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Are Concerns About Leveraged ETFs Overblown?

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Are Concerns About Leveraged ETFs Overblown?*

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Abstract

Leveraged and inverse exchange-traded funds (ETFs) have been heavily criticized for exacerbating volatility in financial markets because it is thought that they mechanically rebalance their portfolios in the same direction as contemporaneous returns. We argue that these criticisms are likely exaggerated because they ignore the effects of capital flows on ETF rebalancing demand. Empirically, we find that capital flows substantially reduce the need for ETFs to rebalance when returns are large in magnitude and, therefore, mitigate the potential for these products to amplify volatility. We also show theoretically that flows can completely eliminate ETF rebalancing in the limit.

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1 Introduction

Leveraged and inverse exchange-traded funds (ETFs) seek to track a multiple of the performance of an underlying index, commodity, currency, or some other benchmark over a specified time frame, which is usually one day. These products have been heavily criticized based on the belief that they exacerbate volatility in financial markets. Commentators have referred to them as "weapons of mass destruction" and claim that they pose "serious threats to market stability" because they "have turned the market into a casino on steroids."[1] Others have claimed that leveraged ETFs "could send volatility through the roof, and prices through the floor[.]"[2] It appears as though policy makers are also concerned about these products, as the Securities and Exchange Commission (SEC) has issued a moratorium on approving exemptive requests for new leveraged and inverse ETFs.[3]

The basis of the commentators' concerns seems to be a common perception that leveraged and inverse ETFs must rebalance their portfolios in the same direction as the contemporaneous return on their underlying assets in order to maintain a constant leverage ratio. Conventional thinking suggests that by purchasing assets following positive returns and selling assets following negative returns, these types of financial products exert additional upward price pressure on the underlying assets following positive returns and additional downward pressure following negative returns (see, e.g., Cheng and Madhavan (2009), Bai, Bond, and Hatch (2014), Tuzun (2014), and Shum et al. (2014)). However, such reasoning is incomplete because it overlooks the effects of capital flows.

We demonstrate theoretically that capital flows can either increase or decrease ETF rebalancing demand because flows alter the size of an ETF, which in turn affects the amount of additional leverage the ETF requires to maintain its target leverage ratio. More specifically, capital flows reduce rebalancing when the flows offset the change in the ETF's assets under management (AUM) that arises from the one-day return on the ETF's underlying portfolio,

[1]See Sorkin (2011).
[2]See Zweig (2009).
[3]See SEC press release 2010-45.

but flows intensify rebalancing when they occur in the opposite direction. From a theoretical perspective, then, capital flows can either increase or decrease the potential for these types of products to amplify volatility.[4]

Empirically, we find that capital flows diminish the potential for leveraged and inverse ETFs to exacerbate volatility. Using a sample of large U.S. equity-based ETFs, we find that capital flows occur frequently and tend to offset the need for ETFs to rebalance their portfolios. Furthermore, the effect of flows on ETF rebalancing demand is strongest when returns are large in magnitude, which is important because ETFs would presumably be most prone to amplify market movements in these cases.

Because ETF rebalancing is likely to have the greatest effect on volatility when the magnitudes of the underlying returns are large, we partition the data according to the size of the daily return on the underlying assets and analyze the relations between returns, capital flows, and ETF rebalancing demand. When returns are large in magnitude, the correlations between capital flows and returns suggest that flows tend to mitigate ETF rebalancing demand. Moreover, estimates from ordinary least squares regressions over the quintiles of returns indicate that, when returns are large in magnitude, the returns on the underlying assets should exert much less influence over ETF rebalancing demand than previously thought. By some estimates, returns generate up to 74% less rebalancing by leveraged and inverse ETFs once capital flows are taken into account. As a consequence, the potential for these types of products to exacerbate volatility should be much lower than many claim.

We also examine the relations between returns, capital flows, and ETF rebalancing demand over different parts of the distributions of capital flows and rebalancing demand. When capital flows occur, the correlations between flows and returns suggest that both capital inflows and outflows tend to mitigate the need for ETFs to rebalance. Furthermore, estimates

[4] We refer to the *potential* for leveraged and inverse ETFs to exacerbate volatility throughout this article because there could be liquidity providers in the market that absorb the additional demand or supply generated when these types of ETFs rebalance their portfolios. Our focus is on how capital flows affect this potential, and we do not address whether this potential is manifested in the market. Additionally, we use the term "rebalancing demand" to refer the amount that an ETF would theoretically need to rebalance to maintain its target leverage ratio.

from quantile regressions over different percentiles of the rebalancing-demand distribution indicate that, when rebalancing demand is strongest, the relation between returns and rebalancing demand is much weaker than it would otherwise be in the absence of capital flows. In some cases, returns wield up to 61% less influence over ETF rebalancing demand after accounting for capital flows.

The effects of capital flows appear to be stronger for ETFs with higher leverage ratios. This is noteworthy because ETFs with higher leverage ratios are more prone to exacerbate volatility than those with lower leverage ratios. For ETFs with a leverage ratio of -1, our evidence suggests that capital flows may aggravate rather than mitigate the potential to amplify volatility. However, these ETFs constitute the smallest segment of our sample, and capital flows seem to have only a small effect on their rebalancing demand.

While our results indicate that capital flows tend to mitigate the potential for leveraged and inverse ETFs to exacerbate volatility, capital flows could be impeded by market frictions, e.g., transaction costs or creation unit constraints. To better understand how market frictions might affect the impact of capital flows on ETF rebalancing, we conduct a numerical simulation that incorporates transaction costs for ETF investors. We find that frictions can considerably reduce the frequency of capital flows. Furthermore, we find that capital flows can aggravate the potential for ETFs to amplify market movements when frictions are large, even though flows reduce rebalancing demand when frictions are small. This is due to the fact that large frictions can have a substantial effect on the size of an ETF because the size is partially determined by capital flows, and the potential to exacerbate volatility is greater when frictions give rise to a larger ETF.

Finally, we show theoretically that, in the absence of frictions, leveraged and inverse ETFs never need to rebalance their portfolios if investors aim to track a multiple of the underlying return over a relatively long horizon by investing in an ETF. Such an objective could naturally arise from investors such as market makers or other intermediaries using leveraged and inverse ETFs to hedge their exposures over a horizon longer than a day. Because rebalancing by an ETF alters investors' exposures to the underlying assets, investors must rebalance their own portfolios by trading shares of the ETF to maintain their desired exposures to the underlying assets.[5] Capital flows arising from such rebalancing by investors completely eliminate the potential for leveraged and inverse ETFs to exacerbate volatility in the limit because they drive the ETFs' leverage ratios back to target levels.

We are not the first to examine the potential for leveraged and inverse ETFs to exacerbate volatility, though, to our knowledge, we are the first to consider the effects of capital flows. A handful of researchers have attempted to assess whether leveraged and inverse ETFs amplify volatility in practice by examining the relation between intra-day returns and end-of-day volatility (see, e.g., Bai, Bond, and Hatch (2014), Trainor (2010), Tuzun (2014), and Shum et al. (2014)). None of these analyses, however, use actual data on ETF holdings or rebalancing activity to estimate the impact of ETFs on volatility. Instead, many of these studies rely on an equation derived by Cheng and Madhavan (2009) to calculate "hypothetical" rebalancing by ETFs, and they do not take into account the effect of capital flows. As our current work demonstrates, capital flows are an important factor to consider when assessing the impact of rebalancing by ETFs on volatility. In our opinion, therefore, the ultimate question of whether leveraged and inverse ETFs exacerbate volatility in practice remains unresolved.

Our paper is also related to the literature that examines the performance of leveraged ETFs (see, e.g., Avellaneda and Zhang (2010), Charupat and Miu (2011), and Tang and Xu (2013)) and the impact of the ETF arbitrage process on the volatility of underlying assets (Ben-David, Franzoni, and Moussawi (2014)) and the return co-movement of underlying assets (Da and Shive (2013)). More generally, our paper contributes to the literature that analyzes capital flows in investment companies. In contrast to Coval and Stafford (2007), who find that capital flows increase institutional price pressure stemming from mutual funds

[5]The SEC and the Financial Industry Regulatory Authority (FINRA) issued an Investor Alert in 2009 cautioning investors that the long-term performance of leveraged and inverse ETFs may differ from the daily performance objective.

during periods of financial distress, we find that capital flows tend to decrease institutional price pressure originating from leveraged and inverse ETFs. This is due to the fact that mutual fund flows are related to performance (see, e.g., Sirri and Tufano (1998)), whereas capital flows for leveraged and inverse ETFs are related to changes in leverage.

The remainder of the paper proceeds as follows. We first demonstrate theoretically in Section 2 that capital flows can either mitigate or aggravate the potential for leveraged and inverse ETFs to exacerbate volatility. We then show empirically that capital flows tend to mitigate the potential for these products to amplify volatility in practice in Section 3. Next, we discuss rationales for capital flows in Section 4 and show that rebalancing by investors can, in the limiting case, completely eliminate the need for ETFs to rebalance. We then numerically evaluate the effects of market frictions in Section 5. Last, Section 6 concludes.

2 Theory

Assume there exists an index or other type of asset that realizes an exogenous return, r_t, from time t to $t+1$. There also exists an ETF that seeks to replicate a multiple, m, of the one-period return on the index. This multiple may be either positive (for a leveraged ETF) or negative (for an inverse ETF). In reality, m typically takes a value of $+2$ or $+3$ for a leveraged ETF and a value of -1, -2, or -3 for an inverse ETF. For our analysis, we only require that $m \notin [0,1]$.[6]

We denote the ETF's time-t AUM by A_t. To replicate m-times the return on the underlying index, the ETF allocates a fraction, m, of its assets to the index and the remaining fraction, $1-m$, to cash. This means that the ETF's time-t index exposure is

$$x_t = mA_t, \tag{1}$$

[6] We use the term "index" for ease of exposition even though an ETF cannot invest directly in an index. In practice, an ETF would invest in the assets that comprise the index, a total return swap on the index, or other type of financial instrument.

and it holds an amount of cash equal to

$$y_t = (1 - m)A_t. \tag{2}$$

The ETF may also experience a capital flow in each period. Let f_t denote the capital flow at time t as a fraction of AUM. In practice, capital flows occur whenever ETF shares are created or redeemed, and they arise from investors wishing to increase or decrease their exposure to the ETF.[7] While investors may choose to alter their exposure for any number of reasons, some of which we discuss below, our results are not dependent on the specific motive behind these capital flows. Rather, capital flows are important because they affect the extent to which the ETF must rebalance its portfolio each period, regardless of the underlying reason for the flows.

Capital flows may either increase or decrease AUM, which evolves according to

$$A_{t+1} = x_t(1 + r_t) + y_t + A_t f_t \tag{3}$$

$$= A_t(1 + mr_t + f_t), \tag{4}$$

where (4) follows from substituting (1) and (2) into (3). Because the evolution of AUM generally does not correspond to the return on the index, the ETF typically must rebalance its portfolio each period to maintain its target leverage ratio. The degree to which the ETF must rebalance its portfolio between t and $t + 1$ is determined by

$$\Delta x_t \equiv x_{t+1} - x_t(1 + r_t). \tag{5}$$

[7] In reality, capital flows are facilitated by authorized participants (APs), as ordinary investors cannot interact directly with an ETF. ETF shares can be created and redeemed by APs only in large blocks called creation units. The size of a creation unit varies between 25,000 and 200,000 shares in today's market, with the most common size being 50,000 ETF shares. While the size of a creation unit can affect capital flows and thus the potential for ETFs to exacerbate volatility, as we briefly discuss in Section 5, here we assume that $f_t \in \mathbb{R}$ to clearly illustrate the role of capital flows on the extent of rebalancing by the ETF.

The following proposition provides an analytical expression for the amount of rebalancing that occurs in the ETF's portfolio.

Proposition 1. *The ETF rebalances its portfolio according to*

$$\Delta x_t = A_t m\big[(m-1)r_t + f_t\big]. \tag{6}$$

Proof. Substitute (1) and (4) into (5). □

According to Proposition 1, an ETF that tracks a multiple of an index and does not experience any capital flows ($f_t = 0$) generally must rebalance its portfolio in the same direction as the index return. This rebalancing places additional upward pressure on prices when index returns are high and additional downward pressure on prices when returns are low. Thus, leveraged and inverse ETFs have the potential to amplify volatility in the market. Because the relation characterized by (6) holds for both leveraged and inverse ETFs, rebalancing by leveraged ETFs does not negate rebalancing by inverse ETFs, and vice versa. Leveraged ETFs rebalance in the same direction as the index return because these ETFs must increase (decrease) their exposure when the return is positive (negative), whereas inverse ETFs rebalance in the same direction as the index return because these ETFs must decrease (increase) their negative exposure when the return is positive (negative). Rebalancing processes similar to (6) have been derived by Cheng and Madhavan (2009) and Jarrow (2010), though these authors ignore the effects of capital flows.

As demonstrated by (6), capital flows may either increase or decrease the amount of rebalancing. Specifically, capital flows reduce the amount of rebalancing by a leveraged ETF when the flows occur in the opposite direction of the index return, but flows intensify rebalancing when they occur in the same direction as the return. The opposite holds true for an inverse ETF, where capital flows mitigate rebalancing when they occur in the same direction as the index return but increase rebalancing when they occur in the opposite direction of the return. As a consequence, capital flows may either diminish or enhance the potential for

ETFs to exacerbate volatility. This is embodied in the following corollary.

Corollary 1. *The extent of rebalancing by the ETF in the same direction as the index return, and therefore the potential for the ETF to exacerbate volatility, increases whenever* $sgn(f_t) = sgn(mr_t)$ *but decreases whenever* $sgn(f_t) = sgn(-mr_t)$.

Capital flows impact the extent to which an ETF must rebalance because flows alter the ETF's AUM, which in turn affects the amount of additional index exposure the ETF requires to maintain its target leverage ratio. For example, a positive index return would ordinarily lead a leveraged ETF to increase its index exposure if the ETF did not experience a capital flow, as described above. A negative capital flow in this case, however, effectively decreases the amount of additional exposure that the ETF must obtain to achieve its target leverage ratio. Therefore, the ETF will not purchase as many additional shares of the underlying index following a positive return. Similarly, a positive capital flow reduces the amount of exposure that a leveraged ETF must shed after a negative return because such a flow increases the ETF's AUM. Thus, the ETF will not sell as many shares of the underlying index following a negative return. Capital flows have the opposite effect when they occur in the reverse direction, and analogous reasoning applies to inverse ETFs. Because capital flows may either reduce or augment the amount of rebalancing undertaken by leveraged and inverse ETFs, capital flows can mitigate or aggravate the potential for these types of financial products to exacerbate volatility.

3 Empirical Analysis

In this section, we empirically investigate the extent to which capital flows occur and how they affect the need for ETFs to rebalance their portfolios. We find that capital flows occur frequently and that they tend to reduce the amount by which ETFs must rebalance. Furthermore, this reduction in rebalancing demand appears to be economically significant. Overall, the evidence indicates that capital flows substantially mitigate the potential for leveraged and inverse ETFs to amplify volatility on a daily basis.

3.1 Data

We use Morningstar to identify the universe of leveraged and inverse ETFs. There are a total of 188 such ETFs as of May 2014. We restrict our analysis to ETFs that track multiples of the daily performance of U.S. equity indices because we are interested in assessing whether leveraged or inverse ETFs have the potential to exacerbate volatility in equity markets. We therefore exclude ETFs that are based on government bond, corporate bond, currency, commodity, and foreign equity indices. After these exclusions, there are 104 ETFs remaining in the sample. We further restrict the sample to relatively large ETFs with a market capitalization of at least $500 million because rebalancing by smaller ETFs may have only a negligible, if any, impact on the volatility of underlying assets. The final sample consists of 31 large domestic equity ETFs and 17,288 ETF-day observations.

We obtain daily data on ETF prices and trading volume from the Center for Research in Security Prices (CRSP) and shares outstanding and daily index price data from Bloomberg for the period starting in June 2006 and ending in May 2014. We adjust these variables to account for stock splits where appropriate. We also hand-collect information about the indices tracked by the ETFs in our sample from public sources such as the websites of the ETF sponsors and index providers. Data regarding creation unit sizes and fees are obtained from Morningstar.

3.2 Descriptive Statistics

We first describe some characteristics of capital flows for all ETF-day observations in our sample. We measure capital flows as the percentage change in the number of ETF shares outstanding. A positive change indicates an inflow (or a net creation of ETF shares) whereas a negative change indicates an outflow (or a net redemption). Capital flows occur on approximately 75% of ETF-days for leverage ratios of -3 and $+3$, on about 54% of ETF-days for leverage ratios of -2, and on roughly and 40% of ETF-days for leverage ratios of -1 and $+2$. The fact that capital flows occur more frequently for ETFs with higher leverage ratios is noteworthy because, according to Proposition 1, the potential for rebalancing to exacerbate volatility is greater for ETFs with higher leverage ratios, and, as we demonstrate below, the greater frequency of flows helps to mitigate this potential.

Table I reports net capital flows—along with other summary statistics—broken down by the ETFs' leverage ratios. The evidence suggests that ETFs with higher leverage ratios experience larger capital flows, as the 10^{th} and 90^{th} percentiles of the flow distributions are larger in magnitude for these ETFs. This, too, is noteworthy because, as Proposition 1 shows, capital flows must be larger if they are to have a meaningful impact on rebalancing by ETFs with higher leverage ratios. Additionally, the average daily trading volume of ETF shares tends to increase with the magnitude of the leverage ratio, and the average trading volume of ETFs with a leverage ratio of -1 is substantially lower than the volume of ETFs with other leverage ratios.

3.3 Relations between Returns and Capital Flows

Next, we examine the relations between capital flows and index returns. Index returns are reported at the end of the day, whereas the number of shares outstanding are reported at the beginning of the day before trading. We therefore measure the capital flow for ETF i on date t as $f_{i,t} \equiv (SO_{i,t+1}/SO_{i,t}) - 1$, where $SO_{i,t}$ denotes the number of shares of ETF i outstanding at t. Returns are measured in the usual way, e.g., $r_{i,t} \equiv (p_{i,t}/p_{i,t-1}) - 1$, where $p_{i,t}$ is the closing value of the index tracked by ETF i on date t.

According to Corollary 1, capital flows mitigate the potential for leveraged and inverse ETFs to amplify volatility when the flows occur in the opposite direction of the return times the leverage ratio, but flows aggravate this potential when they occur in the same direction as the return times the leverage ratio. Figure 1 displays the relations between returns and flows for various leverage ratios. As is evident from the scatter plots, there is a negative relation between returns and flows for leveraged ETFs but a positive relation for inverse ETFs with leverage ratios of -2 and -3. This suggests that flows tend to reduce the extent to which these ETFs must rebalance. Conversely, the negative relation between returns and flows for ETFs with a leverage ratio of -1 suggests that flows may induce these ETFs to undergo greater rebalancing.

Panel A in Table II reports Spearman rank correlation coefficients between capital flows and the magnitude of returns for the first and tenth deciles of returns. We focus our analysis on the tails of the index-return distributions because, in the absence of capital flows, the potential for ETFs to amplify volatility by rebalancing their portfolios is strongest in these cases, as implied by Proposition 1. We calculate Spearman rank correlation coefficients because the prevalence of observations with zero capital flows (i.e., no net creation or redemption activity on a given day) could lead to unreliable estimates of the Pearson product-moment correlation coefficient.

The correlation coefficients indicate that capital flows for leveraged ETFs are positively correlated with the magnitude of negative returns but are negatively correlated with the magnitude of positive returns. Conversely, capital flows for inverse ETFs with -2 and -3 leverage ratios, which constitute the vast majority of the inverse ETF observations in our sample, are negatively correlated with the magnitude of negative returns but are positively correlated with the magnitude of positive returns. Overall, these results suggest that leveraged ETFs tend to experience capital flows in the opposite direction of the underlying index returns whereas inverse ETFs tend to experience capital flows in the same direction as returns. Thus, capital flows appear to drive an ETF's leverage ratio closer to the target leverage ratio when underlying returns are large in magnitude, thereby mitigating the potential for these types of financial products to exacerbate volatility.

We also examine the relations between returns and flows from another angle. Panel B in Table II reports Spearman rank correlation coefficients between capital flows and returns for days on which capital flows occur, i.e., $f_{i,t} \neq 0$. We calculate Spearman rank correlations because the data are not normally distributed. For leveraged ETFs, returns are positively correlated with the magnitude of outflows but are negatively correlated with the magnitude of inflows. For inverse ETFs with leverage ratios of -2 and -3, returns are negatively correlated with the magnitude of outflows but are positively correlated with the magnitude of inflows. Thus, when capital flows occur, they often tend to mitigate the potential for these ETFs to exacerbate volatility. For ETFs with a leverage ratio of -1, returns are positively correlated with the magnitude of outflows but are uncorrelated with inflows, which indicates that outflows tend to aggravate the effects of rebalancing for these ETFs when returns are positive.

3.4 Relations between Returns and Rebalancing

The correlations between capital flows and returns suggest that flows reduce the need for ETFs to rebalance their portfolios. In this subsection, we empirically demonstrate this effect more directly and attempt to quantify the economic impact of capital flows on the potential for ETFs to amplify volatility.

Figure 2 depicts the relations between returns and ETF rebalancing demand, as determined by (6) with A_t normalized to one. The solid lines in these plots represent the amount of rebalancing that ETFs would need to undertake if they did not experience any capital flows. Each data point on the scatter plots represents the rebalancing demand given the observed capital flow. Clearly, capital flows have a substantial effect on ETF rebalancing demand, and thus the potential for ETFs to exacerbate volatility.

In the context of Figure 2, capital flows mitigate (aggravate) rebalancing demand when the rebalancing amount is closer to (farther from) $\Delta x_t = 0$ than the solid line for a given return. The scatter plots indicate that flows often, though not always, reduce ETF rebalancing demand. In many cases, flows generate rebalancing demand in the opposite direction of returns, which, in stark contrast to conventional thinking, could dampen the volatility of the underlying assets.

To more rigorously assess how capital flows affect the relation between returns and rebalancing demand, we perform a segmented regression analysis to analyze the relation between rebalancing demand and returns across the different quintiles of the return distribution. Segmenting the returns allows us to evaluate how returns affect rebalancing demand in different parts of the return distribution, which is important because the potential for ETFs to exacerbate volatility is stronger when returns are larger in magnitude. For each leverage ratio, we estimate the following ordinary least squares regression for the five quintiles of the return distribution:

$$\Delta x_{i,t} = \alpha + \beta r_{i,t} + \varepsilon_{i,t}, \tag{7}$$

where $\Delta x_{i,t}$ is the rebalancing demand of ETF i on date t as predicted by (6) with A_t normalized to one. The estimates of β should be equal to $m(m-1)$ if capital flows do not affect rebalancing demand.

Table III lists the estimates of β. Statistical significance is determined relative to the relation between ETF rebalancing demand and returns when capital flows are ignored. As the table shows, capital flows have a substantial impact on the relation between returns and rebalancing demand when leveraged and inverse ETFs are most prone to exacerbate volatility, i.e., in the first and fifth quintiles of returns. For instance, the influence of returns on ETF rebalancing demand declines by up to 74% for ETFs with a leverage ratio of +3 and by up to 63% for ETFs with a leverage ratio of +2 when capital flows are considered. The influence of returns on rebalancing demand also wanes by up to 40% for ETFs with a leverage ratio of −3 or −2. For ETFs with a leverage ratio of −1, capital flows can either strengthen or weaken the relation between returns and rebalancing demand, though the magnitude of the effect is much smaller.

We also estimate a quantile regression as introduced by Koenker and Bassett (1978) and discussed by Koenker and Hallock (2001) to evaluate how returns affect rebalancing demand across different percentiles of the rebalancing-demand distribution. This is important because the potential for ETFs to amplify volatility is stronger when ETFs undergo more rebalancing. For each leverage ratio, we estimate the following regression for various percentiles of the rebalancing-demand distribution:

$$\Delta x_{i,t} = \tilde{\alpha} + \tilde{\beta} r_{i,t} + \tilde{\varepsilon}_{i,t}. \tag{8}$$

Again, estimates of $\tilde{\beta}$ should be equal to $m(m-1)$ if capital flows do not influence ETF rebalancing demand.

Figure 3 displays the $\tilde{\beta}$ estimates across a range of percentiles of the rebalancing-demand distribution. Lower percentiles represent large negative rebalancing demand, whereas higher percentiles represent large positive rebalancing demand. The estimates from the quantile regressions indicate that the relation between rebalancing demand and returns is much weaker when rebalancing demand is greatest. For example, the relation between returns and rebalancing demand drops by up to approximately 61% for ETFs with a leverage ratio of +3 and by up to roughtly 34% for ETFs with a leverage ratio of −3. Flows have a smaller impact on the relation for ETFs with a leverage ratio of +2 or −2, but the effects are still substantial.

Finally, we quantify the interaction between rebalancing demand and returns to get a sense of how capital flows affect the overall potential for ETFs to exacerbate volatility. The specific statistic we calculate, which we refer to as an efficacy score, is

$$\frac{\sum_i \sum_t \Delta x_{i,t} r_{i,t}}{\sum_i \sum_t |r_{i,t}|} \tag{9}$$

with A_t again normalized to one.

Table IV lists for each leverage ratio the efficacy score as well as the percentage reduction in the efficacy score from the benchmark case with no capital flows. Capital flows cause the efficacy score to drop by approximately 50% for ETFs with a leverage ratio of +3, by roughly 36% for ETFs with a leverage ratio of −3, and by approximately 15% for ETFs with leveraged leverage ratios of +2 and −2. However, the efficacy score increases by about 10% for ETFs with a leverage ratio of −1.

Overall, our results indicate that capital flows have a substantial effect on the potential for leveraged and inverse ETFs to amplify market movements, at least for ETFs with a leverage ratio of $+3$, $+2$, -2, or -3. The evidence is mixed for ETFs with a leverage ratio of -1, but these ETFs constitute the smallest portion of our sample.

Our results also raise questions about the conclusions drawn from several prior studies that attempt to assess whether ETFs amplify market movements in practice. These studies rely on an equation similar to (6) with $f_t = 0$, which is represented by the solid lines in Figure 2, as an estimate of daily rebalancing by ETFs. As the figure illustrates, however, ETF rebalancing demand often differs considerably from the solid lines once capital flows are taken into account.

4 Rationales for Capital Flows

As we demonstrate both theoretically and empirically, capital flows can affect the potential for leveraged and inverse ETFs to exacerbate volatility. So far, though, we have been agnostic about the source of capital flows, as our primary objective is to illustrate the effect of capital flows on the potential for leveraged and inverse ETFs to amplify volatility. There are numerous reasons why investors trade, and identifying the underlying trading motive(s) behind capital flows is beyond the scope of this paper. Nonetheless, in this section we discuss a couple of possible motives for trading that could lead to the empirically-observed relations between returns and capital flows.

One possible explanation could be mean-reversion and/or momentum trading. Under a mean-reversion trading strategy, investors would increase (decrease) their exposure to leveraged ETFs following negative (positive) index returns. Investors would also increase (decrease) their exposure to inverse ETFs following positive (negative) returns. Under a momentum trading strategy, investors would behave in an opposite fashion. Such trading strategies could contribute to the empirical relations that we observe.

The observed relations between returns and capital flows could also originate from investors wishing to track—via an ETF—a multiple of the underlying index over a horizon longer than a day. Because daily rebalancing by an ETF alters investors' exposures to the underlying index, investors must rebalance their own portfolios daily by trading shares of the ETF to maintain their desired exposure to the index over a horizon longer than a day. As we demonstrate below, such trading by investors could contribute to the empirical relations between capital flows and returns. Moreover, capital flows can completely eliminate ETF rebalancing, and therefore eliminate the potential for ETFs to exacerbate volatility, if all investors aim to track a multiple of the underlying index over a long horizon. We describe this mechanism in detail in the remaining portion of this section.

To clearly illustrate the mechanism, we introduce a representative investor who wishes to track m-times the return on the index over a horizon longer than one period and assume that trading by the investor gives rise to capital flows. Although there are likely many investors who desire the exposure provided by a leveraged or inverse ETF over a single day or less, there are liable to be many others who use these types of products for longer-term hedging purposes (e.g., market makers or financial intermediaries).

First consider the case in which the investor does not rebalance his own portfolio to account for changes in the ETF's exposure to the index. In this situation, his one-period return from holding the ETF is equal to

$$R_{t,t+1} \equiv \frac{A_{t+1}}{A_t} - 1 \tag{10}$$

$$= mr_t, \tag{11}$$

where (11) follows from substituting (4) into (10) and setting $f_t = 0$. Thus, the ETF provides the investor with his desired exposure over a single period. However, holding the ETF for more than one period without rebalancing his portfolio generally will not provide the investor with a return that equals m-times the performance of the index over the longer holding period. This mismatch of returns is due to the fact that the return process for the ETF differs from the index return process.

For instance, consider the case where the investor wishes to track m-times the index return over two periods. In this case, the two-period ETF return is given by

$$R_{t,t+2} \equiv \frac{A_{t+2}}{A_t} - 1 \tag{12}$$

$$= (1 + mr_t)(1 + mr_{t+1}) - 1, \tag{13}$$

but m-times the two-period return on the index equals

$$m[(1 + r_t)(1 + r_{t+1}) - 1]. \tag{14}$$

Evidently, the ETF usually will generate a return that differs from m-times the index return. Therefore, the investor generally must rebalance his portfolio every period to successfully track m-times the return on the index over a longer time horizon.

When the investor rebalances, his two period return is given by

$$R_{t,t+2} \equiv \frac{A_{t+2} - A_t f_t - A_{t+1} f_{t+1}}{A_t} - 1 \tag{15}$$

$$= (1 + mr_t)(1 + mr_{t+1}) + mr_{t+1} f_t - 1, \tag{16}$$

where (16) is derived by substituting (4) into (15). The negative flow is included in the return defined by (15) because the flow originates from the investor's portfolio and therefore affects his payoff.

To obtain the desired exposure to the index, the flow at time t must be such that the return from holding the ETF matches m-times the return on the index. Setting (16) equal to (14) and solving for the flow yields

$$f_t = (1 - m)r_t. \tag{17}$$

Not only does this flow process provide the investor with his desired return over multiple periods, but substituting (17) into (6) reveals that trading by the investor completely eliminates the need for the ETF to rebalance its portfolio. This leads to the following corollary.

Corollary 2. *The ETF does not rebalance its portfolio when the investor aims to track a multiple of the underlying return over multiple periods, i.e.,*

$$\Delta x_t = 0. \tag{18}$$

5 Market Frictions

Market frictions that impede capital flows are likely to have a considerable effect on the potential for leveraged and inverse ETFs to amplify market movements. One such friction is a requirement that permits the flow of capital only in large blocks (usually 50,000 shares) called creation units. This requirement forces investors as a group to adjust their ETF holdings in discrete units and may therefore lead to less frequent capital flows. Another friction that could obstruct capital flows is transaction costs incurred by investors to adjust their ETF holdings in their own portfolios. To the extent that these frictions impede capital flows, both could result in greater rebalancing by ETFs and ultimately increase their potential to exacerbate volatility.

In the remainder of this section, we conduct a numerical simulation to better understand the effects of market frictions. While both of the frictions identified above could impact capital flows, for the sake of brevity, we focus solely on transaction costs. In general, we find that frictions can drastically reduce the frequency of capital flows, but flows still tend to mitigate the potential for ETFs to exacerbate volatility. When the frictions are relatively large, however, capital flows can aggravate the potential for ETFs to amplify market movements.

As a first benchmark against which to evaluate the impact of market frictions on ETF rebalancing demand, we select the limiting case where (17) characterizes the flow process and the ETF never rebalances its portfolio. Although this benchmark implicitly relies on the notion that investors seek to track a multiple of the underlying index over a horizon longer than one period, the intuition developed through the simulation should not depend on the specific driver of capital flows. We also consider the rebalancing process defined by (6) with $f_t = 0$ as a second benchmark. The first benchmark enables us to gauge the extent to which frictions might reduce capital flows, and the second benchmark enables us to assess the degree to which capital flows affect ETF rebalancing demand in the presence of frictions.

Because leveraged and inverse ETFs typically aim to track m-times the daily return on an index, capital flows must occur daily to fully mitigate rebalancing by ETFs. As the ultimate source of capital flows is trading by investors, however, this may be prohibitively expensive if the cost of trading is high relative to the amount of each individual investor's investment in the ETF. For example, if an investor allocated \$10,000 to the ETF and paid a \$10 brokerage fee every day to trade the ETF's shares, after one year the total fees would constitute approximately 25% of the initial investment (assuming 250 trading days and ignoring bid-ask spreads). As a consequence, investors are unlikely to trade, and capital flows are therefore unlikely to occur, daily. Rather, capital flows should occur only when the benefit to the investor of trading exceeds the cost.

To capture the effects of transaction costs, we assume that no capital flows occur unless the investor's effective index allocation (through holding the ETF) deviates from his desired allocation by a sufficiently large margin. The investor's index allocation in period t is mA_t/I_t, where I_t denotes the index value at time t. This corresponds to the ETF allocating a constant fraction m of its assets to the index each period. However, (1), (6), and (17) indicate that the investor must maintain an allocation equal to mA_0/I_0 in all periods to achieve his objective of tracking m-times the index performance over a long horizon. This implies that the investor prefers that the ETF allocate a fraction $m(A_0/I_0)/(A_t/I_t)$ of its assets to the index at time t.

The discrepancy between these two portfolios causes the return from holding the ETF to diverge from the investor's desired return, and the magnitude of this divergence depends on the degree to which these portfolios differ. When the investor incurs a transaction cost, capital flows occur only when the benefit of trading outweighs the cost. Therefore, we assume that no capital flows occur unless the discrepancy between the two portfolios reaches a critical threshold, which we denote by $\Gamma \in \mathbb{R}^+$. Thus, a capital flow occurs at time t if and only if

$$\left| m\left(1 - \frac{A_0 I_t(1+r_t)}{I_0 A_t(1+mr_t)}\right) \right| \geq \Gamma. \tag{19}$$

The flow threshold, Γ, is an exogenous reduced-form representation of the transaction costs incurred by investors to trade shares of the ETF. A larger Γ represents higher costs. While we do not explicitly model the source of these costs, Γ could represent, *inter alia*, brokerage fees or bid-ask spreads. In any event, the investor will tolerate a larger deviation between his actual and desired allocations when he incurs a higher cost to rebalance his portfolio.

5.1 Algorithm

When computing the capital flow and rebalancing processes, we assume that the index return follows the historical return on the S&P 500 from January 2007 to May 2014. Details regarding the source of the data are provided in Section 3.1. We compute results for $m \in \{-3, -2, -1, +2, +3\}$.

Algorithm 1 describes the procedure used to compute capital flows in the presence of transaction costs. Inputs for the algorithm include: the flow threshold, Γ; the ETF multiple, m; the index return, r_t; the initial AUM, A_0; the initial index value, I_0; and the initial number of ETF shares outstanding, S_0. We calculate statistics for various combinations of Γ and m. We also normalize A_0, I_0, and S_0 to one. The normalization does not affect the incidence of capital flows or ETF rebalancing, but it does affect the magnitude of our results.

For each date t, the algorithm first checks the extent of the deviation between the investor's actual and desired allocations. If this deviation is relatively small, i.e., if (19) is not satisfied, then there is no capital flow. On the other hand, if (19) is satisifed then there is a capital flow. The capital flow for the ETF in period t is given by

$$f_t = \frac{A_0 I_t (1 + r_t)}{A_t I_0} - (1 + m r_t), \tag{20}$$

which is derived by setting the investor's desired allocation, mA_0/I_0, equal to the ETF's allocation, mA_{t+1}/I_{t+1}, substituting (4) and $I_{t+1} = I_t(1 + r_t)$, and solving for f_t. AUM is then updated according to (4).

Capital flows also affect the number of ETF shares outstanding. Because a positive (negative) flow requires ETF shares to be created (redeeemed), the number of shares outstanding increases (decreases) when the flow is positive (negative). More specifically, the number of outstanding ETF shares evolves according to

$$S_{t+1} = S_t \left(1 + \frac{f_t}{1 + m r_t}\right), \tag{21}$$

as shares created or redeemed as a result of the flow must be created or redeemed at a price equal to the pre-flow NAV.

5.2 Results

Table V reports results related to capital flows and the efficacy of rebalancing when the investor incurs a transaction cost. As predicted by (19), the frequency of capital flows is decreasing in Γ but increasing in $|m|$. Hence, capital flows occur more frequently when transaction costs are lower or the ETF has a higher leverage ratio. For $m = +2$ or -1, capital flows occur less than 15% of the time when Γ is a modest 5%. In many other scenarios, capital flows occur far less than 50% of the time. These results indicate that transaction costs can be a serious impediment to flows.

The reduction in the frequency of capital flows impacts the potential for ETFs to exacerbate volatility, as evidenced by the reduction in the efficacy of rebalancing. We compute two measures of efficacy. The first measure, which we refer to as simple efficacy, normalizes the ratio of the ETF size to the index value, A_t/I_t, to 1 for all t and ignores the path of returns. The second measure, which we refer to as gross efficacy, takes the return path into account and allows the aforementioned ratio to fluctuate over time. Over the entire sample period, the reduction in both efficacy scores is generally decreasing in Γ but increasing in $|m|$. Thus, capital flows reduce the efficacy of rebalancing to a greater extent when transaction costs are lower or the ETF has a higher leverage ratio.

Comparing the two measures of efficacy reveals that the path of returns can have a tremendous effect on the potential for ETFs to amplify market movements when there are frictions that obstruct capital flows. Over the sample period, capital flows reduce simple efficacy more than gross efficacy for leveraged ETFs, but flows decrease gross efficacy more than simple efficacy for inverse ETFs. These differing effects are driven by how the sharp decline in the value of the S&P 500 and subsequent gradual recovery affect the sizes of the ETFs. Because frictions are more of a hindrance to capital flows when returns are smaller in magnitude, and the drop in value of the S&P 500 is steeper than the recovery, capital flows increase (decrease) the size of leveraged (inverse) ETFs during the fall more effectively than they decrease (increase) the size during the rise. Therefore, the reduction in gross efficacy is less (greater) than the reduction in simple efficacy for leveraged (inverse) ETFs over the sample period.

Although capital flows (by construction) always reduce simple efficacy in the simulation, in some cases flows enhance gross efficacy when the frictions are relatively large. Figure 4, which plots the cumulative percentage reduction in the efficacy scores for an ETF with a leverage ratio of +2, indicates that capital flows increase gross efficacy when $\Gamma = 20\%$. The increase in gross efficacy is driven by the relatively infrequent flow of capital in combination with volatile returns. As illustrated by Figure 5, which plots the cumulative percentage changes in the number of shares outstanding for various levels of transaction costs, the large decline in the value of the S&P 500 during 2008-09 leads to a large capital inflow.[8] This results in more assets under management for the ETF relative to the benchmark case with no capital flows. Then, as the index recovers, the ETF undergoes a greater amount of rebalancing on a day-to-day basis because it has more assets under management and frictions prevent capital outflows from reducing AUM until the flow threshold characterized by (19) is reached. Under these circumstances—a long, gradual rise in the value of the index following a sharp decline—capital flows in the presence of large frictions cause the gross efficacy of rebalancing to be higher relative to the case with no capital flows for leveraged ETFs.

6 Concluding Remarks

Leveraged and inverse ETFs have received heavy criticism based on the belief that they exacerbate volatility in financial markets. We show that concerns about these types of products are likely exaggerated. Empirically, we find that capital flows considerably reduce ETF rebalancing demand and, therefore, mitigate the potential for ETFs to amplify volatility. Our analysis has relevant and timely policy implications, as regulators are reportedly considering changes to how ETFs are regulated.[9]

[8]For clarity, the changes in shares outstanding are depicted only for $\Gamma \geq 10\%$. Smaller Γs produce results that more closely track the benchmark case where $\Gamma = 0$.

[9]See Schoeff (2014).

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Table I: Descriptive Statistics. Statistics (mean, standard deviation, 10^{th} percentile, and 90^{th} percentile) for ETF-day observations are reported for the following variables: the market capitalization in millions of dollars, *Mkt. Cap.*; capital flows, $f_{i,t}$; the return on the underlying index, $r_{i,t}$; the return on the market price of the ETF, $R_{i,t}$; the daily trading volume of ETF shares in millions, *Volume*; and the median size of a creation unit, *CU Size*.

	Mkt. Cap.	$f_{i,t}$	$r_{i,t}$	$R_{i,t}$	*Volume*	*CU Size*
	$m = +3$ (5 ETFs and 2550 observations)					
Mean	1059.98	0.0001	0.0011	0.0032	23.371	50,000
St. Dev.	466.51	0.0439	0.0173	0.0500	47.747	
10^{th}	578.55	−0.0443	−0.0169	−0.0499	2.927	
90^{th}	1652.79	0.0438	0.0176	0.0518	43.916	
	$m = +2$ (10 ETFs and 5544 observations)					
Mean	1138.59	−0.0009	0.0008	0.0015	10.797	75,000
St. Dev.	613.95	0.0368	0.0163	0.0315	17.063	
10^{th}	583.53	−0.0167	−0.0162	−0.0315	0.373	
90^{th}	1755.27	0.0124	0.0163	0.0323	25.821	
	$m = -1$ (2 ETFs and 1501 observations)					
Mean	1629.93	0.0019	0.0005	−0.0006	3.293	75,000
St. Dev.	554.70	0.0173	0.0134	0.0132	2.355	
10^{th}	614.94	−0.0107	−0.0143	−0.0142	1.359	
90^{th}	2256.33	0.0189	0.0137	0.0140	5.901	
	$m = -2$ (8 ETFs and 5049 observations)					
Mean	1407.43	0.0013	0.0003	0.0002	19.583	75,000
St. Dev.	921.16	0.0353	0.0241	0.0460	16.063	
10^{th}	554.36	−0.0201	−0.0238	−0.0418	4.648	
90^{th}	2859.84	0.0243	0.0213	0.0466	42.011	
	$m = -3$ (6 ETFs and 2644 observations)					
Mean	776.10	0.0056	0.0004	−0.0015	27.619	50,000
St. Dev.	374.07	0.0974	0.0152	0.0440	42.659	
10^{th}	539.22	−0.0354	−0.0161	−0.0457	5.386	
90^{th}	1082.38	0.0444	0.0155	0.0477	57.688	

Table II: Correlations between Returns and Capital Flows. Panel A reports Spearman rank correlation coefficients between capital flows, $f_{i,t}$, and the magnitudes of contemporaneous returns, $|r_{i,t}|$, for the first and tenth deciles of returns. Panel B reports Spearman rank correlation coefficients between the magnitude of capital flows, $|f_{i,t}|$, and contemporaneous returns, $r_{i,t}$, for inflows and outflows. The number of observations is indicated by N.

	$m=+3$		$m=+2$		$m=-1$		$m=-2$		$m=-3$	
	Panel A: $\lvert r_{i,t}\rvert$									
	1st	10th	1st	10th	1st	10th	1st	10th	1st	10th
$f_{i,t}$	0.334***	−0.070	0.327***	−0.103**	0.188**	−0.035	−0.267***	0.150***	−0.189***	0.345***
	(0.000)	(0.265)	(0.000)	(0.015)	(0.021)	(0.673)	(0.000)	(0.001)	(0.002)	(0.000)
N	255	255	553	554	151	150	503	504	265	264
	Panel B: $\lvert f_{i,t}\rvert$									
	Out	In	Out	In	Out	In	Out	In	Out	In
$r_{i,t}$	0.236***	−0.168***	0.098***	−0.126***	0.178***	−0.015	−0.203***	0.136***	−0.342***	0.140***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.780)	(0.000)	(0.000)	(0.000)	(0.000)
N	959	935	1283	897	280	334	1257	1486	845	1131

p-values in parentheses

$^{*}p<0.10$, $^{**}p<0.05$, $^{***}p<0.01$

Table III: Rebalancing and Returns. Ordinary least squares regression estimates, β, are reported for the relation between ETF rebalancing demand (dependent variable) and contemporaneous returns (independent variable) across the quintiles of returns. The coefficient estimates of the constant term are not reported. Significance is determined relative to the relation between rebalancing demand and returns when there are no capital flows, i.e., $m(m-1)$.

	$m=+3$	$m=+2$	$m=-1$	$m=-2$	$m=-3$
	(1)	(2)	(3)	(4)	(5)
Quintile 1	1.533***	0.742***	2.248**	3.719***	8.820***
	(0.569)	(0.191)	(0.107)	(0.261)	(0.628)
Quintile 2	3.847	1.456	2.012	6.477	9.738
	(1.851)	(0.475)	(0.521))	(0.476)	(2.289)
Quintile 3	1.409*	2.892	1.550	6.339	12.420
	(2.449)	(1.163)	(0.714)	(0.644)	(2.721)
Quintile 4	4.590	1.838	0.993**	5.711	5.964**
	(2.393)	(0.539)	(0.506)	(0.500)	(2.553)
Quintile 5	3.531***	1.512***	1.910	3.602***	7.315***
	(0.438)	(0.133)	(0.133)	(0.412)	(0.821)

standard errors in parentheses

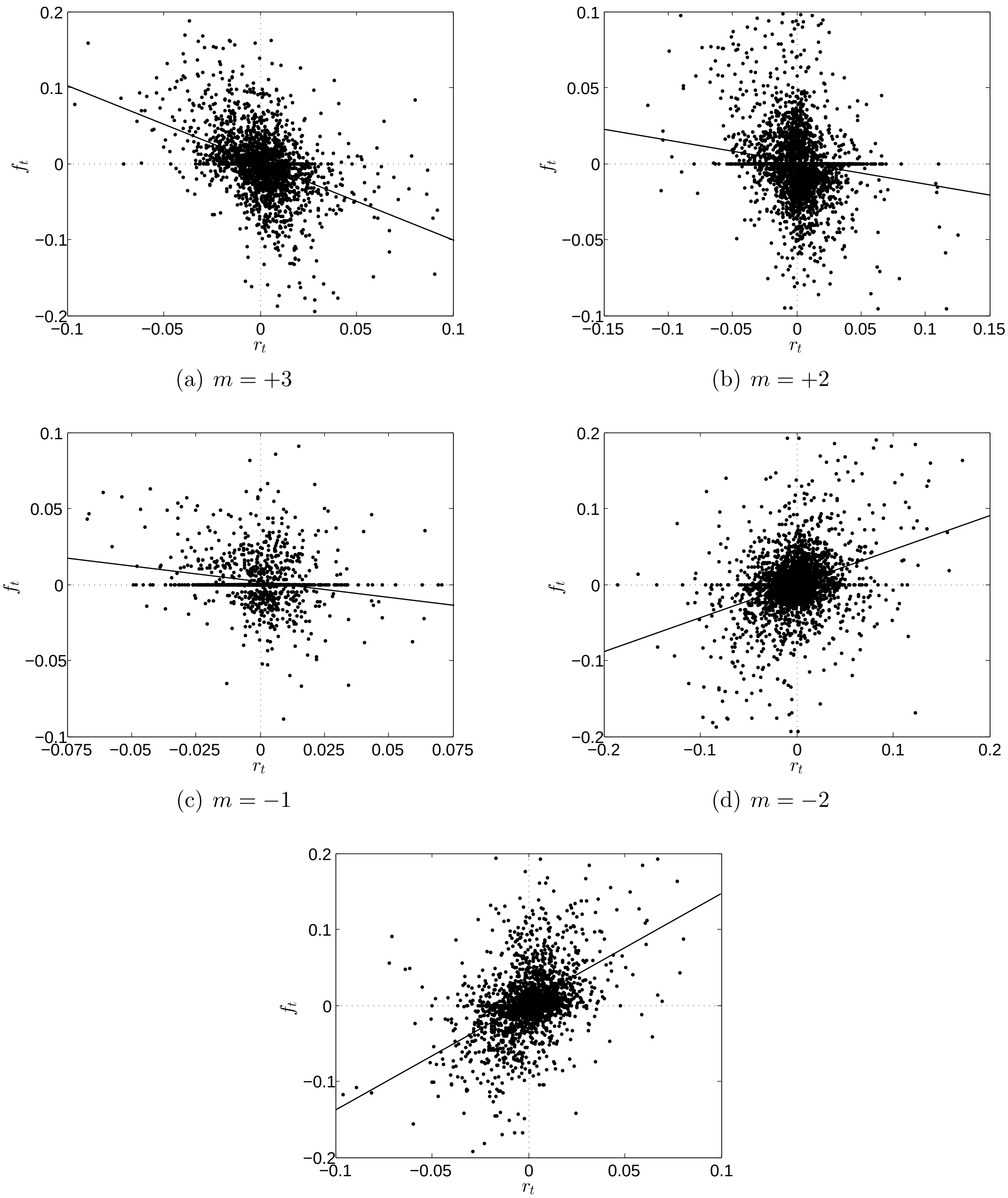
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table IV: Efficacy. Efficacy scores, as defined by (9), are reported with and without capital flows, along with the percentage (100 = 100%) reduction in the efficacy score caused by capital flows.

	$m = +3$	$m = +2$	$m = -1$	$m = -2$	$m = -3$
Efficacy without Flows	0.158	0.050	0.040	0.234	0.261
Efficacy with Flows	0.078	0.043	0.044	0.199	0.168
Efficacy Reduction (%)	50.54	14.31	-10.01	14.94	35.72

Table V: Capital Flows and Efficacy with Transaction Costs. Flow Frequency is the percentage (100 = 100%) of days on which a capital flow occurs. Simple Efficacy Reduction is the percentage reduction in the efficacy score caused by capital flows when A_t/I_t is normalized to 1 for all t. Gross Efficacy Reduction is the percentage reduction in the efficacy score caused by capital flows when A_t/I_t depends on the path of returns.

	Rebalancing Threshold (Γ)			
	1%	5%	10%	20%
	$m = +3$			
Flow Frequency	82.3	41.6	22.8	8.6
Simple Efficacy Reduction	99.9	95.2	84.0	62.8
Gross Efficacy Reduction	99.8	85.9	55.3	−2.3
	$m = +2$			
Flow Frequency	56.1	13.7	4.4	1.4
Simple Efficacy Reduction	97.7	70.6	46.3	24.7
Gross Efficacy Reduction	96.0	51.2	13.7	−16.7
	$m = -1$			
Flow Frequency	56.1	13.3	5.1	1.4
Simple Efficacy Reduction	97.8	71.1	48.5	20.7
Gross Efficacy Reduction	98.2	77.5	61.5	42.5
	$m = -2$			
Flow Frequency	82.2	41.2	22.4	8.8
Simple Efficacy Reduction	99.9	95.2	84.1	63.1
Gross Efficacy Reduction	100.0	95.9	87.0	70.9
	$m = -3$			
Flow Frequency	90.7	61.5	41.1	22.3
Simple Efficacy Reduction	100.0	98.6	95.2	83.9
Gross Efficacy Reduction	100.0	98.7	95.5	85.5



(a) $m = +3$

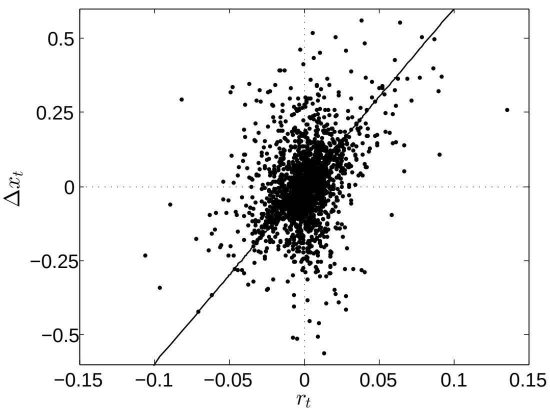
(b) $m = +2$

(c) $m = -1$

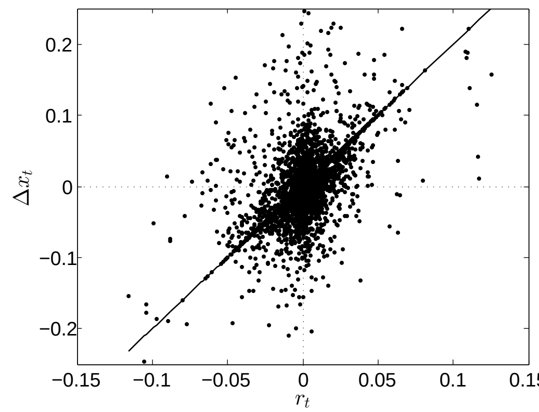
(d) $m = -2$

(e) $m = -3$

Figure 1: Returns and Capital Flows. Capital flows are plotted as a function of contemporaneous returns, along with the line of best fit.



(a) $m = +3$



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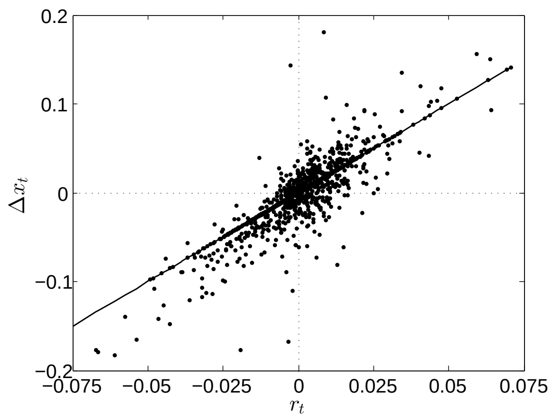

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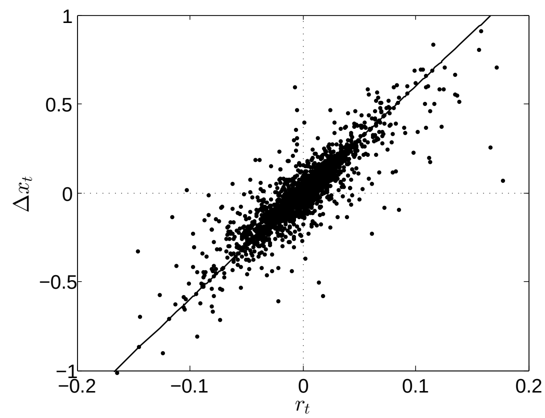
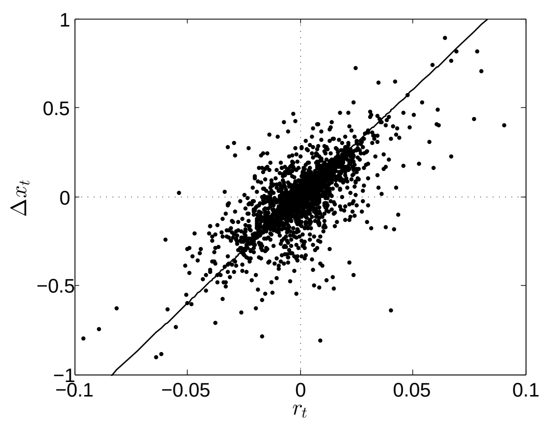
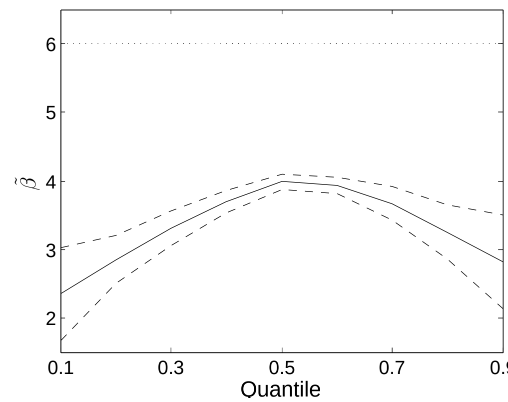


(d) $m = -2$

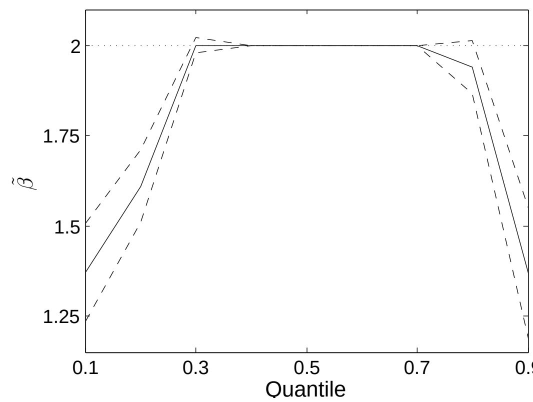


(e) $m = -3$

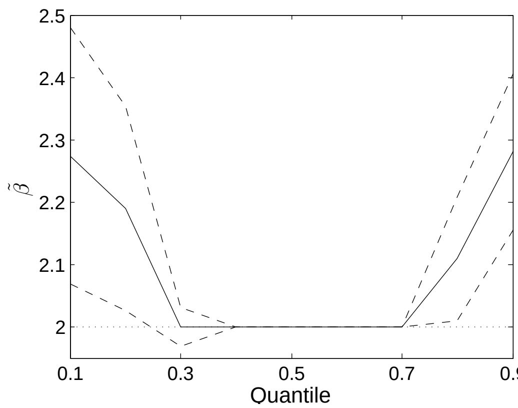
Figure 2: Returns and Rebalancing. ETF rebalancing demand, as determined by (6) with A_t normalized to one, is plotted as a function of contemporaneous returns. The solid line in each plot represents the rebalancing demand when there are no capital flows.



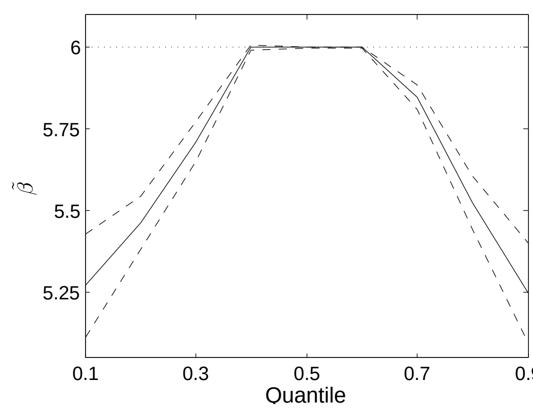
(a) $m = +3$



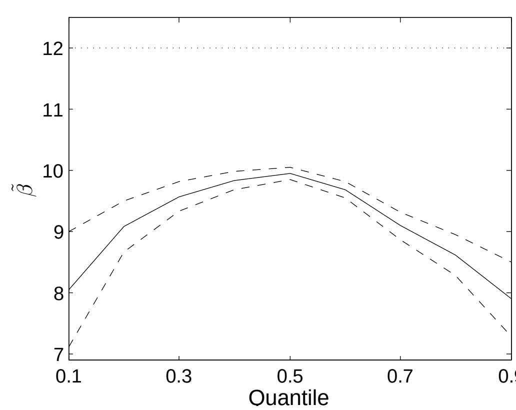
(b) $m = +2$



(c) $m = -1$

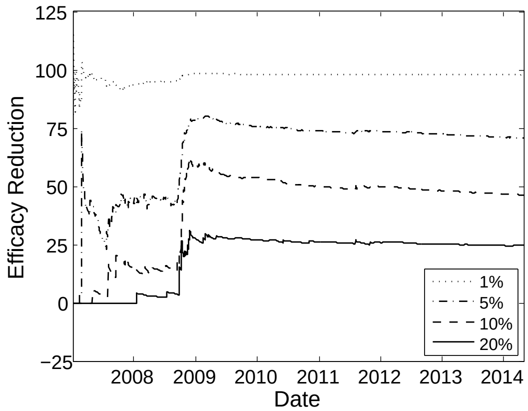


(d) $m = -2$

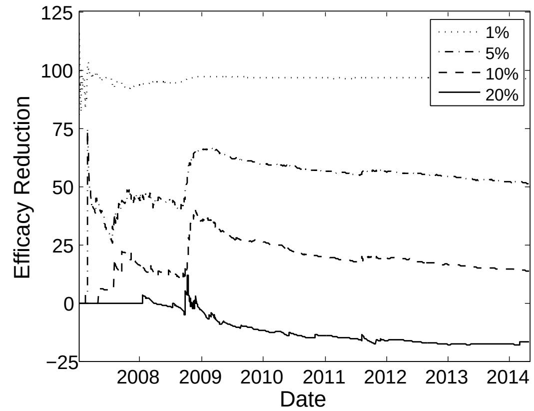


(e) $m = -3$

Figure 3: Quantile Regression Estimates. The solid line represents quantile regression estimates, $\tilde{\beta}$, for the relation between ETF rebalancing demand (dependent variable), as determined by (6) with A_t normalized to one, and contemporaneous returns (independent variable) across the distribution of rebalancing demand. The dashed lines denote 95% confidence bounds, and the dotted line represents the relation between rebalancing demand and returns when there are no capital flows, i.e., $m(m-1)$.



(a) Simple Efficacy



(b) Gross Efficacy

Figure 4: Efficacy. The cumulative percentage ($100 = 100\%$) reduction in the efficacy score caused by capital flows is plotted for various capital flow thresholds, Γ, for a leverage ratio of $m = +2$.

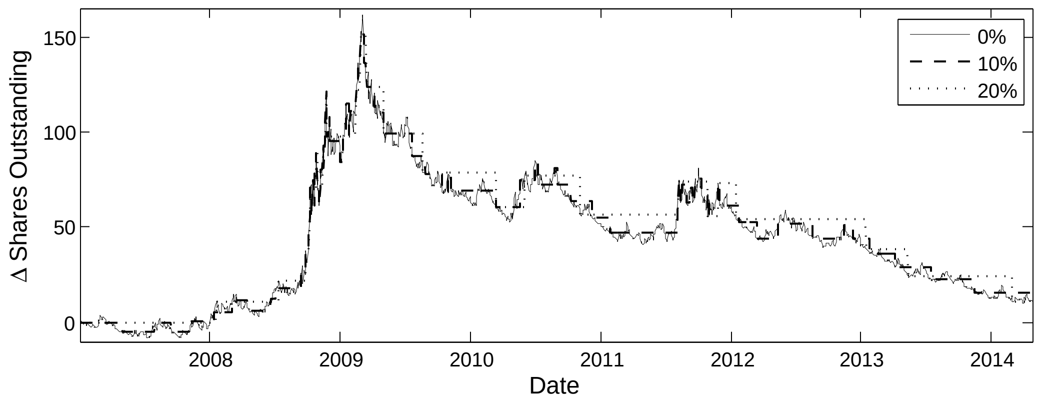


Figure 5: Shares Outstanding. The cumulative percentage (100 = 100%) change in the number of shares outstanding is plotted for various capital flow thresholds, Γ, for a leverage ratio of $m = +2$.

Algorithm 1 compute capital flow process with transaction costs

```
input Γ, m, r_t, A_0, I_0, S_0
for t ← 1 to T do
    if |m(1 − [A_0 I_t(1 + r_t)]/[I_0 A_t(1 + m r_t)])| ≥ Γ then      ▷ threshold given by (19)
        f_t ← A_0 I_t(1 + r_t)/(A_t I_0) − (1 + m r_t)                ▷ flow given by (20)
    else
        f_t = 0
    end if
    A_{t+1} ← A_t(1 + m r_t + f_t)                                    ▷ AUM evolves according to (4)
    S_{t+1} ← S_t(1 + f_t/(1 + m r_t))          ▷ ETF shares outstanding evolve according to (21)
end for
return A_t, f_t ∀t
```