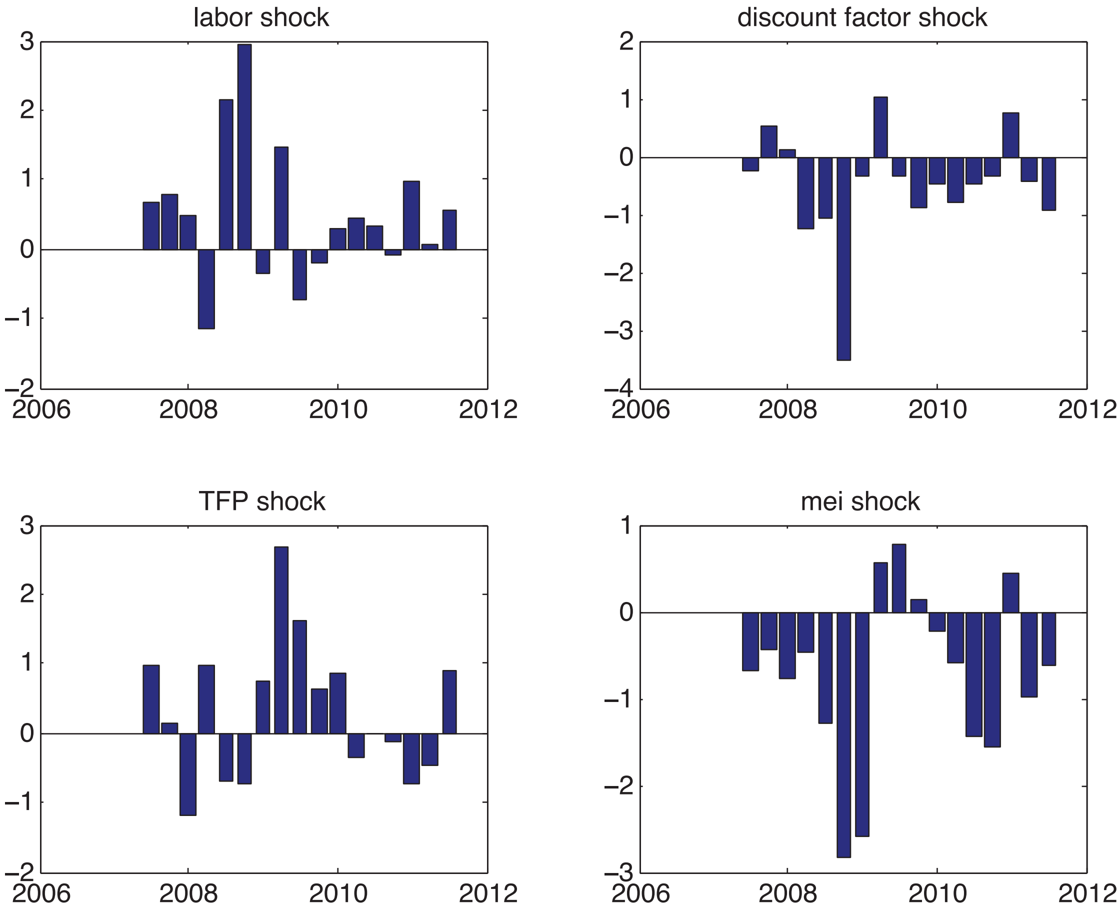


Figure 6a
Impulse Response to Negative Discount Factor Shock
(one standard deviation shock)

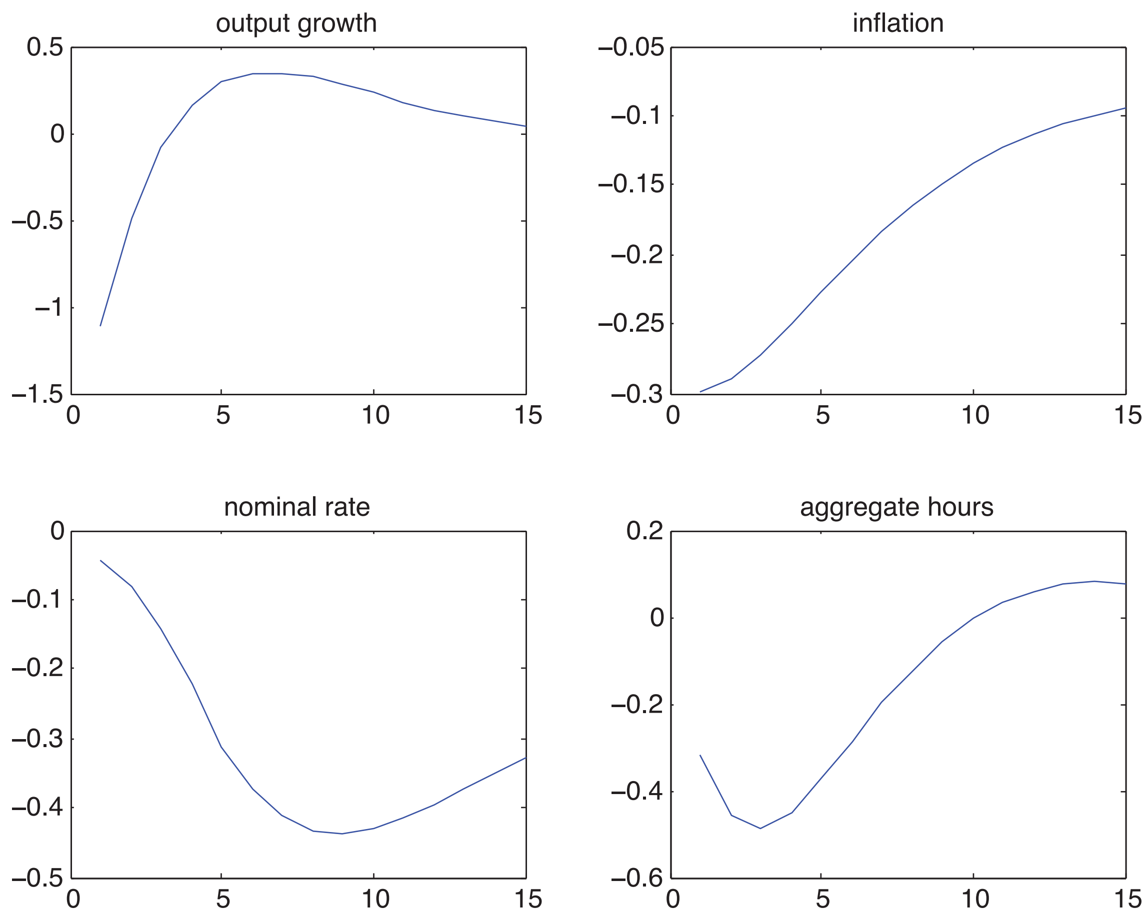
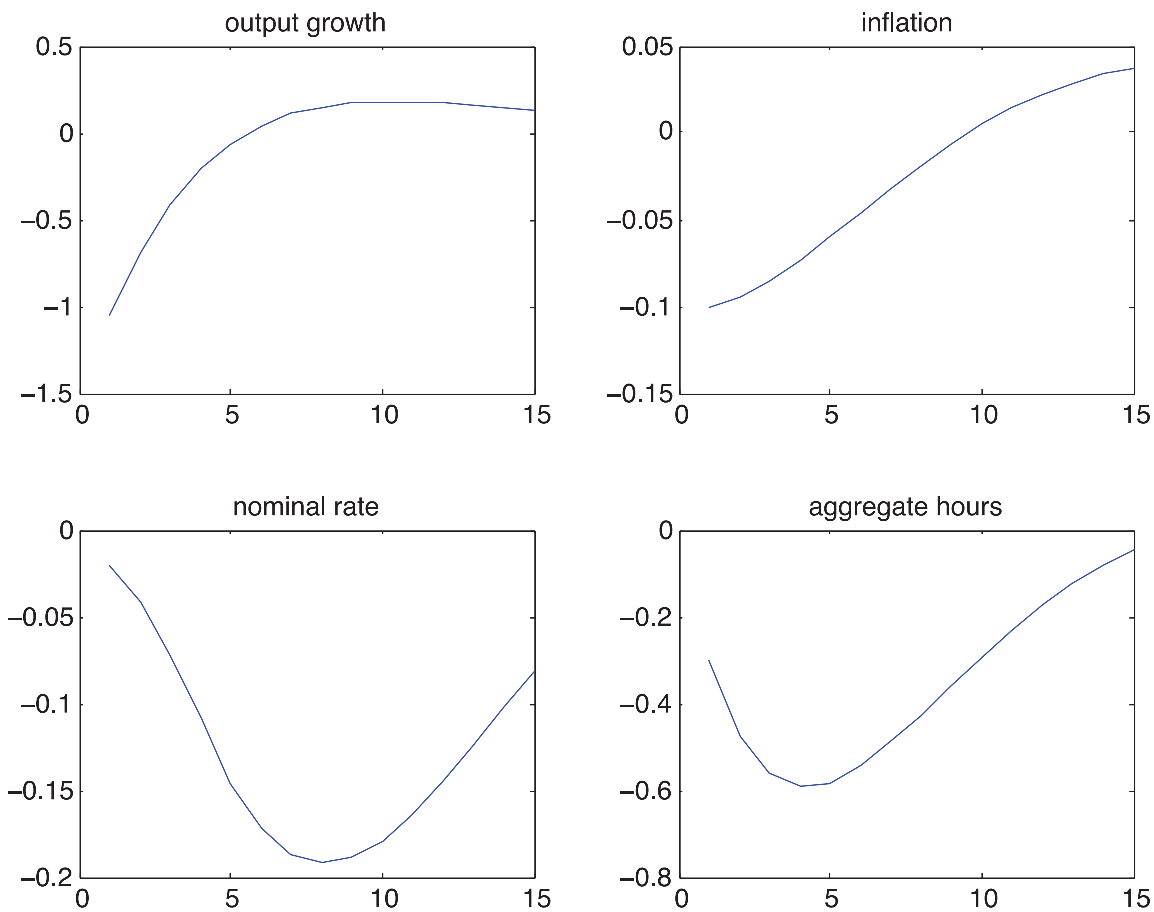


Figure 6b
Impulse Response to Negative MEI Shock
(one standard deviation shock)



Federal Reserve Bank of Chicago

Subject: Summary of Chicago Fed DSGE Model for Academic Researchers

From: Scott Brave Jeffrey R. Campbell Jonas D.M. Fisher Alejandro Justiniano

Date: October 24, 2011

Overview

In this memo, we describe the Chicago Fed's estimated dynamic stochastic general equilibrium model. This framework yields a history of identified structural shocks, which we apply to illuminate recent macroeconomic developments. To aid in the understanding of these results, we follow them with summaries of the model's structure, the data and methodology employed for estimation, and the estimated model's dynamic properties.

In several respects, the Chicago Fed DSGE model resembles many other New Keynesian frameworks. There is a single representative household that owns all firms and provides the economy's labor. Production uses capital, differentiated labor inputs, and differentiated intermediate goods. The prices of all differentiated inputs are "sticky", so standard forward-looking Phillips curves connect wage and price inflation with the marginal rate of substitution between consumption and leisure and marginal cost, respectively. Other frictions include investment adjustment costs and habit-based preferences.

There are, however, several features of the model which distinguish it from these frameworks. For instance, we include both a shock that dominates changes in long-run expected inflation and news shocks which provide short-term guidance regarding the path of the federal funds rate in our Taylor rule. We refer to the former, captured in a shifting intercept in the Taylor rule, as the *inflation drift shock*, and we discipline its fluctuations with data on long-term inflation expectations. We refer to the latter as *forward guidance shocks*, and we measure them with futures market prices.

Another distinguishing feature of the Chicago model is the use of multiple price indices. Alternative available indices of inflation are decomposed into a single model-based measure of consumption inflation and idiosyncratic (series specific) disturbances that allow for persistent deviations from this common component. Estimation uses a factor model with the common factor derived from the DSGE framework.

The model also incorporates a financial accelerator mechanism. We introduce risk-neutral entrepreneurs into the New Keynesian framework who purchase capital goods from capital installers using a mix of internal and external resources. These entrepreneurs optimally choose their rate of capital utilization and rent the effective capital stock to goods producing firms. The dependence on internal resources explicity links fluctuations in the external finance premium, private net worth, and the state of the economy.

To identify parameters governing the financial accelerator, we use multiple credit spreads and data on borrowing by nonfinancial businesses and households. Consistent with our definition of investment, which includes consumer durables and residential investment as well as business fixed investment, we relate the external finance premium to a weighted average of High Yield corporate bond and Asset-backed security spreads, where the weight each receives is derived from the shares of nonfinancial business and household debt in private credit taken from the Flow of Funds. To capture the impact of entrepreneurial leverage on financial conditions, we rely on the growth rate of the ratio of private credit to nominal GDP.

Forecasting Methodology

Constructing forecasts based on this model requires us to assign values to its many parameters. We do so using Bayesian methods to update an uninformative prior with data from 1989:Q2 through 2009:Q3. All of our forecasts condition on the parameters equaling their values at the resulting

Table 1. Model Forecasts Q4 over Q4

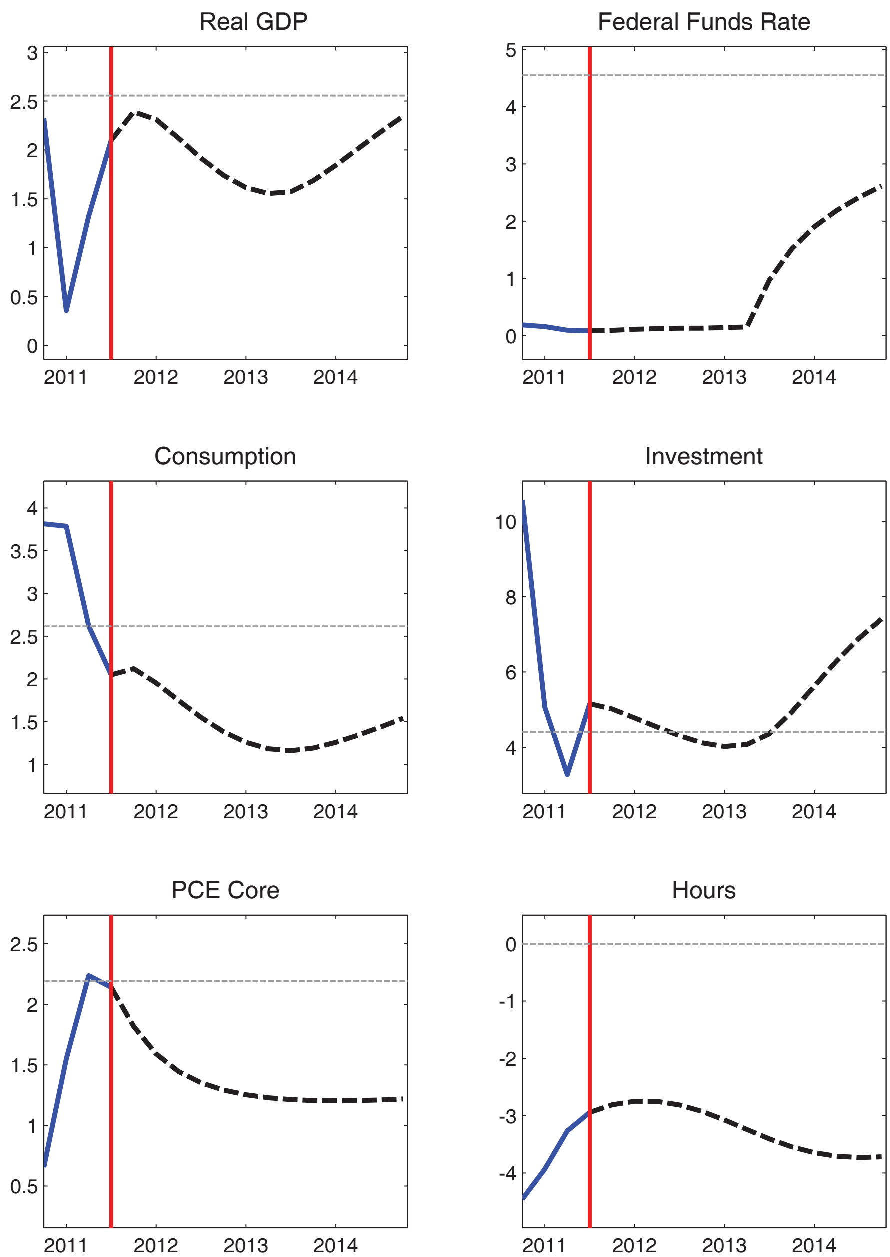
	2010	2011	2012	2013	2014
Real GDP	3.09	1.54	2.02	1.61	2.10
Federal Funds Rate	0.19	0.09	0.13	1.52	2.61
PCE Core	0.95	1.94	1.42	1.23	1.21
Consumption	2.79	2.64	1.66	1.20	1.40
Investment	8.35	4.63	4.44	4.35	6.56
Hours	2.31	1.65	-0.12	-0.62	-0.17

posterior's mode. These parameter values together with the data yield a posterior distribution of the economy's state in the final sample quarter. For the calculation of this initial state's distribution, we add a sequence of *forward guidance shocks* that signal the future path of the Federal Funds rate. These shocks begin arriving in 2009:Q4 and continue to the present. By construction, they match model-based expectations of the policy rate with actual market-based expectations for the first seven quarters of each quarter's forecast horizon. The forecasts begin with 2011:Q4 and extend through 2014:Q4. Our plug for 2011:Q3 GDP growth was set at 2.1 percent.

Table 1 presents data from 2010 and forecasts for the following four years. The first three rows correspond to three key macroeconomic observables, Real GDP growth (Q4-over-Q4), the Federal Funds Rate (Q4 average), and growth of the Core PCE deflator (Q4-over-Q4). The following rows report forecasts of Q4-over-Q4 growth for three model-defined aggregates of importance: Consumption of nondurable goods and non housing services, Investment in durable goods, residential housing, and business equipment and structures, and de-trended Hours worked in nonfarm business.[1] Figure 1 complements this with quarter-by-quarter data and forecasts of these series. The plots' dashed grey lines indicate the series' long-run values. The economy's long-run GDP

[1]Hours are detrended using the Hodrick-Prescott filter with smoothing parameter 1e5.

Figure 1. Quarterly Model Forecasts



growth rate – which we identify with potential growth – equals 2.6 percent.

Our forecast for real GDP growth in 2011 falls short of potential growth by almost 1 percent, and the economy grows near or below potential throughout the forecast horizon. Consequently, de-trended hours do not return to their steady-state by the end of 2014. With resource slack remaining elevated, the forecasted path for core PCE inflation declines at a measured pace from 1.9 percent in 2011 to 1.2 percent in 2014. We show below that this protracted weakness arises from the "financial shocks", a *spread* and *net worth* shock, in our model which have particularly persistent effects on economic activity.

The contractionary forces shaping our forecast have been partially offset by *forward guidance shocks*, which in our model capture policy makers' announcements regarding the path of the federal funds rate over the next seven quarters. Forward guidance has added about 2 percent to four quarter real GDP growth over the last year, and roughly 0.6 percent to core PCE inflation over the same time period. Policy signals have supported consumption and investment growth, as well as hours.

We hard-wire the current values of monetary policy news shocks to match current market expectations, and currently these date the tightening of monetary policy at 2013:Q3. Thereafter, the forecast rate begins to rise as the conventional Taylor rule dynamics take over.

In the last two quarters, our measure of long-term inflation expectations has edged down, and is currently 0.75 percent below its steady-state level. The model interprets this as negative realizations of the *inflation drift shock*. Therefore, the Taylor rule sees expected output growth and inflation as weak enough to merit only the gradual removal of the extraordinary accommodation in place since 2008.

Shock Decompositions

Our analysis identifies the structural shocks responsible for past fluctuations. To summarize this information, we follow a suggestion of Charlie Evans: Fix an object to be forecast, such as Q4-over-Q4 real GDP growth. Then, pick a date in the past and forecast the object conditional on the information as of that date. This is *not* a real-time forecast, because it uses revised data. The model can be used to decompose the associated forecast error into structural shocks. (A detailed explanation of the forecast error decomposition procedure begins below on page 26.) We repeatedly advance the forecast date, decompose the forecast error, and finally plot the results. In total, the model features eleven structural shocks and six idiosyncratic disturbances without structural interpretations. For parsimony's sake, we group the shocks according to the following taxonomy.

Demand These are the structural non-policy shocks that move output and consumption-based inflation in the same direction. The model features four of them. One changes the households' rate of time discount. We call this the Discount shock. Another includes a shock to the sum of government spending, net exports, and changes in the valuation of inventories. The remaining two are financial disturbances. The Spread shock generates fluctuations in the external finance premium beyond the level warranted by current economic conditions, and the Net Worth shock generates exogenous fluctuations in private balance sheets.

Supply Five shocks move real GDP and consumption-based inflation in opposite directions on impact. These *supply shocks* directly change

- Neutral Technology,
- Investment-Specific/Capital-Embodied Technology,
- Markups of Intermediate Goods Producers,
- Markups of Labor Unions, and

Table 2. The Model's Decomposition of Business-Cycle Variance

	Demand	Supply	Monetary Policy	Residual
Real GDP	0.87	0.06	0.07	0.00
Real Consumption	0.88	0.08	0.05	0.00
Real Investment	0.93	0.04	0.04	0.00
Federal Funds Rate	0.83	0.04	0.13	0.00
Core PCE Inflation	0.28	0.55	0.17	0.00

Note: For each variable, the table lists the fraction of variance at frequencies between 6 and 32 quarters attributable to shocks in the listed categories. The numbers may not add to one due to rounding.

– Households' Disutility from Labor

Policy The model's monetary policy follows a Taylor rule with interest-rate smoothing, a time varying intercept, and an *i.i.d.* policy shock. The time varying intercept, or Inflation Drift shock, is disciplined by equating model-based average expected consumer price inflation to a measure of long-term inflation expectations derived from a reduced form affine term structure model. Additionally, we have incorporated *forward guidance* shocks since 2009:Q4, which are revealed to the model's agents one to seven quarters before they affect the federal funds rate. These disturbances allow our forecasting exercise to match model-based expectations with information regarding the path of the federal funds rates over the next seven quarters from futures markets.

Residual We group other shocks that are usually of small importance into a residual category. These include the idiosyncratic, that is series specific, shocks to the various price measures as well as the measurement errors in the credit spread and net worth.

Table 2 reports the fraction of business-cycle variance attributable to shocks in each category for five key variables, the level of Real GDP, Real Consumption, and Real Investment, and the Federal Funds Rate and Core PCE Inflation. Three facts stand out here. First, demand shocks dominate business cycles. Supply shocks account for only 6 percent of GDP's total business-cycle variance, and the non-systematic part of monetary policy shocks also makes only a minor contribution. The accounting for the Federal Funds Rate's variance is similar. Perhaps this is unsurprising, because we classify the shock that directly moves households' rate of time preference as "demand." Inflation fluctuations are dominated by supply shocks, with exogenous shocks to intermediate goods' markups accounting for about three-fourths of supply shocks' 55 percent contribution.

The Model's Specification and Estimation

Our empirical work uses fourteen variables, measured from 1989:Q2 through the present:

- Growth of nominal per capita GDP,
- Growth of nominal per capita consumption, which sums Personal Consumption Expenditures on Nondurable Goods and Services;
- Growth of nominal per capita investment; which sums Business Fixed Investment, Residential Investment, and Personal Consumption Expenditures on Durable Goods
- Per capita hours worked in Nonfarm Business,
- Growth of nominal compensation per hour worked in Nonfarm Business,
- Growth of the implicit deflator for GDP,
- Growth of the implicit deflator for consumption, as defined above,
- Growth of the implicit deflator for investment, as defined above,

- Growth of the implicit deflator for core PCE,
- Growth of the implicit deflator for core CPI,
- The interest rate on Federal Funds,
- Ten-year ahead core CPI forecasts from a reduced form affine term structure model,
- A weighted average of High-Yield corporate and Mortgage-backed bond spreads with the 10-year Treasury and an Asset-backed bond spread with the 5-year Treasury; where the weights equal the shares of nonfinancial business, household mortgage, and household consumer debt in private credit,
- Growth of private credit-to-GDP; which sums household and nonfinancial business credit market debt outstanding and divides by nominal GDP.

We do not directly use data on either government spending, net exports, or the change in the valuation of inventories. Their sum serves as a residual in the national income accounting identity. To construct series measured per capita, we used the civilian non-institutional population 16 years and older. To eliminate level shifts associated with the decennial census, we project that series onto a fourth-order polynomial in time.

Our model confronts these data within the arena of a standard linear state-space model. Given a vector of parameter values, θ, log-linearized equilibrium conditions yield a first-order autoregression for the vector of model state variables, ζ_t.

$$\begin{aligned}\zeta_t &= F(\theta)\zeta_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \Sigma(\theta))\end{aligned}$$

Here, ε_t is a vector-valued innovation built from the model innovations described above. Many of its elements identically equal zero. Table 3 lists the elements of ζ_t. Habit puts lagged nondurable consumption into the list, and investment adjustment costs place lagged investment there. Rules for indexing prices and wages that cannot adjust freely require the state to include lags of inflation and technology growth. Financial frictions place lagged entrepreneurial borrowing and net worth in the state. The list includes the lagged policy rate because it appears in the Taylor rule.

Gather the date t values of the fourteen observable variables into the vector y_t. The model analogues to its elements can be calculated as linear functions of ζ_t and ζ_{t-1}. We suppose that the data equal these model series plus a vector of "errors" v_t.

$$\begin{aligned} y_t &= G(\theta)\zeta_t + H(\theta)\zeta_{t-1} + v_t \\ v_t &= \Lambda(\varphi)v_{t-1} + e_t \\ e_t &\sim N(0, D(\varphi)) \end{aligned}$$

Here, the vector φ parameterizes the stochastic process for v_t. In our application, the only non-zero elements of v_t correspond to the observation equations for the three consumption-based measures of inflation, the GDP deflator, and the spread and private credit-to-GDP measures. The idiosyncratic disturbances in inflation fit the high-frequency fluctuations in prices and thereby allow the price markup shocks to fluctuate more persistently. These errors evolve independently of each other. In this sense, we follow Boivin and Giannoni (2006) by making the model errors "idiosyncratic". The other notable feature of the observation equations concerns the GDP deflator. We model its growth as a share-weighted average of the model's consumption and investment deflators.

We denote the sample of all data observed with Y and the parameters governing data generation with $\Theta = (\theta, \varphi)$. The prior density for Θ is $\Pi(\Theta)$, which resembles that employed by Justiniano, Primiceri, and Tambalotti (2011). Given Θ and a prior distribution for ζ_0, we can use the model solution and the observation equations to calculate the conditional density of Y, $F(Y|\Theta)$. To form the prior density of ζ_0, we apply the Kalman filter. The actual estimation begins

Table 3. Model State Variables

Symbol	Description	Disappears without
C_{t-1}	Lagged Consumption	Habit-based Preferences
I_{t-1}	Lagged Investment	Investment Adjustment Costs
π^p_{t-1}	Lagged Price Inflation	Indexing "stuck" prices to lagged inflation
K_t	Stock of Installed Capital	
A_t	Hicks-Neutral Technology	
a_t	Growth rate of A_t	Autoregressive growth of A_t
a_{t-1}	Lagged Growth Rate of A_t	Indexing "stuck" wages to lagged labor productivity growth
Z_t	Investment-Specific Technology	
z_t	Growth rate of Z_t	Autoregressive growth of Z_t
z_{t-1}	Lagged Growth Rate of Z_t	Indexing "stuck" wages to lagged labor productivity growth
$_t$	Labor-Supply Shock	
b_t	Discount Rate Shock	
$\lambda_{w,t}$	Employment Aggregator's Elasticity of Substitution	Time-varying Wage Markups
$\lambda_{p,t}$	Intermediate Good Aggregator's Elasticity of Substitution	Time-varying Price Markups
B_t	Entrepreneurial Borrowing	Need for external finance
B_{t-1}	Lagged Borrowing	
N_t	Entrepreneurial Net Worth	Risk-neutral entrepreneurs
N_{t-1}	Lagged Net Worth	
ν_t	Spread Shock	
ς_t	Net Worth Shock	
g_t	Government Spending Share Shock	
R_{t-1}	Lagged Nominal Interest Rate	Interest-rate Smoothing
$\varepsilon_{R,t}$	Monetary Policy Shock	
π_t	Inflation Drift Shock	

with 1989:Q2. Bayes rule then yields the posterior density up to a factor of proportionality.

$$P(\Theta|Y) \propto F(Y|\Theta)\Pi(\Theta)$$

We calculate our forecasts with the model's parameter values set to this posterior distribution's mode.

Five Key Equations

This section summarizes the inferred parameters by reporting the estimates of five key equations: the two equations of the financial accelerator capturing the External Finance Premium and the evolution of private Net Worth, and the log-linearized forms of the Taylor Rule, the Price Phillips Curve, and the Wage Phillips Curve.

Financial Accelerator

Financial frictions in the model arise from imperfections in private financial intermediation due to lenders' costly state verification of the returns realized by entrepreneurs' projects. We introduce risk neutral entrepreneurs into the model who at the end of period t purchase capital goods, $\overline{K_t}$, from the capital installers at the price Q_t, using a mix of internal and external resources, given by end of period net worth, N_t, and borrowing B_t, such that $Q_t\overline{K_t} = N_t + B_t$.

In the next period, $t+1$, entrepreneurs optimally choose the rate of utilization, u_{t+1}, and rent the effective capital stock $K_{t+1} = u_{t+1}\overline{K_t}$ to the goods producing firms, receiving in return the gross rental rate of capital ω^k_{t+1}. At the end of period $t+1$ they resell the remaining capital stock, $(1-\delta)\overline{K_t}$ back to the capital producers at the price Q_{t+1}.

External Finance Premium

We assume that the external finance premium –the ratio of the equilibrium return to capital and the expected real interest rate– is an increasing function of the entrepreneurs' leverage ratio, $\frac{\overline{K}_t Q_t}{N_t}$, according to

$$\frac{E_t[1 + r^k_{t+1}]}{E_t[\frac{1+R_t}{\pi_{t+1}}]} = F \quad \frac{\overline{K}_t Q_t}{N_t} \quad e^{\nu_t}$$

with R_t the nominal interest rate, π_{t+1} the gross inflation rate and $F(1) = 1$, $F > 0$, $F > 0$.[2] The spread shock, e^{ν_t}, can be viewed as a disturbance to credit supply, moving the external finance premium beyond the level dictated by entrepreneurial net worth. We parameterize the steady state level of $F \quad \frac{K\,Q}{N}$ as well as its elasticity τ, and estimate them to be 1.93 and 0.05, respectively. The annualized steady state external finance premium is estimated to be 3.16 percent.

Net Worth

The law of motion for entrepreneurial net worth is given by

$$N_t = 0.89 \quad \overline{K}_{t-1} Q_{t-1}[1 + r^k_t] - E_{t-1}[1 + r^k_{t-1}]B_{t-1} \quad + 0.11\Gamma_t + \varsigma_t$$

where Γ_t is the transfer from exiting to new entrepreneurs and ς_t is a shock to net worth that can arise for instance from time-varying survival probabilities for entrepreneurs. The AR(1) laws of motion for the spread and net worth shocks, ν_t and ς_t, are estimated to have independent autoregressive parameters $(0.93, 0.97)$ and volatilities i=$0.34, 0.36$.

[2]Notice that that if entrepreneurs are self-financed, which we rule out in steady state, $F(1) = 1$ and there is no external finance premium.

Taylor Rule

$$R_t - \pi_t = 0.69(R_{t-1} - \pi_t) + 0.31 \times \left(1.36 \frac{1}{4} \sum_{j=0}^{3} (\pi_{t-j} - \pi_t) + 0.2(y_t - y_{t-4}) \right) + \epsilon_t^{mp} + \sum_{s=1}^{7} \xi_{t-s}$$

Besides the lagged interest rate, the variables appearing on the right-hand side are an inflation drift π_t^* term (which in the context of the model can be interpreted as the monetary authority's medium-run desired rate of inflation), the four-quarter average of consumption inflation, the most recent four-quarter output growth rate, the current monetary policy shock (ϵ_t^{mp}), and the seven previous quarters' signals of the current monetary policy stance, ξ_{t-s}^s for $s = 1, \ldots, 7$. (These signals play a prominent role in forecasting, but we do not yet use them during estimation.)

Note that

- Holding the economy's growth rate fixed, the long-run response of R_t to a permanent one-percent increase in inflation is 1.36 percent. Thus, the model satisfies the Taylor principle.
- Since the four-quarter growth rate of output replaces the usual output gap in the rule, it is difficult to compare the estimated coefficient of 0.2 with the typical calibrated output response of 0.5.

Furthermore, inflation drift is perfectly credible in the sense that we equate model-based average expected consumer price inflation over the next forty quarters to ten-year ahead core CPI forecasts derived from a reduced form affine term structure model.

Price Phillips Curve

$$\pi_t^p = 0.88 E_t \pi_{t+1}^p + 0.12 \pi_{t-1}^p + 0.004 s_t + \epsilon_t^p$$

Here, s_t represents intermediate goods producers' common marginal cost. The introduction of inflation drift does not alter the dynamic component of inflation indexation which is linked to the previous quarter's inflation rate.

- The slope of the estimated Phillips Curve is fairly flat compared to some other estimates in the literature. For instance, the associated Calvo probablity of an individual firm not updating its price in a given quarter equals 0.93.
- Producers unable to update their price with all current information are allowed to index their prices to a convex combination of last quarter's inflation rate with the steady-state inflation rate. This places π_{t-1}^p in the Phillips curve. The estimated weight on steady-state inflation is 0.86.

Wage Phillips Curve

The Wage Phillips curve can be written as

$$\pi_t^w + \pi_t^p + j_t - \iota_w \;\; \pi_{t-1}^p + j_{t-1} \;\; = \beta E_t \;\; \pi_{t+1}^w + \pi_{t+1}^p + j_{t+1} - \iota_w \left(\pi_t^p + j_t\right) \; + \kappa_w x_t + \epsilon_t^w,$$

where π_t^w and π_t^p correspond to inflation in real wages and consumption prices respectively, $j_t = z_t + \frac{\alpha}{1-\alpha}\mu_t$ is the economy's technologically determined stochastic trend growth rate, with α equal to capital's share in the production function, z_t the growth rate of neutral technology, and μ_t the growth rate of investment-specific technical change. The term $\pi_{t-1}^p + z_{t-1} + j_t$ arises from indexation of wages to a weighted average of last quarter's productivity-adjusted price inflation and its steady state value. The estimated weight on the steady state equals 0.64. The log-linearized expression for the ratio of the marginal disutility of labor, expressed in consumption units, to the real wage is

$$x_t = b_t + \psi_t + \nu l_t - \lambda_t - w_t,$$

where b_t and ψ_t are disturbances to the discount factor and the disutility of working, respectively, l_t hours, λ_t the marginal utility of consumption and w_t the real wage. Finally, ϵ_t^w is a white noise wage markup shock.

Note that without indexation of wages to trend productivity, this equation says that nominal wage inflation (adjusted by trend growth) depends positively on future nominal wage inflation (also appropriately trend-adjusted), and increases in the disutility of the labor-real wage gap.

The estimated equation is given by

$$\pi_t^w + \pi_t^p + j_t - 0.36 \ \ \pi_{t-1}^p + j_{t-1} \ \ = 0.9984 \times E_t[\pi_{t+1}^w + \pi_{t+1}^p + j_{t+1} - 0.36\,(\pi_t^p + j_t)] + 0.0034 x_t + \epsilon_t^w,$$

The estimated Calvo probability of a wage remaining unadjusted in a given quarter underlying the estimate of $\kappa_w = 0.0034$ equals 0.76.

The Model's Shocks

Our discussion of recent macroeconomic developments above featured the following shocks prominently: The discount rate shock, the "financial" shocks to the external finance premium and private net worth, as well as both anticipated and unanticipated monetary policy and inflation drift shocks. In this section, we provide greater detail on the model's responses to these six shocks.

Figure 2 plots responses to a discount rate shock that increases impatience and tilts desired consumption profiles towards the present. The variables examined are real GDP, the federal funds rate, consumption, investment, inflation, hours worked, the external finance premium (or spread), and the ratio of private credit-to-GDP. The responses are scaled so that the change in GDP after 16 quarters equals one percent.

In a neoclassical economy, this shock would be contractionary on impact. Upon becoming more impatient, the representative household would increase consumption and decrease hours worked. To the extent that the production technology is concave, interest rates and real wages would rise; and regardless of the production technology both real GDP and investment would drop.

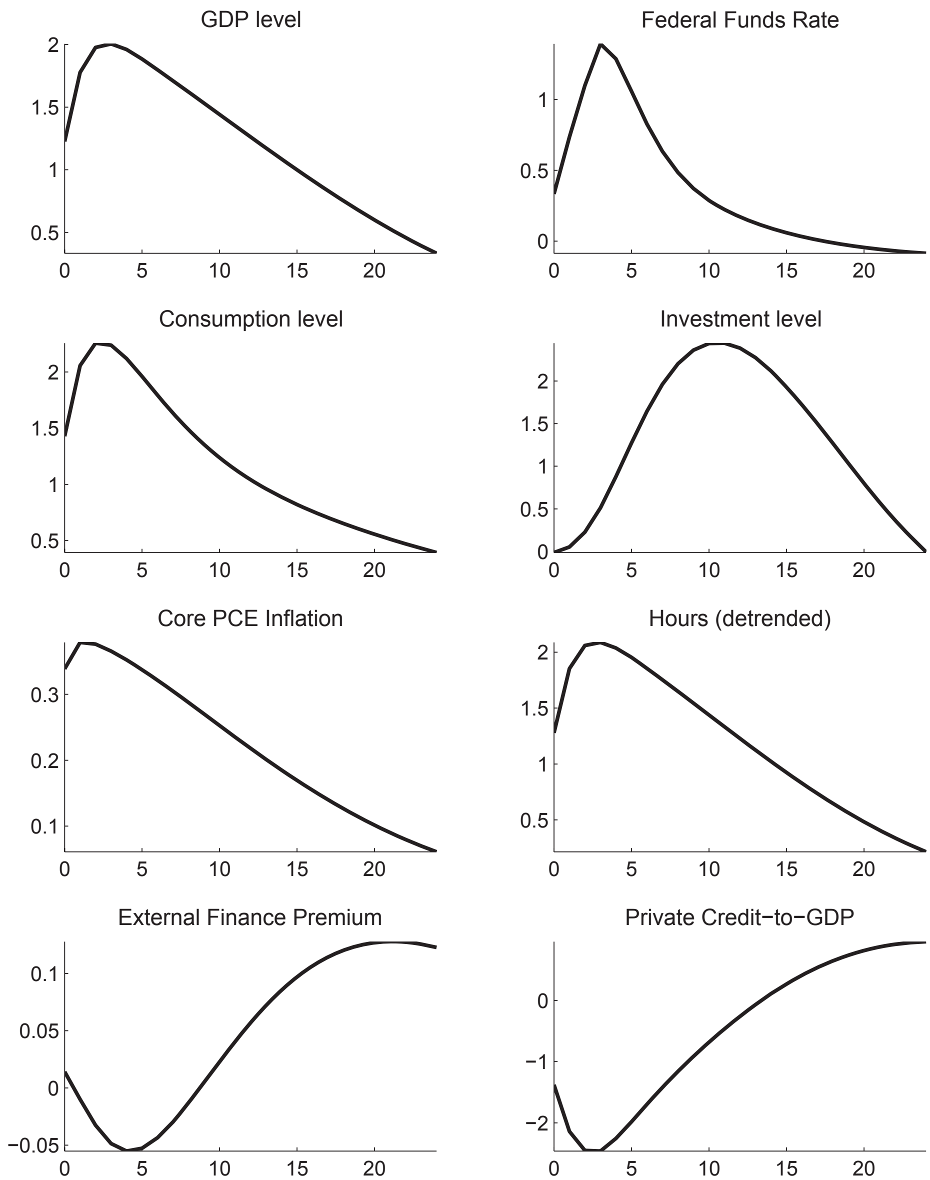
Increasing impatience instead *expands* activity in this New Keynesian economy. As in the neoclassical case, consumption rises on impact. However, investment remains unchanged as adjustment costs penalize the sharp contraction of investment from the neoclassical model. Instead, investment displays a hump-shaped response, exhibiting modest co-movement with consumption with a slight lag. Habit causes the consumption growth to persist for two more quarters before it begins to decline. Market clearing requires either a rise of the interest rate (to choke off the desired consumption expansion) or an expansion of GDP. By construction, the Taylor rule prevents the interest rate from rising unless the shock is inflationary or expansionary. Therefore, GDP must rise. This in turn requires hours worked to increase.

Two model features overcome the neoclassical desire for more leisure. First, some of the labor variants' wages are sticky. For those, the household is obligated to supply whatever hours firms demand. Second, the additional labor demand raises the wages of labor variants with wage-setting opportunities. This rise in wages pushes marginal cost up and lies behind the short-run increase in inflation. After inflation has persisted for a few quarters, monetary policy tightens and real rates rise.

The effect of this shock on financial conditions is somewhat muted, with the large decline in private credit-to-GDP reflecting for the most part fluctuations in the denominator. Close to the end of the monetary policy cycle, the external finance premium and private credit-to-GDP both increase as the demand for credit rises.

Since the discount rate shock moves output and prices in the same direction, a Keynesian analysis would label it a shift in "demand." In the neoclassical sense, it is also a demand shock, albeit a reduction in the demand for future goods. The matching neoclassical supply shock in our model is to the spread shock. A positive shock to it decreases the supply of future goods. Figure 3 plots the

Figure 2. Responses to a Discount Rate Shock



Note: The impulse was scaled to yield a one percent increase in GDP after 16 quarters.

responses to such a shock.[3]

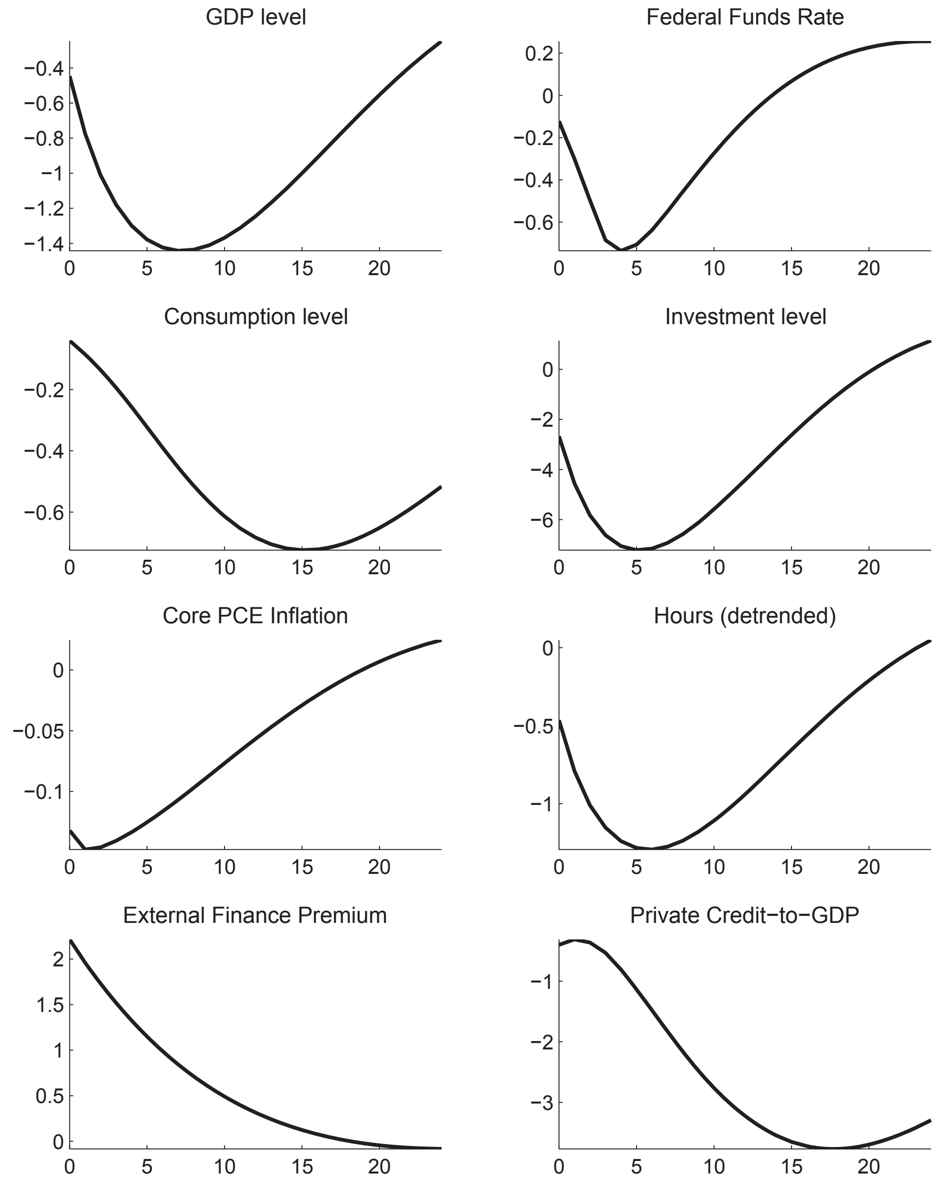
A positive spread shock reduces the supply of credit available to entrepreneurs, who are then forced to shrink their demand for capital. The price of installed capital drops sharply so that the return to capital collapses on impact and is followed by a prolonged contraction in borrowing by entrepreneurs. The decline in borrowing is initially smaller than in net worth, which results in a rising leverage ratio and a further tightening of the external finance premium. Investment and other measures of real activity all decline. Of particular note is the fact that the spread shock generates positive co-movement between consumption and investment on impact. In response to lower activity and inflation, monetary policy eases and real rates move lower.

Increasing the external finance premium thus lowers investment, consumption, hours worked, GDP, and the real interest rate. Two aspects of our model stop consumption from rising on the same shock's impact. First, habit-based preferences penalize an immediate increase in consumption. Second, monetary policy responds to the shock only slowly, so real interest rates are slow to adjust. The investment adjustment costs give the responses of GDP, hours, and investment their hump shape. Although this shock changes the economy's technology for intertemporal substitution – and therefore deserves the neoclassical label "supply" – it makes prices and output move in the same direction. For this reason, it falls into our Keynesian taxonomy's "demand" category.

For the same reason, we consider a positive shock to net worth to be in the "demand" category. It initially reduces entrepreneurs' needs for external financing, so borrowing drops in the first few quarters. Combined with a sustained increase in net worth, this translates into a persistent reduction in the

[3]The interpretation of this shock is not unique. The negative spread shock resembles in nature a positive marginal efficiency of investment (MEI) shock. It could also be interpreted as a shock to the efficiency of channeling funds to entrepreneurs or, more broadly, variations in the supply of credit. Barro and King (1984) and Greenwood, Hercowitz, and Huffman (1988) consider the analogous responses to an MEI shock from a neoclassical model.

Figure 3. Responses to a Spread Shock



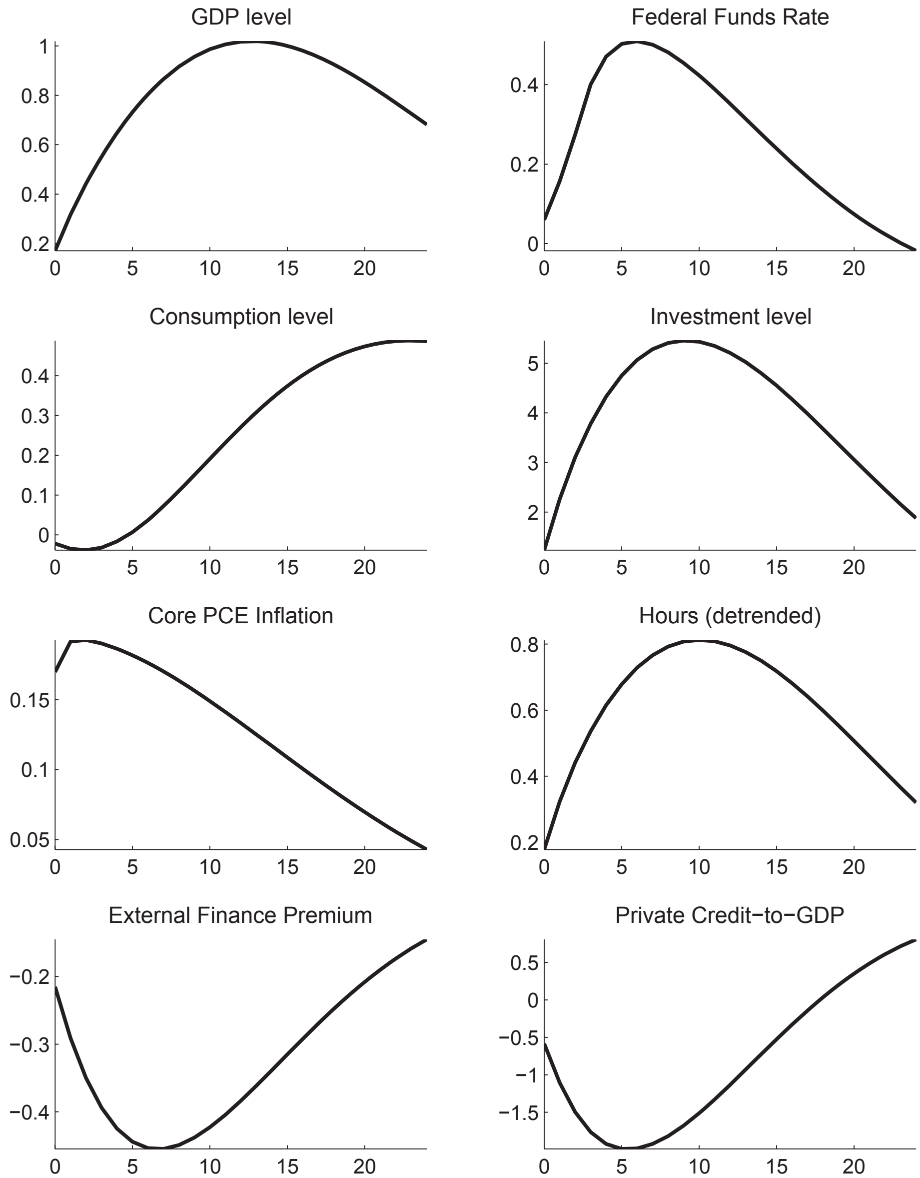
Note: The impulse was scaled to yield a one percent decrease in GDP after 16 quarters.

leverage ratio, moving the external finance premium lower. There is a surge in investment, and consequently GDP and hours, peaking after 8 quarters or so, together with a delayed modest response from consumption. The effects of this shock on real activity are very persistent. With inflation also edging up, monetary policy tightens and real rates rise in response.

As noted in section 2, monetary policy shocks have partially offset negative demand shocks over the last year. Figures 5 and 6 present the impulse response functions for two of these, the unanticipated "contemporaneous" shock and the shock revealed to all agents seven quarters in advance. The responses to the unanticipated shock are standard, but those following an anticipated shock require more explanation. At the announcement date, the expected value of the policy rate seven quarters hence increases by 190 basis points. Because both Phillips curves are forward looking, this expected contraction causes both prices and quantities to fall. This anticipated weakness then feeds through the Taylor rule to create a gradual *easing* of policy.

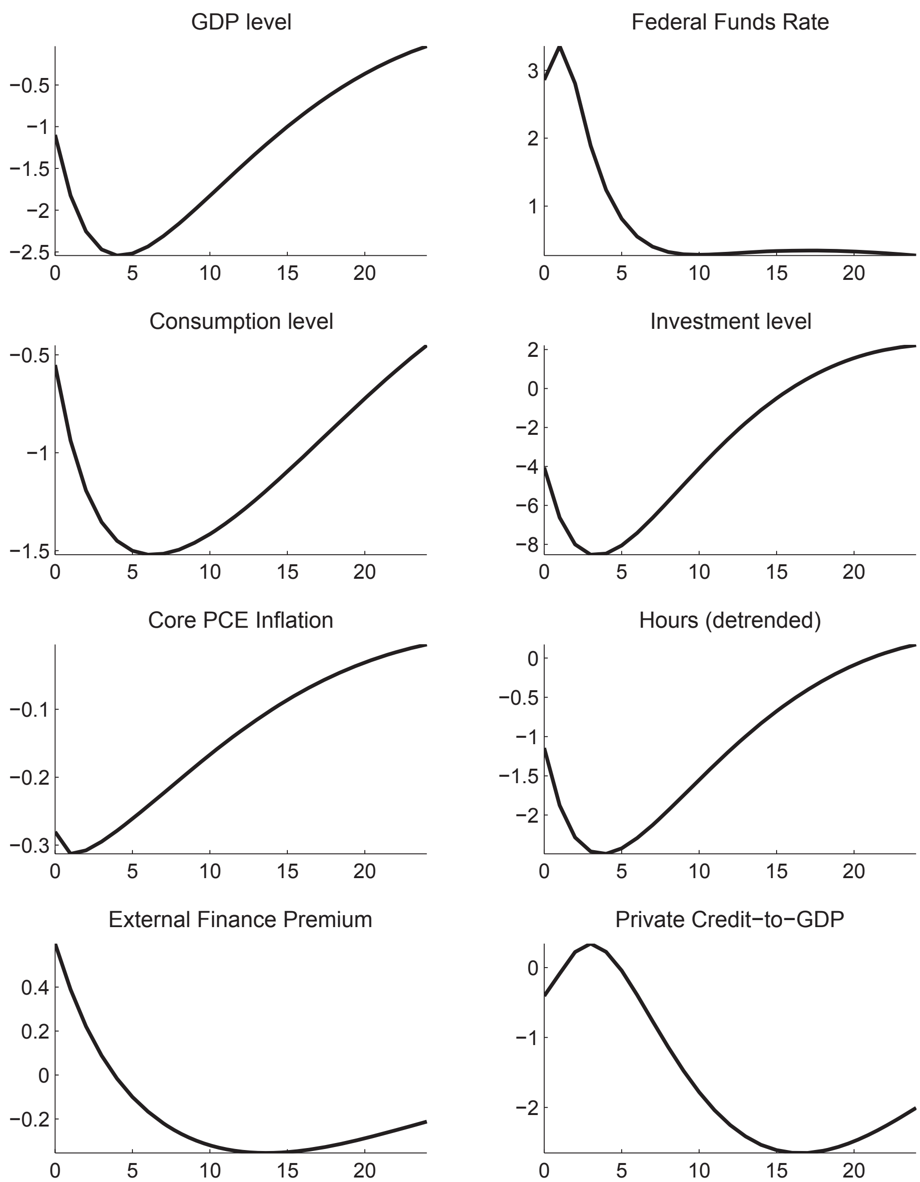
Unlike the two monetary policy shocks above, recent negative realizations of the inflation drift shock have acted to augment the contractionary forces of the model. Figure 7 displays the impulse response functions for a positive inflation drift shock. In response, core PCE inflation jumps on impact, as does expected long-run inflation (not shown). Under the assumption of perfect credibility, higher inflation is achieved without any contemporaneous movement in the federal funds rate. Although monetary policy does eventually tighten to return the real interest rate to its steady-state, lower real rates during the initial transition fuel an increase in consumption, investment, and hours. Therefore, GDP moves up on impact as well. Given the high degree of persistence of this shock, its effects on real activity and inflation dissipate at a glacial pace.

Figure 4. Responses to a Net Worth Shock



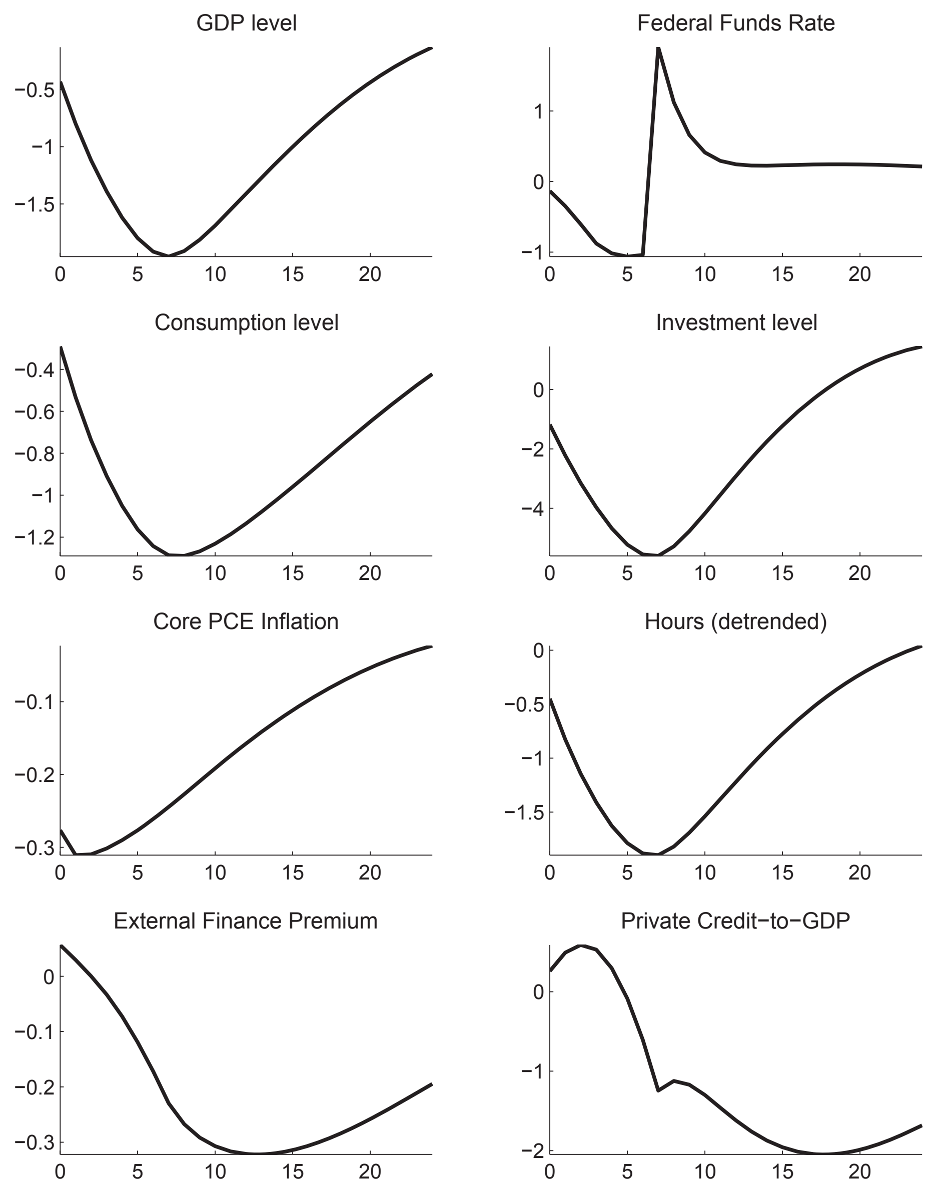
Note: The impulse was scaled to yield a one percent increase in GDP after 16 quarters.

Figure 5. Responses to an Unanticipated Monetary Policy Shock



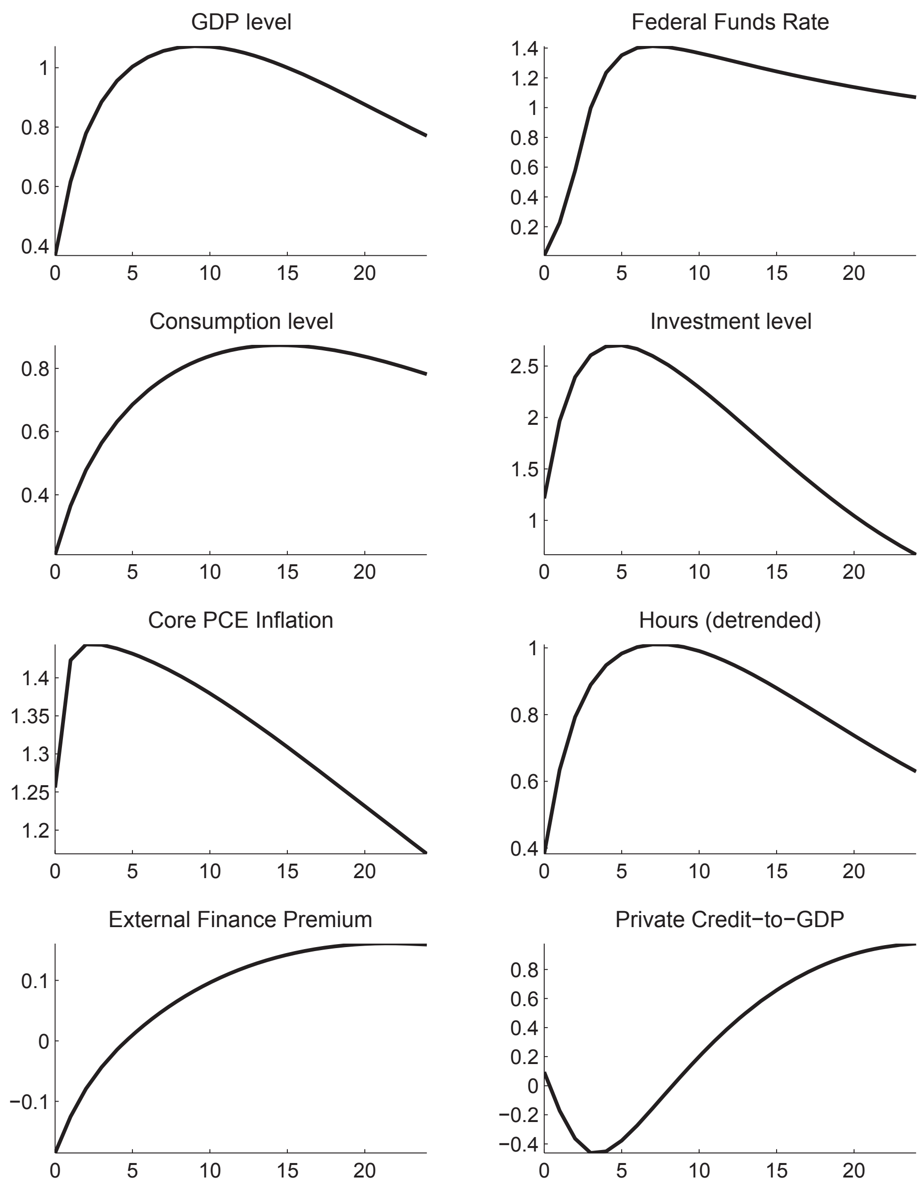
Note: The impulse was scaled to yield a one percent decrease in GDP 16 quarters after that date.

Figure 6. Responses to an Anticipated Monetary Policy Shock



Note: The monetary policy shock is revealed to all agents five quarters before its realization, and the impulse was scaled to yield a one percent decrease in GDP 16 quarters after that date.

Figure 7. Responses to an Inflation Drift Shock



Note: The impulse was scaled to yield a one percent increase in GDP 16 quarters after that date.

Shock Decomposition Methodology

We credit Charles Evans with the original ideas behind this decomposition. For the shock decomposition, we set the model's parameters to their values at the posterior distribution's mode, $\hat{\theta}$. Using *all* available data we use the Kalman smoother to extract sequences of estimated states $\{\hat{\zeta}_t\}_{t=1}^{T}$ and a innovations $\{\hat{\varepsilon}_t\}_{t=1}^{T}$. By construction, these satisfy the estimated transition equation for the state.

$$\hat{\zeta}_t = F(\hat{\theta})\hat{\zeta}_{t-1} + \hat{\varepsilon}_t,$$

To keep this discussion simple, we henceforth suppose that the "error" shocks in v_t equal zero. Incorporating them into the analysis changes the actual calculations only little.

For concreteness, suppose that the forecasted object of interest is Q4-over-Q4 GDP growth for 2010. We position ourselves in 2009:Q4 and calculate

$$\begin{aligned}
\hat{\zeta}_{2010:Q1}^{2009:Q4} &\equiv F(\hat{\theta})\hat{\zeta}_{2009:Q4} \\
\hat{\zeta}_{2010:Q2}^{2009:Q4} &\equiv F(\hat{\theta})\hat{\zeta}_{2010:Q1}^{2009:Q4} \\
&= F^2(\hat{\theta})\hat{\zeta}_{2009:Q4} \\
&\vdots \\
\hat{\zeta}_{2010:Q4}^{2009:Q4} &\equiv F(\hat{\theta})\hat{\zeta}_{2010:Q3}^{2009:Q4}
\end{aligned}$$

These are the "expectations" of the model's states in each quarter of 2010 conditional on the state at the end of 2009 equalling its estimated value.

With these "state forecasts" in hand, we can construct corresponding forecast errors by comparing them with their "realized values" from the Kalman smoother. For the period t state forecasted in 2009:Q4, we denote these with

$$\hat{\eta}_t^{2009:Q4} = \hat{\zeta}_t - \hat{\zeta}_t^{2009:Q4}.$$

These forecast errors are related to the structural shocks by

$$\hat{\eta}_t^{2009:Q4} = \sum_{j=1}^{t-2009:Q4} F^{j-1}(\hat{\theta})\hat{\varepsilon}_{2009:Q4+j}.$$

The shock decomposition is based on four alternative forecasts, $\hat{\zeta}(\iota)_t^{2009:Q4}$ for $t = 2010:Q1, \ldots, 2010:Q4$ and $\iota \in \{\mathcal{D}, \mathcal{S}, \mathcal{M}, \mathcal{R}\}$. Here, ι indexes one of the four groups of structural shocks. For these, let $\hat{\varepsilon}(\iota)_t$ denote a version of $\hat{\varepsilon}_t$ with all shocks except those in group ι set to zero. With these, we construct

$$\begin{aligned} \hat{\zeta}(\iota)_{2010:Q1}^{2009:Q4} &\equiv F(\hat{\theta})\hat{\zeta}_{2009:Q4} + \hat{\varepsilon}(\iota)_{2010:Q1}, \\ &\vdots \\ \hat{\zeta}_{2010:Q4}^{2009:Q4} &\equiv F(\hat{\theta})\hat{\zeta}_{2010:Q3}^{2009:Q4} + \hat{\varepsilon}(\iota)_{2010:Q4}, \end{aligned}$$

and

$$\hat{\eta}(\iota)_t^{2009:Q4} \equiv \hat{\zeta}_t - \hat{\zeta}(\iota)_t^{2009:Q4}.$$

By construction,

$$\hat{\eta}_t^{2009:Q4} = \sum_{\iota \in \{\mathcal{D},\mathcal{S},\mathcal{M},\mathcal{R}\}} \hat{\eta}(\iota)_t^{2009:Q4}.$$

That is, each forecast error can be written as the sum of contributions from each of the shock groups. Using the observation equations, we transform these into components of the forecast error for observable variables.

With this completed, we can then move the forecast date forward to 2010:Q1. The decomposition for that date proceeds similarly, except that we treat growth in 2010:Q1 as data.

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