The great inflation of the 1970s

Fabrice Collard* and Harris Dellas†

October 1, 2003

Abstract

Was the high inflation of the 1970s due to incomplete information about the structure of the economy and the shocks (an honest mistake)? Or, to weak reaction to inflation and/or excessive policy activism (a policy mistake)? We study this question within the NNS model with policy commitment and imperfect information, requiring that the model have satisfactory overall empirical performance during that period. Under this requirement, we find that the model can replicate the behavior of inflation in the 70s in the case of a very substantial productivity slowdown only if the degree of imperfect information is very high and the policy response to inflation is weak but sufficiently large to prevent indeterminacy.

JEL class: E32 E52

Keywords: Inflation, monetary policy, imperfect information, Kalman filter, policy preferences

*CNRS-GREMAQ, Manufacture des Tabacs, bât. F, 21 allée de Brienne, 31000 Toulouse, France. Tel: (33-5) 61–12–85–60, Fax: (33- 5) 61–22–55–63, email: fabrice.collard@gremaq.univ-tlse1.fr, Homepage: http://fabcol.free.fr

†Department of Economics, University of Bern, CEPR, IMOP. Address: VWI, Gesellschaftsstrasse 49, CH 3012 Bern, Switzerland. Tel: (41) 31-6313989, Fax: (41) 31–631-3992, email: harris.dellas@vwi.unibe.ch, Homepage: http://www-vwi.unibe.ch/amakro/dellas.htm

Introduction

The causes of the "great" inflation of the 1970s remain the subject of debate. While there is widespread agreement that *"loose"* monetary policy played a major rule, there is less agreement concerning the factors responsible for such policy. Some[1] have argued that looseness was a reflection of policy opportunism under discretion (see Ireland, 1999, Sargent, 1999). Others that it was a reflection of — mostly unavoidable — policy mistakes that resulted from the combination of bad luck and erroneous information about the structure of the economy and the shocks (see Orphanides, 1999, Lansing, 2001). And, others that it was the result of inadequate concern for inflation, manifested as a low weigh on inflation in the Henderson-McKibbin-Taylor rule (see Clarida, Gertler and Gali, 2000).

The proponents of the first view use the time inconsistency model of Kydland and Prescott, 1978, and Barro and Gordon, 1983 to claim that inflation was the product of a policy inflation bias. According to this theory, in the absence of commitment, monetary authorities systematically attempt to generate inflation surprises as a means of exploiting the expectational Phillips curve and lowering unemployment. Rational agents, though, recognize this incentive and adjust their inflation expectations accordingly. In equilibrium, unemployment does not fall while inflation becomes inefficiently high. Ireland, 1999, claims that the theory is consistent with the behavior of inflation and unemployment in the US during the last four decades. Sargent, 1999, extends the standard time inconsistency model by including a time–varying, perceived slope of the Phillips curve and reaches a similar conclusion.

The proponents of the *"honest mistake, unavoidable"* view recognize too that the pursued monetary policies proved to be much more inflationary than the FED might have anticipated. They attribute this discrepancy to a variety of factors relating to erroneous information about the structure of the economy. One is that the FED was the "victim" of conventional macroeconomic wisdom of the time that claimed the existence of a stable, permanent tradeoff between inflation and unemployment (De Long, 1997). Another is that while the FED may have not believed in the existence of a long term trade off it faced an unobservable, large, negative shift in productivity growth. It has been claimed that even without any significant differences in pre and post Volcker reaction to inflation developments, the large unobserved slowdown in productivity would have led to exceptionally loose monetary policy as the FED operated along what it perceived to be its "*normal*" short run Phillips curve (Orphanides, 1999, 2001).

[1] See Lansing, 2000, for a survey of several theories.

Finally, Clarida, Gali and Gertler, 2000, have argued that the adverse supply shock would not have been sufficient to generate on its own the inflation rates observed during the 1970s. In their view, the great inflation was due to the fact that, during that period, the Federal Reserve pursued a policy that accommodated inflation and *induced* instability in the economy by lowering real interest rates when expected inflation increased.

A few attempts have been made to examine whether the behavior of inflation during that period can be accounted for by standard macroeconomic models (Cristiano and Gust, 1999, Ireland, 1999, Lansing, 2001, Orphanides and Williams, 2002). With the exception of Christiano and Gust, 1999,(who do not deal with the role of imperfect information, though) the models employed tend to consist of a small number of reduced form equations that are estimated or simulated and then used to generate inflation paths. The limitations of such an approach are well known.

We employ the standard New Neoclassical Synthesis (NNS) model. We abstract from issues of time inconsistency by assuming that the policymakers commit to following a standard Henderson-McKibbin-Taylor (henceforth, HMT) rule. We also pay special attention to specifications that allow the model to have a unique equilibrium. While Clarida, Gali and Gertler, 2000, claim that the high inflation in the 70s was due to FED policies that created indeterminacy (while Orphanides, 2001, argues that this was not the case) we think we should first exhaust the possibilities offered by a model with a determinate equilibrium before investigating alternatives.

The main question we address is whether and under what conditions the NNS model with policy commitment can replicate the evolution of inflation and of other important macroeconomic variables following a severe, persistent slowdown in the rate of productivity growth. And if yes, whether the model also meets additional fitness criteria. The importance of evaluating the ability of the model to account for the 1970s on the basis of a larger set of variables and not just inflation cannot be underestimated. In principle, focusing on a single variable offers too little discipline.

We first examine whether the model can generate a "great inflation" under the assumption that the HMT policy rule pursued at the time did not differ from that commonly attributed to the "Volcker- Greenspan" FED (see Clarida, Gali and Gertler, 2000, Orphanides, 2001)). We find that this can be the case if the productivity slowdown is very large and there also exists a very high degree of imperfect information[2]. Nonetheless, while replicating inflation, the model fails to generate sufficient volatility in key macroeconomic variables.

[2] We model imperfect information in the context of the Kalman filter.

We then study rules that involve a weaker reaction to inflation and/or a stronger–weaker reaction to output (relative to the standard, "V-G" Taylor rule). Within the class of *determinate* equilibria we find that three elements are required in order for the model to be successful: *i)* Very weak reaction to *both* inflation and output deviations from trend. Namely, reaction coefficients in the Taylor rule that are in the neighborhood of 1 and 0 respectively; *ii)* A very large — negative — productivity shock (of the order of 10%-15%). And *iii)* substantial imperfect information about the output gap. Under these conditions the model generates a large, persistent, increase in inflation that is comparable to that experienced in the 70s. And it also succeeds in generating overall macroeconomic volatility that matches well that observed in the data.

We have also experimented with HMT rule specifications that lead to indeterminacy (of the type suggested by Clarida, Gali and Gerler, 2000). The results differ depending on the location of the sunspot but, independent of the degree of imperfect information, the model always fared worse relative to the determinate case with weak response to both inflation and output discussed above.

The rest of the paper is organized as follows. Section 1 presents the model. Section 2 discusses the calibration. Section 3 presents the main results. An appendix describes the mechanics of the solution to the model under imperfect information and learning based on the Kalman filter.

1 The model

The set up is the standard NNS model. The economy is populated by a large number of identical infinitely–lived households and consists of two sectors: one producing intermediate goods and the other a final good. The intermediate good is produced with capital and labor and the final good with intermediate goods. The final good is homogeneous and can be used for consumption (private and public) and investment purposes.

1.1 The Household

Household preferences are characterized by the lifetime utility function:[3]

$$\sum_{\tau=0}^{\infty} E_t \beta^{\tau} U\left(C_{t+\tau}, \frac{M_{t+\tau}}{P_{t+\tau}}, \ell_{t+\tau}\right) \tag{1}$$

[3] $E_t(.)$ denotes mathematical conditional expectations. Expectations are conditional on information available at the beginning of period t.

where $0 < \beta < 1$ is a constant discount factor, C denotes the domestic consumption bundle, M/P is real balances and ℓ is the quantity of leisure enjoyed by the representative household. The utility function,$U\left(C, \frac{M}{P}, \ell\right) : \mathbb{R}_+ \times \mathbb{R}_+ \times [0,1] \longrightarrow \mathbb{R}$ is increasing and concave in its arguments.

The household is subject to the following time constraint

$$\ell_t + h_t = 1 \tag{2}$$

where h denotes hours worked. The total time endowment is normalized to unity.

In each and every period, the representative household faces a budget constraint of the form

$$B_{t+1} + M_t + P_t(C_t + I_t + T_t) \quad \leq \quad R_{t-1}B_t + M_{t-1} + N_t + \Pi_t + P_t W_t h_t + P_t z_t K_t \tag{3}$$

where W_t is the real wage; P_t is the nominal price of the domestic final good;.C_t is consumption and I is investment expenditure; K_t is the amount of physical capital owned by the household and leased to the firms at the real rental rate z_t. M_{t-1}) is the amount of money that the household brings into period t, and M_t is the end of period t money holdings. N_t is a nominal lump–sum transfer received from the monetary authority; T_t is the lump–sum taxes paid to the government and used to finance government consumption.

Capital accumulates according to the law of motion

$$K_{t+1} = I_t - \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + (1-\delta)K_t \tag{4}$$

where $\delta \in [0,1]$ denotes the rate of depreciation. The second term captures the existence of capital adjustment costs. $\varphi > 0$ is the capital adjustment costs parameter.

The household determines her consumption/savings, money holdings and leisure plans by maximizing her utility (1) subject to the time constraint (2), the budget constraint (3) and taking the evolution of physical capital (4) into account.

1.2 Final sector

The final good is produced by combining intermediate goods. This process is described by the following CES function

$$Y_t = \left(\int_0^1 X_t(i)^\theta \mathrm{d}i\right)^{\frac{1}{\theta}} \tag{5}$$

where $\theta \in (-\infty, 1)$. θ determines the elasticity of substitution between the various inputs. The producers in this sector are assumed to behave competitively and to determine their demand for each good, $X_t(i)$, $i \in (0,1)$ by maximizing the static profit equation

$$\max_{\{X_t(i)\}_{i\in(0,1)}} P_t Y_t - \int_0^1 P_t(i) X_t(i) \mathrm{d}i \tag{6}$$

subject to (5), where $P_t(i)$ denotes the price of intermediate good i. This yields demand functions of the form:

$$X_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} Y_t \text{ for } i \in (0,1) \tag{7}$$

and the following general price index

$$P_t = \left(\int_0^1 P_t(i)^{\frac{\theta}{\theta-1}} \mathrm{d}i\right)^{\frac{\theta-1}{\theta}} \tag{8}$$

The final good may be used for consumption — private or public — and investment purposes.

1.3 Intermediate goods producers

Each firm i, $i \in (0,1)$, produces an intermediate good by means of capital and labor according to a constant returns–to–scale technology, represented by the Cobb–Douglas production function

$$X_t(i) = A_t K_t(i)^{\alpha} h_t(i)^{1-\alpha} \text{ with } \alpha \in (0,1) \tag{9}$$

where $K_t(i)$ and $h_t(i)$ respectively denote the physical capital and the labor input used by firm i in the production process. A_t is an exogenous stationary stochastic technology shock, whose properties will be defined later. Assuming that each firm i operates under perfect competition in the input markets, the firm determines its production plan so as to minimize its total cost

$$\min_{\{K_t(i),h_t(i)\}} P_t W_t h_t(i) + P_t z_t K_t(i)$$

subject to (9). This leads to the following expression for total costs:

$$P_t S_t X_t(i)$$

where the real marginal cost, S, is given by $\frac{W_t^{1-\alpha} z_t^{\alpha}}{\chi A_t}$ with $\chi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo, 1983, in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability γ) or it does not. In order to maintain long term money neutrality (in the absence of monetary frictions) we also assume that the price set by the firm grows at the steady state rate of inflation. Hence, if a firm i does not reset its price, the latter is given by $P_t(i) = \overline{\pi} P_{t-1}(i)$. A firm i sets its price, $\widetilde{p}_t(i)$, in period t in order to maximize its discounted profit flow:

$$\max_{\widetilde{p}_t(i)} \widetilde{\Pi}_t(i) + E_t \sum_{\tau=1}^{\infty} \Phi_{t+\tau}(1-\gamma)^{\tau-1} \left(\gamma \widetilde{\Pi}_{t+\tau}(i) + (1-\gamma)\Pi_{t+\tau}(i)\right)$$

subject to the total demand it faces

$$X_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\theta-1}} Y_t$$

and where $\widetilde{\Pi}_{t+\tau}(i) = (\widetilde{p}_{t+\tau}(i) - P_{t+\tau}S_{t+\tau})X(i, s^{t+\tau})$ is the profit attained when the price is reset, while $\Pi_{t+\tau}(i) = (\overline{\pi}^{\tau}\widetilde{p}_t(i) - P_{t+\tau}S_{t+\tau})X_{t+\tau}(i)$ is the profit attained when the price is maintained. $\Phi_{t+\tau}$ is an appropriate discount factor related to the way the household values future as opposed to current consumption. This leads to the price setting equation

$$\widetilde{p}_t(i) = \frac{1}{\theta} \frac{E_t \sum_{\tau=0}^{\infty} \left[(1-\gamma)\overline{\pi}^{\frac{1}{\theta-1}}\right]^{\tau} \Phi_{t+\tau} P_{t+\tau}^{\frac{2-\theta}{1-\theta}} S_{t+\tau} Y_{t+\tau}}{E_t \sum_{\tau=0}^{\infty} \left[(1-\gamma)\overline{\pi}^{\frac{\theta}{\theta-1}}\right]^{\tau} \Phi_{t+\tau} P_{t+\tau}^{\frac{1}{\theta-1}} Y_{t+\tau}} \tag{10}$$

Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price.

In each period, a fraction γ of contracts ends, so there are $\gamma(1-\gamma)$ contracts surviving from period $t-1$, and therefore $\gamma(1-\gamma)^j$ from period $t-j$. Hence, from (8), the aggregate intermediate price index is given by

$$P_t = \left(\sum_{i=0}^{\infty} \gamma(1-\gamma)^i \left(\frac{\widetilde{p}_{t-i}}{\overline{\pi}^i}\right)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}} \tag{11}$$

1.4 The monetary authorities

We assume that monetary policy is conducted according to a standard HMT rule. Namely,

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)[k_{\pi} E_t(\widehat{\pi}_{t+1} - \pi) + k_y(\widehat{y}_t - y_t^{\star})]$$

where $\widehat{\pi}_t$ and $\widehat{y}_t$ are actual output and expected inflation respectively and π and $y_t^{\star}$ are the inflation and output targets respectively. The output target is set equal to potential output and the inflation target to the steady state rate of inflation. Potential output is not observable and the monetary authorities learn about shocks to it gradually. The learning process is described in the appendix[4].

There exists disagreement in the literature regarding the empirically relevant values of k_π and k_y for the 1970s. Clarida, Gali and Gertler, 2000, claim that the pre–Volcker, HMT monetary rule involved a policy response to inflation that was too weak. Namely, that $k_\pi < 1$ which led to real indeterminacies and excessive inflation. The estimate the triplet $\{\rho, k_\pi, k_y\} = \{0.75, 0.8, 0.4\}$. Orphanides, 2001, disputes this claim. He argues that the reaction to — expected — inflation was broadly similar in the pre and post–Volcker period, but the reaction to output was stronger in the earlier period. In particular, using real time date, he estimates $\{\rho, k_\pi, k_y\} = \{0.75, 1.6, 0.6\}$

We investigate the consequences of using alternative values for k_π and k_y in order to shed some light on the role of policy preferences relative to that of the degree of imperfect information for the behavior of inflation.

1.5 The government

The government finances government expenditure on the domestic final good using lump sum taxes. The stationary component of government expenditures is assumed to follow an exogenous stochastic process, whose properties will be defined later.

1.6 The equilibrium

We now turn to the description of the equilibrium of the economy.

Definition 1 *An equilibrium of this economy is a sequence of prices* $\{\mathcal{P}_t\}_{t=0}^\infty = \{W_t, z_t, P_t, R_t, P_t(i), i \in (0,1)\}_{t=0}^\infty$ *and a sequence of quantities* $\{\mathcal{Q}_t\}_{t=0}^\infty = \{\{\mathcal{Q}_t^H\}_{t=0}^\infty, \{\mathcal{Q}_t^F\}_{t=0}^\infty\}$ *with*

$$\begin{aligned} \{\mathcal{Q}_t^H\}_{t=0}^\infty &= \{C_t, I_t, B_t, K_{t+1}, h_t, M_t\} \\ \{\mathcal{Q}_t^H\}_{t=0}^\infty &= \{Y_t, X_t(i), K_t(i), h_t(i); i \in (0,1)\}_{t=0}^\infty \end{aligned}$$

such that:

(i) given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^\infty$ *and a sequence of shocks,* $\{\mathcal{Q}_t^H\}_{t=0}^\infty$ *is a solution to the representative household's problem;*

[4] See Ehrmann and Smets, 2003, for a discussion of optimal monetary policy in a related model.

(ii) *given a sequence of prices* $\{\mathcal{P}_t\}_{t=0}^{\infty}$ *and a sequence of shocks,* $\{\mathcal{Q}_t^F\}_{t=0}^{\infty}$ *is a solution to the representative firms' problem;*

(iii) *given a sequence of quantities* $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ *and a sequence of shocks,* $\{\mathcal{P}_t\}_{t=0}^{\infty}$ *clears the markets*

$$\begin{aligned} Y_t &= C_t + I_t + G_t && (12) \\ h_t &= \int_0^1 h_t(i)\,di && (13) \\ K_t &= \int_0^1 K_t(i)\,di && (14) \\ G_t &= T_t && (15) \end{aligned}$$

and the money market.

(iv) *Prices satisfy (10) and (11).*

2 Parametrization

The model is parameterized on US quarterly data for the period 1960:1–1999:4. The data are taken from the Federal Reserve Database.[5] The parameters are reported in table 1.

β, the discount factor is set such that households discount the future at a 4% annual rate, implying β equals 0.988. The instantaneous utility function takes the form

$$U\left(C_t, \frac{M_t}{P_t}, \ell_t\right) = \frac{1}{1-\sigma}\left[\left(\left(C_t^{\eta} + \zeta \frac{M_t}{P_t}^{\eta}\right)^{\frac{\nu}{\eta}} \ell_t^{1-\nu}\right)^{1-\sigma} - 1\right]$$

where ζ capture the preference for money holdings of the household. σ, the coefficient ruling risk aversion, is set equal to 1.5. ν is set such that the model generates a total fraction of time devoted to market activities of 31%. η is borrowed from Chari et al. (2000), who estimated it on postwar US data (-1.56). The value of ζ, 0.0649, is selected such that the model mimics the average ratio of M1 money to nominal consumption expenditures.

γ, the probability of price resetting is set in the benchmark case at 0.25, implying that the average length of price contracts is about 4 quarters. The nominal growth of the economy, μ, is set such that the average quarterly rate of inflation over the period is $\overline{\pi} = 1.2\%$ per quarter. The quarterly depreciation rate, δ, was set equal to 0.025. θ in

[5] URL: `http://research.stlouisfed.org/fred/`

Table 1: Calibration: Benchmark case

Preferences		
Discount factor	β	0.988
Relative risk aversion	σ	1.500
Parameter of CES in utility function	η	-1.560
Weight of money in the utility function	ζ	0.065
CES weight in utility function	ν	0.344
Technology		
Capital elasticity of intermediate output	α	0.281
Capital adjustment costs parameter	φ	1.000
Depreciation rate	δ	0.025
Parameter of markup	θ	0.850
Probability of price resetting	γ	0.250
Shocks and policy parameters		
Persistence of technology shock	ρ_a	0.950
Standard deviation of technology shock	σ_a	0.008
Persistence of government spending shock	ρ_g	0.970
Volatility of government spending shock	σ_g	0.020
Goverment share	g/y	0.200
Nominal growth	μ	1.012

the benchmark case is set such that the level of markup in the steady state is 15%. α, the elasticity of the production function to physical capital, is set such that the model reproduces the US labor share — defined as the ratio of labor compensation over GDP — over the sample period (0.575).

The evolution of technology is assumed to contain two components. One capturing deterministic growth and the other stochastic growth. The stochastic one, $a_t = \log(A_t/A)$ is assumed to follow a stationary AR(1) process of the form

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$

with $|\rho_a| < 1$ and $\varepsilon_{a,t} \rightsquigarrow \mathcal{N}(0, \sigma_a^2)$. We set $\rho_a = 0.95$ and[6] $\sigma_a = 0.008$.

Alternative descriptions of the productivity process may be equally plausible. For instance, productivity growth may have followed a deterministic trend that permanently shifted downward in the late 60s to early 70s. In our model, this would mean that the FED learns about the trend in productivity rather than about the current level of the — temporary — shock to productivity. We are unsure about how our results would be

[6] There is a non-negligible change in the volatility of the Solow residual between the pre and the post Volcker period. That up to 1979:4 is 0.0084 while that after 1980:1 is 0.0062. For the evaluation of the model it is the former period that is relevant. Note that for the government spending shock the difference between the two periods is negligible.

affected by using an alternative process, but, given the state of the art in this area, we do not think that it is possible to identify the productivity process with any degree of confidence.

The government spending shock[7] is assumed to follow an AR(1) process

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g) \log(\overline{g}) + \varepsilon_{g,t}$$

with $|\rho_g| < 1$ and $\varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$. The persistence parameter is set to, ρ_g, of 0.97 and the standard deviation of innovations is $\sigma_g = 0.02$. The government spending to output ratio is set to 0.20.

An important feature of our analysis is that potential output is imperfectly observed by the agents. In other word, it may be written as

$$y_t^\star = y_t^{\mathrm{P}} + \xi_t$$

where y_t^{P} denotes the true potential output and ξ_t a noisy process that satisfies:

i) $E(\xi_t) = 0$ for all t;

ii) $E(\xi_t \varepsilon_{a,t}) = E(\xi_t \varepsilon_{g,t}) = 0$;

iii) and

$$E(\xi_t \xi_k) = \begin{cases} \sigma_\xi^2 & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

In order to facilitate the interpretation of σ_ξ we set its value in relation to the volatility of the technology shock. More precisely, we define ς as $\varsigma = \sigma_\xi / \sigma_a$. Several values are then assigned to ς to gauge the effects of imperfect information in the model.

3 The results

The model is first log–linearized around the deterministic steady state and then solved according to the method outlined in the appendix.

We start by assuming a standard specification for the HMT rule, namely, $\rho = 0.75$, $k_\pi = 1.5$ and $k_y = 0.5$ and vary the degree of uncertainty — the quality of the signal — about potential output[8]. The objective of this exercise is to determine *i)* whether

[7] The –logarithm of the– government expenditure series is first detrended using a linear trend.

[8] To be more precise, we vary the size of ς.

a policy reaction function of the type commonly attributed to the FED during the 80s and 90s would have prevented the occurrence of high and persistent inflation of the type observed in the 70s; and *ii)* the role played by imperfect information in this. This exercise may then prove useful for determining whether the great inflation was mainly the result of bad luck and incomplete information or of inflation bias — the element emphasized by Ireland, 1999 — or insufficiently aggressive reaction to inflation developments — a low k_π, as emphasized by Clarida, Gerler and Gali, 2000. Orphanides, 2001, has defended FED policy during the 70s along these lines by arguing that the FED's reaction to inflation did not differ significantly between the pre and post Volcker era.

We report two sets of statistics. The volatility of H-P filtered actual output, inflation and investment. And the impulse response functions (IRF) of actual output and inflation following a negative technology shock for the perfect information model (Perf. Info.), the imperfect information model with $\varsigma = 1$ (Imp. Info. (I)) and $\varsigma = 10$ (Imp. Info. (II)). The IRF for the inflation rate is annualized and expressed in percentage points. To find the actual rate of inflation following a shock we need to add the response reported in the IRF to the steady state value (which is 4.8%).

There exists considerable uncertainty about the (type and) size of the shock that triggered the productivity slowdown of the 70s. We do not take a position on this. We proceed by selecting a value for the supply shock that can generate a large and persistent increase in the inflation rate. By large, we mean an increase in the inflation rate of the order of 5-6 percentage points (so that the maximum rate of inflation obtained during that period is about 10%-11%). We then feed a series of shocks that include this value into our model and generate the other statistics described above.

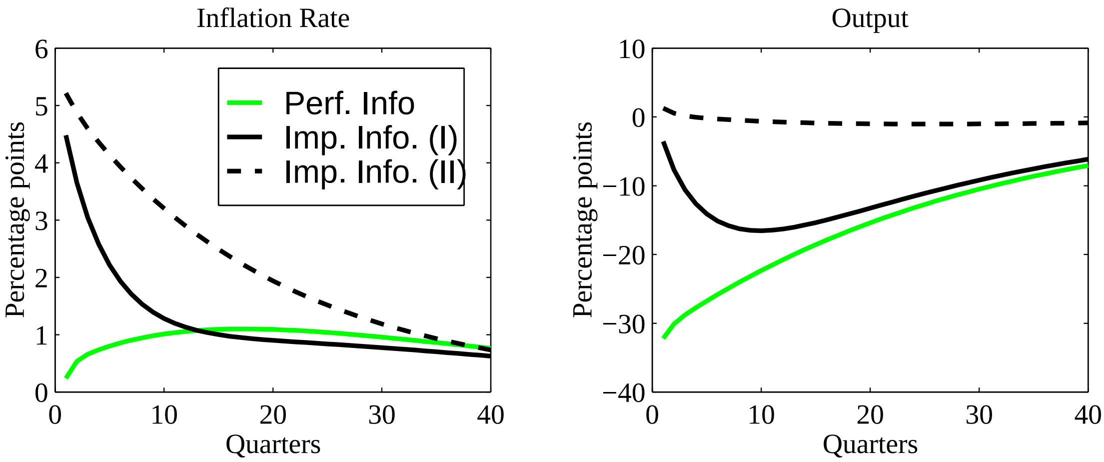
Figure 1, Panel A shows that the model can produce a large and persistent increase in the inflation rate if two conditions are met: The shock is very large (of the order of 30%). And the degree of imperfect information is very high (say, $\varsigma = 10$). Imperfect information is critical because it increases the persistence of inflation while leading to a smoother output path. Moreover, table 2 indicates that the maximum and minimum effects on output and inflation following such a shock are plausible. But the model fails, though, in generating a realistic degree of macroeconomic volatility. In particular, table 3 shows that the model significantly under-predicts volatility.

Increasing the degree of degree of price flexibility (say, from $q = 0.25$ to $q = 1/3$ does not help much. The size of the required supply shock decreases (from 30% to 20%) and volatility increases somewhat but not sufficiently (table 3, panel B). We have run a larger number of experiments involving the same HMT rule and alternative values of the other parameters of the model without managing to improve overall model performance.

We then turned to alternative specifications of the policy rule. Figure 2 and tables 4-5 report the most successful specification encountered (within the range of determinate equilibria). It involves a weak reaction to inflation $k_\pi = 1.01$, passive policy $k_y = 0.01$, somewhat more flexible prices, $q = 1/3$, (an average length of price contracts of about 3 quarters, which is closer to the estimate of Bils and Klenow, 2002), a high degree of imperfect information ($\varsigma = 10$) and a substantial negative productivity shock (15%). Under this parametrization, the model can generate a large and persistence increase in inflation following the productivity slowdown and also macroeconomic volatility that is close to that observed during the sample period.

Figure 1: IRF to a negative technology shock: $((\rho, \kappa_\pi, \kappa_y)$=(0.75,1.50,0.50))

Panel A: q=0.25, -30% shock



Panel B: q=1/3, -20% shock

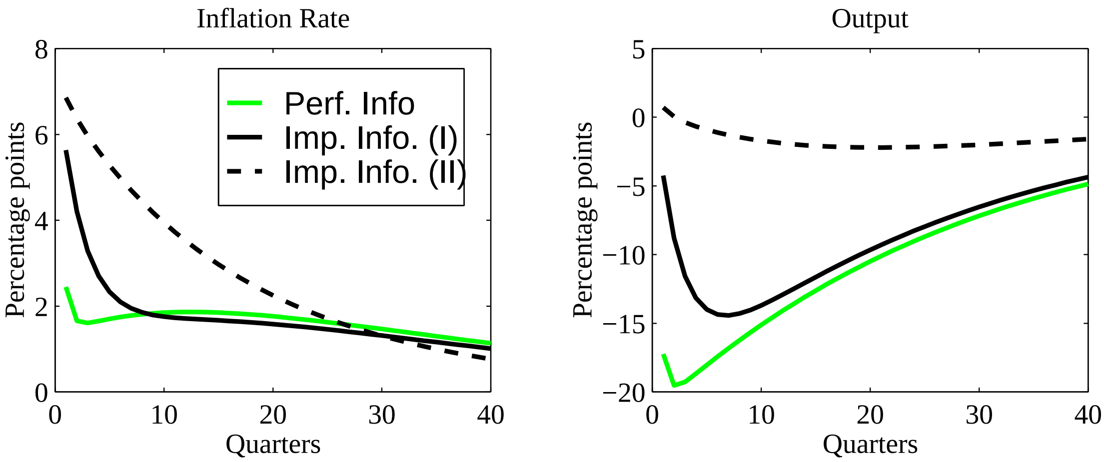


Table 2: Impact and extreme effect of a technology shock $((\rho, \kappa_\pi, \kappa_y)$=(0.75,1.50,0.50))

	Perf. Info		Imp. Info (I)		Imp. Info (II)	
	q=0.25, -30% Shock					
Output	-32.195	-32.195	-3.559	-16.532	1.263	-1.028
Inflation	0.240	1.102	4.481	4.481	5.214	5.214
	q=1/3, -20% Shock					
Output	-21.526	-21.526	-5.717	-14.373	0.601	-2.210
Inflation	-0.107	0.828	4.770	4.770	6.793	6.793

Note: Perfect information, Imperfect information (I) and Imperfect information (II) correspond to ς=0,1,10 respectively, where ς is the amount of noise.

Table 3: Standard Deviations $((\rho, \kappa_\pi, \kappa_y)$=(0.75,1.50,0.50))

	σ_y	σ_i	σ_π
Data	1.639	7.271	0.778
	q=0.25, -30% Shock		
Perf. Info.	2.982	10.675	0.072
Imp. Info. (I)	1.576	5.671	0.409
Imp. Info. (II)	0.376	2.160	0.487
	q=1/3, -20% Shock		
Perf. Info.	2.275	8.117	0.076
Imp. Info. (I)	1.675	6.012	0.514
Imp. Info. (II)	0.548	2.203	0.742

Note: The standard deviations are computed for HP–filtered series. y, i and π are output, investment and inflation respectively. Perfect information, Imperfect information I and Imperfect information II correspond to ς=0,1,10 respectively where ς is the amount of noise.

Figure 2: IRF to a negative technology shock: $((\rho, \kappa_\pi, \kappa_y)$=(0.75,1.01,0.01),q=1/3)

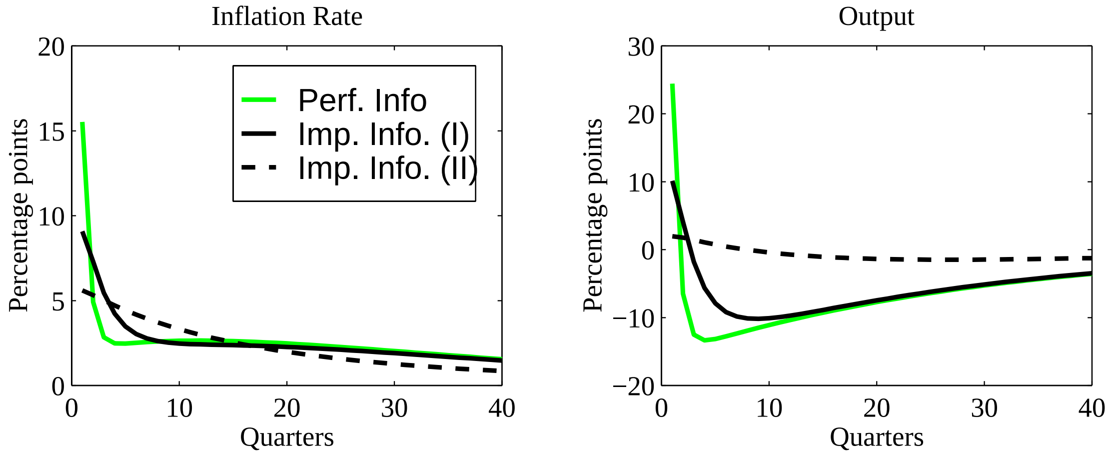


Table 4: Impact and extreme effect of a technology shock (($\rho, \kappa_\pi, \kappa_y$)=(0.75,1.01,0.01))

	Perf. Info		Imp. Info (I)		Imp. Info (II)	
	q=1/3, -15% Shock					
Output	24.440	-13.349	10.132	-10.188	1.960	-1.479
Inflation	15.517	15.517	9.079	9.079	5.595	5.595

Table 5: Standard Deviations (($\rho, \kappa_\pi, \kappa_y$)=(0.75,1.01,0.01))

	σ_y	σ_i	σ_π
Data	1.639	7.271	0.778
	q=1/3, -15% Shock		
Perf. Info.	3.597	15.186	1.467
Imp. Info. (I)	2.906	11.972	1.248
Imp. Info. (II)	1.533	6.178	0.850

Note: The standard deviations are computed for HP–filtered series. y, i and π are output, investment and inflation respectively.

We have also investigated the properties of the model under the policy rule parametrization suggested by Clarida et al. Namely, $\rho = 0.75, \kappa_\pi = 0.80, \kappa_y = 0.40$. This rule leads to an indeterminant equilibrium. We have experimented with different assumptions about the source of the indeterminacy (the location of the sunspot). The most satisfactory results obtain when the sunspot is placed on the marginal utility of consumption. But while the model in this case can easily generate a sufficiently large and persistent effect on inflation even with a much smaller technology shock (see Figures 3, the associated response of output and macroeconomic volatilities are not plausible, independent of the degree of imperfect information. In particular, volatility is very large even when the variance of the sunspot is negligible(see Tables 6-7).

4 Conclusions

Inflation in the US reached high levels during the 1970s, due to a large extent to what proved to be excessively loose monetary policy. There exist several views concerning the conduct of policy at that time. One views it as an honest mistake on the part of a monetary authority whose tolerance of inflation did not differ significantly from that commonly attributed to the authorities in the 80s and 90s. According to this view (Orphanides, 2001), the large decrease in actual output following the persistent downward shift in potential output was interpreted as a decrease in the output gap.

It led to expansionary monetary policy that exaggerated the inflationary impact of the decrease in potential output. Eventually and after a long delay, the FED realized that potential output growth was lower and adjusted policy to bring inflation down. Imperfect information rather than tolerance of inflation played the critical role in the inflation process.

Another leading view is that the FED's reaction rule exhibited a weak response towards inflation (relative to the Volcker–Greenspan (V–G) era) and perhaps more policy activism (Clarida, Gali and Gertler, 2001). The implication of this view is that adoption of the standard (under V–G) Henderson- McKibbin-Taylor rule would have prevented the persistent surge in inflation.

Our findings offer qualified support to the latter view but, at the same time, suggest that the informational problems emphasized by Orphanides may have played an important role. If the reaction to expected inflation is sufficiently large then no matter how large imperfect information may be, the model cannot generate a path of inflation that resembles that actually observed during the seventies and *at the same time* satisfy other fitness criteria. In contrast, were one to accept that the shock that triggered the great inflation were indeed very substantial, a HMT rule with weak reaction to both inflation and output and a great deal of imperfect information could generate behavior that is much more consistent with the data. Interestingly, the reaction to inflation should not be so weak as to lead to indeterminacy. In particular, the model performs very poorly under a monetary policy specification that gives rise to indeterminacies.

References

Barro, Robert and David Gordon, 1983,"Rules, discretion and reputation in a model of monetary policy", Journal of Monetary Economics, 12 (1), 101–21.

Bils, Mark and Peter Klenow, 2002, "Some Evidence on the Importance of Sticky Prices," NBER wp #9069.

Clarida, Richard, Jordi Gali, and Mark Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", Quarterly Journal of Economics, pp. 147–180.

Chari, V., Patrick Kehoe and Ellen McGrattan, 2000, "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rate Changes?" NBER working paper, No 7869.

Christiano, Larry and Christopher Gust,1999, "The Great INflation of the 1970s", mimeo.

DeLong, Bradford, 1997, "America's Peacetime Inflation: The 1970s", In Reducing Inflation: Motivation and Strategy, eds. C. Romer and D. Romer, pp. 247–276. Chicago: Univ. of Chicago Press.

Ehrmann,Michael and Frank Smets, 2003, " Uncertain Potential Output: Implications for Monetary Policy, "Journal of Economic Dynamics and Control, 27, 1611 1638.

Ireland, Peter, 1999, "Does the Time-Consistency Problem Explain the Behavior of Inflation in the United States?" Journal of Monetary Economics, 44(2) pp. 279–91.

Kydland, Finn E., and Edward C. Prescott, 1977, "Rules Rather than Discretion: The Inconsistency of Optimal Plans." Journal of Political Economy 85, pp. 473-491.

Lansing, Kevin J, 2001, "Learning about a Shift in Trend Output: Implications for Monetary Policy and Inflation." Unpublished manuscript. FRB San Francisco.

Orphanides, Athanasios, 1999, "The Quest for Prosperity without Inflation." Unpublished manuscript. Federal Reserve Board, Division of Monetary Affairs.

Orphanides, Athanasios, 2001, "Monetary Policy Rules, Macroeconomic Stability and Inflation: A View from the Trenches," BGFRS.

Orphanides, Athanasios and John C. Williams, 2002, "Imperfect Knowledge, Inflation Expectations, and Monetary Policy," BGFRS.

Sargent, Thomas J, 1999, "The Conquest of American Inflation". Princeton: Princeton Univ. Press.

5 Appendix

The solution of the model under imperfect information with a Kalman filter

Let's consider the following system

$$M_{cc}Y_t = M_{cs}\begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} + M_{ce}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \tag{16}$$

$$M_{ss0}\begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t} \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} + M_{se1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} = M_{sc0}Y_{t+1|t} + M_{sc1}Y_t + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{17}$$

$$S_t = C^0\begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} + C^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} + v_t \tag{18}$$

Y is a vector of n_y control variables, S is a vector of n_s signals used by the agents to form expectations, X^b is a vector of n_b predetermined (backward looking) state variables (including shocks to fundamentals), X^f is a vector of n_f forward looking state variables, finally u and v are two Gaussian white noise processes with variance–covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv') = 0$. $X_{t+i|t} = E(X_{t+i}|\mathcal{I}_t)$ for $i \geqslant 0$ and where $\mathcal{I}_t$ denotes the information set available to the agents at the beginning of period t.

Before solving the system, note that, from (16), we have

$$Y_t = B^0\begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} + B^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \tag{19}$$

where $B^0 = M_{cc}^{-1}M_{cs}$ and $B^1 = M_{cc}^{-1}M_{ce}$, such that

$$Y_{t|t} = B\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \tag{20}$$

with $B = B^0 + B^1$.

5.1 Solving the system

Step 1: We first solve for the expected system:

$$M_{ss0}\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + (M_{ss1} + M_{se1})\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} = M_{sc0}Y_{t+1|t} + M_{sc1}Y_{t|t} \tag{21}$$

Plugging (20) in (21), we get

$$\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} = W \begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \tag{22}$$

where

$$W = -\left(M_{ss0} - M_{sc0}B\right)^{-1}\left(M_{ss1} + M_{se1} - M_{sc1}B\right)$$

After getting the Jordan form associated to (22) and applying standard methods for eliminating bubbles, we get

$$X^f_{t|t} = GX^b_{t|t}$$

From which we get

$$\begin{aligned} X^b_{t+1|t} &= (W_{bb} + W_{bf}G)X^b_{t|t} = W^b X^b_{t|t} && (23) \\ X^f_{t+1|t} &= (W_{fb} + W_{ff}G)X^b_{t|t} = W^f X^b_{t|t} && (24) \end{aligned}$$

Step 2: We go back to the initial system to get and write

Then, (17) rewrites

$$\begin{aligned} M_{ss0}\begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t} \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t} \\ X^f_{t} \end{pmatrix} + M_{se1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} &= M_{sc0}B\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{sc1}B^0\begin{pmatrix} X^b_{t} \\ X^f_{t} \end{pmatrix} \\ &\quad + M_{sc1}B^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \end{aligned}$$

Taking expectations, we have

$$\begin{aligned} M_{ss0}\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} + M_{se1}\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} &= M_{sc0}B\begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_{sc1}B^0\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \\ &\quad + M_{sc1}B^1\begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} \end{aligned}$$

Subtracting, we get

$$M_{ss0}\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} + M_{ss1}\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} = M_{sc1}B^0\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} + \begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{25}$$

which rewrites

$$\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} = W^c\begin{pmatrix} X^b_{t} - X^b_{t|t} \\ X^f_{t} - X^f_{t|t} \end{pmatrix} + M^{-1}_{ss0}\begin{pmatrix} M_e u_{t+1} \\ 0 \end{pmatrix} \tag{26}$$

where, $W^c = -M^{-1}_{ss0}(M_{ss1} - M_{sc1}B^0)$. Hence, considering the second block of the above matrix equation, we get

$$W^c_{fb}(X^b_t - X^b_{t|t}) + W^c_{ff}(X^f_t - X^f_{t|t}) = 0$$

which gives

$$X_t^f = F^0 X_t^b + F^1 X_{t|t}^b$$

with $F^0 = -{W_{ff}^c}^{-1} W_{fb}^c$ and $F^1 = G - F^0$.

Now considering the first block we have

$$X_{t+1}^b = X_{t+1|t}^b + W_{bb}^c (X_t^b - X_{t|t}^b) + W_{bf}^c (X_t^f - X_{t|t}^f) + M^2 u_{t+1}$$

from which we get using (23)

$$X_{t+1}^b = M^0 X_t^b + M^1 X_{t|t}^b + M^2 u_{t+1}$$

with $M^0 = W_{bb}^c + W_{bf}^c F^0$, $M^1 = W^b - M^0$ and $M^2 = M_{ss0}^{-1} M_e$.

We also have

$$S_t = C_b^0 X_t^b + C_t^0 X_t^f + C_b^1 X_{t|t}^b + C_f^1 X_{t|t}^f + v_t$$

from which we get

$$S_t = S^0 X_t^b + S^1 X_{t|t}^b + v_t$$

where $S^0 = C_b^0 + C_f^0 F^0$ and $S^1 = C_b^1 + C_f^0 F^1 + C_f^1 G$

Finally, we get

$$Y_t = B_b^0 X_t^b + B_t^0 X_t^f + B_b^1 X_{t|t}^b + B_f^1 X_{t|t}^f$$

from which we get

$$Y_t = \Pi^0 X_t^b + \Pi^1 X_{t|t}^b$$

where $\Pi^0 = B_b^0 + B_f^0 F^0$ and $\Pi^1 = B_b^1 + B_f^0 F^1 + B_f^1 G$

5.2 Filtering

Since our solution involves terms in $X_{t|t}^b$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that we described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X_{t|t}^b$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectational errors. Therefore, let us define

$$\widehat{X}_t^b = X_t^b - X_{t|t-1}^b$$

and

$$\widehat{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X^b_{t|t}$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X^b_{t|t}$ can be used to infer anything on $X^b_{t|t}$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}^e_t = S_t - S^1 X^b_{t|t}$. The filtering equation then writes

$$X^b_{t|t} = X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) = X^b_{t|t-1} + K(S^0 \widehat{X}^b_t + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X^b_{t|t-1}$, we have

$$\begin{aligned}
\widehat{S}_t &= S^0(X^b_t - X^b_{t|t}) + S^1(X^b_{t|t} - X^b_{t|t-1}) + v_t \\
&= S^0 \widehat{X}^b_t + S^1 K(S^0 \widehat{X}^b_t + v_t) + v_t \\
&= S^\star \widehat{X}^b_t + \nu_t
\end{aligned}$$

where $S^\star = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned}
\widehat{X}^b_{t+1} &= M^0(X^b_t - X^b_{t|t}) + M^2 u_{t+1} \\
&= M^0(X^b_t - X^b_{t|t-1} - X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widehat{X}^b_t - M^0(X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0 \widehat{X}^b_t - M^0 K(S^0 \widehat{X}^b_t + v_t) + M^2 u_{t+1} \\
&= M^\star \widehat{X}^b_t + \omega_{t+1}
\end{aligned}$$

where $M^\star = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widehat{X}^b_{t+1} = M^\star \widehat{X}^b_t + \omega_{t+1} \tag{27}$$

$$\widehat{S}_t = S^\star \widehat{X}^b_t + \nu_t \tag{28}$$

For which the Kalman filter is given by

$$\widehat{X}^b_{t|t} = \widehat{X}^b_{t|t-1} + PS^{\star\prime}(S^\star P S^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widehat{X}^b_t + \nu_t)$$

But since $\widehat{X}^b_{t|t}$ is an expectation error, it is not correlated with the information set in $t-1$, such that $\widehat{X}^b_{t|t-1} = 0$. The prediction formula for $\widehat{X}^b_{t|t}$ therefore reduces to

$$\widehat{X}^b_{t|t} = PS^{\star\prime}(S^\star P S^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widehat{X}^b_t + \nu_t) \tag{29}$$

where P solves

$$P = M^\star P M^{\star\prime} + \Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0'} + M^2\Sigma_{uu}M^{2'}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned}
X^b_{t|t} &= X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) \\
&= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - (S_{t|t-1} - S^1X^b_{t|t-1})) \\
&= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - S^0X^b_{t|t-1})
\end{aligned}$$

Solving for $X^b_{t|t}$, we get

$$\begin{aligned}
X^b_{t|t} &= (I + KS^1)^{-1}(X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\
&= (I + KS^1)^{-1}(X^b_{t|t-1} + KS^1X^b_{t|t-1} - KS^1X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\
&= (I + KS^1)^{-1}(I + KS^1)X^b_{t|t-1} + (I + KS^1)^{-1}K(S_t - (S^0 + S^1)X^b_{t|t-1})) \\
&= X^b_{t|t-1} + (I + KS^1)^{-1}K\widehat{S}_t \\
&= X^b_{t|t-1} + K(I + S^1K)^{-1}\widehat{S}_t \\
&= X^b_{t|t-1} + K(I + S^1K)^{-1}(S^\star\widehat{X}^b_t + \nu_t)
\end{aligned}$$

where we made use of the identity $(I + KS^1)^{-1}K \equiv K(I + S^1K)^{-1}$. Hence, identifying to (29), we have

$$K(I + S^1K)^{-1} = PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}$$

remembering that $S^\star = (I + S^1K)S^0$ and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$, we have

$$K(I+S^1K)^{-1} = PS^{0'}(I+S^1K)'((I+S^1K)S^0PS^{0'}(I+S^1K)'+(I+S^1K)\Sigma_{vv}(I+S^1K)')^{-1}(I+S^1K)S^0$$

which rewrites as

$$\begin{aligned}
K(I + S^1K)^{-1} &= PS^{0'}(I + S^1K)'\left[(I + S^1K)(S^0PS^{0'} + \Sigma_{vv})(I + S^1K)'\right]^{-1} \\
K(I + S^1K)^{-1} &= PS^{0'}(I + S^1K)'(I + S^1K)'^{-1}(S^0PS^{0'} + \Sigma_{vv})^{-1}(I + S^1K)^{-1}
\end{aligned}$$

Hence, we obtain

$$K = PS^{0'}(S^0PS^{0'} + \Sigma_{vv})^{-1} \tag{30}$$

Now, recall that

$$P = M^\star PM^{\star\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^\star = M^0(I + KS^0)$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0'} + M^2\Sigma_{uu}M^{2'}$, we have

$$\begin{aligned}
P &= M^0(I - KS^0)P\left[M^0(I - KS^0)\right]' + M^0K\Sigma_{vv}K'M^{0'} + M^2\Sigma_{uu}M^{2'} \\
&= M^0\left[(I - KS^0)P(I - S^{0'}K') + K\Sigma_{vv}K'\right]M^{0'} + M^2\Sigma_{uu}M^{2'}
\end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0 \left[P - PS^{0\prime}(S^0 P S^{0\prime} + \Sigma_{vv})^{-1} S^0 P \right] M^{0\prime} + M^2 \Sigma_{uu} M^{2\prime} \tag{31}$$

5.3 Summary

We finally end–up with the system of equations:

$$\begin{aligned}
X^b_{t+1} &= M^0 X^b_t + M^1 X^b_{t|t} + M^2 u_{t+1} && (32)\\
S_t &= S^0_b X^b_t + S^1_b X^b_{t|t} + v_t && (33)\\
Y_t &= \Pi^0_b X^b_t + \Pi^1_b X^b_{t|t} && (34)\\
X^f_t &= F^0 X^b_t + F^1 X^b_{t|t} && (35)\\
X^b_{t|t} &= X^b_{t|t-1} + K(S^0(X^b_t - X^b_{t|t-1}) + v_t) && (36)\\
X^b_{t+1|t} &= (M^0 + M^1) X^b_{t|t} && (37)
\end{aligned}$$

which describe the dynamics of our economy.

Figure 3: IRF to a negative -5% technology shock: ($(\rho, \kappa_\pi, \kappa_y)$=(0.75,0.80,0.40))

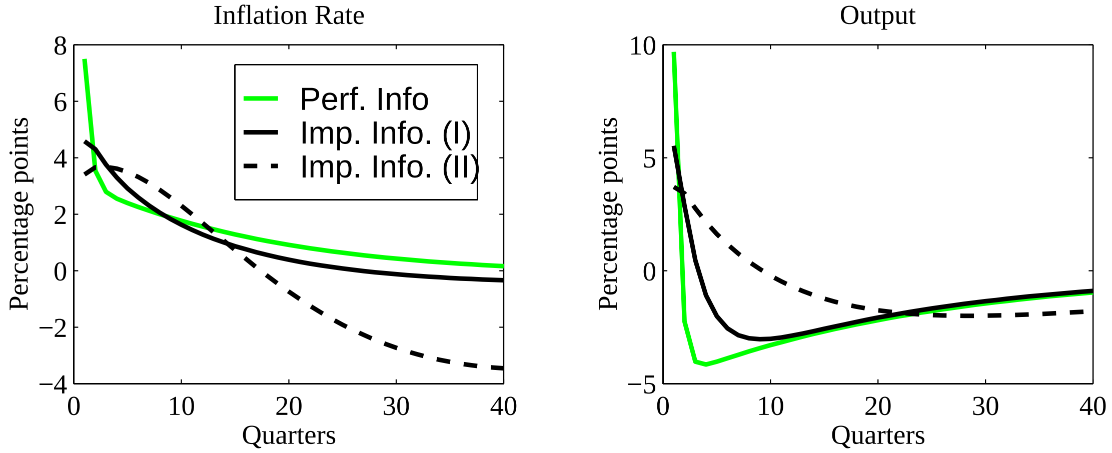


Table 6: Impact and extreme effect of a technology shock ($(\rho, \kappa_\pi, \kappa_y)$=(0.75,0.80,0.40)

	Perf. Info		Imp. Info (I)		Imp. Info (II)	
Output	9.690	-4.148	5.529	-3.032	3.715	-1.993
Inflation	7.504	7.504	4.588	4.588	3.406	3.679

Table 7: Standard Deviations ($(\rho, \kappa_\pi, \kappa_y)$=(0.75,1.01,0.01))

	σ_y	σ_i	σ_π
	q=1/3, -15% Shock $(\rho, \kappa_\pi, \kappa_y)$=(0.75,0.80,0.40)		
Perf. Info.	2.956	11.908	1.607
Imp. Info. (I)	4.283	18.733	2.195
Imp. Info. (II)	18.635	85.925	8.951

Note: The standard deviations are computed for HP–filtered series. y, i and π are output, investment and inflation respectively.