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Abstract

To "ensure depth and liquidity," the European Central Bank in 2010 and 2011 repeatedly intervened in sovereign debt markets through its Securities Markets Programme. These purchases provide a unique natural experiment for testing the effects of large-scale asset purchases on risk premia arising from liquidity concerns. To explore how official intervention influences liquidity premia, we develop a search-based asset-pricing model. Consistent with our model's predictions, we find statistically and economically significant stock and flow effects on sovereign bonds' liquidity premia in response to official purchases.

Keywords: Securities Markets Programme, European Central Bank, bond, liquidity risk, search and matching

JEL classification: D83, E43, E58, G12

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chasing substantial quantities of sovereign debt from member nations via its Securities Markets Programme (SMP). This official intervention had the explicit goal of ensuring "depth and liquidity" in "dysfunctional" markets.[2] The ECB believed that sovereign bond markets in certain euro-area countries were no longer functioning well. The intervention had the specific intention of improving liquidity conditions without changing the fundamental value of the asset or the euro-wide discount rate (which we take as German bund yields). Furthermore, purchases were not pre-announced to the market (in contrast to Federal Reserve and Bank of England programs) and therefore involved at least some exogenous surprise to market participants in each week in which they occurred. SMP purchases of varying sizes occurred in dozens of weeks, often consecutively, in several European sovereign bond markets over the course of almost two years. Some market commentary claimed that the SMP was a failure because a large number of consecutive purchases were deemed necessary, yet peripheral bond spreads often rose soon after the ECB stepped out of the market.

We argue that SMP operations provide an opportunity to learn about large-scale asset purchase programs' effect on liquidity premia. To do so, we first use a term structure model to decompose bond prices into a euro-wide discount rate, default premium, and liquidity premium. From bond prices alone we could not distinguish the last two terms. However, because the interventions were conducted on sovereign bonds in countries with well-functioning, liquid CDS markets, a separate measure of the default premium is available to us from CDS prices. Since CDS prices are not necessarily tied to liquidity conditions in the bond market,

[2] ECB May 2010 Monetary policy decision, see
`http://www.ecb.europa.eu/press/govcdec/otherdec/2010/html/gc100521.en.html`.

we are thus able to combine CDS and bond price data to disentangle fluctuations in the default and liquidity premia for sovereign bonds.[3]

To understand how ECB purchases influenced liquidity premia, we develop a search-based asset-pricing model. Our theory modifies the framework of Duffie, Garleanu and Pedersen (2005, 2007) (DGP hereafter) by explicitly including the risk of default. The model naturally produces a liquidity premium which compensates buyers for the risk that they may not be able to immediately sell the bond in the future, due to search frictions.

We model official intervention as an exogenous reduction in the supply of bonds traded amongst investors, which is caused by the official sector instantaneously purchasing bonds from bond-sellers. We show that this exogenous shock affects the model's steady state and creates transition dynamics in the process. Therefore, we provide a theoretical justification for the existence of a long-run *stock* effect as well as a temporary *flow* effect of official purchases, a key feature missing in standard models.

Furthermore, the model explains why official purchases *should be expected* to occur repetitively over the course of several weeks. In our model, there is an equilibrium price at which some agents are happy to buy and sell bonds. But the pool of bond-sellers is markedly smaller than the entire pool of market participants, and therefore there is a tight limit on how many bonds officials can purchase at the equilibrium price at any given time. This means that

[3]In addition, because the ECB sterilized these operations, the effect of SMP purchases on the euro-wide discount rate (as measured by relevant-maturity German bund yields) should be negligible. That is, we should not expect these operations to influence either the expected path of short rates or the rate of overall European growth. In any case, we are able to control for changes in German rates directly. This highlights a difficulty with performing our analysis on, say, Federal Reserve LSAPs, because there the official intervention *must change the U.S. discount rate*.

numerous consecutive purchases may be required for officials to achieve a particular amount of liquidity improvement.

Using the model as a lens on the data, we provide reduced-form empirical estimates of the SMP's effect on sovereign bonds' liquidity premia in a linear specification that can nest alternative theoretical perspectives. The ECB made SMP purchases of various sizes over the course of almost two years in several European nations. This allows us to exploit both cross-section and time-series features of the data to sharpen our estimates and inference. Our generalized least squares estimates efficiently account for cross-sectional correlation between sovereign bond prices for reasons we fail to capture in our simple model.

We estimate stock and flow effects that are economically and statistically significant. We find that, on average, an official purchase of one percent of sovereign debt outstanding decreases the liquidity premium by 23 basis points on impact. Most of this effect is temporary, and so official purchases tend to "overshoot" on impact and liquidity premia rise after a purchase. However, we find that about 5 basis points of the liquidity premium's decrease remains. Our empirical estimates of the stock and flow effects accord very well with our model's predictions on the effects of official intervention. In particular, our model predicts that "overshooting" is an inherent feature of any successful intervention. The intuition for this is simple – models with frictions involve transition dynamics when shocked to a new steady state.[4]

This paper is related to the large literature on search in asset markets. DGP (2005)

[4]Thus, our model's prediction is likely to be shared amongst a broader class of liquidity models that involve frictions.

introduced a search-based asset-pricing model with risk-neutral investors. DGP (2007) simulated the model and showed how it approximated to first-order the behavior of risk-averse investors. Lagos and Rocheteau (2009) generalized DGP (2005) to allow for multiple types of risk-averse investors who could make continuously-valued purchases. Feldhutter (2012) implements an empirical version of DGP (2005) to obtain maximum likelihood estimates of selling pressure in U.S. corporate bonds from 2004 to 2009, using the detailed TRACE data set that allows the structural parameters to be identified. By contrast, our reduced-form approach is better suited to our more limited data. Furthermore, our estimates provide average effects over a variety of potential liquidity shocks, each of which lead to a different particular relationship between official purchases and the liquidity premium effect in the model, a point we document in simulation studies.

He and Milbradt (2012) embed DGP's model in Leland and Toft's (1996) framework to endogenize the firm's default decision and link bond liquidity conditions to the firm's distance-to-default. Their model predicts that the liquidity premium rises with the probability of default, whereas our model implies that the liquidity premium falls as the probability of default rises.[5] Both models link liquidity premia to the risk that the asset owner is unable to immediately find a buyer when they are desperate to sell. In He and Milbradt (2012) this default leads to a recovery process where investors hold onto more illiquid claims, and so an increased default probability *increases* the expected selling wait-time. In contrast, in

[5]Typically, one thinks of an increase in the probability of default as being associated with an increase in the liquidity premium. This correlation arises naturally in the data as increased default probabilities are often correlated with other forms of market distress and dysfunction. In contrast, in our theory a different mechanism is at play. If these unmodeled forces were significant, we would expect them to swamp the effect our theory predicts going in the opposite direction. However, we do not find this to be the case in our data.

our model the default immediately pays out a recovery value, as in Duffie (1998), and so an increase default probability *decreases* the expected selling wait-time. These contrasting assumptions reflect empirical differences between recovery value payouts for corporate versus sovereign defaults, which we discuss below. He and Milbradt (2012) focus on theoretical properties of their model and provide no estimation. On the other hand, motivated by our simpler framework we control for the default probability in our estimation and find empirical support for the model's prediction that sovereign bonds' liquidity premia fall along with the distance-to-default.

A growing literature investigates the SMP's effects. Ghysels, Idier, Manganelli and Vergote (2012) look at intraday data to investigate the impact effects of SMP purchases and separate them from other high-frequency events. Eser and Schwaab (2013) use daily data and model the factor structure of European sovereign risk in order to identify SMP purchase impact effects separately by country. Both of those empirical papers make use of proprietary ECB data and evaluate overall yield effects, which contain not only the liquidity premia we investigate but also default premia we do not consider (but control for in the estimation). Trebesch and Zettelmeyer (2014) recently used novel data on the particular Greek bonds purchased under the SMP to find significant yield declines in response to ECB purchases. Our estimates dovetail with the average effect seen in Eser and Schwaab's (2013) time-series results and the benchmark results from Trebesch and Zettelmeyer's (2014) cross-sectional analysis.

The three aforementioned papers fit into a larger literature that estimates the effects of large-scale asset purchases. That literature focuses mostly on the United States, including Hamilton and Wu (2011), Gagnon, Raskin and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011), Christensen and Rudebusch (2012), D'Amico, English, Lopez-Salido and Nelson (2012) and D'Amico and King (2013). Joyce, Lasaosa, Stevens and Tong (2011) estimate the effects of the Bank of England's asset purchases in the United Kingdom.

The paper is organized as follows. Section 2 presents the data sources, describes their evolution in the context of European events, and provides a high level description of our preferred yield-curve-based bond liquidity measure (with greater detail relegated to the appendix). Section 3 presents the search-based asset pricing model, and then reports theoretical and numerical results illustrating its mechanisms and showing the liquidity effects of official bond purchases. Section 4 provides estimation results alongside our model-based interpretation of the estimated effects of the ECB's SMP purchases, and summarizes robustness checks of our findings. We then conclude. The appendix contains proofs, detailed discussion of our term-structure approach to measuring bond market liquidity, and robustness checks of our empirical results.

2 Data

We collect data for four European countries: Portugal, Ireland, Italy and Spain.[6] These are countries for which the ECB confirmed to have purchased sovereign debt under its SMP.

[6]We exclude Greece from our main analysis because its bond and CDS prices are outliers to those countries included here. Furthermore, Greece actually defaulted in early 2012. However, we include Greek data in robustness checks and find its inclusion has only modest effects on our main results.

For each country we collected daily CDS spreads and bond price data through February 2012 when the SMP program effectively ended. We first describe the data sources we used, and then give a brief narrative of the evolving European debt crisis over our sample period.

2.1 Sources

Bond prices are from Datastream, and we collected sovereign bond prices per country for all bonds that were outstanding for each day in our sample. With the full set of characteristics of each available bond (maturity date, coupon rate, and coupon payment frequency) we use the methodology detailed in Appendix A.2 to estimate zero-coupon curves for each day. From these we then compute the daily term structure of default probabilities. When doing so, we applied the usual filters to the data: We deleted any bonds that have option-like features or floating coupon payments; we did not include any bills or bonds that were denominated in non-domestic currencies; and we excluded any bond in the estimation as soon as it has less than three months left to maturity.[7] We use German zero-coupon yields as our underlying euro-zone discount rates, constructed using the Bundesbank's estimated Svensson-Nelson-Siegel parameters obtained from the Bundesbank website.[8]

Our source for sovereign CDS spreads is Mark-it Partners and we collected mid-quotes on eight CDS contract maturities: 6 months and 1, 2, 3, 4, 5, 7 and 10 years. All contracts are denominated in U.S. dollars, even though these are designed to offer default risk protection on euro-denominated sovereign bonds. The reason behind this currency mismatch is that

[7] See Gürkaynak, Sack and Wright (2007) for details and a similar approach to estimating zero-coupon yield curves for the U.S.

[8] These are available on `http://www.bundesbank.de/statistik/statistik_zeitreihen.en.php`.

the overwhelming majority of CDS trading on euro-area sovereign bonds occurs in U.S. dollar-denominated contracts.[9] Finally, we obtained 6-month euro-area LIBOR and euro-swap rates from Bloomberg and we use these in our methodology of computing CDS-based default probabilities described in Appendix A.2.

The main source of data on the ECB's SMP purchases comes directly from the SMP entry on the ECB's balance sheet, which is publicly available at the weekly frequency.[10] In addition, the ECB provided some information as to which bonds were purchased and when. For instance, early purchases under the program, beginning in 2010, were only of Greek, Portuguese and Irish bonds. The expansion of the SMP to Spain and Italy began in August 2011. In February 2013, the ECB released a snapshot of how much debt it was holding for each country at the end of December 2012.[11] This snapshot accorded to the country-level weekly purchase data we obtained from Barclays (described immediately below). To date, the ECB has not publicly released any further details on its SMP purchases.

Barclays, who was a significant counterparty to ECB transactions during our data sample, has published weekly country breakdowns of SMP purchases. We use their estimates to allocate total SMP purchases across countries at each point in time.[12] The estimates primarily reflect a rule of proportionality (buying that is proportionate to nations' bond market size), but additionally adjust according to differential market pressures (week-by-week) as

[9] Chen, Fleming, Jackson, Li and Sarkar (2011) analyze all the trades entered into the DTCC warehouse between May and July 2010 and found that only a small fraction of the sovereign single-name trades were in euros. The outstanding volumes of European sovereign CDS is relatively small compared to the stock of outstanding European debt, even though the European CDS market has grown substantially since the onset of the financial crisis, see the data that is available on `www.dtcc.com` .

[10] See `http://www.ecb.int/press/pr/wfs/`.

[11] See `http://www.ecb.europa.eu/press/pr/date/2013/html/pr130221_1.en.html`.

[12] We thank Laurent Fransolet for kindly sharing this data with us.

was observed by the Barclays' trading desk and according to occasional announcements by the ECB of its purchase composition.

2.2 The European Crisis and ECB Actions

In May 2010, sovereign debt markets in several euro-area countries came under extreme pressure due to deteriorating fiscal health. According to contemporaneous statements from the ECB, the functioning of several financial markets became seriously impaired at this time, including money markets, foreign exchange markets, and peripheral European sovereign bond markets. (ECB Monthly Bulletin, June 2010).

Sovereign bond spreads relative to comparable German bunds widened considerably, as Figure 1 shows. Five-year sovereign bond spreads attained record highs and volatility in the bond markets increased sharply. At the same time, according to the ECB, bond market liquidity conditions deteriorated rapidly.

On May 10th (vertical line 1 in Figure 1), the ECB announced several new liquidity support programs. Chief among them was the SMP whose stated intention was to improve bond market liquidity. Aggregate SMP purchases of all countries' bonds, as reported on the ECB's balance sheet, are shown in Figure 2.[13] Figure 3 breaks out these purchases by country using Barclays data, expressing the cumulative purchases as a percentage of each sovereign's total debt outstanding (by aggregating each weekly purchase divided by total outstanding debt at that time). The top panels show that SMP purchases of Irish and

[13]Negative moves in the ECB's SMP balances are attributed to bond redemptions, not bond sales.

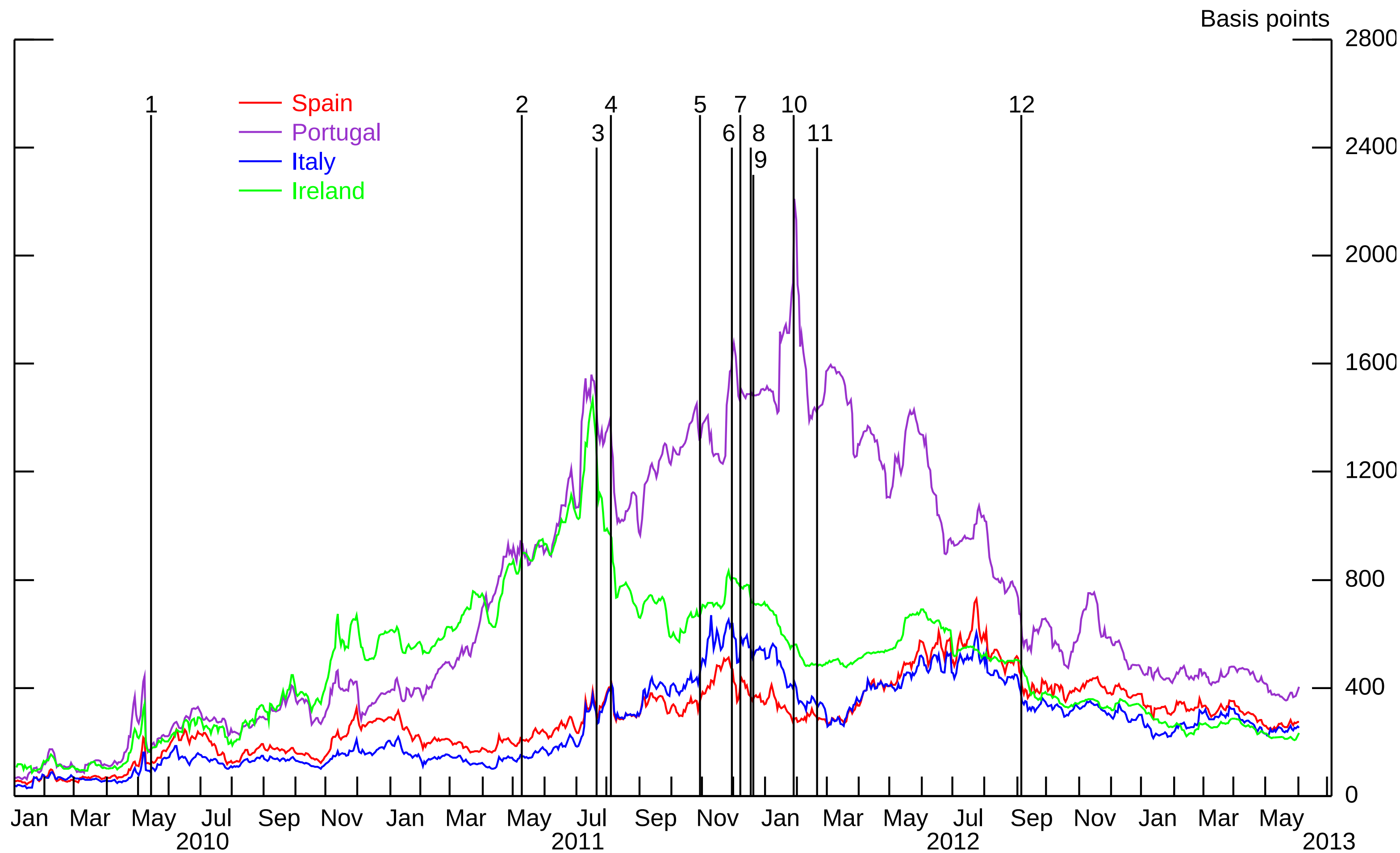

Figure 1: Sovereign Bond Spreads

Notes: Sovereign bond spreads for peripheral European nations, at the five-year maturity. The vertical lines in Figure 1 denote key euro-area events: 1. The announcement of the SMP; 2. First Greek aid package; 3. July European Union leaders summit; 4. The expansion of the SMP to Italy and Spain; 5. October European Union leaders summit; 6. The expansion of the swap line arrangement between the Federal Reserve and the ECB; 7. December 8th ECB meeting wherein they announced additional easing measures especially the longer-term refinancing operations (LTRO); 8. $150 billion in IMF loans released to Greece; 9. The first LTRO; 10. January European Union leasers summit; 11. The second three-year LTRO; and 12. Announcement of Outright Monetary Transactions and the official termination of the SMP.

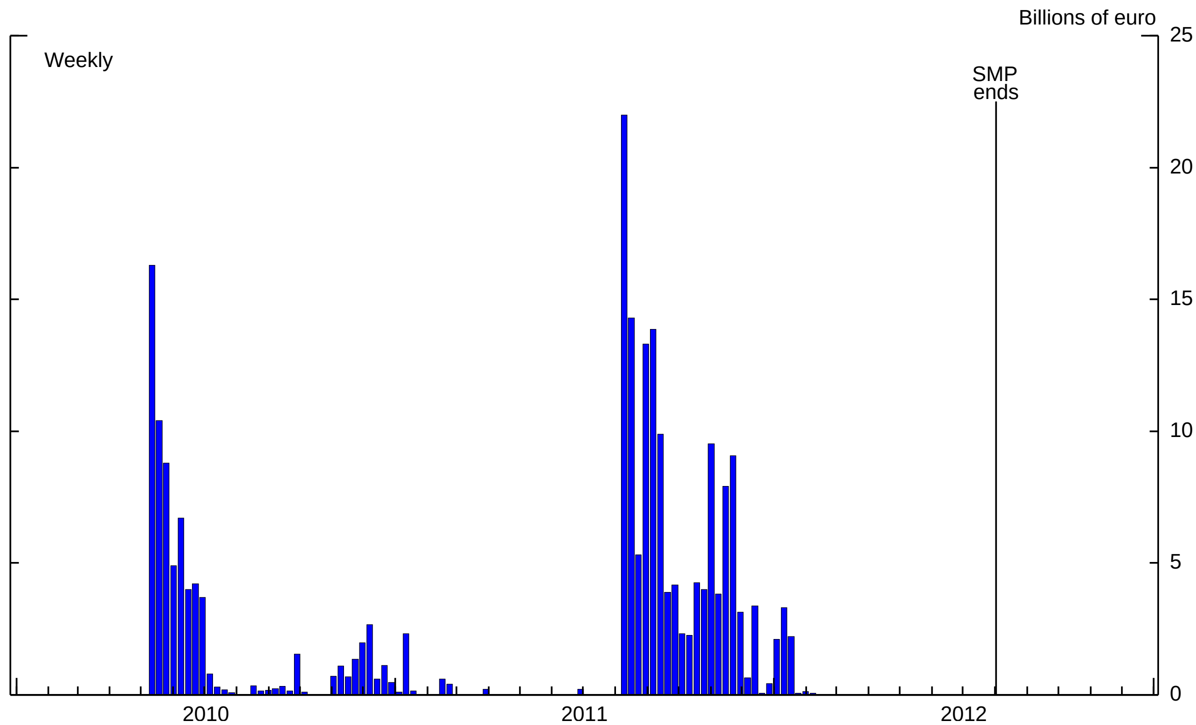

Figure 2: Securities Markets Programme Purchases: Aggregate
Notes: The ECB's sovereign bond purchases under the SMP, in aggregate from the ECB's weekly balance sheet. Bars show the amount purchased each week (in billions of euros).

Portuguese bonds began in May 2010 and within several months amounted to more than ten percent of those nations' total debt outstanding.[14] A long period of relative calm ensued with the ECB periodically conducting modestly-sized consecutive purchases in late 2010 and early 2011.

This calm evaporated with the summer heat of 2011. With further deterioration in Greece necessitating a bailout package (line 2 in Figure 1), peripheral European sovereign bond spreads widened considerably. Tensions, which had previously been somewhat confined to Ireland and Portugal, spread to Italy and Spain. On August 4, sovereign bond spreads

[14]Greek debt was also purchased and is included in the aggregate amounts reported in Figure 2. The ECB purchased a cumulative 14.5% of outstanding Greek sovereign debt through the SMP.

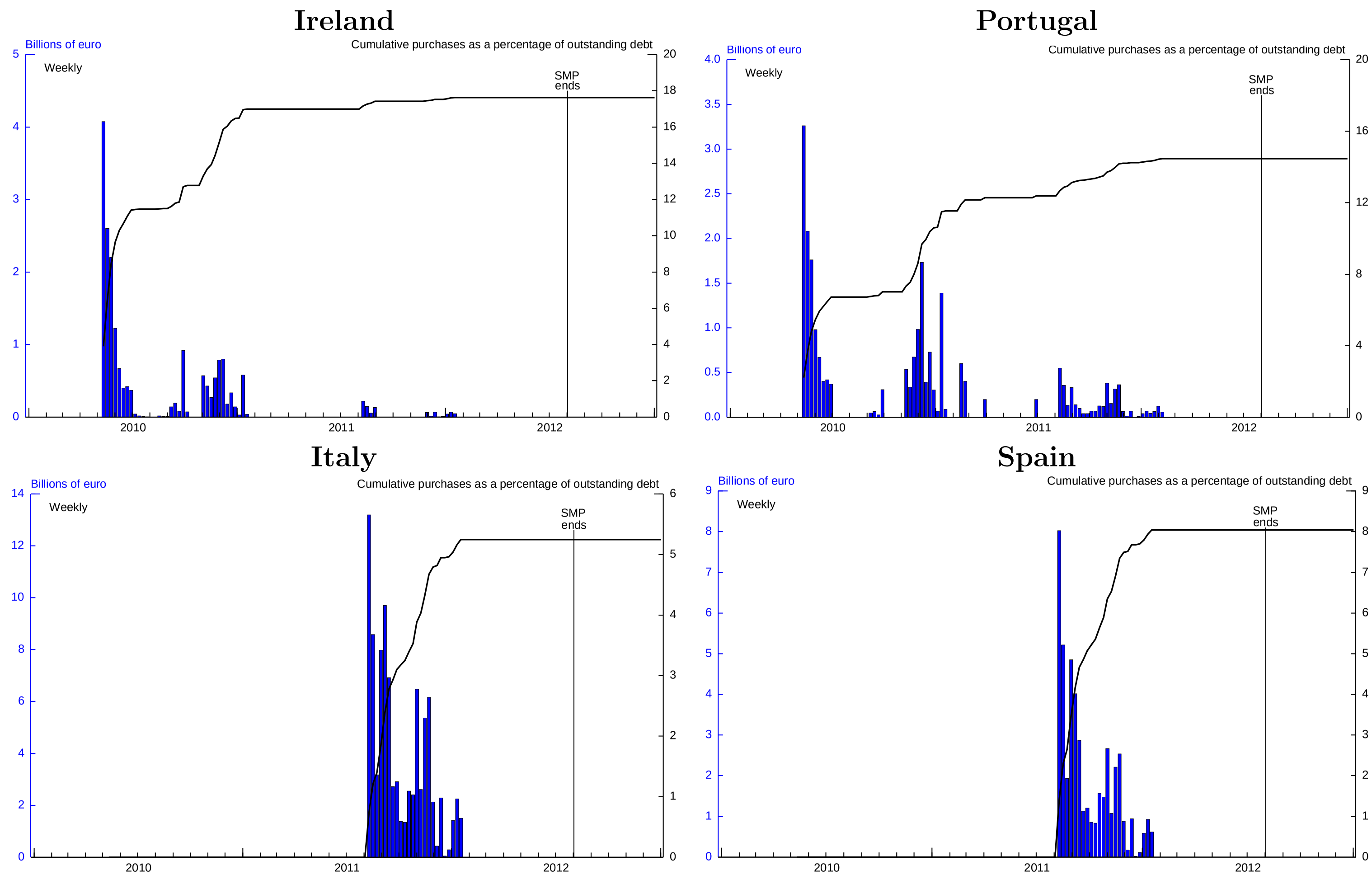

Figure 3: SECURITIES MARKETS PROGRAMME PURCHASES: BY COUNTRY

Notes: The ECB's sovereign bond purchases under the SMP, by country, from Barclays data. Bars show the amount purchased each week (in billions of euros). The line shows the cumulative amount debt purchased through the SMP as a percent of each country's outstanding debt, by aggregating each weekly purchase divided by total outstanding debt at that time.

reached record highs in Italy and Spain. Volatility reached a level last seen in the aftermath of the collapse of Lehman Brothers in September 2008. The ECB announced that liquidity conditions in peripheral European sovereign bond markets deteriorated sharply, and so expanded its SMP purchases (line 4 in Figure 1). Starting in August 2011, the ECB began purchasing Italian and Spanish bonds, as shown in the bottom panels of Figure 3. In short order, the ECB owned more than five percent of the outstanding debt of both Italy and Spain.

Despite repeated purchases beginning in 2011, which were larger (in euros) than the earlier incarnation of the SMP, sovereign yields began to climb once again. It appeared that once the ECB took a step out of the market and did not directly buy bonds, yields rose. Some market commentary cited the inability of the ECB to maintain a fixed level of yields as a sign that the program was fundamentally ineffective. Furthermore, the long string of measures taken by European officials (lines 5 through 11 in Figure 1) were claimed by some commentators to be evidence of the SMP's failure.

The SMP was in existence until September 6, 2012 when the ECB officially announced the program to be terminated (line 12 in Figure 1), although Figure 2 shows that the ECB stopped purchasing bonds long before that in February 2012. The ECB replaced the SMP with the Outright Monetary Transactions program (OMT), which embeds much more conditionality in relation to possible ECB bond purchases, with governments officially having to request bond purchases. As Figure 3 shows, we calculate that the ECB purchased about 18, 14, 8 and 5 percent of Ireland's, Portugal's, Spain's and Italy's outstanding bond supply, respectively, via the SMP. To date, the OMT program has not been activated.

2.3 Measuring Variation in Bond Liquidity

From data on bond and CDS prices, we construct a measure of *yield-curve based* bond liquidity. This measure is an empirical proxy for the liquidity premium that we model in Section 3 and it serves as the dependent variable in our empirical analysis in Section 4. Our procedure for constructing this bond liquidity measure consists of standard yield curve estimation techniques designed to use all available bond and CDS price information. Please refer to Appendix A.2 for the granular details of the estimation – here we primarily give a high-level overview.

Using the framework of Duffie and Singleton (1999) and Pan and Singleton (2008) we separately estimate from bonds and CDS the (time-varying) implied probability that the sovereigns in our sample will default on their outstanding debt. We exploit the fact here that bonds and CDS have known cash flows, and we follow the majority of the literature in assuming a known and identical recovery value of 40 percent in order to estimate these default probabilities.[15] We take German bunds as our risk-free asset in our sovereign bond calculations while we take euro-area swap rates as our risk-free asset in our CDS calculations.

From Duffie and Singleton (1999) we know that bonds are subject to default and liquidity premia, so the default probabilities we estimate from bonds do not measure only the

[15] The bottom panels of Figure A1 in the appendix suggest that our estimates of the default probability change moderately for alternate recovery rate values. However, the time series variation is essentially unchanged over a wide range of recovery rates, implying that our empirical estimates are robust to alternate recovery rate assumptions.

probability of default, but also a liquidity premium. Using bond price data alone, we cannot identify these separate components. However, using CDS data we can construct an empirical estimate of the default probability that is distinct from bond liquidity. Because bonds and CDS on the same sovereign should theoretically reflect the same default event, the probability of default priced by bonds and CDS should be identical. Therefore, by taking the difference between the default probabilities from bonds and CDS, we can identify a measure of the relative liquidity premium that is embedded in bonds and CDS. It is this relative liquidity measure that we use to estimate the effect of ECB bond purchases. In our empirical work to come, we make the identifying assumption that CDS liquidity conditions are uncorrelated with bond market forces such as ECB bond purchases. Under this assumption, any systematic deviation in our *relative liquidity* measure indicates changes in bond market liquidity. Therefore, we can measure fluctuations in *bond liquidity* directly, albeit with noise. From here on we drop the qualifier "relative" for the sake of exposition.

In sum, we can use our liquidity measures, CDS default probabilities, and Germany bund yields to decompose European sovereign bond yields into three separate pieces, as illustrated in Figure 4. The first piece is the euro area-wide discount rate as measured by German rates (the blue area on the bottom of each panel). The second piece is the default premium measured from CDS spreads (the red area in the middle of each panel). The final piece is the liquidity premium identified by our yield-curve-based liquidity measure (the yellow area on the top of each panel). These intuitive graphs trace out the relative importance of default and liquidity premia on peripheral European sovereign bonds over the course of the SMP's

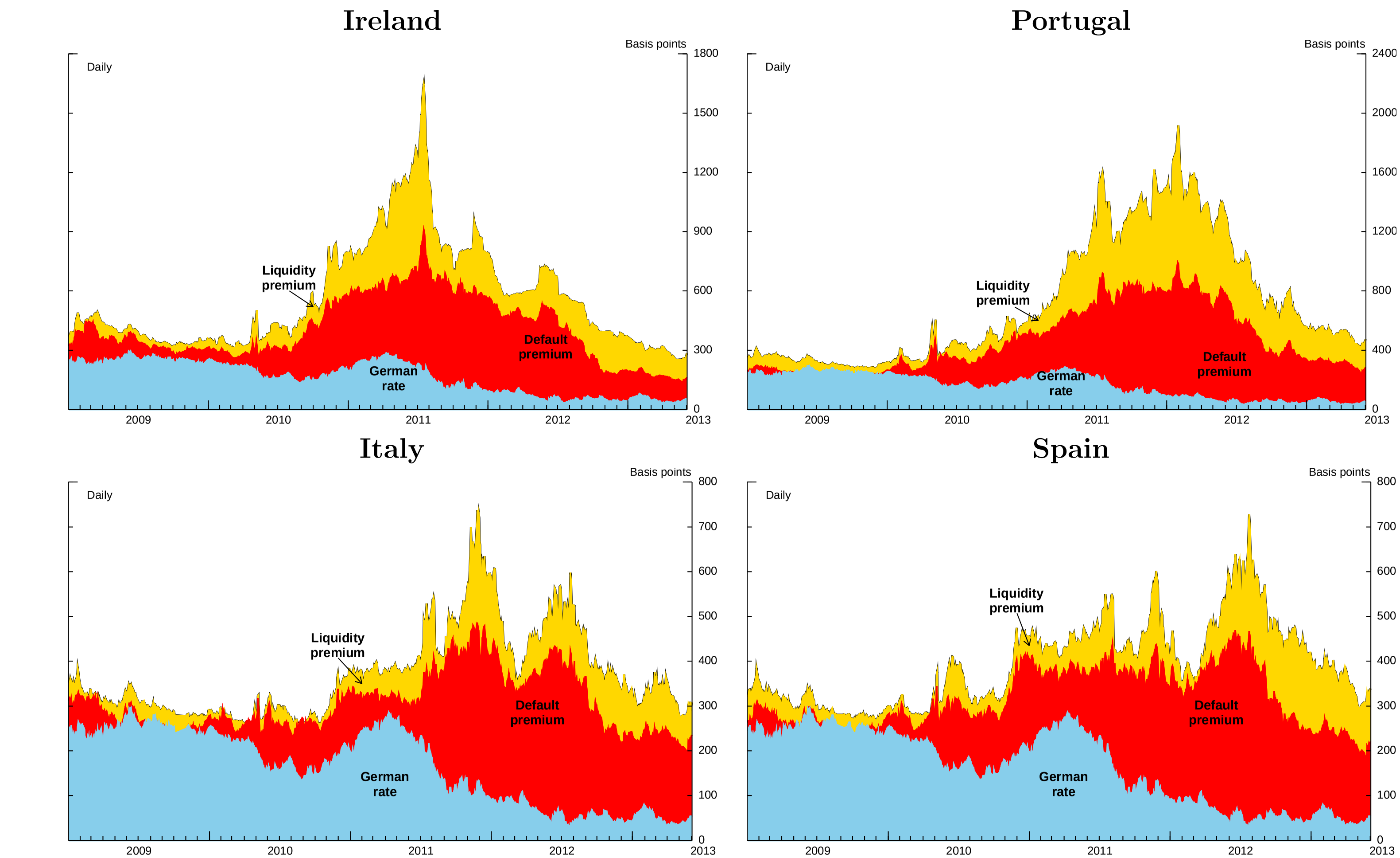

Figure 4: Sovereign Bond Yield Decompositions

Notes: Five-year sovereign bond yields, decomposed into three pieces: The German rate (blue area on the bottom), the default premium (red area in the middle), and the liquidity premium (yellow area on top).

existence.

Towards the end of 2009 and at the beginning of 2010, Italian, Portuguese and Spanish spreads mostly reflected a liquidity premium, while Irish spreads contained comparable default and liquidity premia. After the European debt crisis began in May 2010, liquidity conditions varied dramatically, but to varying degrees and with different timing across nations. By the beginning of 2011, Irish bonds already priced in a sizeable default premium. However, yields kept on climbing, in large part reflecting a terrific increase of the liquidity premium in summer 2011. On the other hand, Portuguese default and liquidity premia both rose by quite similar amounts over this time.

For Italy, a modest default premium opened up after May 2010. In early 2011 this default premium narrowed but the liquidity premium opened up and remained roughly constant until the summer. Then both default and liquidity premia rose dramatically, ushering in the ECB's purchases of Italian debt. Meanwhile, liquidity premia for Spanish bonds fluctuated similarly to Italian bonds', but with default premia playing a greater role. By our estimates, liquidity premia in Ireland and Portugal, at some time during 2011, constituted the largest component of their sovereign bond yields. A sharp decrease in liquidity premia accompanied the SMP's expansion in August 2011.

Figure 4 (as well as the bottom panels in Figure A1) emphasizes that default premia in European sovereign bonds have varied widely across both time and country. This evidence highlights the importance of explicitly modeling the default probability in our theory and empirics.

We argue that our preferred yield-curve-based measure is a less noisy measure of bond liquidity and better fits our model than the well-known CDS-bond basis.[16] Computing the CDS-bond basis for a specific maturity makes results and conclusions dependent on the quality of quotes at exactly that maturity, and a single bad quote could have a substantial impact on results. We opt instead to use the *entire* term structure of bond and CDS prices in an effort to reduce that noise. Nevertheless, our empirical results are qualitatively unchanged if the CDS-bond basis is used instead of our preferred yield-curve-based liquidity measure, as we show in robustness checks.[17]

3 Model

Previous empirical literature suggests that official bond purchases may have *stock* and *flow* effects. How do these effects arise? How could isolated periods of official purchases give rise to a permanent stock effect? Why might official purchases happen consecutively and gradually taper off? Why might purchases temporarily decrease liquidity premia below what is ultimately sustainable? Are there other factors that should be controlled for when

[16]As demonstrated by Duffie (1999), an exact arbitrage pricing relation exists among a risky floating rate bond trading at par, a risk-free par floater of the same maturity, and a CDS contract of the same maturity on the risky bond. Consequently, the CDS-bond basis should be zero at all times. In practice, however, this is not always the case and the basis can be either positive or negative for a variety of reasons. Some commonly stated reasons include one market leading the other in terms of price discovery, the "cheapest-to-deliver" option that is part of standard CDS contracts, liquidity premiums in either the CDS or bond market, counterparty risk, flight-to-safety flows, or an exchange rate effect when the CDS contract is written in a different currency than the reference bonds. In this paper, we do not go into the literature that analyzes this non-zero basis and we refer the interested reader to papers such as Ammer and Cai (2011), Coudert and Gex (2010), and Fontana and Scheicher (2010).

[17]Figure A3 in the appendix plots the CDS-bond basis versus our yield-curve-based liquidity measure and shows a close correspondence between the two measures but also shows that for example for Portugal, the measures do differ at times.

looking for these effects in the data?

To answer these questions we present an asset-pricing model that explains liquidity premia by the risk that an agent cannot immediately sell an asset when she wants to, due to search frictions. Since we have shown that a key feature of our bond and CDS data is variation in default probabilities over time, we explicitly include default in the model. We then interpret official intervention as an exogenous reduction in the supply of bonds, caused by instantaneous official purchases of bonds from agents wishing to sell. The model predicts that these purchases change the steady state (a stock effect) and induce dynamics on the transition path to that new equilibrium (a flow effect). Furthermore, the theory justifies the pattern of official bond purchases that were actually seen, explaining why many separate purchases are necessary and why purchases, on impact, "overshoot" the long-run liquidity premium effect. Finally, the model predicts that an increasing probability of default, all else equal, reduces liquidity premia and should therefore be controlled for in empirical analysis. This theory guides the specification and interpretation of our empirical results in Section 4.

A search-based asset-pricing model is a good framework for the peripheral European bonds we model. These bonds have traditionally been far less liquid than more familiar U.S. and German sovereign bond markets. For instance, whereas turnover in the German bond market is about 25% per month over this period, turnover in the Italian bond market is less than 1% per month. This means that the process of searching for a buyer or seller is quite significant to the prices of the sovereign bonds we study.

An alternative and popular framework is the preferred habitat theory of Vayanos and

Vila (2009), used for instance in D'Amico and King (2013) and others for modeling the Federal Reserve's LSAP program. In that model, it is agents' different preferences for bonds of various maturities that lead to price effects of official purchases. Official purchases in the preferred habitat model necessarily change the underlying frictionless price of the bond purchased, as well as neighboring (in maturity space) bond prices. This assumes *a priori* that the ECB could not have achieved its goal of improving bond market liquidity without affecting euro-area discount rates – indeed, in the preferred habitat model there is no liquidity premium to affect. We prefer to work with a simple framework wherein ECB purchases *could* have had their intended effect, instead of assuming it away from the beginning, and look in the data for such evidence.

3.1 Model

Our search-based asset-pricing model adapts Duffie, Garleanu and Pedersen's (DGP; 2005, 2007) framework which in turn rests on Diamond's (1982) seminal work. Our development mirrors theirs except for the addition of an exogenous default arrival process. As in DGP (2007) we neglect the consideration of market-makers analyzed in DGP (2005). Our primary measure of liquidity is the difference between the equilibrium price and a frictionless price that would prevail if the asset were exchanged in Walrasian spot markets.

We set agents' time preference by a constant discount rate $r > 0$ and assume they are risk-neutral. DGP (2007) show that this framework is a first-order approximation of a model with risk-averse agents and stochastic endowments. An investor is distinguished by whether or not she owns the asset and whether or not her intrinsic type is "high" or "low."[18] A low-type investor experiences a holding cost of $\delta > 0$ per time unit whereas a high-type investor has no such holding cost. A low-type investor switches to being high-type with intensity $\lambda_u > 0$ while a high-type investor switches to being low-type with intensity $\lambda_d > 0$, each as a result of an exogenous Poisson process. DGP (2005) discuss several possible motivations for this construct. For the context of the European debt crisis, we prefer a motivation tied to exogenous liquidity needs. For reasons exogenous to this particular bond market, a private-sector agent may need to liquidate her holdings to raise funds. These exogenous funding shocks can vary across investors and time.

Therefore, the four types of agent in the model are indexed by $\{ho, hn, lo, ln\}$ where h denotes high-type, l denotes low-type, o indicates an asset owner and n indicates an asset non-owner. By definition, shares μ of each type of agents sum to 1;

$$\mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t) = 1, \forall t. \tag{1}$$

The supply of the asset $s \in (0, 1)$ is determined outside the model and restricts the mass of asset-owners,

$$\mu_{ho}(t) + \mu_{lo}(t) = s, \forall t. \tag{2}$$

[18] We maintain this assumption to remain directly comparable to DGP which eases the exposition. Lagos and Rocheteau (2009) generalize the model to allow for finitely many types and a continuous domain of possible holding amounts. They find that trade volume, bid-ask spreads and trading delays are affected by their richer set-up. Our analysis does not explicitly consider these features of the model, hence we opt to follow DGP and leave an extension to Lagos and Rocheteau's framework to future research.

Official intervention will be modeled as exogenous changes to s that we detail below. The asset is infinitely-lived conditional on being in non-default, but we introduce into the model an exogenous default Poisson process with intensity $\lambda_D \geq 0$. Upon default, the asset market closes and owners receive the recovery value $R(t) \geq 0$.

Investors meet other investors according to an exogenous Poisson process with intensity $\lambda > 0$. The search is non-directed and therefore the other investor comes from a uniform distribution across the investor population. Reasonable parameterizations admit equilibrium transactions only when low-type owners (lo) meet high-type non-owners (hn): lo investors wish to sell while the hn investors wish to buy.

Assuming a law of large numbers applies (cf. Duffie and Sun (2007)) the masses' rate of change are given by

$$\begin{aligned}
\dot{\mu}_{lo}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \\
\dot{\mu}_{hn}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t) \\
\dot{\mu}_{ho}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t) \\
\dot{\mu}_{ln}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t).
\end{aligned} \tag{3}$$

These equations are identical to those in DGP (2007) because the default process does not alter the evolution of agent types in the model. The intuition for, say, the first line is as follows. With intensity λ agents of type lo meet other agents – μ_{hn} of them are hn agents with whom lo would like to transact. Meanwhile, lo is met by other agents with intensity λ

– μ_{hn} of them are high-type non-owners with whom, again, *lo* would like to transact. These meetings result in transactions that turn *lo* agents into *ln* agents. At the same time, there are exogenous entries into *lo* from the *ho* pool as well as exogenous exits from *lo* into the *ho* pool. The other lines follow similar explanations. DGP (2005) prove that there is a uniquely stable steady-state solution for this system.

The introduction of default affects the present value of the asset for every agent. Asset owners take account of the possibility of default and associated recovery rate, while non-owners take account of the possibility that they will never be able to buy the asset. We put the value functions for each agent in Appendix A.1 and here present the first-order conditions obtained:

$$\begin{aligned}
\dot{V}_{ln} &= rV_{ln} - \lambda_u \left(V_{hn} - V_{ln}\right) + \lambda_D V_{ln} \\
\dot{V}_{ho} &= rV_{ho} - \lambda_d \left(V_{lo} - V_{ho}\right) - \lambda_D \left(R - V_{ho}\right) - 1 \\
\dot{V}_{hn} &= rV_{hn} - \lambda_d \left(V_{ln} - V_{hn}\right) - 2\lambda\mu_{lo} \left(V_{ho} - P - V_{hn}\right) + \lambda_D V_{hn} \\
\dot{V}_{lo} &= rV_{lo} - \lambda_u \left(V_{ln} - V_{lo}\right) - 2\lambda\mu_{hn} \left(V_{ln} + P - V_{lo}\right) - \lambda_D \left(R - V_{lo}\right).
\end{aligned} \tag{4}$$

We have subsumed the dependence on t of $\dot{V}_{(\cdot)}, V_{(\cdot)}(t), \mu_{(\cdot)}(t)$ and $R(t)$. These equations reduce to those in DGP (2007) when $\lambda_D = 0$. For price determination we use the surplus-splitting rule

$$P = (1 - q)\left(V_{lo} - V_{ln}\right) + q\left(V_{ho} - V_{hn}\right) \tag{5}$$

where $q \in [0,1]$.[19]

Our analytical focus is on assets whose market value decreases when search frictions increase.[20] To ensure this intuitive feature of the model, we adopt condition 1 of DGP (2005). Furthermore, we are interested in situations where the present value of the default-free asset's cash flow is greater than the recovery value obtained upon default. Therefore we maintain the following condition:

Condition 1 *The asset supply and switching intensities are such that*

$$s < \frac{\lambda_u}{\lambda_u + \lambda_d} \quad \text{and} \quad R = \frac{\zeta}{r} \;\text{ for }\; \zeta \in (0,1). \tag{6}$$

The first part of the assumption ensures that in steady state there is less than one unit of asset per high-type agent, and therefore that the asset's discounted cash flow equals the frictionless price (defined below). The second part of the condition implies that the recovery rate is lower than the present value of the asset's cash flow, conditional on it being default-free. Although the following propositions can be modified to condition on whether or not Condition 1 holds, this distracts from the main analysis and so we abstract from it. Our first result is an expression for the equilibrium asset price.

Proposition 1 *For any given initial distribution $\mu(0)$ there exists an unique steady-state*

[19] DGP (2007) discuss how this rule can emerge from various bargaining setups, notably Nash-bargaining.

[20] DGP (2007) note that an asset, for instance U.S. Treasuries, might experience a "scarcity value" instead of a liquidity risk premium.

equilibrium. The price is given by

$$P = \frac{1 + R\lambda_D}{r + \lambda_D} - \frac{\delta}{r + \lambda_D} \frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q) + (1-q)\lambda_D}{r + \lambda_d + \lambda_u + 2\lambda\mu_{lo}(1-q) + 2\lambda\mu_{hn}q + \lambda_D}. \tag{7}$$

Proof: See Appendix A.1

As noted above, default risk does not affect the steady-state distribution of agent types. Hence, when $\lambda_D = 0$, equation (7) reduces to DGP's (2007) price.

The equilibrium price naturally separates into two pieces:

$$\begin{aligned} P &= \text{Frictionless price} - \text{Cost of liquidity risk} \\ &= P_f - P_f L. \end{aligned} \tag{8}$$

Note from (7) that both of these pieces are functions of λ_D. The first term on the right-hand side of the equation in (8) represents the present value of the asset's cash flow, taking into account the possibility of default, which we call the *frictionless price* P_f. The second part of Condition 1 ensures that the default-free frictionless price (i.e. when $\lambda_D = 0$) is greater than any other price for any configuration of the parameters. The point of the model is to analyze the costs of default and liquidity risks and Condition 1 ensures that both costs are always nonnegative. Note that the default-free frictionless price equals $1/r$.

The second term in (8) represents the cost of search frictions as dictated by the *liquidity* *premium*, L;

$$L = \frac{P_f - P}{P_f} = \frac{\delta}{1 + R\lambda_D} \frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q) + (1-q)\lambda_D}{r + \lambda_d + \lambda_u + 2\lambda\mu_{lo}(1-q) + 2\lambda\mu_{hn}q + \lambda_D} \tag{9}$$

This premium expresses the cost of search frictions as a proportion of the frictionless price. These are the correct units for the premium because the frictionless price is the asset value when there is no liquidity risk because $\delta = 0$. For instance, for a liquidity premium $L = 0.1$ we say that agents must be compensated in order to hold the asset's liquidity risk, and this risk weighs down on the price by 10%.

Before we present the proposition, we also state a rather esoteric condition for reference

Condition 2 $\frac{\lambda_d}{1-q} + \lambda\mu_{lo} > (\frac{1}{\zeta} - 1)r$.

This condition is hard to interpret, but generally applies in all but pathological parameterizations of the model (see Appendix A.1 for an argument). The following proposition describes properties of the frictionless price P_f, equilibrium price P, and liquidity premium L.

Proposition 2 *Let $\bar{\lambda}_D$ be some constant that depends on model parameters other than λ_D. Let Condition 1 hold. The frictionless price P_f is*

1. *unchanged as s, λ_d or λ varies*
2. *decreasing in λ_D*

   such that $P_f \xrightarrow[\lambda_D \to 0]{} 1/r$ and $P_f \xrightarrow[\lambda_D \to \infty]{} R$

The liquidity premium L is

3. *increasing in* s

4. *increasing in* λ_d

5. *decreasing in* λ *for all* $\lambda \geq \bar{\lambda}$ *such that* $L \xrightarrow[\lambda \to \infty]{} 0$

6. *decreasing in* λ_D *for all* $\lambda_D \geq \bar{\lambda}_D$ *such that* $L \xrightarrow[\lambda_D \to \infty]{} 0$

The price P *is*

7. *decreasing in* s

8. *decreasing in* λ_d

9. *increasing in* λ

10. *such that* $P \xrightarrow[\lambda_D \to 0]{} P_{df}$ *and* $P \xrightarrow[\lambda_D \to \infty]{} R$

The effects of λ_D *on* L *and* P_f *hold even when* λ *is a bounded function of* λ_D*. If Condition 2 holds,* L *is everywhere decreasing in* λ_D*.*

Proof: See Appendix A.1

Part 1 is obvious from the definition of P_f.[21] Part 2 is also clear from the definition of P_f and intuitively reflects how the probability of default and recovery value enter into the frictionless price for the asset. Our Condition 1 ensures that P_f monotonically decreases towards R as λ_D increases. Parts 4 and 5 (and equivalently 8 and 9) are quite intuitive – a decrease in matching intensity λ or an increase in search pressure λ_d both deteriorate bond market liquidity conditions, forcing the liquidity premium up and the equilibrium bond price down.

[21] The behavior of P_f, L, P in response to changes in δ and λ_u are not used in our analysis, but are clear from the equations above.

An important part of the proposition is part 3.[22] This effect captures the intuition that increases in the supply of bonds rest more and more in the hands of low-type agents. This increases the compensation agents demand for the increased risk that they fall in this undesirable pool of agents. Conversely, a reduction in supply limits the steady-state mass of these agents desperate to sell, meaning that search frictions bear less on the present value of the asset, leading the liquidity premium to fall. The channel through which we model official purchases as having a *stock* effect is by a reduction in s. Therefore, Proposition 2 predicts that SMP purchases permanently decrease the steady-state liquidity premium in euro-area bond prices.

Part 6 says that the liquidity premium falls as the default intensity gets large. If Condition 2 holds, which it does in most reasonable calibrations, L is everywhere decreasing in λ_D. The cost of liquidity in this model stems from the risk that an asset-owner might be unable to sell the asset when they want to in the future. As the asset's default becomes increasingly likely, the "future" over which the asset-owner reckons this liquidity risk shrinks and hence the premium falls. This is because, as in Duffie (1998) and consistent with the Greek default in early 2012, we assume the recovery value is paid out immediately upon default. By contrast, the recent modification of DGP by He and Milbradt (2012) comes to the opposite conclusion, that increasing probabilities of default increase liquidity premia. Their result stems in large part from an assumption that default entails a delay until the recovery value is paid out, which is a feature of corporate defaults such as Lehman Brothers'.

It is key to note that, because of search frictions, the default intensity enters P_f and L

[22]This is equivalent to DGP's (2007) first proposition result for the effect of s on the price.

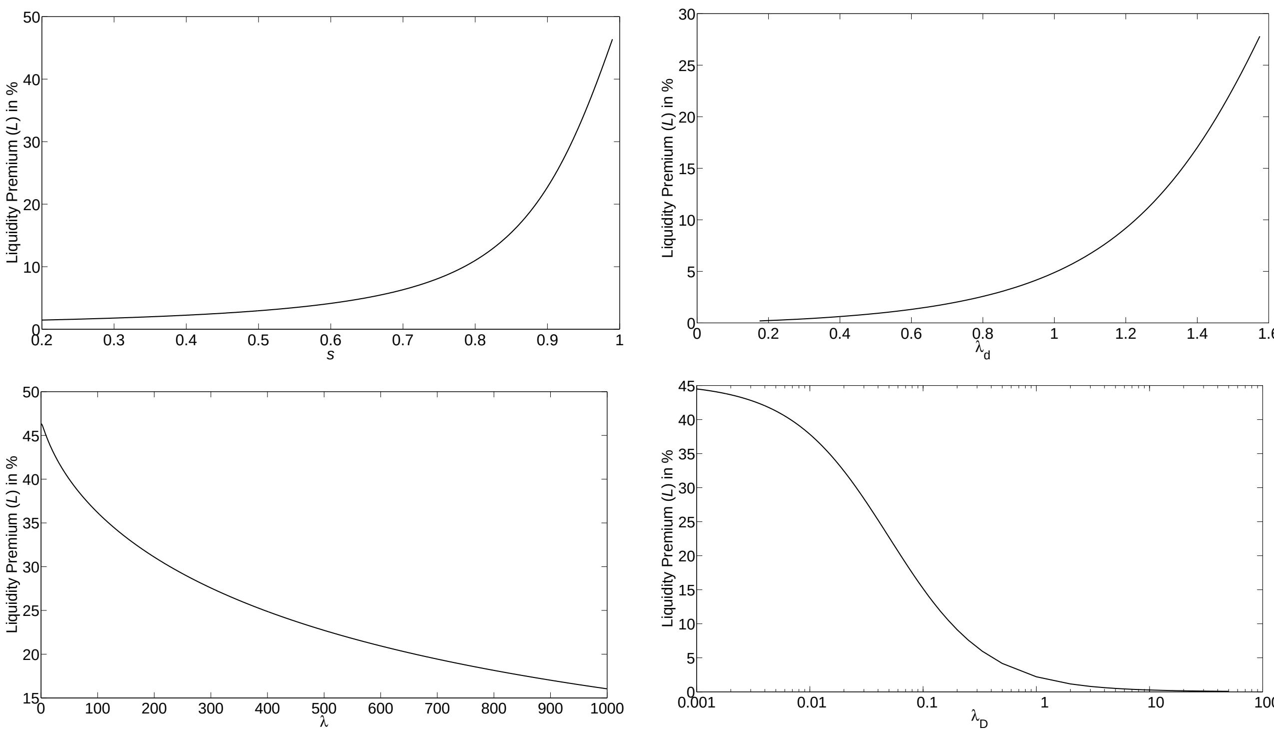

Figure 5: Steady-State Liquidity Premia

Notes: The steady-state liquidity premium, as a function of s, λ_d, λ or λ_D. s is the supply of bonds, reported as a fraction of the maximum value allowed by Assumption 1. λ_d is the intensity at which bond-owners wanting to own the bond become bond-owners wanting to sell the bond (the rate at which *ho* agents become *lo*). λ is the intensity at which agents meet. λ_D is the intensity at which the bond defaults. Other parameters are set at the initial parametrization listed in the first column of Table 1.

separately. It is therefore possible for default risk to systematically affect bond prices via the liquidity premium, apart from the discounting channel at play in P_f. This theoretical consideration supports our combining bond and CDS prices to measure bond market liquidity, which we discussed in Section 2.3.

3.2 Simulations

3.2.1 Steady-state

Now we numerically illustrate Proposition 2 by plotting the relationship of the steady-state liquidity premium to different values of s, λ_d, λ and λ_D. Other parameters are set at the initial parametrization listed in the first column of Table 1 below.

The top left panel of Figure 5 shows that the liquidity premium increases in the supply of bonds s. Whereas for low supply the premium is less than 3%, for high supply the premium is upwards of 40%. This is due to the fact that the pool of bond-sellers μ_{lo} is increasing in s.

The top right panel shows that the liquidity premium increases in the intensity with which agents become low-types (importantly, the rate at which *ho* agents become *lo*). We plot this for values of $\lambda_d < \lambda_u$ for λ_u from Table 1. As λ_d increases, so too does the likelihood the agent will soon need to sell the bond, and so too does the premium demanded by the bond-buyer for this risk.

The bottom left panel of Figure 5 demonstrates that the liquidity premium decreases as λ increases for reasonable average wait-times between matches (the average wait-time between matches is $250/(2\lambda)$ days).

Finally, the bottom right panel shows that an increasing default intensity λ_D leads to a decreasing liquidity premium. This reflects that when default becomes increasingly imminent (λ_D going up) it is increasingly likely that bond-holders will receive the recovery value and the liquidity premium therefore diminishes and ultimately disappears. Note that the premium drops approximately linearly with the logarithm of the default intensity. This informs our estimation below.

3.2.2 Shocks and Transitions

We have built this model in order to understand bond liquidity premia during the European debt crisis. Therefore, it is important to understand the model's predictions for how liquidity shocks propagate through the system. Additionally, we must choose how to represent the ECB's SMP purchases.

We model official intervention as exogenous *lo*-shocks: Immediate (involving no search) official purchases of bonds from *lo* bond-sellers which reduce the overall supply of bonds in the private market. This somewhat mimics DGP's (2007) formulation of a liquidity shock, in that a mass of agents instantaneously switch states – our shock differs in that an exogenous central bank agent has taken bond supply out of the system to create this instantaneous switch, which leads our liquidity shock to create permanent effects.

There are at least four reasons for our conception of a liquidity shock. First, the ECB's objective function is not that of the representative investor. The central bank optimizes different criteria and is not subject to the liquidity pressures we have represented by low-type and high-type investors. Therefore, we regard SMP purchases as exogenous to the model, not undertaken for the profit motives that are captured by our investors' first-order conditions. Second, the ECB is evidently a buy-and-hold investor.[23] This suggests the particular manner in which official intervention should be modeled: An exogenous reduction

[23] This was confirmed by the ECB's February 2013 release in which it classified the SMP holdings as "held-to-maturity".

in the supply of bonds that are available for private market transactions. Third, we assume that the ECB's buying is special – there is no searching required for its purchases. The institution is large, visible and can buy as many bonds as it wishes from the low-type bond sellers, who are immediately able to find the ECB's desk. Fourth, the ECB's intervention was officially intended to fix dysfunction without changing "fundamental" prices: Assuming that officials purchase only at the prevailing market price is consistent with that goal, and moreover the actual practice of ECB asset auctions.[24]

In Table 1 and Figures 6-7 we consider two types of shocks. In Panel A of Table 1 and Figure 6, we consider a fall in λ which increases liquidity premia because bond-sellers meet fewer agents per day. In Panel B of Table 1 and Figure 7, we consider a fall in λ_d which increases liquidity premia because bond-holders become bond-sellers more often. We call the fall in λ a "search friction shock" because an exogenous force has made it more difficult for bond-holders to meet each other over-the-counter. We call the fall in λ_d a "funding risk shock" as one interpretation of why bondholders are more likely to exogenously need to sell their peripheral European bonds in the future. Both of these shocks are liquidity shocks because they cause liquidity premia to rise sharply if unmitigated by official purchases.

[24] There is a range of prices that clear the market but lead to the same distribution of agent masses (in (5) this range is indexed by q). This fact is easily seen by the fact that q plays no role in determining the steady states or evolution of masses μ. The ECB could buy at a price within this range that is higher than what other agents pay. But the liquidity premium would be only mildly reduced because it is the cost δ and masses μ that are key to determining the upper bond of this admissible price range. Only if the ECB offered a price higher than V_{ho} would the liquidity premium fall drastically. But in this case, the premium would drop to zero because every bondholder would immediately sell to the ECB. Additionally, in practice the ECB bought near prices that already prevailed in the market. Therefore, we don't explore how official purchases could be priced to reduce the liquidity premium. We instead assume that the ECB purchases bonds at the equilibrium price P and explore how the quantity of purchases can be used to reduced liquidity premia.

Table 1: Steady-State Values

	A. Search Friction Shock ($\lambda \downarrow$)			B. Funding Risk Shock ($\lambda_d \downarrow$)		
	Initial	Shocked	Official Intervention	Initial	Shocked	Official Intervention
matches per day (# agents)	4	0.2	0.2	4	4	4
match wait-time (days)	0.25	6.25	6.25	0.25	0.25	0.25
sell wait-time (days)	3.4	45.5	16.2	3.4	9.3	2.8
h duration (days)	250	250	250	250	195	195
l duration (days)	125	125	125	125	125	125
L (%)	11.1	33.9	12.2	11.1	29.2	13.2
yield (bp)	394	529	399	394	495	403
liquidity yield spread (bp)	44	179	49	44	144	53
misallocation (%)	1.3	11.7	5.4	0.7	1.5	0.9
reduction in s (%)	–	–	30.4	–	–	5.5

Notes: Columns labeled "Initial" report (important characteristics of) the steady state prior to the liquidity shock. Columns labeled "Shocked" report the steady state after the liquidity shock with no official intervention. Columns labeled "Official Intervention" report steady state after the liquidity shock but when officials intervene. The official intervention is twelve consecutive purchases each spaced five days apart of all bonds available for sale: lo-shocks equal to μ_{lo} each time. The rows are calculated as follows: "matches per day" by $2\lambda/250$, match wait-time by $250/(2\lambda)$, "sell wait-time" by $250/(2\lambda\mu_{hn})$, "h duration" by $250/\lambda_d$, "l duration" by $250/\lambda_u$, "L" by (9), "yield" by $1/P$ for P in (7), "liquidity yield spread" by $1/P - 1/P_f$ for P_f from (7), "misallocation" by μ_{lo}/s, and "reduction in s" from the change s.

In Table 1 the columns labeled "Initial" report (important characteristics of) the steady state prior to the liquidity shock. The columns labeled "Shocked" report the new steady state after the liquidity shock when there is no official intervention. The columns labeled "Official Intervention" report the steady state after the liquidity shock but when official intervention takes place.

The official intervention we have in mind are a series of consecutive lo-shocks – instantaneous official purchases of bonds from lo bond-sellers which reduce the overall supply of bonds. To be concrete, and in line with the information that was available to the market on the ECB's SMP program when it was active, we space these purchases five days apart (once-per-week) and conduct twelve purchases consecutively (about three months' worth), in a rough approximation to SMP purchases we actual saw (recall Figure 2). There is anecdotal evidence that the ECB was not in the market every weekday of the SMP period, and so our assumption is rough approximation to what actually occurred.

We assume that each SMP purchase buys *every bond being sold* at that time (that is, the *lo*-shock equals the size of μ_{lo}). In other words, we suppose that officials do *everything they can do* with their intervention into bond markets. In our simulations we aim to investigate how such purchases counteract a liquidity shock and evaluate to what extent our model intervention looks like the actual SMP experience.

Search Friction Shock

Consider Panel A of Table 1. Prior to the shock, a market participant matches with four agents a day, and is a high-type for one year or is a low-type for six months, on average. A bond-seller must search for 3.4 days on average to sell her bond. The bond's yield is 394 basis points, 44 of which is due to the liquidity premium of 11.1 percent. On average only 1.3 percent of the market is misallocated in the hands of *lo* agents looking to offload their bonds.

Let a search friction shock hit the market. Consider first the results without official intervention. In the new steady state a market participant waits more than 6.25 days on average to match with another agent; a bond-seller must search for more than two months (45.5 days) on average to sell her bond. The bond's yield rises to 529 basis points, 179 of which is due to the liquidity premium of 33.9 percent. After the shock, a sizeable 11.7

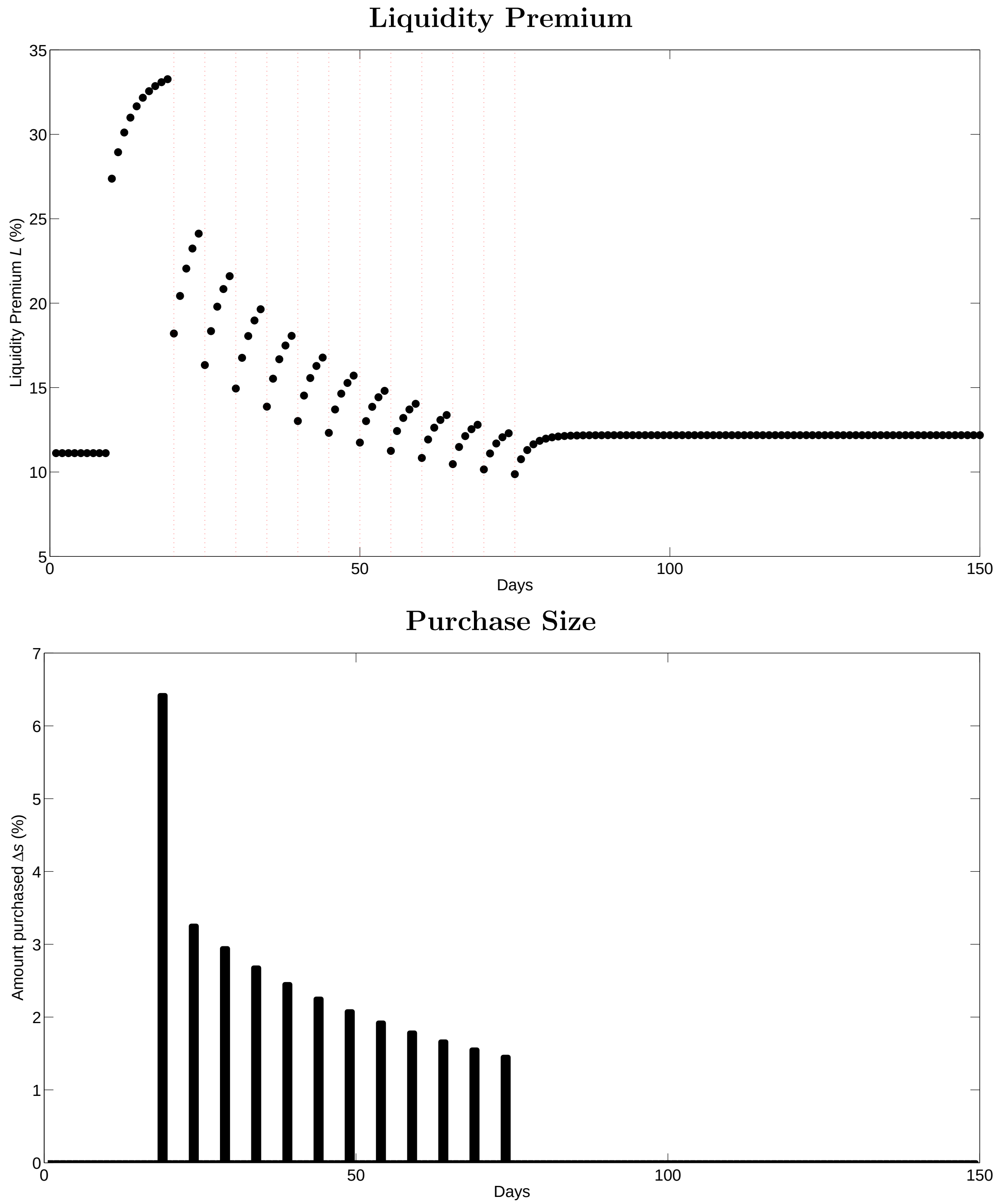

Figure 6: Liquidity Premium and Official Purchases: Search Friction Shock

Notes: The two panels show transition dynamics as the model transitions from the "Initial" steady state to the "Official Intervention" steady state of Table 1 Panel A. *In the top panel,* the liquidity premium for each day is marked by a black dot. The search friction shock ($\lambda \downarrow$) occurs on day 10. The first official purchase of bonds occurs on day 20. Days of official purchases $(20, 25, \ldots, 70, 75)$ are marked by dashed red lines. *In the bottom panel,* the size of each purchase (Δs) is shown.

percent of the market is misallocated.

Now suppose instead that officials intervene with SMP purchases after the shock. The dynamic response of the liquidity premium and size of each SMP purchase is shown in Figure 6. We assume the liquidity shock occurs on day 10, official intervention occurs on days $20, 25, \ldots, 70, 75$, and each SMP purchase buys every bond being sold at that time (all of μ_{lo}). In the top panel, each black dot indicates the liquidity premium for each day and the dotted red lines indicate days with SMP purchases. In the bottom panel, the black bars give the size of each individual SMP purchase as a percent of outstanding debt.

When the shock hits (day 10), the liquidity premium jumps from 11.1 to about 29 percent and rises to just under 35 percent by time officials start intervening. The first SMP purchase (day 20) is a bit more than six percent of the outstanding supply of bonds, which is roughly in line with the size of the first SMP purchases of Irish and Portuguese bonds (recall Figure 3). The liquidity premium falls about sixteen percentage points on impact. Thereafter, the premium rises until officials intervene again. The second SMP purchase (less than half the size of the first purchase) decreases the premium by about nine percentage points on impact (more than half the impact of the first purchase). Subsequent purchases are less than half the size of the first SMP purchase because (a) the steady-state μ_{lo} is decreasing as s decreases, and (b) the purchases' frequency is such that this pool never fills back up before the next purchase. However, by the end of the intervention, the last SMP purchase (about one-fourth the size of the first purchase) decreases the premium by only about two percentage points on impact (less than one-eighth the impact of the first purchase). In this case, the impact-per-purchase-amount varies over the course of the intervention.

The stream of official purchases lowers the liquidity premium with a distinctive up-down pattern. Purchase days push the premium down on impact, but the following days see the premium rise. In this model, every purchase leads to a steady state decrease in the liquidity premium. However, each purchase day pushes the premium temporarily lower than is sustainable (because the μ_{lo} pool is temporarily depleted) and so the liquidity premium rises when officials are not directly buying bonds. This is not due to the inefficacy of official intervention. Instead, the pattern represents the fact that official purchases both lower the steady-state liquidity premium and create transition dynamics as the market equilibrates.

After the official intervention is complete, Table 1 Panel A reports that the steady-state liquidity yield spread is close to where it was prior to the shock (49 versus 44 basis points) and bond-sellers are able to sell their bonds in a little over three weeks (16.2 days) on average. In this calibration the officials purchase 30.4 percent of the outstanding bonds in order to counteract the search friction shock, which is more than the amounts of Irish or Portuguese bonds (about 18 and 15 percent, respectively) we estimate the ECB to have actually purchased. However, the pattern of purchases in Figure 6 is similar to the pattern of purchases seen in the data (Figure 3), with large initial purchases followed by consecutive purchases tapering off.

Funding Risk Shock

Consider Panel B of Table 1. Prior to the shock, a market participant matches with four

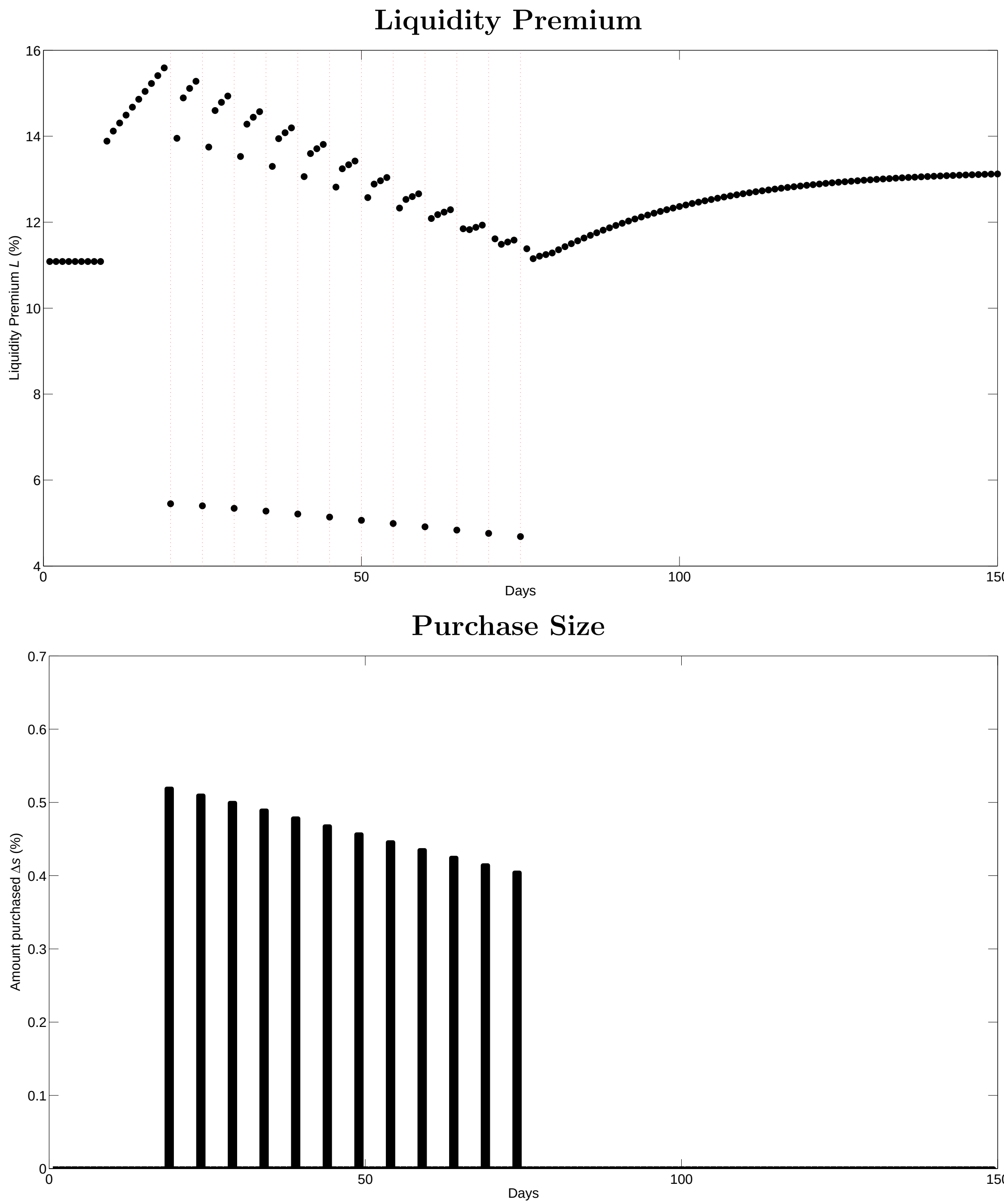

Figure 7: Liquidity Premium and Official Purchases: Funding Risk Shock
Notes: The two panels show transition dynamics as the model transitions from the "Initial" steady state to the "Official Intervention" steady state of Table 1 Panel A. *In the top panel,* the liquidity premium for each day is marked by a black dot. The funding risk shock ($\lambda_d \downarrow$) occurs on day 10. The first official purchase of bonds occurs on day 20. Days of official purchases $(20, 25, \ldots, 70, 75)$ are marked by dashed red lines. *In the bottom panel,* the size of each purchase (Δs) is shown.

other agents each day, and is a high-type for one year or is a low-type for six months, on average. A bond-seller must search for 3.4 days on average to sell her bond. The bond's yield is 394 basis points, 44 basis points of which is due to the liquidity premium of 11.1 percent. On average, only 0.7 percent of the market is misallocated in the hands of *lo* agents looking to offload their bonds.

Let a funding risk shock hit the market. Consider first the results without official intervention. In the new steady state a market participant remains a high-type for three months less than before (190 days); a bond-seller must search for almost two weeks (9.2 days) on average to sell her bond. The bond's yield rises to 495 basis points, 144 of which is due to the liquidity premium of 29.2 percent. After the shock, only about 1.5 percent of the market is misallocated.

Now suppose instead that officials intervene with SMP purchases after the shock. The dynamic response of the liquidity premium and size of each SMP purchase is shown in Figure 7. When the shock hits, the liquidity premium modestly increases from 11.1 to about 14 percent and rises to just under 16 percent by time officials start intervening. The first SMP purchase is about one-half percent of the outstanding supply of bonds, which is roughly in line with the size of the first SMP purchases of Italian and Spanish bonds (recall Figure 3). The liquidity premium falls about eleven percentage points on impact. The following day the premium bounces back about nine percentage points and thereafter rises until officials intervene again. The second SMP purchase is of a similar size to the first purchase and decreases the premium by a little more than ten percentage points on impact, about the same impact-per-purchase-amount as the first purchase. Subsequent purchases grow gradually smaller but their impact-per-purchase-amount remains more or less constant over the intervention.

The stream of official intervention lowers the liquidity premium with a distinctive up-down pattern. Purchase days always sharply drop the premium down on impact, but the following days see the premium bounce back. Each purchase day pushes the premium temporarily lower than is sustainable (because the μ_{lo} pool is temporarily depleted) and so the liquidity premium rises when officials are not directly buying bonds. As with the search friction shock, this pattern represents the fact that official purchases both lower the steady-state liquidity premium and also create transition dynamics as the market equilibrates.

After the official intervention is complete, Table 1 Panel B reports that the steady-state liquidity yield spread is close to where it was prior to the shock (53 versus 44 basis points) and bond-sellers are able to sell their bonds in 2.8 days on average. In this calibration the pattern of purchases in the simulation (Panel B of Figure 7) tapers more gradually than we see in the data (Figure 3). However, officials purchase only 5.5 percent of the outstanding bonds in order to counteract the funding shock, which is near the actual amount of Italian bonds (5.3 percent) and less than the amount of Spanish bonds (8.1 percent) we estimate the ECB to have purchased.

Assumptions in the Model

To make the economic mechanisms at play as clear as possible, our model is quite stylized.

A number of assumptions are made for the sake of clarity, and here we discuss how changing some of them would affect the framework.

First, we have abstracted from dealers, which were studied by DGP (2005) but not present in DGP (2007). None of the model's implications would change if dealers were introduced into the model as in DGP (2005).

Second, we assume that each official purchase is an unanticipated surprise. What is true from the financial press of the time is that the initial forays (May 2010, August 2011) into the markets were surprises. Subsequent weeks were also somewhat surprising – recall the ECB did not confirm it was buying anyone's bonds and admitted to doing so only after the fact via its public balance sheet declaration – but could have been anticipated. Modeling the expectation of future ECB purchases would add a layer of realism and complexity to the model. If agents correctly expected the future amount of purchases, the liquidity premia would fall by more when official purchases began, but the steady state effect would be unchanged. Empirically, this would have the effect of biasing our response estimates towards zero and work *against* finding a response of yields to purchases, which of course we do.

Discussion

In response to two types of liquidity shocks, official intervention that mimicks the ECB's actual SMP purchases are able to lower the liquidity premium back to roughly its pre-shock level. The cumulative sizes and pattern of our simulated interventions can match features of the data, for either the search friction or funding risk shock. If one thought that the actual European debt crisis involved a combination of the two, these features blend and the model predicts SMP purchases of a similar size and pattern to the data.

A robust feature of the model is a prediction that SMP purchases inevitably "overshoot." Liquidity premia fall sharply on impact when purchases are made. Thereafter, the premia rise while officials are not actively buying, even though the steady-state liquidity premium has been effectively reduced through the purchases. This is a simple outcome of the fact that purchases not only change the model's long run behavior, but also move prices and masses away from this new steady state, engendering transition dynamics. It is nearly always the case that these dynamics *increase* liquidity premia relative to the purchases' impact effect.

This is a general feature of models with frictions – shocks which change the steady state create transition dynamics to that new steady state. It is only a stark, frictionless model where one expects prices to immediately adjust to new long-run values. This being the case, the basic mechanism driving our official intervention's "overshoot" would likely be present in other mechanisms of liquidity risk so long as they rest on frictions that hinder economic agents from acting optimally.

Therefore, our model gives a clear rationale for why SMP purchases tapered off, were made repeatedly over consecutive weeks, and saw liquidity premiums (and peripheral bond spreads) rise when the ECB was not actively in the market. These features are not an indication that official intervention is ineffective. In the model, these features result from economic frictions requiring a successful intervention to both purchase bonds repeatedly and temporarily decrease the liquidity premium beyond its new long-run value.

3.3 Empirical Implications

We now complete the link between our theoretical model and the evidence in Section 2. We use our model to represent the behavior of investors in sovereign debt who eat coupon payments and reinvest face value payments back into new sovereign bonds. On the other hand, do not explicitly model CDS. Thus we make a key identifying assumption:

Assumption 1 *CDS liquidity premiums do not systematically vary in response to the bond market factors we have explicitly modeled.*

Our identifying assumption is realistic because search frictions for CDS are unlikely to vary systematically over the life of a bond. To see why, note that when a bond first enters the secondary market, institutional buyers usually take out CDS protection. Whereas search is required if the bond-holder later wants to sell their bond, "selling" CDS takes the form of cash settlement of their existing protection. No search is required because this occurs with the CDS-protection-seller with whom the bondholder has the CDS contract.

Letting the liquidity premium embedded in CDS be written $\tilde{L}$, we have that the difference between CDS and bond liquidity is

$$\hat{L}_t = L_t - \tilde{L}_t \tag{10}$$

with L_t the liquidity term in bonds and $\tilde{L}_t$ that in CDS. Our yield-curve-based empirical measure for $\hat{L}$ was described in Section 2.3. Given our Assumption 1, we interpret systematic fluctuations in $\hat{L}$ as reflecting variation in bonds' liquidity premium, as well as some noise.

A decrease in the bond liquidity premium L causes $\hat{L}$ to decrease. Hence our model makes the following predictions which we bring to the data:

1. $\hat{L}$ decreases in the long-run when SMP purchases take supply out of the market
2. $\hat{L}$ decreases more on impact than in the long-run
3. $\hat{L}$ decreases as the default intensity increases.

While the sign of the effect of SMP purchases on the liquidity premium is pinned down, the magnitude of the effect varies depending on features of the model. For instance, Section 3.2.2 shows that the impact-per-purchase-amount varied depending on both the type of liquidity shock hitting the market as well as the history of previous purchases. Therefore, the model predicts that the structural purchase effects are conditional on features of the data that are not observed. In order to take an agnostic stand on these unidentified features, we choose to not estimate the model structurally.[25] Our reduced-form linear specification estimates the average effects of SMP purchases, providing robustness in dimensions for which we do not have data.

4 Estimates

This section describes our maximum likelihood estimates. The reduced-form linear specification we employ is simple and approximates the structure dictated by our theory. Our

[25] In contrast, Feldhutter (2012) carried out structural maximum likelihood estimation of a variant of DGP (2005), but his question benefited from a greater detailed U.S. corporate bond transaction data set that allowed for the identification of the parameters.

main estimates come from generalized least squares, but our findings are robust to using ordinary least squares instead. Our estimates support the predictions of the model and show robust stock and flow effects of official bond purchases. We then summarize robustness checks of the main findings, which are detailed in the appendix.

4.1 Specification

With our panel data we estimate the equation

$$\Delta \hat{L}_t^j = \beta_{\text{Fix}}^j + \beta_1 \text{Flow}_t^j + \beta_2 \text{Flow}_{t-1}^j + \beta_{\text{Def}} \Delta \text{DefProb}_t^j + \beta_{\text{AR}} \Delta \hat{L}_{t-1}^j + \epsilon_t^j \tag{11}$$

Nations are indexed by the superscript j. We estimate equation (11) in first differences due to evident nonstationarity in the data. The dependent variable $\hat{L}_t^j$ in our main results is our preferred yield-curve-based bond liquidity measure that under our identifying Assumption 1 is a noisy measure of the bond liquidity premium; our results are robust to using the standard CDS-bond basis instead.

We measure SMP purchases in week t by Flow_t^j, converted to a proportion of the amount of country j bonds outstanding.[26] We include the contemporaneous and the first-order lagged effect of Flow_t^j, which enables us to distinguish temporary and permanent effects. We explored using additional lags beyond the first and found them to be insignificant, but we find our qualitative results to be robust to including more lags. We report the Wald statistic

[26] Note that by computing purchases over outstanding debt we take into account bond issuance and redemptions by euro-area sovereigns.

testing the null hypothesis $\beta_1 + \beta_2 = 0$ such that there are no lasting effects on bond liquidity from official purchases.

$\Delta\text{DefProb}_t^j$ is the change in the default probability for country j as derived from CDS prices, and approximately equals the log of the default intensity λ_D. Proposition 2 implies that the bond liquidity premium is decreasing in the default intensity and from our simulations we saw that its effects are roughly log-linear. It is noteworthy that previous empirical studies on the effects of official bond purchases have not included this variable, for a couple of good reasons. One: The periods or countries on which the previous research focuses often include little variation in conventional measures of this variable, and therefore its effects may have been negligible. Two: Without a structural model to rely upon, one might reasonably assume that subtracting bond price from CDS price cancels out the default probability. Indeed, the frictionless part of their prices contain the identical default discount. However, our model predicts that the liquidity premium varies with the default probability due to search frictions in the bond market.

We also include the past change in bond liquidity $\Delta\hat{L}_{t-1}^j$. The main reason to control for the past change in bond liquidity is due to the basic econometric problem of estimating a treatment effect. When doing so, one should consider the following: It could be that bond liquidity itself is a signal used by the ECB to conduct bond purchases in the first place. The intervention might be "timed" such that the liquidity after the intervention is better on average, even if there are no effects of the purchases themselves. If this is the case, we would expect β_{AR} to be negative and β_1, β_2 to be insignificant. In robustness checks, we find

Table 2: Estimation Results

β_1	β_2	β_{Def}	β_{AR}	R^2 (%)	$\beta_1 + \beta_2 = 0$ Wald statistic
−23.06	17.94	−3.49	−0.06	43.96	4.14
2.09	*2.77*	*0.28*	*0.05*		(0.042)
(0.000)	(0.000)	(0.000)	(0.232)		

Notes: Weekly data for January 2010 through April 2012 for Ireland, Portugal, Italy and Spain. Coefficients β_1, β_2 and β_{Def} reflect basis point responses to percentage point changes. Generalized least squares with country fixed-effects. Under the point estimates, heteroskedasticity-robust standard errors are reported in italics and p-values from t-statistics based on the standard normal in parentheses. Below the Wald statistic in parentheses is its p-value based on a $\chi(1)$ distribution. The panel regression is estimated using GLS with country fixed effects.

our results are unaffected by also including additional controls like the German rate and a European financial stress index.

Finally, we include country fixed-effects by means of the $\beta_{\text{Fix,j}}$ coefficients. Nevertheless, we may well be left with error terms ϵ_t^j which are correlated cross-sectionally and so MLE is GLS. We consistently estimate the errors' covariance matrix by the residuals from a first-step ordinary least squares (OLS) estimation. The residuals exhibit negligible autocorrelation and so we report White (1980) standard errors and t-statistic p-values.

4.2 Results

Table 2 reports our benchmark results. These use weekly data from January 2010 through April 2012 for Ireland, Portugal, Italy and Spain, using our yield-curve-based bond liquidity measure as the dependent variable. We focus on the five-year maturity so that our coefficient estimates tell us the average response of the five-year sovereign bond yield. We consider the five-year maturity because market participants report that official purchases were heaviest around this tenor, five-year sovereign CDS are the most liquidly traded, and it roughly captures the maturity preference of ECB purchases as revealed by the Greek debt restructuring.[27]

The estimates of β_1 and β_2 give our main finding. From the highly significant estimate for β_1, we find a 23.1 basis point impact decline of the liquidity premium due to a purchase of one percent of debt outstanding. From the estimate for β_2, we find that 17.9 basis points of this impact effect is temporary. The lasting stock effect is therefore 5.2 basis points per percentage point purchased, which the Wald test says is statistically significant at the 5% level. These estimated temporary and lasting effects correspond to our model's flow and stock effects, respectively.

The stock effect theoretically reflects the fact that ECB purchases permanently reduce the steady-state mass of agents who are stuck with the bond when they wish to sell. In the model it is possible to sharply drive down the bond liquidity premium temporarily by depleting the pool of sellers. We indeed find that the impact effect of ECB purchases is sizeable. Thereafter, the liquidity effects moderate to a modest effect on sovereign bond markets. Putting them into context, our estimates imply that, on average, an SMP purchase of ten percent of outstanding peripheral bonds would have led to a 231 basis point decrease in the liquidity premium on impact, and thereafter the premium would have bounced back 179 basis points, resulting in a long-run liquidity premium decrease of 52 basis points. This seems roughly in line with informal inspection of the evolution of liquidity premia presented

[27] Our results are robust to using nearby maturities instead.

in Figure 4, or simply the bond spreads themselves in response to official intervention.

Note that β_{AR} has a negative point estimate that is insignificant both economically and statistically. Even though we controlled for the possibility that officials' interventions may just have been well-timed, our estimates of SMP purchases' liquidity effects remain large and significant.

Finally, we find a significantly negative estimate for β_{Def}, as suggested by our model. For a one percentage point increase in the country's default probability, the liquidity premium falls 3.5 basis points. Previous studies have often excluded this variable from investigation. Nevertheless, our theory states that it holds a systematic role for bond liquidity and we find support for this in the data.

Altogether our estimates say that ECB intervention had a significant effect on bond market liquidity. Our empirical results benefited from the panel nature of our data set and the fact that European sovereigns have reasonably deep CDS markets that enabled us to get a good read on sovereign bonds' liquidity conditions. The interpretation of our estimates is aided by our theoretical model which describes a clear channel through which official purchases can have a temporary as well as a lasting effect.[28] The empirical evidence supports the theoretical predictions of our simple search-based asset-pricing model, in particular that official purchases and increasing default probabilities lead to decreases in sovereign bonds' liquidity premia.

[28] Nevertheless, our theory suggests that the stock effect is marginally diminishing, which was not captured by our linear empirical specification. Therefore, while our results suggest that past ECB purchases have had a permanent effect on bond market liquidity, care should be taken in extrapolating these effects forward to future potential purchases.

4.3 Robustness

We check the robustness of our main empirical results to a variety of alternative specifications: Using OLS, using the CDS-bond basis as our dependent variable, including additional controls, including additional lags of SMP purchases, including Greek data, and estimating the system in levels. For the sake of exposition, those results are relegated to Appendix A.3 and here we summarize the findings.

Across all alternatives we find that SMP purchases and the default probability significantly decrease bond liquidity premia on impact. In the specifications that difference the data, the flow effect is estimated to be about 19 to 23 basis points while the stock effect is about 3 to 10 basis points. In the levels specification, the flow effect is 7.8 basis points while the stock effect is 4.8 basis points. Across all alternatives a one percent increase in the probability of default leads to a statistically-significant 1.5 to 3.6 basis point reduction in liquidity premia. Broadly speaking, these alternative results support the robustness of our main findings.

5 Conclusion

We investigate how the European Central Bank's purchases of sovereign debt through its Securities Markets Programme have affected peripheral European bond yields, and in particular the liquidity premium component of yields. Following previous literature we consider the possibility of both *stock* and *flow* effects. Importantly, we provide a structural model to show how these arise, which also suggests a role for the probability of default to drive bond liquidity premia.

In our model, official purchases have a lasting effect by reducing the outstanding amount of bonds in the private market, in turn reducing the mass of agents waiting to sell due to search frictions. Official purchases may temporarily deplete this mass of agents, thus requiring the officials to wait with further purchases until more agents naturally look to sell the bond. This suggests that the ECB's repeated entry into sovereign bond markets was a necessary feature of their targeting liquidity premia, not an indication of the SMP's failure. Our empirical estimates find significant flow and stock effects that are consistent with the model's theoretical predictions.

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A Supplemental Appendices

A.1 Analytical Results

In this appendix we give the analytical proofs of the propositions in the main text.

Proof of Proposition 1:

First note that the steady state mass evolutions are unaffected by the introduction of λ_D into the model. Therefore, the first proposition in DGP (2005) applies directly.

Let τ_l denote the arrival (stopping time) of an intrinsic type shift, τ_i denote the arrival of a meeting with another agent and τ_D the arrival of the asset's default. Then the agents' value functions at time t are given by

$$
\begin{aligned}
V_{lo} &= E_t\Big[\int_t^{\tau_l\wedge\tau_i\wedge\tau_D} e^{-r(u-t)}(1-\delta)du + e^{-r(\tau_l-t)}V_{ho}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_i\wedge\tau_D\}} \\
&\qquad + e^{-r(\tau_i-t)}(V_{ln}+P)\mathbf{1}_{\{\tau_i=\tau_l\wedge\tau_i\wedge\tau_D\}} + e^{-r(\tau_D-t)}R(t)\mathbf{1}_{\{\tau_l\wedge\tau_i\wedge\tau_D\}}\Big] \\
V_{ln} &= E_t\Big[e^{-r(\tau_l-t)}V_{hn}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_D\}}\Big] \\
V_{ho} &= E_t\Big[\int_t^{\tau_l\wedge\tau_D} e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_D\}} + e^{-r(\tau_D-t)}R(t)\mathbf{1}_{\{\tau_D=\tau_l\wedge\tau_D\}}\Big] \\
V_{hn} &= E_t\Big[e^{-r(\tau_l-t)}V_{ln}\mathbf{1}_{\{\tau_l=\tau_l\wedge\tau_i\wedge\tau_D\}} + e^{-r(\tau_i-t)}(V_{ho}-P)\mathbf{1}_{\{\tau_i=\tau_l\wedge\tau_i\wedge\tau_D\}}\Big] \qquad \text{(A1)}
\end{aligned}
$$

where dependence on t has been subsumed. Using Leibniz's Rule, the first-order conditions in (4) follow.

Note that (4) is linear in the value functions and price, hence we have

$$\begin{pmatrix} 0 \\ \lambda_D R + 1 \\ 0 \\ \lambda_D R + (1-\delta) \\ 0 \end{pmatrix} = \begin{bmatrix} r+\lambda_u+\lambda_d & -\lambda_u & 0 & 0 & 0 \\ 0 & 0 & -\lambda_d & \lambda_d+\lambda_D+r & 0 \\ -\lambda_d & r+\lambda_d+2\lambda\mu_{lo} & 0 & -2\lambda\mu_{lo} & 2\lambda\mu_{lo} \\ -2\lambda\mu_{hn} & 0 & r+\lambda_u+2\lambda\mu_{hn}+\lambda_D & -\lambda_u & -2\lambda\mu_{hn} \\ -\tilde{q} & -q & \tilde{q} & q & -1 \end{bmatrix} \begin{pmatrix} V_{ln} \\ V_{hn} \\ V_{lo} \\ V_{ho} \\ P \end{pmatrix}$$

where $\tilde{q} \equiv 1 - q$. The solution yields (7).

QED

Proof of Proposition 2:

Begin by noting that the derivative of P_f with respect to s, λ, and δ is zero. Now see that $\partial P_f / \partial \lambda_D = (Rr-1)(\lambda_D + r)^{-2}$. By Condition 1 $R = \zeta / r < 1/r$ and so $Rr - 1$ is negative while $\lambda_D + r$ is positive, hence the derivative is negative.

Turning to L, first note that the asset is divided between low-type and high-type owners so that we have the identity $\mu_{ho} = s - \mu_{lo}$. Second, we have the identity $1 - \mu_{ln} - \mu_{ho} - \mu_{lo} = \mu_{hn}$. Third, in steady state the number of low-type agents is given by $\lambda_d(\lambda_d + \lambda_u)^{-1}$. Combining these three statements with the steady state condition $2\lambda\mu_{hn}\mu_{lo} + \lambda_u\mu_{lo} = \lambda_d\mu_{ho}$ we have the quadratic equation determining μ_{lo} given by $2\lambda\mu_{lo}^2 + (2\lambda\upsilon + \lambda_d + \lambda_u)\mu_{lo} - \lambda_d s = 0$ for $\upsilon = y - s$ and $y = \lambda_u(\lambda_d + \lambda_u)^{-1}$ as in DGP. Equivalently, we can instead solve for $\mu_{hn} = \upsilon + \mu_{lo}$ from

$$2\lambda\mu_{hn}^2 + (\lambda_d + \lambda_u - 2\lambda_\upsilon)\mu_{hn} - (\lambda_d + \lambda_u)\upsilon - \lambda_d s = 0 \tag{A2}$$

which will turn out to be convenient because $\lim_{\lambda\to\infty}\mu_{hn} = \upsilon \neq 0$ (by Condition 1) whereas $\lim_{\lambda\to\infty}\mu_{lo} = 0$. We will be using the derivatives of (A2) with respect to both s and λ:

$$\frac{\partial\mu_{lo}}{\partial s} = \frac{\lambda_d + 2\lambda\mu_{lo}}{4\lambda\mu_{lo} + 2\lambda\upsilon + \lambda_d + \lambda_u} \tag{A3}$$

$$\frac{\partial\mu_{hn}}{\partial s} = \frac{\partial\mu_{lo}}{\partial s} - 1 = \frac{-\lambda_u - 2\lambda\mu_{hn}}{4\lambda\mu_{hn} - 2\lambda\upsilon + \lambda_d + \lambda_u} \tag{A4}$$

$$\frac{\partial\mu_{hn}}{\partial\lambda} = -\frac{2\mu_{hn}(\mu_{hn} - \upsilon)}{4\lambda\mu_{hn} - 2\lambda\upsilon + \lambda_d + \lambda_u} \equiv -\gamma. \tag{A5}$$

Below we show that $\gamma > 0 \;\forall\; \lambda$. Moreover, let us introduce the notation

$$L \equiv \frac{\delta}{\Gamma_0}\frac{\Gamma_1}{\Gamma_2}, \quad \Gamma_{10} \equiv \frac{\Gamma_1}{\Gamma_0}, \quad \Gamma_{12} \equiv \frac{\Gamma_1}{\Gamma_2}.$$

We find that

$$\frac{\partial L}{\partial s} = \frac{\delta}{\Gamma_0}\left(\frac{\partial\Gamma_1}{\partial s}\Gamma_2^{-1} - \Gamma_{12}\frac{\partial\Gamma_2}{\partial s}\right).$$

Now see that $\partial\Gamma_1/\partial s = 2\lambda\tilde{q}(\partial\mu_{lo}/\partial s)$ and $\partial\Gamma_2/\partial s = 2\lambda\tilde{q}(\partial\mu_{lo}/\partial s) + 2\lambda q(\partial\mu_{hn}/\partial s)$. Plugging these into $\partial L/\partial s$ and rearranging terms yields

$$\frac{2\delta\lambda}{\Gamma_0\Gamma_2(4\lambda\mu_{lo} + 2\lambda\upsilon + \lambda_d + \lambda_u)}\left[(1-\Gamma_{12})\tilde{q}\lambda_d + \Gamma_{12}q(\lambda_u + 2\lambda\upsilon) + 2\lambda\mu_{lo}\left(\tilde{q}(1-\Gamma_{12}) + \Gamma_{12}q\right)\right].$$

To sign this expression, note that

$$\Gamma_{12} = \frac{2\lambda\tilde{q}(\mu_{hn} - \upsilon) + r\tilde{q} + \lambda_d + \lambda_D\tilde{q}}{2\lambda\tilde{q}(\mu_{hn} - \upsilon) + 2\lambda\tilde{q}(\upsilon - \tilde{q}\upsilon) + r + \lambda_d + \lambda_D + \lambda_u}.$$

Because $\mu_{hn} = \upsilon + \mu_{lo}$ and $\mu_{lo} \geq 0$ then it follows that $\mu_{hn} - \upsilon \geq 0$. As a sidenote, this shows that $\gamma \geq 0$. Because $q \in [0,1]$ then $\tilde{q} \in [0,1]$ and therefore $\upsilon - \tilde{q}\upsilon \geq 0$. Hence $\Gamma_{12} > 0$. To get a contradiction, assume that $\Gamma_{12} \geq 1$. But then

$$0 \geq 2\lambda\tilde{q}(\upsilon - \tilde{q}\upsilon) + qr + q\lambda_D + \lambda_u \quad \Rightarrow\Leftarrow .$$

Hence $\Gamma_{12} \in (0,1)$. Therefore $1 - \Gamma_{12}$ is positive and the expression for $\partial L/\partial s$ is positive.

The behavior of L for large λ can be directly inferred from DGP's (2007) first proposition and by noting that P_f is unchanged by λ.

Now see that

$$\frac{\partial L}{\partial \lambda_D} = \frac{\delta}{\Gamma_0\Gamma_2}\left[-\frac{\partial \Gamma_0}{\partial \lambda_D}\Gamma_{10} + \frac{\partial \Gamma_1}{\partial \lambda_D} - \frac{\partial \Gamma_2}{\partial \lambda_D}\Gamma_{10}\Gamma_2^{-1}\right]$$

and

$$\frac{\partial \Gamma_0}{\partial \lambda_D} = R, \quad \frac{\partial \Gamma_1}{\partial \lambda_D} = \tilde{q} + 2\lambda\tilde{q}\frac{\partial \mu_{lo}}{\partial \lambda_D} + 2\mu_{lo}\tilde{q}\frac{\partial \lambda}{\partial \lambda_D},$$

$$\frac{\partial \Gamma_2}{\partial \lambda_D} = 1 + 2\lambda\tilde{q}\frac{\partial \mu_{lo}}{\partial \lambda_D} + 2\mu_{lo}\tilde{q}\frac{\partial \lambda}{\partial \lambda_D} + 2\lambda q\frac{\partial \mu_{hn}}{\partial \lambda_D} + 2\mu_{hn}q\frac{\partial \lambda}{\partial \lambda_D}.$$

Before proceeding, note that (A2) implies that

$$\begin{aligned}
\mu_{hn} &= \frac{-(\lambda_d+\lambda_u-2\lambda\upsilon)+\sqrt{(\lambda_d+\lambda_u-2\lambda\upsilon)^2+4(2\lambda)([\lambda_d+\lambda_u]\upsilon+\lambda_d s)}}{2(2\lambda)} \\
&= \frac{\lambda^{-1}(-\lambda_d-\lambda_u)+2\upsilon+\sqrt{4\upsilon^2+R_1(\lambda^{-1})+R_2(\lambda^{-2})}}{4} \\
&\xrightarrow[\lambda\to\infty]{} \frac{4\upsilon}{4}=\upsilon.
\end{aligned}$$

where the second line follows from multiplying by $1=\lambda^{-1}/\lambda^{-1}$ where R_1 is $O(\lambda^{-1})$ and R_2 is $O(\lambda^{-2})$. Limits are therefore finite and the result follows. We can therefore write

$$\frac{\partial L}{\partial \lambda_D}=\frac{\delta}{\Gamma_0\Gamma_2}\left[\tilde{q}-\Gamma_{10}R-\Gamma_{12}+2\left(\mu_{hn}-\upsilon\right)\tilde{q}\eta\left(1-\Gamma_{12}\right)-2\lambda\eta\tilde{q}\gamma\left(1-\Gamma_{12}\right)-2\Gamma_{12}q\eta\left(\mu_{hn}-\lambda\gamma\right)\right] \tag{A6}$$

which is difficult to sign in general when η is nonzero since the fifth and sixth terms are of the opposite sign as the fourth term regardless of the value of η. In the main part of Proposition 2 we have $\eta=0$ and therefore the sign is dictated by $\tilde{q}-\Gamma_{10}R-\Gamma_{12}$. We know that Γ_{12} is positive. However, note

$$\Gamma_{10}R=\frac{\tilde{q}\lambda_D+r\tilde{q}+\lambda_d+2\lambda\mu_{lo}\tilde{q}}{\lambda_D+1/R}=\tilde{q}\frac{\lambda_D+r+\lambda_d/\tilde{q}+2\lambda\mu_{lo}}{\lambda_D+1/R} \tag{A7}$$

and this is not generally-speaking above or below $\tilde{q}$. Hence our inability to sign $\partial L/\partial\lambda_D$ absent further parametric restrictions. However, it is possible to look directly at L and note that it is of the form $O(\lambda)/O(\lambda^2) = O(\lambda^{-1})$ and therefore $L \xrightarrow[\lambda\to\infty]{} 0$.

When $\lambda(\lambda_D)$ is bounded, then L is still of the form $O(\lambda)/O(\lambda^2) = O(\lambda^{-1})$ which is what we needed above in our calculation of λ_D's effects on L and P_f. Those results therefore hold even when λ is a bounded function of λ_D.

To see the sufficiency of the Condition 2's statement $\frac{\lambda_d}{1-q} + \lambda\mu_{lo} > (\frac{1}{\zeta} - 1)r$, consider the derivative $\partial L/\partial\lambda_D$ in (A6) and the expression (A7). $\Gamma_{10}R$ is greater than $\tilde{q}$ when $(\lambda_D + r + \lambda_d\tilde{q} + 2\lambda\mu_{lo})(\lambda_D + 1/R)^{-1}$ is greater than unity. This implies

$$\begin{aligned} r + \frac{\lambda_d}{\tilde{q}} + 2\lambda\mu_{lo} &> 1/R \\ \frac{\lambda_d}{\tilde{q}} + 2\lambda\mu_{lo} &> (\frac{1}{\zeta} - 1)r \end{aligned} \tag{A8}$$

Therefore, when (A8) holds and $\eta = 0$, (A6) has a sign dictated by

$$-\tilde{q}(\Gamma_{10}R - 1) - \Gamma_{12},$$

that $\Gamma_{10}R - 1$ and Γ_{12} are positive, and therefore $\partial L/\partial\lambda_D < 0$.

The results for the price P follow directly from the above results for P_f and L.

QED

Condition 2:

For our data, risk-free rates have been quite low and inflation tame. Hence r will be no more than an annual rate of 0.05. Rules of thumb for bonds' recovery value hover around 40% of par, which implies that ζ is around 0.4. These values would imply the right hand side of the condition is 0.125.

The term $\lambda\mu_{lo}(\lambda)$ is $O(\lambda^{1/2})$ by the following argument. DGP's (2005) proof of their first proposition (characterizing μ_{lo} as the positive root of a quadratic equation) along with the identity $\mu_{hn} = \upsilon + \mu_{lo}$ gives us

$$\begin{aligned}\mu_{hn} &= \frac{-(\lambda_d + \lambda_u - 2\lambda\upsilon) + \sqrt{(\lambda_d + \lambda_u + 2\lambda\upsilon)^2 + 8\lambda\lambda_d s}}{4\lambda} \\ &= \frac{-\lambda_d - \lambda_u}{4\lambda} + \frac{\upsilon}{2} + \frac{\lambda_d + \lambda_u}{4\lambda} + \frac{\upsilon}{2} + \frac{R_1(\sqrt{\lambda})}{4\lambda}\end{aligned}$$

where $R_1(\sqrt{\lambda})$ is $O(\sqrt{\lambda})$. Combining this with the terms gives us

$$\mu_{hn} = \upsilon + R_2(\lambda^{-1/2})$$

where, considering R_1, we can see that $\sqrt{\lambda}R_2(\lambda^{-1/2}) \neq 0$. Therefore, $\mu_{hn} = \upsilon + O(\lambda^{-1/2})$ and then $\mu_{lo} = O(\lambda^{-1/2})$. Thus $\lambda\mu_{lo}$ is $O(\lambda^{1/2})$ and nonzero. Hence the condition is satisfied for even extremely low values of λ like, e.g. 1, where the expected time between matches is about $250/(2 \times 1) = 125$ days. In every simulation shown in the paper (in all figures) we find $\lambda\mu_{lo}$ to be greater than unity.

Alternatively, if one is uncomfortable pinning down the matching intensity, we can instead satisfy the condition by choosing λ_d and q appropriately: These appear because λ_d helps control the steady state mass of low-type investors while the latter contains q the bargaining power of the low-types who are sellers of the asset.

A.2 Estimating Relative Liquidity

In this appendix we describe in detail our estimation approach for extracting probabilities of default and our measure of relative liquidity from sovereign bond prices and sovereign CDS spreads.

Duffie and Singleton (1999) show that one can value bonds that are subject to default and liquidity risk with a default- and liquidity-adjusted zero-coupon rate curve, $y_t(\tau)$;

$$y_t(\tau) = y_{rf,t}(\tau) + \lambda_{D,bonds}(t) \times (1 - R(t)) + L(t) \tag{A9}$$

In essence, a country's zero coupon rate at any maturity is defined as the sum of three components: (i) the risk-free rate, (ii) the product of the hazard rate for default at time t and the fractional loss upon such a default (defined as 1 minus the recovery rate) and (iii) liquidity effects. It is the possibility to identify these different portions of bond yields that motivated the construction of our theoretical model in the main text and allows us to estimate the effects of ECB purchases on bond liquidity. The precise technical description of our estimation procedure is given in the two sections below.

The top two panels of Figure A1 shows the breakdown of, e.g., the Italian and Spanish yield curve on two representative days in our sample into the German yield curve, which we take to be the risk free component (more on this below), and a combined component which encompasses both the default risk on the bonds and our object of interest, the bond liquidity premium. We can make similar pictures for the term structure of CDS spreads for any given day. Unlike Duffie and Singleton (1999) and Pan and Singleton (2008), who simultaneously combine the information from the cross-section as well as the time-series dimension, we extract our estimates of default and liquidity on a day-to-day basis, by separately fitting each day the cross section of sovereign bond prices and sovereign CDS prices. In both fitting procedures we follow Jarrow and Turnbull (1995) and account for the presence of a default event arrival process by means of a Poisson counting process. A default event is characterized as the first event of this counting process which occurs at some time t^* with a probability defined as

$$P[t^* < t + dt | t^* \geq t] = \lambda_{D,t}(dt)dt \tag{A10}$$

This implies that the probability of the default event, occurring within the time interval $[t,\ t+dt)$, conditional on no default having occurred till time t, is proportional to the hazard rate function $\lambda_{D,t}(dt)$ and the length of that time interval, dt. We assume that the hazard rate is a horizon-*dependent* function. For bond prices we assume it is a continuous function, as this better fits with our approach of extracting zero-coupon curves from bond prices, while for CDS spreads we assume it is a piece-wise flat step function as is more commonly done.[29] Nothing changes if we instead assume both bond and CDS spreads have continuous hazard rate functions.

While an easier approach would be to assume that the hazard rate is constant over any

[29]Although we make the hazard rate horizon-dependent, we do assume that it is independent of interest rates and recovery rates.

horizon, i.e. $\lambda_{D,t}(\tau) \equiv \lambda_{D,t}$, we deem such an assumption unnecessarily restrictive, and counterintuitive. One can think of obvious examples why agents would think it is more likely that a country is to default between, say, two and three years from now compared to between today and a year from now, for example if the country has large bond redemption obligations due in two years time and it seems unlikely that the government will be able to meet those at that time.[30]

Before we go into the details of the estimation, we first address the issue of the recovery rate. Our eventual estimate of the relative liquidity premium depends crucially on the scale of the default probability. Since the default risk premium in both bonds and CDS depends on the probability of default and the recovery rate conditional upon default, we can only identify the default rate up to a scaling constant. Throughout the paper we assume the recovery rate to be equal to 40% and the red lines in the bottom two panels of Figure A2 show the estimated default probabilities under this assumption. However, the bottom two panels of Figure A1 also show the default probability that we would estimate for Italy and Spain under different assumptions for the recovery rate, in particular for alternative recovery rate values of 20% and 60% (the black and blue lines, respectively). High recovery rates are associated with high default probabilities and vice versa. Choosing a different recovery rate seems to mainly level shift default probabilities and will therefore not materially impact our relative liquidity measure.

[30]Another approach at making the hazard rate time-varying is proposed in Andritsky (2004) who specifies the hazard rate as a Gumbell distribution and uses it to extract default probabilities from Argentine Eurobonds.

A.2.1 Extracting Default Probabilities from Bond Prices

For each day in the sample, and for each of the countries in our panel separately, we estimate the zero-coupon curve that is embedded in the cross section of available sovereign bond prices for that particular day. We do so by using a modified version of the parametric curve-fitting approach of Nelson and Siegel (1987). This method is very popular among central banks for estimating daily zero coupon rates, see BIS (2005), and often used in practice as well. Assuming that bond prices are default free, which is our assumption for the German bunds that we choose as use our risk free assets in (A9), Nelson and Siegel (1987) stipulate that the zero coupon curve is governed by the following functional form:

$$y_{rf,t}(\tau) = \beta_{1,t} + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{\tau}{c_t}\right)}{\left(\frac{\tau}{c_t}\right)}\right] + \beta_{3,t}\left[\frac{1-\exp\left(-\frac{\tau}{c_t}\right)}{\left(\frac{\tau}{c_t}\right)} - \exp\left(-\frac{\tau}{c_t}\right)\right] \tag{A11}$$

with $y_t(\tau)$ being the time-t zero yield for maturity τ. The Nelson Siegel curve depends on the parameters $\beta_{t,1}$, $\beta_{t,2}$ and $\beta_{t,3}$, while c_t is a constant associated with the equation. The different parts of (A11) are designed to capture the well-known level, slope and curvature which are typically present in term structure data.[31]

We take zero coupon rates derived from German bunds as our risk free asset for the Euro Area. German bunds are arguably default-free and extremely liquidly traded and it is natural to think of an identical marginal bond investor being active in the various European

[31] Note that most central banks that report their zero-coupon curve estimates to the BIS, do not directly use the Nelson-Siegel specification, but instead use the Svensson (1994, 1995) extension which adds a second curvature factor to (A11). This additional curvature factor comes with an additional parameter $\beta_{4,t}$ and has a second constant associated with it, see Svensson (1995) for more details. The German Bundesbank uses the Svensson-Nelson-Siegel model as well, but for ease of exposition we focus solely on the original Nelson-Siegel specification here.

sovereign bond markets due to Europe's shared currency and payment systems. Therefore, we use German debt to back out default/liquidity amalgams from the European sovereign bonds in our sample.

When estimating zero coupon curves for the countries in our sample, we have to account for the fact that these countries bonds' are priced in the market taking into account the probability that the sovereigns issuing these bonds can default on their debt obligations. We do so by using the recovery of face value (RFV; see Duffie, 1998) formulation which stipulates that when default occurs, a bond holder will no longer receive any coupons, but does receive a fractional recovery of the face value on his bonds upon default, with the recovery rate assumed to be equal across all bonds. We value the bond using a binomial tree model which at any point in time weighs the two possible outcomes of default/no default and the accompanying cashflow payments by the (conditional) probability of each outcome occurring. The (discrete) probability of not experiencing a default from time t to a future time point t^* equals $P[t^* > t] = \exp\left(\sum_{\tau=t}^{t^*} \lambda_{D,bonds,t}(\tau)\Delta\tau\right)$. We discount the cashflows using German zero-coupon rates as the risk free term structure and we specify the probability of default/no default during a given period by the Poisson probability in (A10), where we assume a recovery rate of 40%. We specify the time-varying hazard rate for bonds, $\lambda_{D,bonds}(t)$, using the same Nelson-Siegel parametric form as in (A11), but we take the exponent to ensure that it is positive everywhere on its domain; $\lambda_{D,bonds,t}(\tau) = \exp\left(\lambda^*_{D,bonds,t}(\tau)\right)$ with[32]

$$\lambda^*_{D,bonds,t}(\tau) = \delta_1 + \delta_2 \left[\frac{1 - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right)}{\left(\frac{\tau}{c_{\lambda,t}}\right)} \right] + \delta_3 \left[\frac{1 - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right)}{\left(\frac{\tau}{c_{\lambda,t}}\right)} - \exp\left(-\frac{\tau}{c_{\lambda,t}}\right) \right] \quad \text{(A12)}$$

We obtain the default probability estimates by minimizing the sum of squared bond pricing errors and optimizing over the parameters of the hazard rate function.

A.2.2 Extracting Default Probabilities from CDS spreads

We estimate the term structure of default probabilities from CDS spreads by sequentially building this curve up from CDS contracts with increasingly longer maturities. Using a bootstrap approach, we do so by equating the time-t present value of the CDS premium leg to that of the protection leg. The premium leg of a CDS contract is the series of payments of the CDS spread, $Spr_{CDS}(\tau)$, made until maturity τ of the contract or until default occurs. The protection leg is the payment of $(1-r)$ percent of the face value F of the bond upon default. At time t we therefore set

$$\begin{aligned} Price_{PremiumLeg,t} &= Price_{ProtectionLeg,t} \\ F \times Spr_{CDS}(\tau) \times \sum_{k=t}^{\tau-1} B_{t,k} \times P[t^* > k] &= (1 - R(t)) \times F \times \sum_{k=t}^{\tau-1} B(t,k) \times (P[t^* > k] - P[t^* > k+1]) \end{aligned} \quad \text{(A13)}$$

[32]Estimating the constant $c_{\lambda,t}$ alongside the parameters adds additional nonlinearity to the hazard rate curve and complicates the estimation procedure. Therefore, similar as in Diebold and Li (2006) for estimating zero-coupon curves for the U.S., we fix $c_{\lambda,t}$ to a pre-determined constant such that in this case the curvature factor in the hazard rate function reaches it maximum loading at a maturity of 18 months. Our results do not crucially depend on this assumption, however, as different values for the constant yielded comparable results.

where $B(t, k)$ is the time-t discount factor for a k-period horizon. We use euro-area swap rates for these. CDS protection on a European sovereign bond position provided by a typical dealer would arguably reduce the credit risk on such a bond position to roughly that of lending in the swap market. Because of the credit risk of the counterparty dealer selling protection, the default risk on the CDS-protected bond position would presumably not be as as low as that of sovereign bonds. By solving (A13) we estimate the CDS-implied hazard rate (i.e. probability of default) for horizon $\tau = k$. We use the bootstrap methodology to iteratively estimate probabilities of default for increasingly longer horizons, using a piecewise flat hazard rate function and assuming a quarterly CDS premium payment frequency.

CDS protection on a European sovereign bond position provided by a typical dealer would arguably reduce the credit risk on such a bond position to roughly that of lending in the swap market. Because of the credit risk of the counterparty dealer selling protection, the default risk on the CDS-protected bond position would presumably not be as as low as that of sovereign bonds.

Figure A2 plot the default probabilities for Ireland, Italy, Portugal and Spain assuming a forty percent recovery value, the industry standard.

A.3 Robustness

In this Appendix, we present results of a sensitivity analysis on our results that we presented in Table 2 in the main text. The alternative specifications vary as follows.

Specification 1 uses OLS instead of GLS, to explore the sensitivity of our results to the observation-weighting entailed by GLS.

Specification 2 uses the CDS-bond basis (at the five-year maturity) as the dependent variable, instead of our yield-curve-based liquidity measure. For reference, a time-series plots of these two dependent variables for the various countries are provided in Figure A3

Specification 3 returns to using our yield-curve-based liquidity measure, and includes additional control variables. In particular, we include the lagged euro-wide discount rate (the German yield) as well as a composite financial stress index constructed by the Federal Reserve Board. The composite financial stress index is taken as a principal component of equity market, corporate bond market, term spread, and funding market condition variables.

Specification 4 includes additional lags (each of which is found to be insignificant) of $\Delta\hat{L}$ into the regression. To gain precision, we estimate β_{34} on the variable $\Delta\hat{L}_{t-2}+\Delta\hat{L}_{t-3}$ (which is equivalent to imposing that the coefficients β_3 on $\Delta\hat{L}_{t-2}$ and β_4 on $\Delta\hat{L}_{t-3}$ are the same) because this term is significant (t-stat $= 2.22$) and positive $\hat{\beta}_{34} = 4.49$). In this case, the Wald statistic tests the null hypothesis $H_0 : \beta_1 + \beta_2 + \beta_{34} = 0$.

Specification 5 includes Greece in the panel. In this specification it is crucial to use our yield-curve-based liquidity measure because CDS-bond basis is extremely erratic after 2010 and leads all estimates except β_{AR} to be insignificant.

Table A1: Sensitivity Analysis

	β_1	β_2	β_{Def}	β_{AR}	R^2 (%)	$\beta_1 + \beta_2 = 0$ Wald statistic	Notes
(1)	−20.01	15.3	−3.36	−0.07	39.32	2.66	OLS
	2.24	*2.64*	*0.25*	*0.05*		(0.103)	
	(0.000)	(0.000)	(0.000)	(0.104)			
(2)	−19.45	14.47	−3.37	−0.10	41.16	4.83	CDS-bond basis
	2.42	*2.08*	*0.30*	*0.06*		(0.029)	
	(0.000)	(0.000)	(0.000)	(0.114)			
(3)	−22.61	17.07	−3.56	−0.06	44.40	4.67	Controls
	2.18	*2.88*	*0.29*	*0.05*		(0.031)	
	(0.000)	(0.000)	(0.000)	(0.183)			
(4)	−22.86	13.10	−3.44	−0.07	44.86	4.39	Additional lags
	2.06	*2.79*	*0.27*	*0.05*		(0.037)	$H_0 : \beta_1 + \beta_2 + \beta_{34} = 0$
	(0.000)	(0.000)	(0.000)	(0.025)			$\hat{\beta}_{34} = 4.5$ (2.2)
(5)	−19.51	16.56	−3.58	−0.10	43.04	2.64	Include Greece
	2.57	*3.26*	*0.28*	*0.05*		(0.105)	
	(0.000)	(0.000)	(0.000)	(0.056)			
(6)	−7.77	3.01	−2.30	0.61	79.12	2.53	Levels
	2.17	*0.59*	*0.29*	*0.04*		(0.112)	Controls
	(0.000)	(0.000)	(0.000)	(0.000)			GLS
(7)	−8.30	2.25	−1.54	0.65	81.82	3.79	Levels
	2.25	*0.491*	*0.21*	*0.04*		(0.052)	Controls
	(0.000)	(0.000)	(0.000)	(0.000)			OLS

Notes: Weekly data for January 2010 through April 2012 for Ireland, Portugal, Italy and Spain (and Greece in specification 5). Coefficients β_1, β_2 and β_{Def} reflect basis point responses to percentage point changes. Generalized least squares with country fixed-effects. Under the point estimates, heteroskedasticity-robust standard errors are reported in italics and t-statistic p-values based on the standard normal in parentheses. Below the Wald statistic in parentheses is its p-value based on a $\chi(1)$ distribution. Specifications vary as follows: (1) uses OLS instead of GLS. (2) has as the dependent variable the CDS-bond basis instead of our yield-curve-based liquidity measure. (3) includes the lagged change in the German yield and a composite financial stress index as additional controls, using GLS and the yield-curve-based liquidity measure. (4) includes additional lags (each of which is found to be insignificant) and here the Wald statistic tests the null hypothesis $H_0 : \beta_1 + \beta_2 + \beta_{34} = 0$. (5) includes Greece. (6) and (7) estimate the system in levels, where now β_2 multiplies the (lagged) stock of SMP purchases, the default probability enters in levels, and additional controls (the German yield, a composite financial stress index, and country-specific trends) are included; (6) reports GLS and (7) reports OLS.

Specifications 6 and 7 estimate the system in levels. To make this compatible to our difference-based approach, now β_2 multiplies the stock of (lagged) SMP purchases and the default probability enters in levels. Additional controls are included in an attempt to soak up some of the dependent variable's trending behavior; these controls are the lagged German yield, a composite financial stress index, and country-specific trends. Specification 6 reports GLS and specification 7 reports OLS results.

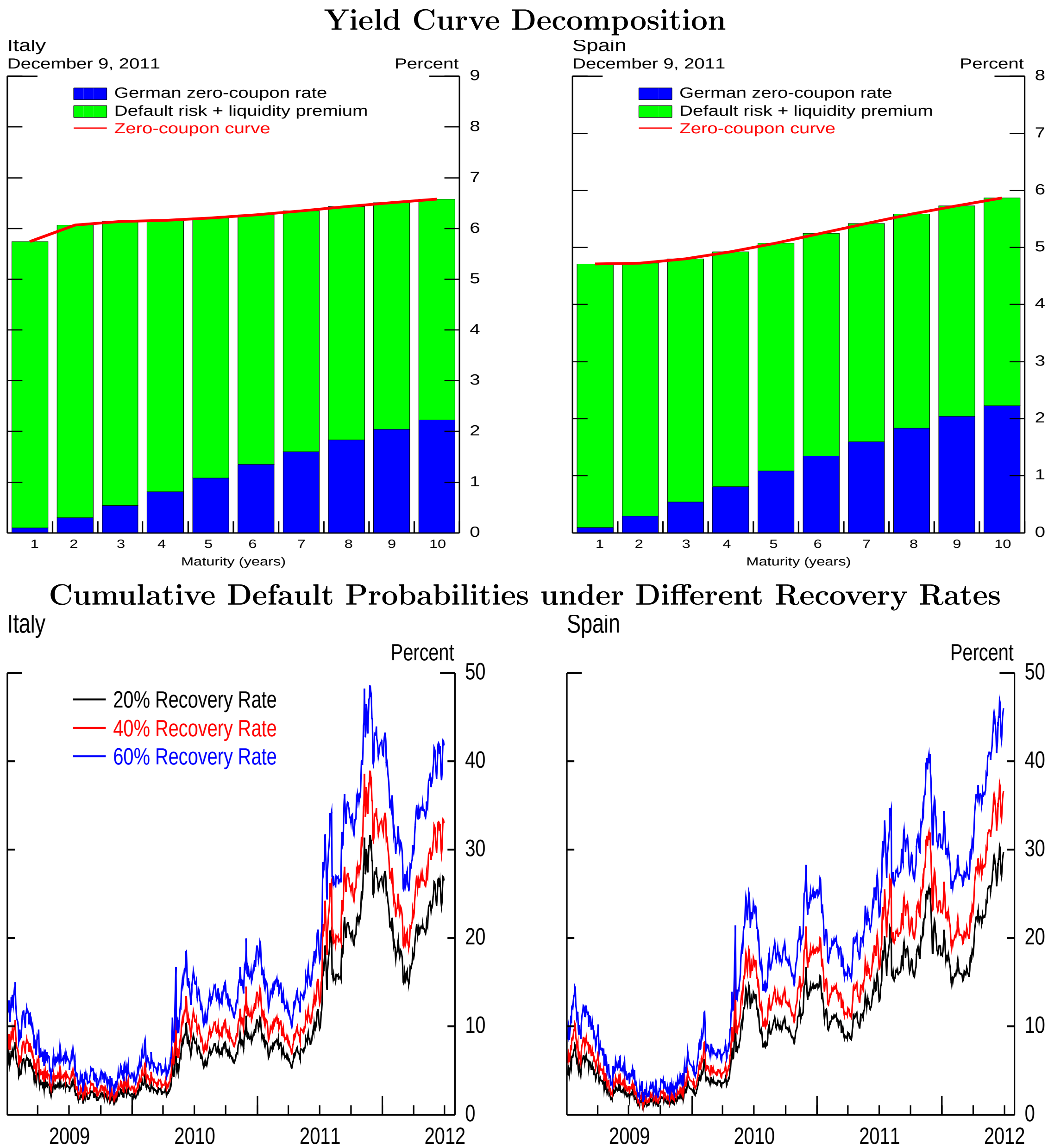

Figure A1: Italian and Spanish Yield Curve Decomposition and Default Probabilities under Various Recovery Rates

Notes: The top panels show the estimated zero coupon yield curve and its decomposition for a representative day in our sample (December 9, 2011) for Italy and Spain. The zero coupon curve (the red line) is the sum of the German zero coupon yield curve (obtained from the Bundesbank, the blue bars) and the sum of default risk and a liquidity premium (the green bars). The bottom panel shows time-series estimates (obtained from CDS spreads, as described in A.2.2) of the cumulative probability that Italy and Spain will default within the 10 years under different assumptions on the recovery rate upon default (under a 20% recovery rate in black, under a 40% recovery rate in red, and under a 60% recovery rate in blue).

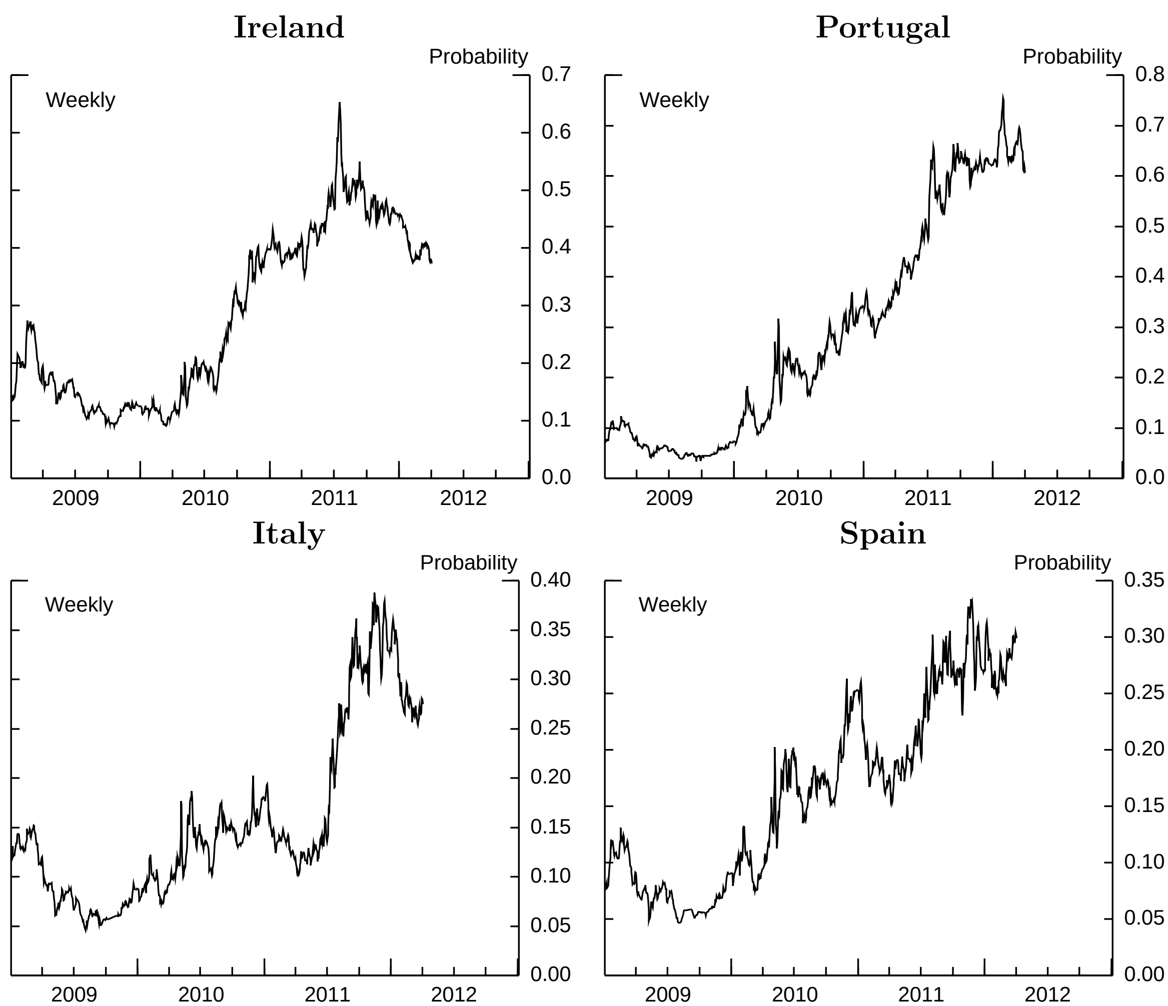

Figure A2: Sovereign Default Probabilities derived from CDS

Notes: Calculated from five-year CDS prices as described in text, assuming a forty percent recovery value.

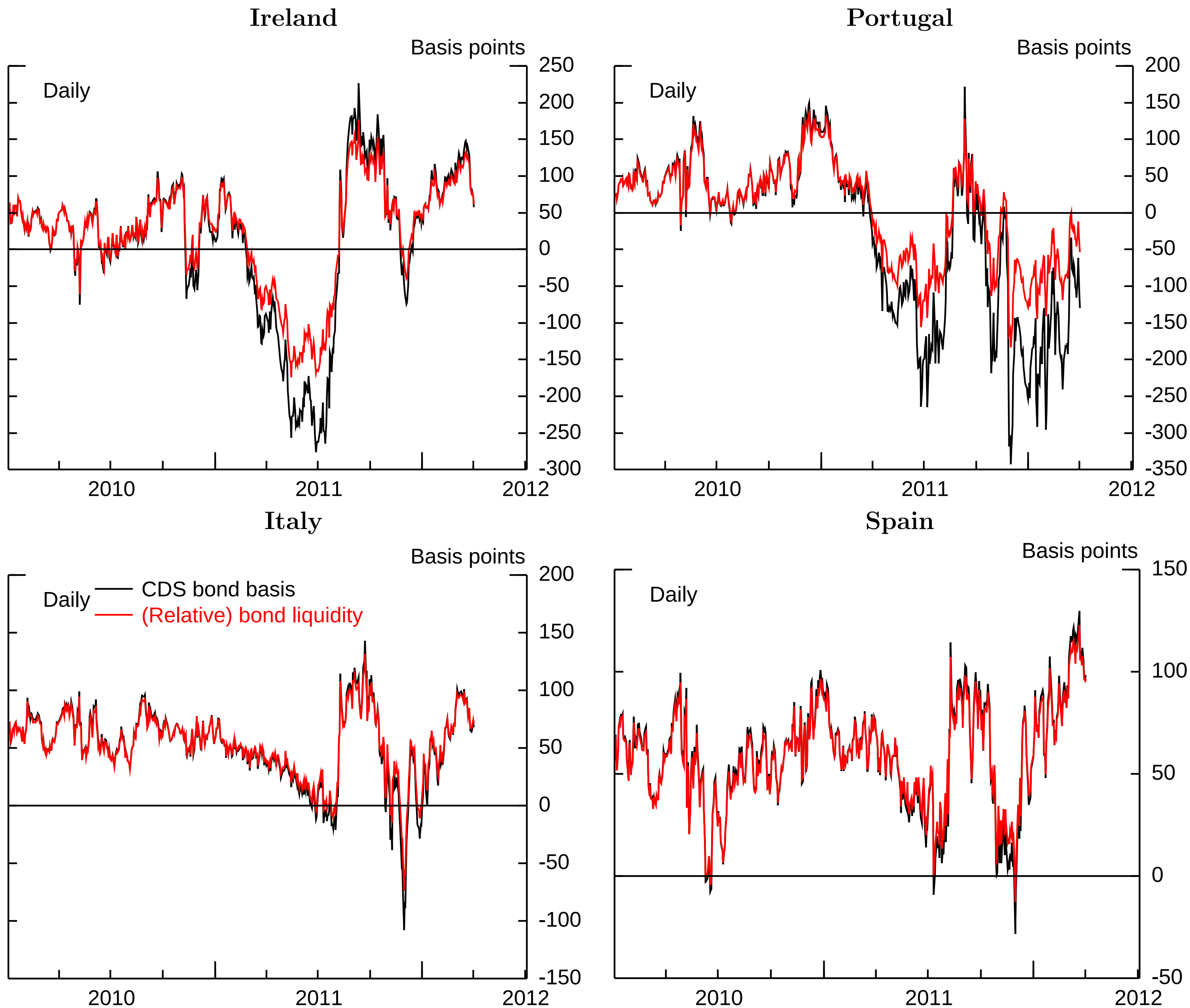

Figure A3: YIELD-CURVE-BASED LIQUIDITY VERSUS CDS-BOND BASIS

Notes: Scatterplot of our yield-curve-based bond liquidity measure (in red) versus the CDS-bond basis (in black). Over our data sample: Ireland, Italy, Portugal and Spain, weekly, from January 2010 to April 2012.