Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Welfare Evaluation in a Heterogeneous Agent Model: How Representative is the CES Representative Consumer?

Maria D. Tito

2015-109

Please cite this paper as:
Tito, Maria D. (2015). "Welfare Evaluation in a Heterogeneous Agent Model: How Representative is the CES Representative Consumer?," Finance and Economics Discussion Series 2015-109. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2015.109.

Welfare Evaluation in a Heterogeneous Agent Model: How Representative is the CES Representative Consumer?

Maria D. Tito*

November 25, 2015

Abstract

The present paper investigates the impact of asymmetric price changes on welfare in a model with heterogeneous consumers. I consider consumer heterogeneity à la Anderson et al. (1992). The standard welfare equivalence between the CES representative consumer and the discrete choice model breaks down in presence of asymmetric price changes. In fact, asymmetric variation in prices produce differential gains among heterogeneous consumers. I show that there exists no feasible Kaldor-Hicks income transfer such that the gains are equally redistributed. Intuitively, in presence of decreasing marginal utility, aggregation creates an insurance mechanism: the CES representative consumer softens the impact of price changes reallocating consumption among the available varieties. Individual consumers, instead, purchase a single product and do not internalize the effects of changes in prices of other available varieties. This result suggests that only symmetric policy-induced price changes minimize the utility losses across heterogeneous consumers.

Key words: Discrete Choice Models, CES Representative Consumer, Asymmetric Price Changes.

JEL classification: D11, D60.

1 Introduction

The increasing availability of micro-level data has challenged the idea of a representative consumer and has fostered the development of systematic frameworks for policy evaluation accounting for consumer heterogeneity. While several econometric methods have been proposed to successfully recover measures of individual well-being, the issue of identifying theory-based aggregate welfare effects remains unsolved.[1]

A common approach is to focus on theoretical settings that satisfy the conditions necessary for the existence of a social welfare function.[2] The CES representative consumer belongs to the group

*Federal Reserve Board, Constitution Ave NW, Washington, DC 20551. Contact: maria.d.tito@frb.gov. I would like to thank Matilde Bombardini, Aaron Flaaen, Keith Head, Colin Hottman, and all participants in the ETSG 2012 for their comments. The views expressed in the paper are those of the author and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

[1]See Slesnick (1998) for a review.

[2]The seminal contribution by Gorman (1953) proved the existence of a social welfare function if all agents have identical homothetic preferences. For successive extensions, see, for example, Eisenberg (1961), Lau (1982), Jorgenson et al. (1982), and Lewbel (1989).

of representative agent models that satisfy the conditions for positive and normative equivalence: Anderson et al. (1992) show that the demand and the welfare of a CES representative consumer can be micro-founded using a Discrete Choice model.

This paper explores in details the normative meaning of a CES representative consumer in a Discrete Choice framework. The choice to focus on the CES representative consumer is mainly motivated by the fact that it forms the basis for the gains from trade in all models based on the Armington (1969) assumption or the Dixit-Stiglitz monopolistic competition.

In this paper, I introduce consumer heterogeneity à la Anderson et al. (1992). Each consumer chooses a single product among the set of available varieties, relying on observable product characteristics and her idiosyncratic taste preferences. I characterize individual utilities to generate the aggregate behaviour of a CES representative consumer.

The normative equivalence between the CES representative consumer and the Discrete Choice Model breaks down in presence of asymmetric price changes. In fact, asymmetric variations in prices produce differential gains among heterogeneous agents. I show that there exists no feasible Kaldor-Hicks income transfer[3] such that the gains are equally redistributed across consumers. Two conditions are at the core of the result, decreasing marginal utility of income and perfect substitutability across varieties. If the varieties are perfectly substitutable, a decreasing marginal utility of income implies that the consumers losing/gaining less from price changes require aggregate compensations larger than the total income collected from those experiencing the biggest welfare improvements. The perfect substitutability across varieties, however, fades through aggregation: while individual consumers purchase a single product and do not internalize the policy effects on the prices of varieties not included in their optimal consumption basket, the CES representative consumer mitigates the impact of variation in prices by reallocating consumption among the available varieties.

Finally, I generalize the main result, linking the condition of a feasible income redistribution to the properties of the individual marginal utility of income.

This paper adds a new perspective to the normative interpretation of the representative consumer, following the comprehensive analysis by Kirman (1992). The most closely related contribution is the redistribution example constructed by Anderson et al. (1992);[4] while Anderson et al. (1992) compare the impact of two asymmetric price changes on the income redistribution across consumers in a setting with two goods, this paper proves the non-feasibility of Kaldor-Hicks income transfers for any asymmetric price change across n commodities.

My work also relates to the empirical contributions quantifying the welfare impact of policy changes across heterogeneous consumers. In particular, my work provides a theoretical foundation to the empirical findings by Sun (2011) and Sheu (2014). Both papers stress the importance of consumer heterogeneity on welfare quantifications in response to trade liberalizations. They show that the empirical estimates of welfare gains from trade are lower when they are based upon the logit random coefficient demand system if compared to those derived under a CES representative consumer model. Sheu (2014) attributes the gap in the welfare calculations to the restrictive substitution structure of the CES model.

The Kaldor-Hicks income transfer complements the set of welfare instruments analyzed by Her-

[3]See Kaldor (1939) and Hicks (1939).

[4]See Anderson et al. (1992), pp. 97-100.

riges and Kling (1999) when quantifying welfare changes in random utility models with non-linear income effects.[5] The measure I propose is a possible solution to the trade-off between computational ease and potential bias described in their paper. In fact, the Kaldor-Hicks transfer are exact measures and relatively easy to compute.[6]

The rest of the paper is organized as follows. Section 2 summarizes the features of Anderson et al. (1992) model. Section 3 characterizes the main result. Section 4 extends the main result to the class of additively- and multiplicatively-separable indirect utility functions. Section 5 describes an application to policy analysis and Section 6 concludes.

2 A Discrete Choice Model of Consumer Behavior

The theoretical framework follows Anderson et al. (1992). I consider an economy populated by a unit mass of statistical independent and identical consumers. Each consumer is endowed with income y that she decides to allocate across n commodities. Commodity i is sold at price p_i. In addition to prices, a consumer's choice is informed by idiosyncratic taste preferences, captured by ε_i. As in Anderson et al. (1992), I also assume that $\varepsilon_i \overset{iid}{\sim} Gumbel\,(\gamma, \mu)$, $i = 1, ..., n$. Thus, each consumer c is identified by a draw of independent Gumbel random variables, $(\varepsilon_1, \ldots, \varepsilon_n)$. Given prices and taste parameters, the consumer allocates her income among the available commodities to maximize her utility

$$\max_{x_i, i=1,\ldots,n} U^c = \max_{x_i, 1,\ldots,n} \sum_{i=1}^{n} [\ln x_i + \varepsilon_i] \qquad \text{s.to} \sum_{i=1}^{n} p_i x_i = y \tag{1}$$

The linearity of the problem implies that the consumer will spend all of her income y on a single commodity. If $x_i = \arg\max U^c$, it is immediate to derive the indirect utility of consumer c

$$V^c = U^c|_{x_i = \arg\max U^c} = \ln y + \varepsilon_i - \ln p_i$$

Different consumer choices characterize a partition of the commodity space. For example, the measure of consumers choosing commodity i is

$$Pr\,(i) = \frac{(p_i)^{-\frac{1}{\mu}}}{\sum_{i=1}^{n} (p_i)^{-\frac{1}{\mu}}}$$

The demand for variety i is, then, obtained multiplying the measure of consumers choosing variety i by the individual demand:

$$X_i = Pr\,(i) \cdot \frac{y}{p_i} \tag{2}$$

Finally, the aggregate indirect utility is given by

$$V = \ln y + \mu \ln \left[p_1^{-\frac{1}{\mu}} + \ldots + p_n^{-\frac{1}{\mu}} \right] \tag{3}$$

[5]They analyze the McFadden's GEV sampler, a linear model, the representative consumer framework and the computation of bounds on the welfare effects.

[6]Its calculation requires only knowledge of prices/tariff changes and trade elasticity estimates.

(2) and (3) represent the demand and the welfare of a CES representative consumer with elasticity of substitution $\sigma = \frac{1-\mu}{\mu}$ and income y.[7]

3 Welfare Effects of Asymmetric Price Changes

The equivalence between the CES representative consumer and a population of idiosyncratic consumers breaks down in presence of asymmetric variations in prices. In fact, asymmetric price changes produce differential gains across consumers choosing different varieties; those gains, however, cannot be reversed transferring income across consumers. Two conditions are sufficient for this result: the perfect substitutability across varieties and the decreasing marginal utility of income. First, because of the perfect substitutability across varieties, individual consumers do not fully internalize the effect of changes in prices of goods that do not enter their optimal consumption basket. Second, in presence of decreasing marginal utility of income, consumers losing/gaining less from price variations require aggregate compensations larger than the total income collected from those experiencing the biggest welfare improvements. Therefore, I argue that the indirect utility of the representative consumer jointly with a measure of dispersion of gains across individuals are not sufficient to characterize welfare in this framework; I propose also to consider whether the Kaldor-Hicks income transfers are feasible.

In order to fix ideas, let us consider an asymmetric price change that leaves the price index unchanged.[8] In particular, let $(\tilde{p}_1, \cdots, \tilde{p}_n)$ be the prices after the change. I will analyze the case with $\tilde{p}_i < p_i$, $\tilde{p}_j > p_j$, and $\tilde{p}_k = p_k$ if $k \neq i, j$[9] such that

$$p_1^{-\frac{1}{\mu}} + \ldots + p_2^{-\frac{1}{\mu}} = \tilde{p}_1^{-\frac{1}{\mu}} + \ldots + \tilde{p}_n^{-\frac{1}{\mu}} \tag{4}$$

Different groups of consumers experience opposite changes in individual welfare.[10] Let us consider the Kaldor-Hicks system of income transfers, i.e. income transfers across consumers with the property to make everyone at least as well off as before the change in prices.

The following theorem summarizes the main contribution of the present paper

Theorem 1. *(*Asymmetric Price Variations and Welfare*). Consider an economy populated by a unit mass of statistical independent and identical consumers solving (1). Suppose that a price change occurs such that the price index remains unchanged, as in equation (4). Then, there does not exist a Kaldor-Hicks system of income transfers such that all consumers are at least as well off as before the change in prices occurred.*

[7]See Anderson et al. (1992) for a proof of this result.

[8]The present analysis extends to the case in which the price index varies. In this case, the total price variation can be decomposed into two components. A first component captures the average decrease in prices; the second component reflect an asymmetric variation in prices. Let $(p_1, \cdots, p_n)$ be an initial vector of prices and let $(\hat{p}_1, \cdots, \hat{p}_n)$ be such that $\hat{p}_i < p_i$ and $\hat{p}_j < p_j$. Define θ such that

$$\theta^{-\frac{1}{\mu}} \left[p_1^{-\frac{1}{\mu}} + \cdots + p_n^{-\frac{1}{\mu}} \right] = \hat{p}_1^{-\frac{1}{\mu}} + \cdots + \hat{p}_n^{-\frac{1}{\mu}}$$

θ represents the average decrease in prices. Let $\tilde{p}_s \equiv \theta^{-\frac{1}{\mu}} p_s^{-\frac{1}{\mu}}$, for $s = 1, \cdots, n$. Therefore, $\forall\, s$, either $\tilde{p}_s \leq \hat{p}_s$ or $\tilde{p}_s \geq \hat{p}_s$.

[9]An extension with k price changes is immediate. It requires the construction of a larger substitution matrix across varieties with elements analogous to what it is derived below.

[10]See Appendix A.1 for a full derivation of the changes in utility.

Proof. Suppose that $\tilde{p}_i < p_i$ and $\tilde{p}_j > p_j$ such that equation (4) is satisfied. Let us characterize the Kaldor-Hicks income transfers across the consumer partition:

- Consumers choosing commodity i before and after the price change. Let A be the set of such consumers. Consumers $c \in A$ experience a higher utility after the price change; the income transfer that leaves their utility unchanged is the level of income, y_T^c, that equates their current utility to the level before the change in prices,

$$\varepsilon_i - \ln p_i + \ln y = \varepsilon_i - \ln \tilde{p}_i + \ln y_T^c$$
$$y_T^c - y = \left[\frac{\tilde{p}_i}{p_i} - 1\right] y$$

  Clearly $y_T^c - y < 0$, for $c \in A$. Since the individual transfer is independent of idiosyncratic factors, the aggregate income transfer is obtained multiplying the individual component by the total measure of consumers in A,

$$T^A = \frac{p_i^{-\frac{1}{\mu}}}{\sum_{s=1}^n p_s^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_i}{p_i} - 1\right] y$$

- Consumers choosing commodity j before and after the price change. Let B be the set of such consumers. Consumers $c \in B$ experience a lower utility after the price change; the transfer to make them as well off as before the change in prices is given by

$$y_T^c - y = \left[\frac{\tilde{p}_j}{p_j} - 1\right] y$$

  The aggregate transfer across consumers $c \in B$ is obtained as follows

$$T^B = \frac{\tilde{p}_j^{-\frac{1}{\mu}}}{\sum_{i=1}^n \tilde{p}_i^{-\frac{1}{\mu}}} \left[\frac{\tilde{p}_j}{p_j} - 1\right] y$$

- Consumers switching from commodity j or to commodity i. Let C_k be the set of consumers choosing commodity j before the price change and switching to commodity $k \neq i$ after the price change; let D_k be the set of consumers choosing commodity $k \neq j$ before the price change and switching to commodity i after the price change; let E be the set of consumers choosing commodity j before the price change and switching to commodity i after the change in prices. Changes in utility across consumers $c \in C_k, D_k, E$ depend on the realization of the taste shocks. Therefore, individual income transfers will also be consumer-specific. For $c \in C_k$, for example, the individual income transfer is given by

$$y_T^c - y = \left[\frac{\tilde{p}_k}{p_j} e^{\varepsilon_j - \varepsilon_k} - 1\right] y$$

Aggregating over consumers $c \in C_k$,

$$T^{C,k} y \frac{\tilde{p}_k}{p_j} \left[\frac{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}} \right]^{-\mu-1} \left[B\left(1-\mu, \mu+1; \frac{p_j^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu} + p_j^{-1/\mu}}\right) - B\left(1-\mu, \mu+1; \frac{\tilde{p}_j^{-1/\mu}}{\sum_{s=1}^{n} \tilde{p}_s^{-1/\mu}}\right) \right]$$
$$+ y \left[\frac{p_k^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu} + p_j^{-1/\mu}} - \frac{p_k^{-1/\mu}}{\sum_{s=1}^{n} \tilde{p}_s^{-1/\mu}} \right]$$

where $B(a, b; x)$ denotes the incomplete Beta function. Expressions for $T^{D,k}$, $k \neq j$, and T^E are equally involved and shown in the appendix.[11]

An income transfer is feasible iff

$$T = T^A + T^B + \sum_{k \neq i,j} T^{C,k} + \sum_{k \neq i,j} T^{D,k} + T^E \leq 0$$

However, for $y = 1$,

$$T^A + T^B + \sum_{k \neq i,j} T^{C,k} + \sum_{k \neq i,j} T^{D,k} + T^E > -\left(\Delta V^A + \Delta V^B + \sum_{k \neq i,j} \Delta V^{C,k} + \sum_{k \neq i,j} \Delta V^{D,k} + \Delta V^E \right)$$

as individual transfers are larger than the underlying changes in prices that affect utility.[12] Since $\Delta V^A + \Delta V^B + \sum_{k \neq i,j} \Delta V^{C,k} + \sum_{k \neq i,j} \Delta V^{D,k} + \Delta V^E = 0$, $T > 0$. A similar result is obtained if $y \neq 1$, since y can be factored out of every transfer. □

Figure 1 shows the intuition why, in presence of perfect substitutability, a decreasing marginal utility of income is a sufficient condition for the result. Since the aggregate change in utility across consumers is zero, it is sufficient to consider two *aggregate consumers* displaying opposite changes in utility. When the marginal utility of income is decreasing, a redistribution requires that the income is reallocated from the consumer with a higher marginal utility to the consumer with a lower marginal utility. Therefore, a Kaldor-Hicks income transfer is never feasible.

4 Generalizations

This section analyzes in more details the sufficient conditions for the infeasibility of Kaldor-Hicks income transfers across heterogeneous consumers. I consider two cases. A first case characterizes the condition for the class of additively-separable indirect utility functions, while the second case focuses on multiplicatively-separable indirect utility functions. The conditions apply to any distribution of taste shocks in the population but require the assumption of perfect substitutability across varieties.

4.1 Additively-Separable Indirect Utility

Assume that the indirect utility function of agent c choosing variety i is given by

$$V^{c,i}\big|_{x_i = \arg\max U^c} = f(y) + \varepsilon_i - g(p_i) \tag{5}$$

[11]See appendix A.2.

[12]See Appendix A.3.

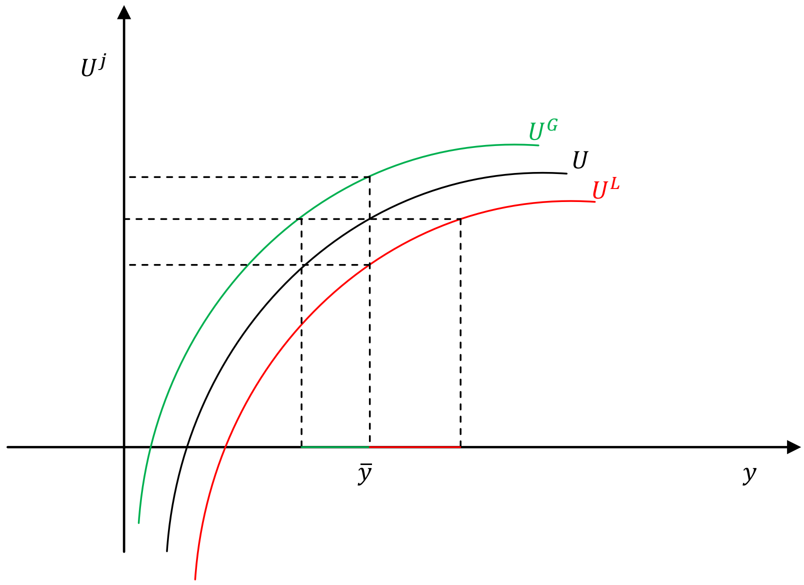

Figure 1: Income transfers across consumers with opposite utility changes and decreasing marginal utilities. The income transfer associated to the consumer who gains (horizontal difference between the black and the green curve) is always lower than the income transfer associated to the consumer who loses (horizaontal difference between the black and the red curve), since all consumers share the same initial income level $\bar{y}$.

where f is an invertible and differentiable function. Let $\Delta V^{c,i}$ denote the utility change for consumer c choosing variety i in presence of a change in prices.

Theorem 2. *Consider a change in prices that satisfies the property of inducing a zero aggregate utility change, i.e.* $\int_{c,i} \Delta V^{c,i} \mathrm{d}\mathbb{P}(c,i) = 0$*, where* $V^{c,i}$ *satisfies (5). Then, a transfer of income across consumer is not feasible if* $\frac{\partial f(y)}{\partial y} \leq 1$, $\forall\, y \geq 0$*, with strict inequality over some set of positive measure.*

Proof. An aggregate income change is not feasible if the individual transfers are larger than the changes in utility,

$$f^{-1}\left[f(y) + \Delta V^c\right] - y \geq \Delta V^c$$

From a first order Taylor approximation around $f(y)$,

$$f^{-1}\left[f(y) + \Delta V^c\right] \approx y + \frac{\Delta V^c}{\frac{\partial f(y)}{\partial y}}$$

The result immediately follows if $\frac{\partial f(y)}{\partial y} \leq 1$, with strict inequality over some set of positive measure. $\square$

If $\frac{\partial f(y)}{\partial y} = 1$, $\forall\, y \geq 0$, Kaldor-Hicks income transfers are feasible.

4.2 Multiplicatively-Separable Indirect Utility

Assume that the indirect utility function of agent c choosing variety i has a multiplicatively-separable form,

$$V^{c,i}\big|_{i=\arg\max U^c} = \ln f(y) + \varepsilon_i - \ln g(p_i) \tag{6}$$

where f is an invertible and differentiable function, as before. Let $\Delta V^{c,i}$ denote the utility change for consumer c choosing variety i in presence of a change in prices.

Theorem 3. *Consider a change in prices that satisfies the property of inducing a zero aggregate utility change, i.e.* $\int_{c,i} \Delta V^{c,i} \mathrm{d}\mathbb{P}(c,i) = 0$*, where* $V^{c,i}$ *satisfies (6). Then, a transfer of income across consumer is not feasible if* $\frac{\partial f(y)}{\partial y} \leq f(y)$*,* $\forall\, y \geq 0$*, with strict inequality over some set of positive measure.*

Proof. An aggregate income change is not feasible if the individual transfers are larger than the changes in utility,

$$f^{-1}\left[f(y) \cdot e^{\Delta V^c}\right] - y \geq \Delta V^c$$

From a first order Taylor approximation around $f(y)$,

$$f^{-1}\left[f(y)\, e^{\Delta V^c}\right] \approx y + \frac{f(y)}{\frac{\partial f(y)}{\partial y}}\left[e^{\Delta V^c} - 1\right]$$

The result follows if $\frac{f(y)}{\frac{\partial f(y)}{\partial y}} \geq 1$. $\square$

Intuitively, if the marginal utility of income is small enough, the redistribution policy implies that transfers are larger than the underlying changes in prices reflected in the utility of individual consumers. Therefore, the aggregate income transfer can never be feasible.

5 Policy Implications

Theorem 1 implies that the aggregate change in utility and a measure of dispersion of utility changes across consumers might not be sufficient to analyze the impact of policy changes on welfare in presence of heterogeneous consumers. We also need to consider two additional factors, whether there are asymmetries in price changes and the properties of individual utility functions.

First, policy-induced price changes would not generate welfare losses if they were evenly distributed across industries. Tables 1-3 analyze, for example, the path of US import tariffs vis-à-vis the world and Canada, the biggest US import partner. Table 1 describes the distribution of tariff changes. I classify each tariff change into one of 3 categories: positive, negative, or zero. While, on average, US import tariffs vs. all countries remain mostly unchanged at the beginning of the period, world tariff reductions represent the largest share of all changes in the late 1990s. Focusing on Canada, tariff changes are, instead, predominantly negative in the early 1990s. The share of tariff increases spikes in 1998 (70%); those increases seem to be quickly overturned the following years. Tables 2-3 report the weighted[13] and unweighted average and standard deviation of tariff changes

[13] Those statistics are constructed using as weights the initial import flow. Similar values are obtained if using the trade flow after the change.

for all countries and Canada. Both tables confirms a substantial dispersion of tariff changes across sectors and countries. On average, tariffs are cut each year – with the exception of 1998. However, in line with the evidence on the distribution of changes by sign, tariff changes tend to be heterogenous: the standard deviation of tariff changes is at the highest level in the late 1990s; it levels off only in 2000. Tables 1-3 suggest that there is a large scope for heterogeneous gains across consumers.

Second, if subsets of varieties are perfectly substitutable and the marginal utility of income satisfies the properties described in theorem 2 or 3, a complete welfare analysis requires also considering whether Kaldor-Hicks income transfers are feasible. Table 4 constructs the Kaldor-Hicks transfers implied by changes in the US-Canada trade policy. I approximate the product share of domestic consumption by using the import share and assume that $\mu = 1/5$.[14] Column (1) shows the *unadjusted* aggregate transfers, i.e. transfers that do not control for the aggregate change in utility. The negative signs suggest that transfers are feasible in most years. How do we interpret the numerical values? Under the assumption of a unit income for the representative consumer, a surplus of 16% remains available in 1991 after redistributing income across consumers. However, the apperent feasibility of the income transfers relies on whether the heterogeneous gains across consumers are all of the same sign - i.e. all consumers gains or lose, although in different measures. In fact, with the exception of 1998, the price index has been decreasing over time. Therefore, the fact that transfers are feasible in all years but in 1998 is not surprising. Column (3) adjusts for the average utility change, as implied by the price index. Under this scenario, I effectively decompose each consumer utility change into an aggregate – i.e., a component that reflects the representative consumer's utility change – and an idiosyncratic – i.e., the change in utility that *exceeds*, on the positive or negative side, the representative consumer's change – components. The adjusted transfer (column (3)) is, then, constructed by redistributing income across heterogeneous consumers in order to equate the *excess* utility changes. The results in column 3 are much different than those reported in column (1): adjusted income transfers are, in general, not feasible, with the exception of 1992.[15] In 1991, a deficit of 20% is necessary to equally redistribute the *excess* gains across consumers. Table 4 suggests that Kaldor-Hicks transfers are useful tools of welfare analysis in presence of policy changes inducing heterogenous gains across consumers.

6 Conclusions

This paper analyzes the normative meaning of a representative consumer. In particular, I focus on the CES representative consumer since it forms the basis for the gains from Trade in models based on the Armington (1969) assumption or the Dixit-Stigliz monopolistic competition.

I show that the welfare equivalence between the CES representative consumer and the discrete choice model breaks down in presence of asymmetric price changes. In fact, asymmetric variations in prices produce differential gains among heterogeneous consumers. I prove that there exists no feasible Kaldor-Hicks income transfer such that the utility gains are equally redistributed across consumers. This result extends to the class of additively- and multiplicatively-separable indirect utility functions under some conditions on the marginal utility of income.

[14] This implies that the trade elasticity $\sigma = 4$.

[15] The feasibility of the Kaldor-Hicks transfers in 1992 is related to two factors, the growth in imports and the coarse approximation of import shares to domestic consumption.

I propose an application of the theoretical result to the US Trade Policy. Since empirical evidence suggests that most policy changes induce heterogeneous gains across consumers, the issue of whether those gains can be equally redistributed across consumers must be given attention. This paper proposes Kaldor-Hicks income transfers as instruments to evaluate and address this concern.

References

Anderson, S. P., A. De Palma, and J. F. Thisse (1992). *Discrete choice theory of product differentiation.* MIT press.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production (une théorie de la demande de produits différenciés d'après leur origine)(una teoría de la demanda de productos distinguiéndolos según el lugar de producción). *Staff Papers-International Monetary Fund*, 159–178.

Eisenberg, E. (1961). Aggregation of utility functions. *Management Science 7*(4), 337–350.

Gorman, W. M. (1953). Community preference fields. *Econometrica: journal of the Econometric Society*, 63–80.

Herriges, J. A. and C. L. Kling (1999). Nonlinear income effects in random utility models. *Review of Economics and Statistics 81*(1), 62–72.

Hicks, J. R. (1939). The foundations of welfare economics. *The Economic Journal*, 696–712.

Jorgenson, D., L. J. Lau, T. M. Stoker, R. Basmann, and G. Rhodes (1982). The transcendental logarithmic model of aggregate consumer behavior. *Advances in econometrics*.

Kaldor, N. (1939). Welfare propositions of economics and interpersonal comparisons of utility. *The Economic Journal*, 549–552.

Kirman, A. P. (1992). Whom or what does the representative individual represent? *The Journal of Economic Perspectives*, 117–136.

Lau, L. J. (1982). A note on the fundamental theorem of exact aggregation. *Economics Letters 9*(2), 119–126.

Lewbel, A. (1989). Exact aggregation and a representative consumer. *The Quarterly Journal of Economics*, 621–633.

Sheu, G. (2014). Price, quality, and variety: Measuring the gains from trade in differentiated products. *American Economic Journal: Applied Economics 6*(4), 66–89.

Slesnick, D. T. (1998). Empirical approaches to the measurement of welfare. *Journal of Economic Literature*, 2108–2165.

Sun, Y. S. (2011). Quantifying the gains from trade across countries with consumer heterogeneity! Technical report, Working Paper.

Table 1: US Tariff Changes: Signs

	All Countries			Canada		
Year	Negative	Zero	Positive	Negative	Zero	Positive
1990	4.73	94.54	0.73	66.98	32.73	0.29
1991	4.18	95.76	0.06	63.95	35.63	0.42
1992	4.18	95.66	0.15	65.00	34.98	0.03
1993	4.28	95.34	0.38	61.75	37.96	0.29
1996	52.41	45.32	2.26	36.17	62.29	1.55
1997	50.50	43.41	6.10	33.74	56.43	9.83
1998	52.99	36.69	10.32	7.09	22.67	70.24
1999	60.58	39.19	0.22	71.67	28.33	-
2000	28.09	71.88	0.03	0.23	99.77	-

Table 2: US Tariff Changes: Summary Statistics

	Unweighted		Weighted	
Year	Avg	Sd	Avg	Sd
1990	-0.04	0.43	-0.04	0.29
1991	-0.05	0.48	-0.03	0.26
1992	-0.04	0.31	-0.04	0.25
1993	-0.05	0.57	-0.04	0.32
1996	-0.12	1.80	-0.06	1.80
1997	-0.14	4.68	-0.10	2.06
1998	0.05	4.80	0.26	2.47
1999	-0.70	2.72	-0.82	2.59
2000	-0.10	0.26	-0.06	0.20

Notes: Average and Standard Deviation of US-Canada Tariff changes. Tariff changes are weighted by the previous period trade flows in the last two columns.

Table 3: US-Canada Tariff Changes: Summary Statistics

Year	Unweighted Avg	Unweighted Sd	Weighted Avg	Weighted Sd
1990	-0.71	1.07	-0.19	0.56
1991	-0.56	0.69	-0.16	0.37
1992	-0.62	0.79	-0.19	0.49
1993	-0.52	0.69	-0.15	0.36
1996	-0.25	1.05	-0.05	0.49
1997	-0.21	6.20	0.001	2.05
1998	3.16	5.95	1.00	2.58
1999	-3.94	9.24	-1.24	3.10
2000	-0.002	0.08	-0.001	0.02

Notes: Average and Standard Deviation of US-Canada Tariff changes. Tariff changes are weighted by the previous period trade flows in the last two columns.

Table 4: Tariff Changes and Welfare

Year	(1) Transfer	(2) Avg. Price Change[1]	(3) Adj. Transfer[2]
1991	-0.161	-0.36%	0.200
1992	-0.201	-0.76%	-0.005
1993	-0.155	-0.81%	0.052
1997	-0.065	-0.82%	0.289
1998	1.026	4.16%	0.024

[1] Change in the Price Index, approximating the (opposite of) the utility change for the representative consumer. I assume that $\mu = 1/5$.
[2] Income Transfer across consumers to equally redistribute the utility gains, adjusting for the average change in utility.
Notes: Measure of compensations across consumers. In the calculations, $\mu = 1/5$.

A Mathematical Appendix

A.1 Welfare Effects of Asymmetric Price Changes

A price change satisfying (4) affects the choices of consumers. In particular, there are 5 areas to analyze:

- Consumers choosing commodity i before and after the price change. Let A be the set of such consumers. Since $\tilde{p}_i < p_i$, consumers who chose commodity i before the price change will not modify their choice; in fact,

$$\varepsilon_i - \ln \tilde{p}_i > \varepsilon_i - \ln p_i \geq \max_{k \neq i} \{\varepsilon_k - \ln p_k\}$$

  The measure of such consumers coincides with the fraction of individuals choosing commodity i before the price change

$$Pr(A) = \frac{p_i^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + \ldots + p_n^{-\frac{1}{\mu}}}$$

  Each consumer experience a change in utility captured by the difference in prices, $\Delta V^c = \ln p_i - \ln \tilde{p}_i$, $c \in A$. In aggregate,

$$\Delta V^A = (\ln p_i - \ln \tilde{p}_i) \frac{p_i^{-\frac{1}{\mu}}}{p_1^{-\frac{1}{\mu}} + \ldots + p_n^{-\frac{1}{\mu}}}$$

- Consumers choosing commodity j before and after the price change. Let B be the set of such consumers. Since $\tilde{p}_j > p_j$, only consumers with sufficiently high valuation for good j will buy it after the change in prices,

$$\varepsilon_j - \ln p_j \geq \varepsilon_j - \ln \tilde{p}_j \geq \max_{k \neq j} \{\varepsilon_k - \ln \tilde{p}_k\}$$

  The measure of consumers $c \in B$ coincides with the fraction of individuals choosing commodity j after the price change

$$Pr(B) = \frac{\tilde{p}_j^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \ldots + \tilde{p}_n^{-\frac{1}{\mu}}}$$

  Similarly to the previous case, the change in price is reflected into a change in the individual utility, $\Delta V^c = \ln p_j - \ln \tilde{p}_j$, for $c \in B$. Aggregating across consumers, the overall utility change is given by

$$\Delta V^B = (\ln p_j - \ln \tilde{p}_j) \frac{\tilde{p}_j^{-\frac{1}{\mu}}}{\tilde{p}_1^{-\frac{1}{\mu}} + \ldots + \tilde{p}_n^{-\frac{1}{\mu}}}$$

- Consumers choosing commodity j before the price change and switching to commodity $k \neq i$,

$$\begin{aligned} & \varepsilon_j - \ln p_j \geq \max_{k \neq j} \{\varepsilon_k - \ln p_k\} \quad \text{before the price change} \\ \exists\, k \neq i: \quad & \varepsilon_k - \ln \tilde{p}_k \geq \max_s \{\varepsilon_s - \ln \tilde{p}_s\} \quad \text{after the price change} \end{aligned}$$

Let C_k be the set of those consumers. Each consumer $c \in C_k$ experiences a different utility change which depends upon the realizations of $(\varepsilon_k, \varepsilon_j)$. In particular, $\Delta V^c = \varepsilon_k - \varepsilon_j + \ln p_j - \ln \tilde{p}_k$, for $c \in C_k$. Aggregating across all consumers $c \in C_k$, the change in utility is obtained as follows

$$\Delta V^{C,k} = \int_{-\infty}^{\infty} \int_{\varepsilon_j + \ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\varepsilon_j + \ln \frac{p_k}{p_j}} \left(\varepsilon_k - \varepsilon_j + \ln \frac{p_j}{\tilde{p}_k}\right) Pr\left[\varepsilon_k + \ln \frac{\tilde{p}_1}{\tilde{p}_k}\right] \cdots Pr\left[\varepsilon_k + \ln \frac{\tilde{p}_n}{\tilde{p}_k}\right] f(\varepsilon_j) f(\varepsilon_k) \mathrm{d}\varepsilon_k \mathrm{d}\varepsilon_j$$

Let $y \equiv \varepsilon_k - \varepsilon_j$,

$$\begin{aligned}
\Delta V^{C,k} &= \int_{-\infty}^{\infty} \int_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} \left(y + \ln \frac{p_j}{\tilde{p}_k}\right) Pr\left[\varepsilon_j + y + \ln \frac{\tilde{p}_1}{\tilde{p}_k}\right] \cdots Pr\left[\varepsilon_j + y + \ln \frac{\tilde{p}_n}{\tilde{p}_k}\right] f(\varepsilon_j) f(\varepsilon_j + y) \mathrm{d}y \mathrm{d}\varepsilon_j \\
&= \int_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} \left(y + \ln \frac{p_j}{\tilde{p}_k}\right) \int_{-\infty}^{\infty} Pr\left[\varepsilon_j + y + \ln \frac{\tilde{p}_1}{\tilde{p}_k}\right] \cdots Pr\left[\varepsilon_j + y + \ln \frac{\tilde{p}_n}{\tilde{p}_k}\right] f(\varepsilon_j) f(\varepsilon_j + y) \mathrm{d}\varepsilon_j \mathrm{d}y \\
&= \int_{\ln \tilde{p}_k - \ln \tilde{p}_j}^{\ln p_k - \ln p_j} \left(y + \ln \frac{p_j}{\tilde{p}_k}\right) \frac{1}{\mu} \frac{\exp\left[-\frac{y}{\mu}\right]}{\left[1 + \exp\left[-\frac{y}{\mu}\right] \frac{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2} \mathrm{d}y \\
&= \left| \frac{\tilde{p}_k^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}} \frac{y + \ln \frac{p_j}{\tilde{p}_k}}{1 + \exp\left(-\frac{y}{\mu}\right) \frac{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}} \right|_{\ln \tilde{p}_k - \ln \tilde{p}_j}^{\ln p_k - \ln p_j} - \left| \mu \frac{\tilde{p}_k^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}} \ln \left[\frac{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}} + \exp\left(\frac{y}{\mu}\right)\right] \right|_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} \\
&= -\frac{\tilde{p}_k^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}} \frac{\ln p_j - \ln \tilde{p}_j}{\frac{\sum_{s=1}^n \tilde{p}_s^{-1/\mu}}{\tilde{p}_j^{-1/\mu}}} + \mu \frac{\tilde{p}_k^{-1/\mu}}{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}} \left[\ln \sum_{s=1}^n \tilde{p}_s^{-1/\mu} - \ln \left(\sum_{s \neq j} \tilde{p}_s^{-1/\mu} + p_j^{-1/\mu}\right)\right]
\end{aligned}$$

where

$$\int_{-\infty}^{\infty} \prod_{s \neq j,k} Pr\left[\varepsilon_j + y + \ln \frac{\tilde{p}_s}{\tilde{p}_k}\right] f(\varepsilon_j) f(\varepsilon_j + y) \mathrm{d}\varepsilon_j = \frac{1}{\mu} \frac{\exp\left[-\frac{y}{\mu}\right]}{\left[1 + \exp\left[-\frac{y}{\mu}\right] \frac{\sum_{s \neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2}$$

- Consumers choosing commodity $k \neq j, i$ before the price change and switching to commodity i after the price change,

$$\begin{aligned}
\varepsilon_k - \ln p_k &\geq \max_s \{\varepsilon_s - \ln p_s\} \quad \text{before the price change} \\
\varepsilon_i - \ln \tilde{p}_i &\geq \max_s \{\varepsilon_s - \ln \tilde{p}_s\} \quad \text{after the price change}
\end{aligned}$$

Let D_k be the set of those consumers. Similarly to what described above, consumers $c \in D_k$ experience changes in utility that depend on the realization of the idiosyncratic shocks $(\varepsilon_k, \varepsilon_i)$

$$\Delta V^c = \varepsilon_i - \varepsilon_k + \ln p_k - \ln \tilde{p}_i, \qquad c \in D_k$$

In aggregate,

$$\Delta V^{D,k} = -\frac{p_k^{-1/\mu}}{\sum_{s \neq i} p_s^{-1/\mu}} \frac{\ln p_i - \ln \tilde{p}_i}{\frac{\sum_{s=1}^n p_s^{-1/\mu}}{p_i^{-1/\mu}}} + \mu \frac{p_k^{-1/\mu}}{\sum_{s \neq i} p_s^{-1/\mu}} \left[\ln \left(\sum_{s \neq i} p_s^{-1/\mu} + \tilde{p}_i^{-1/\mu}\right) - \ln \sum_{s=1}^n p_s^{-1/\mu}\right]$$

- Consumers switching from commodity j to commodity i. Let E be the set of those consumers.

Consumers $c \in E$ are identified by the following conditions

$$\ln p_j - \ln p_i \leq \varepsilon_j - \varepsilon_i \leq \ln p_j - \ln \tilde{p}_i$$
$$\ln p_j - \ln \tilde{p}_i \leq \varepsilon_j - \varepsilon_i \leq \ln \tilde{p}_j - \ln \tilde{p}_i$$

The total utility change over E is given by

$$\Delta V^E = \frac{p_j^{-1/\mu}}{\sum_{s\neq i} p_s^{-1/\mu}} \frac{\ln \tilde{p}_i - \ln p_i}{\frac{\sum_{s=1}^n p_s^{-1/\mu}}{p_i^{-1/\mu}}} + \mu \frac{p_j^{-1/\mu}}{\sum_{s\neq i} p_s^{-1/\mu}} \left[\ln \left(\sum_{s\neq i} p_s^{-1/\mu} + \tilde{p}_i^{-1/\mu} \right) - \ln \sum_{s=1}^n p_s^{-1/\mu} \right]$$
$$- \frac{\tilde{p}_i^{-1/\mu}}{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}} \frac{\ln p_j - \ln \tilde{p}_j}{\frac{\sum_{s=1}^n \tilde{p}_s^{-1/\mu}}{\tilde{p}_j^{-1/\mu}}} + \mu \frac{\tilde{p}_i^{-1/\mu}}{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}} \left[\ln \sum_{s=1}^n \tilde{p}_s^{-1/\mu} - \ln \left(\sum_{s\neq i} \tilde{p}_s^{-1/\mu} + p_j^{-1/\mu} \right) \right]$$

It is easy to verify that the aggregate change in utility coincides with the change in utility of the representative consumer,

$$\Delta V^A + \Delta V^B + \sum_{k\neq i,j} \Delta V^{C,k} + \sum_{k\neq i,j} \Delta V^{D,k} + \Delta V^E = 0$$

A.2 Income Transfers across Switching Consumers

Aggregating across consumers $c \in C_k$

$$T^{C,k} = \int_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} y \left[\frac{\tilde{p}_k}{p_j} e^{-x} - 1 \right] \frac{1}{\mu} \frac{\exp\left[-\frac{x}{\mu}\right]}{\left[1 + \exp\left[-\frac{x}{\mu}\right] \frac{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2} \mathrm{d}x$$
$$= y \frac{\tilde{p}_k}{p_j} \int_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} \frac{1}{\mu} \frac{\exp\left[-x\left(\frac{1}{\mu}+1\right)\right]}{\left[1 + \exp\left[-\frac{x}{\mu}\right] \frac{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2} \mathrm{d}x + \left. \frac{y}{\frac{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}} + \exp\left[\frac{x}{\mu}\right]} \right|_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}}$$
$$= y \frac{\tilde{p}_k}{p_j} \int_{\ln \frac{\tilde{p}_k}{\tilde{p}_j}}^{\ln \frac{p_k}{p_j}} \frac{1}{\mu} \frac{\exp\left[-x\left(\frac{1}{\mu}+1\right)\right]}{\left[1 + \exp\left[-\frac{x}{\mu}\right] \frac{\sum_{s\neq j} \tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2} \mathrm{d}x + y \left[\frac{p_k^{-1/\mu}}{\sum_{s\neq j} \tilde{p}_s^{-1/\mu} + p_j^{-1/\mu}} - \frac{p_k^{-1/\mu}}{\sum_{s=1}^n \tilde{p}_s^{-1/\mu}} \right]$$

where $x \equiv \varepsilon_k - \varepsilon_j$. Let $\exp\left[-\frac{x}{\mu}\right] \equiv t$. Then, $\mathrm{d}t = -\frac{1}{\mu} \exp\left[-\frac{x}{\mu}\right] \mathrm{d}x$ and

$$t \to \left(\frac{\tilde{p}_k}{\tilde{p}_j}\right)^{-\frac{1}{\mu}} \quad \text{as} \quad x \to \ln \frac{\tilde{p}_k}{\tilde{p}_j}$$
$$t \to \left(\frac{p_k}{p_j}\right)^{-\frac{1}{\mu}} \quad \text{as} \quad x \to \ln \frac{p_k}{p_j}$$

Thus, using the proposed variable change,

$$-\int_{\left(\frac{\tilde{p}_k}{\tilde{p}_j}\right)^{-\frac{1}{\mu}}}^{\left(\frac{p_k}{p_j}\right)^{-\frac{1}{\mu}}} \frac{t^{\mu}}{\left[1+t\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^2}\mathrm{d}t = \left[\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^{-\mu-1} B\left(1-\mu,\mu+1,\frac{1}{t\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}+1}\right)\Bigg|_{\left(\frac{\tilde{p}_k}{\tilde{p}_j}\right)^{-\frac{1}{\mu}}}^{\left(\frac{p_k}{p_j}\right)^{-\frac{1}{\mu}}}$$

$$= \left[\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^{-\mu-1} B\left(1-\mu,\mu+1;\frac{p_j^{-1/\mu}}{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}+p_j^{-1/\mu}}\right) - \left[\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^{-\mu-1} B\left(1-\mu,\mu+1;\frac{\tilde{p}_j^{-1/\mu}}{\sum_{s=1}^{n}\tilde{p}_s^{-1/\mu}}\right)$$

where $B(a,b;x)$ denotes the incomplete Beta function. Thus,

$$T^{C,k} = y\frac{\tilde{p}_k}{p_j}\left[\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_k^{-1/\mu}}\right]^{-\mu-1}\left[B\left(1-\mu,\mu+1;\frac{p_j^{-1/\mu}}{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}+p_j^{-1/\mu}}\right)-B\left(1-\mu,\mu+1;\frac{\tilde{p}_j^{-1/\mu}}{\sum_{s=1}^{n}\tilde{p}_s^{-1/\mu}}\right)\right] + y\left[\frac{p_k^{-1/\mu}}{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}+p_j^{-1/\mu}}-\frac{p_k^{-1/\mu}}{\sum_{s=1}^{n}\tilde{p}_s^{-1/\mu}}\right]$$

Similarly for $c\in D_k$ and $c\in E$,

$$T^{D,k} = y\frac{\tilde{p}_i}{p_k}\left[\frac{\sum_{s\neq i}p_s^{-1/\mu}}{p_k^{-1/\mu}}\right]^{\mu-1}\left[B\left(\mu+1,1-\mu;\frac{\tilde{p}_i^{-1/\mu}}{\sum_{s\neq i}p_s^{-1/\mu}+\tilde{p}_i^{-1/\mu}}\right)-B\left(\mu+1,1-\mu;\frac{p_i^{-1/\mu}}{\sum_{s=1}^{n}p_s^{-1/\mu}}\right)\right] + y\left[\frac{p_k^{-1/\mu}}{\sum_{s\neq i}p_s^{-1/\mu}+\tilde{p}_i^{-1/\mu}}-\frac{p_k^{-1/\mu}}{\sum_{s=1}^{n}p_s^{-1/\mu}}\right]$$

$$\begin{aligned} T^{E} &= y\frac{\tilde{p}_i}{p_j}\left[\frac{\sum_{s\neq i}p_s^{-1/\mu}}{p_j^{-1/\mu}}\right]^{\mu-1}\left[B\left(\mu+1,1-\mu;\frac{\tilde{p}_i^{-1/\mu}}{\sum_{s\neq i}p_s^{-1/\mu}+\tilde{p}_i^{-1/\mu}}\right)-B\left(\mu+1,1-\mu;\frac{p_i^{-1/\mu}}{\sum_{s=1}^{n}p_s^{-1/\mu}}\right)\right] \\ &+ y\frac{\tilde{p}_i}{p_j}\left[\frac{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}}{\tilde{p}_i^{-1/\mu}}\right]^{-\mu-1}\left[B\left(1-\mu,\mu+1;\frac{p_j^{-1/\mu}}{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}+p_j^{-1/\mu}}\right)-B\left(1-\mu,\mu+1;\frac{\tilde{p}_j^{-1/\mu}}{\sum_{s=1}^{n}\tilde{p}_s^{-1/\mu}}\right)\right] \\ &+ y\left[\frac{p_j^{-1/\mu}}{\sum_{s\neq i}p_s^{-1/\mu}+\tilde{p}_i^{-1/\mu}}-\frac{p_j^{-1/\mu}}{\sum_{s=1}^{n}p_s^{-1/\mu}}\right]+y\left[\frac{\tilde{p}_i^{-1/\mu}}{\sum_{s\neq j}\tilde{p}_s^{-1/\mu}+p_j^{-1/\mu}}-\frac{\tilde{p}_i^{-1/\mu}}{\sum_{s=1}^{n}\tilde{p}_s^{-1/\mu}}\right] \end{aligned}$$

A.3 Aggregate Income Transfer

Income transfers are always larger than the underlying price changes that affect individual utilities

- For consumers $c\in A$

$$T^{A} > -\Delta V^{A} \quad \text{since} \quad \left[\frac{\tilde{p}_i}{p_i}-1\right] > \ln\frac{\tilde{p}_i}{p_i}$$

Similarly, for consumers $c\in B$.

- For consumers $c \in C_k$,

$$T^{C,k} > -\Delta V^{C,k} \quad \text{since} \quad \left[\frac{\tilde{p}_k}{p_j} e^{\varepsilon_j - \varepsilon_k} - 1\right] > \left(\varepsilon_j - \varepsilon_k + \ln \frac{\tilde{p}_k}{p_j}\right)$$

  Similarly, for consumers $c \in D_k, E$.

Therefore,

$$T^A + T^B + \sum_{k \neq i,j} T^{C,k} + \sum_{k \neq i,j} T^{D,k} + T^E > -\left(\Delta V^A + \Delta V^B + \sum_{k \neq i,j} \Delta V^{C,k} + \sum_{k \neq i,j} \Delta V^{D,k} + \Delta V^E\right) = 0$$