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# The Risky Steady State and the Interest Rate Lower Bound*

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**Abstract**

Even when the policy rate is currently not constrained by its effective lower bound (ELB), the possibility that the policy rate will become constrained in the future lowers today's inflation by creating tail risk in future inflation and thus reducing expected inflation. In an empirically rich model calibrated to match key features of the U.S. economy, we find that the tail risk induced by the ELB causes inflation to undershoot the target rate of 2 percent by as much as 45 basis points at the economy's risky steady state. Our model suggests that achieving the inflation target may be more difficult now than before the Great Recession, if the recent ELB experience has led households and firms to revise up their estimate of the ELB frequency.

JEL: E32, E52

Keywords: Deflationary Bias, Disinflation, Inflation Targeting, Risky Steady State, Tail Risk, Zero Lower Bound.

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# 1 Introduction

This paper characterizes the risky steady state in an empirically rich sticky-price model with occasionally binding effective lower bound (ELB) constraints on nominal interest rates. The risky steady state is the "point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date" (Coeurdacier, Rey, and Winant (2011)). The risky steady state is an important object in dynamic macroeconomic models: This is the point around which the economy fluctuates, the point where the economy eventually converges to when all headwinds and tailwinds dissipate.

We first use a stylized New Keynesian model to illustrate how, and why, the risky steady state differs from the deterministic steady state. We show that inflation and the policy rate are lower, and output is higher, at the risky steady state than at the deterministic steady state. This result obtains because the lower bound constraint on interest rates makes the distribution of firm's marginal costs of production asymmetric; the decline in marginal costs caused by a large negative shock is larger than the increase caused by a positive shock of the same magnitude. As a result, the ELB constraint reduces expected marginal costs for forward-looking firms, leading them to lower their prices even when the policy rate is not currently constrained. Reflecting the lower inflation rate at the risky steady state, the policy rate is lower at the risky steady state than at the deterministic steady state. In equilibrium, the expected real rate is lower at the risky steady than at the deterministic steady state, and the output gap is positive as a result. These qualitative results are consistent with those in Adam and Billi (2007) and Nakov (2008) on how the ELB risk affects the economy near the ELB constraint under optimal discretionary policy.

We then turn to the main exercise of our paper, which is to explore the quantitative importance of the wedge between the risky and deterministic steady states in an empirically rich DSGE model calibrated to match key features of the U.S. economy. We find that the wedge between the deterministic and risky steady states is non-trivial in our calibrated empirical model. Inflation is about 25 basis points lower than the target inflation of 2 percent at the risky steady state, with 18 basis points attributable to the ELB constraint as opposed to other nonlinearities of the model. The policy rate and the output gap are 50 basis points lower and 0.3 percentage points higher, respectively, at the risky steady state than at the deterministic steady state. The magnitude of the wedge depends importantly on the frequency of hitting the ELB, which in turn depends importantly on the level of the long-run equilibrium policy rate. If the policy rate at the deterministic steady state is 40 basis points lower than our baseline of 3.75 percent, then the deflationary bias would increase to more than 50 basis points, with the ELB risk contributing 45 basis points to the overall deflationary bias.

The observation that inflation falls below the inflation target in the policy rule at the risky steady state is different from the well-known fact that the average inflation falls below the target rate in the model with the ELB constraint. The decline in inflation arising from a contractionary shock can be exacerbated when the policy rate is at the ELB, while the rise in inflation arising from an expansionary shock is tempered by a corresponding increase in the policy rate. As a result, the distribution of inflation is negatively skewed and the average inflation falls below the median. This fact is intuitive and has been well known in the profession for a long time (Coenen, Orphanides, and Wieland (2004) and Reifschneider and Williams (2000)). The risky steady state inflation is different from the average inflation; it is the rate of inflation that would prevail at the economy's steady state when agents are aware of risks. It is worth mentioning that the average inflation falls below the target even in perfect-foresight models or backward-looking models where the inflation rate eventually converges to its target. On the other hand, for the risky steady state inflation to fall below the inflation target, it is crucial that price-setters are forward-looking and take tail risk in future marginal costs into account in their pricing decisions.

Our result that the ELB constraint has enduring effects on the economy even after liftoff provides a cautionary tale for policymakers aiming to overcome the problem of persistently low inflation. In particular, according to our model, inflation at the risky steady state is tightly linked to how frequently the policy rate will be constrained by the ELB in the future. Thus, our model suggests that achieving the inflation target may be more difficult now than before the Great Recession, if the recent lower bound experience, together with the recent downward assessment of the long-run growth rate of the economy and long-run equilibrium policy rate, have made the private sector to increase its assessment of the likelihood of hitting the ELB in the future.

The question of how the possibility of returning to the ELB affects the economy has remained largely unexplored. The majority of the literature adopts the assumption that the economy will eventually return to an absorbing state where the policy rate is permanently away from the ELB constraint, and analyzes the dynamics of the economy, and the effects of various policies, when the policy rate is *at* the ELB (Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011)). While an increasing number of studies have recently departed from the assumption of an absorbing state, the focus of these studies is mostly on how differently the economy behaves at the ELB versus away from the ELB, instead of how the ELB risk affects the economy away from the ELB.[1] With the federal funds rate finally raised from the ELB constraint after staying there for seven years, and with the pace of the policy tightening expected to be gradual, the question of how the possibility of returning to the ELB affects the economy is as relevant as ever.[2]

Our paper builds on the work by Adam and Billi (2007) and Nakov (2008) who first

[1]For example, Gavin, Keen, Richter, and Throckmorton (2015) and Keen, Richter, and Throckmorton (2015) ask how differently technology and anticipated monetary policy shocks affect the economy when the policy rate is constrained than when it is not, respectively. Schmidt (2013) and Nakata (2013a) ask how differently the government should conduct fiscal policy when the policy rate is at the ELB than when it is not.

[2]According to a special question in the Survey of Primary Dealers on December 2015, a median respondent attached 20 percent probability to the event that the federal funds rate returns to the ELB within two years after liftoff.

observed that the possibility of returning to the ELB has consequences for the economy even when the policy rate is currently away from the ELB. Our work differs from these papers in two substantive ways. First, while they pointed out the anticipation effect of returning to the ELB on the economy when the policy rate is *near* the ELB and the economy is away from the steady state, our work shows that the possibility of returning to the ELB has consequences for the economy even when the policy rate is well above the ELB and the economy is at the steady state. Second, while they studied the effects of the ELB risk in a highly stylized model, we quantify the magnitude of the effects of the ELB risk in an empirically rich, calibrated model.

Our work complements a body of work that explores the so-called deflationary steady state in sticky price models. A seminal work of Benhabib, Schmitt-Grohe, and Uribe (2001) shows the existence of a deflationary steady state with the policy rate stuck at the lower bound and inflation below target in a standard sticky-price model with a Taylor rule. Some have recently studied deflationary steady states with zero nominal interest rates in other interesting models with a nominal friction and a Taylor rule (Benigno and Fornaro (2015); Eggertsson and Mehrotra (2014); Schmitt-Grohe and Uribe (2013)). Bullard (2010) argues that these deflationary steady states are relevant in understanding the Japanese economy over the last two decades as well as what may happen in other advanced economies. The steady state we focus on is similar to the deflationary steady state in that both entail below-target inflation. However, unlike in the deflationary steady state, the nominal interest rate is above the ELB in the risky steady state.

Finally, this paper is related to recent papers that have emphasized the importance of the effect of risk on steady states in various nonlinear dynamic models. de Groot (2014) and Gertler, Kiyotaki, and Queralto (2012) discuss how the degree of risk affects the balance sheet conditions of financial intermediaries at the steady state. Coeurdacier, Rey, and Winant (2011), Devereux and Sutherland (2011) and Tille and van Wincoop (2010) study optimal portfolio choices at the risky steady state in open-economy models. Our work is similar to theirs in analyzing the effect of risk on the steady state. However, while the wedge between the deterministic and risky steady states is driven by the nonlinearity of smooth differentiable functions in their models, the wedge is driven by an inequality constraint in our model. Reflecting the difference in the types of nonlinearity involved, the solution methods employed are different. While these authors solve their models by using local approximation methods that take advantage of differentiability of policy functions, we use a global method to solve the model.[3]

The rest of the paper is organized as follows. After a brief review of the concept of the risky steady state in section 2, Section 3 analyzes the risky steady state in a stylized New Keynesian economy. Section 4 quantifies the wedge between the deterministic and risky

[3]See de Groot (2013) and Meyer-Gohde (2015) for recent progress in computing the risky steady state in nonlinear differentiable economies.

steady states in an empirically rich DSGE model. After putting our analysis in the context of the current policy debate in Section 5, section 6 concludes.

## 2 The Risky Steady State: Definition

The risky steady state is defined generically as follows.

Let $\Gamma_t$ and $S_t$ denote vectors of endogenous and exogenous variables, respectively, in the model under investigation. Let $f(\cdot,\cdot)$ denote a vector of policy functions mapping the values of endogenous variables in the previous period and today's realizations of exogenous variables into the values of endogenous variables today.[4] That is,

$$\Gamma_t = f(\Gamma_{t-1}, S_t) \tag{1}$$

The risky steady state of the economy, $\Gamma_{RSS}$, is given by a vector satisfying the following condition.

$$\Gamma_{RSS} = f(\Gamma_{RSS}, S_{SS}) \tag{2}$$

where $S_{SS}$ denote the steady state of $S_t$.[5] That is, the risky steady state is where the economy will eventually converge as the exogenous variables settle at their steady state. In this risky steady state, the agents are aware that shocks to the exogenous variables can occur, but the current realizations of those shocks are zero. On the other hand, the deterministic steady state of the economy, $\Gamma_{DSS}$, is defined as follows:

$$\Gamma_{DSS} = f_{PF}(\Gamma_{DSS}, S_{SS}) \tag{3}$$

where $f_{PF}(\cdot,\cdot)$ denotes the vector of policy functions obtained under the perfect-foresight assumption.

## 3 The Risky Steady State in a Stylized Model with the ELB

### 3.1 Model

We start by characterizing the risky steady state in a stylized New Keynesian model. Since the model is standard, we only present its equilibrium conditions here. The details of the model are described in the Appendix A.

$$C_t^{-\chi_c} = \beta \delta_t R_t \mathrm{E}_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \tag{4}$$

[4] Note that the policy function does not need to depend on the entire set of the endogenous variables in the prior period. It may not depend on any endogenous variables in the prior period at all, as in the stylized model presented in the next section.

[5] There is no distinction between deterministic and risky steady states for $S_t$ because $S_t$ is exogenous.

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{5}$$

$$\frac{Y_t}{C_t^{\chi_c}}\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)\frac{\Pi_t}{\bar{\Pi}}-(1-\theta)-\theta w_t\right]=\beta\delta_t \mathrm{E}_t \frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\varphi\left(\frac{\Pi_{t+1}}{\bar{\Pi}}-1\right)\frac{\Pi_{t+1}}{\bar{\Pi}} \tag{6}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\frac{\Pi_t}{\bar{\Pi}}-1\right]^2 Y_t \tag{7}$$

$$Y_t = N_t \tag{8}$$

$$R_t = \max\left[R_{ELB}, \quad \frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right] \tag{9}$$

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1}-1)+\epsilon_t^\delta \tag{10}$$

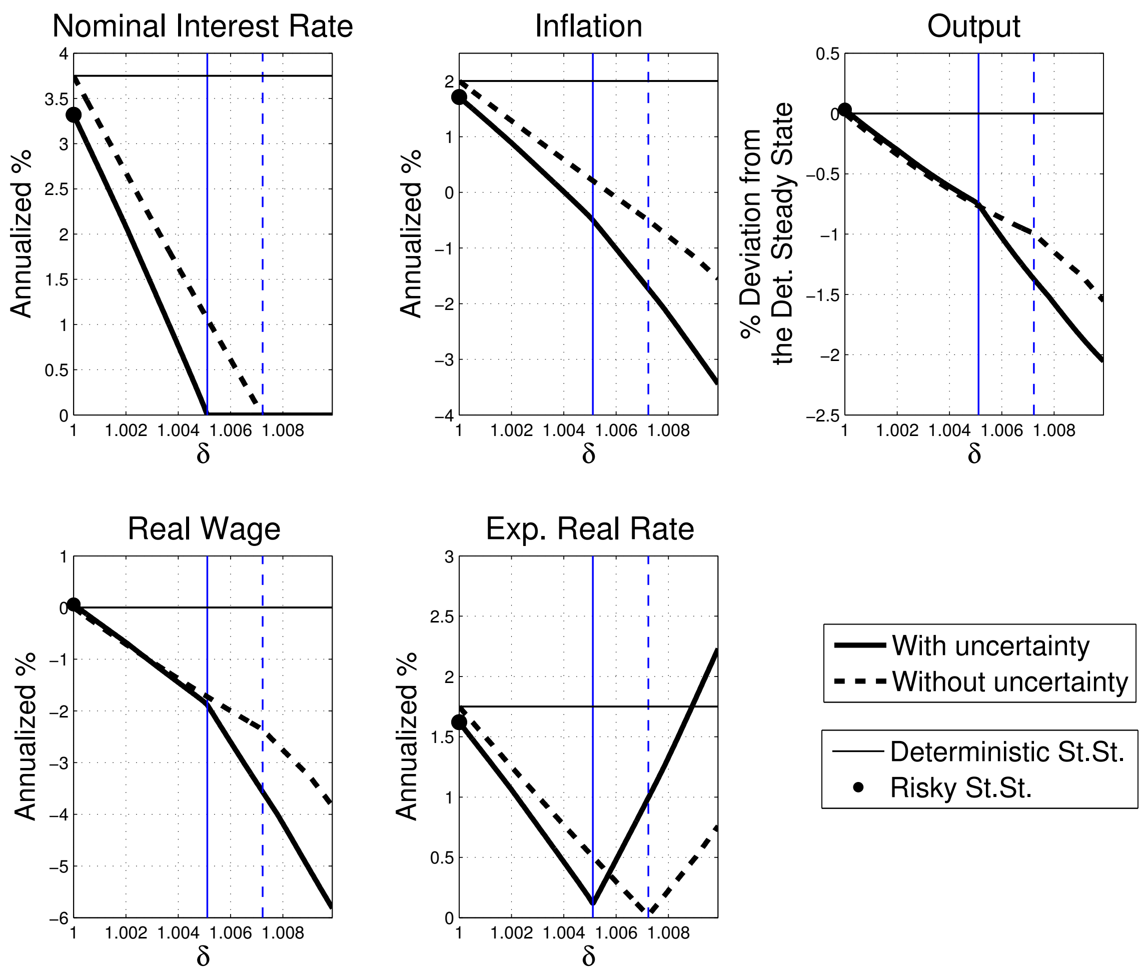
$C$, $N$, $Y$, $w$, $\Pi$, and $R$ are consumption, labor supply, output, real wage, inflation, and the policy rate, respectively. $\delta$ is an exogenous shock to the household's discount rate, and follows an AR(1) process with mean one, as shown in equation 10. Equation 4 is the consumption Euler equation, Equation 5 is the intratemporal optimality condition of the household, Equation 6 is the optimality condition of the intermediate good producing firms relating today's inflation to real marginal cost today and expected inflation tomorrow (forward-looking Phillips Curve), Equation 7 is the aggregate resource constraint capturing the resource cost of price adjustment, and Equation 8 is the aggregate production function. Equation 9 is the interest-rate feedback rule where $\bar{\Pi}$ and $\bar{Y}$ are the central bank's objectives for inflation and output.

A recursive equilibrium of this stylized economy is given by a set of policy functions for $\{C(\cdot),\ N(\cdot),\ Y(\cdot),\ w(\cdot),\ \Pi(\cdot),\ R(\cdot)\}$ satisfying the equilibrium conditions described above. The model is solved with a global solution method described in detail in the Appendix C. Table 1 lists the parameter values used for this exercise.

Table 1: **Parameter Values for the Stylized Model**

| Parameter | Description | Parameter Value |
|---|---|---|
| $\beta$ | Discount rate | $\frac{1}{1+0.004365}$ |
| $\chi_c$ | Inverse intertemporal elasticity of substitution for $C_t$ | 1.0 |
| $\chi_n$ | Inverse labor supply elasticity | 1 |
| $\theta$ | Elasticity of substitution among intermediate goods | 11 |
| $\varphi$ | Price adjustment cost | 200 |
| $400(\bar{\Pi}-1)$ | (Annualized) target rate of inflation | 2.0 |
| $\phi_\pi$ | Coefficient on inflation in the Taylor rule | 1.5 |
| $\phi_y$ | Coefficient on the output gap in the Taylor rule | 0 |
| $R_{ELB}$ | The effective lower bound | 1 |
| $\rho$ | AR(1) coefficient for the discount factor shock | 0.8 |
| $\sigma_\epsilon$ | The standard deviation of shocks to the discount factor | $\frac{0.24}{100}$ |
| | *The implied prob. that the policy rate is at the lower bound | 10% |

Figure 1: Policy Functions from the Stylized Model




*The dashed black lines ("Without uncertainty" case) show policy functions obtained under the perfect-foresight assumption (i.e., $\sigma_\epsilon = 0$).

### 3.2 Dynamics and the risky steady state

Before analyzing the risky steady state of the model, it is useful first to look at the dynamics of the model. Solid black lines in Figure 1 show the policy functions for the policy rate, inflation, output, and the expected real interest rate. Dashed black lines show the policy function of the model obtained under the assumption of perfect foresight. Under the perfect-foresight case, the agents in the model attach zero probability to the event that the policy rate will return to the ELB when the policy rate is currently away from the ELB. Under both versions of the model, an increase in the discount rate makes households want to save more for tomorrow and spend less today. Thus, as $\delta$ increases, output, inflation, and the policy rate decline. When $\delta$ is large and the policy rate is at the ELB, an additional increase in the discount rate leads to larger declines in inflation and output than when $\delta$ is small and the policy rate is not at the ELB, as the adverse effects of the increase in $\delta$ are not countered by a corresponding reduction in the policy rate.

When the policy rate is at the ELB, the presence of uncertainty reduces inflation and output. This is captured by the fact that the solid lines are below the dashed lines for inflation and output in the figure. The non-neutrality of uncertainty is driven by the ELB constraint. If the economy is buffeted by a sufficiently large expansionary shock, then the policy rate will adjust to offset some of the resulting increase in real wages. If the economy is hit by a contractionary shock, regardless of the size of the shock, the policy rate will stay at the ELB and the resulting decline in real wages will not be tempered. Due to this asymmetry, an increase in uncertainty reduces the expected real wage, which in turn reduces inflation as price-setters are forward-looking and thus inflation today depends on the expected real wage. With the policy rate constrained at the ELB, a reduction in inflation leads to an increase in the expected real rate, pushing down consumption and output today. These adverse effects of uncertainty at the ELB are studied in detail in Nakata (2013b).

When the policy rate is away from the ELB, the presence of uncertainty reduces inflation and the policy rate, but increases output. If the economy is hit by a sufficiently large contractionary shock, the policy rate will hit the ELB and the resulting decline in real wages will not be tempered. If the economy is hit by an expansionary shock, regardless of the size of the shock, the policy rate will adjust to partially offset the resulting increase in real wages. Thus, the presence of uncertainty, by generating the possibility that the policy rate will return to the ELB, reduces the expected real wage and thus today's inflation. When the policy rate is away from the ELB, its movement is governed by the Taylor rule. Since the Taylor principle is satisfied (i.e., the coefficient of inflation is larger than one), the reduction in inflation comes with a larger reduction in the policy rate. As a result, the expected real rate is lower, and thus consumption and output are higher, with uncertainty than without uncertainty.

Table 2: **The Risky Steady State in the Stylized Model**

| | Inflation | Output* | Policy rate |
|---|---|---|---|
| Deterministic steady state | 2 | 0 | 3.75 |
| Risky steady state | 1.71 | 0.03 | 3.32 |
| (Wedge) | (−0.29) | (0.03) | (−0.43) |
| Risky steady state w/o the ELB | 1.99 | −0.02 | 3.72 |
| (Wedge) | (−0.01) | (−0.02) | (−0.03) |

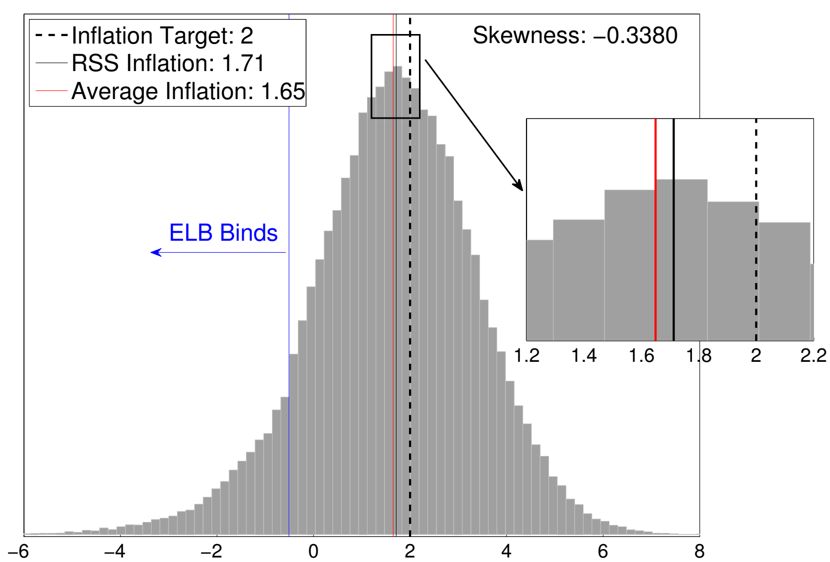
*Output is expresed as a percentage deviation from the determistic steady state.

While these effects are stronger the closer the policy rate is to the ELB, they remain nontrivial even at the economy's risky steady state. In the stylized model of this section, in which the policy functions do not depend on any of the model's endogenous variables from the previous period, the risky steady state is given by the vector of the policy functions evaluated at $\delta = 1$. That is, inflation, output, and the policy rate at the risky steady state are given by the intersection of the policy functions for these variables and the left vertical axes. As shown in Table 2, inflation and output are 29 basis points lower and 0.03 percentage points higher at the risky steady state than at the deterministic steady state, respectively. The risky steady state policy rate is 43 basis point lower than its deterministic counterpart.

In our model, the ELB constraint is not the only source of nonlinearity. Our specifications of the utility function and the price adjustment cost also make the model nonlinear, and thus explain some of the wedge between the deterministic and risky steady states. To understand the extent to which these other nonlinearities matter, Table 2 also reports the risky steady state in the version of the model without the ELB constraint. Overall, the differences between the deterministic and risky steady states would be small were it not for the ELB constraint. Inflation and the policy rate at the risky steady state are only 1 and 3 basis points below those at the deterministic steady state, respectively. Output at the risky steady state is about 2 basis points below that at the deterministic steady state. Thus, the majority of the overall wedge between the deterministic and risky steady states is attributed to the nonlinearity induced by the ELB constraint, as opposed to other nonlinearities of the model.

### 3.3 The risky steady state and the average

Figure 2: Unconditional Distribution of Inflation in the Stylized Model




*RSS stands for the Risky Steady State.

It is important to recognize that the risky steady state is different from the average. Let's take inflation as an example. The risky steady state inflation is the point around which inflation fluctuates and coincides with the median of its unconditional distribution in the model without any endogenous state variables like the one analyzed here. On the other hand, the average inflation is the average of inflation in all states of the economy. Provided that the probability of being at the ELB is sufficiently large, the unconditional distribution of inflation is negatively skewed and therefore the risky steady state inflation is higher than the average inflation, as depicted in Figure 2. The observation that the ELB constraint pushes down the average inflation below the median by making the distribution of inflation negatively skewed is intuitive and has been well known for a long time (Coenen, Orphanides, and Wieland (2004) and Reifschneider and Williams (2000)). This holds true even when price-setters form expectations in a backward-looking manner. The result that the ELB risk lowers the median of the distribution below the target is less intuitive and requires that price-setters are forward-looking in forming their expectations.

The magnitude of the wedge between the deterministic and risky steady states depends importantly on the probability of being at the ELB. The blue line in the left panel of Figure 3 illustrates this point for inflation. In this figure, we vary the standard deviation of the discount rate shock to induce changes in the probability of being at the ELB. According to the blue line, a higher probability of being at the ELB is associated with a larger deflationary bias at the risky steady state. In this stylized model, the risky steady state increases from 29 basis points to 38 basis points when the probability of being at the ELB increases from 10 percent to 12 percent. Similarly, the average inflation is lower when the probability of being at the ELB is higher, as shown by the red line. As discussed earlier, when the ELB probability is sufficiently high, the average inflation is below the risky steady state inflation. However, when the ELB probability is sufficiently low, the average inflation is above the risky steady state inflation, which reflects the fact that other nonlinear features of the model make the unconditional distribution of inflation slightly positively skewed.

Figure 3: Conditional and Unconditional Averages of Inflation in the Stylized Model

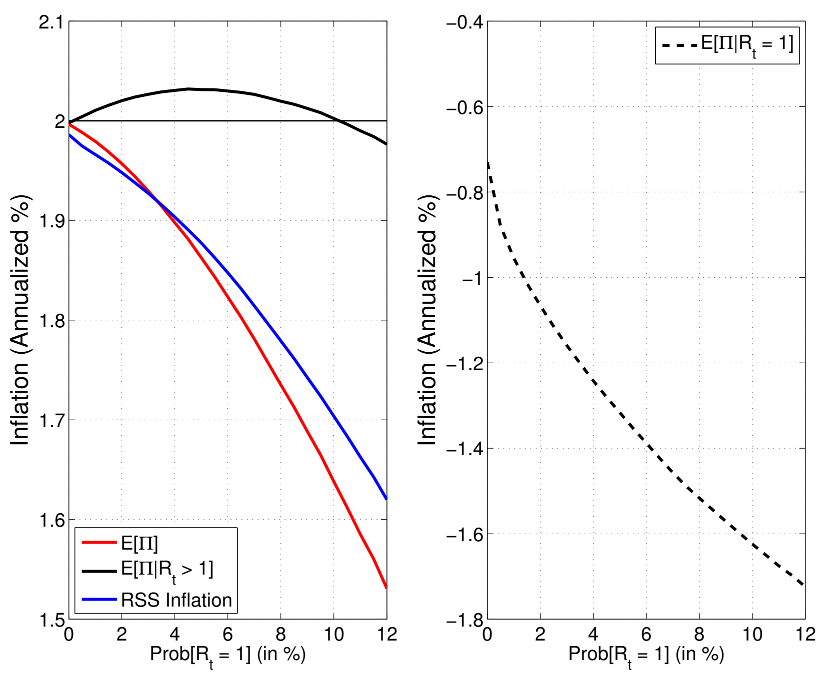



Figure 3 also plots the conditional averages of inflation away from the ELB (the black line in the left panel) and the conditional average of inflation at the ELB (the dashed black line in the right panel). Not surprisingly, the conditional average of inflation away from the ELB is higher than the unconditional average of inflation, which in turn is higher than the conditional average of inflation at the ELB. The conditional average of inflation at the ELB monotonically declines with the probability of being at the ELB, as the risky steady state inflation and the unconditional average inflation do.

In contrast, the conditional average of inflation away from the ELB is non-monotonic; It increases when the ELB probability is low and declines when the probability is high. As a result, the conditional average of inflation away from the ELB is above the target rate of 2 percent when the ELB probability is sufficiently low. This happens because, when the ELB probability is sufficiently low, the conditional distribution of inflation away from the ELB excludes the lower tail of the unconditional distribution, which is centered around a level that is only slightly below 2 percent. However, the conditional average of inflation away from the ELB is below the target rate when the lower bound risk is sufficiently high and the unconditional distribution of inflation is centered around a point sufficiently below 2 percent. As described in the Appendix H, the conditional average of inflation away from the ELB is always above the target rate of 2 percent in the perfect-foresight version of the model with the ELB. Thus, the importance of the ELB risk manifests itself in the below-target conditional average of inflation away from the ELB.

While the risky steady state inflation cannot be measured in the data, the conditional average of inflation away from the ELB can be. Thus, the importance of the lower bound risk in the data manifests itself in the extent to which the conditional average of inflation away from the ELB falls below the target rate of inflation. We will later examine the average inflation away from the lower bound in several advanced economies in Section 5.

### 3.4 The risk-adjusted Fisher relation

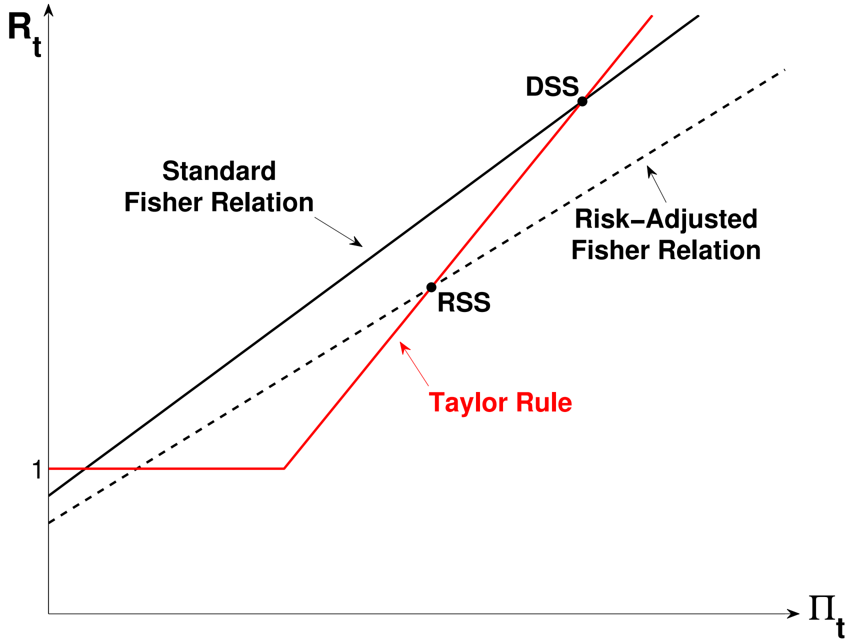
One way to understand the discrepancy between deterministic and risky steady states is to examine the effect of the ELB risk on the Fisher relation. Let $R_{DSS}$ and $\Pi_{DSS}$ be the deterministic steady state policy rate and inflation. In the deterministic environment, the consumption Euler equation evaluated at the steady state becomes

$$R_{DSS} = \frac{\Pi_{DSS}}{\beta} \tag{11}$$

after dropping the expectation operator from the consumption Euler equation and eliminating the deterministic steady-state consumption from both sides of the equation. This relation is often referred to as the Fisher relation.

In the stochastic environment, the consumption Euler equation evaluated at the (risky)

Figure 4: The Risk-Adjusted Fisher Relation and the Taylor Rule




†DSS stands for "deterministic steady state," and RSS stands for "risky steady state."

steady state can be written as

$$R_{RSS} = \frac{\Pi_{RSS}}{\beta} \cdot \frac{1}{\mathrm{E}_{RSS}[\left(\frac{C_{RSS}}{C_{t+1}}\right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}}]} \tag{12}$$

where $R_{RSS}$, $\Pi_{RSS}$, and $C_{RSS}$ are the risky steady-state policy rate, inflation, and consumption. $\mathrm{E}_{RSS}[\cdot]$ is the conditional expectation operator when the economy is at the risky steady state today. In the stylized model with one shock and without any endogenous state variables, $\mathrm{E}_{RSS}[\cdot] := E_t[\cdot|\delta_t = 1]$. We will refer to Equation 12 as the *risk-adjusted Fisher relation*. Relative to the standard Fisher relation, there is an adjustment term that reflects the discrepancy between today's economic conditions and the expected economic conditions next period. This term captures the effect of the tail risk in future economic conditions induced by the ELB constraint on household expectations. Notice that the adjustment term is less than one,

$$\frac{1}{\mathrm{E}_{RSS}[\left(\frac{C_{RSS}}{C_{t+1}}\right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}}]} < 1, \tag{13}$$

because of the fat tail on the lower end of the distributions of future inflation and consumption induced by the ELB constraint.

In equilibrium, the steady state is given by the intersection of the line representing the Fisher relation and the line representing the Taylor rule. Since the risk-adjustment term is less than one, the line representing the risk-adjusted Fisher relation crosses the line representing the Taylor rule at a point below the line for the standard Fisher relation crosses it, as shown in Figure 4. Thus, inflation and the policy rate are lower at the risky steady state than at the deterministic steady state.[6]

# 4 The Risky Steady State in an Empirical Model with the ELB

We now quantify the magnitude of the wedge between the deterministic and risky steady states in an empirically rich model calibrated to match key features of the U.S. economy.

## 4.1 Model

Our empirical model adds four additional features on top of the stylized New Keynesian model of the previous section: (i) a non-stationary productivity process, (ii) consumption habits, (iii) sticky wages, and (iv) an interest-rate smoothing term in the interest-rate feedback rule. Since these features are standard, we relegate the detailed description of them to the Appendix B and only show the equilibrium conditions of the model here. Let $\tilde{Y}_t = \frac{Y_t}{A_t}$, $\tilde{C}_t = \frac{C_t}{A_t}$, $\tilde{w}_t = \frac{w_t}{A_t}$, and $\tilde{\lambda}_t = \frac{\lambda_t}{A_t^{-\chi_c}}$ be the stationary representations of output, consumption, real wage, and marginal utility of consumption, respectively, where $A_t$ is a (deterministic) non-stationary productivity path. The stationary equilibrium is characterized by the following system of equations:

$$\tilde{\lambda}_t = \frac{\beta}{a^{\chi_c}} \delta_t R_t \mathrm{E_t} \tilde{\lambda}_{t+1} \left(\Pi^p_{t+1}\right)^{-1} \tag{14}$$

$$\tilde{\lambda}_t = (\tilde{C}_t - \frac{\zeta}{a} \tilde{C}_{t-1})^{-\chi_c} \tag{15}$$

$$\frac{N_t \tilde{w}_t}{\tilde{\lambda}_t^{-1}} \left[ \varphi_w \left( \frac{\Pi^w_t}{\bar{\Pi}^w} - 1 \right) \frac{\Pi^w_t}{\bar{\Pi}^w} - (1 - \theta^w) - \theta^w \frac{N_t^{\chi_n}}{\tilde{\lambda}_t \tilde{w}_t} \right] = \frac{\beta \varphi_w}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{N_{t+1} \tilde{w}_{t+1}}{\lambda_{t+1}^{-1}} \left( \frac{\Pi^w_{t+1}}{\bar{\Pi}^w} - 1 \right) \frac{\Pi^w_{t+1}}{\bar{\Pi}^w} \tag{16}$$

$$\Pi^w_t = \frac{\tilde{w}_t}{\tilde{w}_{t-1}} \Pi^p_t \tag{17}$$

$$\frac{\tilde{Y}_t}{\tilde{\lambda}_t^{-1}} \left[ \varphi_p \left( \frac{\Pi^p_t}{\bar{\Pi}^p} - 1 \right) \frac{\Pi^p_t}{\bar{\Pi}^p} - (1 - \theta^p) - \theta^p \tilde{w}_t \right] = \frac{\beta \varphi_p}{a^{\chi_c - 1}} \delta_t \mathrm{E}_t \frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}} \left( \frac{\Pi^p_{t+1}}{\bar{\Pi}^p} - 1 \right) \frac{\Pi^p_{t+1}}{\bar{\Pi}^p} \tag{18}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\varphi_p}{2} \left[ \frac{\Pi^p_t}{\bar{\Pi}^p} - 1 \right]^2 \tilde{Y}_t + \frac{\varphi_w}{2} \left[ \frac{\Pi^w_t}{\bar{\Pi}^w} - 1 \right]^2 \tilde{w}_t N_t \tag{19}$$

$$\tilde{Y}_t = N_t \tag{20}$$

[6]The other intersection of the risk-adjusted Fisher relation and the Taylor-rule equation indicates that, in a deflationary equilibrium, inflation is higher at the risky steady state than at the deterministic steady state. We have confirmed that this is indeed the case using a semi-loglinear model with a three-state discount rate shock. We will leave an in-depth examination of the risky steady state in a deflationary equilibrium to future research.

and

$$R_t = \max\left[1, R_t^*\right] \tag{21}$$

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R} \left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{(1-\rho_r)\phi_y} \tag{22}$$

and the following processes for the discount rate:

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_t^\delta \tag{23}$$

$\zeta$ is the degree of consumption habits in the household's utility function and $a$ is the trend growth rate of productivity. $\varphi_p$ and $\varphi_w$ are the price and wage adjustment costs. $\rho_R$ is the weight on the lagged shadow policy rate in the truncated interest-rate feedback rule. $\bar{\Pi}^p$ and $\bar{\Pi}^w$ are price and wage inflation rates in the determistic steady state. In the truncated interest-rate feedback rule, $\bar{R}$ and $\bar{Y}$ are the policy rate and output (normalized by $A_t$) at the deterministic steady state and are functions of the structural parameters. $\tilde{Y}_t/\bar{Y}$ is the deviation of the stationarized output from its deterministic steady state and will be referred to as the output gap in this paper.[7] In the Appendix G, we also study the risky steady state in several extended models which add more frictions and shocks to the empirical model of this section.

## 4.2 Calibration

We calibrate our model to match key features of the output gap, inflation, and the policy rate in the U.S. over the past two decades, which are shown in Figure 5. We focus on this relatively recent past because long-run inflation expectations were low and stable and the ELB was either a concern or a binding constraint to the Federal Reserve during this period. As shown in Figure 6, the median of CPI inflation forecasts 5-10 years ahead in the Survey of Professional Forecasters, a commonly used measure of long-run inflation expectations, declined to 2.5 percent in late 1990s and has been relatively stable since then.[8] Also, the concern for the ELB surged in the U.S. in the second half of 1990s when the Bank of Japan lowered the policy rate to the lower bound for the first time in the Post WWII history among major advanced economies.[9]

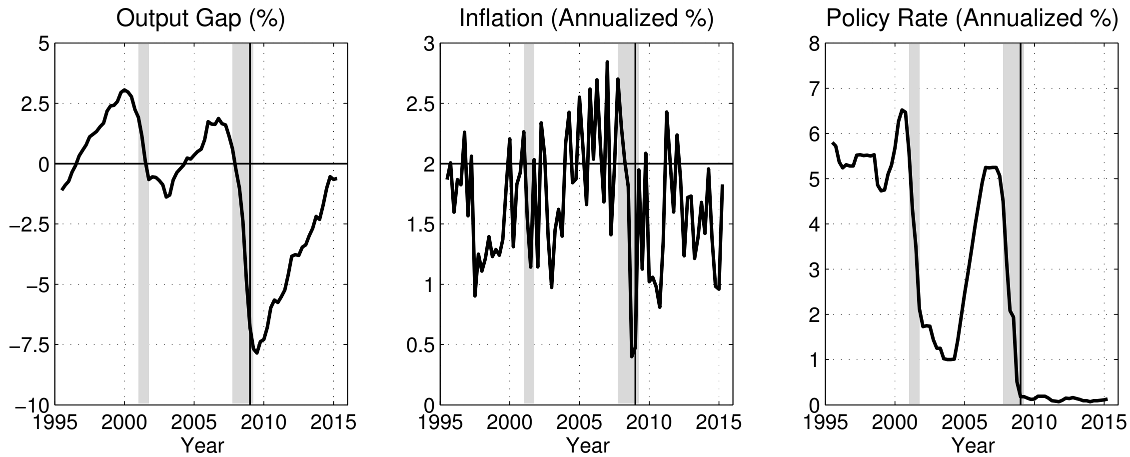
We set the time discount rate to 0.99875 so that the contribution of the discount rate to

[7]Note that our output gap is not the flexible-price output gap that is the deviation of the (normalied) output from its flexible-price counterpart.

[8]The long-run inflation expectations measured by PCE inflation is available only from 2007. The average differential between CPI and PCE inflation rates over the past two decades is about 50 basis points. Thus, the stability of CPI inflation expectations at 2.5 percent can be interpreted as the stability of PCE inflation expectations at 2 percent.

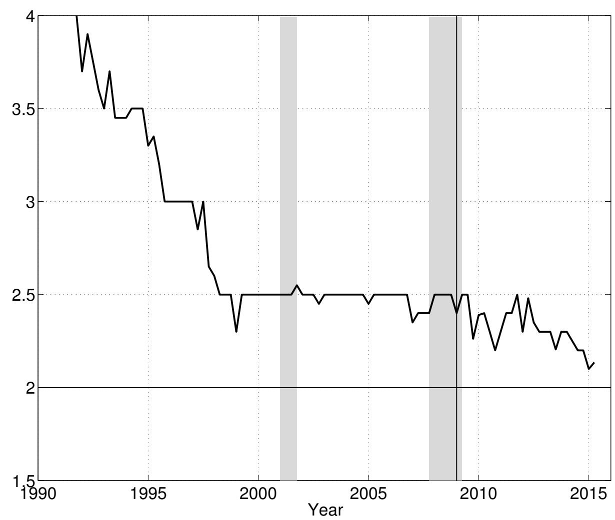
[9]Some of the earliest research on the ELB were initiated within the Federal Reserve System in this period. See, for example, Clouse, Henderson, Orphanides, Small, and Tinsley (2003), Reifschneider and Williams (2000), and Wolman (1998).

Figure 5: Output Gap, Inflation, and Policy Rate†




†The measure of the output gap is based on the FRB/US model and is available upon request. The inflation rate is computed as the annualized quarterly percentage change (log difference) in the personal consumption expenditure core price index (St. Louis Fed's FRED). The quarerly average of the (annualized) federal funds rate is used as the measure for the policy rate (St. Louis Fed's FRED). Vertical lines mark the year when ELB started binding. Horizontal lines represent target values for the respective variables.

Figure 6: Long-Run Inflation Expectations†




†Calculations based on: Federal Reserve Board, Survey of Professional Forecasters, accessed November 2015, https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/.

the deterministic steady state real rate is 50 basis points. We set the target inflation in the interest-rate feedback rule to 2 percent as this is the FOMC's official target rate of inflation. We set the trend growth rate of productivity to 1.25 percent so that the policy rate is 3.75 percent at the economy's deterministic steady state.

In the household utility function, the degree of consumption habits, the inverse Frisch labor elasticity, and the inverse intertemporal elasticity of substitution are set to 0.5, 0.5 and 1, respectively. These are all within the range of standard values found the literature.

Table 3: **Parameter Values for the Empirical Model**

| Parameter | Description | Parameter Value |
|---|---|---|
| $\beta$ | Discount rate | 0.99875 |
| $a$ | Trend growth rate of productivity | $\frac{1.25}{400}$ |
| $\chi_c$ | Inverse intertemporal elasticity of substitution for $C_t$ | 1.0 |
| $\zeta$ | Degree of consumption habits | 0.5 |
| $\chi_n$ | Inverse labor supply elasticity | 0.5 |
| $\theta_p$ | Elasticity of substitution among intermediate goods | 11 |
| $\theta_w$ | Elasticity of substitution among intermediate labor inputs | 4 |
| $\varphi_p$ | Price adjustment cost | 1000 |
| $\varphi_w$ | Wage adjustment cost | 300 |
| Interest-rate feedback rule | | |
| $400(\Pi - 1)$ | (Annualized) target rate of inflation | 2.0 |
| $\rho_R$ | Interest-rate smoothing parameter in the Taylor rule | 0.8 |
| $\phi_\pi$ | Coefficient on inflation in the Taylor rule | 3 |
| $\phi_y$ | Coefficient on the output gap in the Taylor rule | 0.25 |
| $400(R_{ELB} - 1)$ | (Annualized) effective lower bound | 0.13 |
| Shocks | | |
| $\rho_d$ | AR(1) coefficient for the discount factor shock | 0.85 |
| $\sigma_{\epsilon,\delta}$ | The standard deviation of shocks to the discount factor | $\frac{0.69}{100}$ |

Following Erceg and Lindé (2014), the parameters governing the steady-state markups for intermediate goods and the intermediate labor inputs are set to 11 and 4 and the parameters governing the price adjustment costs for prices and wages to 1000 and 300. In a hypothetical log-linear environment, these values would correspond to 90 and 85 percent probabilities that prices and wages cannot adjust each quarter in the Calvo version of the model, respectively. High degrees of stickiness in prices and wages help the model to match the moderate decline in inflation in the data while the federal funds rate was constrained at the ELB.

The coefficients on inflation and the output gap in the interest-rate feedback rule are set to 3 and 0.25. The coefficient on the output gap, 0.25, is standard. The coefficient on inflation is a bit higher compared to the values commonly used in the literature. A higher coefficient serves two purposes. First, it reduces the volatility of inflation relative to the volatility of the output gap. Second, a higher value makes the existence of the equilibrium more likely.[10] Erceg and Lindé (2014) argue that an inflation coefficient of this magnitude is consistent with an IV-type regression estimate of this coefficient based on a recent sample. The interest-rate smoothing parameter for the policy rule is set to 0.8. This high degree of interest-rate smoothing helps in increasing the expected duration of the lower bound episodes, improving the model's implication in this dimension. The ELB on the policy rate is set to 0.13 percent, the average of the annualized federal funds rate during the recent lower bound episode (from 2009:Q1 to 2015:Q2).

The persistence of the discount rate shock is set to 0.85. This is a bit higher than the

[10] Richter and Throckmorton (2015) show that the model with occasionally binding ELB constraints may not have minimum-state-variable solutions when this coefficient is low even if the Taylor-principle is satisfied.

common value of 0.8 used in most existing studies of models with occasionally binding lower bound constraints.[11] A higher persistence of the shock helps in increasing the expected duration of being at the ELB, as the higher interest-rate smoothing parameter in the policy rule does. The standard deviation of the discount rate shock is chosen so that the standard deviation of the policy rate in the model matches with that in the data.

Table 4 shows the key statistics for the output gap, inflation and the policy rate in the model and in the data. The measure of the output gap is based on the estimate of potential output based from the FRB/US model. As for the measure of inflation, we use core PCE Price Index inflation.

Table 4: **Key Moments**

| Moment | Variable | Model | Data† (1995Q3–2015Q2) |
|---|---|---|---|
| St.Dev.(·) | Output gap | 3.0 | 2.9 |
| | Inflation | 0.31 | 0.52 |
| | Policy rate | 2.34 | 2.34 |
| E(X\|ELB) | Output gap | −3.7 | −4.2 |
| | Inflation | 1.21 | 1.48 |
| | Policy rate | 0.13 | 0.13 |
| ELB | Frequency | 13.8% | 36% |
| | Expected/Actual Duration | 8.6 quarters | 26 quarters |

†The measure of the output gap is based on the FRB/US model. Inflation rate is computed as the annualized quarterly percentage change (log difference) in the personal consumption expenditure core price index. The quarerly average of the (annualized) federal funds rate is used as the measure for the policy rate.

The standard deviation of the output gap in the model is 3.0, which is in line with the sample standard deviation from the data. The conditional mean of the output gap at the ELB in the model is -3.7 percent, which is above, but similar to, the estimate from the data. The standard deviation of inflation in the model is 0.31 percent, which is lower than what's observed in the data, while the ELB conditional mean of inflation in the model is 1.21 percent, which is somewhat lower than what's observed in the data. It should be noted that, given that inflation was remarkably stable during the recent lower-bound episode in the U.S., there is a tension between matching the unconditional standard deviation of inflation and the conditional average of inflation at the ELB. Since the conditional mean of inflation is the average of the left-tail of the unconditional distribution of inflation, an increase in the standard deviation of inflation would necessarily imply a decrease in the conditional mean of inflation at the ELB. In our case, if we increase the standard deviation of inflation in our model to get closer to what's observed in the data, the conditional mean of inflation at the ELB would decline further away from its empirical counterpart.[12] Our calibration reflects

[11]See, for example, Adam and Billi (2007), Nakov (2008), and Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015).

[12]A similar tension exists for matching the standard deviation of the output gap and the conditional mean

a compromise of staying reasonably close to the data in these two dimension. It is worth noting that our estimate of the deflationary bias would be higher if we ignored the observed stability of inflation at the ELB and adjusted our calibration to match the standard deviation of inflation, as a lower conditional average of inflation at the ELB induces firms to lower prices by more when away from the ELB. Thus, our estimate of the deflationary bias can be seen as a conservative estimate.

As previously mentioned, the standard deviation of the discount rate shock was chosen so that the standard deviation of the policy rate in the model matches with that in the data, which is 2.34 percent. The model-implied unconditional probability of being at the ELB and the expected ELB duration are about 14 percent and 2 years, respectively. While these numbers are substantially higher than those in other existing models with occasionally binding ELB constraints, they are substantially lower than the empirical counterparts over the past two decades in the U.S. In particular, the duration of the recent ELB experience is seen by the model as surprisingly long. Consistent with this interpretation, the data on liftoff expectations shows that market participants have underpredicted how long the policy rate will be kept at the ELB throughout the recent ELB episode, as described in the Appendix E.

### 4.3 Results

Table 5: **The Risky Steady State in the Empirical Model**

| | Inflation | Output gap | Policy rate |
|---|---|---|---|
| Deterministic steady state | 2 | 0 | 3.75 |
| Risky steady state | 1.74 | 0.30 | 3.26 |
| (Wedge) | (−0.26) | (0.30) | (−0.49) |
| Risky steady state w/o the ELB | 1.92 | 0.05 | 3.56 |
| (Wedge) | (−0.08) | (0.05) | (−0.19) |
| $E[\cdot \mid R_t > R_{ELB}]$ | 1.78 | 0.85 | 3.85 |

Table 5 shows the risky and deterministic steady state values of inflation, the output gap, and the policy rate from our empirical model. For this model, the risky steady state is computed by simulating the model for a long period while setting the realization of the exogenous disturbances to zero. All (stationarized) endogenous variables eventually converge in that simulation, and that point of convergence is the risky state of the economy. By construction, the deterministic steady state of inflation is given by the target rate of inflation and the output gap is zero at the deterministic steady state. As explained earlier, parameter values ($\beta$, $\chi_c$ and $a$) are chosen so that the deterministic steady state of the policy rate is 3.75 percent.

Consistent with our earlier analyses based on a stylized model, inflation and the policy rate are lower, and the output gap is higher, at the risky steady state than at the deterministic

of the output gap at the ELB, but to a lesser extent.

steady state. Inflation falls 26 basis points below the target rate of inflation at the risky steady state. This is large given the small standard deviation of inflation. The policy rate at the risky steady state falls 49 basis points below its deterministic counterpart. While this is a small number relative to its standard deviation, it is nevertheless significant in light of recent discussions among economists and policymakers regarding the long-run equilibrium policy rate.[13] Finally, the output wedge between the deterministic and risky steady states is small, with the output gap standing at 0.30 percentage point at the risky steady state.

As explained in the previous section, the discrepancy between the deterministic and risky steady states is not only driven by the lower bound constraint on policy rates, but is also affected by other nonlinear features of the model. To isolate the effects of the lower bound constraint, the fourth line of Table 5 shows the risky steady state of the model without the lower bound constraint. Inflation, the output gap, and the policy rate are 1.92, 0.05, and 3.56 percent, respectively. Thus, most of the wedge between the deterministic and risky steady states in the model with the ELB constraint is attributed to the nonlinearity associated with the ELB constraint, as opposed to other nonlinear features of the model. For inflation, the ELB risk accounts for 18 basis points of the overall deflationary bias.

## 4.4 Sensitivity Analyses

### 4.4.1 Long-Run Interest Rates

There are substantial uncertainties surrounding the level of the long-run real equilibrium interest rate. Many economists recently have argued that various structural factors—including a lower trend growth rate of productivity, demographic trends, and global factors—have contributed to a persistent downward trend in the long-run equilibrium interest rate.[14] A lower long-run equilibrium interest rate means that the probability of hitting the ELB is higher, which ceteris paribus increases the magnitude of the undershooting of the inflation target at the risky steady state.

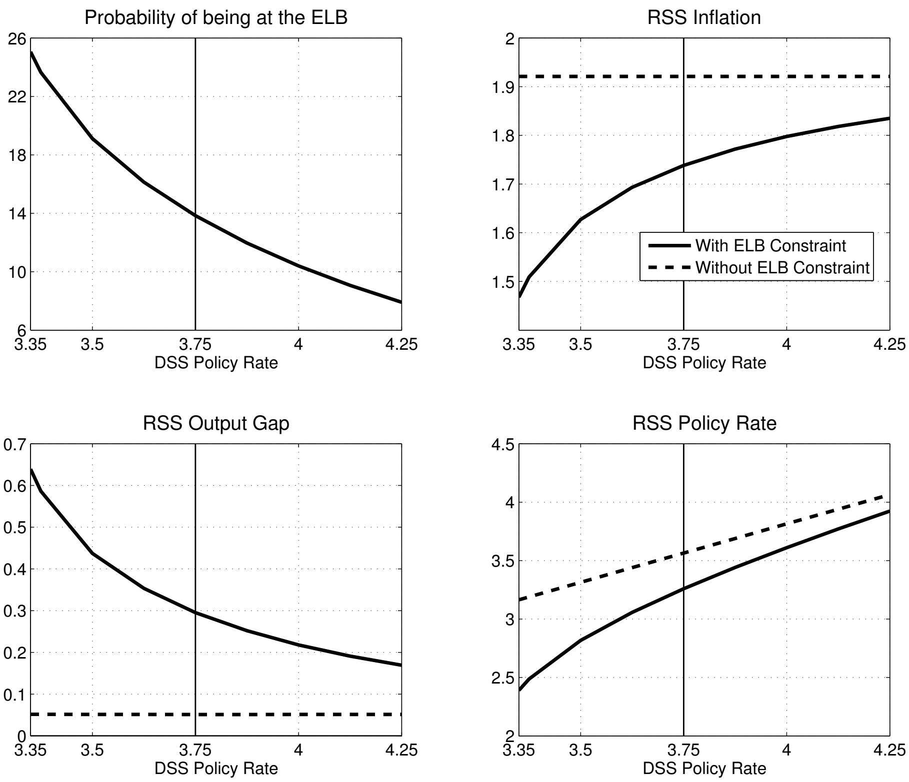
Figure 7 shows how sensitive the risky steady state of our empirical model is to alternative assumptions about the deterministic steady state interest rate. In this exercise, we vary the long-run deterministic steady-state policy rate by varying the trend growth rate. As shown in the top-left panel, the probability of the policy rate being at the ELB increases as the deterministic steady state policy rate declines. With the deterministic steady state policy rate at 3.35 percent, the probability of being at the ELB is approximately 25 percent. A higher probability of being at the ELB increases the wedge between the deterministic and risky steady states. With the deterministic steady state policy rate at 3.35 percent, the risky steady state inflation, output, and the policy rates are 1.47 percent, 0.64 percent, and 2.39 percent.[15] Since the risky steady state does not depend on the deterministic steady state

[13] See, for example, Hamilton, Harris, Hatzius, and West (2015) and Rachel and Smith (2015).

[14] See, for example, the Council of Economic Advisers (2015) and IMF (2014).

[15] Note that an increase in the output gap does not necessarily mean an increase in the level of output

Figure 7: Long-Run Interest Rates and the Risky Steady State†




†DSS stands for "deterministic steady state," and RSS stands for "risky steady state." Vertical lines mark the DSS policy rate in the baseline calibration.

policy rate in the model without the ELB constraint, as shown in the dashed lines, a large fraction of the overall wedge is explained by the ELB risk when the deterministic steady state policy rate is lower. For inflation, the ELB risk accounts for 45 basis points of the overall deflationary bias of 53 basis points.[16]

### 4.4.2 Policy Parameters

We have shown that, at the risky steady state, inflation falls below the target rate of 2 percent by a nontrivial amount in our empirical model. In our model where the prices and wages are indexed to the target rate of inflation, such undershooting of the inflation target is undesirable. A natural question to ask is what the central bank can do to mitigate the deflationary bias.[17]

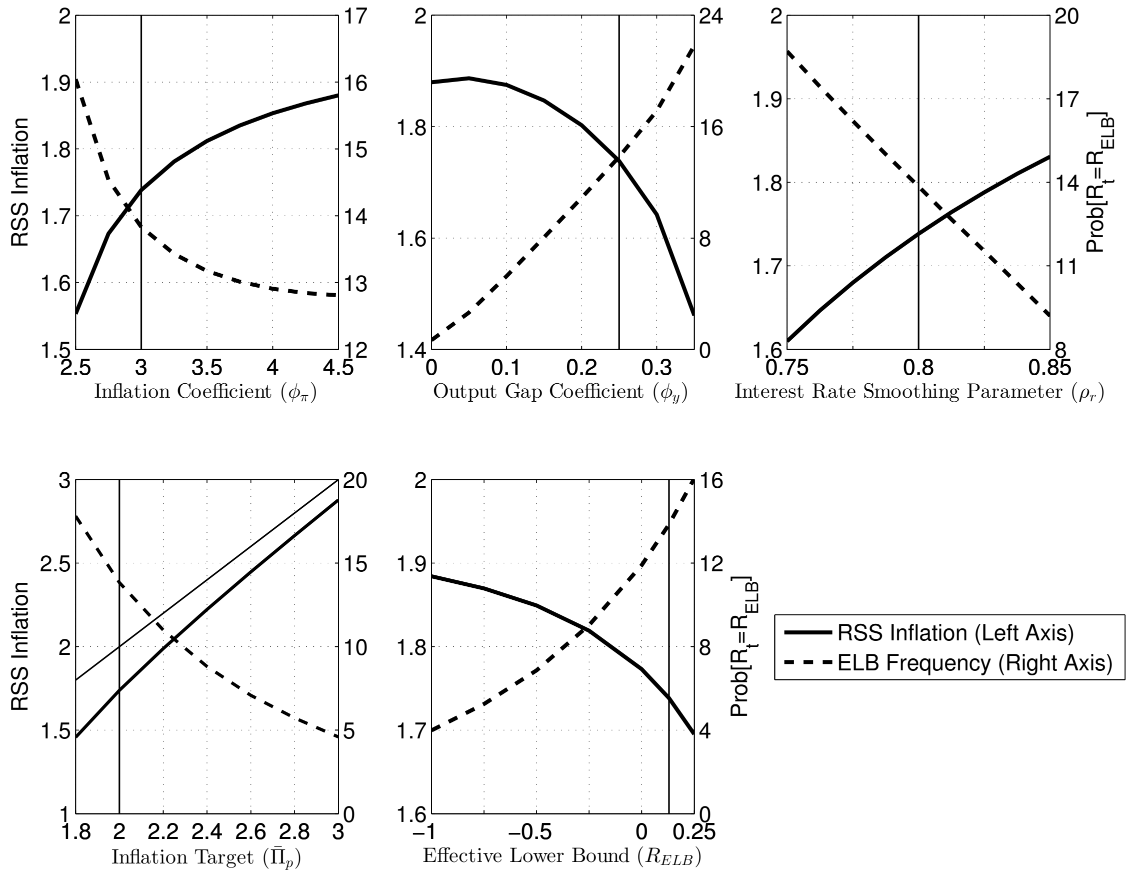
Figure 8 showd how the probability of being at the ELB and the risky steady-state

because output measures are stationarized by the trend growth rate.

[16]Hamilton, Harris, Hatzius, and West (2015) argue that any value between 0 and 2 percent is a plausible value for the long-run real rate. Thus, the long-run nominal rate of 2.39 percent—or equivalently, the long-run real rate of 0.39 percent—in this example is within their plausible range.

[17]In this paper, we examine this issue only in the context of a Taylor-type rule. Under price-level targeting or nominal income targeting, which are known to mitigate the decline in inflation at the ELB, the deflationary bias is likely to be smaller than under a conventional Taylor-type rule.

Figure 8: Alternative Policy Parameters†




†In each panel, the vertical line marks the baseline parameter value. In the bottom-left panel, the thin black line shows the 45 degree line.

inflation rate depends on parameters governing the interest-rate feedback rule. According to the top-left panel, a higher coefficient on inflation reduces the probability of being at the ELB, raising the risky steady state closer to the inflation target. On the other hand, a higher coefficient on the output gap increases the probability of being at the ELB and reduces RSS inflation. These results hold because a more aggresive response to inflation reduces both the volatility of inflation and the policy rate in equilibrium, and are consistent with the analytical results from a two-state shock model in Nakata and Schmidt (2014).

The top-right panel shows that a higher interest-rate smoothing parameter reduces the probability of being at the ELB, thus reducing the deflationary bias at the risky steady state. When the policy rate depends more on the lagged policy rate, the policy rate adjusts by less in response to fluctuations in the demand shock. Thus, higher inertia lowers the standard deviation of the policy rate, leading to a lower ELB frequency.

Not surprisingly, a higher inflation target reduces the probability of being at the ELB and reduces the difference in inflation between deterministic and risky steady states, as shown in the bottom-left panel. With the inflation target at 1.8 percent, the deflationary bias at the risky steady state is about 35 basis points. With the inflation target at 3 percent, the deflationary bias at the risky steady state is about 12 basis points. This exercise demonstrates the importance of taking into account the lower bound risk in the cost-benefit analysis of raising the inflation target.[18] Finally, the last panel demonstrates that, if the effective lower bound is lower, the ELB binds less often and thus the deflationary bias is lower at the risky steady state.

# 5 Discussion

Why should we care about what a theoretical model has to say about the ELB risk and the resulting deflationary bias? In this section, we argue that we should care about it for two reasons. First, the model with the ELB risk is consistent with the undershooting of the inflation target observed in some economies even before the policy rate became constrained by the ELB constraint. Second, the model with the ELB risk provides a cautionary tale for policymakers aiming to achieve their inflation objectives in the current environment of low long-run equilibrium real interest rates.

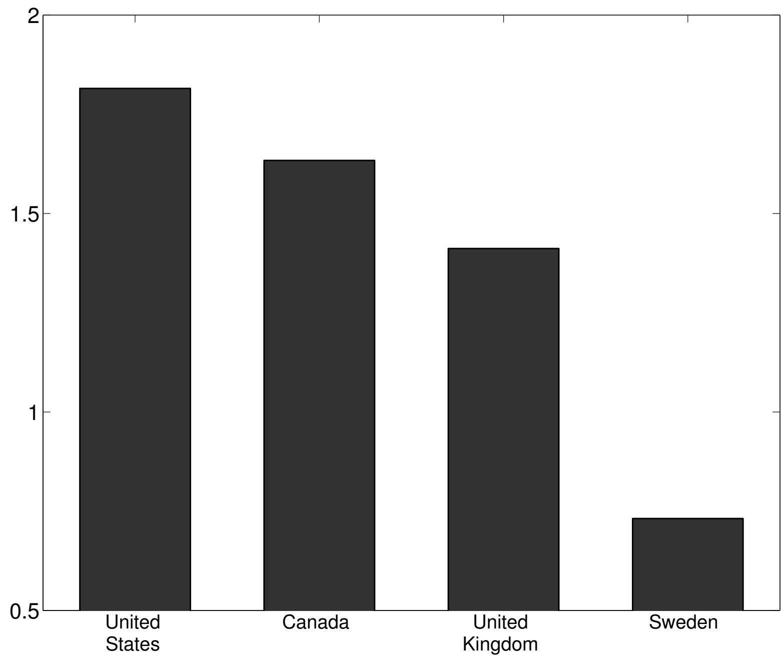
## 5.1 Low inflation before the Great Recession

Figure 9 shows the conditional averages of inflation over the past two decades when policy rates were not constrained by the ELB in four advanced economies—economies that have recently faced the ELB constraint for the first time since WWII and target 2% inflation symmetrically. According to the figure, the conditional averages of inflation away from the ELB are below the target rate of 2 percent in all these economies. In the U.S and Canada, inflation averaged about 20 and 40 basis points below 2 percent while the policy rate was not constrained by the ELB. In the UK, the conditional average of inflation away from the ELB is 60 basis points below 2 percent. In Sweden, the average inflation rate is below 1 percent while the policy rate was above the ELB. In the Appendix F, we show that the undershooting of the inflation target is robust to alternative starting dates of sample and the difference between the conditional average and the target rate is statistically significant for all four economies.

While there are many explanations for this systematic undershooting of the inflation targets before the ELB became a binding constraint—such as positive supply shocks from emerging economies and persistent slack in the economies—it is interesting to note that this undershooting is consistent with the prediction of the model with the ELB risk. As argued earlier, one key prediction of the model with the ELB risk is that the conditional average of inflation away from the ELB falls below the target rate of 2 percent, provided that the probability of being at the ELB is sufficiently high. In our empirical model, the deflationary

[18]The computation of the optimal inflation target is often conducted under the assumption of perfect-foresight. See, for example, Williams (2009) and Coibion, Gorodnichenko, and Wieland (2012).

Figure 9: Conditional Averages of Inflation Away From the ELB†




† The figure shows the average of the annualied quarterly inflation rate over the past two decades (since 1995Q3) for each country during non-ELB quarters—when the country's policy rate was not constrained by the ELB. For the U.S, the measure of inflation is based on core PCE Price Index. For other economies, the measure of inflation is ased on core CPI Index. ELB-binding quarters are from 2009Q1 to present in the U.S., from 2009:Q2 to 2010:Q1 in Canada, from 2009:Q2 to present in UK, from 2009:Q4 to 2010:Q2 and from 2014:Q4 to present in Sweden.
Data for inflation sourced from: for United States, Federal Reserve Bank of St. Louis, Personal Consumption Expenditures: Chain-type Price Index Less Food and Energy, accessed December 2015, https://research.stlouisfed.org/fred2/; for all other countries, OECD (2015), Inflation (Core CPI) (indicator), accessed November 2015, DOI: 10.1787/eee82e6e-en.

bias is indeed large enough to push the conditional average of inflation away from the ELB below the inflation target, as shown in the last row of Table 5. [19]

Since the risk of hitting the ELB was widely seen as unlikely before the Great Recession, we do not think that this ELB risk is a good explanation for the target undershooting in pre-ELB era in reality. However, we think the ability of our model to generate the undershooting in pre-ELB era is attractive. As explained in Section 3.3 and Appendix H, a version of our model with the ELB constraint, but without the ELB risk, does not have this feature. More research is certainly needed to better understand the sources of low inflation in advanced economies and what policymakers can do to address this problem.

## 5.2 An implication for the future

It is quite likely that the perceived probability of hitting the ELB is higher now than before the Great Recession. The Great Recession made clear that the economy can be hit by

[19] Note that this undershooting of the inflation target while the policy rate is above the ELB is not consistent with the deflationary steady state of the sticky-price economy (Benhabib, Schmitt-Grohe, and Uribe (2001) and Armenter (2014)). In the deflationary steady state, inflation is below the target, but the policy rate is at the ELB.

shocks that are substantially larger than the macro shocks that hit the economy during the Great Moderation. Moreover, several years of disappointing output growth in the aftermath of the Great Recession have led many analysts to revise down their estimates of the trend growth rate of productivity and long-run nominal interest rates. As shown in Section 4.4.1 a lower long-run equilibrium interest rate would imply a higher ELB frequency.

As our earlier sensitivity analyses demonstrated, the size of the deflationary bias at the risky steady state increases with the probability of being at the ELB. Thus, our model provides a cautionary tale for policymakers: Achieving the target rate of inflation may have become more difficult now than before the Great Recession if the recent ELB experience has led the private sector to revise its assessment of the likelihood of ELB events.

# 6 Conclusion

In this paper, we have examined the implications of the ELB risk—the possibility that the policy rate will be constrained by the ELB in the future—for the economy when the policy rate is currently not constrained. Using an empirically rich DSGE model calibrated to capture key features of the U.S. economy over the past two decades, we have shown that the ELB risk causes inflation to fall below the target rate of 2 percent by about 20 basis points at the risky steady state. The deflationary bias induced by the ELB risk at the risky steady state can be as much as 45 basis points under alternative plausible assumptions about the long-run growth rate of the economy and monetary policy parameters. Our analysis suggests that achieving the inflation target may be more difficult now than before the Great Recession, if the recent lower bound episode has led the private sector to increase their assessment of the ELB risk.

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# For Online Publication: Technical Appendix

## A Details of the Stylized Model

This section describes a stylized DSGE model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and government policies.

### A.1 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_t,N_t,B_t} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s\Big] \Big[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n}\Big] \tag{A.1}$$

subject to the budget constraint

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t \tag{A.2}$$

or equivalently

$$C_t + \frac{B_t}{R_t P_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + \Phi_t \tag{A.3}$$

where $C_t$ is consumption, $N_t$ is the labor supply, $P_t$ is the price of the consumption good, $W_t$ ($w_t$) is the nominal (real) wage, $\Phi_t$ is the profit share (dividends) of the household from the intermediate goods producers, $B_t$ is a one-period risk free bond that pays one unit of money at period t+1, and $R_t^{-1}$ is the price of the bond.

The discount rate at time t is given by $\beta\delta_t$ where $\delta_t$ is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. This shock follows an AR(1) process:

$$\delta_t - 1 = \rho(\delta_{t-1} - 1) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon) \tag{A.4}$$

This increase in $\delta_t$ is a preference imposed by the household to increase the relative valuation of future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

### A.2 Firms

There is a final good producer and a continuum of intermediate goods producers indexed by $i \in [0, 1]$. The final good producer purchases the intermediate goods $Y_{i,t}$ at the intermediate price $P_{i,t}$ and aggregates them using CES technology to produce and sell the final good

$Y_t$ to the household and government at price $P_t$. Its problem is then summarized as

$$\max_{Y_{i,t}, i\in[0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{A.5}$$

subject to the CES production function

$$Y_t = \Big[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di\Big]^{\frac{\theta}{\theta-1}}. \tag{A.6}$$

Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{i,t} = N_{i,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[20] by setting the price of its own good. We can assume that each firm receives a production subsidy $\tau$ so that the economy is fully efficient in the steady state.[21] In our baseline, however, we set $\tau = 0$. Price changes are subject to quadratic adjustment costs.

$$\max_{P_{i,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\prod_{s=0}^{t-1} \delta_s\Big] \lambda_t \Big[P_{i,t} Y_{i,t} - (1-\tau) W_t N_{i,t} - P_t \frac{\varphi}{2} [\frac{P_{i,t}}{P_{i,t-1}\bar{\Pi}} - 1]^2 Y_t\Big] \tag{A.7}$$

such that

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t.^{22} \tag{A.8}$$

$\lambda_t$ is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\Big[\prod_{s=0}^{t-1} \delta_s\Big]\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

## A.3 Government policies

It is assumed that the monetary authority determines nominal interest rates according to a Taylor rule

$$R_t = \max\left[1, \quad \frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right] \tag{A.9}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ and $\bar{Y}$ is the steady state level of output. This equation will be modified in order to do an extensive sensitivity analysis of policy inertia and other rule specifications.

[20] Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level $P_t$ which we take care of further along in the document.

[21] $(\theta-1) = (1-\tau)\theta$ which implies zero profits in the zero inflation steady state. In a welfare analysis, this would extract any inflation bias from the second-order approximated objective welfare function. $\tau$ therefore represents the size of a steady state distortion (see Chapter 5 Appendix, Galí (2008)).

[22] This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \Big[\int_0^1 P_{i,t}^{1-\theta} di\Big]^{\frac{1}{1-\theta}}$.

### A.4 Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$Y_t = C_t + \int_0^1 \frac{\varphi}{2}\Big[\frac{P_{i,t}}{P_{i,t-1}\bar{\Pi}} - 1\Big]^2 Y_t di \tag{A.10}$$

$$N_t = \int_0^1 N_{i,t} di \tag{A.11}$$

and

$$B_t = 0. \tag{A.12}$$

### A.5 Recursive equilibrium

Given $P_0$ and a two-state Markov shock process establishing $\delta_t$ and $\gamma_t$, an equilibrium consists of allocations $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}, G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{i,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that (i) given the determined prices and policies, allocations solve the problem of the household, (ii) $P_{i,t}$ solves the problem of firm $i$, (iii) $R_t$ follows a specified rule, and (iv) all markets clear.

Combining all of the results from (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$C_t^{-\chi_c} = \beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1} \tag{A.13}$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \tag{A.14}$$

$$\frac{Y_t}{C_t^{\chi_c}}\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} - (1-\theta) - \theta(1-\tau)w_t\right] = \beta\delta_t \mathrm{E_t}\frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\varphi\left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\bar{\Pi}} \tag{A.15}$$

$$Y_t = C_t + \frac{\varphi}{2}\left[\frac{\Pi_t}{\bar{\Pi}} - 1\right]^2 Y_t \tag{A.16}$$

$$Y_t = N_t \tag{A.17}$$

$$R_t = \max\left[1, \quad \frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right] \tag{A.18}$$

Equation A.13 is the consumption Euler equation, Equation A.14 is the intratemporal optimality condition of the household, Equation A.15 is the optimal condition of the intermediate good producing firms (forward-looking Phillips Curve) relating today's inflation to real marginal cost today and expected inflation tomorrow, Equation A.16 is the aggregate resource constraint capturing the resource cost of price adjustment, and Equation A.17 is the aggregate production function. Equation A.18 is the interest-rate feedback rule.

## B Details of the Empirical Model

This section describes an extension of the stylized model with a representative household, a final good producer, a continuum of intermediate goods producers with unit measure, and the government.

## B.1 Household markets

### B.1.1 Labor packer

The labor packer buys labor $N_{h,t}$ from households at their monopolistic wage $W_{h,t}$ and resells the packaged labor $N_t$ to intermediate goods producers at $W_t$. The problem can be written as

$$\max_{N_{h,t},h\in[0,1]} W_t N_t - \int_0^1 W_{h,t} N_{h,t} df \tag{B.1}$$

subject to the following CES technology

$$N_t = \left[\int_0^1 N_{h,t}^{\frac{\theta^w - 1}{\theta^w}} dh\right]^{\frac{\theta^w}{\theta^w - 1}}. \tag{B.2}$$

The first order condition implies a labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t.^{23} \tag{B.3}$$

$\theta^w$ is the wage markup parameter.

### B.1.2 Household

The representative household chooses its consumption level, amount of labor, and bond holdings so as to maximize the expected discounted sum of utility in future periods. As is common in the literature, the household enjoys consumption and dislikes labor. Assuming that period utility is separable, the household problem can be defined by

$$\max_{C_{h,t},w_{h,t},B_{h,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \left[\frac{(C_{h,t} - \zeta C_{t-1}^a)^{1-\chi_c}}{1-\chi_c} - A_t^{1-\chi_c} \frac{N_{h,t}^{1+\chi_n}}{1+\chi_n}\right] \tag{B.4}$$

subject to the budget constraint

$$P_t C_{h,t} + R_t^{-1} B_{h,t} \leq W_{h,t} N_{h,t} - W_t \frac{\varphi_w}{2} \left[\frac{W_{h,t}}{a W_{h,t-1} \left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + B_{h,t-1} + P_t \Phi_t - P_t T_t \tag{B.5}$$

or equivalently

$$C_{h,t} + \frac{B_{h,t}}{R_t P_t} \leq w_{h,t} N_{h,t} - w_t \frac{\varphi_w}{2} \left[\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w} \left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t + \frac{B_{h,t-1}}{P_t} + \Phi_t - T_t \tag{B.6}$$

and subject to the labor demand schedule

$$N_{h,t} = \left[\frac{W_{h,t}}{W_t}\right]^{-\theta^w} N_t. \tag{B.7}$$

[23] This implies that the labor packer will set the wage of the packaged labor to $W_t = \left[\int_0^1 W_{h,t}^{1-\theta^w} dh\right]^{\frac{1}{1-\theta^w}}$.

–or equivalently

$$N_{h,t} = \left[\frac{w_{h,t}}{w_t}\right]^{-\theta^w} N_t. \tag{B.8}$$

where $C_{h,t}$ is the household's consumption, $N_{h,t}$ is the labor supplied by the household, $P_t$ is the price of the consumption good, $W_{h,t}$ ($w_{h,t}$) is the nominal (real) wage set by the household, $W_t$ ($w_t$) is the market nominal (real) wage, $\Phi_t$ is the profit share (dividends) of the household from the intermediate goods producers, $B_{h,t}$ is a one-period risk free bond that pays one unit of money at period t+1, $T_t$ are lump-sum taxes or transfers, and $R_t^{-1}$ is the price of the bond. $C_{t-1}^a$ represents the aggregate consumption level from the previous period that the household takes as given. The parameter $0 \leq \zeta < 1$ measures how important these external habits are to the household. Because we are including wage indexation, measured by the parameter $\iota_w$, we assume the household takes as given the previous period wage inflation, $\Pi_{t-1}^w$, where $\Pi_t^w = \frac{W_t}{aW_{t-1}} = \frac{w_t P_t}{a w_{t-1} P_{t-1}} = \frac{w_t}{a w_{t-1}} \Pi_t^p$.

The discount rate at time t is given by $\beta\delta_t$ where $\delta_t$ is the discount factor shock altering the weight of future utility at time t+1 relative to the period utility at time t. $\delta_t$ is assumed to follow an AR(1) process

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_t^\delta \quad \forall t \geq 2 \tag{B.9}$$

and $\delta_1$ is given. The innovation $\epsilon_t^\delta$ is normally distributed with mean zero and standard deviation $\sigma_\delta$. It may therefore be interpreted that an increase in $\delta_t$ is a preference imposed by the household to increase the relative valuation of the future utility flows, resulting in decreased consumption today (when considered in the absence of changes in the nominal interest rate).

$A_t$ is a non-stationary total factor productivity shock that also augments labor in the utility function in order to accommodate the necessary stationarization of the model later on. See the next section for more details on this process.

## B.2 Producers

### B.2.1 Final good producer

The final good producer purchases the intermediate goods $Y_{f,t}$ at the intermediate price $P_{f,t}$ and aggregates them using CES technology to produce and sell the final good $Y_t$ to the household and government at price $P_t$. Its problem is then summarized as

$$\max_{Y_{f,t}, f\in[0,1]} P_t Y_t - \int_0^1 P_{f,t} Y_{f,t} di \tag{B.10}$$

subject to the CES production function

$$Y_t = \left[\int_0^1 Y_{f,t}^{\frac{\theta^p - 1}{\theta^p}} di\right]^{\frac{\theta^p}{\theta^p - 1}}. \tag{B.11}$$

$\theta^p$ is the price markup parameter.

### B.2.2 Intermediate goods producers

There is a continuum of intermediate goods producers indexed by $f \in [0,1]$. Intermediate goods producers use labor to produce the imperfectly substitutable intermediate goods according to a linear production function ($Y_{f,t} = A_t N_{f,t}$) and then sell the product to the final good producer. Each firm maximizes its expected discounted sum of future profits[24] by setting the price of its own good. Any price changes are subject to quadratic adjustment costs. $\varphi_p$ will represent an obstruction of price adjustment, the firm indexes for prices—measured by $\iota_p$—and takes as given previous period inflation $\Pi^p_{t-1}$, and $\bar{\Pi}^p$ represents the monetary authority's inflation target.

$$\max_{P_{f,t}} \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s\right] \lambda_t \left[P_{f,t}Y_{f,t} - W_t N_{f,t} - P_t \frac{\varphi_p}{2}\left[\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p} P_{f,t-1}} - 1\right]^2 Y_t\right] \tag{B.12}$$

such that

$$Y_{f,t} = \left[\frac{P_{f,t}}{P_t}\right]^{-\theta^p} Y_t.^{25} \tag{B.13}$$

$\lambda_t$ is the Lagrange multiplier on the household's budget constraint at time t and $\beta^{t-1}\left[\prod_{s=0}^{t-1}\delta_s\right]\lambda_t$ is the marginal value of an additional profit to the household. The positive time zero price is the same across firms (i.e. $P_{i,0} = P_0 > 0$).

$A_t$ represents total factor productivity which follows a random walk with drift:

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t. \tag{B.14}$$

$a$ is the unconditional rate of growth of productivity. $a_t$ is a productivity shock following an AR(1) process:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon^A_t. \tag{B.15}$$

where $\epsilon^A_t$ is normally distributed with mean zero and standard deviation $\sigma_A$. This growth factor will imply that some of the variables will acquire a unit root, meaning the model will have to be stationarized. Monetary policy will also have to accommodate this growth factor as well.

## B.3 Government policies

It is assumed that the monetary authority determines nominal interest rates according to a truncated notional inertial Taylor rule augmented by a speed limit component.

$$R_t = \max\left[1, R^*_t\right] \tag{B.16}$$

[24] NOTE: Each period, as it is written below, is in *nominal* terms. However, we want each period's profits in *real* terms so the profits in each period must be divided by that period's price level $P_t$ which we take care of further along in the document.

[25] This expression is derived from the profit maximizing input demand schedule when solving for the final good producer's problem above. Plugging this expression back into the CES production function implies that the final good producer will set the price of the final good $P_t = \left[\int_0^1 P_{f,t}^{1-\theta^p} di\right]^{\frac{1}{1-\theta^p}}$.

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R} \left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{Y_t}{A_t\bar{Y}}\right)^{(1-\rho_r)\phi_y} \exp(\epsilon_t^R) \tag{B.17}$$

where $\Pi_t^p = \frac{P_t}{P_{t-1}}$ is the inflation rate between periods $t-1$ and $t$, $\bar{R} = \frac{\bar{\Pi}^p a^{\chi_c}}{\beta}$ (see the section on stationarization to see why), and $\epsilon_t^R$ represents white noise monetary policy shocks with mean zero and standard deviation $\sigma_R$.

## B.4 Market clearing conditions

The market clearing conditions for the final good, labor and government bond are given by

$$\begin{aligned} Y_t = C_t + \int_0^1 \frac{\varphi_p}{2}\left[\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p} P_{f,t-1}} - 1\right]^2 Y_t df + ... \\ ... + \int_0^1 w_t \frac{\varphi_w}{2}\left[\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 N_t dh \end{aligned} \tag{B.18}$$

$$N_t = \int_0^1 N_{f,t} di \tag{B.19}$$

$$C_t^a = C_t = \int_0^1 C_{h,t} dh \tag{B.20}$$

and

$$B_t = \int_0^1 B_{h,t} dh = 0. \tag{B.21}$$

## B.5 An equilibrium

Given $P_0$ and stochastic processes for $\delta_t$, an equilibrium consists of allocations $\{C_t,\ N_t,\ N_{f,t},\ Y_t,\ Y_{f,t},\ G_t\}_{t=1}^{\infty}$, prices $\{W_t, P_t, P_{f,t}\}_{t=1}^{\infty}$, and a policy instrument $\{R_t\}_{t=1}^{\infty}$ such that

(i) allocations solve the problem of the household given prices and policies

$$\partial C_{h,t} : (C_{h,t} - \zeta C_{t-1}^a)^{-\chi_c} - \lambda_t = 0 \tag{B.22}$$

$$\begin{aligned} \partial w_{h,t} : \theta^w A_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{w_t}\left(\frac{w_{h,t}}{w_t}\right)^{-\theta^w(1+\chi_n)-1} \\ +(1-\theta^w)\lambda_t \left(\frac{w_{h,t}}{w_t}\right)^{-\theta^w} N_t \\ -\lambda_t w_t \varphi_w \left(\frac{w_{h,t}}{a w_{h,t-1}} \frac{\Pi_t^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right) N_t \frac{\Pi_t^p}{a w_{h,t-1}\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} \\ +\beta\delta_t \mathrm{E}_t \lambda_{t+1} w_{t+1} \varphi_w \left(\frac{w_{h,t+1}}{a w_{h,t}} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}} - 1\right) N_{t+1} \frac{w_{h,t+1}}{a w_{h,t}^2} \frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}} = 0 \end{aligned} \tag{B.23}$$

$$\partial B_{h,t} : -\frac{\lambda_t}{R_t P_t} + \beta\delta_t \mathrm{E}_t \frac{\lambda_{t+1}}{P_{t+1}} = 0 \tag{B.24}$$

(ii) $P_{f,t}$ solves the problem of firm $i$

By making the appropriate substitution (the intermediate goods producer's constraints in place of $Y_{f,t}$ and subsequently in for $N_{f,t}$) and by dividing each period's profits by that period's price level $P_t$ so as to put profits in real terms (and thus make profits across periods comparable) we get the following:

$$\partial P_{f,t} : \lambda_t \frac{Y_t}{P_t}\Big[\frac{P_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p} P_{f,t-1}}\varphi_p\left(\frac{P_{f,t}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p} P_{f,t-1}} - 1\right) - (1-\theta^p)\left(\frac{P_{f,t}}{P_t}\right)^{-\theta^p}$$
$$-\theta^p \frac{w_t}{A_t}\left(\frac{P_t}{P_{f,t}}\right)^{1+\theta^p}\Big] = \beta\delta_t \mathrm{E_t}\frac{\lambda_{t+1}Y_{t+1}}{P_{t+1}}\left[P_{t+1}\varphi_p\left(\frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p} P_{f,t}} - 1\right)\frac{P_{f,t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p} P^2_{f,t}}\right] \tag{B.25}$$

(iii) $P_{f,t} = P_{j,t} \quad \forall i \neq j$

$$\begin{aligned}\frac{Y_t}{\lambda_t^{-1}}\left[\varphi_p\left(\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p}} - 1\right)\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p}} - (1-\theta^p) - \theta^p\frac{w_t}{A_t}\right] &= ...\\ ... = \beta\delta_t\mathrm{E_t}\frac{Y_{t+1}}{\lambda_{t+1}^{-1}}\varphi_p\left(\frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p}} - 1\right)&\frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p}}\end{aligned} \tag{B.26}$$

(iv) $R_t$ follows a specified rule

and

(v) all markets clear.

Combining all of the results derived from the conditions and exercises in (i)-(v), a symmetric equilibrium can be characterized recursively by $\{C_t, N_t, Y_t, w_t, \Pi^p_t, R_t\}_{t=1}^{\infty}$ satisfying the following equilibrium conditions:

$$\lambda_t = \beta\delta_t R_t \mathrm{E_t}\lambda_{t+1}\left(\Pi^p_{t+1}\right)^{-1} \tag{B.27}$$

$$\lambda_t = (C_t - \zeta C_{t-1})^{-\chi_c} \tag{B.28}$$

$$\begin{aligned}\frac{N_t}{\lambda_t^{-1}}\left[\varphi_w\left(\frac{\Pi^w_t}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi^w_{t-1}\right)^{\iota_w}} - 1\right)\frac{\Pi^w_t}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi^w_{t-1}\right)^{\iota_w}} - (1-\theta^w) - \theta^w\frac{A_t^{1-\chi_c}N_t^{\chi_n}}{\lambda_t w_t}\right] &= ...\\ ... = \beta\delta_t\mathrm{E_t}\frac{N_{t+1}}{\lambda_{t+1}^{-1}}\varphi_w\left(\frac{\Pi^w_{t+1}}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi^w_t\right)^{\iota_w}} - 1\right)&\frac{\Pi^w_{t+1}}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi^w_t\right)^{\iota_w}}\frac{w_{t+1}}{w_t}\end{aligned} \tag{B.29}$$

$$\Pi^w_t = \frac{w_t}{a w_{t-1}}\Pi^p_t \tag{B.30}$$

$$\begin{aligned}\frac{Y_t}{\lambda_t^{-1}}\left[\varphi_p\left(\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p}} - 1\right)\frac{\Pi^p_t}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_{t-1}\right)^{\iota_p}} - (1-\theta^p) - \theta^p\frac{w_t}{A_t}\right] &= ...\\ ... = \beta\delta_t\mathrm{E_t}\frac{Y_{t+1}}{\lambda_{t+1}^{-1}}\varphi_p\left(\frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p}} - 1\right)&\frac{\Pi^p_{t+1}}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi^p_t\right)^{\iota_p}}\end{aligned} \tag{B.31}$$

$$Y_t = C_t + \frac{\varphi_p}{2}\left[\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}} - 1\right]^2 Y_t + \frac{\varphi_w}{2}\left[\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2 w_t N_t \tag{B.32}$$

$$Y_t = A_t N_t \tag{B.33}$$

$$R_t = \max\left[1, R_t^*\right] \tag{B.34}$$

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R}\left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi}\left(\frac{Y_t}{A_t\bar{Y}}\right)^{(1-\rho_r)\phi_y}\exp(\epsilon_t^R) \tag{B.35}$$

and given the following processes ($\forall t \geq 2$):

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_t^\delta \tag{B.36}$$

and

$$\ln(A_t) = \ln(a) + \ln(A_{t-1}) + a_t. \tag{B.37}$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_t^A. \tag{B.38}$$

### B.6 A stationary equilibrium

Let $\tilde{Y}_t = \frac{Y_t}{A_t}$, $\tilde{C}_t = \frac{C_t}{A_t}$, $\tilde{w}_t = \frac{w_t}{A_t}$, and $\tilde{\lambda}_t = \frac{\lambda_t}{A_t^{-\chi_c}}$ be the stationary representations of output, consumption, real wage, and marginal utility of consumption respectively. The stationary symmetric equilibrium can now be characterized by the following system of equations.

$$\tilde{\lambda}_t = \frac{\beta}{a^{\chi_c}}\delta_t R_t \mathrm{E}_t \tilde{\lambda}_{t+1}\left(\Pi_{t+1}^p\right)^{-1}\exp(-\chi_c\epsilon_{t+1}^A) \tag{B.39}$$

$$\tilde{\lambda}_t = (\tilde{C}_t - \tilde{\zeta}\tilde{C}_{t-1}\exp(-\epsilon_t^A))^{-\chi_c}, \quad \tilde{\zeta} = \frac{\zeta}{a} \tag{B.40}$$

$$\begin{aligned}\frac{N_t\tilde{w}_t}{\tilde{\lambda}_t^{-1}}\left[\varphi_w\left(\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right)\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - (1-\theta^w) - \theta^w\frac{N_t^{\chi_n}}{\tilde{\lambda}_t\tilde{w}_t}\right] = ...\\ ... = \frac{\beta\varphi_w}{a^{\chi_c-1}}\delta_t\mathrm{E}_t\frac{N_{t+1}\tilde{w}_{t+1}}{\lambda_{t+1}^{-1}}\left(\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}} - 1\right)\frac{\Pi_{t+1}^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_t^w\right)^{\iota_w}}\exp\left(\left(1-\chi_c\right)\epsilon_{t+1}^A\right)\end{aligned} \tag{B.41}$$

$$\Pi_t^w = \frac{\tilde{w}_t}{\tilde{w}_{t-1}}\Pi_t^p\exp\left(\epsilon_t^A\right) \tag{B.42}$$

$$\begin{aligned}\frac{\tilde{Y}_t}{\tilde{\lambda}_t^{-1}}\left[\varphi_p\left(\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}} - 1\right)\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}} - (1-\theta^p) - \theta^p\tilde{w}_t\right] = ...\\ ... = \frac{\beta\varphi_p}{a^{\chi_c-1}}\delta_t\mathrm{E}_t\frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}}\left(\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_t^p\right)^{\iota_p}} - 1\right)\frac{\Pi_{t+1}^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_t^p\right)^{\iota_p}}\exp\left(\left(1-\chi_c\right)\epsilon_{t+1}^A\right)\end{aligned} \tag{B.43}$$

$$\tilde{Y}_t = \tilde{C}_t + \frac{\varphi_p}{2}\left[\frac{\Pi_t^p}{\left(\bar{\Pi}^p\right)^{1-\iota_p}\left(\Pi_{t-1}^p\right)^{\iota_p}} - 1\right]^2\tilde{Y}_t + \frac{\varphi_w}{2}\left[\frac{\Pi_t^w}{\left(\bar{\Pi}^w\right)^{1-\iota_w}\left(\Pi_{t-1}^w\right)^{\iota_w}} - 1\right]^2\tilde{w}_t N_t \tag{B.44}$$

$$\tilde{Y}_t = N_t \tag{B.45}$$

and

$$R_t = \max\left[1, R_t^*\right] \tag{B.46}$$

where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}}\right)^{\rho_R} \left(\frac{\Pi_t^p}{\bar{\Pi}^p}\right)^{(1-\rho_r)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}}\right)^{(1-\rho_r)\phi_y} \exp\left(\epsilon_t^R\right) \tag{B.47}$$

and given the following processes ($\forall t \geq 2$):

$$(\delta_t - 1) = \rho_\delta(\delta_{t-1} - 1) + \epsilon_t^\delta \tag{B.48}$$

and

### B.7 Stationary deterministic steady-state values

For each variable, $X_t$, we denote its corresponding *stationary* deterministic steady-state value as $\bar{X}$. The following is a list of analytical expressions for the stationary steady states for each of the variables of the model.

$$\bar{\Pi}^p = \bar{\Pi}^p, \quad \text{(this parameter is set exogenously by the monetary authority)}$$

$$\bar{\Pi}^w = \bar{\Pi}^p$$

$$\bar{R} = \frac{a^{\chi_c}\bar{\Pi}^p}{\beta}$$

$$\bar{w} = \frac{\theta_p - 1}{\theta_p}$$

$$\bar{C} = \left(\frac{\bar{w}\left(\theta_w - 1\right)}{\theta_w\left(1 - \tilde{\zeta}\right)^{\chi_c}}\right)^{\frac{1}{\chi_c + \chi_n}}$$

$$\bar{\lambda} = \left[\left(1 - \tilde{\zeta}\right)\bar{C}\right]^{-\chi_c}$$

$$\bar{N} = \bar{Y} = \bar{C}$$

# C Solution Method

We describe our solution method using the stylized model analyzed in the main text. The extension of the method to the empirical model is straightforward.

The problem is to find a set of policy functions, $\{C(\cdot),\, N(\cdot),\, Y(\cdot),\, w(\cdot),\, \Pi(\cdot),\, R(\cdot)\}$, that solves the following system of *functional* equations.

$$C(\delta_t)^{-\chi_c} = \beta\delta_t R(\delta_t) E_t C(\delta_{t+1})^{-\chi_c} \Pi(\delta_{t+1})^{-1} \tag{C.1}$$

$$w(\delta_t) = N(\delta_t)^{\chi_n} C(\delta_t)^{\chi_c} \tag{C.2}$$

$$\frac{N(\delta_t)}{C(\delta_t)^{\chi_c}}\Big[\varphi\Big(\frac{\Pi(\delta_t)}{\bar{\Pi}}-1\Big)\frac{\Pi(\delta_t)}{\bar{\Pi}}-(1-\theta)-\theta w(\delta_t)\Big]...$$

$$... = \beta\delta_t E_t \frac{N(\delta_{t+1})}{C(\delta_{t+1})^{\chi_c}}\varphi\Big(\frac{\Pi(\delta_{t+1})}{\bar{\Pi}}-1\Big)\frac{\Pi(\delta_{t+1})}{\bar{\Pi}} \tag{C.3}$$

$$Y(\delta_t) = C(\delta_t) + \frac{\varphi}{2}\Big[\frac{\Pi(\delta_t)}{\bar{\Pi}}-1\Big]^2 Y(\delta_t) \tag{C.4}$$

$$Y(\delta_t) = N(\delta_t) \tag{C.5}$$

$$R(\delta_t) = \max\left[1, \frac{\bar{\Pi}}{\beta}\left[\frac{\Pi(\delta_t)}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{Y(\delta_t)}{\bar{Y}}\right]^{\phi_y}\right] \tag{C.6}$$

Substituting out $w(\cdot)$ and $N(\cdot)$ using equations (C.2) and (C.5), this system can be reduced to a system of four functional equations for $C(\cdot)$, $Y(\cdot)$, $\Pi(\cdot)$, and $R(\cdot)$.

$$C(\delta_t)^{-\chi_c} = \beta\delta_t R(\delta_t) E_t C(\delta_{t+1})^{-\chi_c} \Pi(\delta_{t+1})^{-1} \tag{C.7}$$

$$\frac{Y(\delta_t)}{C(\delta_t)^{\chi_c}}\Big[\varphi\Big(\frac{\Pi(\delta_t)}{\bar{\Pi}}-1\Big)\frac{\Pi(\delta_t)}{\bar{\Pi}}-(1-\theta)-\theta Y(\delta_t)^{\chi_n} C(\delta_t)^{\chi_c}\Big]...$$

$$... = \beta\delta_t E_t \frac{Y(\delta_{t+1})}{C(\delta_{t+1})^{\chi_c}}\varphi\Big(\frac{\Pi(\delta_{t+1})}{\bar{\Pi}}-1\Big)\frac{\Pi(\delta_{t+1})}{\bar{\Pi}} \tag{C.8}$$

$$Y(\delta_t) = C(\delta_t) + \frac{\varphi}{2}\Big[\frac{\Pi(\delta_t)}{\bar{\Pi}}-1\Big]^2 Y(\delta_t) \tag{C.9}$$

$$R(\delta_t) = \max\left[1, \frac{\bar{\Pi}}{\beta}\left[\frac{\Pi(\delta_t)}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{Y(\delta_t)}{\bar{Y}}\right]^{\phi_y}\right] \tag{C.10}$$

Following the idea of Christiano and Fisher (2000) and Gust, López-Salido, and Smith (2012), I decompose these policy functions into two parts using an indicator function: One in which the policy rate is allowed to be less than zero, and the other in which the policy rate is assumed to be zero. That is, for any variable Z,

$$Z(\cdot) = \mathbb{1}_{\{R(\cdot)\geq 1\}} Z_{unc}(\cdot) + (1-\mathbb{1}_{\{R(\cdot)\geq 1\}}) Z_{ELB}(\cdot). \tag{C.11}$$

The problem then becomes finding a set of a *pair* of policy functions, $\{[C_{unc}(\cdot), C_{ELB}(\cdot)], [Y_{unc}(\cdot), Y_{ELB}(\cdot)], [\Pi_{unc}(\cdot), \Pi_{ELB}(\cdot)], [R_{unc}(\cdot), R_{ELB}(\cdot)]\}$ that solves the system of functional equations above. This method can achieve a given level of accuracy with a considerable less number of grid points relative to the standard approach.[26]

The time-iteration method starts by specifying a guess for the values policy functions take on a finite number of grid points. The values of the policy function that are not on any of the grid points are interpolate or extrapolated linearly. Let $X(\cdot)$ be a vector of policy functions that solves the functional equations above and let $X^{(0)}$ be the initial guess of such policy functions.[27] At the $s$-th iteration and at each point of the state space, we solve the

[26] A systematic analysis of the benefits of using the Christiano-Fisher approach is available upon request.

[27] For all models and all variables, we use flat functions at the deterministic steady-state values as the initial guess.

system of nonlinear equations given by equations (C.7)-(C.10) to find today's consumption, output, inflation, and the policy rate, given that $X^{(s-1)}(\cdot)$ is in place for the next period. In solving the system of nonlinear equations, we use Gaussian quadrature to evaluate the expectation terms in the consumption Euler equation and the Phillips curve, and the value of future variables not on the grid points are evaluated with linear interpolation. The system is solved numerically by using a nonlinear equation solver, `dneqnf`, provided by the IMSL Fortran Numerical Library. If the updated policy functions are sufficiently close to the guessed policy functions, then the algorithm ends. Otherwise, using the updated policy functions just obtained as the guess for the next period's policy functions, we iterate on this process until the difference between the guessed and updated policy functions is sufficiently small ($\left\|vec(X^s(\delta) - X^{s-1}(\delta))\right\|_\infty <$ 1E-11 is used as the convergence criteria). The solution method can be extended to models with multiple exogenous shocks and endogenous state variables in a straightforward way.

For the stylized model, we used equally spaced 201 grid points on the interval between $[1-4.5\sigma_\delta, 1+4.5\sigma_\delta]$ and 9 grid points for the Gaussian quadrature. For the empirical model, we used 27 grid points for the discount rate shock on the interval between $[1-4.5\sigma_\delta, 1+4.5\sigma_\delta]$, 15 grid points for the lagged consumption on the interval between -12 and 8 percent from the steady state (normalized) consumption, 15 grid points for the lagged real wage on the interval between -2.5 and 2 percent from the steady state (normalized) real wage, and 15 grid points on the lagged shadow policy rate on the interval between -8 and 10 annualized percent. For the empirical model, we use 31 grid points for the Gaussian quadrature.

# D Solution Accuracy

In this section, we report the accuracy of our numerical solutions for the stylized and empirical models. Following Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) and Maliar and Maliar (2015), we evaluate these residuals functions along a simulated equilibrium path. The length of the simulation is 100,000.

## D.1 Stylized model

For the stylized model, there are two residual functions, one associated with the consumption Euler equation and the other associated with the sticky-price equilibrium condition.

$$\mathrm{R}_{1,t} = \left|1 - C_t^{\chi_c}\beta\delta_t R_t \mathrm{E_t} C_{t+1}^{-\chi_c}\Pi_{t+1}^{-1}\right| \tag{D.1}$$

$$\mathrm{R}_{2,t} = \left|\left(\frac{\Pi_t}{\bar{\Pi}} - 1\right)\frac{\Pi_t}{\bar{\Pi}} - \left[\frac{(1-\theta)+\theta(1-\tau)w_t}{\varphi} - \frac{C_t^{\chi_c}}{Y_t}\beta\delta_t \mathrm{E_t}\frac{Y_{t+1}}{C_{t+1}^{\chi_c}}\left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\right]\right| \tag{D.2}$$

With our log-utility specification, the residual $\mathrm{R}_{1,t}$ measures the difference between the chosen consumption today and today's consumption consistent with the optimization behavior of the household, as a percentage of the chosen consumption. The residual $\mathrm{R}_{2,t}$ is given by the difference between $(\Pi_t - \bar{\Pi})\Pi_t/\bar{\Pi}$ and the sum of the term involving today's real wages and the term involving the expectations. Given that the standard deviation of $\Pi_t$ is about 0.0075 ($\cong$ 31 basis points), $\Pi_t/\bar{\Pi}$ is always close to one and thus the $(\Pi_t - \bar{\Pi})\Pi_t/\bar{\Pi}$ is roughly equal to $(\Pi_t - \bar{\Pi})$. $(\Pi_t - \bar{\Pi})$ is the deviation of inflation from the target rate of inflation. Thus, the difference between this term and the sum of the term involving today's real wages and the term involving the expectations measures how much the chosen inflation rate differs from the inflation rate consistent with the optimization behavior of firms.

Table D.1: **Solution Accuracy**

| | Mean[$\log_{10}(\mathrm{R}_{k,t})$] | 95th-percentile of [$\log_{10}(\mathrm{R}_{k,t})$] |
|---|---|---|
| Stylized Model | | |
| $k = 1$: Euler equation error | −6.5 | −6.0 |
| $k = 2$: Sticky-price equation error | −7.5 | −6.9 |
| | | |
| Empirical Model | | |
| $k = 1$: Euler equation error | −4.3 | −3.7 |
| $k = 2$: Sticky-price equation error | −4.7 | −4.2 |
| $k = 3$: Sticky-wage equation error | −4.5 | −3.9 |

Reflecting the large number of grid points used to solve the stylized model, the approximation errors are very small. The average errors on the consumption Euler equation is $0.3*10^{-4}$ percent ($= 10^{-6.5}$) with the 95th percentile being $1.0 * 10^{-4}$ percent ($= 100 * 10^{-6.0}$). The average errors on the sticky price equation is 0.001 basis points ($= 400 * 100 * 10^{-7.5}$) with the 95th percentile being 0.005 basis points ($= 400 * 100 * 10^{-6.9}$).

## D.2 Empirical Model

For the empirical model, there are three residual functions of interest. As in the stylized model, the first and second residual functions are associated with the consumption Euler equation and the sticky-price equilibrium condition, respectively. The third residual function is associated with the sticky-price and sticky-wage equilibrium conditions.

$$\mathrm{R}_{1,t} = \left| 1 - \frac{\beta}{a^{\chi_c}\tilde{\lambda}_t}\delta_t R_t \mathrm{E}_t \tilde{\lambda}_{t+1}\left(\Pi^p_{t+1}\right)^{-1} \right| \tag{D.3}$$

$$\mathrm{R}_{2,t} = \left| \left(\frac{\Pi^w_t}{\bar{\Pi}^w} - 1\right)\frac{\Pi^w_t}{\bar{\Pi}^w} - \Big[\frac{1-\theta^w}{\varphi_w} + \frac{\theta^w}{\varphi_w}\frac{N_t^{\chi_n}}{\tilde{\lambda}_t \tilde{w}_t} + \frac{\tilde{\lambda}_t^{-1}}{N_t \tilde{w}_t}\frac{\beta}{a^{\chi_c - 1}}\delta_t \mathrm{E}_t \frac{N_{t+1}\tilde{w}_{t+1}}{\lambda_{t+1}^{-1}}\left(\frac{\Pi^w_{t+1}}{\bar{\Pi}^w} - 1\right)\frac{\Pi^w_{t+1}}{\bar{\Pi}^w}\Big] \right| \tag{D.4}$$

$$\mathrm{R}_{3,t} = \left| \left(\frac{\Pi^p_t}{\bar{\Pi}^p} - 1\right)\frac{\Pi^p_t}{\bar{\Pi}^p} - \Big[\frac{(1-\theta^p) + \theta^p \tilde{w}_t}{\varphi_p} + \frac{\tilde{\lambda}_t^{-1}}{\tilde{Y}_t}\frac{\beta}{a^{\chi_c - 1}}\delta_t \mathrm{E}_t \frac{\tilde{Y}_{t+1}}{\tilde{\lambda}_{t+1}^{-1}}\left(\frac{\Pi^p_{t+1}}{\bar{\Pi}^p} - 1\right)\frac{\Pi^p_{t+1}}{\bar{\Pi}^p}\Big] \right| \tag{D.5}$$

Table D.1 shows the average and the 95th percentile of the residuals for the three equilibrium conditions. The average size of the Euler equation errors is 0.005 percent ($= 10^{-4.3}$) with the 95th percentile of the errors being 0.02 percent ($= 10^{-3.7}$). These are larger than those in the stylized model, reflecting the coarser grid used in the solution of the empirical model. However, this degree of accuracy fares well in comparison with what's reported in other studies such as Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) and Maliar and Maliar (2015).

The average size of the sticky-price equation errors is 0.8 basis points ($= 400*100*10^{-4.7}$) with the 95th percentile being 2.7 basis points ($= 400 * 100 * 10^{-4.2}$). These are again larger than those in the stylized model, reflecting the coarser grid used in the solution of the empirical model. While the size of the error is not ideal, it is acceptable as the approximation errors of this magnitude do not materially affect the main claim regarding the quantitative importance of the deflationary bias. The instances of large approximation errors are only observed in the area of the state space where the discount rate shock is high and the policy rate is constrained by the effective lower bound.

The sticky-wage equation errors are somewhat larger than the sticky-price equation errors. The average size is 1.3 basis points ($= 400 * 100 * 10^{-4.5}$) with the 95th percentile being 4.9 basis points ($= 400 * 100 * 10^{-3.9}$). However, given that the standard deviation of the wage inflation is more than two times as large as that of the price inflation (66 versus 31 basis points), the sticky-wage equation errors are smaller in relative terms than the sticky-price equation errors. As in the sticky-price equation error, the instances of larger approximation errors are only observed in the area of the state space where the discount rate shock is high and the policy rate is constrained by the effective lower bound.

# E Expected Time Until the Liftoff

In this section, we present the survey-based measures of the expected time until the liftoff to support the claim that the market participants consistently underestimated the duration of the lower bound episode since the federal funds rate hit the lower bound in late 2008. The surveys we examine are (i) the Blue Chip Surveys, (ii) the Survey of Professional Forecasters, and (iii) the Primary Dealers Survey.

The evidence from all three surveys is consistent with the claim that the market participants have consistently underestimated the duration of the lower bound episode. In particular, for the first two years of the lower bound episode, the market participants expected that the federal funds rate to stay at the ELB only for additional few quarters.[28]

## E.1 Blue Chip Surveys

The Blue Chip Surveys consists of two monthly surveys, the Blue Chip Economic Indicators Survey and the Blue Chip Financial Forecasts Survey. These two surveys ask their participants (about 50 financial institutions for each survey) their forecast paths of various macroeconomic variables, including the 3-month Treasury Bill rate in the Economic Indicators Survey and the federal funds rate in the Financial Forecasts Survey. The near-term forecast horizon is up until the end of next calendar year and the frequency of the projection is quarterly. Thus, the forecast path of the Treasury rate or the federal funds rate can tell us the expected time until the liftoff when the participants expect the first liftoff to occur within two years.

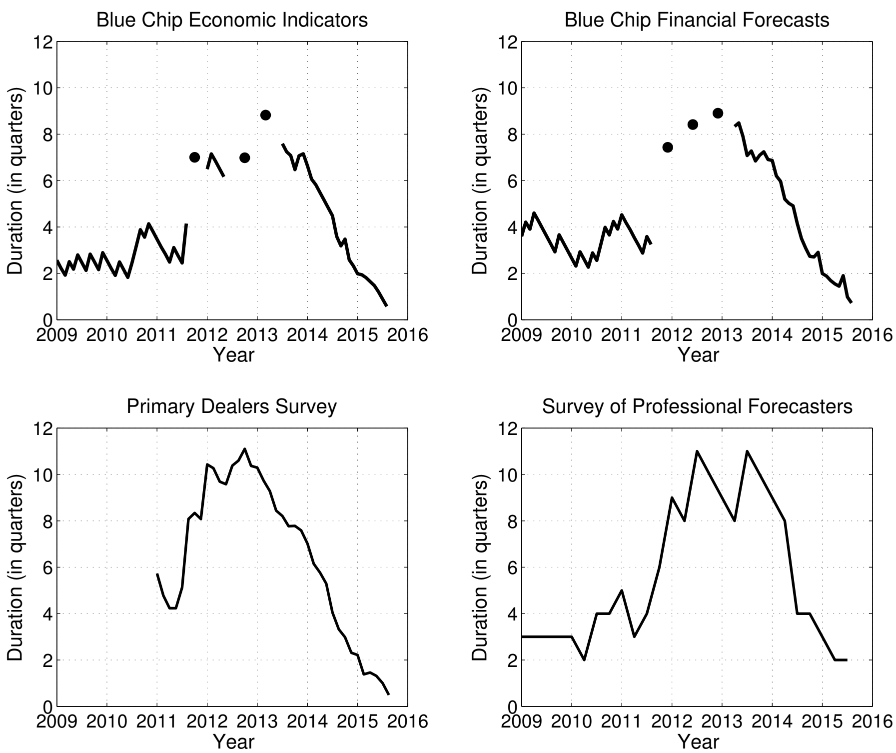
Twice a year, the surveys ask longer-run projections of certain variables in the special question section (March and October for the Economic Indicators and June and December for the Financial Forecasts). The longer-run forecasts are in annual frequency for next 5 to 6 years. Towards the end of the lower bound episode, the Surveys also asked the participants to provide the expected liftoff date in the special questions section.

For each survey, we combine these various pieces of information in the following way to construct a series for the expected period until the liftoff. First, we use the average probability distribution over the timing of the liftoff to compute the expected time until the liftoff whenever that information is available. Second, if the probability distribution is not available, we use the information from the near-term forecasts. The time of liftoff is defined

[28] While not shown, the expected duration of the lower bound episode based on the expected policy path implied by the federal funds rate futures is also consistent with this claim.

to be the first quarter when the median federal funds rate forecast exceeds 37.5 basis points. Finally, when the policy rate is projected to stay at the ELB until the end of the near-term forecast horizon, we use the information from the long-run projections if the Survey has that information and leave the series blank when the Long-Range section is not available.

Figure E.1: Expected Time Until Liftoff†




† Data sourced from: Blue Chip Economic Indicators Survey (from January 2009 to August 2015); Blue Chip Financial Forecasts Survey (from January 2009 to August 2015); Federal Reserve Bank of New York, Primary Dealer Survey, accessed September 2015, https://www.newyorkfed.org/markets/primarydealer˙survey˙questions.html; Federal Reserve Board, Survey of Professional Forecasters, accessed September 2015, https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

The top two panels in figure E.1 show the evolutions of the expected period until liftoff based on the Blue Chip Economic Indicators Survey and the Blue Chip Financial Forecasts Survey. According to both panels, the market participants expected the lower bound episode to be transitory in the early stage of the lower bound episode. The expectation shifted in the second half of 2011, with the expectated duration of staying at the ELB exceeding 2 years. Starting in late 2012 or early 2013, the survey respondents gradually reduced their expectation for the additional duration of the lower bound episode.

## E.2 Primary Dealers Survey

The Primary Dealers Survey (the PD Survey in the remainder of the text), conducted by the Federal Reserve Bank of New York, asks primary dealers about their policy expectations eight times a year. The survey asks its participants their probability distribution over the liftoff timing (quarter or FOMC meeting). We compute the expected time until the liftoff using the average probability distribution over the liftoff timing. The results of the PD Survey are publicly available since January 2011.

The bottom-left panel of figure E.1 shows the evolution of the expected period until the liftoff based on the PD Survey. Consistent with the measures based on the Blue Chip survey, the expected duration of the additional period of the lower bound episode increase markedly in the second half of 2011. The expected duration hovers around 10 quarters during 2012, and has declined steadily since then.

### E.3 Survey of Professional Forecasters

The Survey of Professional Forecasters (SPF) is a quarterly survey of about 40 individuals in academia, financial industries, and policy institutions, administered by the Federal Reserve Bank of Philadelphia. Like the Blue Chip Surveys, the SPF asks its participants their projections of various macroeconomic variables, including the 3-month Treasury rate. For the near-term projection that extends to the end of the next calendar year, the forecasts are available in quarterly frequency. For the longer horizon, the forecast is available in annual frequency.

The bottom-right panel of figure E.1 shows the evolution of the expected period until liftoff based on the SPF. Consistent with the Blue Chip Surveys and the Primary Dealers Survey, the SPF shows that the market anticipated the lower bound episode to last for only about one additional year until the second half of 2011. The expected duration averages about 9 quarters in 2012 and 2013. The expected duration started declining in the second half of 2013 and has come down to 2 quarters in February 2015.

## F Low inflation prior to the ELB episodes

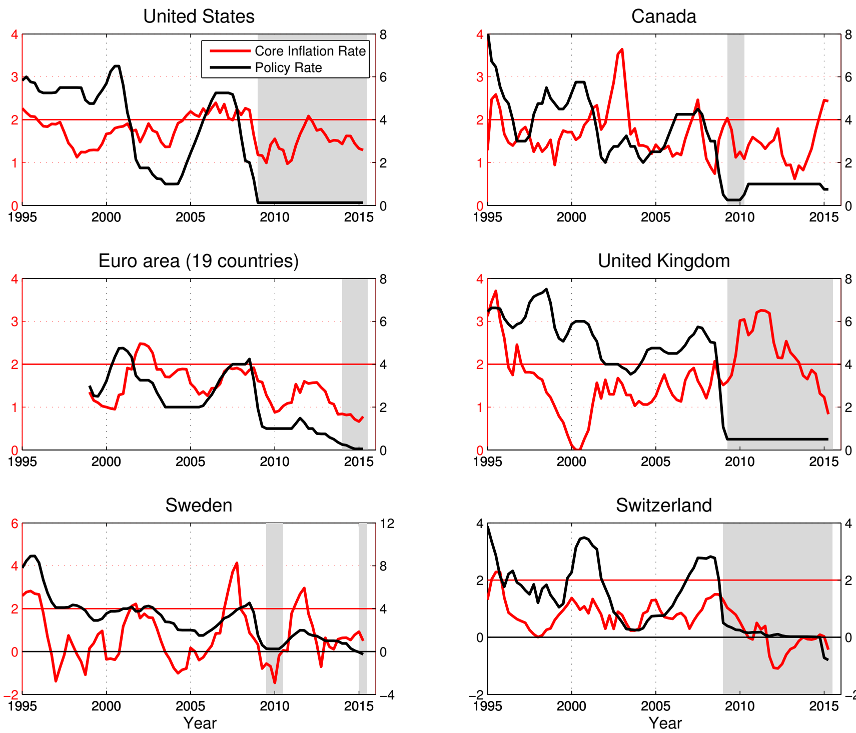
Figure F.1 shows the evolution of inflation and policy rate past two decades in six select economies (U.S., Canada, Euro Area, U.K., Sweden, and Switzerland). We exclude Japan because the policy rate in Japan has been at the ELB for most of the past two decades and thus the Japanese economy is better described by the deflationary equilibrium that fluctuates around a deflationary steady state in the context of sticky-price models.[29] In all of the six economies, the policy rate became constrained at the ELB during the recent global recession for the first time in the post WWII era and the central bank has an explicit inflation target of 2 percent.

Figure F.2 shows the average inflation in these six economies over the period when the policy rate was above the ELB. Since the sample period is short, the figure shows how the conditional average depends on the starting date of the sample as well as the confidence band for plus/minus two standard errors.

According to the figure, inflation averaged below the inflation target while the policy rate was away from ELB the all six countries. In the U.S. and Canada, inflation averaged about 20 and 30 basis point below the 2 percent target while the ELB was not binding. The conditional averages in these two countries are closer to 2 percent and the confidence band includes 2 percent when the starting date of the sample is around 2000. Thus, we cannot completely rule out the possibility that the observed undershooting of the inflation target is statistically insignificant. However, the figures for the U.S. and Canada demonstrate the overall tendency for the central banks to undershoot the long-run target by non-trivial amount in these two countries. In the Euro Area, the United Kingdom, Sweden, and Switzerland, the conditional

[29] Aruoba, Cuba-Borda, and Schorfheide (2014) provide an empirical support for this view.

Figure F.1: Policy Rates and Inflation in Six Economies†




†Shaded regions mark the ELB era. Red horizontal lines represent inflation target rate.

average inflation away from the ELB are substantially below the target rate of 2 percent, regardless of the starting date of the sample.

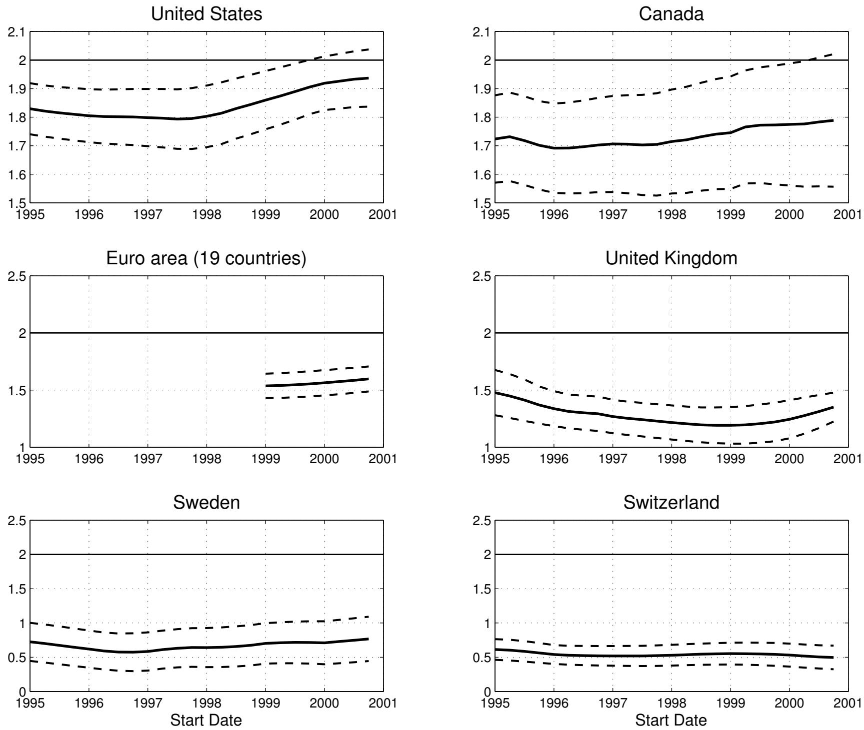
# G More Sensitivity Analyses

## G.1 Price Indexation

In this subsection, we examine the implications of price indexation for the risky steady state. Figures G.1 and G.2 show how the key features of the model vary with the degree of price indexation. Note that, as we vary the degree of price indexation, we adjust the standard deviation of the discount rate shock so that the standard deviation of the policy rate in the model remains unchanged at 2.34.

As shown in Figure G.1, the volatility of inflation increases, and the volatility of the output gap decreases, with the degree of price indexation. As the volatility of the policy rate stays constant, the expected duration of the ELB episode increases as the degree of price indexation increases. According to Figure G.2, inflation, output, and the policy rate at the risky steady state increases, decreases, and increases with the degree of price indexation, respectively. However, the magnitudes are small. Thus, price-setters do not need to be fully forward-looking for the ELB risk to reduce the risky steady state inflation by non-trivial amount.

## Figure F.2: Average Inflation Rates Away From the Lower Bound with Alternative Starting Dates[†]




[†]Horizontal lines mark the inflation target rate. Dashed lines represent the 95% confidence interval (2 standard errors) of the average inflation, which is shown by the solid black lines.

## Figure G.1: Moments with Alternative Degrees of Price Indexation

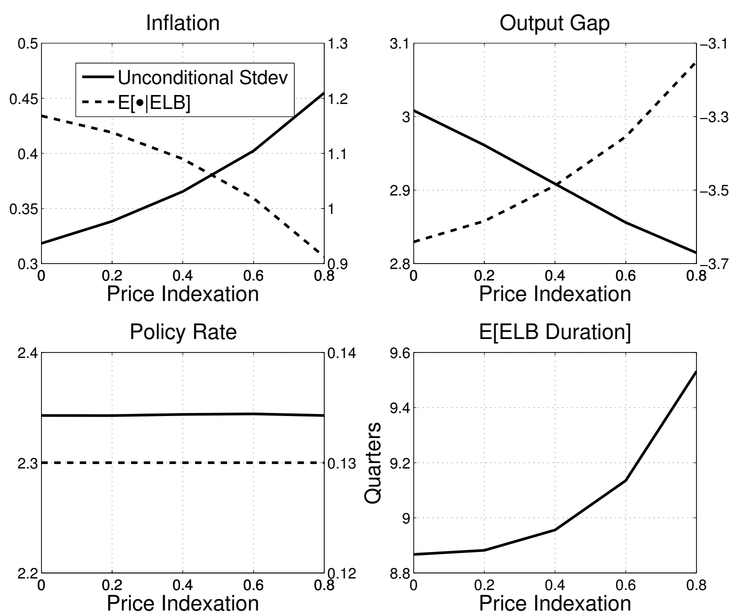
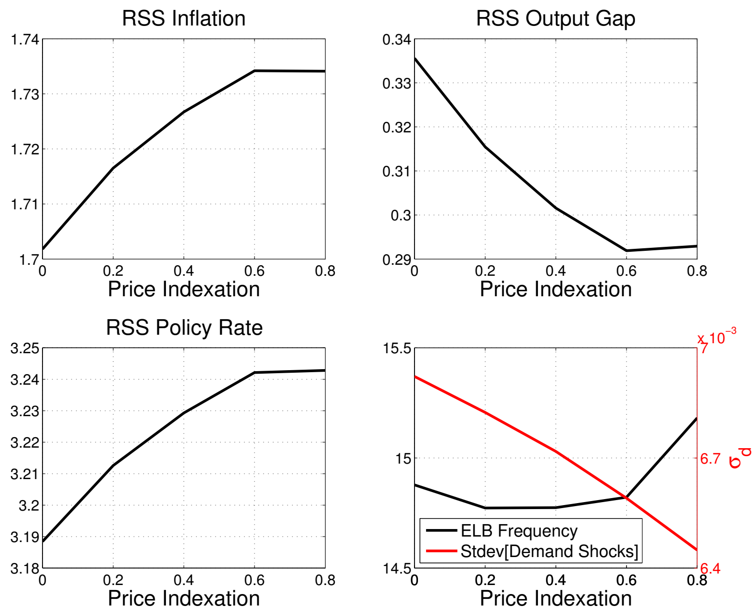


Figure G.2: Risky Steady State with
Alternative Degrees of Price Indexation




## G.2 Wage Indexation

In this subsection, we examine the implications of wage indexation for the risky steady state. Figures G.3 and G.4 show how the key features of the model vary with the degree of wage indexation. Note that, as we vary the degree of wage indexation, we adjust the standard deviation of the discount rate shock so that the standard deviation of the policy rate in the model remains unchanged at 2.34.

As shown in Figure G.3, the volatilities of inflation, the output gap, and the policy rate as well as the expected duration of the ELB episode increase with the degree of wage indexation. According to Figure G.2, inflation, output, and the policy rate at the risky steady state decreases, increases, and decreases with the degree of wage indexation, respectively. The effects of wage indexation on the risky steady state is quantitatively large. For example, inflation is about 1.03 percentage point and the policy rate is about 2.17 percent at the risky steady state when the degree of wage indexation is 0.4. Thus, even when wage-setters are not fully forward-looking, the wedge between the risky and deterministic steady states can be large.

Figure G.3: Moments with
Alternative Degrees of Wage Indexation

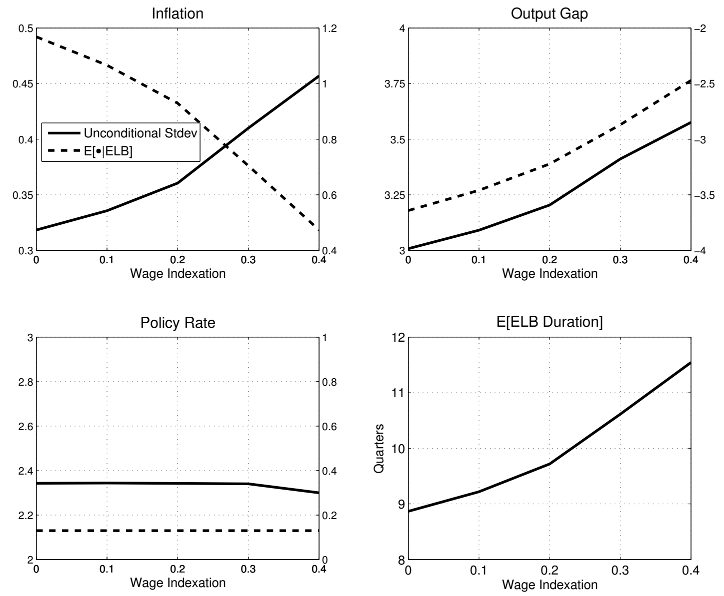
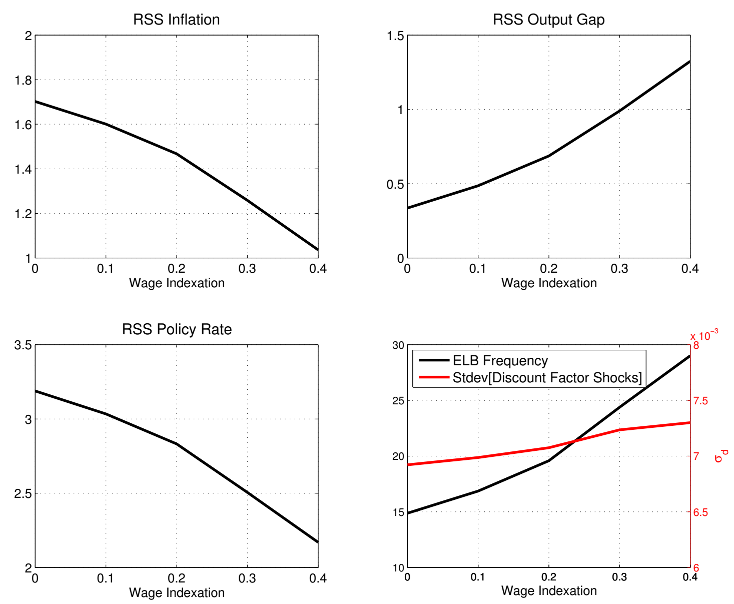



Figure G.4: Risky Steady State with
Alternative Degrees of Wage Indexation




## G.3 A Model with Monetary Policy Shocks

In this subsection, we introduce monetary policy shocks into the baseline empirical model. The standard deviation of the monetary policy shock is set to the standard deviation of the residuals in the interest-rate feedback rule computed using the U.S. data before the federal funds rate hit the ELB ($\sigma_r = \frac{0.14}{100}$). The standard deviation of the discount rate shock is adjusted so that the standard deviation of the policy rate in the model remains unchanged at 2.34.

As shown in Table G.1, The moments and risky steady states of this model are similar to those of the baseline empirical model. This reflects the fact that the magnitude of the monetary policy shock is very small, accounting for only less than 5 percent of the total variation in the policy rate.

Table G.1: A Model with Monetary Policy Shocks

(a) Moments

| Moment | Variable | Model (original) | Model (w/ MP shocks) | Data (1995Q3–2015Q2) |
|---|---|---|---|---|
| St.Dev.(·) | Output gap | 3.0 | 2.9 | 2.9 |
| | Inflation | 0.31 | 0.29 | 0.52 |
| | Policy rate | 2.34 | 2.34 | 2.34 |
| E(X\|ELB) | Output gap | −3.7 | −3.3 | −5.2 |
| | Inflation | 1.21 | 1.28 | 1.48 |
| | Policy rate | 0.13 | 0.13 | 0.13 |
| ELB | Frequency | 13.8% | 13.5% | 36% |
| | Expected/Actual Duration | 8.6 quarters | 6.3 quarters | 26 quarters |
| • | $100\sigma_{\epsilon,\delta}$ | 0.69 | 0.67 | • |

(b) Risky Steady State†

| | Inflation | Output Gap | Policy Rate |
|---|---|---|---|
| DSS | 2 | 0 | 3.75 |
| RSS (w/ $\sigma_r = \frac{0.14}{100}$ ) | 1.75 | 0.27 | 3.28 |
| (Wedge) | (−0.25) | (0.27) | (−0.47) |
| RSS (original) | 1.74 | 0.30 | 3.26 |
| (Wedge) | (−0.26) | (0.30) | (−0.49) |

†DSS stands for 'Deterministic Steady State' while RSS stands for 'Risky Steady State.'

## G.4 A Model with TFP Shocks

In this subsection, we introduce TFP shocks into the baseline empirical model in such a way that the standard deviations of output, inflation, and the policy rate are roughly unchanged from the baseline empirical model. To do so, we adjust a few parameters ($\varphi_p = 700$ and $\phi_\pi = 3.5$) and choose the size of the standard deviation of the TFP shock so that the volatility of output explained by the TFP shock accounts for 10 percent, as opposed to 25 percent, of the total output volatility. This model is used in the non-technical summary of this paper.

As shown in Table G.2, the moments and risky steady states of this model is broadly similar to those of the baseline empirical model.

Table G.2: A Model with TFP Shocks

(a) Moments

| Moment | Variable | Model (original) | Model (w/ TFP shocks) | Data (1995Q3–2015Q2) |
|---|---|---|---|---|
| St.Dev.(·) | Output gap | 3.0 | 3.1 | 2.9 |
| | Inflation | 0.31 | 0.42 | 0.52 |
| | Policy rate | 2.34 | 2.34 | 2.34 |
| E(X\|ELB) | Output gap | −3.7 | −3.4 | −4.2 |
| | Inflation | 1.21 | 1.18 | 1.48 |
| | Policy rate | 0.13 | 0.13 | 0.13 |
| ELB | Frequency | 13.8% | 15.9% | 36% |
| | Expected/Actual Duration | 8.6 quarters | 9.0 quarters | 26 quarters |
| • | $100\sigma_{\epsilon,\delta}$ | 0.69 | 0.66 | • |

(b) Risky Steady State†

| | Inflation | Output Gap | Policy Rate |
|---|---|---|---|
| DSS | 2 | 0 | 3.75 |
| RSS (w/ TFP shocks) | 1.71 | 0.32 | 3.04 |
| (Wedge) | (−0.29) | (0.32) | (−0.71) |
| RSS (original) | 1.74 | 0.30 | 3.26 |
| (Wedge) | (−0.26) | (0.30) | (−0.49) |

†DSS stands for 'Deterministic Steady State' while RSS stands for 'Risky Steady State.'

# H Distribution of Inflation in a Perfect-Foresight Version of the Stylized Model

Figure H.1 shows the unconditional distribution of inflation in the perfect-foresight version of the model. The distribution is positively skewed, reflecting the fact that the nonlinearity associated with the ELB is dominated by the other nonlinear features of the model. As a result, the unconditional average of inflation is slightly above 2 percent. The standard deviation of the discount rate shock is the same as the one for the stochastic model. With this standard deviation, the probability of being at the ELB is x percent in the perfect-foresight model.

Figure H.2 shows how the unconditional average of inflation and the conditional averages of inflation at and away from the ELB vary with the probability of being at the ELB in the model with the ELB constraint solved under the assumption of perfect-foresight.

When the probability of being at the ZLB is sufficiently small, the unconditional average of inflation is slightly positive, reflecting the positive skewness implied by other nonlinear features of the model besides the ELB. However, when the probability of being at the ELB is sufficiently high, the distribution becomes negatively skewed and the average inflation becomes below 2 percent. The conditional average of inflation away from the ELB monotonically increases with the probability of being at the ELB. In the perfect-foresight version of the model with the ELB, the median of inflation is always 2 percent. As the ELB frequency increases, there are more masses on the areas of the distribution where the ELB binds and the conditional average of inflation in those areas decrease with the probability of being at the ELB. The conditional average of inflation away from the ELB is by definition the average inflation in the region that excludes those areas with low inflation. Thus, as the probability of the ELB frequency increases, the conditional average of inflation at the ELB increases.

Figure H.1: Unconditional Distribution of Inflation
in a Perfect-Foresight Version of the Stylized Model

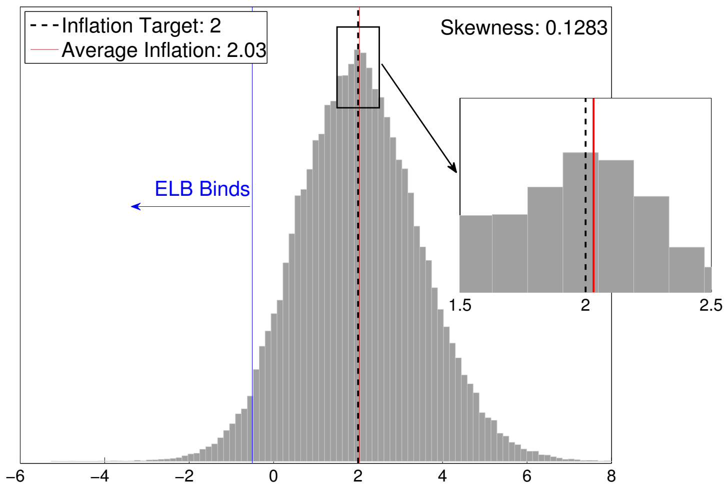
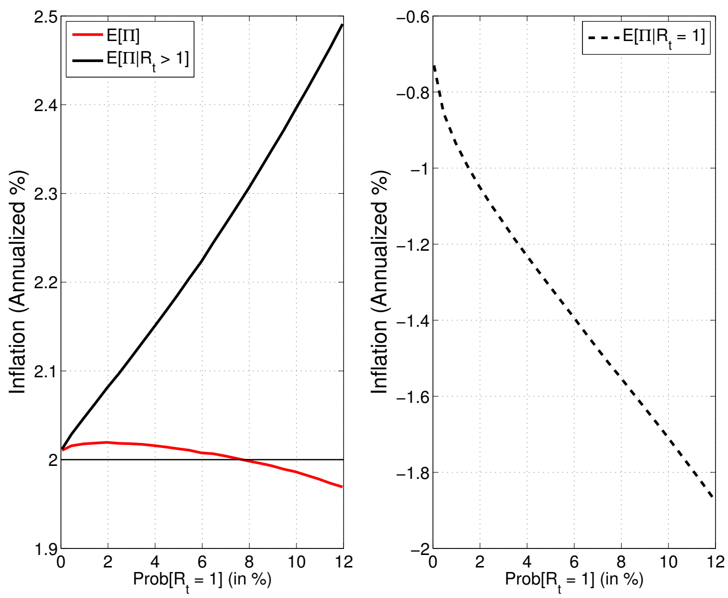



Figure H.2: Conditional and Unconditional Averages of Inflation
in a Perfect-Foresight Version of the Stylized Model




As discussed in the main text, the increase is non-monotonic in the stochastic model with the ELB; The conditional average of inflation away from the ELB initially increases, but later declines, with the ELB frequency. Unlike in the perfect-foresight version of the model, an increase in the probability of being at the ELB shifts down the distribution of inflation, as the forward-looking price-setters take into account the ELB risk in their pricing decisions. This force dominates the other force to increase the conditional average of inflation away from the ELB, and lowers the conditional average of inflation away from the ELB. Provided that the ELB frequency is sufficiently high, the conditional average of inflation away from the ELB is below 2 percent in the stochastic model where price-setters take the ELB risk into account, while it is above 2 percent in the perfect-foresight version of the model that abstracts from the ELB risk. As a result, the fact that the conditional average fell below 2 percent when the policy rate is away from the ELB can be seen as the manifestation of the ELB risk, an idea that is pursued in Section 5.