Ramsey Monetary Policy and International Relative Prices*

Ester Faia†
Universitat Pompeu Fabra

Tommaso Monacelli‡
IGIER, Università Bocconi and CEPR

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Abstract

We analyze welfare maximizing monetary policy.in a dynamic two-country model with price stickiness and imperfect competition. In this context, a typical terms of trade externality affects policy competition between independent monetary authorities. Unlike the existing literature, we remain consistent to a public finance approach by an explicit consideration of all the distortions that are relevant to the Ramsey planner. This strategy entails two main advantages. First, it allows an accurate characterization of optimal policy in an economy that evolves around a steady-state which is not necessarily efficient. Second, it allows to describe a full range of alternative dynamic equilibria when price setters in both countries are completely forward-looking and household's preferences are not restricted. In this context, we study optimal policy both in the long-run and along a dynamic path, and we compare optimal commitment policy under Nash competition and under cooperation. By deriving a second order accurate solution to the policy functions, we also characterize the welfare gains from commitment as well as from international policy cooperation.

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†Correspondence to Universitat Pompeu Fabra, Ramon Trias Fargas 25, Barcelona, Spain. E-mail: ester.faia@econ.upf.es, homepage: http://www.econ.upf.es/~faia/.

‡Correspondence to IGIER Universita' Bocconi, Via Salasco 3/5, 20136 Milan, Italy. Email: tommaso.monacelli@uni-bocconi.it, Tel: +39-02-58363330, Fax: +39-02-58363332, Web page: www.igier.uni-bocconi.it/monacelli.

1 Introduction

In the classic approach to the study of optimal policy in dynamic economies (Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1992)), and in a typical public finance spirit, a social planner maximizes household's welfare subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy, and via an explicit consideration of all the distortions that characterize the long-run and cyclical behavior of the economy. Relative to the corresponding closed economy literature, this kind of approach has received much less attention in the analysis of optimal monetary and exchange rate arrangements for open economies.[1]

In this paper we characterize welfare maximizing monetary policy in a dynamic two-country world where financial markets are complete, policymakers act under commitment and compete in a Nash equilibrium. Both economies are characterized by two distortions: goods markets are monopolistic competitive and producers face quadratic costs of adjusting prices. However, and relative to a cooperative setting enforced by a world Ramsey planner, openness adds a further inefficiency typical of the equilibrium outcome under a Nash equilibrium. This inefficiency stems from the monopoly power that each country can exert on its own terms of trade, and therefore from a policy competition motive that this necessarily entails.[2]

We argue that policy competition on the terms of trade shapes the optimal setting of policy along two dimensions. First, in affecting the wedge between the marginal rate of substitution and the marginal rate of transformation. In an open economy, in fact, where consumer and producer prices necessarily differ, such a wedge does not only depend on the fact that markups are time-varying (due to monopolistic competition coupled with sticky prices), but also on the dynamic behavior of the terms of trade. Second, policymakers in different countries compete in affecting the cross-country allocation of wealth. Each policymaker faces the incentive to try to appreciate its own terms of trade and therefore increase the real income of its own residents for any given level of consumption and labor effort.

The central insight of our analysis is that both these policy competition motives - on distortions and on wealth - constitute a source of deviation from a price stability policy. Only when policy is set by a world Ramsey planner the two countries are able to coordinate their actions in such a way

[1]Recently there has been a resurgence of interest for a Ramsey approach in the analysis of optimal policy in dynamic sticky-price models. Important contributions are Schmitt-Grohe and Uribe (2002), Siu (2002), and Khan, King and Wolman (2003). For an analysis of optimal fiscal policy in a small open economy see Schmitt-Grohe and Uribe (2003).

[2]See Corsetti and Pesenti (2000), Tille (2001), Benigno and Benigno (2003), Sutherland (2002). The idea that terms of trade spillovers generate an externality and therefore room for international (monetary and/or fiscal) policy coordination is already discussed (although within ad-hoc models) in Canzoneri and Henderson (1991), Persson and Tabellini (1995) and dates back in the trade literature at least to Johnson (1954). Chari and Kehoe (1990) discuss the specific role of terms of trade distortions for the optimal fiscal policy in a two-country general equilibrium model.

to replicate very closely the equilibrium dynamics that would prevail under purely flexible prices.

Optimal monetary policy in an international setting has recently become synonymous with New Open Economy Macroeconomics (henceforth NOEM). The NOEM has featured two types of approach at the matter. One class of models, exemplified by the work of Obstfeld and Rogoff (1995), Corsetti and Pesenti (2002) and Devereux and Engel (2002), has taken the form of elegant but more stylized frameworks in which the analysis of optimal policy is simplified by the assumption that prices (or wages) are predetermined one-period. This assumption is restrictive, for it typically gives rise to a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role. It is by now well understood that this entails a major consequence in that it neglects the sense in which (time consistent) discretionary policies are suboptimal in dynamic environments with forward-looking price (and/or wage) decisions.[3]

In order to overcome the above limitation, a second class of models in the NOEM literature involves the specification of forward-looking price setting mechanisms, either in the Calvo-Yun (1996) form or like in Rotemberg (1982). Examples comprise, among others, Benigno and Benigno (2002), Mc Callum and Nelson (2000), Smets and Wouters (2002). In this framework, and following the lead of Woodford (2003), the typical approach to the study of optimal policy builds on a two-step procedure that involves, at first, taking a log-linear approximation of the competitive equilibrium conditions, and then deriving a quadratic approximation to the households' utility function. This strategy aims at reducing the optimal policy problem to a tractable linear-quadratic specification. However, for this approach to deliver an accurate representation of welfare associated to alternative policies, specific conditions must be satisfied that guarantee that the economy evolves around an efficient allocation (namely a first-best steady state). Benigno and Benigno (2003) show that, in open economies, these conditions take the form of restrictions on households' preferences, such as log-utility and unitary international elasticity of substitution. Yet precisely these assumptions already constrain the form of the optimal policy to coincide, somewhat artificially, with the one that implements the flexible price allocation. Furthermore, if not satisfied, the same conditions do not allow to study each country's policymaker's problem independently, forcing to ignore those equilibria that emerge under policy competition and to restrict the analysis only to world social planner's policy design problems.

The strategy employed in the present paper differs from both streams of the recent NOEM. It differs from the first stream in that it employs producers' price setting decisions that are completely forward-looking. It differs from the second stream in that it allows to describe the form of the optimal policy in a way that is independent of the underlying specification of preferences. In particular, our approach allows us to study optimal policy in an economy that evolves around a

[3] Goodfriend and King (1997), Woodford (2003).

steady-state which is not necessarily efficient. In that, it differs crucially from a recurrent strategy in the recent New-Keynesian literature (which is at the heart of the aforementioned linear-quadratic approach) of forcing fiscal policy to offset second order effects of stochastic uncertainty on the mean levels of variables.[4]

From a methodological point view, it is also of some interest that we reverse the logic of the linear-quadratic approach. In fact, we employ standard log-linear approximation methods to describe the policy function only when the optimal Ramsey allocation has been already characterized. On the other hand, when computing relative *conditional* welfare of alternative policy equilibria, we resort to a second order approximation of the same policy function. This is necessary to account for the natural effect of stochastic volatility on the first moments of critical variables, as well as for the transitional dynamics that characterize the economy in its adjustment towards the long-run steady-state of the Ramsey policy.

Recently, and within the NOEM paradigm, several papers have re-considered the issue of the international optimal monetary policy design problem. Benigno and Benigno (2003) adopt a one-period predetermined price framework and then turn to the linear-quadratic specification of the optimal policy problem when price setting is forward-looking. Clarida, Gali and Gertler (2002) follow a similar strategy. Devereux and Engel (2003) compare equilibria with producer and local currency pricing but in an essentially static setting. The same feature is in Sutherland (2002), where the effect on welfare of financial markets imperfections is analyzed. Pappa (2002) defines a two-country fully dynamic general equilibrium setup but resorts to a linear-quadratic approach. More in line with the present analysis is the one of Kollmann (2003) and Bergin and Tchakarov (2003), who study optimizing linear interest rate rules and perform welfare calculations based on a second order approximation of the model's equilibrium conditions. Our paper differs crucially in that it characterizes equilibrium allocations under the Ramsey policy, without restricting the form of the policy function to the arguments of a pre-specified (log-linear) interest rate rule.

[4]See, for instance, Rotemberg and Woodford (1997), and a long list of papers thereafter. More precisely, in an open economy, the presence (as we shall see) of an additional terms of trade distortion, pushing inflationary incentives in the opposite direction relative to the markup distortion, does not necessarily require the use lump-sum subsidies. What it takes for the (zero-inflation) steady-state allocation to coincide with the efficient one is that the two competing distortions exactly neutralize each other. However this happens to be the case exactly under the knife-edge conditions of log-utility and unitary elasticity of substitution. This is the reason why an accurate second order approximation of welfare is feasible only when such conditions are met. More recently, Benigno and Woodford (2003) take a step towards overcoming this problem and show (although within a closed economy model) how to preserve a quadratic approximation of the household's welfare objective in the case in which the economy fluctuates around a non-efficient steady-state. This per se requires taking a second order approximation also of (some of) the underlying equilibrium conditions.

2 The Model

The world economy consists of two countries which are symmetric in size. We label them Home and Foreign. Each economy is populated by infinite-lived agents, whose total measure is normalized to unity.

2.0.1 Domestic Households

Let's denote by $C_t \equiv [(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta-1}{\eta}}$ a composite consumption index of domestic and imported bundles of goods, where α is the balanced-trade steady state share of imported goods (i.e., an inverse measure of home bias in consumption preferences), and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. Each bundle is composed of imperfectly substitutable varieties (with elasticity of substitution $\varepsilon > 1$). Optimal allocation of expenditure within each variety of goods yields:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \; ; C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{1}$$

where $C_{H,t} \equiv \int_0^1 [C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ and $C_{F,t} \equiv \int_0^1 [C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign bundles yields:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t ; \; C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2}$$

where $P_t \equiv [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the CPI index.

We assume the existence of complete markets for state-contingent money claims expressed in units of domestic currency.[5] Let $s^t = \{s_0, ...s_t\}$ denote the history of events up to date t, where s_t is the event realization at date t. The date 0 probability of observing history s^t is given by $\rho(s^t)$. The initial state s^0 is given so that $\rho(s^0) = 1$. Henceforth, and for the sake of simplifying the notation, let's define the operator $E_t\{.\} \equiv \sum_{s_{t+1}} \rho(s^{t+1}|s^t)$ as the mathematical expectation over all possible states of nature conditional on history s^t. Agents maximize the following expected discounted sum of utilities over possible paths of consumption and labor:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \tag{3}$$

where N_t denotes labor hours. We assume that period utility is separable in its arguments. At the beginning of time t the households receive a nominal labor income of $W_t N_t$. To insure their

[5] Given that, in our setting, the law of one price holds continually, the unit of denomination of the payoffs of state-contingent assets is not strictly relevant. Alternatively, e.g., in the case in which deviations from the law of one price are due to consumer currency pricing, as in Devereux and Engel (2003), the distinction between nominal and real payoffs would be relevant for the specification of the equilibrium.

consumption pattern against random shocks at time t they decide to spend $\nu_{t+1,t}B_{t+1}$ in nominal state contingent securities where $\nu_{t,t+1} \equiv \nu(s^{t+1}|s^t)$ is the pricing kernel of the state contingent portfolio. Each state contingent asset B_{t+1} pays one unit of domestic currency at time $t+1$ and in state s^{t+1}. Hence the sequence of budget constraints, after considering the optimal expenditure conditions (1) and (2), assumes the following form:

$$P_tC_t + \sum_{s^{t+1}} \nu_{t+1,t}B_{t+1} \leq W_tN_t + \tau_t + B_t + \int_0^1 \Gamma_t(i) \tag{4}$$

where τ_t are government net transfers of domestic currency and $\Gamma_t(i)$ are the profits of monopolistic firm i, whose shares are owned by the domestic residents.[6] Households choose the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and bonds $\{B_{t+1}\}_{t=0}^{\infty}$, taking as given the set of processes $\{P_t,\ W_t,\ \upsilon_{t+1,t}\}_{t=0}^{\infty}$ and the initial wealth B_0 so as to maximize (3) subject to (4).

For any given state of the world, the following set of efficiency conditions must hold:

$$U_{c,t}\frac{W_t}{P_t} = -U_{n,t} \tag{5}$$

$$\beta\frac{P_t}{P_{t+1}}\frac{U_{c,t+1}}{U_{c,t}} = \nu_{t+1,t} \tag{6}$$

$$\lim_{j\to\infty} E_t\left\{\nu_{t+j,t}B_{t+j}\right\} = 0 \tag{7}$$

where $U_{x,t}$ defines the first order derivative of utility with respect to its argument $x = C, N$. Our separability assumption implies $U_{cn,t} = U_{nc,t} = 0$. Equation (5) equates the CPI-based real wage to the marginal rate of substitution between consumption and leisure. Optimality requires that the first order conditions (5), (6) and the no-Ponzi game condition (7) are simultaneously satisfied. The conditional expected return on the state contingent asset is given by $R_{n,t}$, so that $R_{n,t}^{-1} \equiv E_t\left\{\nu_{t+1,t}\right\}$.
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2.1 Law of One Price, the Real Exchange Rate and Risk Sharing

We assume throughout that the *law of one price* holds, implying that $P_F(i) = \mathcal{E}\ P_F^*(i)$ for all $i \in [0,1]$, where $\mathcal{E}$ is the nominal exchange rate, i.e., the price of foreign currency in terms of home currency, and $P_F^*(i)$ is the price of foreign good i denominated in foreign currency. Let's denote by B^F foreign households' holdings of the state contingent bond denominated in domestic currency. The budget constraint of the foreign representative household will read:

$$P_t^*C_t^* + \sum_{s^{t+1}} \nu_{t+1,t}\frac{B_{t+1}^F}{\mathcal{E}_t} \leq W_t^*N_t^* + \tau_t^* + \frac{B_t^F}{\mathcal{E}_t} + \int_0^1 \Gamma_t^*(i) \tag{8}$$

[6]Each domestic household owns an equal share of the domestic monopolistic firms.

The efficiency condition for bonds' holdings is

$$\beta \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \frac{U^*_{c^*,t+1}}{U^*_{c^*,t}} = \nu_{t+1,t} \tag{9}$$

Foreign demand for domestic variety i must satisfy:

$$\begin{aligned} C^*_{H,t}(i) &= \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon} C^*_{H,t} \\ &= \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon} \alpha^* \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} C^*_t \end{aligned} \tag{10}$$

The remaining efficiency conditions characterizing the foreign economy are then exactly symmetric to the ones of the domestic economy described above.

2.2 The Domestic Production Sector

Each monopolistic firm i produces a homogenous good according to:

$$Y_t(i) = A\ N_t(i) \tag{11}$$

The cost minimizing choice of labor input implies:

$$\frac{W_t}{P_{H,t}} = mc_t A_t \tag{12}$$

where mc denotes the real marginal cost.

Changing output prices is subject to some costs. We follow Rotemberg (1982) and model the cost of adjusting prices for each firm i equal to:

$$\psi_t(i) \equiv \frac{\theta}{2} \left(\frac{P_{H,t}(i)}{P_{H,t}} - 1 \right)^2 \tag{13}$$

where the parameter θ measures the degree of price stickiness. The higher the θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$ prices are flexible.

The cost of price adjustment renders the domestic producer's pricing problem dynamic. Each producer chooses the price $P_{H,t}(i)$ of variety i to maximize its total market value:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \lambda_t \ \frac{D_t(i)}{P_{H,t}} \right\} \tag{14}$$

subject to the constraint

$$Y_t(i) \leq \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} (C_{H,t} + C^*_{H,t}) \tag{15}$$

where $\beta^t \lambda_t$ measures the marginal utility value to the representative producer of additional profits expressed in domestic currency, and where

$$\frac{D_t(i)}{P_{H,t}} \equiv \frac{P_{H,t}(i) Y_t(i)}{P_{H,t}} - \frac{W_t}{P_{H,t}} N_t - \frac{\theta}{2}\left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right)^2$$

The first order condition of the above problem reads

$$\begin{aligned} 0 \quad = \quad & \lambda_t \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \frac{C_t^W}{P_{H,t}} \left((1-\varepsilon) + \varepsilon \frac{W_t}{A_t P_{H,t}} \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-1}\right) \\ & -\lambda_t \theta \left(\frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1\right) \frac{1}{P_{H,t-1}(i)} \; + \; \beta \lambda_{t+1} \theta \left(\frac{P_{H,t+1}(i)}{P_{H,t}(i)} - 1\right) \frac{P_{H,t+1}(i)}{P_{H,t}(i)^2} \end{aligned} \tag{16}$$

Let's define $\widetilde{p}_{H,t} \equiv \frac{P_{H,t}(i)}{P_{H,t}}$ as the relative price of domestic variety i and $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ as the gross domestic *producer inflation rate*. It is useful to see that the above condition can be rewritten as

$$\begin{aligned} 0 \quad = \quad & \lambda_t C_t^W \widetilde{p}_{H,t}{}^{-\varepsilon} \left((1-\varepsilon) + \varepsilon \frac{W_t}{A_t P_{H,t}}\right) - \\ & \lambda_t \theta \left(\pi_{Ht,1} \frac{\widetilde{p}_{H,t}}{\widetilde{p}_{H,t-1}} - 1\right) \frac{\pi_H, t}{\widetilde{p}_{H,t-1}} \\ & + \beta \lambda_{t+1} \theta \left(\pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H.t}} - 1\right) \pi_{H,t+1} \frac{\widetilde{p}_{H,t+1}}{\widetilde{p}_{H,t}{}^2} \end{aligned} \tag{17}$$

3 Equilibrium in the Home Economy

We focus our attention on a *symmetric* equilibrium where all domestic producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies that

$$\widetilde{p}_{H,t} = 1, \text{ for all t} \tag{18}$$

$$N_t(i) = N_t, \text{ for all } i, \ t \tag{19}$$

$$\Upsilon_t(i) = \Upsilon_t, \text{ for all } i, \ t \tag{20}$$

In such an equilibrium equation (17) will simplify to

$$\lambda_t \pi_{H,t}(\pi_{H,t}-1) = \beta E_t \left\{ \lambda_{t+1}\pi_{H,t+1}(\pi_{H,t+1}-1) \right\} + \frac{\lambda_t \varepsilon A_t N_t}{\theta}\left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{21}$$

The total net supply of bonds must satisfy

$$B_t + B_t^* = 0 \tag{22}$$

Market clearing for domestic variety i must satisfy:

$$\begin{aligned} Y_t(i) &= C_{H,t}(i) + C^*_{H,t}(i) + \psi_t(i) \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)C_t + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} \alpha^* C_t^* \right] + \psi_t(i) \end{aligned} \tag{23}$$

for all $i \in [0,1]$ and t. Plugging (23) into the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y(i)^{1-\frac{1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}$, and recalling that $P_{H,t} = \mathcal{E}_t P^*_{H,t}$, we can express the resource constraint as

$$A_t N_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)\ C_t + \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-\eta} \alpha^* C_t^* + \psi_t \tag{24}$$

4 Deriving the Relevant Constraints

As mentioned before, the optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. Our next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey (1983). There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman (2003) adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe (2002) to analyze a problem of joint determination of optimal monetary and fiscal policy.

Let's first proceed by defining a series of relationships linking real quantities to the relevant relative prices in our framework. The terms of trade is the relative price of imported goods:

$$\mathcal{T}_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{25}$$

Let's define the CPI to PPI ratio as:

$$\Phi_t \equiv \frac{P_t}{P_{H,t}} = [(1-\alpha) + \alpha \mathcal{T}_t^{1-\eta}]^{\frac{1}{1-\eta}} \tag{26}$$

This relative price, which is monotonically increasing in the terms of trade, is key to our analysis. By representing the wedge between producer and consumer prices it captures a distortion naturally linked to the openness dimension of the economy.

Notice also that the terms of trade and the real exchange rate can be related as follows:

$$\begin{aligned} \mathcal{T}_t &= \frac{P_{F,t}\ \Phi_t}{P_t} \\ &= Q_t\ \Phi_t \frac{P^*_{F,t}}{P^*_t} \\ &= Q_t\ \frac{\Phi_t}{\Phi^*_t} \end{aligned} \tag{27}$$

where $\Phi^*_t \equiv \frac{P^*_t}{P^*_{F,t}}$.

We now wish to rewrite the relative prices Φ_t and Φ^*_t as a function or real allocations only. By combining (26), (27) and (38) one can write

$$\begin{aligned} \Phi_t &\equiv \Phi(C_t, \Phi^*_t) \\ &= \left(\frac{1-\alpha\,(\Phi^*_t)^{\eta-1}\left(\frac{U_{c,t}}{\kappa U^*_{c^*,t}}\right)^{\eta-1}}{(1-\alpha)} \right)^{\frac{1}{\eta-1}} \end{aligned} \tag{28}$$

and similarly:

$$\begin{aligned} \Phi^*_t &\equiv \Phi(C^*_t, \Phi_t) \\ &= \left(\frac{1-\alpha^*\Phi_t^{\eta-1}\left(\frac{\kappa U^*_{c^*,t}}{U_{c,t}}\right)^{\eta-1}}{(1-\alpha^*)} \right)^{\frac{1}{\eta-1}} \end{aligned} \tag{29}$$

The CPI level can be linked to the domestic price level and aggregate consumption as follows

$$P_t = P_{H,t}\ \Phi_t$$

Let's define gross CPI inflation as $\pi_t \equiv \frac{P_t}{P_{t-1}}$. This can be linked to domestic producer inflation and aggregate relative consumption as follows:

$$\pi_t = \pi_{H,t} \frac{\Phi_t}{\Phi_{t-1}} \tag{30}$$

The condition on optimal bond investment can then be rearranged accordingly. By taking conditional expectations of (6) we obtain

$$U_{c,t} = \beta E_t \left\{ R_t U_{c,t+1} \right\} \tag{31}$$

where

$$R_t = E_t \left\{ \frac{R_t^n P_t}{P_{t+1}} \right\} \tag{32}$$

is the CPI-based gross real interest rate.

Let's operate on the budget constraint of the Home consumers. By substituting the government budget constraint, which implies $\tau_t = 0$ all t, into equation (4), we obtain (in terms of units of domestic currency):

$$B_0 + \sum_{t=0}^{\infty} \sum_{s^t} z_{0,t} \left[W_t N_t + \Gamma_t \right] = \sum_{t=0}^{\infty} \sum_{s^t} z_{0,t} P_t C_t \tag{33}$$

where the price system $z_{0,t}$ is obtained after iteration of equation (6) and can be expressed as

$$z_{0,t} = \beta^t \rho_t \frac{U_{c,t}}{P_t} \frac{P_0}{U_{c,0}} \tag{34}$$

Notice, next, that aggregate real profits can be written as:

$$\frac{\Gamma_t}{P_t} = \frac{(1 - mc_t) A_t N_t - \frac{\theta}{2} (\pi_{H,t} - 1)^2}{\Phi_t} \tag{35}$$

By summing over all possible states s_t in equation (34), substituting (35), (5) and (40) into (33) and imposing the transversality condition (7) we obtain the present value budget constraint for domestic households (expressed in real terms):

$$\widetilde{B}_0 + E_0 \sum_{t=0}^{\infty} \beta^t U_{c,t} \left[\frac{A_t N_t - \frac{\theta}{2} (\pi_{H,t} - 1)^2}{\Phi_t} \right] = E_0 \sum_{t=0}^{\infty} \beta^t U_{c,t} C_t \tag{36}$$

where $\widetilde{B}_0 \equiv \frac{B_0}{P_0} U_{c,0}$. This equation states that the sum of initial financial wealth plus expected present discounted net income must match the expected presented discounted value of consumption.

We proceed in a similar fashion for the Foreign household. The price system $z_{0,t}^F$ (expressed in domestic currency) obtained from the forward iteration of (9) can be written:

$$z^F_{0,t} = \left(\beta^t \rho_t \frac{U^*_{c^*,t}}{P^*_t}\frac{P^*_0}{U^*_{c^*,0}}\right)\frac{\mathcal{E}_0}{\mathcal{E}_t} \equiv z^*_{0,t}\frac{\mathcal{E}_0}{\mathcal{E}_t} = z_{0,t} \tag{37}$$

Equating with (34) implies the following condition

$$\kappa \frac{U^*_{c^*,t}}{U_{c,t}} = \frac{\mathcal{E}_t P^*_t}{P_t} \equiv Q_t \tag{38}$$

where Q_t is the real exchange rate and $\kappa \equiv \frac{\mathcal{E}_0 P^*_0 U_{c,0}}{P_0 U^*_{c^*,0}}$ is a condition capturing the initial distribution of wealth.[7] Notice that we are assuming that such initial distribution of wealth is taken as given from the view point of the policy maker. This amounts to implicitly assuming that the initial trade of assets is undertaken before the policy plan is defined at time zero.[8]

By taking conditional expectations of both sides of (37) and proceeding with similar substitutions to the ones operated in the Home case we obtain[9]

$$-\widetilde{B}_0\kappa^{-1} + E_0\sum_{t=0}^{\infty}\beta^t U^*_{c^*,t}\left[\frac{A^*_t N^*_t - \frac{\theta}{2}(\pi^*_{F,t}-1)^2}{\Phi^*_t}\right] = E_0\sum_{t=0}^{\infty}\beta^t U^*_{c^*,t}C^*_t \tag{39}$$

In the following we assume that, prior to policy implementation, the initial wealth is given (i.e., $\widetilde{B}_0$ is given) and that it is allocated symmetrically across countries, which amounts to assuming $\kappa = 1$.

Next we need to rearrange the optimality conditions for the production sector. This requires, at first, to express the real marginal cost and the real wage in terms of aggregate real quantities. Hence by combining (5) and (12) we can write

$$mc_t = -\frac{U_{n,t}}{U_{c,t}A_t}\Phi_t \tag{40}$$

This implies that the aggregate condition for optimal pricing (21) can be rewritten as

$$\begin{aligned} U_{c,t}\pi_{H,t}(\pi_{H,t}-1) &= \beta U_{c,t+1}E_t\left\{\pi_{H,t+1}(\pi_{H,t+1}-1)\right\} + \\ &\quad \frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(-\frac{U_{n,t}}{U_{c,t}A_t}\Phi_t - \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{41}$$

[7] See also Chari, Kehoe and McGrattan (2003).

[8] However, Devereux and Engel (2003) show that in a setup with complete markets and law of one price a constant κ and equal to 1 is an equilibrium also in the case of policy chosen *before* the allocation of wealth. Sutherland (2002) shows instead that the timing of bond trading matters in a setup with imperfectly integrated financial markets.

[9] In particular one should note that

$$\widetilde{B}^*_0 = \frac{B^F_0}{\mathcal{E}_0}\frac{U_{c,0}\ \kappa^{-1}}{P_0}$$

Since equilibrium requires $B^F_0 = -B_0$ we obtain $\widetilde{B}^*_0 = -\widetilde{B}_0\kappa^{-1}$.

An analogous condition will hold in Foreign:

$$\begin{aligned} U^*_{c^*,t}\pi^*_{F,t}(\pi^*_{F,t}-1) &= \beta U^*_{c^*,t}E_t\left\{\pi^*_{F,t+1}(\pi^*_{F,t+1}-1)\right\} + \\ & \frac{U^*_{c^*,t}\varepsilon A^*_t N^*_t}{\theta}\left(-\frac{U^*_{n,t}}{U^*_{c^*,t}A^*_t}\Phi^*_t - \frac{\varepsilon-1}{\varepsilon}\right) \end{aligned} \tag{42}$$

Finally let's turn to the domestic goods market equilibrium condition (24). This can be rewritten as

$$\begin{aligned} A_t N_t &= (1-\alpha)\, C_t \Phi_t^\eta + Q_t^\eta \Phi_t^\eta \alpha^* C_t^* + \psi_t \\ &= (1-\alpha)\, C_t \Phi_t^\eta + \kappa^\eta \left(\frac{U^*_{c^*,t}}{U_{c,t}}\right)^\eta \Phi_t^\eta \alpha^* C_t^* + \frac{\theta}{2}\left(\pi_{H,t}-1\right)^2 \end{aligned} \tag{43}$$

Symmetrically, the resource constraint for Foreign will read:

$$A^*_t N^*_t = (1-\alpha^*)\, C^*_t\left(\Phi^*_t\right)^\eta + \kappa^{-\eta}\left(\frac{U_{c,t}}{U^*_{c^*,t}}\right)^\eta \left(\Phi^*_t\right)^\eta \alpha C_t + \frac{\theta}{2}\left(\pi^*_{F,t}-1\right)^2 \tag{44}$$

It is of independent interest to notice that the present value budget constraint must be part of the policy maximization problem. In fact, and unlike the closed economy case, it is not implicitly satisfied by a combination of the government budget constraint and the resource feasibility constraint. This dimension characterizes specifically the policy maximization problem in an open economy as opposed to the corresponding closed-economy case.

In the following, we formulate a proposition that establishes a mapping between the minimal form summarized by conditions (41), (43), (42), (44), (36), (39) expressed above and the set of allocations describing the (imperfectly) competitive equilibrium in the world economy.

Proposition 1. *For a given initial* $\widetilde{B}_0$, *any equilibrium allocation* $\{C_t,\, Y_t,\, N_t,\, mc_t,\, Q_t,\, \pi_{H,t},\, C_{F,t},\, C_{H,t}\}_{t=0}^{\infty}$ *satisfying equations (2), (5)-(7), (12), (21), (24) in Home, along with a symmetric set of conditions holding for Foreign, also satisfies equations (41), (43) and (36). By reverse, using allocations* $\{C_t,\, N_t,\, \pi_{H,t}\}_{t=0}^{\infty}$ *and* $\{C^*_t,\, N^*_t,\, \pi^*_{F,t}\}_{t=0}^{\infty}$ *that satisfy equations (41), (43), (36 and (42), (44), (39), it is possible to construct all the remaining real allocations, nominal variables and policy instruments for Home and Foreign.*

Proof. See Appendix A.

5 Steady-State Constrained Optimal Policy

When looking at the optimal monetary policy design in the long-run a distinction between the *constrained* and the *unconstrained* optimal inflation rate is warranted. The former is the inflation rate that maximizes households' instantaneous utility under the constraint that the steady state conditions are imposed ex-ante. In analogy with the terminology of the neoclassical growth model, and as in King and Wolman (1997), we can define this as the policy maker's *golden rule*. However it is important to recall that in dynamic economies with discounted utility the golden rule does not in general coincide with the unconstrained optimal long-run rate of inflation, which is the one to which the planner would like the economy to converge to if allowed to undertake its optimization unconditionally. We will return to this distinction below.

5.1 Nash Golden Rule

We start here by characterizing the golden rule steady state associated to Nash competition between the two policy makers. The economy is characterized by three distortions. The first two, *market power* and *price stickiness*, are common to both the closed and the open economy version of our model. The price stickiness distortion, summarized by the quadratic term in inflation in the resource constraint, is obviously minimized at zero net inflation (i.e., $\pi_H = 1$). On the other hand, the market power distortion, stemming from the level of activity being inefficiently low, calls for a higher level of output and consumption and hence for a *positive* rate of inflation. King and Wolman (1997), in the context of a closed economy, show that once the tension between these two distortions is taken into account the welfare maximizing steady state inflation rate must necessarily be positive.[10]

Relative to a closed economy, and as emphasized in Corsetti and Pesenti (2000), what distinguishes the analysis of the steady-state optimal policy in an open economy is the presence of an additional distortion. This stems from the possibility, in the presence of rigid nominal prices, of strategically affecting the equality between the marginal rate of substitution and the marginal rate of transformation via a manipulation of the terms of trade. This is immediately apparent from a visual inspection of the steady-state version of equation (40).

The interesting aspect of such a distortion is that it creates room for policy competition. To understand this argument it is instructive to derive the markup function from the competitive equilibrium of the domestic open economy. By combining the steady-state version of (5) and (12) we can write

$$\frac{-U_n(N)}{U_c(C)} = \frac{1}{\mu}\frac{P_H}{P} = \frac{1}{\mu(\pi_H, N)}\left[\Phi(\mathcal{T})\right]^{-1}$$

where $\mu(\pi_H, N)$ derives from the steady-state version of (21) as

[10] This argument is correct, though, to the extent that a money distortion associated to the presence of transaction frictions, which would drive incentives towards the Friedman rule and hence a negative steady state inflation rate, is ignored.

$$\mu(\pi_H, N) = \frac{\varepsilon N}{\theta \pi_H (\pi_H - 1)(1-\beta) + (\varepsilon - 1) N} \tag{45}$$

Hence efficiency in any given steady state of the economy requires

$$\mu(\pi_H, N)\Phi(\mathcal{T}) = 1 \tag{46}$$

To gain intuition on how the international relative price distortion (summarized by the wedge $\Phi(\mathcal{T})$ which is increasing in $\mathcal{T}$) interacts with the markup distortion from the view point of a given country, let's assume, for the sake of argument, that producer prices are fully flexible, so that the markup is always equal to a constant value of $\frac{\varepsilon}{\varepsilon-1}$. We therefore temporarily abstract from the price-stickiness distortion. The only distortions left interacting are the ones summarized by the markup and by the openness wedge $\Phi(\mathcal{T})$. By making use of (26), we can rewrite a relationship between the desired terms of trade and the markup which reads

$$\mathcal{T} = \left(\frac{\alpha}{\mu^{\eta-1} - (1-\alpha)} \right)^{\frac{1}{\eta-1}} \tag{47}$$

Notice that, independently of the values of η and α, $\mathcal{T}$ is always decreasing in μ. One should view equation (47) as an iso-efficiency condition. Hence, and in order to keep the economy at the maximum welfare in steady-state, a higher markup calls for more *appreciated* terms of trade. The intuition is simple. The presence of imperfectly competitive output markets makes desirable to expand output towards the perfectly competitive efficient level. While in an closed economy with flexible prices this is always welfare improving, in an open economy the same rise in output requires (in equilibrium) also a depreciation of the terms of trade, which hurts the purchasing power of domestic consumers. Equation (47) shows that, at the margin and for *any given level of foreign consumption*, it is optimal to have the terms of trade *depreciate less,* or equivalently let output expand less relatively to the case of an imperfectly competitive closed economy.

However, such a strategic incentive characterizes also the optimal reaction function in Foreign. To formalize the policy game let's define, *for any given* Φ^* *function*, the *golden rule* Nash steady state for Home as the triplet

$$\{\pi_H, C, N\}^{gr} \equiv \arg\max\{U(C, N)\} \tag{48}$$

subject to a (steady state) pricing-implementability condition

$$\pi_H(\pi_H - 1)(1-\beta) \leq \frac{\varepsilon N}{\theta} \left(\frac{-U_n(N)\ \Phi(C, \Phi^*)}{U_c(C)} - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{49}$$

and to a (steady state) feasibility constraint

$$N \leq (1-\alpha)\ C\ \left[\Phi(C,\Phi^*)\right]^{\eta} + \kappa^{\eta}\left(\frac{U^*_{c^*}(C^*)}{U_c(C)}\right)^{\eta}\ \left[\Phi(C,\Phi^*)\right]^{\eta}\alpha^* C^* + \frac{\theta}{2}\left(\pi_H - 1\right)^2 \tag{50}$$

where it is understood that $\Phi^*$is taken as given from the view point of the monetary authority in Home but is instead chosen optimally by the policymaker in Foreign.[11]

First order conditions of this problem (which are reported in *Appendix B*) define Home policymaker's reaction function for any given level of Foreign consumption. An exactly symmetric problem characterizes the reaction function of the policymaker in Foreign. The solution to the joint system of equations pins down the Nash equilibrium.

Figure 1 plots (in *dashed line*) the solution under the Nash game for a selected number of variables as a function of the (inverse) home bias parameter α. This parameter is a natural index of openness in our context. In the simulations, we set the vector $[\sigma,\ \gamma,\ \eta] = [1\ ,\ 3,\ 2]$, while we maintain that $\alpha = \alpha^*$. This is our benchmark parameterization. Hence we see that for $\alpha \to 0$ the rate of (producer) inflation that maximizes steady-state welfare is positive and coincides exactly with the one of the corresponding closed economy with sticky prices and monopolistic competition. As α turns positive, i.e., both economies become open to trade, the desired steady-state inflation rate decreases below the one of the corresponding closed economy. The intuition is simple. As explained above, policy competition calls for a strategic reduction in consumption (relative to the closed economy case) in order to obtain an appreciation of the terms of trade and a reduction in work effort. For any given level of foreign consumption, this is welfare improving. However, in a Nash-game where policy objectives collide, terms of trade and work effort remain constant, while consumption ends up being slightly lower. This entails, for any level of openness (as measured by α), a lower steady-state rate of inflation.

5.2 Golden Rule under Cooperation

It is apparent from the analysis so far that the two policymakers may coordinate their actions in order to avoid a strategic manipulation of the terms of trade and therefore enjoy higher consumption for any given level of work effort. Under Cooperation, we assume that the planner chooses simultaneously the two triplets $\{\pi_H, C, N\}$ and $\{\pi^*_F, C^*, N^*\}$ in order to maximize average utility $\frac{U(C,N)+U(C^*,N^*)}{2}$, subject to the constraints (49), (50) and to the corresponding equations for Foreign. *Appendix B* shows first order conditions for this problem.

[11] Notice that one needs not include the budget constraint in the steady state optimal policy problem for this is implied by the combination of the government and resource constraint. This can be seen by noticing that in the non-stochastic steady state $\Phi = \Phi^* = 1$. By substituting into (50) one obtains

$$N - \frac{\theta}{2}(\pi_H - 1)^2 - C + N^* - \frac{\theta}{2}(\pi^*_F - 1)^2 - C^* = 0$$

This expression states that excess real income (over consumption) in Home must match negative excess real income in Foreign.

Figure 2 (in *solid line*) shows the outcome of the optimal cooperative policy game. Notice that once again the optimal inflation rate lies below the one of the corresponding closed economy. Yet the inflation rate under policy cooperation lies monotonically above the one prevailing under Nash competition. Intuitively, under cooperation, the planner induces both policymakers to avoid any strategic reduction in consumption aimed at appreciating the terms of trade. In equilibrium, the terms of trade and work effort are unchanged, and consumption is higher relative to the Nash outcome. This, ceteris paribus, entails also a relatively higher inflation rate.

Is the result of a Nash deflationary bias robust to parameter values? We show here that it is not. Intuitively, and as emphasized, although in a different framework, also by Sutherland (2002) and Benigno and Benigno (2001), the gains from strategically manipulating the terms of trade should be decreasing in the elasticity of substitution between domestic and foreign goods, for this is a measure of the strength of the expenditure switching effect. We therefore employ an alternative parameterization, with high labor supply elasticity and low international elasticity of substitution. This implies setting $[\sigma, \gamma, \eta] = [1, 1, 0.7]$. We see that in this case there exist values for the degree of openness such that the inflation rate under Nash lies *above* the one under policy cooperation. It is only when the economy becomes extremely open (i.e., the degree of home bias is extremely small) that the Nash deflationary bias result re-emerges.

The role played by openness and the elasticity of substitution is critical also for evaluating associated welfare gains from policy cooperation. *Figure 2* computes percentage utility gains under policy cooperation relative to the implied utility level under a Nash equilibrium. Hence we see that cooperation does not necessarily lead to higher welfare. When both α, the economy's openness to trade, and η, the elasticity of substitution between Home and Foreign goods, are small, then policy cooperation may even lead to a reduction in welfare.

The above discussions suggests stating the following results:

Result 1 *(Open economy deflationary bias)***.** *In an open economy with price adjustment costs and monopolistic competition, the (producer) inflation rate that maximizes steady-state utility lies monotonically below the one of the corresponding closed economy. This holds under both policy competition and cooperation.*

The interesting aspect of Result 1 is that, while in a closed economy with sticky prices and monopolistic competition price stability cannot implement the steady-state maximization of welfare, it can indeed do so when the economy is open, due to the additional effect of the international relative price distortion that pushes the efficient inflation rate downwards.[12]

[12]Notice that this result differs from the one of Nash *inflationary* bias obtained in Cooley and Quadrini (2003). The reason lies in the structure of their model, which features flexible prices and an unambiguos *positive* output effect of real appreciations. In our context, and crucially, prices are sticky (so that monetary authorities can exert a direct effect on the terms of trade) and the effect of international relative price movements on output strictly depends on the value of the elasticity of substitution between domestic and foreign goods. In particular, under our preferred

Result 2 (*Nash deflationary bias*). *In the golden rule steady state, the rate of inflation associated to a Nash equilibrium lies below the one under Cooperation. However, this happens only for sufficiently high values of the elasticity of substitution between domestic and foreign goods.*

Result 3 (*Welfare gains from cooperation*). *In the golden rule steady state, welfare gains from cooperation arise only for sufficiently low values of home bias and sufficiently high values of the elasticity of substitution between Home and Foreign goods.*

As a final remark on the steady state analysis, it is worth recalling that, while our discussion of the deflationary bias is reminiscent of the results in Corsetti and Pesenti (2001), it also differs in two respects. First, it concerns the optimal policy in a given steady state as opposed to the optimal response to shocks in an essentially static model with one period predetermined prices.[13] Second, it concerns the steady state of a dynamic economy with discounting. This necessarily renders the solution to the policy games analyzed above only constrained optimal. Yet before turning to this issue we need to characterize the optimal policy design in a fully dynamic context.

6 Optimal Monetary Policy under Commitment

We now turn from a steady-state perspective to the specification of a general set-up for the optimal policy conduct in a dynamic context. We begin our analysis by assuming that *ex-ante commitment* is feasible. In this section we take full advantage of our characterization of the equilibrium conditions (in each country) in terms of a minimal set of relations involving only the choice of allocations for consumption and labor input along with the inflation instrument.

6.1 Nash-Commitment

Definition 1. *Let's define* $\mathcal{U}(C_t,\ N_t,\ \pi_{H,t},\ \Omega) \equiv U(C_t, N_t) + \Omega \left[U_{c,t} \left(C_t - \frac{A_t N_t - \frac{\theta}{2}(\pi_{H,t}-1)^2}{\Phi_t} \right) \right]$ *where* Ω *is a (constant) multiplier on constraint (36). Let* $\{\lambda_{p,t},\ \lambda_{f,t}\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (41) and (43) respectively. Let* $\widetilde{B}_0$ *be given. Then for given allocations* $\{C_t^*\}_{t=0}^{\infty}$ *and stochastic processes* $\{A_t,\ A_t^*\}_{t=0}^{\infty}$*, plans for the control variables* $\{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$ *and for the costate variables* $\{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\Lambda_t^n \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ and $\Xi_t^n \equiv \{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}}\ Max_{\{\Xi_t^n\}_{t=0}^{\infty}}\ E_0\{\sum_{t=0}^{\infty} \beta^t E_t\{\mathcal{U}(C_t, N_t, \pi_{H,t}, \Omega) \tag{51}$$

parameterization of $\eta > 1$ with log utility, real appreciations exert a *negative* effect on output.

[13] In particular, the result in Corsetti and Pesenti (2001) can be qualified as a systematic deflationary bias when policy is conducted under discretion in a one-period model (with pre-determined prices).

$$+\lambda_{p,t}\left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1)-\beta U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1}-1)+\frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(\frac{U_{n,t}\ \Phi_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right]$$
$$+\lambda_{f,t}\left[A_t N_t-(1-\alpha)\ C_t\Phi_t^{\eta}-\kappa^{\eta}\left(\frac{U^*_{c^*,t}}{U_{c,t}}\right)^{\eta}\Phi_t^{\eta}\alpha^* C_t^*-\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\}\}\ -\ \Omega\widetilde{B}_0$$

A series of observations on the nature of this policy problem are in order. Notice, first, that the distinctive feature of the commitment problem under Nash competition is that the Home policymaker *does not internalize* that the relative price Φ_t depends also on the level of consumption in Foreign (see equation (28) above), and that, symmetrically, the relative price Φ_t^* is affected by its own consumption choice.

In addition, it is important to notice that, as a consequence of the initial stock of wealth $\widetilde{B}_0$ being exogenously supplied, the multiplier Ω is taken as given in each policymaker's maximization problem. In other words, the initial stock of wealth does not depend on the anticipation about the future implementation of policy.[14] This does not mean, however, that policymakers do not compete on the expected future allocation of wealth. As it is clear from equations (36) and (39), the expected future stream of wealth in each country depends on the evolution of Φ_t (and Φ_t^*). Hence each policymaker would like to manipulate Φ_t in order to increase the expected future real income of its own households.

More generally, it follows that we can identify two dimensions in which the strategic manipulation of international relative prices matters for international policy competition. First, policy makers compete in eliminating wedges that affect the equality between the marginal rate of substitution and the marginal rate of transformation. This aspect is clearly captured by equation (40) (and the corresponding equation for Foreign). To see this one should notice that the relative price Φ_t would be identically equal to 1 in the closed economy version of (40). Second, policymakers compete over the relative allocation of wealth. As suggested above, in fact, Φ_t affects the present value budget constraints (36) and (39), and therefore the future determination of wealth for any given initial (relative) wealth $\widetilde{B}_0$ (and therefore for a given shadow price Ω attached to the private present value budget constraint). Both these two motives are potential sources of deviations from a price stability policy when policymakers compete in an international setting.

[14] See Schmitt-Grohe and Uribe (2003) for a small open economy model in which the probability of future "policy reform" is not negligible and therefore the determination of $\widetilde{B}_0$ (and Ω) is endogenous. Another way of interpreting this kind of approach is that it tends to shift the attention from the strategic interaction of governments (policymakers) to the one of private agents.

6.1.1 Non-recursivity and Initial Conditions

As a result of the constraint (41) exhibiting future expectations of control variables, the maximization problem as spelled out in (51) is intrinsically non-recursive.[15] As first emphasized in Kydland and Prescott (1980), and then developed by Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) costate variables. Such variables, that we denote χ_t and χ_t^* for Home and Foreign respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in our case the forward-looking Phillips curve constraint features a simple one period expectation, the same costate variables have to obey the laws of motion:[16]

$$\chi_{t+1} = \lambda_{p,t} \tag{52}$$

$$\chi_{t+1}^* = \lambda_{p,t}^* \tag{53}$$

A particularly important point concerns the definition of the appropriate initial conditions for χ_t and χ_t^*. Marcet and Marimon (1999) show that for the modified (recursive) Lagrangian in (51) to generate a global optimum under time zero commitment it must hold:

$$\chi_0 = 0 = \chi_0^* \tag{54}$$

The above condition states that there is no value to the policy planner, in either country and as of time zero, attached to prior commitments. Commitment, in this context, bears exactly the meaning that while it would be technically feasible for the planner (in each country) to satisfy (54) for all $t > 0$, it would also be suboptimal to do so.

In *Appendix C* we show how to reformulate the optimal plan in an equivalent recursive stationary form. First order conditions for time $t \geq 1$ for the choice of C_t, N_t and $\pi_{H,t}$ imply respectively:

$$\begin{aligned} 0 \;=\; & U_{c,t} + U_{cc,t}\ \pi_{H,t}(\pi_{H,t}-1)\ (\lambda_{p,t}-\chi_t) + \frac{\lambda_{p,t}N_t}{\theta}\left(\epsilon\ U_{n,t}\Phi_{c,t} + (\varepsilon-1)A_tU_{cc,t}\right) \\ & -\lambda_{f,t}\ (1-\alpha)\left(\Phi_t^{\eta} + \eta C_t\Phi_t^{\eta-1}\Phi_{c,t}\right) - \lambda_{f,t}\alpha^* C_t^*\kappa^{\eta}U_{c,t}^{*\eta}\left(\eta\Phi_t^{\eta-1}\Phi_{c,t}U_{c,t}^{-\eta} - \eta U_{c,t}^{-\eta-1}U_{cc,t}\Phi_t^{\eta}\right) \\ & -\Omega(A_tN_t - \frac{\theta}{2}(\pi_{H,t}-1)^2)\left(U_{cc,t}\Phi_t^{-1} - \Phi_{c,t}\Phi_t^{-2}U_{c,t}\right) + \Omega\left(U_{cc,t}C_t + U_{c,t}\right) \end{aligned} \tag{55}$$

[15] See Kydland and Prescott (1977), Calvo (1978). As such the system does not satisfy per se the principle of optimality, according to which the optimal decision at time t is a time invariant function only of a small set of state variables.

[16] The laws of motion of the additional costate variables would take a more general form if the expectations horizon in the forward looking constraint(s) featured a more complicated structure, as, for instance, in the case of constraints in present value form. See Marcet and Marimon (1999).

$$0 = U_{n,t} + \frac{\lambda_{p,t}\varepsilon\Phi_t}{\theta}\left(U_{n,t} + N_t U_{nn,t}\right) + \lambda_{p,t}\frac{\varepsilon - 1}{\theta}U_{c,t}A_t + \lambda_{f,t}A_t - \Omega\frac{U_{c,t}A_t}{\Phi_t} \tag{56}$$

$$U_{c,t}(2\pi_{H,t} - 1)\left(\lambda_{p,t} - \chi_t\right) - \theta(\pi_{H,t} - 1)\left(\lambda_{f,t} - \Omega\frac{U_{c,t}A_t}{\Phi_t}\right) = 0 \tag{57}$$

The system of efficiency conditions in Home is completed by the law of motion (52), the initial condition (54) and by the constraints (41) and (43) holding with equality. Notice also that first order conditions evaluated at time $t = 0$ differ for what concerns equation (55), which must feature the additional term $-\Omega\frac{B_0}{P_0}U_{cc,0}$.

Once defined a completely symmetric problem for the policy maker in Foreign, we can state the following definition of a Nash equilibrium:

Definition 2 *(Nash equilibrium under commitment) The set of processes* $\Lambda_t^n \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$, $\Lambda_t^{n*} \equiv \{\lambda_{p,t}^*, \lambda_{f,t}^*\}_{t=0}^{\infty}$, $\Xi_t^n \equiv \{C_t,\ \pi_{H,t},\ N_t\}_{t=0}^{\infty}$, $\Xi_t^{n*} \equiv \{C_t^*,\ \pi_{F,t}^*,\ N_t^*\}_{t=0}^{\infty}$ *fully describe a Nash equilibrium under commitment if they solve the system of equations (55) - (57), equations (41), (43), (36) holding with equality, along with a similar set of conditions jointly holding for Foreign.*

6.1.2 Optimal Response to Shocks around the Long-Run Steady-State

We are now in the position to analyze the dynamic features of the optimal commitment policy under Nash competition. In this section we interpret optimal policy in the sense of optimal stabilization in response to shocks. To this end, we proceed in the following way. First, we compute (for both countries) the stationary allocations that characterize the deterministic steady state of the first order conditions (55)-(57) (and symmetric ones for Foreign). We then compute a log-linear approximation of the respective policy functions in the neighborhood of the same steady state.

The spirit of this exercise deserves some further comments. In concentrating on (log-linear) dynamics in the neighborhood of the steady state associated to the efficiency conditions, we deviate from the initial condition (54) in the fact that we set the initial value of the lagged lagrange multipliers equal to their deterministic steady state values, i.e.:

$$\chi_0 = \overline{\chi}_0\ ;\ \ \chi_0^* = \overline{\chi}_0^* \tag{58}$$

It is important to understand that this strategy, as in Khan, King and Wolman (2003), corresponds to focussing on a particular dimension. Namely, optimal stabilization policy in response to bounded shocks that hit in the neighborhood of the *long-run* steady state. This amounts to implicitly assuming that such a steady state has been already reached after the implementation of the optimal policy plan as of time zero. While ignoring such transitional issues may be harmless for the sake of a local dynamic analysis, it may (eventually) critically impinge any relative welfare consideration of the policy regimes under scrutiny. We will return to the welfare issues below.

In conducting our simulations we employ the following form of the period utility: $U(C_t, N_t) = \frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\gamma}N_t^{1+\gamma}$. The time unit is meant to be quarters. The discount factor β is equal to 0.99. The degree of risk aversion σ is 1, the inverse elasticity of labor supply γ is equal to 3. As a benchmark value (see below for a discussion) we set $\eta = 2$. As in Bergin and Tchakarov (2003), and consistent with estimates by Ireland (2001), we set the price stickiness parameter θ equal to 50. The elasticity ε between varieties produced by the monopolistic sector is 6. The (inverse) degree of home bias α, identified by the share of foreign imported goods in the domestic consumption basket, is set to a default value of 0.4. This implies the existence of a mild home bias, which is assumed to be symmetric across countries ($\alpha = \alpha^*$).

Figure 3 compares impulse responses of selected variables to a one percent rise in Home productivity in the case of *Nash-commitment* with the same responses under (domestic) *inflation targeting*. The figure is illustrative of the inefficiency associated to policy competition in our context. Since productivity is higher in Home, the adjustment to the equilibrium requires an increase in the demand of domestic relative to foreign goods. This is achieved by means of a terms of trade depreciation, captured by a rise in the CPI to PPI ratio Φ (recall that $\Phi_t = \Phi(\mathcal{T}_t)$). The only equilibrium is the one in which the same terms of trade depreciation is achieved via an increase in prices in both countries, Home and Foreign. In fact, and due to risk sharing, both countries face the incentive to increase prices to tilt the terms of trade in their own favor (and hence achieve a relatively higher real income). However, since Home is the country in which productivity is relatively higher, the increase in domestic (producer) prices falls short the increase in foreign (producer) prices. This explains why, for a given nominal adjustment in the exchange rate, *the terms of trade depreciate more in a Nash equilibrium* relatively to the inflation targeting case. In the resulting dynamics, since aggregate consumption must rise equally in both countries due to risk sharing, the rise in employment exceeds the one that obtains under inflation targeting.

It is also interesting to notice that a Nash equilibrium generates a response of the price level that resembles the one in response to a cost-push shock. The novel aspect of our results is that the same dynamics are obtained in response to a productivity shock, which is not aimed per se (like in many recent New Keynesian studies of optimal monetary policy) to induce the artificial effect of exogenously drifting the economy away from the efficient allocation. The fact that productivity shocks are a source of price variability under the optimal policy is here an endogenous outcome of the competition on international relative prices.[17]

Figure 4 investigates whether policy competition on the allocation of wealth constitutes per se a motive for deviation from price stability. In the figure we plot impulse responses of the domestic price level to a Home positive productivity shock for alternative values of the Lagrange multiplier

[17] For an analysis of the optimal policy setting in response to this type of shocks see Woodford (2003) and Clarida et al. (1999). For open economy models with one period predetermined prices see Sutherland (2001).

Ω. The case $\Omega = 0$ corresponds to a hypothetical situation in which the household's present value budget does not exert any additional constraining effect for welfare maximization than the one already implied by the feasibility constraint and the Phillips curve constraint. With $\Omega = 0$ the policymaker in either country would not face any incentive to stimulate movements in the price level in order to affect Φ and in turn the real income of households. The figure clearly illustrates that the holding of the present value budget constraint, which is the case captured by values of $\Omega > 0$, constitutes per se a further motive of deviation from price stability. Policymakers, in fact, not only compete on affecting the wedge between the marginal rate of substitution and transformation, but also on trying to increase their own residents' wealth by relatively appreciating the terms of trade. For any given level of consumption and employment, in fact, a fall in Φ_t (for Home, a rise in $P_{H,t}$ for any given $P_{F,t}$) rises real income, as can be directly seen from equation (36).

Figure 5 illustrates how the incentive to generate price movements vary with a critical parameter, namely the elasticity of substitution between domestic and foreign goods. The figure displays impulse responses (under Nash-commitment) of the same selected variables to a productivity shock for alternative values of $\eta = [1, 2, 3]$. The first case corresponds to the benchmark case of Cobb-Douglas preferences typically employed in the linear-quadratic approach to the study of optimal policy for open economies. The literature lacks a consensus on the value of this parameter. Harrigan (1993) and Trefler and Lai (1999) suggest an empirical value as high as 5. Collard and Dellas (2002) derive an estimated value of 2.5. In their quantitative (theoretical) studies, Backus, Kehoe and Kydland (1992) explore a range of η between 0 and 5. Chari et al (2002) set $\eta = 1.5$, while Bergin and Tchakarov (2003) set $\eta = 5$. Overall, there seems to exist both empirical and theoretical support for the hypothesis that the value of η lies above unity. The figure highlights the coincidence of the Nash-optimal response with a price stability strategy only in the particular case of $\eta = 1$. In this knife-edge case, and as pointed out in Corsetti and Pesenti (2001), the income effect of the required terms of trade depreciation (given the relatively higher productivity in Home) exactly balances the incentive to switch expenditure towards Home goods.[18] In general, the higher the elasticity of substitution, the larger (at the margin) the incentive for the policymaker to induce a strategic rise in the (producer) price level to try to generate a relative appreciation of its own residents' real income and purchasing power.

6.2 Cooperation-Commitment

Under cooperation, a social planner explicitly recognizes the channel of interdependence that works through the relative prices $\Phi_t(C_t, C_t^*)$ and $\Phi_t(C_t^*, C_t)$. Below we define the world Ramsey planner problem, under the assumption that the same planner aims at maximizing the average level of utility of the two countries. We also assume that both countries receive equal weight in the planner's

[18] See also Benigno and Benigno (2003).

objective function.

Let's define the *world Ramsey period utility objective* as:

$$\mathcal{U}_t^w(C_t, C_t^*, N_t, N_t^*, \pi_{H,t}, \pi_{F,t}^*, \Omega^w) \equiv \left\{ \frac{U(C_t, N_t) + U_t^*(C_t^*, N_t^*)}{2} \right\} +$$

$$\Omega^w \left[U_{c,t} \left(C_t - \frac{A_t N_t - \frac{\theta}{2}(\pi_{H,t} - 1)^2}{\Phi_t} \right) + U_{c^*,t}^* \left(C_t^* - \frac{A_t^* N_t^* - \frac{\theta}{2}(\pi_{F,t}^* - 1)^2}{\Phi_t^*} \right) \right]$$

where Ω^w is a constant multiplier on the sum of the constraints (36) and (39). Then the Ramsey maximization problem can be defined as follows.

Definition 3. *Let* $\left\{ \lambda_{p,t}, \lambda_{f,t}, \lambda_{p,t}^*, \lambda_{f,t}^* \right\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (41), (42), (43) and (44) respectively. Let* $\widetilde{B}_0$ *be given and* $\kappa = 1$. *For any given stochastic processes* $\{A_t, A_t^*\}_{t=0}^{\infty}$, *the set of allocations* $\{C_t, \pi_{H,t}, N_t, C_t^*, \pi_{F,t}^*, N_t^*,\}$ *and of costate variables* $\{\lambda_{p,t}, \lambda_{f,t}, \lambda_{p,t}^*, \lambda_{f,t}^*\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

Choose $\Lambda_t^c \equiv \left\{ \lambda_{p,t}, \lambda_{f,t}, \lambda_{p,t}^*, \lambda_{f,t}^* \right\}_{t=0}^{\infty}$ and $\Xi_t^c \equiv \{C_t, \pi_{H,t}, N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t^c\}_{t=0}^{\infty}} \; Max_{\{\Xi_t^c\}_{t=0}^{\infty}} \; E_0\{\sum_{t=0}^{\infty} \beta^t E_t\{\mathcal{U}_t^w(C_t, C_t^*, N_t, N_t^*, \pi_{H,t}, \pi_{F,t}^*, \Omega^w) \tag{59}$$

$$+\lambda_{p,t} \left[U_{c,t}\pi_{H,t}(\pi_{H,t} - 1) - \beta U_{c,t+1}\pi_{H,t+1}(\pi_{H,t+1} - 1) + \frac{U_{c,t}\varepsilon A_t N_t}{\theta} \left(\frac{U_{n,t} \; \Phi_t}{U_{c,t} A_t} + \frac{\varepsilon - 1}{\varepsilon} \right) \right]$$
$$+\lambda_{f,t} \left[A_t N_t - (1 - \alpha) \; C_t \Phi_t^{\eta} - \kappa^{\eta} \left(\frac{U_{c^*,t}^*}{U_{c,t}} \right)^{\eta} \Phi_t^{\eta} \alpha^* C_t^* - \frac{\theta}{2} \left(\pi_{H,t} - 1 \right)^2 \right]$$
$$+\lambda_{p,t}^* \left[U_{c^*,t}^* \; \pi_{F,t}^*(\pi_{F,t}^* - 1) - \beta U_{c^*,t+1}^* \pi_{F,t+1}^*(\pi_{F,t+1}^* - 1) + \frac{U_{c^*,t}^* \; \varepsilon A_t^* N_t^*}{\theta} \left(\frac{U_{n,t}^* \; \Phi_t^*}{U_{c^*,t}^* A_t^*} + \frac{\varepsilon - 1}{\varepsilon} \right) \right]$$
$$+\lambda_{f,t}^* \left[A_t^* N_t^* - (1 - \alpha^*) \; C_t^* \left(\Phi_t^* \right)^{\eta} - \kappa^{-\eta} \left(\frac{U_{c,t}}{U_{c^*,t}^*} \right)^{\eta} \left(\Phi_t^* \right)^{\eta} \alpha C_t - \frac{\theta}{2} \left(\pi_{F,t}^* - 1 \right)^2 \right] \}\} \; - \; 2\Omega^w \widetilde{B}_0$$

We defer to *Appendix D* the description of the first order conditions corresponding to this plan. The discussion on the non-recursivity structure of the problem follows exactly the logic applied above to the re-definition of the Nash-commitment policy setup. In practice, this will entail specifying an equivalent recursive stationary program in the new world planner's state space defined by $\{A_t, A_t^*, \; \chi_t, \; \chi_t^*\}$.

6.2.1 Optimal Response to Shocks around the Ramsey Steady-State: Nash vs. Cooperation

In *Figure 6* we show impulse responses to a normalized one percent increase in home productivity and compare selected variables under *Nash-commitment* versus *Cooperation-commitment*. Under policy cooperation, the planner coordinate the responses of both policy makers to achieve the required terms of trade depreciation only by means of a nominal exchange rate depreciation. In other words, it is optimal for the Ramsey planner to have both countries targeting the flexible price allocation. This results in a dampened dynamic of the CPI-PPI ratio Φ under cooperation. The crucial aspect is that this is now compatible with a smooth path of the price level (the response of the price level, measured in percent deviation from steady state, barely deviates from zero) and with a smoother response of employment, for any given variation in consumption (in turn equalized across countries).

It is also interesting to notice, under the Ramsey cooperative regime, that while the response of the price level resembles the one that would obtain in a closed economy under the optimal policy (see for instance King and Wolman, 1997), so does not the response of employment. For an intuition, consider the (equilibrium) real marginal cost equation (40). As already emphasized, the closed economy version of that equation obtains in the case $\Phi_t = 1$. Hence, in a closed economy, it is optimal, in response to a rise in productivity, to fully absorb the rise in productivity by means of an equal increase in consumption (and output), while keeping employment constant. In an open economy, the equilibrium requires a rise in Φ_t (i.e., a real depreciation). Hence, coordinating on stable prices (i.e., constant real marginal costs), requires a rise in consumption which is smaller than the one in productivity and, under the benchmark parameterization, also a rise in employment.[19]

7 Welfare Analysis and Dynamic Features of the Ramsey Policy

We now turn to a characterization of the dynamic properties of the alternative policy regimes analyzed so far. We illustrate our numerical analysis in terms of cyclical properties of selected variables and welfare levels associated to each policy arrangements. We report results under three alternative parameter scenarios: 1) *High home bias,* in which the value of $\alpha = \alpha^*$ is set to 0.1; 2) *Zero home bias* in which $\alpha = \alpha^* = 0.5$; 3) Low elasticity of substitution, in which $\eta = 0.1$.

Some observations on the computation of welfare are in order. For these are dynamic economies, the setting (under commitment) of the initial conditions for the co-state variables χ_t and χ_t^* becomes a relevant issue. In fact, it may be important to capture possible welfare effects that may characterize the transition from the time 0 implementation of the policy announcement to the long-run steady state of the (Ramsey) policy. Consistent with this view, it is important to focus on the *conditional* expected discounted utility of the representative agent.

The same line of reasoning implies that our solution methodology needs to be amended. So far, we have considered only a log-linear approximation method to the policy functions implied

[19]The size of the rise in employment will be, in turn, a function of the elasticity η (with a smaller η implying a smaller rise) and of the labor supply elasiticity $\frac{1}{\gamma}$ (with a higher elasticity requirying a smaller rise in employment).

under each monetary regime.[20] This was consistent with conducting our impulse response analysis in the neighborhood of the (long-run) deterministic Ramsey steady state. However, when the time zero implementation of the optimal commitment policy is considered, one needs to account for the possibility that the economy takes time to adjust to the new steady state. If the final steady state is sufficiently far away from the time of the initial implementation of the policy, first order approximation methods may deliver inaccurate results.

This argument notwithstanding, there is a second critical reason that dictates the use of higher order solution methods for the evaluation of welfare. Namely, that in an economy like ours, in which the existing distortions are not neutralized by special assumptions on the behavior of another (complementary) policy instrument (e.g., fiscal subsidies), stochastic volatility may well affect the first moments of variables that are critical for household's welfare. More simply put, in a first order approximation of the model's solution, the expected value of a variable is always equal to its non-stochastic steady state. As a result, the effects of volatility on mean values of variables is by construction neglected. See Kim and Kim (2002) for a discussion on the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.

We proceed in the following steps. First, we compute a second order approximation of the policy function(s) around the long-run deterministic steady-state implied by each policy regime under scrutiny.[21] Then we assume that both economies are subject to a stationary distribution

[20]The reason we proceeded this way was simply expository. We wanted to maintain that a characterization of the equilibrium allocations under the optimal policy is feasible without resorting to a second order approximation of the model. In fact the (log-linearized) system of efficiency conditions of either the Nash policy game or of the Ramsey cooperative equilibrium characterize the optimal allocation in an already accurate way. Hence they can be solved using standard algorithms usually applied to linear systems of difference equations.

[21]The set of optimality conditions of the Ramsey plan can be described as follows:

$$E_t \ f(Y_{t+1}, Y_t, X_{t+1}, X_t) = 0 \tag{60}$$

where E_t denotes the mathematical expectations operator, conditional on information available at time t, Y_t is the vector of endogenous non-predetermined variables and $X_t \equiv [x_{1,t}, x_{2,t}]$ is the state vector. Here $x_{1,t}$ denotes the vector of (pseudo) co-state variables $[\chi_t, \ \chi_t^*]$, while $x_{2,t}$ is the vector of exogenous variables $[A_t, A_t^*]$ which follows a stochastic process.

$$x_{2,t+1} = F x_{2,t} + \overline{\eta}\sigma\varepsilon_{t+1}; \quad \varepsilon_t \sim i.i.d.N(0, \Sigma) \tag{61}$$

The scalar σ and $\overline{\eta}$ are known parameters. The solution to the model is of the form:

$$Y_t = g(X_t, \sigma) \tag{62}$$

$$X_{t+1} = h(X_t, \sigma) + \bar{\eta}\sigma\varepsilon_{t+1} \tag{63}$$

where Y_t is the vector of control variables, equation (62) is the policy function and equation (63) is the transition function. We compute a second order expansion of the functions $g(x_t, \sigma)$ and $h(x_t, \sigma)$. Schmitt-Grohe and Uribe (2002) show, crucially, that, up to a second order, the coefficients of the policy functions attached to terms that are linear in the state vector x_t are independent of the size of the volatility of the shock(s). To evaluate numerically the first and second order derivatives of the policy functions we employ the Matlab codes compiled by Schmitt-Grohe and Uribe, available at the website http://www.econ.duke.edu/˜grohe/.

of (productivity shocks) and generate, for a given initial condition $\chi_t = 0 = \chi_t^*$ as of time zero, artificial time series of length $T_p = 500$ periods. We compute mean, standard deviation and implied presented discounted utility for any given random draw. We then iterate the computation $T_n = 1000$ times and average across experiments.[22]

Along the lines of Lucas (1987), the measure of welfare cost (of business cycles) that we associate to each policy is the proportional upward shift in the consumption process that would be required to make the representative household indifferent between its random consumption allocation and a nonrandom consumption allocation with the same mean. Hence such measure is defined as the fraction Δ that satisfies:

$$E_0 \sum_{t=0}^{\infty} \beta^t U((1+\Delta)C_t, N_t) = \sum_{t=0}^{\infty} \beta^t U(E_0\,(C_t, N_t)) \tag{64}$$

In *Table 1* we report second moments of selected variables under Nash competition and Cooperation. In particular, we report the cyclical properties of the CPI to PPI ratio (our key relative price) meant as a proxy of the terms trade. Hence we see that, across parameter scenarios, and relative to cooperation, Nash competition is invariably a source of enhancement of inflation volatility. In particular this happens in a low home bias scenario. Intuitively, under this scenario, economies are more open to trade, and therefore, at the margin, more open to a policy competition motive over their terms of trade.

In the same Table we also report the measure Δ of the welfare costs associated to the cyclical properties of under alternative policy scenarios. Hence it is clear that cooperation delivers welfare gains relative to a Nash competition arrangement. For cyclical properties of the CPI to PPI ratio that remain similar across policy scenarios, we see that under Cooperation both consumption and employment are less volatile than under Nash. However, such welfare numbers remain quite small. In absolute terms, and across all policy scenarios, the upward shift in consumption needed to make the household indifferent between a random and a nonrandom allocation range between a minimum of 0.0113 percent (achieved under Cooperation with high home bias) to a maximum of 0.0157 percent (achieved under Nash competition with low home bias). In relative terms, welfare gains from Cooperation are also rather small.

It is of some interest, however, to see that, in our exercise, such gains depend on the comparative statics on two critical parameters that identify "openness" and that affect the relationship between the terms of trade and the CPI to PPI ratio, namely α and η . Hence our exercise indicate that welfare gains from cooperation are minimized when the elasticity of substitution between domestic and foreign goods η is small, and maximized when the home bias is low (i.e., α is close

[22] We employ a standard parameterization for the innovations to the productivity processes, and assume $Var(\varepsilon_t^a) = Var(\varepsilon_t^{a^*}) = (0.01)^2$, with persistence $\rho^a = \rho^{a^*} = 0.9$.

to 0.5). Intuitively, these two scenarios correspond, respectively, to a case in which the policy competition motive is either reduced in scope or, alternatively, magnified.

8 Conclusions

We have laid out a typical public finance framework for the analysis of welfare maximizing monetary policy within an economy characterized by three distortions: market power, rigidity in the adjustment of producer prices and international terms of trade externality. The main advantage of our approach, relative to the existing literature, is that it allows to characterize optimal policy in an open economy while still maintaining all the relevant distortions completely spelled out. Hence the presence of forward looking price setting as well as of a general form of household's preferences over the consumption of domestic and foreign goods does not represent a constraint for the characterization of the mechanics of the optimal commitment policy.

Despite the generality of the approach, our modelling framework remains restrictive in three main dimensions. First, in assuming that the law of one price for traded goods holds continually. Second, in allowing households to obtain full risk sharing via international financial markets. Third, in not allowing households to invest in physical capital. Amending on all these features should aim at generating less trivial dynamics of the current account than the ones generated here via the only movements in the trade balance. Such dynamics may be of first order importance for the welfare evaluation along two dimensions. First, they would more critically affect the transition from the time of policy implementation to the long-run steady state of the new policy. Second, they would impinge on the transition from one policy regime to another. For instance from Nash-competition to cooperation, or from the optimal commitment policy to a fixed exchange rate arrangement. We are currently investigating all these issues in our ongoing work.

A Proof of Proposition 1

The proof of part A just follows from the substitutions and the rearrangements done in section 3. As for part B, given the sequence $\{C_t,\ N_t,\ \pi_{H,t}, C_t^*, N_t^*, \pi_{F,t}^*\}_{t=0}^{\infty}$ that satisfy equations (41), (42), (43) and (44) and for given A_t it is possible to construct all of the remaining equilibrium (real) allocation, price and policy variables.

For given productivity processes A_t, A_t^* and using the optimal allocations for labor N_t, N_t^* satisfying the Ramsey plan we can obtain the optimal allocation for output, real marginal cost and real wage:

$$A_t N_t = Y_t \tag{65}$$
$$A_t^* N_t^* = Y_t^* \tag{66}$$

$$mc_t = A_t U_{n,t} \tag{67}$$
$$mc_t^* = A_t^* U_{n,t}^* \tag{68}$$
$$\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}; \frac{U_{n,t}^*}{U_{c,t}^*} = \frac{W_t^*}{P_t^*} \tag{69}$$

For given value κ of the risk sharing condition and using the optimal allocations for consumption, C_t, C_t^*,satisfying the Ramsey plan we can back up the relative prices Φ_t and Φ_t^* as a function or real allocations only. By combining (26), (27) and (38) one can write

$$\begin{aligned} \Phi_t &\equiv \Phi(C_t, \Phi_t^*) \\ &= \left(\frac{1 - \alpha \left(\Phi_t^*\right)^{\eta-1} \left(\frac{U_{c,t}}{\kappa U_{c,t}^*}\right)^{\eta-1}}{(1-\alpha)} \right)^{\frac{1}{\eta-1}} \end{aligned} \tag{70}$$

and similarly:

$$\begin{aligned} \Phi_t^* &\equiv \Phi(C_t^*, \Phi_t) \\ &= \left(\frac{1 - \alpha^* \Phi_t^{\eta-1} \left(\frac{\kappa U_{c,t}^*}{U_{c,t}}\right)^{\eta-1}}{(1-\alpha^*)} \right)^{\frac{1}{\eta-1}} \end{aligned} \tag{71}$$

Using the inflation rates obtained by the optimal plans and the relative prices obtained above to back up inflation rates:

$$\pi_t = \pi_{H,t} \frac{\Phi_t}{\Phi_{t-1}} \tag{72}$$

$$\pi_t^* = \pi_{H,t}^* \frac{\Phi_t^*}{\Phi_{t-1}^*} \tag{73}$$

Optimal demands for domestic and foreign goods can be obtained from:

$$C_{H,t} = (1-\alpha)(\frac{P_{H,t}}{P_t})^{-\eta}(C_t); C_{F,t} = \alpha(\frac{P_{F,t}}{P_t})^{-\eta}(C_t) \tag{74}$$

Optimal demands are obtainable as well for the foreign country.

Finally, the optimal nominal interest rate set by the monetary authority is obtained from:

$$R_t^n = \frac{R_t}{\pi_t}$$

with $\pi_t = \frac{P_t}{P_{t-1}}$.

B Optimal Policy in the Steady State

B.1 Nash Golden Rule

Let λ_p and λ_f be the Lagrange multipliers associated to the steady-state constraints (49) and (50) respectively. Hence one can set up the Lagrangian:

$$\begin{aligned} \mathcal{L} &= U(C,N) + \lambda_p \left\{ U_c \pi_H (\pi_H - 1)(1-\beta) + \frac{U_c \varepsilon N}{\theta} \left(\frac{U_n \ \Phi(C,\Phi^*)}{U_c} + \frac{\varepsilon - 1}{\varepsilon} \right) \right\} \\ &+ \lambda_f \left\{ N - (1-\alpha) \ C \ \left[\Phi(C,\Phi^*)\right]^\eta - \kappa^\eta \left(\frac{U_{c^*}^*(C^*)}{U_c(C)} \right)^\eta \left[\Phi(C,\Phi^*)\right]^\eta \alpha^* C^* - \frac{\theta}{2} (\pi_H - 1)^2 \right\} \end{aligned}$$

First order necessary conditions for this problem read as follows

- (C)

$$U_c + \lambda_p \frac{\varepsilon N U_n}{\theta} \left(\frac{\Phi_c U_c - U_{cc}\Phi}{U_c^2} \right) - \lambda_f (1-\alpha) \left(\Phi^\eta + C\eta \Phi^{\eta-1} \Phi_c \right) - \\ -\lambda_f \alpha^* C^* \eta \left(\frac{\Phi^{1-\eta} - (1-\alpha)}{\alpha} \right)^{\frac{2\eta-1}{1-\eta}} \frac{\Phi^{-\eta}\Phi_c}{\alpha} = 0 \tag{75}$$

- (N)

$$U_n + \lambda_p \frac{\varepsilon \Phi}{\theta U_c}(U_n + N U_{nn}) + \lambda_p \frac{\varepsilon - 1}{\theta} + \lambda_f = 0 \tag{76}$$

- (π_H)

$$\pi_H \lambda_p (1-\beta)(2\pi_H - 1) - \lambda_f \theta (\pi_H - 1) = 0 \tag{77}$$

While Φ is given by the steady state version of (28), we have that $\Phi_c \equiv \frac{\partial \Phi(C,\Phi^*)}{\partial C}$ is given by

$$\Phi_c = -\frac{\alpha}{1-\alpha}\Phi^{2-\eta}U_c^{\eta-2}U_{cc}\left(\frac{\Phi^*}{\kappa U_{c^*}^*}\right)^{\eta-1} \tag{78}$$

In order to define the set of conditions that maximize steady state utility one should add the constraints (49) and (50) holding with equality. A similar set of conditions define the optimal steady state policy of Foreign.

B.2 Golden Rule under Cooperation

In the case of a steady-state in which Cooperation is implemented by a planner the Lagrangian will read

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\{U(C,N)+U(C^*,N^*)\} \\
&+\frac{1}{2}\lambda_p\left\{U_c\pi_H(\pi_H-1)(1-\beta)+\frac{U_c\varepsilon N}{\theta}\left(\frac{U_n\ \Phi(C,\Phi^*)}{U_c}+\frac{\varepsilon-1}{\varepsilon}\right)\right\} \\
&+\frac{1}{2}\lambda_f\left\{N-(1-\alpha)\ C\ \ [\Phi(C,\Phi^*)]^\eta-\kappa^\eta\left(\frac{U_{c^*}^*}{U_c}\right)^\eta[\Phi(C,\Phi^*)]^\eta\,\alpha^*C^*-\frac{\theta}{2}(\pi_H-1)^2\right\} \\
&+\frac{1}{2}\lambda_p^*\left\{U_{c^*}^*\pi_F^*(\pi_F^*-1)(1-\beta)+\frac{U_{c^*}^*\varepsilon N^*}{\theta}\left(\frac{U_{n^*}^*\ \Phi^*(C^*,\Phi)}{U_{c^*}^*}+\frac{\varepsilon-1}{\varepsilon}\right)\right\} \\
&+\frac{1}{2}\lambda_f^*\left\{N^*-(1-\alpha^*)\ C\ \ [\Phi^*(C^*,\Phi)]^\eta-\kappa^{-\eta}\left(\frac{U_c}{U_{c^*}^*}\right)^\eta[\Phi^*(C^*,\Phi)]^\eta\,\alpha C-\frac{\theta}{2}(\pi_F^*-1)^2\right\}
\end{aligned}$$

First order necessary conditions for this problem read as follows:

- (C)

$$\begin{aligned}
0 &= U_c-\lambda_pU_{cc}\pi_H(\pi_H-1)(1-\beta)-\lambda_p\frac{\varepsilon}{\theta}NU_n\Phi_c-\lambda_p(\varepsilon-1)\frac{NU_{cc}}{\theta} \\
&\quad -\lambda_f(1-\alpha)\left(\Phi^\eta+C\eta\Phi^{\eta-1}\Phi_c\right)-\lambda_f\alpha^*C^*\kappa^\eta(U_c^*)^\eta\left[\eta\Phi^{\eta-1}\Phi_cU_c^{-\eta}-\eta U_c^{-\eta-1}U_{cc}\Phi^\eta\right] \\
&\quad -\lambda_p^*\frac{\varepsilon N^*U_n^*\Phi_c^*}{\theta}-\lambda_f^*\eta(\Phi^*)^{\eta-1}\Phi_c^*(1-\alpha^*)C^* \\
&\quad -\lambda_f^*\alpha\kappa^{-\eta}(U_c^*)^{-\eta}\left[\eta(\Phi^*)^{\eta-1}\Phi_c^*CU_c^\eta+(\Phi^*)^\eta\left(U_c^\eta+\eta U_c^{\eta-1}U_{cc}C\right)\right]
\end{aligned} \tag{79}$$

- (N)

$$U_n-\lambda_p\frac{\varepsilon\Phi}{\theta}(U_n+NU_{nn})-\lambda_pU_c\frac{\varepsilon-1}{\theta}+\lambda_f=0 \tag{80}$$

- (π_H)

$$\pi_H-U_c\lambda_p(1-\beta)(2\pi_H-1)-\lambda_f\theta(\pi_H-1)=0 \tag{81}$$

Notice that the expression for Φ_c now reads

$$\Phi_c = -\frac{\alpha}{1-\alpha}\Phi^{2-\eta}\left(\frac{1}{\kappa U_c^*}\right)^{\eta-1}\left(U_c^{\eta-2}U_{cc}\left(\Phi^*\right)^{\eta-1} + \left(\Phi^*\right)^{\eta-2}U_c^{\eta-1}\Phi_c^*\right) \quad (82)$$

The expression for Φ_c^* reads:

$$\Phi_c^* \equiv \frac{\partial \Phi^*}{\partial C} = -\left(\Phi^*\right)^{2-\eta}\frac{\alpha^*}{1-\alpha^*}\left(U_c^*\right)^{\eta-1}\kappa^{\eta-1}\left(\Phi^{\eta-2}\Phi_c U_c^{1-\eta} - U_c^{-\eta}U_{cc}\Phi^{\eta-1}\right) \quad (83)$$

Recall that under a Nash equilibrium we have that $\Phi_c^* = \Phi_{c^*} = 0$. In order to define the set of conditions that maximize steady state utility one should add the constraints (49) and (50) holding with equality. The set of efficiency order conditions is completed by an analogous set of equations for Foreign.

C The Stationary Policy problem

Here we derive the stationary form of the policy problem under Nash commitment. We illustrate the argument only for the Home policymaker's problem, for the problem in Foreign is exactly symmetric. Let's consider the optimal plan as formulated in equation (51) in the text. By applying the law of iterated expectations and by grouping expectations and multipliers that share the same date one obtains:

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}}\; Max_{\{\Xi_t\}_{t=0}^{\infty}} E_0\{\mathcal{U}(C_0, N_0, \pi_{H,0}, \Omega)$$

$$+\lambda_{p,0}\left[U_{c,0}\pi_{H,0}(\pi_{H,0}-1) + \frac{U_{c,0}\varepsilon A_0 N_0}{\theta}\left(\frac{U_{n,0}\Phi_0}{U_{c,0}A_0} + \frac{\varepsilon-1}{\varepsilon}\right)\right]$$
$$+\lambda_{f,0}\left[A_0N_0 - (1-\alpha)C_0\Phi_0^{\eta} - \kappa^{\eta}\left(\frac{U_{c^*,0}^*}{U_{c,0}}\right)^{\eta}\Phi_0^{\eta}\alpha^*C_0^* - \frac{\theta}{2}\left(\pi_{H,0}-1\right)^2\right]$$
$$+\beta\{U(C_1,N_1) + (\lambda_{p,1} - \beta\lambda_{p,0})(U_{c,1}\pi_{H,1}(\pi_{H,1}-1)) + \lambda_{p,1}\left(\frac{U_{c,1}\varepsilon A_1 N_1}{\theta}\left(\frac{U_{n,1}\Phi_1}{U_{c,1}A_1} + \frac{\varepsilon-1}{\varepsilon}\right)\right)$$
$$+\lambda_{f,1}\left[A_1N_1 - (1-\alpha)C_1\Phi_1^{\eta} - \kappa^{\eta}\left(\frac{U_{c^*,1}^*}{U_{c,1}}\right)^{\eta}\Phi_1^{\eta}\alpha^*C_1^* - \frac{\theta}{2}\left(\pi_{H,1}-1\right)^2\right] + ...\}\} \;-\; \Omega\widetilde{B}_0$$

Notice that this problem is not time-invariant due to the fact that the constraints at time zero lack the term $-\beta\lambda_{p,-1}(U_{c,0}\pi_{H,0}(\pi_{H,0}-1))$. For this reason we amplify the state space to introduce a new (pseudo) costate variable χ_t and define a new policy functional $\mathcal{W}(C_t, N_t, \pi_{H,t}, \chi_t, \Omega) \equiv \mathcal{U}(C_t, N_t, \pi_{H,t}, \Omega) - \chi_t(U_{c,t}\pi_{H,t}(\pi_{H,t}-1))$. We then write the optimal policy plan in the following form:

Choose $\Lambda_t \equiv \{\lambda_{p,t}, \lambda_{f,t}\}_{t=0}^{\infty}$ and $\Xi_t \equiv \{C_t,\, \pi_{H,t},\, N_t\}_{t=0}^{\infty}$ to

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}}\ Max_{\{\Xi_t\}_{t=0}^{\infty}}\ E_0\{\sum_{t=0}^{\infty}\beta^t E_t\{\mathcal{W}(C_t, N_t, \pi_{H,t}, \chi_t, \Omega) \tag{84}$$

$$+\lambda_{p,t}\left[U_{c,t}\pi_{H,t}(\pi_{H,t}-1)+\frac{U_{c,t}\varepsilon A_t N_t}{\theta}\left(\frac{U_{n,t}\ \Phi_t}{U_{c,t}A_t}+\frac{\varepsilon-1}{\varepsilon}\right)\right]$$
$$+\lambda_{f,t}\left[A_t N_t-(1-\alpha)\ C_t\Phi_t^{\eta}-\kappa^{\eta}\left(\frac{U^*_{c^*,t}}{U_{c,t}}\right)^{\eta}\Phi_t^{\eta}\alpha^* C_t^*-\frac{\theta}{2}\left(\pi_{H,t}-1\right)^2\right]\}\}-\Omega\widetilde{B}_0$$

with law of motion for the new costate

$$\chi_{t+1}=\lambda_{p,t}$$

and initial condition

$$\chi_0=0$$

Following Marcet and Marimon (1999), one can show that this new maximization program is now saddle point stationary in the amplified state space $\{A_t,\ \chi_t\}$. First order conditions of this problem exactly replicate conditions (55)-(57) in the text. An exactly symmetric argument is applied to the design of the policy problem in Foreign, which will involve specifying an amplified state space $\{A_t^*,\ \chi_t^*\}$, with law of motion $\chi_{t+1}^*=\lambda_{p,t}^*$ and initial condition $\chi_0^*=0$.

D First order conditions of the Cooperation-Commitment problem

First order conditions for the Ramsey problem under Cooperation at time $t\geq 1$ read:

- (C_t) :

$$\begin{aligned}
0 \;=\; & \frac{1}{2}U_{c,t}+U_{cc,t}\ \pi_{H,t}(\pi_{H,t}-1)\ (\lambda_{p,t}-\chi_t)+\frac{\lambda_{p,t}N_t\varepsilon}{\theta}U_{n,t}\Phi_{c,t}+\lambda_{p,t}\left(\frac{\varepsilon-1}{\theta}\right)A_t N_t U_{cc,t} & (85)\\
& -\lambda_{f,t}\ (1-\alpha)\left(\Phi_t^{\eta}+\eta C_t\Phi_t^{\eta-1}\Phi_{c,t}\right)-\lambda_{f,t}\alpha^* C_t^*\kappa^{\eta}\left(U^*_{c^*,t}\right)^{\eta}\left[\eta\Phi_t^{\eta-1}\Phi_{c,t}U_{c,t}^{-\eta}-\eta U_{c,t}^{-\eta-1}U_{cc,t}\Phi_t^{\eta}\right] & \\
& +\lambda_{p,t}^*\left(\frac{\varepsilon N_t^* U_{n,t}^*\Phi_{c,t}^*}{\theta}\right)-\lambda_{f,t}^*\left(\eta\left(\Phi_t^*\right)^{\eta-1}\Phi_{c,t}^*\left(1-\alpha^*\right)C_t^*\right) & \\
& -\lambda_{f,t}^*\alpha\kappa^{-\eta}\left(U^*_{c^*,t}\right)^{-\eta}\ \left[\eta\left(\Phi_t^*\right)^{\eta-1}\Phi_{c,t}^* C_t U_{c,t}^{\eta}+\left(\Phi_t^*\right)^{\eta}\left(U_{c,t}^{\eta}+\eta U_{c,t}^{\eta-1}U_{cc,t}C_t\right)\right] & \\
& +\Omega^w(A_t N_t-\frac{\theta}{2}(\pi_{H,t}-1)^2)\ \left(U_{cc,t}\Phi_t^{-1}-\Phi_{c,t}\Phi_t^{-2}U_{c,t}\right)-\Omega^w\left(U_{cc,t}C_t+U_{c,t}\right)+ & (86)\\
& +\Omega^w(A_t^* N_t^*-\frac{\theta}{2}(\pi_{F,t}^*-1)^2)\ U^*_{c^*,t}\ \left(-\Phi_{c,t}^*\Phi_t^{*-2}\right) & (87)
\end{aligned}$$

- (N_t) :

$$U_{n,t} + \frac{\lambda_{p,t}\varepsilon\Phi_t}{\theta}\left(U_{n,t} + N_t U_{nn,t}\right) + \lambda_{p,t}\left(\frac{\varepsilon-1}{\theta}\right)U_{c,t}A_t + \lambda_{f,t}A_t + \Omega^w \frac{U_{c,t}A_t}{\Phi_t} = 0 \tag{88}$$

- $(\pi_{H,t})$:

$$U_{c,t}(2\pi_{H,t}-1)\left(\lambda_{p,t}-\chi_t\right) - \theta(\pi_{H,t}-1)\left(\lambda_{f,t} + \Omega^w \frac{U_{c,t}A_t}{\Phi_t}\right) = 0 \tag{89}$$

The expression for $\Phi^*_{c,t} \equiv \frac{\partial \Phi^*_t}{\partial C_t}$ reads:

$$\Phi^*_{c,t} = -\left(\frac{\alpha^*}{1-\alpha^*}\right)\left(\Phi^*_t\right)^{2-\eta}\left(U^*_{c,t}\right)^{\eta-1}\kappa^{\eta-1}\left[\Phi_t^{\eta-2}\Phi_{c,t}U_{c,t}^{-(\eta-1)} - U_{c,t}^{-\eta}U_{cc,t}\Phi_t^{\eta-1}\right] \tag{90}$$

The set of analogous conditions for Foreign variables at time $t \geq 0$ will involve an expression for $\Phi_{c^*,t} \equiv \frac{\partial \Phi_t}{\partial C^*_t}$:

$$\Phi_{c^*,t} = -\left(\frac{\alpha}{1-\alpha}\right)\Phi_t^{2-\eta}\left(U_{c,t}\right)^{\eta-1}\kappa^{1-\eta}\left[\left(\Phi^*_t\right)^{\eta-2}\Phi^*_{c^*,t}\,\left(U^*_{c,t}\right)^{-(\eta-1)} - \left(U^*_{c,t}\right)^{-\eta}U^*_{cc,t}\left(\Phi^*_t\right)^{\eta-1}\right] \tag{91}$$

In addition all constraints must hold with equality. Also, when evaluated at time $t = 0$, condition (85) must feature the additional term $-\Omega_0^w \frac{B_0}{P_0}$.

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Table 1

Volatility and Welfare under Alternative Policy Regimes

	High Home Bias		Low Home Bias		Low Elasticity	
	Nash	**Cooperation**	**Nash**	**Cooperation**	**Nash**	**Cooperation**
Consumption	1.553	1.443	1.834	1.695	1.561	1.491
Labor	0.227	0.196	0.184	0.100	0.155	0.142
PPI Inflation	0.053	0.005	0.072	0.006	0.041	0.009
CPI/PPI Ratio	0.698	0.699	0.240	0.237	1.552	1.541
Welfare cost Δ	0.0122	0.0113	0.0157	0.0146	0.0118	0.0118

Note: Standard deviations are in %. The welfare cost (in %) is the proportional upward shift in the consumption process that would make the representative household indifferent between its random consumption allocation and a nonrandom consumption allocation with the same mean

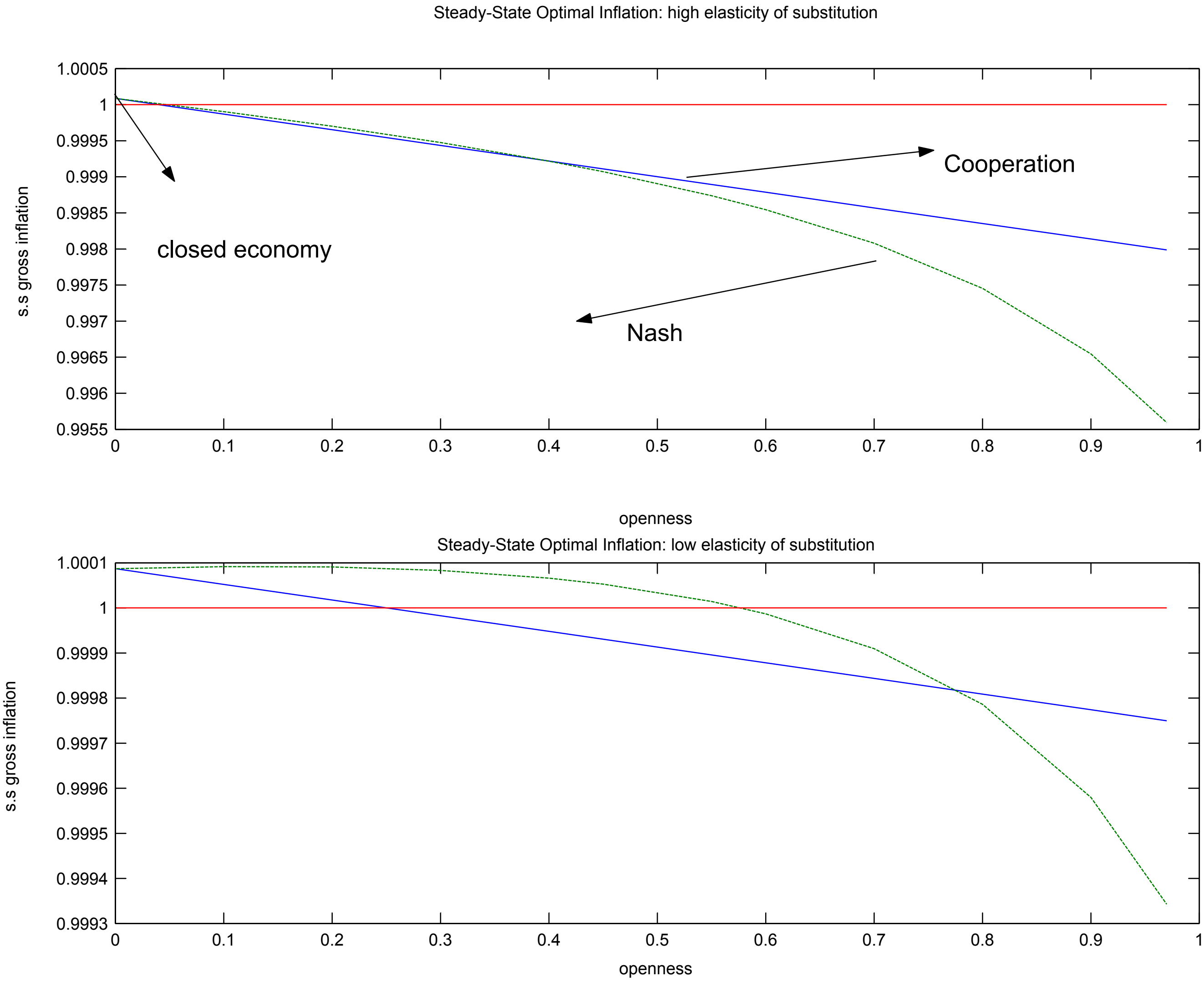
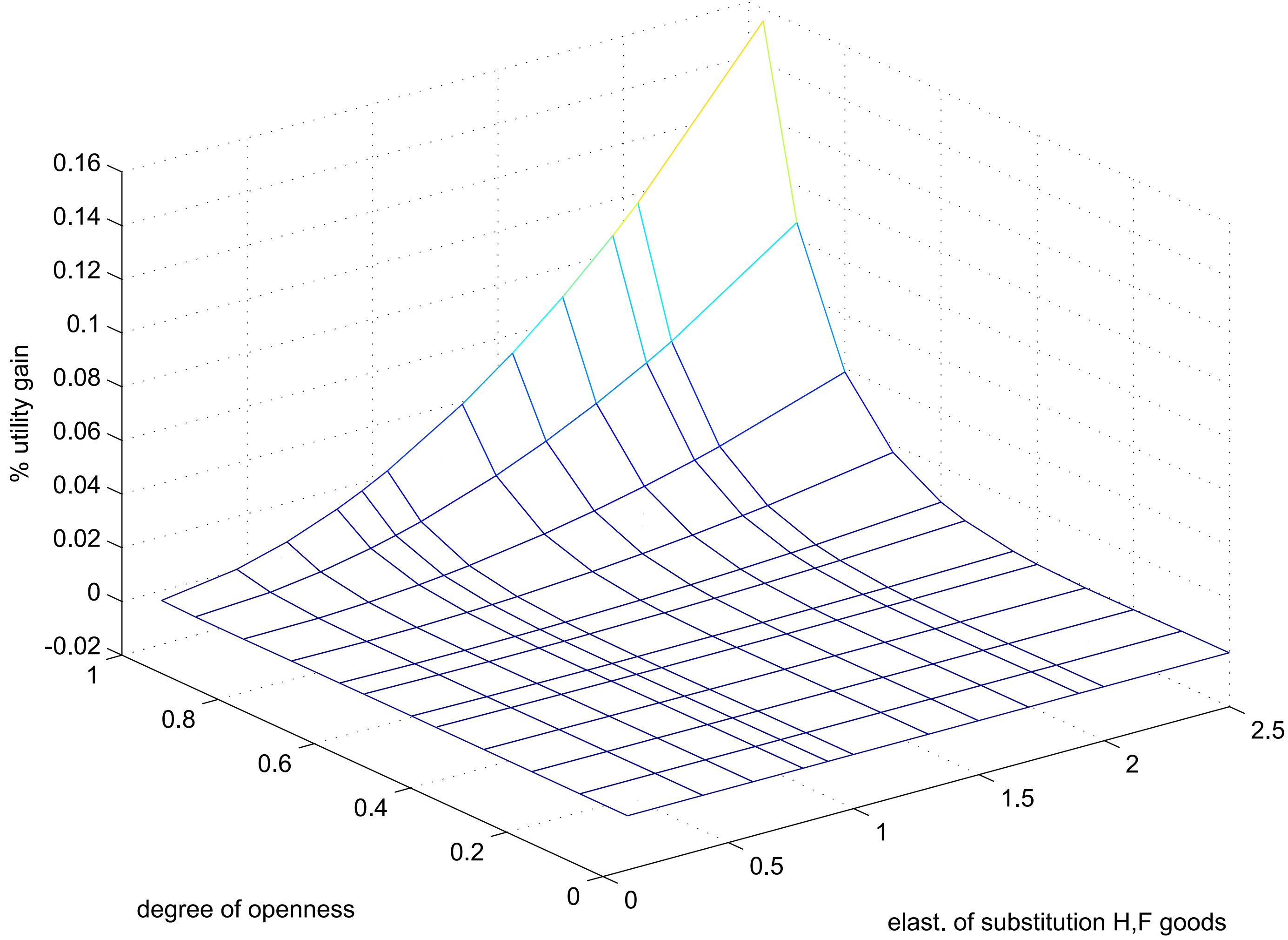


Figure 1. Effect of Varying Openness on Steady-State Optimal Inflation Rate

Figure 2. % Gains from Policy Cooperation: Effect of Varying Openness and Elast.of Substitution



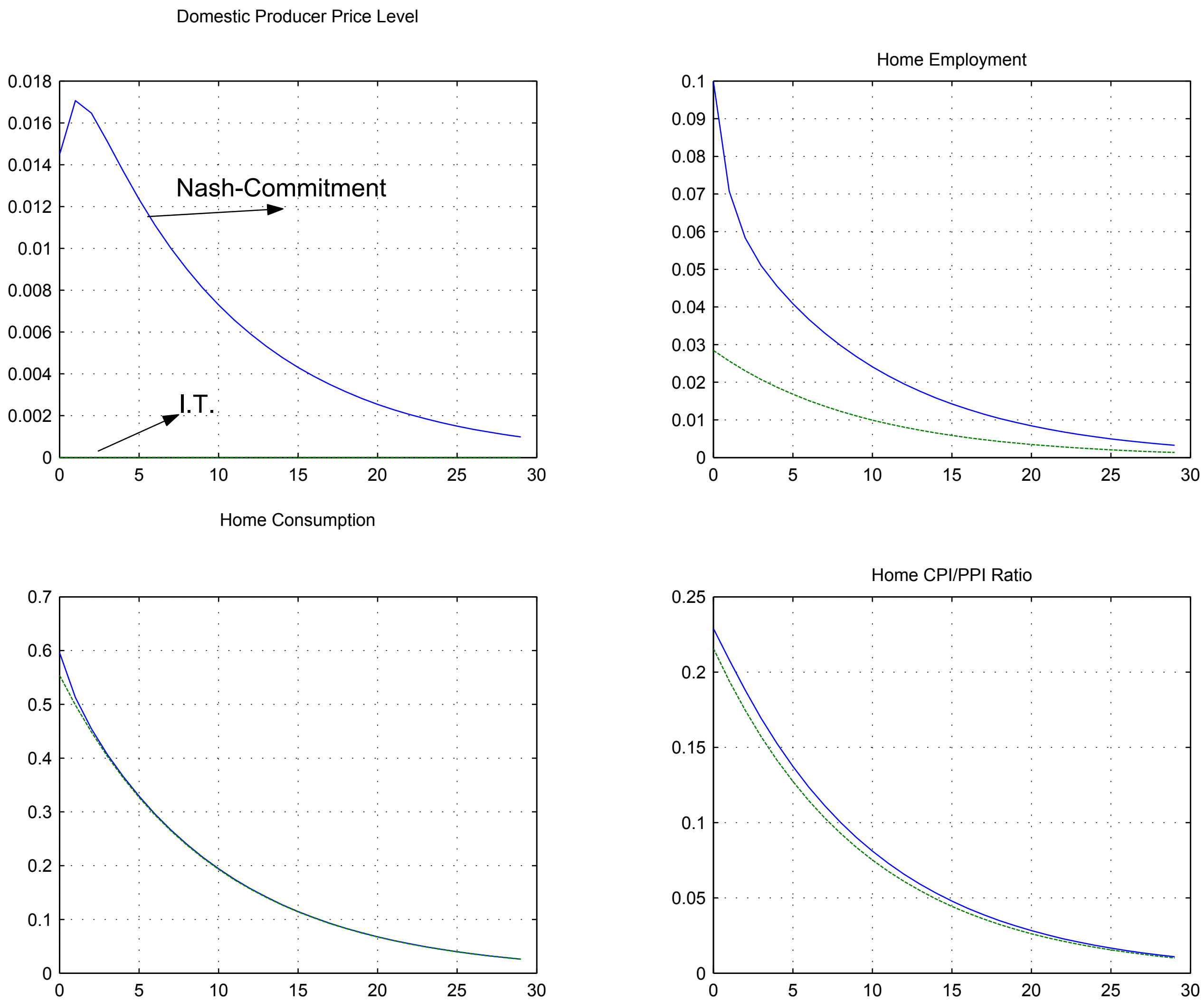


Figure 3. Responses to a Home Productivity Shock: Nash-Commitment vs. Inflation Targeting

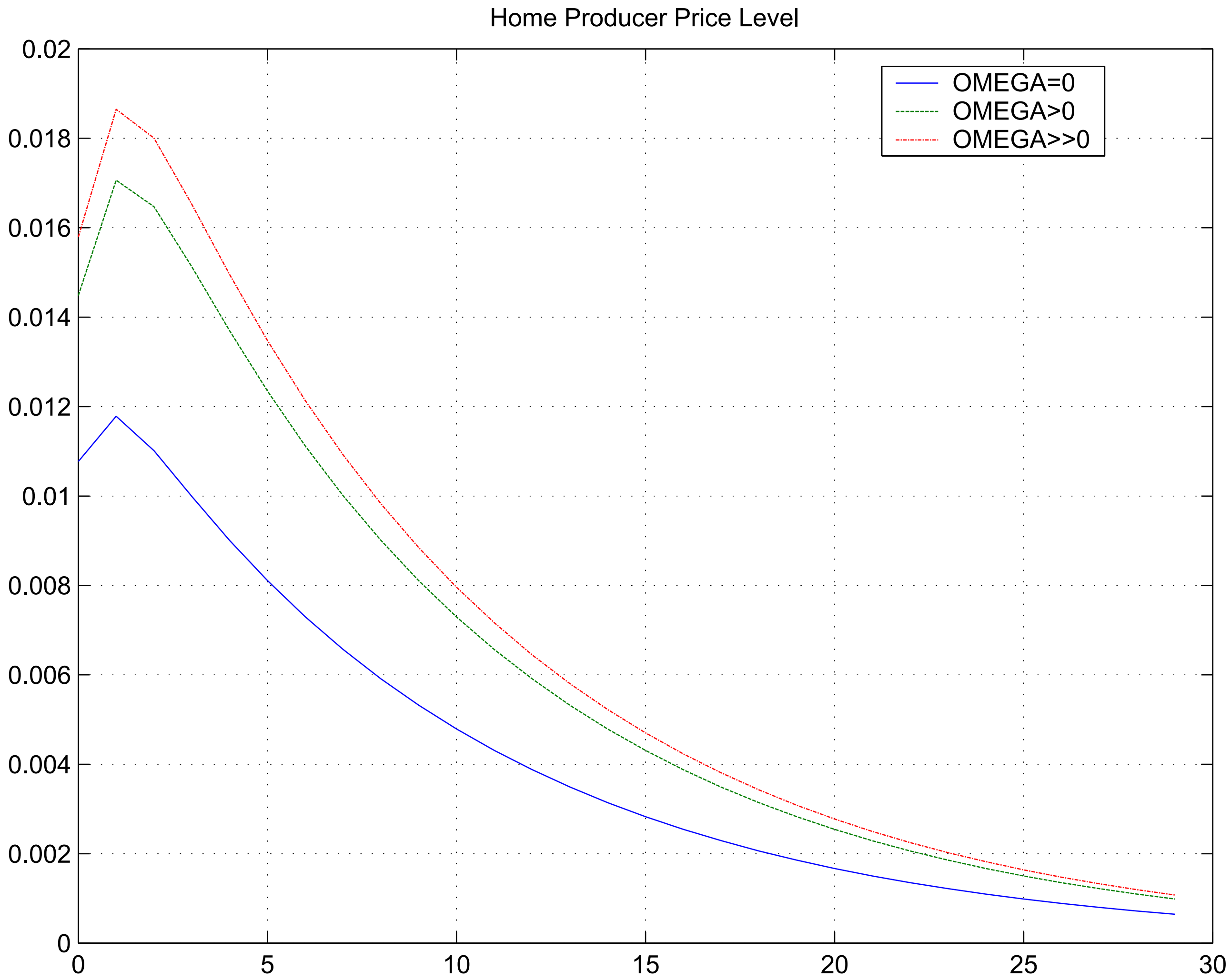


Figure 4. Effect on the Response of the Price Level of Policy Competition on Wealth

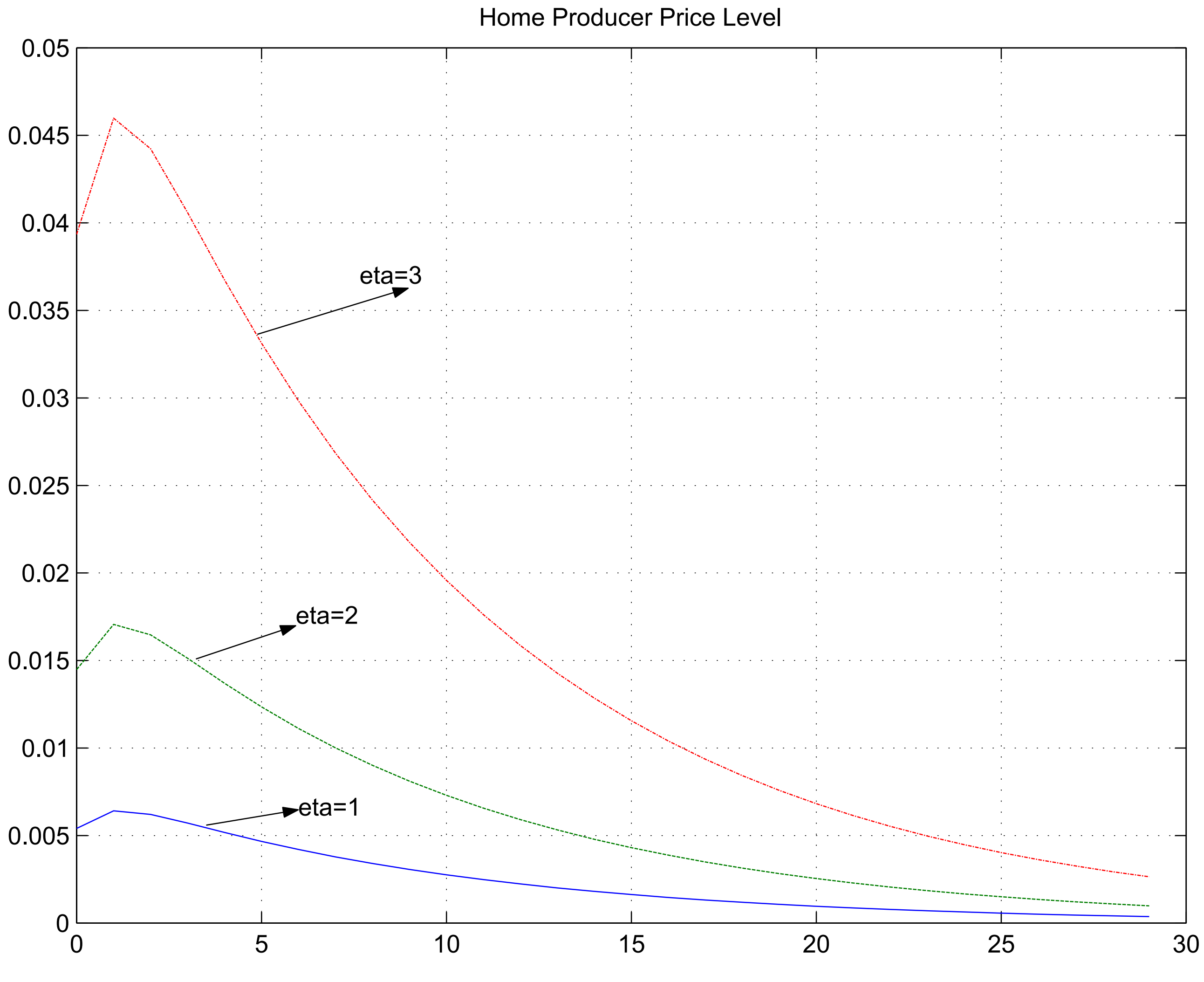


Figure 5. Response of the Price Level to a Home Productivity Shock: Effect of Varying the Elasticity of Substitution

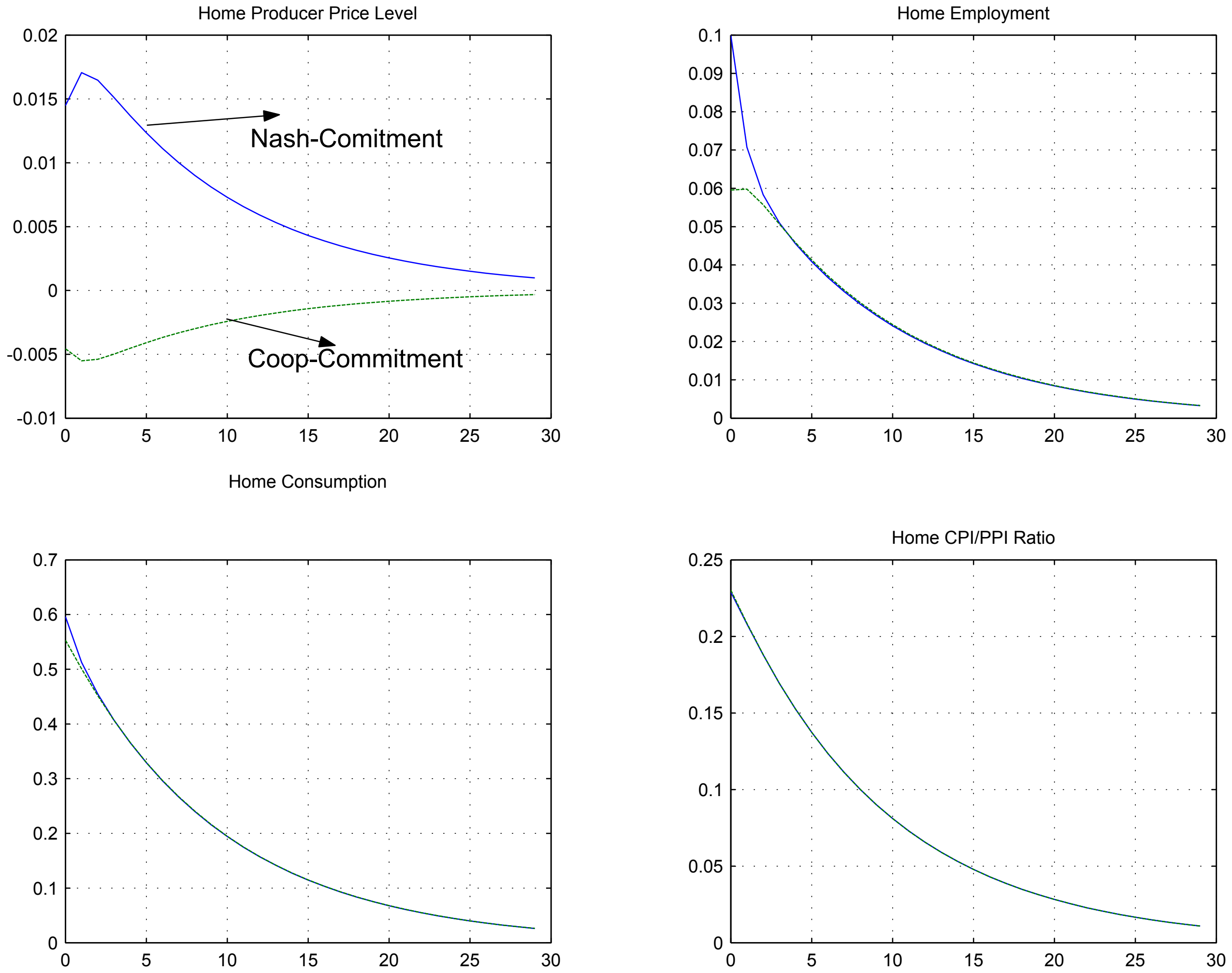


Figure 6. Responses to a Productivity Shock: Nash-Commitment vs. Cooperation-Commitment