

**Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.**

Variance Disparity and Market Frictions

Yang-Ho Park

2019-059

Please cite this paper as:

Park, Yang-Ho (2019). "Variance Disparity and Market Frictions," Finance and Economics Discussion Series 2019-059. Washington: Board of Governors of the Federal Reserve System, <https://doi.org/10.17016/FEDS.2019.059>.

NOTE: Staff working papers in the Finance and Economics Discussion Series (FEDS) are preliminary materials circulated to stimulate discussion and critical comment. The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors. References in publications to the Finance and Economics Discussion Series (other than acknowledgement) should be cleared with the author(s) to protect the tentative character of these papers.

Variance Disparity and Market Frictions

Yang-Ho Park

Federal Reserve Board

July 10, 2019

This paper introduces a new model-free approach to measuring the expectation of market variance using VIX derivatives. This approach shows that VIX derivatives carry different information about future variance than S&P 500 (SPX) options, especially during the 2008 financial crisis. I find that the segmentation is associated with frictions such as funding illiquidity, market illiquidity, and asymmetric information. When they are segmented, VIX derivatives contribute more to the variance discovery process than SPX options. These findings imply that VIX derivatives would offer a better estimate of expected variance than SPX options, and that a measure of segmentation may be useful for policymakers as it signals the severity of frictions.

JEL Classification: G01; G13; G14

Keywords: economic uncertainty; illiquidity; asymmetric information; implied variance; VIX derivative

This paper was previously circulated under the title “On the Relation between S&P 500 Options and VIX Derivatives,” and revised while the author was visiting the Federal Reserve Bank of Boston. I am very grateful for comments and suggestions from Yacine Aït-Sahalia (editor), Celso Brunetti, Erik Heitfield, Scott Mixon, Matthew Pritsker, Fabio Trojani, Clara Vega, and seminar participants at the Federal Reserve Board, the 2015 Paris Financial Management conference, the 2016 Lancaster Financial Econometrics and Empirical Asset Pricing conference, 2016 Research in Behavioral Finance conference, the 2017 Society for Nonlinear Dynamics and Econometrics symposium, the 2017 Annual Society for Financial Econometrics conference, and the 2017 European Financial Association conference. This paper has also benefited from the excellent research assistance of Sathya Ramesh and Robertson Wang. The analysis and conclusions set forth are those of the author and do not indicate concurrence by the Board of Governors or other members of its research staff. Send correspondence to Yang-Ho Park, Risk Analysis Section, Board of Governors of the Federal Reserve System, 20th & C Streets, NW, Washington, DC 20551, Telephone: (202) 452-3177, e-mail: yangho.park@gmail.com.

1 Introduction

Economic uncertainty, or market variance, plays a key role in finance. Uncertainty can affect asset returns if economic agents prefer an early resolution of uncertainty (Bansal and Yaron, 2005). Uncertainty also affects corporate investment and hiring (Bloom, 2009) and thus, can have predictive information about the business cycle (Bekaert and Hoerova, 2014). Perceptions of uncertainty often manifest in derivative prices. In particular, stock index options have been widely used to infer the market's expectation of future uncertainty.

In this paper, I introduce an alternative approach to measuring the market's expectation of uncertainty using a new class of derivatives: VIX futures and options (collectively, VIX derivatives). VIX derivatives reference the VIX index, which is in turn derived from SPX option prices. As a result, the VIX derivatives-implied variance (VIV) should be consistent with the SPX options-implied variance (SIV) if the two markets are well integrated. However, I find significant gaps between the VIV and SIV, especially in the wake of the Lehman Brothers' bankruptcy. The gaps (henceforth referred to as variance disparity) suggest that some trading impediments may have deterred the integration of the two markets. That noted, the goal of this paper is to understand the roles played by illiquidity and asymmetric information in the variance market segmentation and draw implications of the segmentation for practitioners and policymakers.¹

My analysis indicates that funding liquidity, as measured by the London inter-bank offered rate (LIBOR)–overnight index swap (OIS) spread, is a key driver of variance disparity.² This result is associated with the margins required in options and futures trading because margins can impair market makers' ability to provide liquidity in derivatives markets and arbitrageurs' ability to exploit price differentials between the two markets. Importantly, margins are subject to daily marking-to-market, so market makers and arbitrageurs may shy away from the markets when concerned about a margin call and an unwanted liquidation of their positions at a loss. Overall, the importance of funding liquidity can be explained by the Gârleanu

¹ In her presidential address, O'Hara (2003) points to liquidity and asymmetric information as two essential frictions that should be incorporated into asset pricing models.

² Such an interest rate spread has been widely adopted as a proxy for funding liquidity in empirical research (Hameed, Kang, and Viswanathan, 2010; Boyson, Stahel, and Stulz, 2010; and Karolyi, Lee, and Van Dijk, 2012).

and Pedersen (2011) model which shows that when heterogeneous agents face margin constraints, the price gaps between two identical assets should depend on the shadow cost of capital.

My analysis also suggests that market illiquidity, as measured by bid–ask spreads, is another significant source of variance disparity. This result is consistent with empirical evidence that market illiquidity deters convergence between two equivalent asset prices (see, for example, Roll, Schwartz, and Subrahmanyam, 2007; Chordia, Roll, and Subrahmanyam, 2008; and Deville and Riva, 2007). Related to this finding, Oehmke (2011) provides a theoretical model which shows that market illiquidity results in gradual arbitrage.

It should be emphasized that funding and market liquidity proxies are separately important for explaining variance disparity. Funding liquidity and market liquidity can mutually reinforce each other (Brunnermeier and Pedersen, 2009). Moreover, Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010) and Hameed, Kang, and Viswanathan (2010) have provided empirical evidence that the capital constraint faced by market makers is a key determinant of bid–ask spreads in the stock market. Given the endogenous relation between the two liquidity factors, it is possible that one type of liquidity might serve as a mediating channel through which the other type of liquidity drives variance disparity. However, a multivariate regression analysis indicates that both liquidity factors have a direct effect on variance disparity, even when the Lehman Brothers crisis period is excluded.

In addition to liquidity factors, I provide evidence that informed trading about future variance contributes to variance disparity. The VIX derivatives market provides a new way for informed traders to capitalize on their expectation of future variance. Importantly, the VIX derivatives market is subject to different margins and liquidity than the SPX options market; therefore, informed traders may prefer one market to the other. Several variance discovery analyses, including those by Gonzalo and Granger (1995) and Hasbrouck (1995), suggest that VIX derivatives are far more informative about future variance than SPX options. This result suggests that informed trading may cause variance disparity as information is incorporated into VIX derivative prices before SPX option prices.

To confirm the effect of informed trading on variance disparity, I use the volume ratio of VIX futures to SPX options (F/O) as a measure of informed trading with

respect to future variance. Option writers are required to post far greater margins than option buyers.³ For example, in my sample period, margins on option sales are, on average, one order of magnitude larger than those on option purchases. Because of such asymmetric option margins, when expecting a lower future variance, informed traders may find it easier to sell VIX futures instead of delta-neutral SPX options. Therefore, I conjecture that a higher F/O may be associated with a lower future variance. Consistent with this expectation, I demonstrate that as the volume ratio rises, VIX derivatives (which are more informative) tend to imply lower levels of variance than SPX options (which are less informative), resulting in further deviations between the VIV and SIV.

This paper contributes to the finance literature in several ways. First, it is related to the extensive literature studying no-arbitrage violations in various financial markets, including papers on interest rate parity violations (Coffey, Hrungrung, and Sarkar, 2009; Baba and Packer, 2009; Fong, Valente, and Fung, 2010; and Mancini Griffoli and Rinaldo, 2012); American Depositary Receipt (ADR) parity violations (Gagnon and Karolyi, 2010; and Pasquariello, 2014); credit default swap (CDS)–bond parity violations (Gârleanu and Pedersen, 2011; and Bai and Collin-Dufresne, 2013); and TIPS (Treasury Inflation-Protected Securities)–Treasury bond parity violations (Fleckenstein, Longstaff, and Lustig, 2014). Some of these existing papers focus on identifying important impediments to arbitrage, similar to this paper. For example, Gagnon and Karolyi (2010) attribute ADR parity violations to the holding cost measured by idiosyncratic risk; Gârleanu and Pedersen (2011) impute CDS–bond parity violations to funding liquidity; and Roll, Schwartz, and Subrahmanyam (2007) assess whether transaction cost deters convergence between future markets and cash markets. However, the current paper studies the role of liquidity and asymmetric information in the segmentation of variance trading markets.

Second, this paper contributes to the literature on the informational role of derivatives. Chakravarty, Gulen, and Mayhew (2004), Holowczak, Simaan, and Wu (2006), and Muravyev, Pearson, and Broussard (2013) compare the price informativeness of stock options to that of the underlying stocks, while Hasbrouck (2003) and Blanco, Brennan, and Marsh (2005) study the informational role of stock index futures and CDS, respectively. However, to the best of my knowledge, this paper

³ It is because option writers impose greater counterparty risk on central clearing houses than option buyers.

is the first to study the variance discovery process between SPX options and VIX derivatives.

Third, this paper is related to the literature on a volume-based measure of informed trading with respect to future stock prices. For example, Pan and Poteshman (2006) report that put–call volume ratios are predictive of future stock returns and attribute this finding to informed trading in options markets. Johnson and So (2012) and Ge, Lin, and Pearson (2016) find that option-to-stock volume ratios contain predictive information for stock returns and attribute their results to informed trading in options markets. However, my paper makes a unique contribution to the literature by introducing a volume-based measure of informed trading with respect to future variance.

The remainder of this paper is organized as follows: In Section 2, I introduce two equivalent measures of variance implied by SPX options and VIX derivatives and show firsthand evidence of variance disparity; in Section 3, I introduce methodology to study the determinants of variance disparity; in Section 4, I examine the impact of liquidity on variance disparity; in Section 5, I investigate the impact of informed trading on variance disparity; and I conclude in Section 6.

2 Variance disparity

2.1 SPX options-implied variance (SIV)

It is well documented that a risk-neutral measure of variance can be replicated by a static portfolio of stock options. Early work by Carr and Madan (1998), Britten-Jones and Neuberger (2000), and Demeterfi, Derman, Kamal, and Zou (1999) introduced model-free formulas under the assumption of continuous stock prices. Subsequent researchers, such as Jiang and Tian (2005) and Carr and Wu (2009), extended this idea to cases in which stock prices are driven by both diffusion and jump components.

To begin with, I assume a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ and information filtration $\{\mathcal{F}_t\}$. Let the stock price, S_t , take the following stochastic differential equation under the \mathbb{Q} measure:

$$\frac{dS_t}{S_t} = r_t dt + \sigma_t dB_t^{\mathbb{Q}} + \int_{\mathbb{R}} (\exp(x) - 1) [J^{\mathbb{Q}}(dx, dt) - \nu_t^{\mathbb{Q}}(dx)dt], \quad (1)$$

where r_t is the risk-free rate, σ_t is the instantaneous diffusive volatility, $B_t^{\mathbb{Q}}$ is a standard Brownian motion under the \mathbb{Q} measure, $J^{\mathbb{Q}}(dx, dt)$ is a random jump measure, and $\nu_t^{\mathbb{Q}}(dx)$ is a jump compensator for the log price.

Given Equation (1), return variance, or annualized quadratic variation, may be expressed as the sum of integrated variance and jump variation as follows:

$$\mathcal{V}(t, T) = \frac{1}{T-t} \left[\underbrace{\int_t^T \sigma_s^2 ds}_{\text{Integrated Variance}} + \underbrace{\int_t^T \int_{\mathbb{R}} x^2 J^{\mathbb{Q}}(dx, ds)}_{\text{Jump Variation}} \right], \quad (2)$$

where $\mathcal{V}(t, T)$ denotes the return variance over the $(t, T]$ horizon. The risk-neutral expectation of return variance is given by

$$E_t^{\mathbb{Q}}[\mathcal{V}(t, T)] = \frac{1}{T-t} E_t^{\mathbb{Q}} \left[\int_t^T \lambda_s^{\mathbb{Q}} ds \right], \quad (3)$$

where

$$\lambda_s^{\mathbb{Q}} = \sigma_s^2 + \int_{\mathbb{R}} x^2 \nu_s^{\mathbb{Q}}(dx).$$

Here $\lambda_s^{\mathbb{Q}}$ is referred to as the \mathbb{Q} -spot variance. Jiang and Tian (2005) and Carr and Wu (2009) show that Equation (3) can be approximated using the prices of out-of-the-money (OTM) SPX options up to a high-order error term as follows:

$$E_t^{\mathbb{Q}}[\mathcal{V}(t, T)] \approx \frac{2 \exp(r_t(T-t))}{T-t} \left[\int_{F_t(T)}^{\infty} \frac{C_t(T, K)}{K^2} dK + \int_0^{F_t(T)} \frac{P_t(T, K)}{K^2} dK \right], \quad (4)$$

where $F_t(T)$ is the SPX future price at time t and $C_t(T, K)$ and $P_t(T, K)$ are the SPX call and put prices, respectively, with a maturity of T and a strike price of K at time t .⁴

2.2 VIX derivatives-implied variance (VIV)

Let $VF_t(T)$ denote the VIX future price with a maturity of T at time t . By construction, the VIX future price is the same as the risk-neutral expectation of the time- T VIX index: $VF_t(T) = E_t^{\mathbb{Q}}[VIX_T]$. By Jensen's inequality, it follows that the squared VIX future price is less than or equal to the risk-neutral expectation of a

⁴ Ait-Sahalia, Karaman, and Mancini (2018) compared synthetic variance swaps implied by SPX options and actual over-the-counter variance swaps and found that the high-order error term may be nontrivial.

forward-starting return variance:

$$\begin{aligned}
VF_t(T)^2 &= \left(E_t^{\mathbb{Q}}[VIX_T]\right)^2 \\
&\leq E_t^{\mathbb{Q}}[VIX_T^2] \\
&= E_t^{\mathbb{Q}}[E_T^{\mathbb{Q}}[\mathcal{V}(T, T + 30d)]] \\
&= E_t^{\mathbb{Q}}[\mathcal{V}(T, T + 30d)],
\end{aligned} \tag{5}$$

where $30d$ stands for 30 calendar days and $\mathcal{V}(T, T + 30d)$ denotes the return variance starting on date T with a fixed 30-day window.

The difference between the risk-neutral expectation of a forward-starting variance (with a fixed 30-day window) and the squared VIX future price is called a convexity adjustment term. This term is associated with the variance of the time- T VIX index, which can be backed out from a cross-section of the OTM VIX option prices.

Proposition 1. *Under no arbitrage, the variance of the time- T VIX index, which I denote by $\text{var}_t(VIX_T)$, may be expressed in terms of a cross-section of the VIX option prices with different strike prices but with the same maturity of T :*

$$\text{var}_t(VIX_T) = 2 \exp(r_t(T - t)) \left[\int_{VF_t(T)}^{\infty} VC_t(T, K) dK + \int_0^{VF_t(T)} VP_t(T, K) dK \right], \tag{6}$$

where $VC_t(T, K)$ and $VP_t(T, K)$ are the VIX call and put prices with a maturity of T and a strike price of K at time t , respectively.

See Appendix A for the proof. Adding the convexity adjustment term to the squared VIX future price, I can infer the risk-neutral expectation of a forward-starting return variance with a fixed 30-day window as follows:

$$E_t^{\mathbb{Q}}[\mathcal{V}(T, T + 30d)] = VF_t(T)^2 + 2 \exp(r_t(T - t)) \left[\int_{VF_t(T)}^{\infty} VC_t(T, K) dK + \int_0^{VF_t(T)} VP_t(T, K) dK \right]. \tag{7}$$

To evaluate the relative magnitude of the convexity adjustment term in measurements of return variance, I define a convexity ratio, $CVRT_t(T)$, as

$$CVRT_t(T) = \frac{\text{var}_t(VIX_T)}{E_t^{\mathbb{Q}}[\mathcal{V}(T, T + 30d)]}, \tag{8}$$

where the numerator and the denominator are given by Equations (6) and (7), re-

spectively. Table 1 shows the summary statistics of the convexity ratios divided into short and long maturities (less than three months and more than three months, respectively). Figure 1 shows the histograms of the short-term and long-term convexity ratios. The average short-term convexity ratio is 9%, implying that VIX options contribute 9% of the total VIV measurement. In general, the long-term convexity ratios are larger than the short-term ones, suggesting that accounting for convexity adjustment is particularly important for long-term VIV measures.

2.3 Firsthand evidence of variance disparity

Let $SIV_t(T)$ and $VIV_t(T)$ denote the variance measures implied by SPX options and VIX derivatives, respectively, with a maturity of T ; that is, $SIV_t(T) = E_t^{\mathbb{Q}}[\mathcal{V}(t, T)]$ and $VIV_t(T) = E_t^{\mathbb{Q}}[\mathcal{V}(T, T+30d)]$. Note that the two variance measures are differently associated with over-the-counter variance swaps.⁵ $SIV_t(T)$ can be thought of as a synthetic strike price for a variance swap starting today with a tenor of $T - t$, whereas $VIV_t(T)$ can be considered as a synthetic strike price for a forward variance swap that starts at time T with a fixed 30-day tenor.

Although the SIV and VIV are not directly comparable because of their different time horizons, it is possible to replicate a VIV measure with a maturity of T using two SIV measures with maturities of T and $T + 30d$ as follows:

$$\begin{aligned} \widetilde{VIV}_t(T) &= E_t^{\mathbb{Q}}[\mathcal{V}(T, T + 30d)] \\ &= \left(\frac{T + 30d - t}{30d}\right) E_t^{\mathbb{Q}}[\mathcal{V}(t, T + 30d)] - \left(\frac{T - t}{30d}\right) E_t^{\mathbb{Q}}[\mathcal{V}(t, T)] \\ &= \left(\frac{T + 30d - t}{30d}\right) SIV_t(T + 30d) - \left(\frac{T - t}{30d}\right) SIV_t(T), \end{aligned} \quad (9)$$

where $\widetilde{VIV}_t(T)$ denotes a replicated version of the VIV measure computed from the SIV measures. In practice, the two SIV measures in Equation (9) may not be available for a given maturity, so I interpolate them from the available SIV data using a cubic smoothing spline and replicate the VIV measures from the interpolated SIV measures.

Figure 2 shows the time series plots of the original and replicated VIV measures

⁵ A variance swap is an OTC swap that exchanges the future realized variance of the underlying asset for a prespecified strike price at the expiration date. Because no money changes hands at the initiation of a variance swap contract, a no-arbitrage condition requires that the strike price be equal to the risk-neutral expectation of the variance of the underlying asset over the horizon.

with a three-month maturity. Panels A and B correspond to the entire sample period and a subsample during the Lehman Brothers crisis, respectively. Although the two VIV measures remain close to each other for most of the sample period (see Panel A), massive differences are observed in the wake of the Lehman Brothers' bankruptcy (see Panel B).⁶

I acknowledge that deviations in the two VIV measures do not necessarily indicate real-world arbitrage opportunities for two main reasons. First, the original VIV measure is not replicable by a static VIX derivatives position because the squared VIX future price is not a tradable asset, although the convexity adjustment term is replicable by an equally weighted portfolio of OTM VIX options. Second, the replicated VIV measure is subject to truncation and interpolation errors because of the limited availability of strike prices and maturities.

Nevertheless, frictions can drive the segmentation of variance trading markets because they can make it difficult for sophisticated investors to implement relative value trading to exploit price differentials between the two markets or for market makers to intermediate the markets. Therefore, the goal of this paper is to understand the roles played by frictions in the variance market segmentation and draw implications of the segmentation for practitioners and policymakers.

3 Methodology

3.1 Stochastic variance models

This subsection introduces three kinds of stochastic variance models: the market integration (SV2) model, the market segmentation (SV4) model, and the market segmentation with error corrections (SV4-EC) model. These models are designed to shed light on whether the SPX options and VIX derivatives markets are integrated or segmented and which of the two markets is more informative about future variance.

⁶ The replicated VIV measures (dotted line) had some negative values in October 2008. The negative values arise because the near-term SIV measures are too large compared with the long-term SIV measures.

3.1.1 Market integration (SV2) model

Existing studies have found that a two-factor variance structure is essential to describe the term structure of return variance (see Gallant, Hsu, and Tauchen, 1999; Christoffersen, Heston, and Jacobs, 2009; Aït-Sahalia, Amengual, and Manresa, 2015; and others). Furthermore, the literature has shown that modeling the logarithm of variance fits the data better than modeling variance itself (Jones, 2003; Aït-Sahalia and Kimmel, 2007; and Durham, 2013). I thus accommodate these two important findings in the joint modeling of the SIV and VIV. The model that follow is similar to the term structure models of variance swap rates (see, for example, Aït-Sahalia, Karaman, and Mancini, 2018; Filipović, Gourier, and Mancini, 2016; and Amengual and Xiu, 2018).

In the SV2 model, the SIV and VIV data are generated by the same two-factor stochastic variance model. Let v_{1t} and u_{1t} denote the short-run and long-run variance factors, respectively. I assume that $X_t = (v_{1t}, u_{1t})'$ takes the following stochastic differential equation under the \mathbb{Q} measure:

$$\begin{aligned} dv_{1t} &= -\kappa_{v_1} v_{1t} dt + \sigma_{v_1} dB_{1t}^{\mathbb{Q}} \\ du_{1t} &= -\kappa_{u_1} u_{1t} dt + \sigma_{u_1} dB_{2t}^{\mathbb{Q}}, \end{aligned} \tag{10}$$

where κ_{v_1} and κ_{u_1} are the persistence parameters, σ_{v_1} and σ_{u_1} are the variance-of-variance parameters, and $B_{1t}^{\mathbb{Q}}$ and $B_{2t}^{\mathbb{Q}}$ are independent standard Brownian motions under the \mathbb{Q} measure. I also assume that the SIV and VIV are determined by the same form of the \mathbb{Q} -spot variance process:

$$\lambda_{1t}^{\mathbb{Q}} = \exp(\mu_1 + v_{1t} + u_{1t}), \tag{11}$$

where μ_1 captures the risk-neutral long-term level of the variance measures.⁷

Let $M(X_t, t, T; \phi)$ denote the moment-generating function of X_T at time t under the \mathbb{Q} measure: $M(X_t, t, T; \phi) = E_t^{\mathbb{Q}}[\exp(\phi' X_T)]$. The model prices of the SIV and VIV, which I denote by $SIV_t(T; \theta)$ and $VIV_t(T; \theta)$, respectively, are obtained by

⁷ Modeling of the \mathbb{Q} -spot variance process is introduced by Filipović, Gourier, and Mancini (2016).

integrating out the moment-generating function with $\phi = (1, 1)'$ as follows:

$$\begin{aligned} SIV_t(T; \theta) &= \frac{\exp(\mu_1)}{T-t} \int_t^T M(X_t, t, \tau; \phi = (1, 1)') d\tau \\ VIV_t(T; \theta) &= \frac{\exp(\mu_1)}{30d} \int_T^{T+30d} M(X_t, t, \tau; \phi = (1, 1)') d\tau, \end{aligned} \tag{12}$$

where θ denotes the set of model parameters.⁸ Note that the pricing formula of the SIV has the same integrand (the moment-generating function) as that of the VIV.

3.1.2 Market segmentation (SV4) model

In the SV4 model, the SIV and VIV are driven by different two-factor stochastic variance models. That is, I assume that only the SIV data are driven by the model specified in Equations (10) and (11). To separately model the dynamics of the VIV data, I introduce another pair of short-run and long-run variance factors, v_{2t} and u_{2t} . The new variance pair, $(v_{2t}, u_{2t})'$, is assumed to follow the stochastic differential equation below:

$$\begin{aligned} dv_{2t} &= -\kappa_{v_2} v_{2t} dt + \sigma_{v_2} dB_{3t}^{\mathbb{Q}} \\ du_{2t} &= -\kappa_{u_2} u_{2t} dt + \sigma_{u_2} dB_{4t}^{\mathbb{Q}}, \end{aligned} \tag{13}$$

where κ_{v_2} and κ_{u_2} are the persistence parameters, σ_{v_2} and σ_{u_2} are the variance-of-variance parameters, and $B_{3t}^{\mathbb{Q}}$ and $B_{4t}^{\mathbb{Q}}$ are independent standard Brownian motions under the \mathbb{Q} measure. I assume that the VIV is determined by the following \mathbb{Q} -spot variance process:

$$\lambda_{2t}^{\mathbb{Q}} = \exp(\mu_2 + v_{2t} + u_{2t}), \tag{14}$$

where μ_2 captures the risk-neutral long-term level of the VIV. The model price of the VIV can be similarly obtained as in Equation (12).

The SV4 model is a two-market model in which each market has a two-factor variance structure; no market has a four-factor variance structure. There is no interaction between the pair (v_{1t}, u_{1t}) and the pair (v_{2t}, u_{2t}) . Thus, despite the multi-market nature, the dynamics of v_{1t} and u_{1t} (v_{2t} and u_{2t}) can be separately estimated only using the SIV (VIV) data.

⁸ An analytic solution to the moment-generating function is provided in Appendix B.1.

3.1.3 Market segmentation with error corrections (SV4-EC) model

In the SV4-EC model, error correction mechanisms are added to the short-run variance dynamics. Let $X_t = (v_{1t}, u_{1t}, v_{2t}, u_{2t})'$, where v_{1t} and u_{1t} drive the SIV and v_{2t} and u_{2t} drive the VIV. I assume that X_t takes the following stochastic differential equation under the \mathbb{Q} measure:

$$\begin{aligned}
 dv_{1t} &= -\kappa_{v_1} v_{1t} dt + \gamma_{v_1} (v_{1t} - v_{2t}) dt + \sigma_{v_1} dB_{1t}^{\mathbb{Q}} \\
 du_{1t} &= -\kappa_{u_1} u_{1t} dt + \sigma_{u_1} dB_{2t}^{\mathbb{Q}} \\
 dv_{2t} &= -\kappa_{v_2} v_{2t} dt + \gamma_{v_2} (v_{1t} - v_{2t}) dt + \sigma_{v_2} dB_{3t}^{\mathbb{Q}} \\
 du_{2t} &= -\kappa_{u_2} u_{2t} dt + \sigma_{u_2} dB_{4t}^{\mathbb{Q}},
 \end{aligned} \tag{15}$$

where γ_{v_1} and γ_{v_2} capture the speed of convergence between the two short-run variance factors and the other model parameters are similarly defined as in the SV4 model. I also assume that the SIV and VIV are determined by the \mathbb{Q} -spot variance processes specified in Equations (11) and (14), respectively. Note that the SV4-EC model nests the SV4 model, with the restrictions: $\gamma_{v_1} = 0$ and $\gamma_{v_2} = 0$.

The error correction terms $\gamma_{v_1} (v_{1t} - v_{2t})$ and $\gamma_{v_2} (v_{1t} - v_{2t})$ allow for convergence between the two short-run variance factors and, thus, between the SIV and VIV. If one market is informationally efficient, it will not respond to deviations from the other market. However, if neither of the two markets is informationally efficient and information diffusion occurs in both directions, γ_{v_1} and γ_{v_2} should be negative and positive, respectively. Importantly, the magnitudes of the speed-of-convergence parameters allow us to compare price informativeness between the two markets. The larger the speed of convergence in magnitude, the less informationally efficient the market.

In a discrete-time version of the SV4-EC model, the conditional variance will be affected by the cross-market lagged variance state as well as its own lagged variance state.⁹ This cross-market interaction is not conceptually new. In many of the multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models, conditional variance and covariance are allowed to influence one another through their cross-market lagged values (as well as their cross-market squared innovation terms). For example, authors such as Karolyi (1995), Kearney and Patton (2000), and Ma-

⁹ A discrete-time model can be obtained by applying an Euler approximation to the continuous-time model specified in Equation (15).

lik and Ewing (2009) study volatility transmission phenomena across closely related markets using the BEKK (Baba, Engle, Kraft, and Kroner, 1989) model, a type of multivariate GARCH models.

Let $M(X_t, t, T; \phi)$ denote the moment-generating function of X_T at time t under the \mathbb{Q} measure.¹⁰ The model prices of the SIV and VIV are obtained by integrating out the moment-generating function with different boundary conditions as follows:

$$\begin{aligned} SIV_t(T; \theta) &= \frac{\exp(\mu_1)}{T-t} \int_t^T M(X_t, t, \tau; \phi = (1, 1, 0, 0)') d\tau \\ VIV_t(T; \theta) &= \frac{\exp(\mu_2)}{30d} \int_T^{T+30d} M(X_t, t, \tau; \phi = (0, 0, 1, 1)') d\tau, \end{aligned} \tag{16}$$

where θ denotes the set of model parameters. Note that because of the interaction between v_{1t} and v_{2t} , the SV4-EC model should be jointly estimated using both the SIV and VIV data, unlike the SV4 model.

3.2 Estimation method

The model prices of the SIV and VIV are nonlinear functions of the state vector, and therefore, the standard Kalman filtering is inappropriate. Instead, I implement unscented Kalman filtering, a variant of the standard Kalman filtering in which a set of sample points is carefully chosen to represent the true mean and covariance of the state vector. This method is accurate up to the third order for a Gaussian state vector and the second order for a non-Gaussian state vector, and has been widely used in the option pricing literature (see Carr and Wu, 2007; Trolle and Schwartz, 2009a,b; and Mencía and Sentana, 2012). In particular, Christoffersen, Dorion, Jacobs, and Karoui (2014) show that unscented Kalman filtering is superior to extended Kalman filtering in the application of interest rate derivatives. Refer to Appendix C for further details on unscented Kalman filtering and Appendix D for the data used in the estimation.

3.3 Parameter estimates and model comparison

Table 2 presents parameter estimates for all three models across three sample periods. Panels A, B, and C correspond to the full sample period (July 1, 2006 to August

¹⁰ A quasi-analytic solution to the moment-generating function is provided in Appendix B.2.

31, 2014), the pre-crisis period (July 1, 2006 to August 31, 2008), and the post-crisis period (December 1, 2008 to August 31, 2014), respectively. The table also reports two kinds of model comparison criteria: Akaike information criteria (AIC) and Schwarz information criteria (SIC). These criteria are based on the log likelihoods of the estimation and various penalty scores, which depend on the number of free parameters. The lower the information criteria, the better the model.

The SV4 model is strongly preferred to the SV2 model based on the information criteria, regardless of the sample periods. In the SV4 model, the parameter estimates for the SIV are notably different from those for the VIV. Specifically, the SIV dynamics have a lower long-run mean ($\mu_1 < \mu_2$), lower degrees of persistence ($\kappa_{v_1} > \kappa_{v_2}$ and $\kappa_{u_1} > \kappa_{u_2}$), and higher levels of variance-of-variance ($\sigma_{v_1} > \sigma_{v_2}$ and $\sigma_{u_1} > \sigma_{u_2}$) than the VIV dynamics. Overall, the two derivative markets appear to imply different variance dynamics, a result consistent with that of Bardgett, Gourier, and Leippold (2013) that the SPX options and VIX derivatives markets contained different information on variance during the market distress.

The SV4-EC model is strongly preferred to the SV4 model based on the information criteria. Note that γ_{v_1} and γ_{v_2} are estimated to be negative and positive, respectively, for every sample period considered. These signs suggest that information diffuses in both directions, although the speed of convergence depends on the directions. Importantly, γ_{v_1} is larger than γ_{v_2} in absolute terms, suggesting that deviations between the SIV and VIV are largely resolved in the SPX options market rather than in the VIX derivatives market (this issue will be further discussed in Section 5.1.2).

3.4 Introducing a measure of variance disparity

I introduce a measure of variance disparity using the full-sample estimation result of the SV4 model. Let $\hat{\theta}_{SIV}$ ($\hat{\theta}_{VIV}$) denote the parameter set that is obtained by estimating a two-factor stochastic variance model using the SIV (VIV) data alone. Thus, $\hat{\theta}_{SIV}$ and $\hat{\theta}_{VIV}$ represent the two distinct variance dynamics implied by the SPX options and VIX derivatives markets, respectively.

To measure the extent of variance disparity, I first calculate a model price for the VIV using the parameter $\hat{\theta}_{VIV}$, which I denote by $VIV_t(T; \hat{\theta}_{VIV})$. This model price

represents a fair price for the VIV in the VIX derivatives market. I calculate another model price for the same VIV observation using the parameter $\hat{\theta}_{SIV}$, which I denote by $VIV_t(T; \hat{\theta}_{SIV})$. The second model price represents a fair price for the VIV in the SPX options market. I then define a basis in the VIV as the logarithmic difference between the two model prices as follows:

$$VBASIS_t(T) \equiv \log(VIV_t(T; \hat{\theta}_{VIV})) - \log(VIV_t(T; \hat{\theta}_{SIV})), \quad (17)$$

where $VBASIS_t(T)$ denotes a basis in $VIV_t(T)$. A positive $VBASIS_t(T)$ means that the VIX derivatives market would price the VIV observation higher than the SPX options market, while a negative $VBASIS_t(T)$ means that the VIX derivatives market would price the VIV observation lower than the SPX options market.

Similarly, I can define a basis in the SIV as the logarithmic difference between its two model prices as follows:

$$SBASIS_t(T) \equiv \log(SIV_t(T; \hat{\theta}_{VIV})) - \log(SIV_t(T; \hat{\theta}_{SIV})), \quad (18)$$

where $SBASIS_t(T)$ denotes a basis in $SIV_t(T)$. Again, a positive or negative $SBASIS_t(T)$ means that the VIX derivatives market would price the SIV observation higher or lower, respectively, than the SPX options market.

For a given date, there are multiple VIV and SIV observations, so I average the bases to obtain a measure of variance disparity:

$$BASIS_t = \frac{1}{K_t + N_t} \left(\sum_{i=1}^{K_t} VBASIS_t(T_i) + \sum_{i=1}^{N_t} SBASIS_t(T_i) \right), \quad (19)$$

where $BASIS_t$ denotes an average of the bases at date t , and K_t and N_t denote the number of VIV observations and the number of SIV observations, respectively, at time t .

Figure 3 shows the time evolution of the variance disparity measure, $BASIS_t$. A remarkable finding shown in the figure is that the bases skyrocketed to about 100% in the wake of the Lehman Brothers' bankruptcy. In particular, positive bases during the crisis mean that investors were willing to pay higher prices for VIX derivatives relative to SPX options. There were other ups and downs in the bases, although the magnitudes were smaller than those observed right after the Lehman Brothers' bankruptcy. Overall, the figure shows wide fluctuations in the bases over time, with

the largest during the Lehman Brothers crisis.

4 Liquidity as a driver of variance disparity

This section investigates whether measures of funding and market liquidity can explain time variation in the bases.

4.1 Role of funding liquidity

I conjecture that limited capital might explain time variation in the bases. It is known that market makers tend to take negative net holdings of SPX options and positive net holdings of VIX futures (see, for example, Bollen and Whaley, 2004; and Gârleanu, Pedersen, and Poteshman, 2009). However, their ability to hedge non-zero derivative positions can be impaired by limited capital. Recently, Barras and Malkhozov (2016) and Fournier and Jacobs (2016) find evidence that market makers' risk bearing capacity affects their willingness to provide liquidity in option markets and, thus, option prices. In addition, arbitrageurs are also subject to limited capital because a margin is required for any arbitrage position in futures and options markets.

To understand the role of limited capital in variance disparity, I take the LIBOR–OIS spread (*LIBOIS*) as a measure of funding liquidity. Such an interest rate spread between risky and risk-free debt has been widely adopted as a proxy for funding friction (see, for example, Hameed, Kang, and Viswanathan, 2010; Boyson, Stahel, and Stulz, 2010; and Karolyi, Lee, and Van Dijk, 2012). More formally, Gârleanu and Pedersen (2011) show that when agents are constrained by margins, an interest rate spread between uncollateralized and collateralized loans captures the shadow cost of capital.

Figure 4 shows the time series plot of *LIBOIS*. Before August 9, 2007, *LIBOIS* averaged about 10 basis points. The next day, *LIBOIS* rose to about 40 basis points and continued to climb to 365 basis points until the peak of the financial crisis. Starting from the fourth quarter of 2009, *LIBOIS* returned to pre-crisis levels.

Panel A of Table 3 shows the univariate regression results of the bases (*BASIS*) onto the funding liquidity proxy for two sample periods. In the subsample analy-

sis (ex-crisis), I exclude the three-month window corresponding to the peak of the Lehman Brothers crisis to determine if the result is driven by the Lehman Brothers' bankruptcy. All explanatory variables are standardized in the regression, so each coefficient can be interpreted as a change in the dependent variable in response to a one standard deviation change in the explanatory variable. The summary statistics and correlation matrix for explanatory variables are provided in Table 4.

In the full sample, *LIBOIS* (the funding liquidity measure) alone can explain 29% of the variation in *BASIS*. However, in the ex-crisis sample, the explanatory power is significantly reduced to 3%. Nevertheless, all of the results are statistically significant at the 1% level for both samples. Note that *LIBOIS* is positively associated with the bases in both of the sample periods. These positive signs suggest that as funding conditions deteriorate, investors are willing to pay higher prices for VIX derivatives relative to SPX options.

I also consider other interest rate spreads, such as the LIBOR–general collateral repo spread and the LIBOR–Treasury bill spread, and the results (not shown in this paper) suggest that other interest rate spreads have an explanatory power similar to or slightly weaker than *LIBOIS*. Overall, regardless of the choice of an interest rate spread, funding liquidity is associated with the bases, consistent with the Gârleanu and Pedersen (2011) model which states that when heterogeneous agents face margin constraints, price gaps between two identical assets should depend on the shadow cost of capital.

4.2 Role of market liquidity

Several existing studies suggest that market illiquidity deters convergence between two equivalent asset prices (see, for example, Roll, Schwartz, and Subrahmanyam, 2007; Deville and Riva, 2007; Chordia, Roll, and Subrahmanyam, 2008; and Oehmke, 2011). I thus conjecture that market illiquidity might be another important driver of the bases.

As a proxy for market illiquidity, I obtain weekly moving averages of three relative bid–ask spreads corresponding to SPX options (*SPRDS*), VIX futures (*SPRDF*), and VIX options (*SPRDV*).¹¹ Their time series plots are provided in Figure 5. The

¹¹ When taking a weekly moving average of the relative bid–ask spreads for each derivatives

three bid–ask spreads all increased in the wake of the Lehman Brothers’ bankruptcy, although the increases were not as dramatic as those seen for *LIBOIS*.

Panel B of Table 3 shows the results of regressing the bases onto the three bid–ask spread measures only. In the full sample, the three market liquidity proxies together can explain 31% of the variation in *BASIS*. In the ex-crisis sample, the explanatory power is significantly reduced to 4%. Statistical significance is obtained at different levels. *SPRDS* is statistically significant at the 1% level in the full sample and at the 5% level in the ex-crisis sample. *SPRDF* is statistically significant at the 1% level in both the full and ex-crisis samples. *SPRDV* is statistically significant at the 1% level in the full sample only.

Note that the coefficients on the bid–ask spreads always have positive signs. These signs suggest that as market liquidity deteriorates, VIX derivatives tend to imply higher levels of variance than SPX options. That said, market liquidity is another key driver of variance disparity, a finding that is consistent with the conventional wisdom that transaction cost impedes convergence between two identical assets.

4.3 Joint role of funding liquidity and market liquidity

In the framework of Brunnermeier and Pedersen (2009), funding and market liquidity can mutually reinforce each other. There is also empirical evidence that the capital constraint faced by market makers is a key determinant of bid–ask spreads in the stock market (Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes, 2010; and Hameed, Kang, and Viswanathan, 2010). Consistent with this endogenous relation, *LIBOIS* is positively associated with the three bid–ask spread measures, with correlations between 0.39 and 0.47 (see Table 4). As a result, market liquidity might be a mediating channel through which funding liquidity drives variance disparity. Although less likely, market liquidity alone might be the true source of variance disparity.

Panel C of Table 3 shows the joint effect of the funding and market liquidity factors on the bases. The combined explanatory power goes up to 37% in the full sample. Again, the explanatory power is significantly decreased if the peak of the crisis is excluded from the regression. Nonetheless, *LIBOIS* is still statistically significant

market, I include only the contracts with trading volume of more than 10 contracts on that date.

at the 1% level in the full sample and at the 5% level in the ex-crisis sample. *SPRDS* is also statistically significant at the 1% level in the full sample and at the 5% level in the ex-crisis sample. *SPRDF* is statistically significant at the 5% level in both the full and ex-crisis samples. Overall, the results suggest that funding liquidity and market liquidity are both critical drivers of variance disparity.

Holding cost—defined as the cost that occurs while an arbitrage position remains open—is known as one of the greatest impediments to arbitrage (Pontiff, 2006). In particular, Gagnon and Karolyi (2010) find that idiosyncratic risk, a measure of holding cost, is a key driver of the parity violations in ADRs and other cross-listed stocks. Holding cost is also relevant for futures and options because arbitrageurs facing a margin call may need to liquidate their positions prematurely at a loss. I thus include two stochastic risk proxies as controls in the regression: the VIX and the SKEW indexes.¹² These indexes are taken as proxies for arbitrage holding cost because they are associated with the variance risk and jump risk, respectively, inherent in the markets in this study. Panel D of Table 3 shows that while the holding cost proxies have some effects on the bases, funding liquidity and market liquidity remain statistically significant. Interestingly, after the holding cost proxies are accounted for, *LIBOIS* and *SPRDS* have even larger marginal effects on the bases, with stronger statistical significance.

4.4 Robustness to hedging pressure

There is a large literature suggesting that agents’ hedging pressure can distort the prices of futures and options. Hirshleifer (1989, 1990), Bessembinder (1992), and de Roon, Nijman, and Veld (2000) are good examples for futures markets. Bollen and Whaley (2004) and Gârleanu, Pedersen, and Poteshman (2009) show that option prices are also affected by demand pressure. More directly related to this work, Mixon and Onur (2015) show that demand from asset managers puts upward pressure on VIX futures prices, and Cheng (2018) finds that a falling risk premium in VIX futures when risk rises is associated with a falling hedging demand. In addition, rebalancing demand arising from VIX-linked exchange-traded products (ETPs) can impose additional pressure in VIX futures market.

¹² These data are obtained directly from the Chicago Board of Options Exchange.

Given the importance of hedging demand in derivatives, it is possible that variance disparity may have been driven by some agents' hedging pressure in VIX futures. To check such possibility, I obtain hedging pressure measures based on the disaggregated Commitments of Traders report from the Commodity Futures Trading Commission, where the hedging pressure measure is defined as each group's net open interest divided by the overall open interest.¹³ These data show that variance disparity is somewhat related to the hedging pressure from asset managers and leverage funds, but not from dealers (the results are not reported in this paper). More importantly, the significance of illiquidity measures that I have found earlier is robust to the hedging pressure of any investor group.

5 Informed trading as a driver of variance disparity

This section proceeds in three steps. First, I provide evidence of informed trading with respect to future variance. Second, I introduce a measure of informed trading using the volume data on VIX futures and SPX options. Third, I examine whether the informed trading measure can explain time variation in the bases in addition to funding and market liquidity.

5.1 Evidence of informed trading

5.1.1 Nonparametric analysis

I provide evidence of informed trading about future variance by running a vector error correction (VEC) model between the replicated and original constant-maturity VIV measures:

$$\Delta y_t = \alpha(\beta' y_{t-1} + c_0) + \sum_{j=1}^p A_j \Delta y_{t-j} + \varepsilon_t, \quad (20)$$

where $y_t = (\log(\widetilde{VIV}_t(T)), \log(VIV_t(T)))'$, $\alpha = (\alpha_1, \alpha_2)'$ is the speed-of-adjustment vector, β is the co-integrating relation vector, c_0 captures an intercept in the co-

¹³ Note that some dealers' ETN-related trading is captured through dealers' positions, and ProShares VIX ETFs-related trading is captured through asset managers' positions.

integrating vector, A_j is the autoregressive coefficient matrix, ε_t is an independently and identically distributed normal shock, and the autoregressive order, p , is determined using a likelihood ratio test.¹⁴ Table 5 displays the summary statistics of the square root of the two equivalent VIV measures for three chosen maturities: one, three, and five months. I refer to this subsection as a nonparametric analysis because the model-free measures of return variance are directly used to illuminate informed traders' behavior without any parametric model assumption.

Table 6 shows the estimates of the speed-of-adjustment parameters for two sub-periods: the pre-crisis period and the post-crisis period.¹⁵ Note that α_1 is negative for every maturity in both periods, indicating that SPX option prices adjust to eliminate the departure from VIX derivative prices. Similarly, α_2 is positive for every maturity in both the pre-crisis and post-crisis periods, suggesting that VIX derivative prices also adjust to eliminate the departure from SPX option prices.

Two common approaches to measuring the relative contribution of each market to price discovery can be found in Gonzalo and Granger (1995) and Hasbrouck (1995). Following the Gonzalo and Granger (1995) approach, I define the portion of variance discovery that is attributable to VIX derivatives, which I denote by GG , as

$$GG = \frac{\alpha_1}{\alpha_1 - \alpha_2}. \quad (21)$$

The GG metric measures the relative contribution of innovations in the original VIV measure to innovations in efficient prices.

Hasbrouck (1995) defines the relative contribution to price discovery as each market's contribution to the total variance of innovations in efficient prices, and calls it information share. Using the speed-of-adjustment parameters and the variance matrix of errors in the VEC model, I define the lower and upper bounds on information share of VIX derivatives, which I denote by HAS_L and HAS_U , respectively, as

$$\begin{aligned} HAS_L &= \frac{\alpha_1^2(\sigma_2^2 - \sigma_{12}^2/\sigma_1^2)}{\alpha_2^2\sigma_1^2 - 2\alpha_1\alpha_2\sigma_{12} + \alpha_1^2\sigma_2^2} \\ HAS_U &= \frac{(\alpha_1\sigma_2 - \alpha_2\sigma_{12}/\sigma_2)^2}{\alpha_2^2\sigma_1^2 - 2\alpha_1\alpha_2\sigma_{12} + \alpha_1^2\sigma_2^2}, \end{aligned} \quad (22)$$

¹⁴ The replicated and original VIV measures appear to be integrated and co-integrated with each other (see Appendix E).

¹⁵ Note that I exclude the three-month period corresponding to the peak of the Lehman Brothers crisis (September 1, 2008 to November 30, 2008) because the replicated VIV measures sometimes had negative values in that period.

where σ_1^2 , σ_{12} , and σ_2^2 constitute the variance matrix of ε_t .

The variance discovery ratios are shown in the last three columns of Table 6. The *GG* metric ranges from 76% to 94% in the pre-crisis period and from 42% to 96% in the post-crisis period, depending on the maturities. A similar result can be found based on the information shares, although they have wide ranges between the lower and upper bounds. Overall, it is clear that VIX derivatives contribute more to the variance discovery process than SPX options. In short, the tail wags the dog.

The large informational role of VIX derivatives may be explained by a literature arguing that leverage is a key driver of informed trading behavior (Black, 1975; and Easley, O'Hara, and Srinivas, 1998). Moreover, Ge, Lin, and Pearson (2016) present empirical evidence of the importance of leverage in informed trading. My result that VIX derivatives are more informative than SPX options is consistent with the existing papers because the former require much smaller margins than the latter. For example, VIX futures margins in my sample period are, on average, 24.7%, whereas SPX options margins are at least as large as their full prices if the maturities are less than nine months (see Appendix F for details on futures and options margins).

However, a surprising finding is that VIX derivatives played a large role in assimilating information even during the pre-crisis period. Existing research argues that liquidity matters in informed trading because it helps informed traders hide their private information (Easley, O'Hara, and Srinivas, 1998; and Anand and Chakravarty, 2007). However, although the VIX derivatives market grew dramatically in recent years, they were thinly traded in the beginning of the sample period (see Figure 6). Nonetheless, despite the low trading volume during this period, the VIX derivative market is found to have been a preferred venue for informed trading. Overall, it seems that stealth is not as important as leverage for informed trading in the variance trading markets.

I should acknowledge some methodological limitations in this nonparametric analysis. First, the VIV measures appear to have a unit root (see Appendix E), but the sample period went through a number of the crisis events, including the Lehman Brothers crisis, the Greek debt crisis, and the U.S. debt ceiling crisis. Such exogenous, perhaps structural, shocks tend to make the standard unit root tests falsely conclude the existence of a unit root even if the true data-generating process is stationary (Peron, 1989). Second, the model-free measures of variance are subject to measurement

errors because we do not observe a full continuum of strike prices for SPX and VIX options. Authors such as Jiang and Tian (2007) and Andersen, Bondarenko, and Gonzalez-Perez (2015) demonstrate that measurement errors in the VIX index are nontrivial. Third, the nonparametric analysis was unable to accommodate the peak of the crisis because the replicated VIV measures sometimes had negative values.

5.1.2 Parametric analysis

This subsection is intended to complement the previous nonparametric analysis using the SV4-EC model. Note that the parametric analysis here can account for the concerns raised about the nonparametric analysis to some degree. First, the stochastic variance models introduced in this paper assume that the variance process is stationary rather than nonstationary. Second, measurement errors in the SIV and VIV are accounted for in the observation equations (see Appendix C). Third, the parametric analysis can be applied to the full sample because the SIV and VIV data are directly fed into the estimation method.

In the SV4-EC model, the speed-of-convergence parameters are indicative of the magnitude of information transmission. Following the Gonzalo and Granger (1995) idea, I quantify the relative contribution of VIX derivatives to the variance discovery as

$$PDR = \frac{\gamma_{v_1}}{\gamma_{v_1} - \gamma_{v_2}}, \quad (23)$$

where *PDR* stands for the parametric variance discovery ratio. Following this formula, *PDR* is 73.2% in the full sample, 71.5% in the pre-crisis sample, and 66.7% in the post-crisis sample. These numbers suggest that information about future variance is largely revealed in VIX derivatives rather than in SPX options, consistent with the results of the preceding nonparametric analysis.

Recall that the SV4-EC model allows for error corrections between the two short-run variance factors. I also estimate an alternative error correction specification in which error corrections occur between the two long-run variance factors. Although the estimation result is not reported in this paper, I demonstrate that deviations between the two long-run variance factors are resolved mainly in the SPX option prices rather than in the VIX derivative prices, further confirming the previous results.

5.2 Introducing a measure of informed trading

The stock option literature has found that the unsigned volume ratio of options to stocks is associated with informed trading with respect to stock prices (Roll, Schwartz, and Subrahmanyam, 2010; Johnson and So, 2012; and Ge, Lin, and Pearson, 2016). Specifically, short sale constraints on stocks drive greater informed trading in options relative to stocks especially during economic downturns. As a result, there is a well-documented negative relationship between option-to-stock volume ratios and future stock returns.

Drawing from the existing literature, I conjecture that the volume ratio of VIX futures to SPX options, or F/O , may be a good proxy for informed trading with respect to future variance. Option writers impose greater counterparty risk on central clearing houses than option buyers, and therefore, the former are required to post far greater margins than the latter. In my sample period, margins on option sales are, on average, at least 10 times larger than those on option purchases (see Appendix F for further details on the asymmetric option margins). As a result, when expecting a lower future variance, informed traders may find it easier to sell VIX futures instead of delta-neutral SPX options. That said, a higher F/O may be associated with a lower future variance.

The top panel of Figure 7 shows the time series plot of the logarithm of F/O . It is apparent that F/O has an upward trend in the sample period. This secular trend can be, in part, attributed to the fact that VIX futures have become a popular hedge against stock market downturns and macroeconomic uncertainty since the 2008 financial crisis. The growing interest in variance trading has further motivated the introduction of VIX-linked ETPs, which are designed to attract retail investors.¹⁶ The sponsors and issuers of the VIX-linked ETPs mostly use VIX futures to hedge the variance risks that they have taken in their products. The dotted vertical line in the figure refers to January 29, 2009 when the first, and most actively traded, VIX-linked ETP (ticker: VXX) was introduced. A stronger upward trend is observed after this issuance.

These market developments indicate that the secular trend in F/O is unlikely to be associated with informed trading. To define a measure of informed trading,

¹⁶ See Whaley (2013) and Alexander and Korovilas (2013) for the introduction to the VIX-linked ETPs.

I need to remove the secular trend in F/O to isolate the time variation pertaining to informed trading. This task is done by running a linear time trend regression as $\log(F/O_t) = a + bt + \varepsilon_t$. I then take the standardized fitted residuals as a measure of informed trading (IT). The IT measure is depicted in the bottom panel of Figure 7. Unlike F/O , the IT measure appears to be stationary, while having substantial fluctuations over time.

5.3 Effect of informed trading on variance disparity

Arrivals of low-variance information may strengthen informed trading in VIX futures relative to SPX options, while causing more negative bases between the VIV (which is more informative) and the SIV (which is less informative). I thus hypothesize a negative relationship between the informed trading measure and the bases.

Panel A of Table 7 shows the univariate regression results of the bases onto the IT measure. Consistent with my expectation, the coefficients on the IT measure are negative in both the full and ex-crisis samples, with the statistical significance at the 1% level. Panel B of Table 7 shows the multivariate regression results of the bases onto the IT measure, the liquidity measures, and the holding cost proxies. The coefficient and statistical significance of the IT measure are not materially affected by the inclusion of the liquidity and holding cost proxies.

In an unreported table, I show that the results highlighted in this paper are robust to other detrending methods, such as a quadratic time trend and a partial linear time trend, which permits a linear time trend only after the introduction of the first VIX-linked ETP. To summarize, informed trading regarding future variance is a contributor to variance disparity in addition to liquidity and holding cost.

6 Conclusion

Derivative markets provide rich ground for understanding the market's perceptions of future market variance or economic uncertainty. While it is well established that a measure of expected market variance can be inferred from SPX options, I introduce an alternative model-free approach to measuring the market's expectation of future variance from VIX derivatives. VIX derivative prices are ultimately derived from SPX

option prices, so the former should convey similar information about future variance to the latter if the two derivatives markets are well integrated.

However, this paper documents significant gaps between two variance measures implied by S&P 500 (SPX) options and VIX derivatives, and attributes these gaps to illiquidity and asymmetric information. Specifically, as funding liquidity and market liquidity deteriorate, VIX derivatives tend to imply higher levels of market variance than SPX options. Moreover, informed trading about future variance is more active in VIX derivatives than in SPX options, causing further gaps. While frictions have deterred the integration of variance derivatives markets, several variance discovery analyses points to the superior information role of VIX derivatives relative to SPX options. This result is even true before the 2008 financial crisis when they were thinly traded.

My findings have two implications for investors and policymakers. First, market variance determines asset returns as well as corporate investment and hiring decisions. Investors and policymakers also keep track of variance risk premiums because they are informative of investors' risk aversion or sentiment. Given the importance of market variance or economic uncertainty, my analyses suggest that VIX derivatives would offer a more reliable estimate of expected market variance than SPX options especially in crisis periods when frictions are severe.

Second, policymakers have long been interested in measuring the severity of frictions because they can cause and exacerbate fire sales and market freeze during crisis periods. For example, Pasquariello (2014) argues that commonality of no-arbitrage violations across multiple markets can be useful for monitoring the stability of the financial system. Given the empirical link between variance disparity and frictions, a measure of variance disparity can serve as an indicator of the severity of frictions in financial markets.

References

- AÏT-SAHALIA, Y., D. AMENGUAL, AND E. MANRESA (2015): “Market-based estimation of stochastic volatility models,” *Journal of Econometrics*, 187(2), 418–435.
- AÏT-SAHALIA, Y., M. KARAMAN, AND L. MANCINI (2018): “The term structure of variance swaps and risk premia,” Working paper, Princeton University.
- AÏT-SAHALIA, Y., AND R. KIMMEL (2007): “Maximum likelihood estimation of stochastic volatility models,” *Journal of Financial Economics*, 83, 413–452.
- AÏT-SAHALIA, Y., AND A. W. LO (1998): “Nonparametric estimation of state-price densities implicit in financial asset prices,” *Journal of Finance*, 53, 499–547.
- (2000): “Nonparametric risk management and implied risk aversion,” *Journal of Econometrics*, 94, 9–51.
- ALEXANDER, C., AND D. KOROVILAS (2013): “Volatility exchange-traded notes: Curse or cure?,” *Journal of Alternative Investments*, 16(2), 52–70.
- AMENGUAL, D., AND D. XIU (2018): “Resolution of policy uncertainty and sudden declines in volatility,” *Journal of Econometrics*, 203(2), 297–315.
- ANAND, A., AND S. CHAKRAVARTY (2007): “Stealth trading in options markets,” *Journal of Financial and Quantitative Analysis*, 42(1), 167–187.
- ANDERSEN, T. G., O. BONDARENKO, AND M. T. GONZALEZ-PEREZ (2015): “Exploring return dynamics via corridor implied volatility,” *Review of Financial Studies*, 28(10), 2902–2945.
- BABA, N., AND F. PACKER (2009): “From turmoil to crisis: Dislocations in the FX swap market before and after the failure of Lehman Brothers,” *Journal of International Money and Finance*, 28(8), 1350–1374.
- BABA, Y., R. ENGLE, D. KRAFT, AND K. KRONER (1989): “Multivariate simultaneous generalized ARCH,” Working paper, University of California at San Diego.
- BAI, J., AND P. COLLIN-DUFRESNE (2013): “The CDS-bond basis,” Working paper, Georgetown University and Swiss Finance Institute.

- BANSAL, R., AND A. YARON (2005): “Risks for the long run: A potential resolution of asset pricing puzzles,” *Journal of Finance*, 59(4), 1481–1509.
- BARDGETT, C., E. GOURIER, AND M. LEIPPOLD (2013): “Inferring volatility dynamics and risk premia from the S&P 500 and VIX markets,” Working paper, Swiss Finance Institute.
- BARRAS, L., AND A. MALKHOZOV (2016): “Does variance risk have two prices? Evidence from the equity and option markets,” *Journal of Financial Economics*, 121(1), 79–92.
- BEKAERT, G., AND M. HOEROVA (2014): “The VIX, the variance premium and stock market volatility,” *Journal of Econometrics*, 193, 181–192.
- BESSEMBINDER, H. (1992): “Systematic risk, hedging pressure, and risk premiums in futures markets,” *Review of Financial Studies*, 5(4), 637–667.
- BLACK, F. (1975): “Fact and fantasy in the use of options,” *Financial Analysts Journal*, 31(4), 36–41.
- BLANCO, R., S. BRENNAN, AND I. W. MARSH (2005): “An empirical analysis of the dynamic relation between investment-grade bonds and credit default swaps,” *Journal of Finance*, 60(5), 2255–2281.
- BLOOM, N. (2009): “The impact of uncertainty shocks,” *Econometrica*, 77(3), 623–685.
- BOLLEN, N., AND R. WHALEY (2004): “Does net buying pressure affect the shape of implied volatility functions?,” *Journal of Finance*, 59, 711–753.
- BOYSON, N. M., C. W. STAHEL, AND R. M. STULZ (2010): “Hedge fund contagion and liquidity shocks,” *Journal of Finance*, 65(5), 1789–1816.
- BRITTEN-JONES, M., AND A. NEUBERGER (2000): “Option prices, implied price processes, and stochastic volatility,” *Journal of Finance*, 55, 839–866.
- BRUNNERMEIER, M. K., AND L. H. PEDERSEN (2009): “Market liquidity and funding liquidity,” *Review of Financial Studies*, 22(6), 2201–2238.

- CARR, P., AND D. MADAN (1998): “Towards a theory of volatility trading,” in *Volatility: New Estimation Techniques for Pricing Derivatives*. RISK Publications.
- (2001): “Optimal positioning in derivative securities,” *Quantitative Finance*, 1, 19–37.
- CARR, P., AND L. WU (2007): “Stochastic skew in currency options,” *Journal of Financial Economics*, 86, 213–247.
- (2009): “Variance risk premiums,” *Review of Financial Studies*, 22, 1311–1341.
- CHAKRAVARTY, S., H. GULEN, AND S. MAYHEW (2004): “Informed trading in stock and option markets,” *Journal of Finance*, 59(3), 1235–1257.
- CHENG, I.-H. (2018): “The VIX Premium,” *Forthcoming in Review of Financial Studies*.
- CHORDIA, T., R. ROLL, AND A. SUBRAHMANYAM (2008): “Liquidity and market efficiency,” *Journal of Financial Economics*, 87(2), 249–268.
- CHRISTOFFERSEN, P., C. DORION, K. JACOBS, AND L. KAROUI (2014): “Non-linear Kalman filtering in affine term structure models,” *Management Science*, 60, 2248–2268.
- CHRISTOFFERSEN, P., S. HESTON, AND K. JACOBS (2009): “The shape and term structure of the index option smirk: Why multifactor stochastic volatility models work so well,” *Management Science*, 55, 1914–1932.
- COFFEY, N., W. B. HRUNG, AND A. SARKAR (2009): “Capital constraints, counterparty risk, and deviations from covered interest rate parity,” Staff report, Federal Reserve Bank of New York.
- COMERTON-FORDE, C., T. HENDERSHOTT, C. M. JONES, P. C. MOULTON, AND M. S. SEASHOLES (2010): “Time variation in liquidity: The role of market-maker inventories and revenues,” *Journal of Finance*, 65(1), 295–331.
- DE ROON, F. A., T. E. NIJMAN, AND C. VELD (2000): “Hedging pressure effects in futures markets,” *Journal of Finance*, 55(3), 1437–1456.

- DEMETERFI, K., E. DERMAN, M. KAMAL, AND J. ZOU (1999): “More than you ever wanted to know about volatility swaps,” Research Notes, Goldman Sachs Quantitative Strategies.
- DEVILLE, L., AND F. RIVA (2007): “Liquidity and arbitrage in options markets: A survival analysis approach,” *Review of Finance*, 11(3), 497–525.
- DURHAM, G. B. (2013): “Risk-neutral modelling with affine and nonaffine models,” *Journal of Financial Econometrics*, 11, 650–681.
- EASLEY, D., M. O’HARA, AND P. S. SRINIVAS (1998): “Option volume and stock prices: Evidence on where informed traders trade,” *Journal of Finance*, 53(2), 431–465.
- FILIPOVIĆ, D., E. GOURIER, AND L. MANCINI (2016): “Quadratic variance swap models,” *Journal of Financial Economics*, 119(1), 44–68.
- FLECKENSTEIN, M., F. A. LONGSTAFF, AND H. LUSTIG (2014): “The TIPS-Treasury bond puzzle,” *Journal of Finance*, 69(5), 2151–2197.
- FONG, W.-M., G. VALENTE, AND J. K. FUNG (2010): “Covered interest arbitrage profits: The role of liquidity and credit risk,” *Journal of Banking and Finance*, 34(5), 1098–1107.
- FOURNIER, M., AND K. JACOBS (2016): “Inventory risk, market-maker wealth, and the variance risk premium: Theory and evidence,” Working paper, HEC Montreal and the University of Houston.
- GAGNON, L., AND G. A. KAROLYI (2010): “Multi-market trading and arbitrage,” *Journal of Financial Economics*, 97(1), 53–80.
- GALLANT, A. R., C. HSU, AND G. TAUCHEN (1999): “Using daily range data to calibrate volatility diffusions and extract the forward integrated variance,” *Review of Economics and Statistics*, 81, 617–631.
- GÂRLEANU, N., AND L. PEDERSEN (2011): “Margin-based asset pricing and deviations from the law of one price,” *Review of Financial Studies*, 24(6), 1980–2022.
- GÂRLEANU, N., L. PEDERSEN, AND A. POTESHMAN (2009): “Demand-based option pricing,” *Review of Financial Studies*, 22(10), 4259–4299.

- GE, L., T.-C. LIN, AND N. D. PEARSON (2016): “Why does the option to stock volume ratio predict stock returns?,” *Journal of Financial Economics*, 120(3), 601–622.
- GONZALO, J., AND C. GRANGER (1995): “Estimation of common long-memory components in cointegrated systems,” *Journal of Business and Economic Statistics*, 13(1), 27–35.
- HAMEED, A., W. KANG, AND S. VISWANATHAN (2010): “Stock market declines and liquidity,” *Journal of Finance*, 65(1), 257–293.
- HASBROUCK, J. (1995): “One security, many markets: Determining the contributions to price discovery,” *Journal of Finance*, 50(4), 1175–1199.
- HASBROUCK, J. (2003): “Intraday price formation in U.S. equity index markets,” *Journal of Finance*, 58(6), 2375–2400.
- HIRSHLEIFER, D. (1989): “Determinants of hedging and risk premia in commodity futures markets,” *Journal of Financial and Quantitative Analysis*, 24(3), 313–331.
- (1990): “Hedging pressure and futures price movements in a general equilibrium model,” *Econometrica*, 58(2), 411–428.
- HOLOWCZAK, R., Y. E. SIMAAN, AND L. WU (2006): “Price discovery in the US stock and stock options markets: A portfolio approach,” *Review of Derivatives Research*, 9(1), 37–65.
- JIANG, G., AND Y. TIAN (2005): “The model-free implied volatility and its information content,” *Review of Financial Studies*, 18, 1305–1342.
- (2007): “Extracting model-free volatility from option prices: An examination of the VIX index,” *Journal of Derivatives*, 14, 35–60.
- JOHANSEN, S. (1988): “Statistical analysis of cointegration vectors,” *Journal of Economic Dynamics and Control*, 12(2), 231–254.
- (1991): “Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models,” *Econometrica*, 59(6), 1551–1580.

- JOHNSON, T. L., AND E. C. SO (2012): “The option to stock volume ratio and future returns,” *Journal of Financial Economics*, 106(2), 262–286.
- JONES, C. (2003): “The dynamics of stochastic volatility: Evidence from underlying and options markets,” *Journal of Econometrics*, 116, 181–224.
- KAROLYI, G. A. (1995): “A multivariate GARCH model of international transmissions of stock returns and volatility: The case of the United States and Canada,” *Journal of Business and Economic Statistics*, 13(1), 11–25.
- KAROLYI, G. A., K.-H. LEE, AND M. A. VAN DIJK (2012): “Understanding commonality in liquidity around the world,” *Journal of Financial Economics*, 105(1), 82–112.
- KEARNEY, C., AND A. J. PATTON (2000): “Multivariate GARCH modeling of exchange rate volatility transmission in the European monetary system,” *Financial Review*, 35(1), 29–48.
- KWIATKOWSKI, D., P. C. PHILLIPS, P. SCHMIDT, AND Y. SHIN (1992): “Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?,” *Journal of Econometrics*, 54, 159–178.
- MALIK, F., AND B. T. EWING (2009): “Volatility transmission between oil prices and equity sector returns,” *International Review of Financial Analysis*, 18(3), 95–100.
- MANCINI GRIFFOLI, T., AND A. RANALDO (2012): “Limits to arbitrage during the crisis: Funding liquidity constraints and covered interest parity,” Working paper, University of St. Gallen.
- MENCÍA, J., AND E. SENTANA (2012): “Valuation of VIX derivatives,” *Journal of Financial Economics*, 108, 367–391.
- MIXON, S., AND E. ONUR (2015): “Volatility derivatives in practice: Activity and impact,” Staff report, Commodity Futures Trading Commission.
- MURAVYEV, D., N. D. PEARSON, AND J. P. BROUSSARD (2013): “Is there price discovery in equity options?,” *Journal of Financial Economics*, 107(2), 259–283.

- NEWKEY, W. K., AND K. D. WEST (1987): “A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix,” *Econometrica*, 55, 703–708.
- OEHMKE, M. (2011): “Gradual arbitrage,” Working paper, Columbia Business School.
- O’HARA, M. (2003): “Presidential address: Liquidity and price discovery,” *Journal of Finance*, 58(4), 1335–1354.
- PAN, J., AND A. M. POTESHMAN (2006): “The information in option volume for future stock prices,” *Review of Financial Studies*, 19(3), 871–908.
- PASQUARIELLO, P. (2014): “Financial market dislocations,” *Review of Financial Studies*, 27(6), 1868–1914.
- PERRON, P. (1989): “The great crash, the oil price shock, and the unit root hypothesis,” *Econometrica*, 57(6), 1361–1401.
- PONTIFF, J. (2006): “Costly arbitrage and the myth of idiosyncratic risk,” *Journal of Financial Economics*, 42(1), 35–52.
- ROLL, R., E. SCHWARTZ, AND A. SUBRAHMANYAM (2007): “Liquidity and the law of one price: The case of the futures-cash basis,” *Journal of Finance*, 62(5), 2201–2234.
- (2010): “O/S: The relative trading activity in options and stock,” *Journal of Financial Economics*, 96(1), 1–17.
- TROLLE, A. B., AND E. S. SCHWARTZ (2009a): “A general stochastic volatility model for the pricing of interest rate derivatives,” *Review of Financial Studies*, 22(5), 2007–2057.
- (2009b): “Unspanned stochastic volatility and the pricing of commodity derivatives,” *Review of Financial Studies*, 22(11), 4423–4461.
- WHALEY, R. E. (2013): “Trading volatility: At what cost?,” *Journal of Portfolio Management*, 40(1), 95–108.

Appendix A Proof for Proposition 1

Let me start with the basic mathematical result (Carr and Madan, 2001):

$$f(S) = f(S_0) + f'(S_0)(S - S_0) + \int_{S_0}^{\infty} f''(K) \max(S - K, 0) dK + \int_0^{S_0} f''(K) \max(K - S, 0) dK, \quad (\text{A.1})$$

where $f(S)$ is a twice differentiable function. Assuming $f(S) = S^2$ and replacing S and S_0 with VIX_T and $VF_t(T)$, respectively, I obtain

$$\begin{aligned} VIX_T^2 &= VF_t(T)^2 + 2VF_t(T)(VIX_T - VF_t(T)) \\ &+ 2 \int_{VF_t(T)}^{\infty} \max(VIX_T - K, 0) dK + 2 \int_0^{VF_t(T)} \max(K - VIX_T, 0) dK. \end{aligned} \quad (\text{A.2})$$

Taking the risk-neutral expectation on Equation (A.2) leads to

$$\begin{aligned} E_t^{\mathbb{Q}}[VIX_T^2] &= VF_t(T)^2 + 2VF_t(T)(E_t^{\mathbb{Q}}(VIX_T) - VF_t(T)) \\ &+ 2 \int_{VF_t(T)}^{\infty} E_t^{\mathbb{Q}}[\max(VIX_T - K, 0)] dK + 2 \int_0^{VF_t(T)} E_t^{\mathbb{Q}}[\max(K - VIX_T, 0)] dK. \end{aligned} \quad (\text{A.3})$$

Substituting the relations,

$$\begin{aligned} VF_t(T) &= E_t^{\mathbb{Q}}(VIX_T), \\ VC_t(T, K) &= \exp(-r_t(T-t)) E_t^{\mathbb{Q}}[\max(VIX_T - K, 0)], \text{ and} \\ VP_t(T, K) &= \exp(-r_t(T-t)) E_t^{\mathbb{Q}}[\max(K - VIX_T, 0)], \end{aligned}$$

into Equation (A.3), I obtain

$$\begin{aligned} \text{var}_t(VIX_T) &= E_t^{\mathbb{Q}}[VIX_T^2] - VF_t(T)^2 \\ &= 2 \exp(r_t(T-t)) \left[\int_{VF_t(T)}^{\infty} VC_t(T, K) dK + \int_0^{VF_t(T)} VP_t(T, K) dK \right]. \end{aligned} \quad (\text{A.4})$$

This equation means that the variance of VIX_T can be replicated by an equally weighted continuum of the OTM VIX options. Q.E.D.

Appendix B Moment-generating functions

Appendix B.1 Market integration (SV2) model

Given the model specified in Equation (10), the moment-generating function of X_t evaluated at $\phi = (1, 1)'$ takes an exponentially affine form:

$$M(X_t, t, T; \phi = (1, 1)') = \exp(\alpha(s) + \alpha_{v_1}(s)v_{1t} + \alpha_{u_1}(s)u_{1t}), \quad (\text{B.1})$$

where $s = T - t$ and the coefficients are given by

$$\begin{aligned} \alpha(s) &= \frac{\sigma_{v_1}^2}{4\kappa_{v_1}} (1 - \exp(-2\kappa_{v_1}s)) + \frac{\sigma_{u_1}^2}{4\kappa_{u_1}} (1 - \exp(-2\kappa_{u_1}s)) \\ \alpha_{v_1}(s) &= \exp(-\kappa_{v_1}s) \\ \alpha_{u_1}(s) &= \exp(-\kappa_{u_1}s). \end{aligned} \quad (\text{B.2})$$

Appendix B.2 Market segmentation with error corrections (SV4-EC) model

Given the model specified in Equation (15), the moment-generating function of X_t takes an exponentially affine form as follows:

$$M(X_t, t, T; \phi) = \exp(\alpha(s) + \alpha_{v_1}(s)v_{1t} + \alpha_{u_1}(s)u_{1t} + \alpha_{v_2}(s)v_{2t} + \alpha_{u_2}(s)u_{2t}), \quad (\text{B.3})$$

where $s = T - t$ and the coefficients, $\alpha(s)$, $\alpha_{v_1}(s)$, $\alpha_{u_1}(s)$, $\alpha_{v_2}(s)$, and $\alpha_{u_2}(s)$, satisfy the system of ordinary differential equations as follows:

$$\begin{aligned} \dot{\alpha}(s) &= \frac{1}{2}(\sigma_{v_1}\alpha_{v_1}(s))^2 + \frac{1}{2}(\sigma_{v_2}\alpha_{v_2}(s))^2 + \frac{1}{2}(\sigma_{u_1}\alpha_{u_1}(s))^2 + \frac{1}{2}(\sigma_{u_2}\alpha_{u_2}(s))^2 \\ \dot{\alpha}_{v_1}(s) &= -(\kappa_{v_1} - \gamma_{v_1})\alpha_{v_1}(s) + \gamma_{v_2}\alpha_{v_2}(s) \\ \dot{\alpha}_{u_1}(s) &= -\kappa_{u_1}\alpha_{u_1}(s) \\ \dot{\alpha}_{v_2}(s) &= -(\kappa_{v_2} + \gamma_{v_2})\alpha_{v_2}(s) - \gamma_{v_1}\alpha_{v_1}(s) \\ \dot{\alpha}_{u_2}(s) &= -\kappa_{u_2}\alpha_{u_2}(s), \end{aligned} \quad (\text{B.4})$$

where the boundary conditions are given as $\alpha(0) = 0$ and $[\alpha_{v_1}(0), \alpha_{u_1}(0), \alpha_{v_2}(0), \alpha_{u_2}(0)] = \phi'$.

Appendix C Unscented Kalman filtering

Unscented Kalman filtering requires us to recast the model under study into a state space form that comprises observation equations and state equations. With respect to observation equations, I assume that the logarithmic SIV and VIV observations have constant measurement errors as follows:

$$\begin{aligned}\log(SIV_t(T)) &= \log(SIV_t(T; \theta)) + \sigma_{e_1} \xi_{1,t} \\ \log(VIV_t(T)) &= \log(VIV_t(T; \theta)) + \sigma_{e_2} \xi_{2,t},\end{aligned}\tag{C.1}$$

where σ_{e_1} and σ_{e_2} capture the size of measurement errors and $\xi_{1,t}$ and $\xi_{2,t}$ are independent standard normal random variables.

Let me specify the physical dynamics of the state vector in the SV4-EC model as

$$\begin{aligned}dv_{1t} &= -\kappa_{v_1} v_{1t} dt + (\zeta_{v_1} + \eta_{v_1} v_{1t}) dt + \gamma_{v_1} (v_{1t} - v_{2t}) dt + \sigma_{v_1} dB_{1t}^{\mathbb{P}} \\ du_{1t} &= -\kappa_{u_1} u_{1t} dt + \sigma_{u_1} dB_{2t}^{\mathbb{P}} \\ dv_{2t} &= -\kappa_{v_2} v_{2t} dt + (\zeta_{v_2} + \eta_{v_2} v_{2t}) dt + \gamma_{v_2} (v_{1t} - v_{2t}) dt + \sigma_{v_2} dB_{3t}^{\mathbb{P}} \\ du_{2t} &= -\kappa_{u_2} u_{2t} dt + \sigma_{u_2} dB_{4t}^{\mathbb{P}},\end{aligned}\tag{C.2}$$

where the two terms, $(\zeta_{v_1} + \eta_{v_1} v_{1t})$ and $(\zeta_{v_2} + \eta_{v_2} v_{2t})$, capture the variance risk premiums in v_{1t} and v_{2t} , respectively.¹⁷ Note that zero variance risk premiums are assumed in the two long-run variance factors because they are estimated to be insignificant. By applying an Euler approximation to Equation (C.2), I can define discrete-time state equations, which will be used when I update the state vector. Trolle and Schwartz (2009a,b) and Christoffersen, Dorion, Jacobs, and Karoui (2014) provide a detailed procedure for unscented Kalman filtering.

Appendix D Data

The data set comprises the daily prices of SPX options, VIX futures, and VIX options. The SPX and VIX options data come from OptionMetrics and the futures data are obtained from Thomson Reuters Datastream. The SPX options market closes 15 minutes after the closing of the SPX cash market. To address the nonsynchronous trading hours, I back out the SPX spot price for each of the first three pairs of at-the-

¹⁷ The physical dynamics of the state vector are similarly defined in the SV2 and SV4 models.

money SPX put and call options by using the put-call parity and take an average of the three extracted spot prices. This idea originates from Aït-Sahalia and Lo (1998, 2000).

The sample period is restricted by the short history of VIX futures and options. The VIX futures market started on March 26, 2004, and the VIX options market opened about two years later on February 24, 2006. Because the trading of VIX options was illiquid in the very beginning, my sample period spans from July 1, 2006 to August 31, 2014.

The options market has experienced vibrant changes in the trading environment. The Chicago Board of Options Exchange (CBOE) has recently introduced weekly and end-of-month options, but they are not included in my analysis because they were not accounted for in the computation of the VIX index until October 6, 2014.¹⁸ The CBOE introduced overnight trading hours for SPX options and VIX derivatives, but only regular trading hours are considered in my analysis.¹⁹ Option prices are taken as the bid–ask midpoint at the close of regular trading hours: 3:15 p.m.

To eliminate inaccurate or illiquid options, various data filters are applied. Specifically, both SPX and VIX options are deleted when the mid price is less than five cents, the Black-Scholes implied volatility is empty in OptionMetrics, the deviation between the best bid and offer prices is larger than five dollars, or the lower bound constraint is violated. I also exclude SPX options with fewer than eight days or more than one year to maturity, as well as VIX options with fewer than eight days or more than 11 months to maturity.²⁰ The selection criteria yield a sample of 9,813 SIV observations and 8,079 VIV observations.

¹⁸ Including weekly options would put too much weight on fitting the short end of the term structure of variance.

¹⁹ The trading hours were extended on March 9, 2015 for SPX options; June 23, 2014 for VIX futures; and March 2, 2015 for VIX options.

²⁰ Note that VIX options with 11 months to maturity cover the horizon between 11 months and 12 months. Thus, the term structure horizon covered by the VIV data matches that covered by the SIV data.

Appendix E Unit root and co-integration tests

Here I test whether the replicated and original VIV measures are stationary or integrated by implementing the KPSS (Kwiatkowski, Phillips, Schmidt, and Shin, 1992) and ADF (Augmented Dickey Fuller) tests. Table A1 reports the test results for the logarithm of the replicated and original VIV measures, with an optimal lag chosen using the Akaike information criteria. The null hypothesis of the KPSS test is that a time series is stationary against the existence of a unit root. The KPSS test rejects the null hypothesis of stationarity at the 1% level for both the pre-crisis and post-crisis periods, regardless of the maturities considered, and the ADF test fails to reject the null hypothesis that the time series has a unit root. The two test results point to the existence of a unit root in the replicated and original VIV measures.

I next run the Johansen (1988, 1991) co-integration test. The Johansen maximum eigenvalue test assesses the null hypothesis that the number of co-integrating relations, the rank of $\alpha\beta'$, is equal to r against $r + 1$. As shown in Table A1, we can reject the null of $r = 0$ but cannot reject the null of $r = 1$ for every maturity in both the pre-crisis and post-crisis periods at the 1% level. Thus, the replicated and original VIV measures appear to have a single co-integrating relation in both the pre-crisis and post-crisis periods.

Appendix F Margins

A margin is one of the most important tools to protect a central clearing house from the counterparty risk arising from derivative transactions.²¹ There are two types of margins: initial margins and maintenance margins. Initial margins are collected to cover the potential future loss that may arise in the event of a counterparty's default. Maintenance margins, however, are the smallest amount of the margin that should be maintained throughout the life of a margin account. If the balance of a margin account drops below the maintenance margin, the investor will be asked to post an additional margin to bring the balance back to the initial margin. Margins thus play a crucial role in guaranteeing contractual obligations by ensuring that both realized

²¹ CBOE options and CFE (CBOE futures exchange) futures are centrally cleared by the Options Clearing Corporation.

and future losses will be covered.

VIX future margins are set in terms of dollars (not a percentage) and differ by the type of margin account.²² For example, the initial margin for a customer's speculative account was set at 3,300 dollars on June 19, 2014. This dollar amount can be translated into a relative initial margin of 31% relative to the day's VIX index after the contract size of 1,000 is accounted for. For my sample period, relative initial margins (dollar margins divided by the VIX index) range from 7.2% to 51.8%, with a mean of 24.7%.²³ Note that margins are symmetric between long and short positions in VIX futures.

In contrast to VIX futures margins, SPX options margins differ between long and short positions. For an option buyer, the initial margin is equivalent to 100% of the option price if the maturity is less than nine months or 75% otherwise. This simple margin calculation is because an option buyer is not subject to any contractual obligation as long as the option price is paid in full. In contrast, the initial margin calculation is rather complicated for an option writer. Specifically, the initial margin for a call writer is defined as whichever is the larger of (i) the call price plus 15% of the underlying index level minus the OTM amount or (ii) the call price plus 10% of the underlying index level. Here, the term, *15% of the underlying index level*, is associated with a stress scenario in the underlying index price, assumed by the central clearing house. Similarly, the initial margin for a put writer is defined as whichever is the larger of (i) the put price plus 15% of the underlying index level minus the OTM amount or (ii) the put price plus 10% of the strike price.

Let me illustrate the calculation of a margin call. On August 29, 2014, one of the most actively traded SPX calls, with a strike price of 2,040 and a maturity of less than nine months, had a closing price of 6.9 dollars. On that day, the SPX index closed at about 2,003. The initial margin for a buyer is the call price, 6.9 dollars. In this case, the initial margin equals the margin call. In contrast, the initial margin for

²² VIX futures margin data are available at <http://cfe.cboe.com/margins/cfe-margins>.

²³ Starting on June 27, 2013, margins were set differently depending on the maturity of VIX futures. In general, margins are higher for short-term contracts than long-term contracts because the former are more volatile than the latter. When computing the relative margins, I use only the largest margin for that day.

a writer is calculated as

$$\max(\underbrace{6.9}_{\text{call price}} + \underbrace{0.15 \times 2,003}_{\text{stress scenario}} - \underbrace{(2,040 - 2,003)}_{\text{OTM amount}}, \underbrace{6.9 + 0.10 \times 2,003}_{\text{minimum margin}}) = 207.2 \text{ dollars.}$$

The margin call is then $207.2 - 6.9 = 200.3$ dollars as the option sale price can be applied to the margin account. Thus, the sell-side margin call is about 29 times larger than the buy-side margin call.

I calculate the sellers' margin call for every SPX option in my sample and find that sell-side margin calls are, on average, 145.5 dollars in my sample, while option prices are, on average, 12.3 dollars.²⁴ Note that the average sell-side margin call is about one order of magnitude larger than the average buy-side margin call. This asymmetric margin structure arises because option sellers impose far greater counterparty risk on a central clearing house than option buyers.

²⁴ Here I account for only the OTM SPX options that are used to compute the SIV measures.

Table 1: **Summary statistics of convexity ratios**

This table presents the summary statistics for the convexity ratio, $CVRT_t(T)$, which is defined as the ratio of the convexity adjustment to the VIV measure as follows:

$$CVRT_t(T) = \frac{\text{var}_t(VIX_T)}{VIV_t(T)},$$

where the numerator and the denominator are defined in Equations (6) and (7), respectively. The short and long terms are defined as less than three months and more than three months, respectively.

Maturity	1%	5%	Median	95%	99%	Mean	Std.	Skew.	Kurt.
Short term	0.020	0.033	0.09	0.16	0.18	0.09	0.038	0.11	2.33
Long term	0.082	0.097	0.16	0.20	0.22	0.16	0.032	-0.31	2.83
Total	0.023	0.044	0.13	0.19	0.21	0.13	0.047	-0.25	2.30

Table 2: **Parameter estimates and information criteria**

This table shows the parameter estimates and the information criteria. Panels A, B, and C correspond to the full sample period (July 1, 2006 to August 31, 2014), the pre-crisis period (July 1, 2006 to August 31, 2008), and the post-crisis period (December 1, 2008 to August 31, 2014), respectively. Standard errors are in parentheses. AIC and SIC stand for Akaike information criteria and Schwarz information criteria, respectively.

	Panel A: Full sample period			Panel B: Pre-crisis period			Panel C: Post-crisis period		
	SV2	SV4	SV4-EC	SV2	SV4	SV4-EC	SV2	SV4	SV4-EC
μ_1	-3.33 (0.02)	-3.44 (0.03)	-4.26 (0.05)	-3.20 (0.03)	-3.75 (0.05)	-5.04 (0.43)	-2.99 (0.02)	-2.95 (0.02)	-3.20 (0.02)
μ_2		-3.17 (0.03)	-2.98 (0.03)		-3.15 (0.05)	-2.57 (0.11)		-2.74 (0.02)	-2.79 (0.02)
κ_{v_1}	4.48 (0.02)	5.89 (0.05)	4.29 (0.21)	5.39 (0.05)	9.78 (0.17)	8.16 (0.46)	4.97 (0.03)	6.34 (0.08)	5.08 (0.28)
κ_{v_2}		4.45 (0.02)	4.16 (0.08)		5.31 (0.06)	3.84 (0.17)		5.08 (0.05)	3.73 (0.15)
σ_{v_1}	1.75 (0.02)	2.29 (0.04)	4.77 (0.11)	2.23 (0.05)	2.57 (0.08)	5.51 (0.24)	1.68 (0.03)	2.24 (0.05)	4.68 (0.13)
σ_{v_2}		1.75 (0.04)	1.80 (0.04)		1.78 (0.09)	2.10 (0.12)		1.81 (0.05)	1.75 (0.06)
ζ_{v_1}	-2.41 (0.74)	-1.04 (1.22)	7.53 (2.08)	-1.63 (2.19)	-2.77 (2.58)	1.48 (4.70)	-5.47 (1.08)	-4.89 (1.43)	0.33 (2.40)
ζ_{v_2}		-2.08 (0.98)	-3.51 (0.95)		-1.51 (2.29)	-0.42 (2.08)		-4.26 (1.74)	-1.57 (1.32)
η_{v_1}	-0.26 (0.92)	-1.22 (0.67)	10.05 (1.62)	-6.96 (5.68)	-18.55 (5.03)	-1.70 (8.04)	-3.35 (1.46)	-10.12 (2.12)	-4.75 (3.09)
η_{v_2}		-0.21 (0.97)	1.61 (1.15)		-2.39 (4.01)	2.70 (4.43)		-3.19 (2.19)	5.47 (1.78)
κ_{u_1}	0.39 (0.01)	0.52 (0.01)	0.25 (0.01)	0.29 (0.01)	0.53 (0.02)	0.07 (0.02)	0.74 (0.01)	0.80 (0.01)	0.60 (0.01)
κ_{u_2}		0.24 (0.01)	0.23 (0.01)		0.21 (0.01)	0.14 (0.01)		0.62 (0.02)	0.54 (0.02)
σ_{u_1}	0.62 (0.01)	1.05 (0.01)	0.73 (0.01)	0.53 (0.02)	0.89 (0.04)	0.55 (0.02)	0.77 (0.02)	0.94 (0.02)	0.61 (0.01)
σ_{u_2}		0.58 (0.01)	0.48 (0.01)		0.55 (0.02)	0.41 (0.02)		0.73 (0.02)	0.52 (0.01)
γ_{v_1}			-34.60 (1.42)			-32.47 (2.01)			-30.94 (1.70)
γ_{v_2}			12.67 (0.34)			12.96 (0.75)			15.48 (0.60)
σ_{e_1}	0.105 (0.000)	0.060 (0.000)	0.054 (0.000)	0.090 (0.001)	0.035 (0.000)	0.029 (0.000)	0.083 (0.000)	0.050 (0.000)	0.044 (0.000)
σ_{e_2}	0.031 (0.000)	0.029 (0.000)	0.029 (0.000)	0.025 (0.000)	0.022 (0.000)	0.024 (0.000)	0.032 (0.000)	0.028 (0.000)	0.028 (0.000)
log(L)	30,277	34,946	36,746	8,506	10,531	11,063	23,954	27,163	28,859
AIC	-60,535	-69,860	-73,455	-16,993	-21,030	-22,091	-47,891	-54,295	-57,682
SIC	-60,485	-69,770	-73,354	-16,955	-20,961	-22,014	-47,843	-54,211	-57,587

Table 3: **Effects of liquidity on variance disparity**

This table shows the results of regressing the variance disparity measure, *BASIS*, on the liquidity proxies. The full sample spans from July 1, 2006 to August 31, 2014. In the ex-crisis sample analysis, I exclude the three-month window corresponding to the peak of the Lehman Brothers crisis. *LIBOIS* denotes the LIBOR–OIS spread, *SPRDS* denotes the relative bid–ask spread of SPX options, *SPRDF* denotes the relative bid–ask spread of VIX futures, *SPRDV* denotes the relative bid–ask spread of VIX options, *VIX* denotes the CBOE VIX index, and *SKEW* denotes the CBOE SKEW index. Newey and West (1987) robust *t*-statistics with an optimal lag are shown in parentheses. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Sample period	const.	<i>LIBOIS</i>	<i>SPRDS</i>	<i>SPRDF</i>	<i>SPRDV</i>	<i>VIX</i>	<i>SKEW</i>	adj. R^2	Nobs
Panel A: Funding liquidity									
Full sample	3.20*** (4.45)	4.14*** (4.10)						0.29	2050
Ex-crisis sample	4.92*** (12.36)	0.97*** (3.48)						0.03	1987
Panel B: Market liquidity									
Full sample	-21.28*** (-3.30)		3.57*** (3.76)	1.52*** (3.45)	1.28*** (3.36)			0.31	2050
Ex-crisis sample	-0.96 (-0.34)		0.63** (2.31)	0.91*** (2.80)	0.15 (0.52)			0.04	1987
Panel C: Funding and market liquidity									
Full sample	-11.44*** (-2.73)	2.67*** (4.47)	2.50*** (3.91)	0.82** (2.26)	0.29 (0.89)			0.37	2050
Ex-crisis sample	-0.20 (-0.07)	0.79** (2.46)	0.61** (2.34)	0.67** (2.01)	-0.01 (-0.03)			0.06	1987
Panel D: All explanatory variables									
Full sample	-5.79 (-0.78)	4.23*** (5.38)	2.85*** (4.75)	0.83** (2.24)	0.34 (1.07)	-2.31*** (-3.64)	-0.22 (-0.74)	0.41	2050
Ex-crisis sample	11.66* (1.82)	1.43*** (3.43)	0.87*** (3.49)	0.57* (1.65)	0.01 (0.02)	-1.17*** (-2.79)	-0.60** (-2.06)	0.09	1987

Table 4: **Summary statistics and correlation matrix**

BASIS denotes a measure of variance disparity, *LIBOIS* denotes the LIBOR–OIS spread, *SPRDS* denotes the relative bid–ask spread of SPX options, *SPRDF* denotes the relative bid–ask spread of VIX futures, *SPRDV* denotes the relative bid–ask spread of VIX options, *VIX* denotes the CBOE VIX index, *SKEW* denotes the CBOE SKEW index, and *IT* denotes the informed trading measure (which is defined as a detrended volume ratio of VIX futures to SPX options).

	<i>BASIS</i>	<i>LIBOIS</i>	<i>SPRDS</i>	<i>SPRDF</i>	<i>SPRDV</i>	<i>VIX</i>	<i>SKEW</i>	<i>IT</i>
Panel A: Summary statistics								
Mean	0.07	0.35	0.29	0.01	0.22	0.22	120.48	-0.00
Median	0.06	0.16	0.28	0.01	0.21	0.19	119.92	0.06
Min.	-0.22	0.06	0.16	0.00	0.13	0.10	0.00	-4.35
Max.	1.02	3.64	0.81	0.02	0.38	0.81	143.26	3.44
Std.	0.08	0.43	0.06	0.00	0.04	0.10	6.47	1.00
Skew.	4.52	3.47	2.32	1.52	0.60	2.11	-2.63	-0.34
Kurt.	44.46	19.24	16.84	6.26	3.22	8.66	61.32	3.75
AR(1)	0.79	1.00	0.96	0.96	0.97	0.98	0.76	0.58
Panel B: Correlation matrix								
<i>BASIS</i>	1.00	0.54	0.46	0.28	0.23	0.34	-0.16	-0.10
<i>LIBOIS</i>		1.00	0.39	0.43	0.47	0.78	-0.34	-0.18
<i>SPRDS</i>			1.00	0.03	-0.04	0.41	-0.00	-0.04
<i>SPRDF</i>				1.00	0.43	0.34	-0.29	0.03
<i>SPRDV</i>					1.00	0.35	-0.26	-0.16
<i>VIX</i>						1.00	-0.31	-0.30
<i>SKEW</i>							1.00	0.03
<i>IT</i>								1.00

Table 5: **Summary statistics of the replicated and original VIV measures**

This table presents the summary statistics of the square root of the replicated and original VIV measures with one-, three-, and five-month maturities, which I denote by $\widetilde{VIV}_t(T)$ and $VIV_t(T)$, respectively. The replicated VIV measure is computed from a pair of the SIV measures. In contrast, the original VIV measure is computed directly from the prices of VIX futures and options. Panels A and B correspond to the pre-crisis period (July 1, 2006 to August 31, 2008) and the post-crisis period (December 1, 2008 to August 31, 2014), respectively.

Variable	Mean	Median	Min.	Max.	Std.	Skew.	Kurt.	AR(1)
Panel A: Pre-crisis period								
Replicated VIV measures:								
$\sqrt{\widetilde{VIV}_t(1)}$	0.19	0.19	0.11	0.29	0.05	0.11	1.53	0.98
$\sqrt{\widetilde{VIV}_t(3)}$	0.20	0.20	0.13	0.28	0.05	0.13	1.41	0.98
$\sqrt{\widetilde{VIV}_t(5)}$	0.20	0.19	0.14	0.39	0.04	0.27	1.87	0.97
Original VIV measures:								
$\sqrt{VIV_t(1)}$	0.20	0.21	0.12	0.29	0.05	0.00	1.47	0.99
$\sqrt{VIV_t(3)}$	0.21	0.22	0.14	0.29	0.05	-0.01	1.37	0.99
$\sqrt{VIV_t(5)}$	0.21	0.22	0.15	0.29	0.04	0.00	1.38	0.99
Panel B: Post-crisis period								
Replicated VIV measures:								
$\sqrt{\widetilde{VIV}_t(1)}$	0.23	0.21	0.13	0.60	0.09	1.42	5.03	0.98
$\sqrt{\widetilde{VIV}_t(3)}$	0.26	0.24	0.15	0.55	0.08	0.93	3.68	0.99
$\sqrt{\widetilde{VIV}_t(5)}$	0.27	0.26	0.17	0.52	0.07	0.70	3.05	0.98
Original VIV measures:								
$\sqrt{VIV_t(1)}$	0.24	0.21	0.13	0.61	0.09	1.35	4.81	0.99
$\sqrt{VIV_t(3)}$	0.26	0.25	0.15	0.54	0.08	0.84	3.48	0.99
$\sqrt{VIV_t(5)}$	0.28	0.27	0.17	0.49	0.07	0.53	2.76	0.99

Table 6: **Informational role of VIX derivatives: VEC analysis**

This table presents the variance discovery analysis between the replicated and original VIV measures, based on the VEC model as defined in Equation (20). The replicated VIV measure is computed from a pair of the SIV measures. In contrast, the original VIV measure is computed directly from the prices of VIX futures and options. The *GG* (Gonzalo and Granger, 1995) metric measures the relative contribution of innovations in the original VIV measure to innovations in efficient prices. HAS_L and HAS_U denote the lower and upper bounds on the information share (Hasbrouck, 1995) of VIX derivatives, respectively. Panels A and B correspond to the pre-crisis period (July 1, 2006 to August 31, 2008) and the post-crisis period (December 1, 2008 to August 31, 2014), respectively.

Maturity	Speeds of adjustment		Variance discovery ratios		
	α_1	α_2	<i>GG</i>	HAS_L	HAS_U
Panel A: Pre-crisis period					
1 month	-0.37	0.02	0.94	0.19	1.00
3 month	-0.28	0.06	0.83	0.47	0.94
5 month	-0.16	0.05	0.76	0.43	0.86
Panel B: Post-crisis period					
1 month	-0.15	0.21	0.42	0.05	0.92
3 month	-0.19	0.09	0.67	0.21	0.93
5 month	-0.26	0.01	0.96	0.59	1.00

Table 7: Effects of informed trading on variance disparity

This table shows the results of regressing the variance disparity measure, *BASIS*, on the *IT* measure and control variables. The full sample spans from July 1, 2006 to August 31, 2014. In the ex-crisis sample analysis, I exclude the three-month window corresponding to the peak of the Lehman Brothers crisis. *IT* denotes the informed trading measure (which is defined as a detrended volume ratio of VIX futures to SPX options), *LIBOIS* denotes the LIBOR–OIS spread, *SPRDS* denotes the relative bid–ask spread of SPX options, *SPRDF* denotes the relative bid–ask spread of VIX futures, *SPRDV* denotes the relative bid–ask spread of VIX options, *VIX* denotes the CBOE VIX index, and *SKEW* denotes the CBOE SKEW index. Newey and West (1987) robust *t*-statistics with an optimal lag are shown in parentheses. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Sample period	const.	<i>IT</i>	<i>LIBOIS</i>	<i>SPRDS</i>	<i>SPRDF</i>	<i>SPRDV</i>	<i>VIX</i>	<i>SKEW</i>	adj. R^2	Nobs
Panel A: Informed trading measure only										
Full sample	6.57*** (16.07)	-0.77*** (-3.38)							0.01	2050
Ex-crisis sample	5.95*** (21.84)	-0.69*** (-3.32)							0.02	1987
Panel B: All explanatory variables										
Full sample	-4.73 (-0.63)	-0.62*** (-2.84)	4.29*** (5.45)	2.90*** (4.82)	0.94** (2.55)	0.25 (0.79)	-2.58*** (-3.96)	-0.25 (-0.84)	0.41	2050
Ex-crisis sample	13.74** (2.08)	-0.87*** (-4.53)	1.41*** (3.52)	0.89*** (3.61)	0.73** (2.15)	-0.13 (-0.44)	-1.46*** (-3.47)	-0.66** (-2.17)	0.11	1987

Table A1: Unit root and co-integration test results

This table presents the unit-root and co-integration test results for the logarithm of replicated and original VIV measures, which I denote by $\widetilde{VIV}_t(T)$ and $VIV_t(T)$, respectively. The replicated VIV measure is computed from a pair of SIV measures, which are extracted from the SPX option prices. In contrast, the original VIV measure is computed directly from the prices of VIX futures and options. The KPSS test evaluates the null hypothesis that the time series is stationary against the existence of a unit root. In contrast, the ADF test assesses the null of the existence of a unit root. The Johansen maximum eigenvalue test assesses the null hypothesis that the number of co-integrating relations is equal to r against $r + 1$. The pre-crisis period (Panel A) and the post-crisis period (Panel B) are defined as July 1, 2006 to August 31, 2008 and December 1, 2008 to August 31, 2014, respectively. The symbols *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Maturity	KPSS test		ADF test		Johansen test	
	$\log \widetilde{VIV}$	$\log VIV$	$\log \widetilde{VIV}$	$\log VIV$	$H_0 : r = 0$	$H_0 : r = 1$
Panel A: Pre-crisis period						
1 month	4.56***	4.81***	-0.73	-0.75	92.36***	2.01
3 month	4.61***	5.11***	-0.83	-0.88	77.44***	1.61
5 month	4.96***	5.34***	-0.99	-0.94	41.15***	1.65
Panel B: Post-crisis period						
1 month	4.59***	4.58***	0.30	0.26	204.38***	9.77**
3 month	5.61***	5.95***	0.62	0.67	155.42***	7.36
5 month	6.82***	8.35***	0.85	0.87	154.16***	5.69

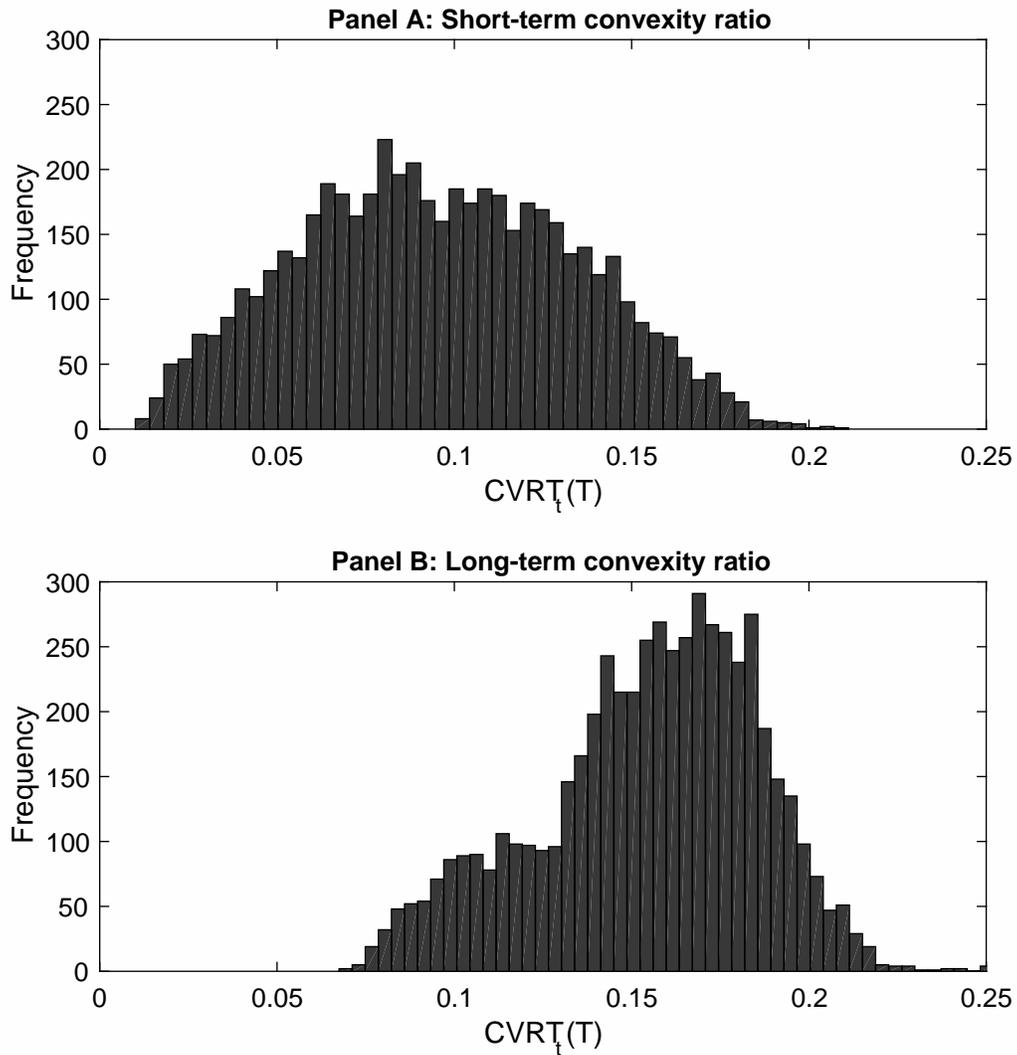


Figure 1: Histograms of the convexity ratios. The convexity ratio, $CVRT_t(T)$, is defined as the ratio of the convexity adjustment to the VIV measure. Panels A and B correspond to short-term contracts (less than three months) and long-term contracts (more than three months), respectively.

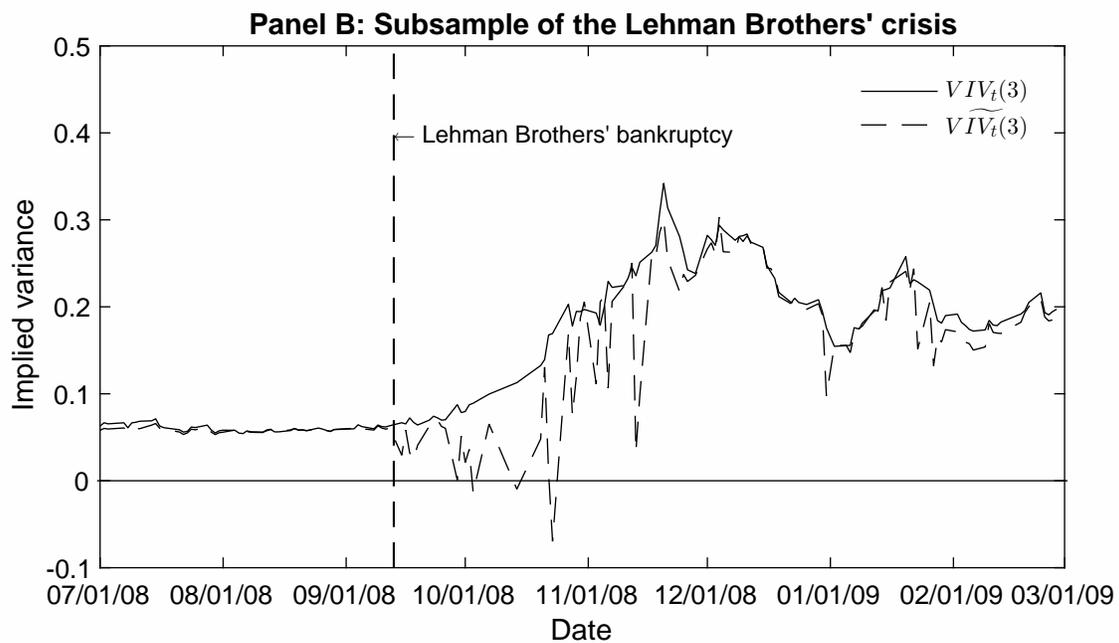
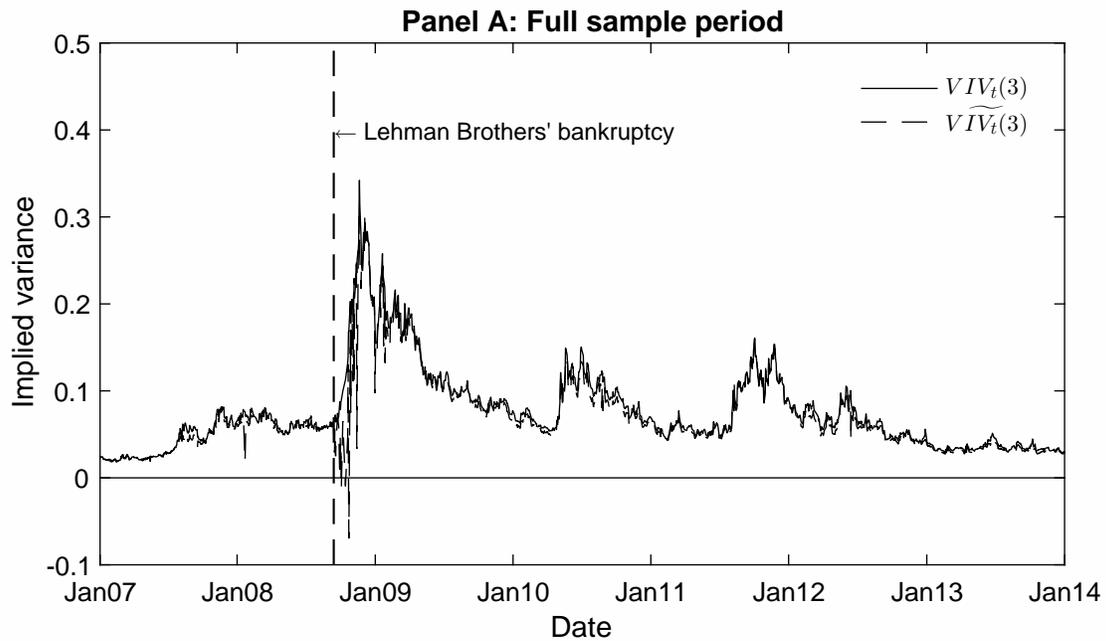


Figure 2: Time series plots of the original and replicated VIV measures with a three-month maturity. The original VIV measure (solid line) is calculated directly from the VIX derivative prices. In contrast, the replicated VIV measure (dotted line) is computed from a pair of the SIV measures.

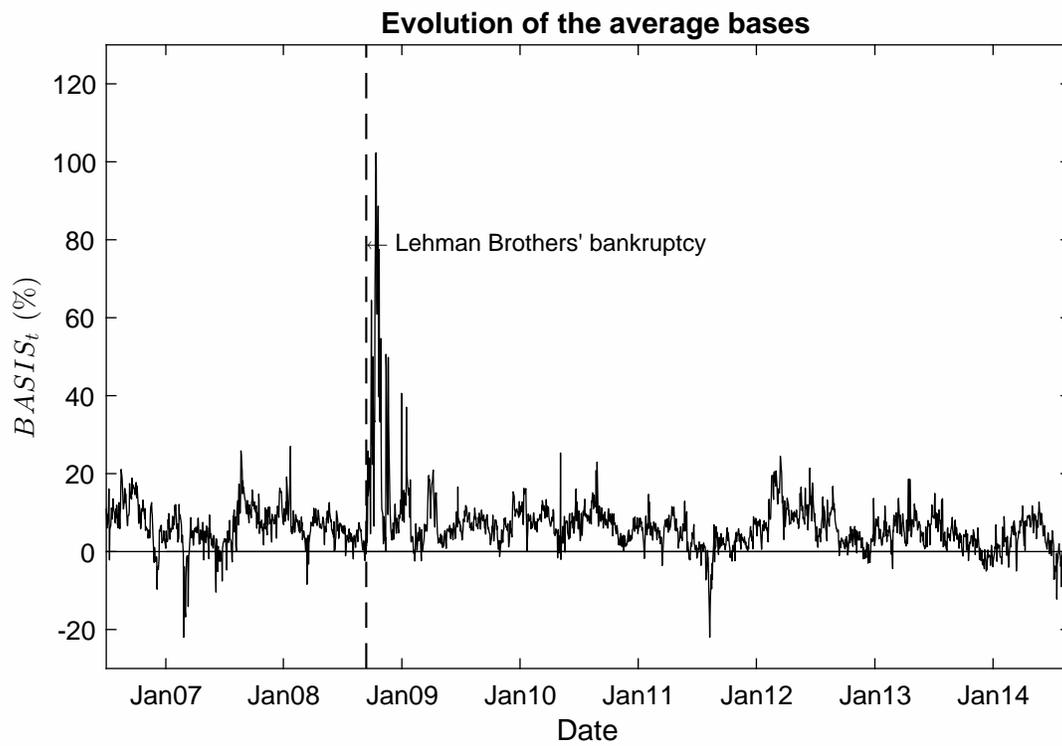


Figure 3: Time series plot of the average bases.

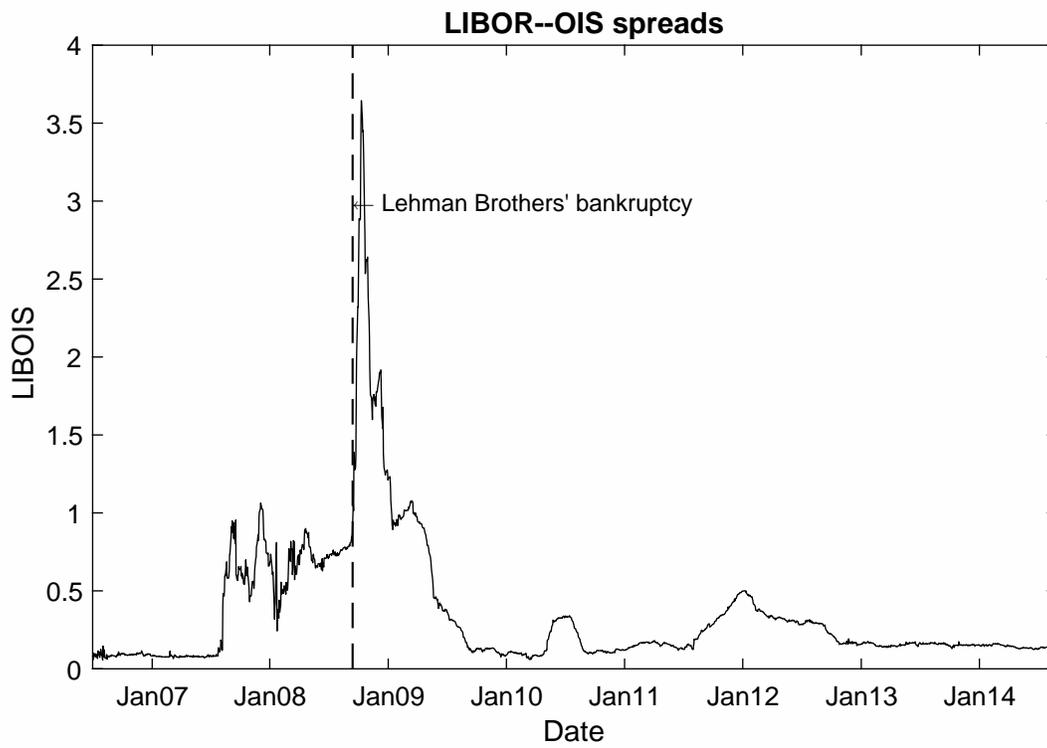


Figure 4: Time series plot of LIBOR–OIS spreads.

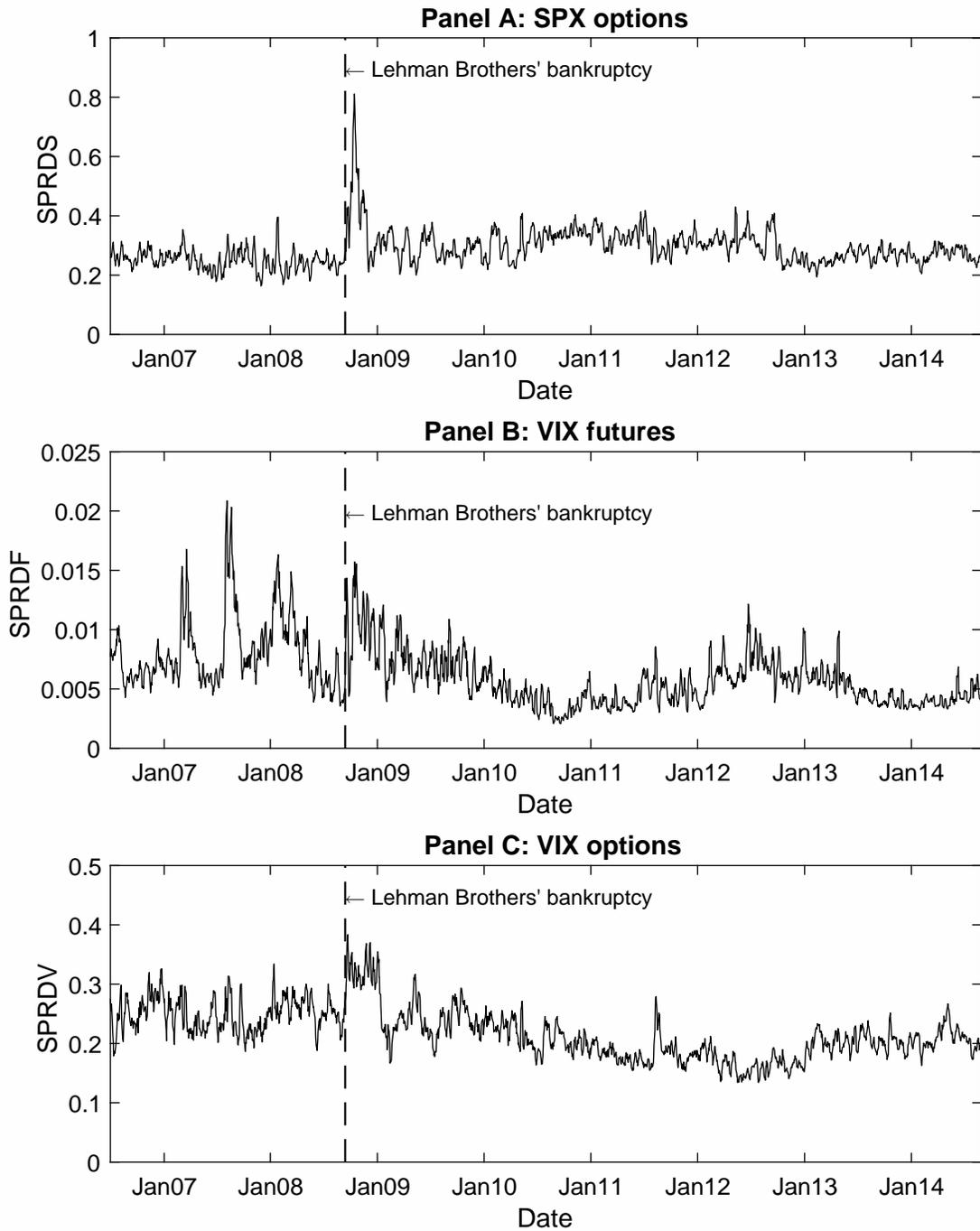


Figure 5: Time series plots of the weekly moving averages of the relative bid-ask spreads. Panels A, B, and C correspond to SPX options, VIX futures, and VIX options, respectively.

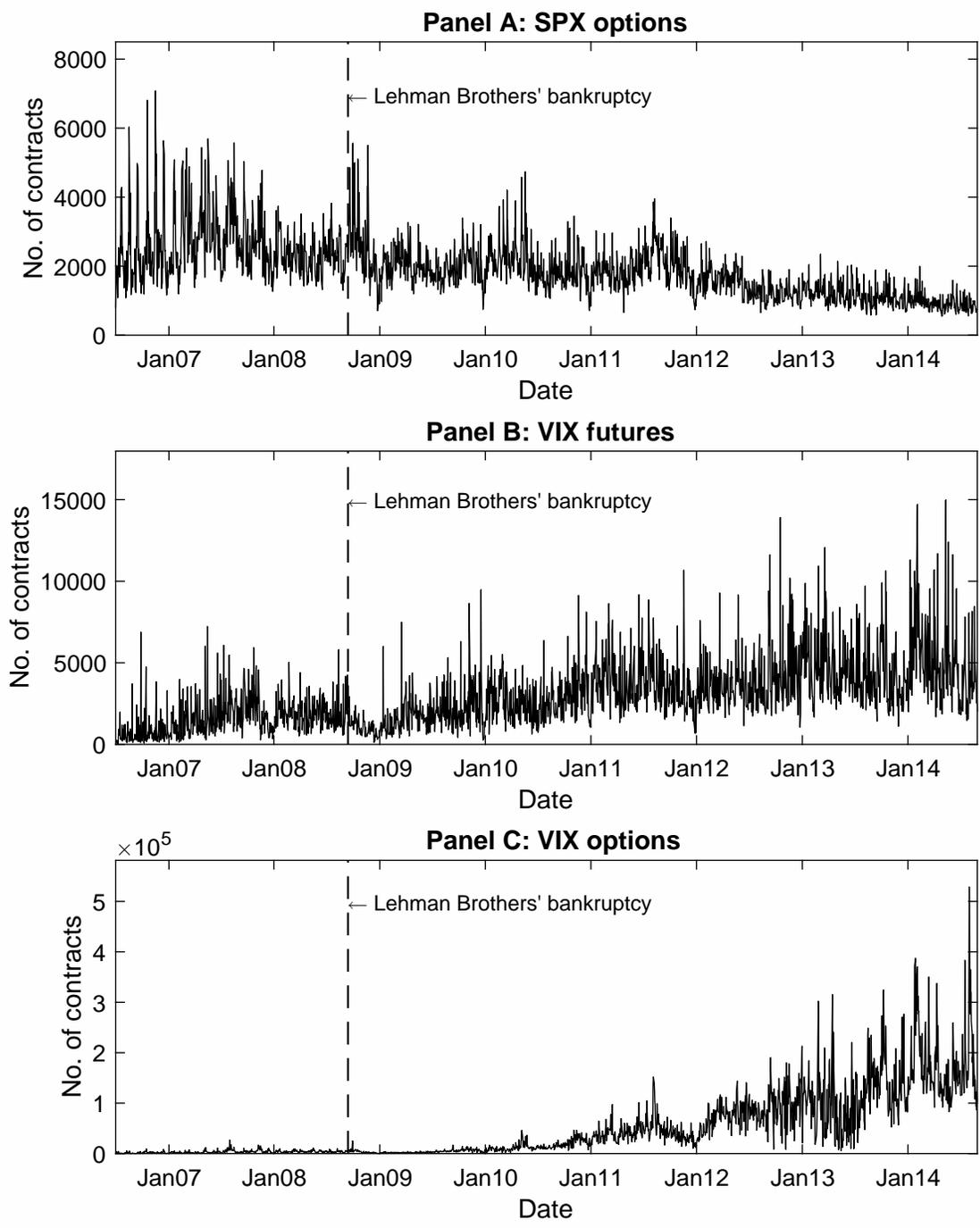


Figure 6: Time series plots of trading volume. Panels A, B, and C correspond to SPX options, VIX futures, and VIX options, respectively.

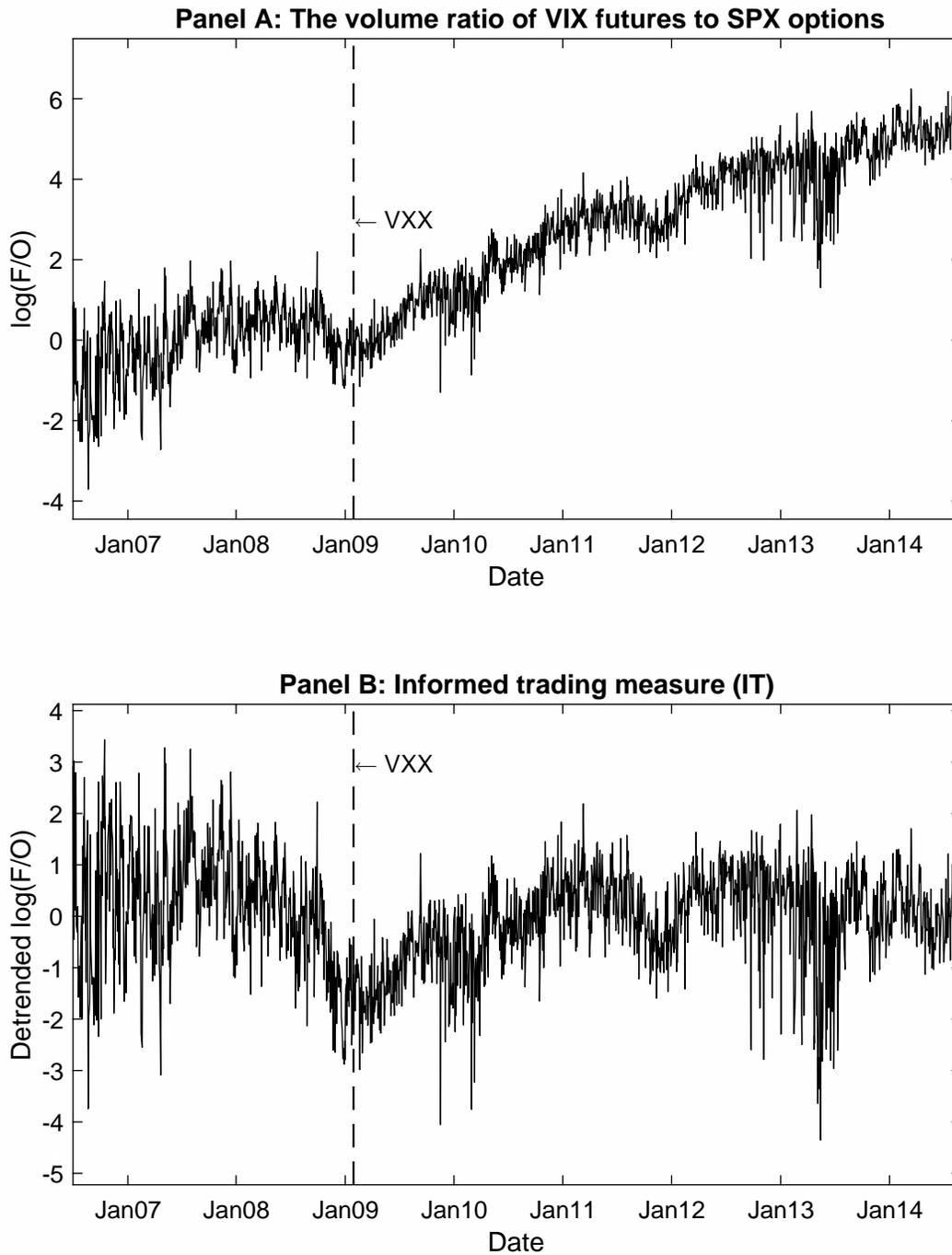


Figure 7: Time series plots of the volume ratio of VIX futures to SPX options (F/O) and the informed trading (IT) measure. The IT measure (Panel B) is obtained by detrending the volume ratio of VIX futures to SPX options (Panel A). The dotted vertical line refers to January 29, 2009 when the first, and most actively traded, VIX-linked ETP (ticker: VXX) was introduced.