Estimation of Latent Variables for the FRB/US Model

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November 17, 2014

In FRB/US, latent supply-side variables such as potential output and the natural rate of unemployment are estimated using a multivariate state-space model similar to that of Fleischman and Roberts (2011).[1] This note provides an overview of the approach and some recent estimates, using data through the second quarter of 2014.

Overview

In FRB/US, potential output is built up using a production function. The key business-sector potential output measure, XGPOT, is based on a three-factor Cobb-Douglas production function with capital services, trend energy use, and trend quality-adjusted hours as inputs; trend multifactor productivity captures a standard Solow residual.[2] XGPOT is then related to potential GDP (XGDPT) through a sequence of accounting identities.

FRB/US potential GDP can thus be thought of as built up from a set of underlying building blocks related to potential labor input (LURNAT, QLFPR, QLEOR, QLWW, and LQUALT), capital services (KS), trend multi-factor productivity (MFPT), and wedges between business-sector output and potential GDP (among them, UXBT). Several of these components also have time-varying drift terms (HQLFPR, HQLWW, HMFPT, and HUXBT). Some of these components are derived from published data, including capital services (KS) and labor quality (LQUALT), which are based on BLS estimates. Estimation of many of the other key FRB/US supply-side components (LURNAT, QLFPR, QLEOR, QLWW, MFPT, and UXBT), along with their drift terms (HQLFPR, HQLWW, HMFPT, and HUXBT), uses a reduced-form state-space model that draws on the model of Fleischman and Roberts (2011).

[1] Charles Fleischman and John M. Roberts, "From Many Series, One Cycle: Improved Estimates of the Business Cycle from a Multivariate Unobserved Components Model," Federal Reserve Board FEDS working paper no. 2011-46 (April). http://www.federalreserve.gov/pubs/feds/2011/201146/201146pap.pdf

[2] Details on the FRB/US model are available in other documents on this website.

Our decision to rely on an auxiliary model to estimate trends reflects a compromise across several factors. While it would be best to estimate the trends simultaneously with the estimation of the model itself, that is not possible in a model the size of FRB/US. A piecemeal approach would be feasible, in which each trend would be estimated separately. That approach could use models that are consistent with the FRB/US specifications, but would lead to trends that may not be mutually consistent, and would lose the statistical efficiency benefits of estimating the trends as a system. On balance, we concluded that the benefits of systems estimation outweighed the costs of using individual equations that differed from those in FRB/US. [3]

To minimize the differences between the equations in the reduced-form state-space model and the FRB/US equations, we have modified the equations of Fleischman and Roberts. First, the observation equations in the Fleischman-Roberts model were modified to more closely resemble those in FRB/US. In particular, in the observation equations in Fleischman and Roberts, there is a moving-average relationship between the gap between an observable (X_{it}) and its trend (X_{it}^*) and the output gap (cyc_t):

$$X_{it} = X_{it}^* + \lambda_i(L)\, cyc_t + u_{it}.$$

By contrast, most of the relevant FRB/US equations have a "partial-adjustment" formulation. To make the observation equations of the state-space model more similar to the FRB/US equations, a simple partial-adjustment mechanism was used for the LFPR, LUR, and LEP equations:

$$X_{it} = X_{it}^* + \lambda_{i0}\, cyc_t + \varphi_i\, (X_{it-1} - X_{it-1}^*) + u_{it}.$$

Reflecting the more-rapid adjustment process for the workweek (LWW), a slightly different partial-adjustment specification is used:

$$X_{it} = \theta_{i0}\, cyc_t + \theta_{i1}\, (cyc_t - cyc_{t-1}) + \varphi_i\, (X_{it-1} - X_{it-1}^*) + X_{it}^* + u_{it}.$$

[3] We have found in practice that the tensions between the FRB/US equations and state-space model do not raise important practical problems.

We also modified the inflation model of the Fleischman-Roberts model to include trend inflation (PTR). Introducing trend inflation into the model allows it to line up better with the inflation specification in FRB/US. See the Appendix for details on the inflation specification.

Two other modifications of the Fleischman-Roberts model were made: (1) Introducing capital services, labor-quality, and the trend energy-output ratio (VEOA) so that a FRB/US-consistent estimate of MFPT could be derived. (2) Forcing the drift terms of the various supply-side latent variables to be mean-reverting at a 5 percent quarterly rate, as is imposed in FRB/US, rather than following a random walk, as in Fleischman-Roberts.

Results

Except for the modifications noted in the previous section, the reduced-form state-space model follows the specification in Fleischman-Roberts. A summary description is included in the Appendix. In particular, the eight observable variables are real GDP (FRB/US mnemonic XGDP), real business sector output (XB), the income-side counterpart of XB, the labor-force participation rate (LFPR), the unemployment rate (LUR), private-sector employment (LEP), the workweek (LWW), and inflation as measured by the change in the log of the price index for consumer prices other than food and energy taken from the national accounts (PCXFE).

Figures 1 and 2 show estimates of the output gap and various trends from the model. The output gap presented in Figure 1 is the FRB/US output gap, XGAP2, which is defined using the model's measure of definition of output excluding measurement error, XGDO. The FRB/US estimates of measurement error are also based on the state-space model, so that FRB/US output gap is the same as that of the state-space model. Thus, the output gap is 100*log (XGDO/potential output). The overall pattern of the output gap is similar to that of Fleischman and Roberts (2011). In particular, the output gap falls to nearly -8 percent of potential output in 2009Q3. The most recent estimate, for 2014:Q2, puts the output gap at -1¾ percent of potential output. The middle panel shows the model's estimate of the natural rate of unemployment. From more than 7 percent in the mid-1980s, the model's estimate of the natural rate fell through the mid-1990s, to as low as 4¾ percent, before edging back up to a range of 5¼ to 5¾ percent by the late 1990s, where it remained until very recently. The most recent reading, for 2014:Q2, is 5 percent.

The bottom panel shows the model's estimate of the trend labor-force participation rate, and Figure 2 shows other supply-side components.

- **Labor-force participation.** The filtering model ascribes most of the drop in the LFPR since 2008 to the LFPR trend.
- **Output per hour.** Movements in output per hour are dominated by the trend and account for a relatively small portion of overall output gap fluctuations. As of 2014:Q2, output per hour was just slightly below the model's estimate of trend.
- **Workweek.** The filtering model was surprised to the upside by the level of the workweek during the recession, and as a consequence, the trend that had been falling prior to 2008 flattened out for a while. More recently, the trend has resumed its downward movement.
- **Private-vs.-household employment gap.** Private employment (LEP) fell more steeply in the recent recession than household employment (LEH). The model ascribes most of that drop to cyclical factors. Historically, private-sector employment is more cyclical than the household measure for a number of reasons, including cyclical patterns in multiple jobholding.[4] The trend ratio fell sharply from the mid-1990s to the eve of the recession and has been falling more gradually since.
- **GDP vs. business output.** On average, real GDP rises more slowly than real business sector output, largely reflecting slower growth in the government sector than elsewhere in the economy. During the recession, the ratio of GDP to business output rose, as government sector output helped stabilize overall GDP in relative terms. The model ascribes most of the increase in the ratio during the recession to cyclical factors, and indicates a continued downward movement in the trend in recent years.

[4] If a person with two (private-sector) jobs loses one, private employment falls by one, but household employment is unchanged, because the person still has a job.

Appendix: Summary of FRB/US Supply-Side State-Space Model

The generic partial adjustment model is:[5]

$$X_{it} = \lambda_{i0}\, cyc_t + \varphi_i\, (X_{it-1} - X_{it-1}^*) + X_{it}^* + u_{it}. \quad (1)$$

For the cycle (*cyc*), we assume an AR(2) specification:

$$cyc_t = \rho_1\, cyc_{t-1} + \rho_2\, cyc_{t-2} + \eta_t \quad (2)$$

The seven labor-market and output variables are: the log of real Gross Domestic Product (XGDP*)*; the log of real business output measured on the product side (XBP; referred to as XB in FRB/US); the log of real business output measured on the income side (XBI, not in FRB/US); the log of business sector employment (LEP*)*; the log of the business-sector workweek (LWW); the log of the labor-force participation rate (LFPR); and the log of the employment rate: $ER = \log[1 - (LUR/100)]$. In what follows, italics are used to indicate logs. The discussion below also makes reference to "XGDI"—the income-side analogue of XGDP—although because it is redundant, it is not included in the estimation. The log of the civilian working-age population is subtracted from each of *XGDP*, *XGDI*, *XBP*, *XBI*, and *LEP*, on the grounds that variations in working-age population are not a source of stochastic variation. Inflation, as measured by the price index for consumer spending excluding food and energy (PCXFE), is also included in the model.

The specific versions of the observation equation 1 for the four output variables are:

$$XGDP_t = cyc_t + XGDO_t^* + u_{1t} \quad (3)$$

$$XGDI_t = cyc_t + XGDO_t^* + u_{2t} \quad (4)$$

$$XBP_t = \lambda_{10}\, cyc_t + XBO_t^* + u_{3t} \quad (5)$$

$$XBI_t = \lambda_{10}\, cyc_t + XBO_t^* + u_{4t} \quad (6)$$

XGDO—for gross domestic output—is the common component of GDP and GDI, and *XBO* is the common component of XBP and XBI. The effect of the cycle on *GDP* and *GDI* is

[5] This appendix draws heavily on the discussion in Fleischman and Roberts (2011).

contemporaneous with a coefficient of one. Making the effect of the cycle on *GDP* and *GDI* contemporaneous reflects the common views that overall output is the best contemporaneous business-cycle indicator. Restricting the coefficient to be one is a normalization that has the effect of making the units of *cyc* the same as those of an output gap.[6] Equations 5 and 6 also restrict the effect of the cycle on business output to be contemporaneous and the same across the income and product measures, but allow the coefficient on the cycle to be freely estimated; we would expect it to be greater than one because the non-business portions of GDP (such as government) are not very cyclical.

As noted in the introduction, the supply-side model does not use the FRB/US specifications for key observables. Rather, it uses simplified partial adjustment equations. The specific versions of equation 1 for the labor-market variables are:

$$LEP_t = \lambda_{20}\, cyc_t + \varphi_2\,(LEP_{it-1} - LEPPOT_{it-1}) + LEPPOT_t + u_{5t}. \quad (7)$$

$$LWW_t = \lambda_{30}\, cyc_t + \lambda_{31}\,\Delta\, cyc_t + \varphi_3\,(LWW_{it-1} - QLWW_{it-1}) + QLWW_t + u_{6t}. \quad (8)$$

$$ER_t = \lambda_{40}\, cyc_t + \varphi_4\,(ER_{it-1} - ER_{it-1}^{*}) + ER_t^{*} + u_{7t}. \quad (9)$$

$$LFPR_t = \lambda_{50}\, cyc_t + \varphi_5\,(LFPR_{it-1} - QLFPR_{it-1}) + QLFPR_t + u_{8t}. \quad (10)$$

The reduced-form Phillips curve is:

$$\Delta PCXFE_t = a_1\,\Delta PCXFE_{t-1} + (1-a_1)\,PTR_t + \beta_{11}(L)drpe_{t-1} + \beta_{12}(L)\times d85_t \times drpe_{t-1} + \beta_2(\mathrm{L})\,drpi_t + \theta\,(0.50\, cyc_t + 0.33\, cyc_{t-1} + 0.17\, cyc_{t-2}) + u_{9t} \quad (12)[7]$$

PTR measures long-run inflation expectations, captured empirically by a survey. The sum of the coefficients on lagged inflation and *PTR* is constrained to equal one. The relative price of energy ($drpe_{t-1}$) enters with a six-quarter moving average; *drpi* is the relative change in prices of imported goods; *d85* is a dummy variable that allows the effect of relative energy prices on inflation to fall after 1985. The cyclical effect enters as a moving average.

[6] Starting with the November 2014 public release, the FRB/US output gap, XGAP2, is measured with XGDO in the numerator; the estimate of XGDO is taken from the state-space model. As a consequence, XGAP2 now corresponds to the state-space model's *cyc*. See Jean-Philippe Laforte and John Roberts's FEDS Note on the "November 2014 Update of the FRB/US Model" for more details on the revisions to the FRB/US model.

[7] For convenience, the equation numbers are the same as those in Fleischman and Roberts (2011).

Structural relationships are imposed among the trend components in equations 5 through 10 based on a decomposition of business-sector potential output, XBO_t^*, into the trends in business sector hours, LHP^*, and productivity, OPH^*:

$$XBO_t^* = LHP_t^* + OPH_t^* \tag{13}$$

Trend OPH is in related to MFPT through the following, imposing FRB/US structure:[8]

$$OPH_t^* = (1 / 0.965)\, MFPT_t + 0.725\, LQUALT_t + 0.275\, (KS_t - LHP_t^*) + (0.035 / 0.965)\, VEOA_t \tag{13′}$$

The trend in business sector hours is the sum of trend business-sector employment, *LEPPOT*, and trend business workweek, *QLWW*:[9]

$$LHP_t^* = LEPPOT_t + QLWW_t. \tag{14}$$

Trend business employment is assumed to be related to trend household employment by:

$$LEPPOT_t = ER_t^* + QLFPR_t + ESR_t^*, \tag{16}$$

where ER^* is the trend employment rate and is thus a transformation of LURNAT:

$$\text{LURNAT}_t = 100\, [1 - exp(ER_t^*)] \tag{18}$$

and the difference in employment trends as measured by the employment sector ratio (ESR^*).

Potential output in the aggregate economy is potential gross domestic output (GDO_t^*):

$$XGDO_t^* = XBO_t^* + UXBT, \tag{19}$$

where *UXBT* is the trend output sector ratio—the trend in the (log of the) ratio of XGDO to XBO.

[8] In principle, there should also be an adjustment for trend oil imports, EMPT. In practice, this omission makes little difference to the results.

[9] Where the state-space measures correspond to FRB/US concepts, FRB/US mnemonics are used. That's not the case, for example, with QLHP, which corresponds to a different concept than LHP^*.

As can be seen in equations 13, 13′, 14, 16, 17, and 19, the set of six trends—*QLFPR*, ER^* (and thus LURNAT), *QLWW*, *MFPT*, and the output and employment sector ratios, OSR^* and ESR^*—are the blocks upon which the other trends are built. These basic trend components are assumed to follow a random walk with drift; the drift terms are assumed to follow an AR(1) process with $\rho = 0.95$:

$$Z_t^* = Z_{t-1}^* + \gamma_{Zt} + \varepsilon_{Zt} \quad (20)$$

$$\gamma_{Zt} = 0.95\ \gamma_{Zt-1} + 0.05\ \gamma_{0Z} + \nu_{Zt} \quad (21)$$

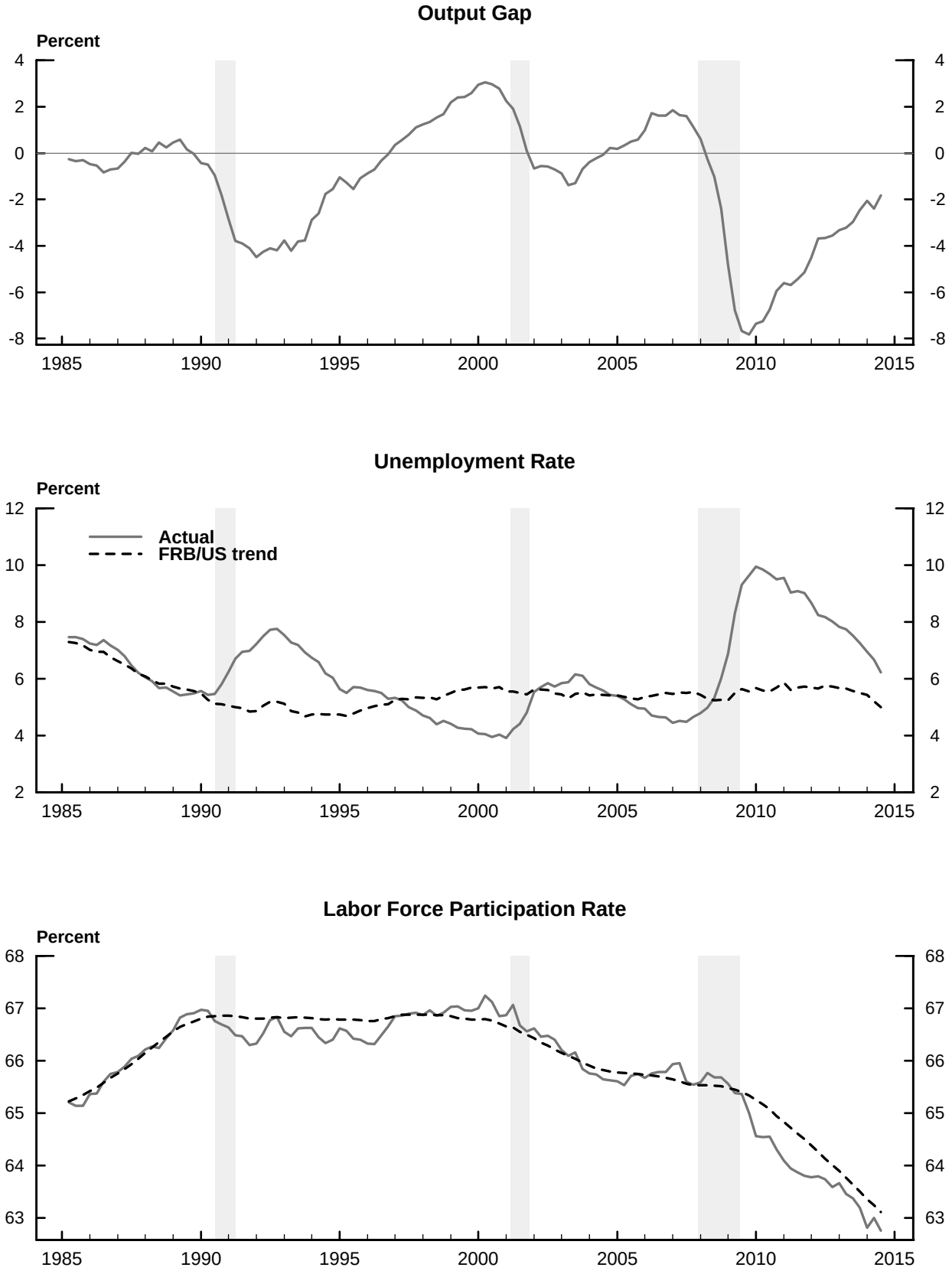
One exception is the trend employment rate, ER^*, which is assumed to follow a random walk without drift—that is, $\gamma_{ERt} = 0$ for all t.

The estimates of the FRB/US supply side are built up from these six basic trends. *QLFPR*, ER^* (which is related to LURNAT), *QLWW*, *UXBT*, and *MFPT* are used directly, after obvious transformations. LEPPOT is derived from equation 16, XBT from equations 13 and 13′, and XGDPT from equation 19. The FRB/US trend QLEOR is backed out according to:

QLEOR= 1 - LURNAT/100 – [(LEPPOT + LEFT + LEST)/(QLFPR*N16)],

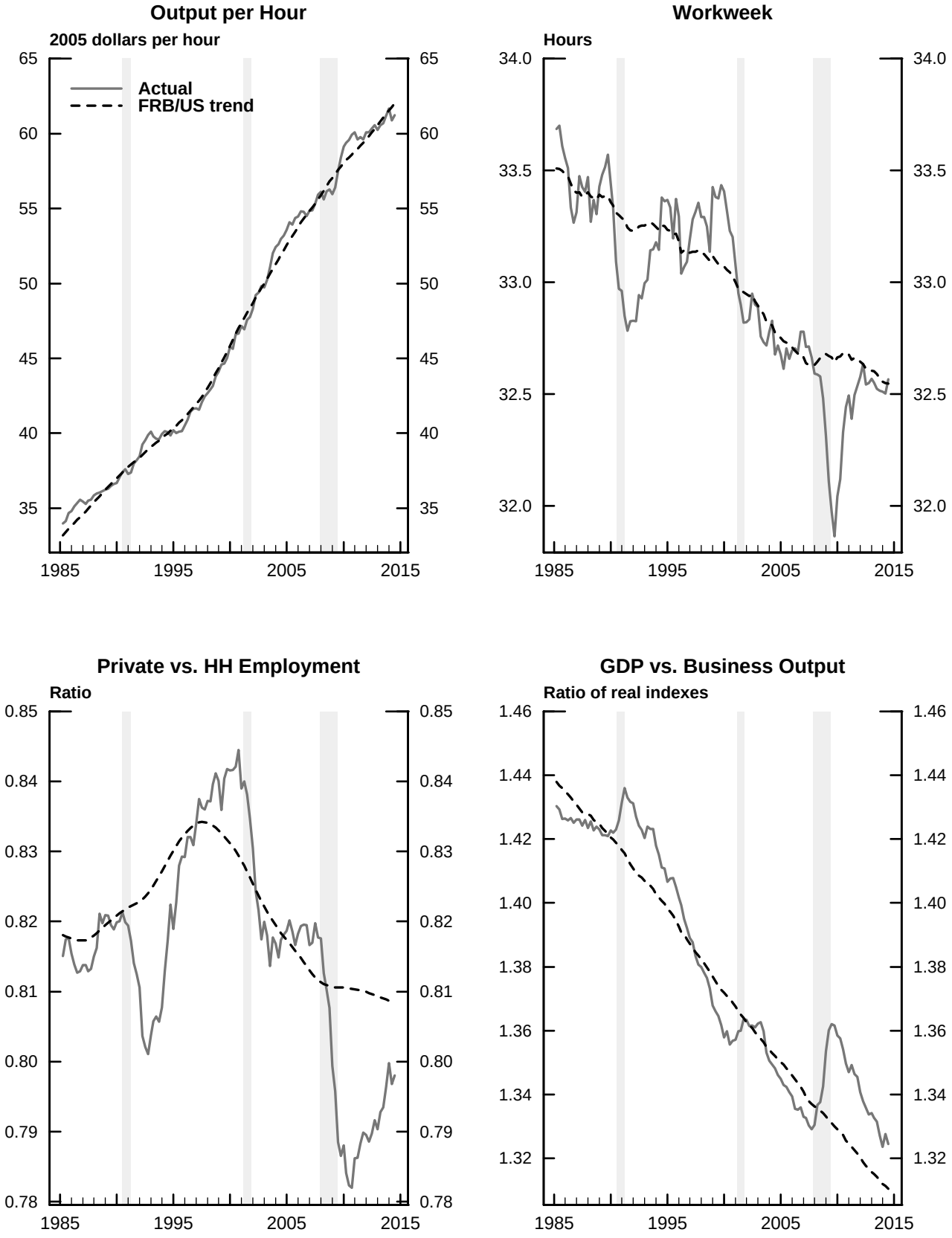
where LEFT and LEST are FRB/US trends for government employment.

Figure 1: Estimates of FRB/US Latent Variables



Shaded regions indicate NBER recessions.

Figure 2: Estimates of FRB/US Latent Variables, continued



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