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Identifying the Stance of Monetary Policy at the Zero Lower Bound: A Markov-switching Estimation Exploiting Monetary-Fiscal Policy Interdependence

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Abstract

In this paper, I propose an econometric technique to estimate a Markov-switching Taylor rule subject to the zero lower bound of interest rates. I show that incorporating a Tobit-like specification allows to obtain consistent estimators. More importantly, I show that linking the switching of the Taylor rule coefficients to the switching of the coefficients of an auxiliary uncensored Markov-switching regression improves the identification of an otherwise unidentifiable prevalent monetary regime. To illustrate the proposed estimation technique, I use U.S. quarterly data spanning 1960:1-2013:4. The chosen auxiliary Markov-switching regression is a fiscal policy rule where federal revenues react to debt and the output gap. Results show that there is evidence of policy co-movements with debt-stabilizing fiscal policy more likely accompanying active monetary policy, and vice versa.

Keywords: Markov-switching coefficients, zero lower bound, monetary-fiscal policy interactions

JEL Classification Numbers: C34, E52, E63

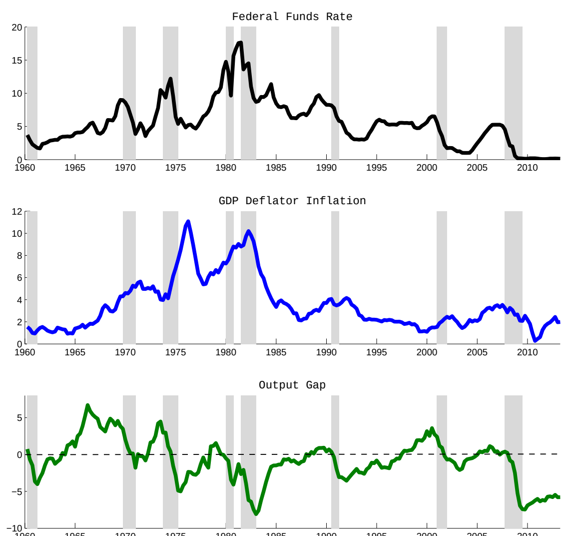
*The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System.

1 Introduction

The forward guidance provided by the Federal Open Market Committee in its statements from December 2012 to June 2014 indicates that a highly accommodative stance of monetary policy remains appropriate to support continued progress toward maximum employment and price stability. In particular, the forward guidance in place sets exceptionally low federal funds rates between 0 and 1/4 percent, defining an effective lower bound.

At least since Clarida et al. (2000), we have known that the monetary-policy regime can change. One could infer the stance of monetary policy, as measured by the strength of the reaction of the federal funds rate with respect to inflation deviations from target, by estimating a Markov-switching coefficients Taylor rule and obtaining the prevalent regime. Unfortunately, because of the current effective lower bound, the federal funds rate does not react to fluctuations in the inflation rate and the (CBO-implied) output gap, as shown in Figure 1. This introduces an important censoring problem in the estimation of monetary-policy rules and poses identification problems to the estimation of the prevalent regime.

Figure 1: Evolution of the Interest Rate, the GDP Price Deflator Inflation and the CBO-implied Output Gap



Notes: Shaded areas are NBER recession periods.

In this paper, I exploit monetary-fiscal policy interdependence and develop an estimation method for an interest rate rule with Markov-switching coefficients that is robust to the effective lower bound. The devised estimation technique provides the probability that, at or just after exiting the effective lower bound, the central bank adopts a hawkish or a dovish regime, hence providing an estimate of the current stance of monetary policy.

Since the work of Tobin (1958), it is known that the inadequate estimation of a censored regression produces inconsistent estimators. In this paper, I show that estimating a Markov-switching regression using the Hamilton (1989) filter ignoring the censoring problem produces inconsistent estimators of the Markov-switching regression coefficients and the transition probabilities. Moreover, I show that, even when a censored regression specification is introduced in the estimation, the filtered probabilities fail to identify the prevalent regime over the censored part of the sample.

There is a way to solve, at least partially, the identification problem of the prevalent regime over the censored part of the sample. The solution involves the joint estimation of the censored Markov-switching regression and an uncensored auxiliary Markov-switching regression whose switching is correlated with the switching of the coefficients of the censored equation. In particular, I show that as the correlation between the states driving the switching of the coefficients of the two regressions increases, identification of the prevalent regime of the censored Markov-switching regression is more precise.

The present work fits in the literature of estimating Taylor rules with Markov-switching coefficients. Bae et al. (2012), for example, estimate a forward-looking Taylor rule for the period spanning 1956 to 2005 and identify regimes that roughly correspond to the terms of the Federal Reserve chairs. Murray et al. (2013) estimate a real-time forward-looking two-state Markov-switching Taylor rule to make inference about the periods when the Taylor principle was present. They find that the Fed consistently adhered to the Taylor principle before 1973 and after 1984, but did not follow the Taylor principle from 1980 to 1984.

Markov-switching monetary policy regimes have also been considered within the context of dynamic stochastic general equilibrium (DSGE) models. Eo (2009) estimates a Markov-switching DSGE model with recurring regime changes in the monetary policy rule coefficients, the technology coefficients, and the coefficients characterizing nominal price rigidities. In an application to postwar U.S. data, he finds stronger support for regime switching in monetary policy than in technology or nominal rigidities. Davig and Doh (2008) estimate a Markov-switching New Keynesian model that allows shifts in the monetary policy reaction coefficients and shock volatilities. Using U.S. data, they find that a more-aggressive monetary policy regime was in place after the Volcker disinflation and before 1970 than during the Great Inflation of the 1970s. Bianchi (2013) estimates a two-state model and finds that monetary policy has fluctuations between a Hawk and a Dove regime, with the latter prevalent in the 1970s and during the recent crisis.

Another strand of the literature estimates the monetary policy rule along with a fiscal policy rule. For example, Davig and Leeper (2006, 2011) estimate two-state Markov-switching monetary and fiscal policy rules to evaluate the presence of regimes of monetary or fiscal dominance. In their specification of the Markov-switching processes, two independent states drive the evolution of the monetary and fiscal policy rule coefficients. They find that monetary and fiscal policies fluctuate between active and passive behavior. In a Markov-switching DSGE framework, Bianchi (2012) specifies and estimates a model with monetary and fiscal policy rules whose coefficients' switching is driven by a single state. His estimates show that the monetary/fiscal policy mix has evolved over time and identifies three distinct regimes.

I apply the proposed estimation technique to a two-state Markov-switching forward-looking Taylor rule using quarterly data spanning 1960:1-2013:4. Interest rates at or below 0.25 percent are classified as censored, and the lower bound is set to that value. For the Markov-switching uncensored auxiliary regression, I take a fiscal policy rule where federal revenues respond to debt deviations from target and to the output gap.

Results imply that the estimated correlation between the switching states of the two policy rules is 0.82. Moreover, the null hypothesis of independent switching between the coefficients of the monetary and fiscal policy rules is rejected at conventional significance levels. The estimated coefficients allow us to classify the monetary/fiscal policy mix into four regimes according to the response of the interest rate to inflation and the reaction of revenues to debt: (i) a regime of weak interest rate response to inflation and weak tax response to debt, that I denominate regime F, for fiscal; (ii) a regime of weak interest rate response to inflation and strong tax response to debt, that I denominate regime I, for indeterminate; (iii) a regime of strong interest rate response to inflation and weak tax response to debt, that I denominate regime E, for explosive; (iv) a regime of strong interest rate response to inflation and strong tax response to debt, that I denominate regime M, for monetary. The estimated ergodic regime probabilities are: 43% for regime F, 7% for regime I, 2% for regime E, and 48% for regime M. The transition probabilities for the policy rule coefficients imply that regime M is expected to last about 11 quarters, regime F, about 20 quarters, regime I, 1.6 quarters, and E, 1 quarter.

The model's smoothed probabilities imply that in the fourth quarter of 2013 the economy was more likely in regime F, where the stance of monetary policy was accommodative and fiscal policy was paying more attention to output stabilization rather than to debt stabilization.

This document is structured as follows: in Section 2, I present the specification of a Markov-switching Taylor Rule at the zero lower bound. Section 3 develops the estimation procedure and the Monte Carlo exercise that justifies it. Results of the estimation appear in Section 4. Section 5 puts the results in context with the historical narrative on monetary and fiscal policy. Finally, Section 6 concludes.

2 A Markov-switching Taylor Rule at the Zero Lower Bound

I am interested in estimating the following two-state Markov-switching regression model of a monetary policy rule with a smoothing component:[1]

$$R_t^* = \rho_{S_{m,t}} R_{t-1} + \left(1 - \rho_{S_{m,t}}\right) \left(R_{S_{m,t}} + \alpha^{\pi}_{S_{m,t}} \pi_t + \alpha^{y}_{S_{m,t}} y_t\right) + \sigma_{S_{\sigma_R,t}} u_t \tag{1}$$

$$R_t = \max\left(\underline{R}, R_t^*\right) \tag{2}$$

$$S_{m,t} = 1, 2; \qquad S_{\sigma_R,t} = 1, 2,$$

where R_t^* is the underlying policy rate in period t, R_t is the observed policy rate in period t, π_t is a measure of the inflation rate in period t, y_t is a measure of the output gap in period t, and $u_t \sim \mathbb{N}(0,1)$ is a monetary policy shock to the policy rate. The observed interest rate is bounded from below by $\underline{R} \geq 0$.

$S_{m,t}$ and $S_{\sigma_R,t}$ are 2-state, possibly correlated, first-order Markov switching processes.

[1]More regimes could be allowed in the specification of the monetary policy rule for both the switching policy rule coefficients and the standard deviation of the shocks. However, to keep the exposition simple and to maintain a closer connection with the simulation exercise and the empirical part, I focus on a two-regime specification.

Their transition probabilities are

$$\mathbb{P}\left(S_{m,t} = j_m | S_{m,t-1} = j'_m\right) = p_{j_m j'_m} \tag{3}$$
$$\mathbb{P}\left(S_{\sigma_R,t} = j_{\sigma_R} | S_{\sigma_R,t-1} = j'_{\sigma_R}\right) = p_{j_{\sigma_R} j'_{\sigma_R}}. \tag{4}$$

Bae et al. (2012) show that Equation (1) is the empirical counterpart of a forward-looking monetary policy rule with a smoothing component subject to regime changes. A result of this specification is that the inflation rate and the output gap are correlated with the error term.

I show that, to estimate consistently this Markov-switching regression model, it is not enough to incorporate in the estimation the censored part of the process. In particular, inference about the prevalent regime over the censored period is inaccurate. The next section specifies a system of equations with interdependent Markov-switching coefficients. Interdependent switching is the key to identification of the prevalent regime of the economy over the censored part of the sample.

3 Estimation Procedure

This section sets up a system of equations with interdependent Markov-switching coefficients where one of the equations is censored, and develops the proposed estimation technique to identify the prevalent regime of the censored equation.

3.1 Setup

Consider the following Markov-switching regression model with a censored dependent variable:[2]

$$y^*_{1t} = x'_{1t}\beta_{1,S_{1t}} + \sigma_{1,S_{1t}} u_{1t}, \quad S_{1t} = 1, 2, \tag{5}$$
$$y_{1t} = \max(y_{1L}, y^*_{1t}), \tag{6}$$
$$y_{2t} = x'_{2t}\beta_{2,S_{2t}} + \sigma_{2,S_{2t}} u_{2t}, \quad S_{2t} = 1, 2, \tag{7}$$
$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \sim \text{iid } \mathbb{N}(\mathbf{0}_{2\times 1}, I_2) \tag{8}$$
$$\beta_{1,S_{1t}} = \sum_{j_1=1}^{2} \beta_{1,j_1} \tilde{S}_{1,j_1,t}; \qquad \sigma_{1,S_{1t}} = \sum_{j_1=1}^{2} \sigma_{1,j_1} \tilde{S}_{1,j_1,t} \tag{9}$$
$$\beta_{2,S_{2t}} = \sum_{j_2=1}^{2} \beta_{2,j_2} \tilde{S}_{2,j_2,t}; \qquad \sigma_{2,S_{2t}} = \sum_{j_2=1}^{2} \sigma_{2,j_2} \tilde{S}_{2,j_2,t}, \tag{10}$$

[2]For simplicity of exposition, I assume that the state that drives the switching in the conditional mean parameters also drives the switching in the standard deviation of the shocks. I will relax this assumption to conduct the estimation of the Markov-switching model of the federal funds rate given in specification (1)-(4).

where

$$\tilde{S}_{i,j_i,t} = \begin{cases} 1, & \text{if } S_{it} = j_i; j_i = 1,2; i = 1,2 \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

and where y_{1t} and y_{2t} are 1×1; x_{1t} and x_{2t} are $k_1 \times 1$ and $k_2 \times 1$, respectively, vectors of explanatory variables. I assume that y_{1t} conditional on S_{1t}, and x_{1t} are covariance stationary. The same holds for y_{2t} conditional on S_{2t}, and for x_{2t}. Following Kim (2009), to allow for non-zero correlation between S_{1t} and S_{2t}, I introduce the following four-state Markov-switching process S_t:

$$S_t = (S_{2t} - 1)\, 2 + S_{1t}, \qquad S_{it} = 1, 2, i = 1, 2, \tag{12}$$

where the transition probabilities are given by

$$\begin{aligned} \mathbb{P}\left(S_t = j | S_{t-1} = j'\right) &= \mathbb{P}\left(S_{1t} = j_1, S_{2t} = j_2 | S_{1,t-1} = j_1', S_{2,t-1} = j_2'\right) \\ &= p_{jj'}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} j &= (j_2 - 1) J_1 + j_1, \\ j' &= (j_2' - 1) J_1 + j_1', \end{aligned}$$

with $\sum_{j=1}^{J} p_{jj'} = 1$. The marginalized transition probabilities for S_{1t} and S_{2t} are given by

$$p_{1,j_1 j_1'} = \mathbb{P}\left(S_{1t} = j_1 | S_{1,t-1} = j_1'\right), \tag{14}$$

$$p_{2,j_2 j_2'} = \mathbb{P}\left(S_{2t} = j_2 | S_{2,t-1} = j_2'\right), \tag{15}$$

which can be obtained using the derivation in Kim (2009).

I assume that the explanatory variables x_{1t} and x_{2t} are uncorrelated with the error terms of their respective equations, u_{1t} and u_{2t}. In case of correlation with the error terms, the approaches in Kim (2004) or Kim (2009) can be added to the system above.

Notice that the errors of Equations (5) and (7) are independent. The dependence between y_{1t} and y_{2t} occurs only through the dependent switching of the coefficients of both equations.

3.2 Maximum Likelihood Estimation

Let $y_t = [y_{1t} \quad y_{2t}]'$, $x_t = [x_{1t}' \quad x_{2t}']'$. Let $\mathfrak{F}_{i,t-1} = \sigma\left(x_{i1}, x_{i2}, \ldots, x_{it}, y_{i0}, y_{i1}, \ldots, y_{i,t-1}\right)$ for $i = 1, 2$ be the sigma-algebras generated by the vectors of exogenous random variables of Equations (5)-(7), and let $\mathfrak{F}_{t-1} = \sigma\left(x_1, x_2, \ldots, x_t, y_0, y_1, \ldots, y_{t-1}\right)$ be the sigma-algebra generated by the vectors of all exogenous random variables. Let $\theta = [\theta_1' \quad \theta_2' \quad vec(\tilde{p})']'$ be the vector of parameters of the model, where

$$\begin{aligned} \theta_1 &= \left[\beta_{1,1}' \quad \beta_{1,2}' \quad \sigma_{1,1} \quad \sigma_{1,2}\right]', \\ \theta_2 &= \left[\beta_{2,1}' \quad \beta_{2,2}' \quad \sigma_{2,1} \quad \sigma_{2,2}\right]', \end{aligned}$$

and $\tilde{p}$ is a $4{\times}4$ matrix of transition probabilities given in (13). For consistent and efficient estimation of the model (5)-(13), I maximize the log-likelihood function, $\mathfrak{L}_\theta(\theta; Y_T) = \ln f_Y(Y_T; \theta)$, with respect to θ by applying the conventional Hamilton (1989) filter, where $Y_t = \{y_s\}_{s=1}^{t}$. The filter allows obtaining $f_Y(Y_T;\theta)$ as follows:

$$
\begin{aligned}
f_Y(Y_T;\theta) &= \prod_{t=1}^{T} f_y(y_t|\mathfrak{F}_{t-1};\theta) \\
&= \prod_{t=1}^{T}\sum_{j=1}^{4} f_{y|S}(y_t|S_t=j,\mathfrak{F}_{t-1};\theta)\mathbb{P}(S_t=j|\mathfrak{F}_{t-1};\theta),
\end{aligned}
$$

where

$$
f_{y|S}(y_t|S_t=j,\mathfrak{F}_{t-1};\theta) = f_{y_1|S_1}(y_{1t}|S_{1t}=j_1,\mathfrak{F}_{1,t-1};\theta_1)f_{y_2|S_2}(y_{2t}|S_{2t}=j_2,\mathfrak{F}_{2,t-1};\theta_2),
$$

with

$$
\begin{aligned}
f_{y_1|S_1}(y_{1t}|S_{1t}=j_1,\mathfrak{F}_{1,t-1};\theta_1) &= \left[\Phi\left(\frac{y_{1L}-x_{1t}'\beta_{1,j_1}}{\sigma_{1,j_1}}\right)\right]^{1[y_{1t}=y_{1L}]}\times \\
&\quad\times\left[\frac{1}{\sigma_{1,j_1}}\phi\left(\frac{y_{1t}-x_{1t}'\beta_{1,j_1}}{\sigma_{1,j_1}}\right)\right]^{1[y_{1t}>y_{1L}]}, \\
f_{y_2|S_2}(y_{2t}|S_{2t}=j_2,\mathfrak{F}_{2,t-1};\theta_2) &= \frac{1}{\sigma_{2,j_2}}\phi\left(\frac{y_{2t}-x_{2t}'\beta_{2,j_2}}{\sigma_{2,j_2}}\right),
\end{aligned}
$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ denote the distribution and density functions, respectively, of the standard normal distribution, and

$$
\mathbb{P}(S_t=j|\mathfrak{F}_{t-1};\theta) = \sum_{j'=1}^{4} p_{jj'}\mathbb{P}(S_{t-1}=j'|\mathfrak{F}_{t-1};\theta). \tag{16}
$$

Once y_t is realized at the end of time t, the filtered probability of S_t in (16) is updated as

$$
\mathbb{P}(S_t=j|\mathfrak{F}_t;\theta) = \frac{f_{y|S}(y_t|S_t=j,\mathfrak{F}_{t-1};\theta)\mathbb{P}(S_t=j|\mathfrak{F}_{t-1};\theta)}{f_y(y_t|\mathfrak{F}_{t-1};\theta)}. \tag{17}
$$

3.3 Why Is Interdependent Switching Necessary?

In this section, I discuss the need to introduce the auxiliary Equation (7) and correlated states S_{1t} and S_{2t}. I show that if censoring is ignored, the estimates of $\beta_{1,S_{1t}}$, $\sigma_{1,S_{1t}}$, $p_{j_1,j_1'}$ are biased, and that inference about the prevalent regime S_{1t} is not accurate. When censoring is introduced in the specification, the biases in $\beta_{1,S_{1t}}$, $\sigma_{1,S_{1t}}$, and $p_{j_1,j_1'}$ are corrected, but inference about the prevalent regime remains inaccurate. I finally show that when the system (5)-(13) is estimated jointly, discrimination of the prevalent regime is possible.

To show the potential estimation problem and the features of the proposed solution, I perform a Monte Carlo experiment where the model is specified as in (5)-(13) with $J_1 =$

$J_2 = 2$, $k_1 = k_2 = 1$, $T = 200$, and

$$
\begin{aligned}
\beta_{1,j_1} &= \begin{cases} 0.5 & \text{if } j_1 = 1 \\ 1.5 & \text{if } j_1 = 2 \end{cases}, \qquad \sigma_{1,j_1} = \begin{cases} 0.05 & \text{if } j_1 = 1 \\ 0.05 & \text{if } j_1 = 2 \end{cases}, \\
\beta_{2,j_2} &= \begin{cases} 0 & \text{if } j_2 = 1 \\ 0.1 & \text{if } j_2 = 2 \end{cases}, \qquad \sigma_{2,j_2} = \begin{cases} 0.005 & \text{if } j_2 = 1 \\ 0.005 & \text{if } j_2 = 2 \end{cases}, \\
x_{1t} &\sim \begin{cases} \mathcal{U}(1,2) & \text{if } t \leq 150 \\ \mathcal{U}(-1,0) & \text{if } 151 \leq t \leq 200 \end{cases}, \\
x_{2t} &\sim \mathcal{U}(0,1), \\
\tilde{p} &= \begin{bmatrix} 0.3 & 0.2 & 0.2 & 0.1 \\ 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 \\ 0.6 & 0.7 & 0.7 & 0.8 \end{bmatrix}, \\
\operatorname{corr}(S_{1t}, S_{2t}) &= 0.67, \\
y_{1L} &= 0.
\end{aligned}
\tag{18}
$$

In the benchmark specification, censoring of y^*_{1t} occurs over the final 25% of the sample. I implement this censoring by switching x_{1t} to a different distribution, as shown in (18). I choose a cluster of periods where censoring occurs to illustrate the severity of the problem at obtaining the estimates of the prevalent regime.

In the Monte Carlo analysis, I obtain estimates of β_{1,j_1}, σ_{1,j_1}, $p_{1,j_1 j_1'}$ for $j_1 = 1, 2$, and the smoothed estimate of $\mathbb{P}(S_{1t} = j_1)$ for $t = 1, 2, \ldots, 200$ under three scenarios:

(i) Ignoring both censoring of y_{1t} and joint switching between S_{1t} and S_{2t}.

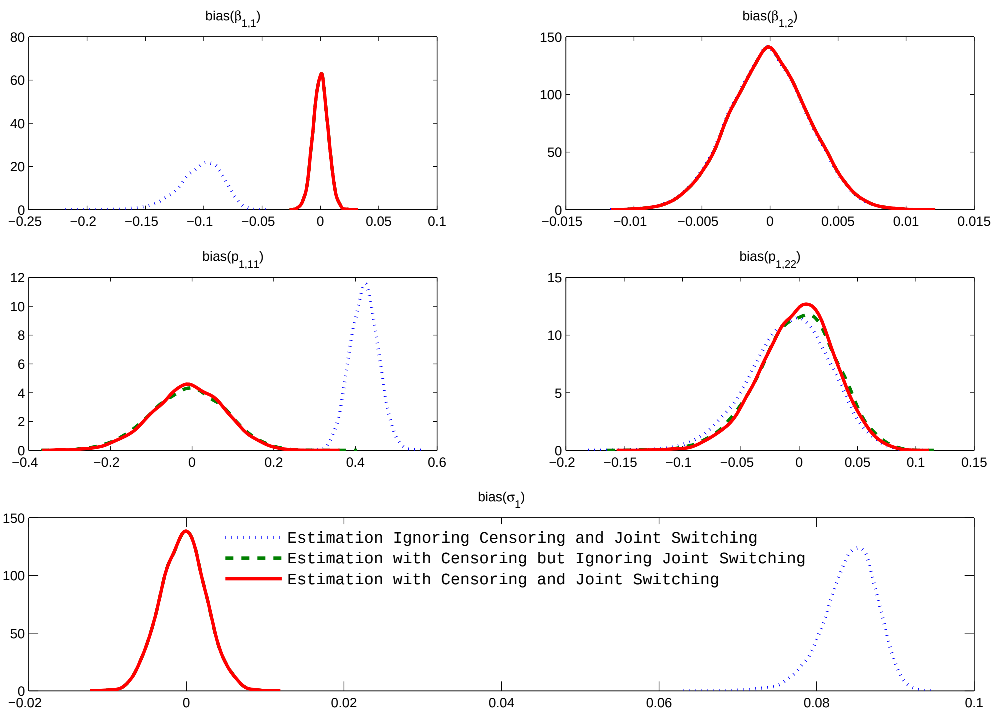
(ii) Allowing for censoring of y_{1t}, but ignoring joint switching between S_{1t} and S_{2t}.

(iii) Allowing for both censoring of y_{1t} and joint switching between S_{1t} and S_{2t}.

Appendices A and B obtain the likelihood functions for cases (i) and (ii), respectively. The likelihood function for case (iii) was obtained in Section 3.2. I simulate and estimate the model 10,000 times.

Figure 2 shows the bias in the estimates of the parameters under the three scenarios listed above. The results show that the estimation under scenario (i) yields biased estimates as would have been expected. In particular, the estimates of $\beta_{1,1}$ is downward biased, while the estimates of σ_1 and $p_{1,11}$ are upward biased. The downward bias in $\beta_{1,1}$ is due to the estimation attributing to a low slope coefficient the fact that $y_{1t} = 0$ in the final 25% of the sample. The persistent censoring implies an upward bias in $p_{1,11}$, the probability of remaining in the low-β state. A higher standard deviation of shocks is also needed to reconcile the fact that $y_{1t} = 0$ while x_{1t} takes negative values. On the other hand, $\beta_{1,2}$ and $p_{1,22}$ do not seem to suffer from a bias problem. The information that the estimation obtains from the uncensored part of the sample seems enough to obtain accurate estimates of these parameters.

Figure 2: Parameter Bias in the Benchmark Case



Estimation under scenarios (ii), which incorporates censoring, and (iii), which incorporates censoring and joint switching, yield unbiased parameter estimates, as expected. Thus, estimation under scenario (iii) is not necessary for unbiasedness.

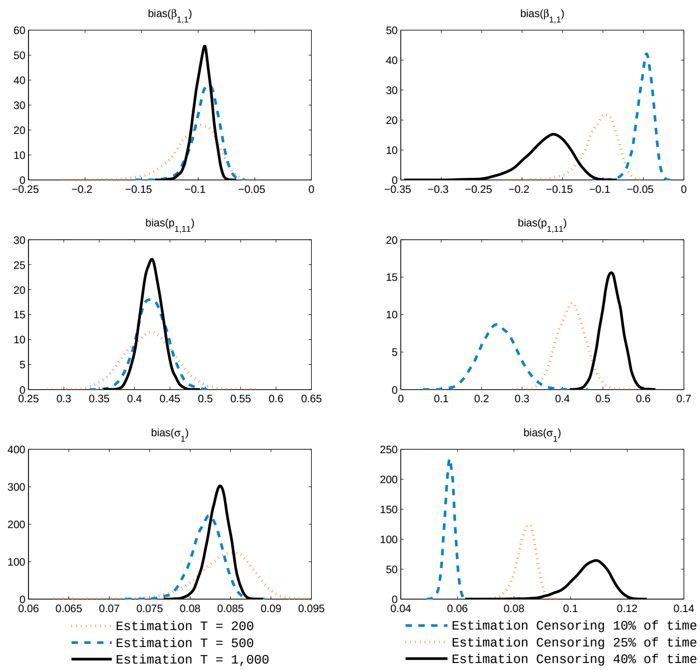
I also analyze the effect of increasing the sample size and of changing the length of censoring at the end of the sample. Figure 3 reports the changes on the biases of $\beta_{1,1}$, σ_1, and $p_{1,11}$ estimated under scenario (i). Increasing the sample size does not reduce the bias of the estimates, suggesting that there is a problem of consistency. On the other hand, and as expected, reducing the length of censoring over the final part of the sample reduces the biases.

3.3.1 Discriminating the Prevalent Regime

I now investigate the ability of the estimation strategies to identify correctly the prevalent regime. In the Monte Carlo exercise, I have set the standard deviation of the shock to the censored variable, σ_1, so that there is an almost perfect discrimination of the states over the uncensored part of the sample. Hence, to evaluate the capabilities of the three estimation scenarios at discriminating correctly the prevalent regimes, I focus on the censored part of the sample only.

To measure the ability of the estimation techniques to identify the prevalent regimes, I use the area under the Receiver Operating Characteristic (ROC) curve. The ROC curve is a plot that assesses the performance of a binary classifier system as its discrimination threshold is changed. The ROC curve was first developed by electrical engineers and radar engineers during World War II to evaluate their capabilities to detect enemy objects in battlefields, and

Figure 3: Parameters Biases in the Benchmark Case Ignoring Censoring and Joint Switching - Effects of Sample Size and Censoring Length



was introduced in psychology to account for perceptual detection of signals (see Peterson et al., 1954; Swets, 1979). The use of ROC curves in medicine to assess diagnostic test performance has been described by Lusted (1971). In our case, I obtain smoothed estimates of $\mathbb{P}\left(S_{1t}=1\right)$, vary the discrimination threshold between 0 and 1, and evaluate the ability of the smoothed estimates of $\mathbb{P}\left(S_{1t}=1\right)$ to classify correctly the prevalent regime, which is given by the simulated states.

The ROC curve plots the fraction of true positives out of the total of actual positives, called true positive rate (TPR), against the fraction of false positives, called false positive rate (FPR), at various threshold settings. TPR is also known as Sensitivity, and FPR is known as one minus the Specificity or true negative rate. Given a cut-off value $q \in [0,1]$, a realization of $\{S_{1t}\}_{t=1}^{200}$, and smoothed estimates of $\mathbb{P}\left(S_{1t}=1\right)$, I can tabulate a contingency table like Table 1. Varying the cut-off value $q \in [0,1]$ allows obtaining Sensitivity values that can be plotted against 1-Specificity values, which is the ROC curve.

A perfectly discriminating variable would have Sensitivity and Specificity both equal to 1. If a cut-off value existed to produce such a test, then Sensitivity would be 1 for any non-zero values of 1-Specificity. The ROC curve would start at the origin (0,0), go vertically up the y-axis to (0,1), and then horizontally across to (1,1) (see Bewick et al., 2004). On the other hand, a completely random guess would give a point along a diagonal line that starts at the origin (0,0) and go diagonally to (1,1). In that no-discrimination case, the discriminating variable would produce a TPR equal to its FPR, or Sensitivity = 1-Specificity.

The performance of a discriminating variable can be quantified by calculating the area under the ROC curve. An ideal discriminating variable would have an area under the ROC

Table 1: Contingency Table

		Regime	
		$S_{1t} = 1$	$S_{1t} = 2$
Predicted Regime	$\mathbb{P}\left(S_{1t} = 1\right) \geq q$	True Positive	False Positive
	$\mathbb{P}\left(S_{1t} = 1\right) < q$	False Negative	True Negative
		Sensitivity $= \frac{\sum_{t=1}^{200} \mathbb{1}_{\{\mathbb{P}(S_{1t}=1)\geq q\}}}{\sum_{t=1}^{200} \mathbb{1}_{\{S_{1t}=1\}}}$	*Specificity* $= \frac{\sum_{t=1}^{200} \mathbb{1}_{\{\mathbb{P}(S_{1t}=1)< q\}}}{\sum_{t=1}^{200} \mathbb{1}_{\{S_{1t}=2\}}}$

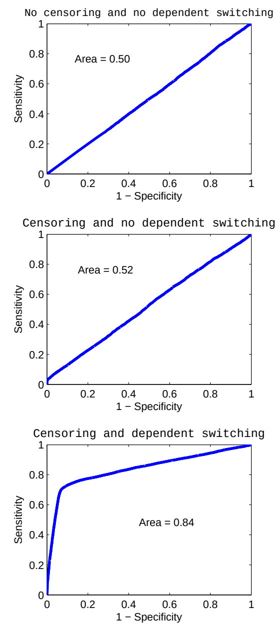
curve of 1, whereas a random guess would have an area under the ROC curve of 0.5.

Figure 4 plots the ROC curves and reports the areas under the ROC curves for the estimation of model (18) under the three scenarios mentioned before: (i) Ignoring both censoring of y_{1t} and joint switching between S_{1t} and S_{2t}; (ii) Allowing for censoring of y_{1t} but ignoring joint switching between S_{1t} and S_{2t}; (iii) Allowing for both censoring of y_{1t} and joint switching between S_{1t} and S_{2t}. The figure shows that, over the censored part of the sample, the only estimation scenario that allows for some degree of discrimination of the prevalent regime is the one with censoring of y_{1t} and joint switching between S_{1t} and S_{2t}. The areas under the ROC curves for the scenarios that do not allow for joint switching are very close to 0.5, whereas the area under the ROC curve for the joint switching scenario is about 0.84. This indicates that, to identify the prevalent regime over the censored part of the sample, an auxiliary uncensored Markov-switching regression whose coefficients switch in a correlated manner with the coefficients of the variable of interest is needed.[3]

Figure 5 plots the ROC curves and reports the areas under the ROC curves for the effects of changing the sample size, T, the frequency of censoring, and the correlation between S_{1t} and S_{2t}. The results show that the estimation procedure that incorporates censoring and dependent switching is not affected in its discrimination ability when the sample size increases to $T = 500$ or $T = 1,000$. Changing the proportion of the sample that is subject to censoring to 10% or 40% does not change the performance of the estimation technique, either. Finally, the results show that eliminating the correlation between S_{1t} and S_{2t} annihilates the ability of the estimation technique to identify the prevalent regime, while a perfect correlation between the latent states implies a nearly perfect discrimination, as it would have been

[3] It turns out that if we allow the standard deviation of the shock of censored variable, σ_1, to switch, there is lack of discrimination of the state driving the evolution of this standard deviation. The lack of discrimination is partially solved if in specification (18) of the Monte Carlo exercise I allow the standard deviation σ_1 to switch between regimes as a function of a latent state that drives the switching of β_1. In that case, the ROC curves show that discrimination of the prevalent regime improves when a joint estimation with an auxiliary Markov-switching regression is performed. Additional simulations (not shown here) illustrate that the higher the correlation between the latent state driving the standard deviation, σ_1, and the latent states S_1 or S_2, the better the discrimination of the prevalent regime of the standard deviation.

Figure 4: Area under the ROC Curve in the Benchmark Case



expected. This exercise highlights the importance of an auxiliary regression with Markov-switching coefficients whose state is correlated with the state of the coefficients of the censored variable. The higher the correlation between switching states, the better the discrimination of the prevalent regime of the coefficients of interest over the censoring period.

3.3.2 Why Does Joint Switching Help Identify the Prevalent Regime over the Censoring Period?

Under case (ii), that is, allowing for censoring but not for joint switching, I use the Tobit-like specification for the density function $f_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1)$ that appears in Appendix B. In this case, as shown in the previous section, the Hamilton filter is unable to identify the prevalent regime S_{1t} over the censoring period. The cause of the lack of discrimination lies in the fact that

$$\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1t}; \theta_1) = \frac{f_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1)\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1,t-1}; \theta_1)}{f_{y_1}(y_{1t}|\mathfrak{F}_{1,t-1}; \theta_1)} \tag{19}$$

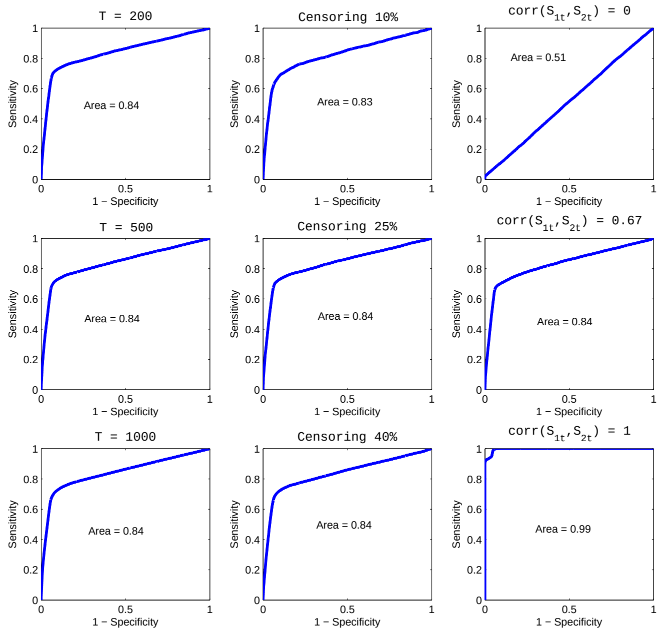
may not vary enough over the censored sample. In particular,

$$f_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1) = \Phi\left(\frac{y_{1L} - x'_{1t}\beta_{1,j_1}}{\sigma_{1,j_1}}\right)$$

will show little variation over the censored sample if $\frac{y_{1L}-x'_{1t}\beta_{1,j_1}}{\sigma_{1,j_1}}$ is too small or too large. If that is the case, $\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1t}; \theta_1)$ will have very little variation.

In contrast, incorporating joint switching in the estimation allows to write the updated

Figure 5: Area under the ROC Curve in the Benchmark Case with Censoring and Dependent Switching - Effects of Sample Size, Censoring Frequency and Correlation between States



probability $\mathbb{P}(S_{1t} = j_1 | \mathfrak{F}_t; \theta)$, using (17), as

$$\begin{aligned}
\mathbb{P}(S_{1t} = j_1 | \mathfrak{F}_t; \theta) &= \sum_{j_2=1}^{2} \mathbb{P}(S_{1t} = j_1, S_{2t} = j_2 | \mathfrak{F}_t; \theta) \\
&= \sum_{j_2=1}^{2} \frac{f_{y|S_1,S_2}(y_t|\mathfrak{F}_{t-1}, S_{1t} = j_1, S_{2t} = j_2; \theta)\mathbb{P}(S_{1t} = j_1, S_{2t} = j_2|\mathfrak{F}_{t-1}; \theta)}{f_y(y_t|\mathfrak{F}_{t-1}; \theta)} \\
&= \sum_{j_2=1}^{2} \frac{f_{y_1|S_1}(\cdot) f_{y_2|S_2}(\cdot)\mathbb{P}(S_{1t} = j_1, S_{2t} = j_2|\mathfrak{F}_{t-1}; \theta)}{f_y(y_t|\mathfrak{F}_{t-1}; \theta)} \\
&= \frac{f_{y_1|S_1}(y_{1t}|\mathfrak{F}_{1,t-1}, S_{1t} = j_1; \theta_1)\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{t-1}; \theta)}{f_{y_1}(y_{1t}|\mathfrak{F}_{t-1}; \theta)} \\
&\times \sum_{j_2=1}^{2} \frac{f_{y_2|S_2}(y_{2t}|\mathfrak{F}_{2,t-1}, S_{2t} = j_2; \theta_2)\mathbb{P}(S_{2t} = j_2|S_{1t} = j_1, \mathfrak{F}_{2,t-1}; \theta)}{f_{y_2|y_1}(y_{2t}|y_{1t}, \mathfrak{F}_{t-1}; \theta)} \\
&= \frac{f_{y_1|S_1}(y_{1t}|\mathfrak{F}_{1,t-1}, S_{1t} = j_1; \theta_1)\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{t-1}; \theta)}{f_{y_1}(y_{1t}|\mathfrak{F}_{t-1}; \theta)} \\
&\times \sum_{j_2=1}^{2} \mathbb{P}\left(S_{2t} = j_2|\mathfrak{F}_{2t}; \theta_2\right) \frac{\mathbb{P}(S_{2t} = j_2|S_{1t} = j_1, \mathfrak{F}_{2,t-1}; \theta)}{\mathbb{P}(S_{2t} = j_2|\mathfrak{F}_{2,t-1}; \theta_2)} \frac{f_{y_2}(y_{2t}|\mathfrak{F}_{2,t-1}; \theta_2)}{f_{y_2|y_1}(y_{2t}|y_{1t}, \mathfrak{F}_{t-1}; \theta)},
\end{aligned} \tag{20}$$

where the step from the next-to-last to the last equation uses the definition

$$\mathbb{P}(S_{2t}=j_2|\mathfrak{F}_{2,t};\theta_2)=\frac{f_{y_2|S_2}(y_{2t}|\mathfrak{F}_{2,t-1},S_{2t}=j_2;\theta_2)\mathbb{P}(S_{2t}=j_2|\mathfrak{F}_{2,t-1};\theta_2)}{f_{y_2}(y_{2t}|\mathfrak{F}_{2,t-1};\theta_2)}.$$

If S_{1t} and S_{2t} are uncorrelated, the last line of (20) is equal to one. In that case, inference about S_{1t} obtained from (20) would be the same as inference about S_{1t} obtained from (19). Hence, discrimination about the prevalent regime would be unfeasible. It is the additional information given by the degree of interdependence between the latent states what allows a better inference about the prevalent regime of S_{1t}.

4 Estimating a Markov-Switching Taylor Rule at the Zero Lower Bound

In this section, I apply the proposed technique to estimate a Taylor rule with Markov-switching coefficients including the sample period after the financial crisis, where the federal funds rate has been at the effective lower bound.

4.1 Selecting the Auxiliary Regression

To implement the estimation procedure presented in Section 3, I need an auxiliary Markov-switching regression that is not subject to censoring and whose switching could be correlated with the switching of the coefficients of the Taylor rule.

Gonzalez-Astudillo (2013) estimates time-varying monetary and fiscal policy rules whose coefficients are driven by correlated latent factors and finds a non-negligible degree of interdependence between the coefficients of the policy rules. This finding is related to the literature on monetary-fiscal policy interactions initiated by Leeper (1991) and followed by Davig and Leeper (2006) and Chung et al. (2007), among others. Along these lines, I propose a fiscal policy rule with Markov-switching coefficients to be the auxiliary regression. I will test for interdependence between the switching of the Taylor rule coefficients and the coefficients of the proposed fiscal policy rule to confirm that this is an adequate choice.

4.2 Setting up the System to be Estimated

The system to be estimated in order to consistently obtain the estimates the Markov-switching coefficients of the Taylor rule, as well as to make inference about the prevalent regime, is given by

$$R_t^*=\rho^R_{S_{m,t}}R_{t-1}+\left(1-\rho^R_{S_{m,t}}\right)\left(R_{S_{m,t}}+\alpha^\pi_{S_{m,t}}\pi_t+\alpha^y_{S_{m,t}}y_t\right)+\sigma^R_{S_{\sigma^R,t}}u_t^R, \tag{I}$$

$$R_t=\max\left(\underline{R},R_t^*\right) \tag{II}$$

$$\tau_t=\rho^\tau_{S_{f,t}}\tau_{t-1}+\left(1-\rho^\tau_{S_{f,t}}\right)\left(\tau_{S_{f,t}}+\gamma^b_{S_{f,t}}b_{t-1}+\gamma^y_{S_{f,t}}y_t\right)+\sigma^\tau_{S_{\sigma^\tau},t}u_t^\tau, \tag{III}$$

where R_t is the policy rate t, π_t is the inflation rate in period t, y_t is the output gap in period t, and $u_t^R \sim \mathbb{N}(0,1)$ is a monetary policy shock to the federal funds rate. The observed interest rate is bounded from below by $\underline{R} = 0.25$. In the auxiliary equation, τ_t is a measure of federal government receipts net of transfers in period t, b_{t-1} is a measure of federal government debt in period $t-1$, and $u_t^\tau \sim \mathbb{N}(0,1)$ is a fiscal policy shock to receipts net of transfers.

I introduce dependent switching between $S_{m,t}$ and $S_{f,t}$ by specifying the following 4-state Markov-switching process S_t:

$$S_{mf,t} = (S_{f,t} - 1)\,2 + S_{m,t}, \qquad S_{i,t} = 1, 2, \qquad \text{for } i = f, m,$$

where the transition probabilities are given by:

$$\begin{aligned}\mathbb{P}\left(S_{mf,t} = j | S_{mf,t-1} = j'\right) &= \mathbb{P}\left(S_{m,t} = j_m, S_{f,t} = j_f | S_{m,t-1} = j'_m, S_{f,t-1} = j'_f\right) \\ &= p_{jj'},\end{aligned}$$

and

$$\begin{aligned} j &= (j_f - 1)\,2 + j_m, \\ j' &= \left(j'_f - 1\right)2 + j'_m, \end{aligned}$$

with $\sum_{j=1}^{4} p_{jj'} = 1$. I denote as P_{mf} the 4×4 transition probability matrix of $S_{mf,t}$.

The standard deviations of the monetary and fiscal policy rule shocks change according to the 2-state Markov-switching regimes $S_{\sigma^R,t}$ and $S_{\sigma^\tau,t}$, respectively. Their 2×2 transition probability matrices are denoted as P_{σ_R} and P_{σ_τ}.

With this specification for the number of regimes of the states S_m, S_f, S_{σ^R}, and S_{σ_τ}, the model yields 16 transition probabilities to be estimated: $12 = (16-4)$ in P_{mf}, $2 = (4-2)$ in P_{σ^R}, and $2 = (4-2)$ in P_{σ^τ}. Allowing for more regimes in S_m, S_f, S_{σ^R}, or S_{σ^τ} would imply an increasing number of transition probabilities to be estimated that could result in an unfeasible estimation, in particular if the latent states are correlated.

In this setup, the policy rules have endogenous explanatory variables, namely the inflation rate and the output gap, so that I implement the two-step maximum likelihood estimation procedure proposed by Kim (2009).

4.3 Data

I use quarterly data from 1960:1 to 2013:4. The policy rate is the federal funds rate. Inflation is the percentage change over the last four quarters of the price level given by the GDP price deflator.[4] The output gap is the log difference between real GDP and the Congressional Budget Office's measure of potential real GDP. These variables are obtained from the FRED database. Receipts net of transfers corresponds to the seasonally adjusted quarterly current receipts of the federal government from which the current transfer payments

[4] I use the GDP price deflator instead of CPI to make the estimation comparable with that of Davig and Leeper (2006), and the fact that estimated Markov-switching DSGE models use the price deflator as observable to estimate policy rules (see for example Bianchi, 2012).

have been deducted. These variables are obtained from NIPA Table 3.2, lines 1 and 22, respectively. Debt is the market value of privately held gross federal debt at the end of the quarter. This variable comes from the Federal Reserve Bank of Dallas. To correct for endogeneity, I use M2 growth, given by the percentage change over the last four quarters of seasonally adjusted M2, and commodity price inflation, given by the percentage change over the last four quarters of the commodity price index. These last two variables are also obtained from FRED. Appendix C describes the construction of the data series in more detail.

4.4 Estimation Results

This section analyzes the results of the estimation by performing a set of hypothesis tests to choose a parsimonious model in terms of switching parameters and the specification of joint switching between the coefficients of the two policy rules.

To correct for the endogeneity of regressors, I perform a two-stage constant-parameter estimation where the inflation rate and the output gap are regressed against a set of instruments that include: four lags of the inflation rate, four lags of the output gap, four lags of M2 growth, and four lags of inflation of the commodity price index. The (standardized) residuals from these regressions appear as additional regressors with Markov-switching coefficients in the specifications of Equations (I) and (III).

4.4.1 Finding a Parsimonious Specification

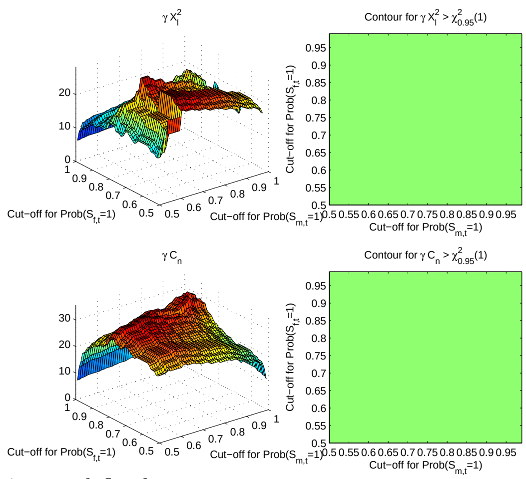
To find a parsimonious specification of the model (I)-(III), I estimate an unconstrained model where all the coefficients are subject to switching and test for independence between S_m and S_f, the states of the monetary and fiscal policy rule coefficients, respectively.[5] To contrast the hypothesis of independent switching, I use the conventional independence chi-square test based on 2×2 contingency tables. Tavaré and Altham (1983) modify the conventional test of independence based on contingency tables for the case when the data are generated by first-order Markov sequences.

To implement the independence tests, I obtain the smoothed probabilities $\mathbb{P}(S_{m,t} = 1)$ and $\mathbb{P}(S_{f,t} = 1)$ from the unconstrained model and write 2×2 contingency tables varying in the range $[0.5, 1]$ the threshold at which it is decided that $S_{m,t} = 1$ or $S_{f,t} = 1$. With each of these contingency tables, I calculate the two test statistics for independence between S_m and S_f obtained by Tavaré and Altham (1983) which, under the null, are distributed as a chi-square with one degree of freedom.

Figure 6 shows the value of the statistics for testing the null hypothesis of independence between S_m and S_f for different thresholds of $\mathbb{P}(S_{m,t} = 1)$ and $\mathbb{P}(S_{f,t} = 1)$ on the left hand side, and the contour of the figure on the left for values of the statistics greater than the critical value that corresponds to a chi-square with one degree of freedom. Both test statistics reject the null hypothesis of independence between S_m and S_f at the 5% level of significance for all the values of the thresholds. Thus, I conclude that S_m and S_f should be specified with a joint 4×4 transition probability matrix that needs to be estimated.

[5] A full description of the results obtained from the estimation of the unconstrained model appears in Appendix D.

Figure 6: Independence Test between S_m and S_f



Notes: The tests statistics are defined as:

$$X_l^2 = \left\{ \ln\left(\frac{n_1 n_4}{n_3 n_2}\right) \right\}^2 \Bigg/ \left(\frac{1}{np_1} + \frac{1}{np_2} + \frac{1}{np_3} + \frac{1}{np_4}\right)$$

$$C_n = \frac{n\left(n_1 n_4/n^2 - n_2 n_3/n^2\right)^2}{n^{-4}\left(n_1+n_2\right)\left(n_1+n_3\right)\left(n_2+n_4\right)\left(n_3+n_4\right)},$$

where $p_j = \mathbb{P}\left(S=j\right)$, and $S{=}(S_f-1)2+S_m$ and $j=(j_f-1)2+j_m$. Also, $n_j = \sum_{t=1}^{n} \mathbb{1}_{\{S_t=j\}}$. Additionally, $\gamma = (1-\mu\lambda)(1+\mu\lambda)$, where μ and λ are the nonunit eigenvalues of P_m and P_f, the 2×2 transition probability matrices of S_m and S_f, respectively.

With respect to which should be specified as switching coefficients, the test statistic of the likelihood ratio test does not reject policy rules whose intercepts and persistence coefficients are invariant between regimes. The value of the log-likelihood function of the model where all the coefficients are allowed to switch is -53.39, while the model with fixed intercepts and persistence coefficients yields a log-likelihood function value equal to -56.20. The value of the statistic is 5.62, lower than the critical value implied by the chi-square distribution with four degrees of freedom at the 5% level of significance, which is 9.49. To test for switching regimes on the remaining coefficients, I use the z-statistics of the difference between the coefficients of the two regimes. The statistics reject the null hypotheses of constant coefficients.

4.4.2 Obtaining Estimates of the Parsimonious Specification

In the final specification, I estimate the system of Markov-switching Equations (I)-(III) with intercepts and smoothing coefficients fixed between regimes and four switching states: $S_m = 1, 2$, $S_f = 1, 2$, $S_{\sigma^R} = 1, 2$, and $S_{\sigma^\tau} = 1, 2$, where S_m and S_f have a joint transition probability matrix, denoted as P_{mf}, that corresponds to the four-regime state $S_{mf} = (S_f - 1)2 + S_m$, with $S_i = 1, 2$ for $i = m, f$. Results of the estimation under this specification appear in Table 2.

Table 2: Parameter Estimates

Monetary Policy Rule

Parameters	$j_m = 1$	$j_m = 2$
$\alpha^\pi_{j_m}$	0.53	1.67
	(4.27)	(4.70)
$\alpha^y_{j_m}$	1.61	0.38
	(7.14)	(-7.48)
R_{j_m}	2.59	
	(5.65)	
$\rho^R_{j_m}$	0.88	
	(60.68)	
	$j_{\sigma^R} = 1$	$j_{\sigma^R} = 2$
$\sigma^R_{j_{\sigma^R}}$	0.23	1.16
	(10.64)	(8.24)

For $j_m = 1$ and $j_{\sigma^R} = 1$, values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero. For $j_m = 2$ and $j_{\sigma^R} = 2$, values in parenthesis are z-statistics of the null hypothesis that the difference between the coefficients of the two regimes is zero.

Fiscal Policy Rule

Parameters	$j_f = 1$	$j_f = 2$
$\gamma^b_{j_f}$	0.05	0.13
	(2.74)	(4.24)
$\gamma^y_{j_f}$	0.29	0.08
	(4.35)	(-5.58)
τ_{j_f}	1.76	
	(10.20)	
$\rho^\tau_{j_f}$	0.90	
	(61.80)	
	$j_{\sigma^\tau} = 1$	$j_{\sigma^\tau} = 2$
$\sigma^\tau_{j_{\sigma^\tau}}$	0.09	0.48
	(14.25)	(5.93)

For $j_f = 1$ and $j_{\sigma^\tau} = 1$, values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero. For $j_f = 2$ and $j_{\sigma^\tau} = 2$, values in parenthesis are z-statistics of the null hypothesis that the difference between the coefficients of the two regimes is zero.

The estimated smoothing coefficient of the monetary policy rule implies that about 12% of the adjustment of the interest rate with respect to the target occurs every quarter. The monetary policy rule coefficients on inflation take the values 0.53 and 1.67, depending on the regime. The monetary policy rule coefficients on the output gap take the values 1.61 and 0.38, depending on the regime. Notice that when the monetary authority is hawkish on inflation, less attention is given to the output gap in comparison to the regime when the monetary authority is dovish. With respect to volatility, the standard deviation of the interest rate takes the values 0.23% in the low volatility regime and 1.16% in the high volatility regime.

In regard to the fiscal policy rule, the smoothing coefficient implies that about 10% of the adjustment with respect to the target occurs every quarter. The fiscal policy rule coefficients on debt take the values 0.05 and 0.13, depending on the regime. The fiscal policy rule coefficients on the output gap take the values 0.29 and 0.08, depending on the regime. Notice that when the fiscal authority pays less attention to debt dynamics, more weight is given to the output gap.[6] With respect to volatility, the standard deviation of the real per capita quarterly revenues net of transfers takes the values \$90 (real 2005 \$) in the low volatility regime and \$480 in the high volatility regime.[7]

Before analyzing the estimates of the transition probabilities and the smoothed probabilities, I make precisions about the labels of the Markov-switching regimes. The labels correspond to the identification conditions imposed in the estimation of the Markov-switching regressions: $\alpha_2^\pi \geq \alpha_1^\pi$, $\gamma_2^b \geq \gamma_1^b$, $\sigma_2^R \geq \sigma_1^R$, and $\sigma_2^\tau \geq \sigma_1^\tau$.

Leeper (1991) labels the monetary and fiscal policy regimes according to the strength of the response of the policy instrument to the targets. Roughly speaking, a strong (weak) response of interest rates to inflation is called an 'Active' ('Passive') monetary policy regime, while a strong (weak) response of taxes to debt is called a 'Passive' ('Active') fiscal policy regime. Hence, there are four possible combinations of regimes, depending on the strength of the response of the policy instruments to their targets. I label the four possible regimes as follows:

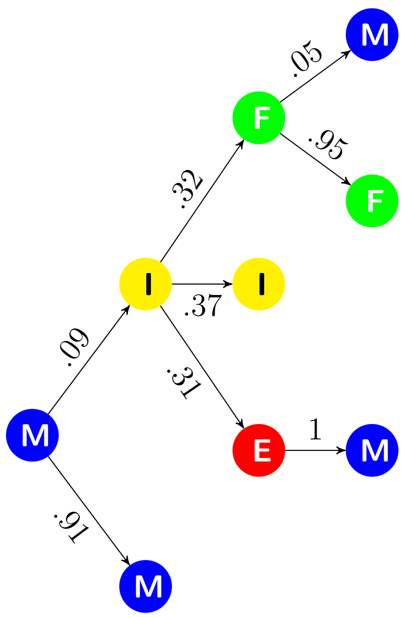
- $S_{mf} = 1$: $(\alpha_1^\pi, \gamma_1^b) \Leftrightarrow$ **F** regime,
- $S_{mf} = 2$: $(\alpha_2^\pi, \gamma_1^b) \Leftrightarrow$ **E** regime,
- $S_{mf} = 3$: $(\alpha_1^\pi, \gamma_2^b) \Leftrightarrow$ **I** regime,
- $S_{mf} = 4$: $(\alpha_2^\pi, \gamma_2^b) \Leftrightarrow$ **M** regime,

Here F stands for 'fiscal', a regime where the fiscal authority is reacting weakly to debt deviations from target and the monetary authority is reacting weakly to inflation deviations from target. M stands for 'monetary', a regime where the monetary authority is reacting strongly to inflation deviations from target and the fiscal authority is reacting strongly to debt deviations from target. I stands for 'indeterminate', a regime where the monetary authority is reacting weakly to inflation deviations from target and the fiscal authority is reacting strongly to debt deviations from target. E stands for 'explosive', a regime where

[6]Previous approaches to the estimation of Markov-switching fiscal policy rules as in Davig and Leeper (2006) have used fiscal variables, namely tax receipts and debt, as a ratio of GDP. I also estimated the model proposed here with the fiscal variables as ratios of GDP. Results show that there is not evidence of regime switching in the coefficients linked to debt and output stabilization. The reason behind these results is that, with data until 2013:Q4, the process for federal receipts over GDP is downward trending, which implies a very high persistence coefficient of about 0.98, making it difficult for the technique to estimate precisely the other coefficients that are multiplied by 0.02 in the policy rule.

[7]Heteroscedastic policy shocks in Markov-switching monetary policy rules have been found by Davig and Leeper (2006) and Bianchi (2012), and in Markov-switching fiscal policy rules, by Davig and Leeper (2006), Bianchi (2012) and Fernandez-Villaverde et al. (2011).

Figure 7: Estimated Transition Probabilities, Ergodic Probabilities and Correlation between States



$$P_{mf} = \begin{bmatrix} 0.95 & 0 & 0.32 & 0 \\ 0 & 0 & 0.31 & 0 \\ 0 & 0 & 0.37 & 0.09 \\ 0.05 & 1 & 0 & 0.91 \end{bmatrix}$$

$$\mathbb{P}(F) = .43$$
$$\mathbb{P}(E) = .02$$
$$\mathbb{P}(I) = .07$$
$$\mathbb{P}(M) = .48$$

$$\text{corr}(S_m, S_f) = 0.82$$

the monetary authority is reacting strongly to inflation deviations from target and the fiscal authority is reacting weakly to debt deviations from target. According to Leeper (1991), regimes M and F could deliver determinacy of the equilibrium in a local-linear version of a dynamic stochastic general equilibrium model, depending on the values of the coefficients. Along the same lines, in regime I there would be indeterminacy of the equilibrium, while in regime E, except for a particular case, there would be no equilibrium with bounded debt.

To better understand the transitional dynamics between the four regimes described above, Figure 7 presents the probability tree implied by the estimated transition probability matrix, along with the ergodic regime probabilities, and the implied correlation between S_m and S_f.

The estimated transition probabilities imply that regime M is expected to last about 11 quarters, regime F, about 20 quarters, regime I, 1.6 quarters, and E, 1 quarter. The ergodic probability of regime M is about 48%, while the ergodic probability of regime F is about 43%. Taken together, the ergodic probabilities of regimes I and E add to about 9%. The probability tree shows that if the economy starts in regime M, the only possibilities would be to stay in regime M with probability 91%, or to move to regime I with probability 9%. If the economy moves to regime I, the possibilities would be to move to regime E with probability 31%, to stay in regime I with probability 37%, or to move to regime F with probability 32%. If the economy moves to regime E, the only possibility is to then move to regime M. If the economy moves to regime F, it can stay in regime F with probability 95%, or it can move to regime M with probability 5%. Notice that the results rule out the possibility of moving from regime M to F directly. Finally, the implied correlation between the state driving the switching of the monetary policy rule coefficients and the state driving the switching of the fiscal policy rule coefficients is 0.82.

Using the algorithm of Kim (1994), I obtain the smoothed probabilities for each of the four states. The evolutions of the smoothed probabilities appear in Figure 8. The results show a high complementarity between regimes M and F. According to the smoothed probabilities, regime M was more likely in place during the 1960s, the 1980s, the second half of the 1990s, and a short period between 2005 and 2007. On the other hand, regime F was more likely in place during a large portion of the 1970s, the first half of the 1990s, a short period between 2003 and 2005, and a period between 2008 and the end of the sample. With respect to regimes I and E, there are short periods for both at the beginning of the 1980s. I will put these results in context with the narrative of monetary-fiscal policymaking in the next section.

In regard to the smoothed probabilities for interest rate and tax volatilities, Figure 9 plots the evolution of these probabilities along with the transition probability matrices. The transition probability matrix for interest rate volatility indicates that the low volatility regime is expected to last about 17 quarters, while the high volatility regime, about 10 quarters. On the other hand, the transition probability matrix for net receipts indicates that the low-volatility regime is expected to last about 14 quarters, while the high-volatility regime, about 2.6 quarters. The smoothed probability for the high-volatility regime of interest rates indicates that highly volatile interest rates were in place between around 1965 and 1975, the first half of and the end of the 1980s, a few years during the first half of the 2000s, and the 2008-2009 years. On the other hand, the smoothed probability for the high volatility regime of taxes net of transfers shows a few spikes. The estimates indicate that two short high volatility regimes were likely present in the 1970s, a couple more in the 1980s, one at the beginning of the 1990s, at least three in the first half of the 2000s, another during the year 2009, and a final one at the end of the sample. I will put these results in context with the narrative of monetary-fiscal policymaking in the next section.

Finally, Figure 10 shows the evolution of realized and predicted interest rates. The model performs reasonably well to predict the interest rate. In particular, at the end of the sample the underlying interest rate was below zero and increased gradually.

5 Narrative of the Results

This section puts the results from Figures 8 and 9 of the benchmark estimation of Section 4.4.2 in context with the historical narrative on monetary and fiscal policy.

According to Hetzel (2008), good conduct of monetary policy dominated the policy mix during the 1960s. He compares Fed Chairman William Martin to Fed Chairmen Paul Volcker and Alan Greenspan in that Martin believed that raising short-term interest rates in an expansion was a way to preempt inflation. Despite the Tax Reduction Act of 1964 that cut income tax rates across the board by approximately 20%, fiscal policy remained supportive of monetary policy during the 1960s. A fiscal regime starts to emerge during the 1970s, possibly due to the expansionary tax reforms of 1971, 1975 and 1976. Hetzel (2008) emphasizes the weak reaction of interest rates to inflation during the 1970s due to the focus of the central bank on promoting employment and the belief that inflation was a nonmonetary phenomenon. In Hetzel's narrative, the 1980s saw the commitment of the Federal Reserve to money targets, allowing the Federal Open Market Committee to raise interest rates by whatever extent necessary to lower inflation. In general, a monetary regime was in place during this decade, except for a couple of very short fiscal regimes due, most likely, to the

Figure 8: Smoothed Probabilities - Policy Regimes

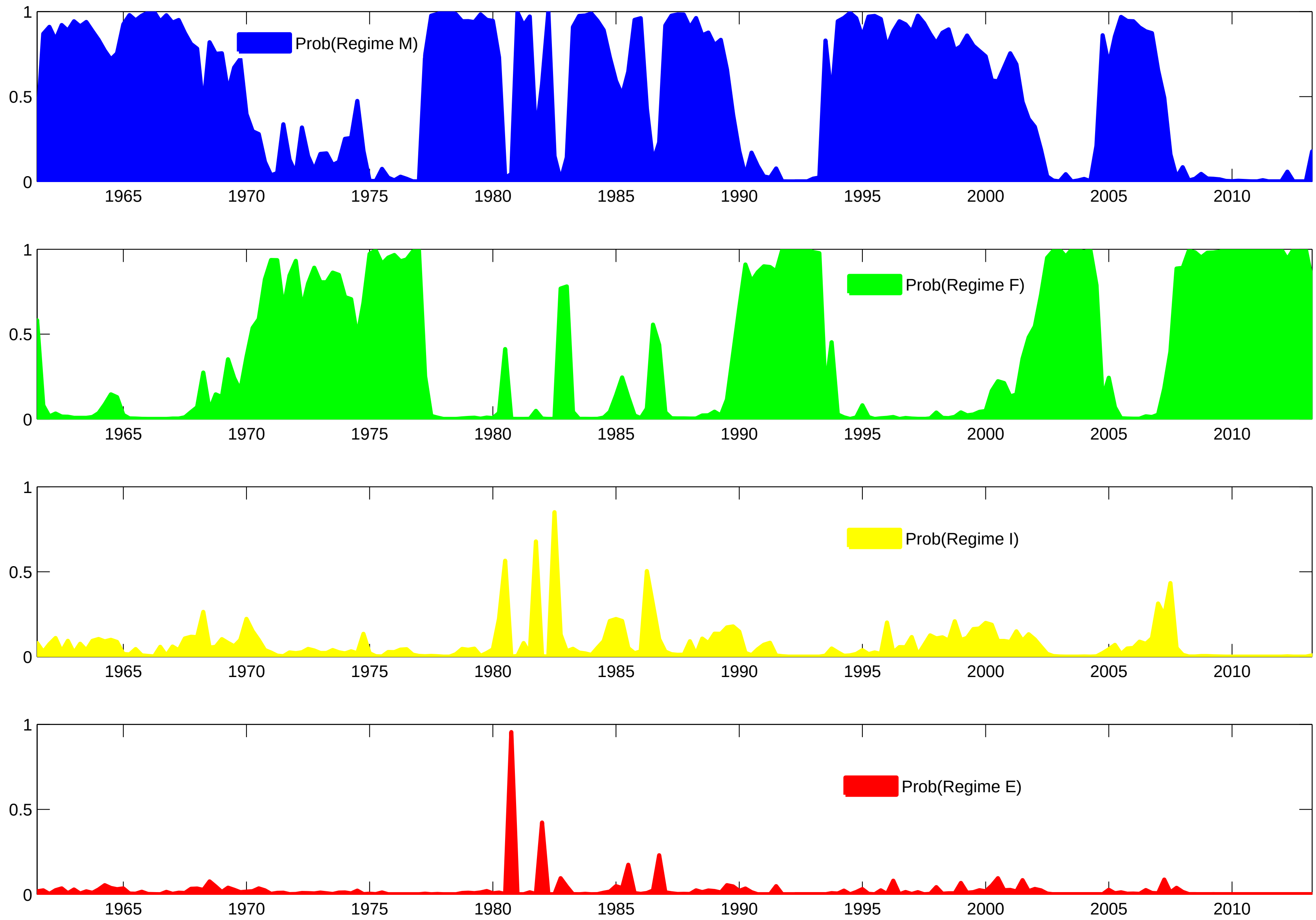
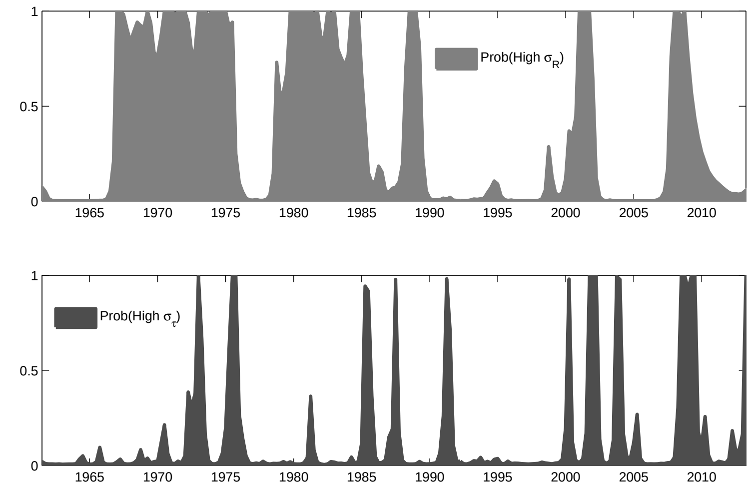


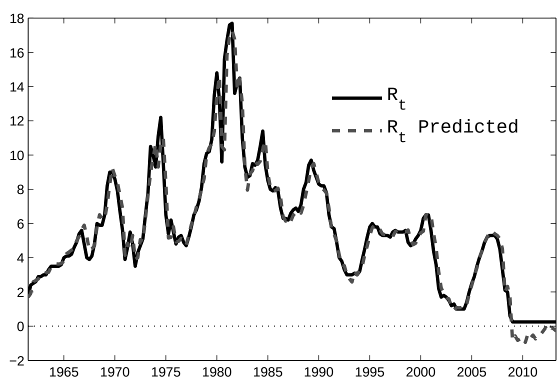
Figure 9: Smoothed Probabilities - Volatility Regimes



$$P_{\sigma^R} = \begin{bmatrix} 0.94 & 0.10 \\ 0.06 & 0.90 \end{bmatrix}$$

$$P_{\sigma^\tau} = \begin{bmatrix} 0.93 & 0.38 \\ 0.07 & 0.62 \end{bmatrix}$$

Figure 10: Observed and Predicted Interest Rate



expansionary tax reforms of 1981 and 1986. After tightening monetary policy at the end of the 1980s to counteract concerns about inflation, the results show the prevalence of a fiscal regime at the beginning of the 1990s due, possibly, to the combination of policies in reaction to the early 1990s recession. The "covert inflation targeting" of the 1990s (see Mankiw, 2001) and the deficit reduction act of 1993 make a monetary regime more likely during the second half of this decade. The rapid decline in interest rates during the first half of the 2000s and the expansionary tax reforms during that period put the economy, most likely, in a fiscal regime. A monetary regime starts to emerge after 2005 to avoid inflation pressures and the fact that economic activity was boosting tax revenues. This monetary regime lasts until the second half of 2007 when the central bank adopts a more dovish regime due to recessionary concerns. Once the recession hit in 2008, the model finds that a fiscal regime is much more likely to have been in place until the end of the sample in the fourth quarter of 2013.

With respect to volatility, interest rates experienced, most likely, a long period of high interest rate volatility between 1970 and the first half of the 1980s. Then, interest rate volatility decreases except for the stock market crashes of 1989 (Black Monday) and 2000 (Dot-com Bubble). Finally, volatility increases during the recent financial crisis and has, eventually, declined. On the other hand, federal receipts net of transfers experience spikes in volatility that coincide with some of the tax reforms that I listed in the previous paragraph, and are of very short duration. In particular, there are spikes that coincide with the tax reforms of the 1970s, the 1980s, the deficit reduction act of 1993, the numerous tax reforms of the 2000s, and the recovery act of 2009. At the end of the sample, it is likely that a high volatility regime is in place due to, possibly, the budget sequestration.

6 Concluding Remarks

This paper devised an estimation technique for a Markov-switching Taylor rule at the effective lower bound. The estimation method allows obtaining consistent estimates of the switching coefficients and the transition probabilities. Importantly, it also permits identification of the prevalent regime of monetary policy. Results show that monetary and fiscal authorities switch between policy regimes in a correlated manner.

The results of the paper suggest that, in modeling monetary policy at the zero lower bound, it is useful to endow agents with information about fiscal policymaking so that they can draw reasonable inferences on the monetary policy regime. Inferring the monetary-fiscal policy regime after lift off has implications on economic activity well before the lift-off date, as pointed out by Melosi and Bianchi (2013). The estimation technique proposed in this paper is a tool that agents can use to infer the monetary-fiscal policy regime under the effective lower bound of interest rates.

Appendix

A Estimation Ignoring Censoring and Joint Switching

Here the estimation consists of maximizing the log-likelihood function $\mathfrak{L}_{\theta_1}(\theta_1; Y_{1T}) = \ln g_{Y_1}(Y_{1T}; \theta_1)$, with respect to θ_1 by applying the Hamilton filter. The filter allows obtaining $g_{Y_1}(Y_{1T}; \theta_1)$ as follows:

$$\begin{aligned} g_{Y_1}(Y_{1T}; \theta_1) &= \prod_{t=1}^{T} g_{y_1}(y_{1t}|\mathfrak{F}_{1,t-1}; \theta_1) \\ &= \prod_{t=1}^{T} \sum_{j_1=1}^{J_1} g_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1)\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1,t-1}; \theta_1), \end{aligned}$$

where

$$g_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1) = \frac{1}{\sigma_{1,j_1}}\phi\left(\frac{y_{1t} - x_{1t}'\beta_{1,j_1}}{\sigma_{1,j_1}}\right), \tag{21}$$

and

$$\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1,t-1}; \theta_1) = \sum_{j_1'=1}^{J} p_{j_1 j_1'}\mathbb{P}(S_{1,t-1} = j_1'|\mathfrak{F}_{1,t-1}; \theta_1). \tag{22}$$

Once y_{1t} is realized at the end of time t, the filtered probability of S_{1t} in (22) is updated as

$$\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1t}; \theta_1) = \frac{g_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1)\mathbb{P}(S_{1t} = j_1|\mathfrak{F}_{1,t-1}; \theta_1)}{g_{y_1}(y_{1t}|\mathfrak{F}_{1,t-1}; \theta_1)}.$$

To obtain the smoothed probabilities $\mathbb{P}\left(S_{1t}|\mathfrak{F}_T; \theta_1\right)$ of the prevalent regime S_{1t} I use the smoothing algorithm in Kim (1994).

B Estimation Ignoring Joint Switching

Here the estimation consists of maximizing the log-likelihood function $\mathfrak{L}_{\theta_1}(\theta_1; Y_{1T}) = \ln f_{Y_1}(Y_{1T}; \theta_1)$, with respect to θ_1 by applying the Hamilton filter. The filter allows obtaining $f_{Y_1}(Y_{1T}; \theta_1)$ as in Appendix A, where I replace $g_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1)$ in (21) with:

$$\begin{aligned} f_{y_1|S_1}(y_{1t}|S_{1t} = j_1, \mathfrak{F}_{1,t-1}; \theta_1) = &\left[\Phi\left(\frac{y_{1L} - x_{1t}'\beta_{1,j_1}}{\sigma_{1,j_1}}\right)\right]^{1[y_{1t}=y_{1L}]} \\ &\times \left[\frac{1}{\sigma_{1,j_1}}\phi\left(\frac{y_{1t} - x_{1t}'\beta_{1,j_1}}{\sigma_{1,j_1}}\right)\right]^{1[y_{1t}>y_{1L}]}. \end{aligned}$$

C Data Construction

The transformation of the data is as follows:

- R_t: It is the quarterly federal funds rate ('FF' or 'Effective Federal Funds Rate' in the FRED database) until 2008:3. Starting 2008:4, the rate is fixed at 0.25%.

- τ_t: In the benchmark estimation, it is the real per capita quarterly federal receipts net of transfers (NIPA Table 3.2, line 1 minus line 22). The GDP deflator (described below) is used to deflate the series to (thousand) dollars of 2005, and the total population ('POP' or 'Total Population: All Ages including Armed Forces Overseas' in the FRED database) is used to transform the series to per capita terms. In the estimation with fiscal data over GDP, it is the quarterly federal receipts net of transfers divided by quarterly nominal GDP ('GDP' or 'Gross Domestic Product' in the FRED database).

- b_{t-1}: In the benchmark estimation, it is the average over the last four quarters of the real per capita stock of market value of 'Privately held gross federal debt' hosted by the Federal Reserve Bank of Dallas under Market Value of U.S. Government Debt. The GDP deflator is used to deflate the series to dollars of 2005, and the total population is used to transform the series to per capita terms. In the estimation with fiscal data over GDP, it is the stock of market value of 'Privately held gross federal debt' divided by quarterly GDP.

- π_t: It is the annual inflation rate of the GDP deflator ('GDPDEF' or 'Gross Domestic Product: Implicit Price Deflator' in the FRED database).

- y_t: It is the log difference between quarterly GDP ('GDPC1' or 'Real Gross Domestic Product' in the FRED database) and quarterly CBO potential GDP ('GDPPOT' or 'Real Potential Gross Domestic Product' in the FRED database).

D Unconstrained Specification

The unconstrained specification allows all the coefficients to switch between regimes and also allows for correlation between the state driving the evolution of the monetary policy rule coefficients, S_m, and the state driving the evolution of the standard deviation of the monetary policy shock, S_{σ_R}. Table 3 shows the parameter estimates and the smoothed probabilities appear in Figure 11, as well as the correlation between the monetary and the fiscal states, and the transition and ergodic probabilities.

$$P_{mf} = \begin{bmatrix} 0.94 & 0 & 0 & 0.04 \\ 0.06 & 0 & 0 & 0 \\ 0 & 0 & 0.49 & 0.09 \\ 0 & 1 & 0.51 & 0.87 \end{bmatrix}$$

$$\mathbb{P}(F) = .37$$
$$\mathbb{P}(E) = .02$$
$$\mathbb{P}(I) = .09$$

Table 3: Parameter Estimates - Unconstrained Specification

Monetary Policy Rule

Parameters	$j_m = 1$	$j_m = 2$
$\alpha^{\pi}_{j_m}$	0.51	2.42
	(3.92)	(5.88)
$\alpha^{y}_{j_m}$	1.73	0.56
	(2.81)	(−4.77)
R_{j_m}	3.42	1.99
	(4.45)	(2.26)
$\rho^{R}_{j_m}$	0.87	0.93
	(36.56)	(39.02)
	$j_{\sigma^R} = 1$	$j_{\sigma^R} = 2$
$\sigma^{R}_{j_{\sigma^R}}$	0.25	1.07
	(11.86)	(7.88)

For $j_m = 1$ and $j_{\sigma^R} = 1$, values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero. Except for R_{j_m} and ρ_{j_m}, where the values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero, for $j_m = 2$ and $j_{\sigma^R} = 2$ values in parenthesis are z-statistics of the null hypothesis that the difference between the coefficients of the two regimes is zero.

Fiscal Policy Rule

Parameters	$j_f = 1$	$j_f = 2$
$\gamma^{b}_{j_f}$	0.02	0.13
	(2.15)	(6.04)
$\gamma^{y}_{j_f}$	0.40	0.13
	(14.21)	(−8.08)
τ_{j_f}	2.80	1.50
	(24.85)	(7.36)
$\rho^{\tau}_{j_f}$	0.75	0.91
	(26.40)	(40.85)
	$j_{\sigma^\tau} = 1$	$j_{\sigma^\tau} = 2$
$\sigma^{\tau}_{j_{\sigma^\tau}}$	0.08	0.44
	(15.07)	(5.87)

For $j_f = 1$ and $j_{\sigma^\tau} = 1$, values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero. Except for τ_{j_f} and $\rho^{\tau}_{j_f}$, where the values in parenthesis are z-statistics of the null hypothesis that the coefficient is zero, for $j_f = 2$ and $j_{\sigma^\tau} = 2$ values in parenthesis are z-statistics of the null hypothesis that the difference between the coefficients of the two regimes is zero.

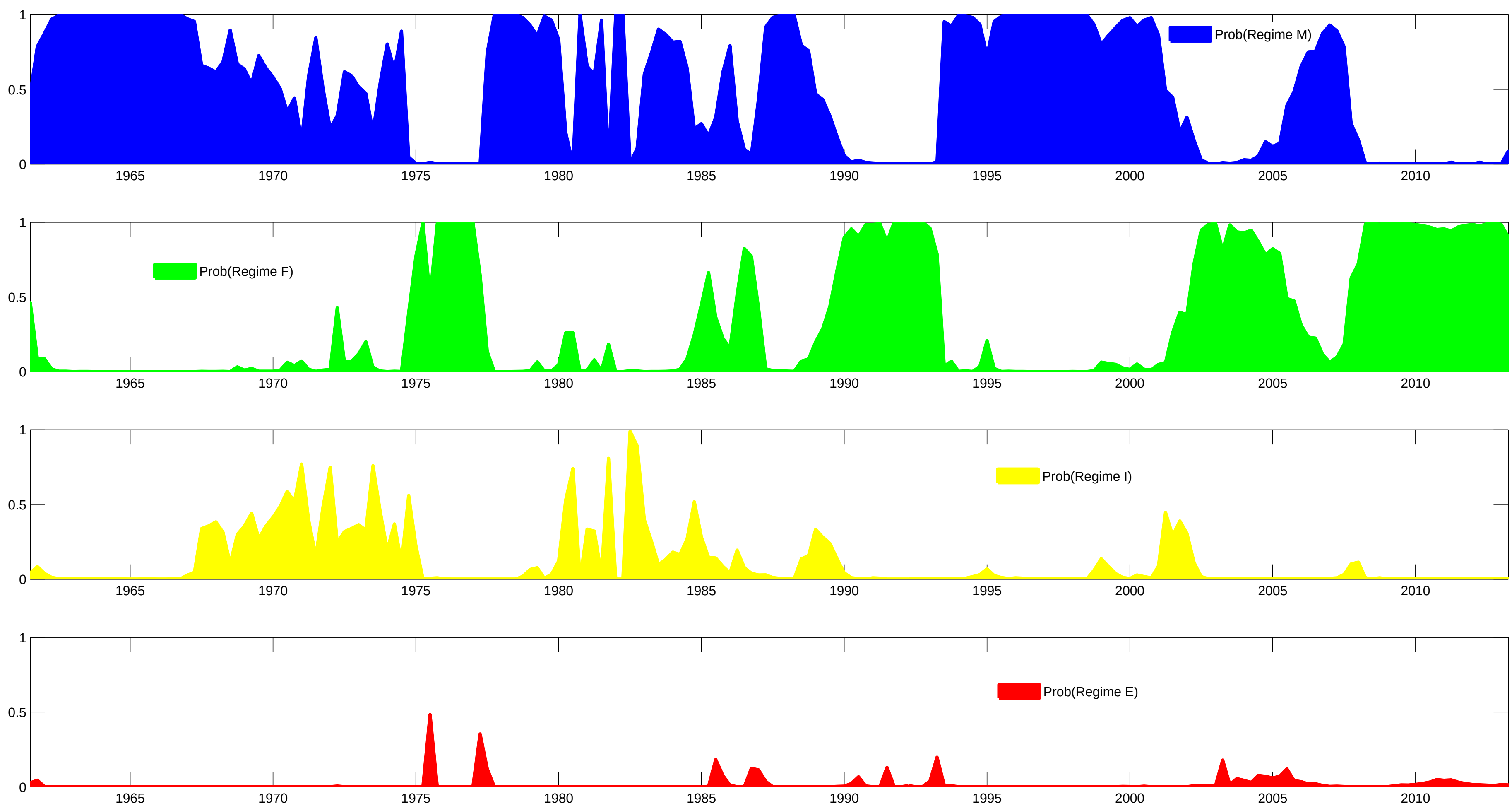
$$\mathbb{P}(M) = .52$$

$$\mathrm{corr}(S_m, S_f) = 0.78$$

As can be seen, the parameter estimates of the unconstrained specification are very similar, in general, to the estimates of the constrained estimation shown in Table 2. The most significant differences are with respect to the monetary policy rule coefficient on inflation, α^{π}, in regime 2, which results in a higher value in the unconstrained estimation, and with respect to the fiscal policy rule on output, γ^{y}, in regime 1, which also results in a higher value in the unconstrained estimation. The evolutions of the regime probabilities are also similar across the two specifications, with a high degree of complementarity between the M and F regimes. One notable difference is with respect to the probability of regime I in the early part of the 1970s, where the unconstrained estimation gives more likelihood to this regime than does the constrained estimation.

From the 8×8 transition probability matrix associated to a composite state formed by S_m, S_f, and S_{σ_R}, we can obtain the implied 4×4 joint transition probability matrix associated to the composite state formed by S_m and S_f, the implied correlation between these two states, as well as the ergodic probabilities of regimes M, F, I, and E. The most significant differences between the constrained and the unconstrained transition probability matrices are that the probability of going to a monetary regime having started in a fiscal regime is zero in the latter, while it was positive in the former; and that the probability of going to a fiscal regime having started in a monetary regime is nonzero in the latter, while it was zero in the former. The ergodic regime probabilities are similar to the constrained case, and

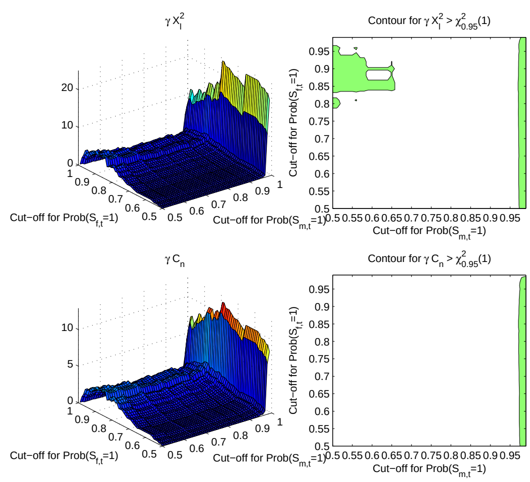
Figure 11: Smoothed Probabilities - Policy Regimes in the Unconstrained Estimation



the implied correlation between states is still high. All told, the unconstrained estimation does not imply significantly different results. In particular, the model still predicts a fiscal regime at the end of the sample and a high correlation between the monetary and fiscal policy states.

Using the unconstrained specification, I test for independence between the state driving the evolution of the monetary policy rule coefficients, S_m, and the state driving the evolution of the standard deviation of the monetary policy shock, S_{σ_R}. Figure 12 shows the value of the Tavaré and Altham (1983) statistics for testing the null hypothesis of independence between S_m and S_{σ^R} for different thresholds of $\mathbb{P}\left(S_{m,t}=1\right)$ and $\mathbb{P}\left(S_{\sigma^R,t}=1\right)$ on the left hand side, and the contour of the figure on the left for values of the statistics greater than the critical value that corresponds to a chi-square distribution with one degree of freedom. Both test statistics fail to reject the null hypothesis of independence between S_m and S_{σ^R} at the 5% level of significance for the majority of possible thresholds. I conclude that S_m and S_{σ^R} can be specified with separate transition probability matrices that need to be estimated. This supports the choice of the constrained specification of Section 4.4.2.

Figure 12: Independence Test between S_m and S_{σ_R}



Notes: The tests statistics are defined as:

$$X_l^2 = \left\{ \ln\left(\frac{n_1 n_4}{n_3 n_2}\right)\right\}^2 \Bigg/ \left(\frac{1}{np_1}+\frac{1}{np_2}+\frac{1}{np_3}+\frac{1}{np_4}\right)$$

$$C_n = \frac{n\left(n_1 n_4/n^2 - n_2 n_3/n^2\right)^2}{n^{-4}\left(n_1+n_2\right)\left(n_1+n_3\right)\left(n_2+n_4\right)\left(n_3+n_4\right)},$$

where $p_{j_{m\sigma^R}} = \mathbb{P}\left(S_{m\sigma^R} = j_{m\sigma^R}\right)$, and $S_{m\sigma^R} = (S_{\sigma^R}-1)J_m + S_m$ and $j_{m\sigma^R} = (j_{\sigma^R}-1)J_m + j_m$. Also, $n_{j_{m\sigma^R}} = \sum_{t=1}^{n} \mathbb{1}_{\{S_{m\sigma^R,t} = j_{m\sigma^R}\}}$. Additionally, $\gamma = (1-\mu\lambda)(1+\mu\lambda)$, where μ and λ are the nonunit eigenvalues of P_m and P_{σ^R}, the transition probability matrices of S_m and S_{σ^R}, respectively.

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