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FORWARD EXCHANGE RATES AS ESTIMATES OF EXCHANGE RATE EXPECTATIONS: A COMMENT

by

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Forward Exchange Rates as Estimates of Exchange Rate Expectations: A Comment*

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In a recent note[1/] Jeremy Siegel applied the statistical theorem -- that the expectation of a positive random variable with positive variance will be greater than the reciprocal of the expectation of the inverse of the random variable -- to demonstrate that

> Even in the simple case where investors are risk-neutral, ... , it can be shown that the forward price of the foreign currency is not an unbiased estimator of the anticipated future price. (304)

In particular, he found from theoretical considerations alone that

> the expected value of the anticipated future spot price [of foreign currency in terms of domestic currency] is greater than the forward price .... (304)

Unfortunately, his analysis contains several problems such that whether or not this particular result is correct, it is not implied by his argument. The procedure in this comment will be to set forth what I believe to be the correct analysis and then to summarize the problems in the first section of Siegel's paper.

*The author has benefited from discussions with Russell Boyer, Lance Girton, Dale Henderson, and Donald Tucker on various points in this paper.

1/ "Risk, Interest, and Forward Exchange," Quarterly Journal of Economics, LXXXVI (May 1972).

The analysis will be based upon a two region model of the world with two currencies. To simplify terminology, call the domestic country's currency "pounds" and the foreign country's currency "dollars." The spot price of dollars will be represented by c_o and the forward price of dollars to be delivered at time t is c_t^f. Let e_o and e_t^f be the spot and forward price of pounds in terms of dollars where $e_o = (c_o)^{-1}$ and $e_t^f = (c_t^f)^{-1}$. Let c_t $(= e_t^{-1})$ be a random variable with a positive variance and a subjective probability function that is attributed to all participants in the exchange markets. c_t is the spot rate that is anticipated at time t in the future.

It is useful to summarize the relationship between E(e), E(c) and the market forward rate graphically. A rectangular hyperbola that represents the equations $e_t c_t = 1$ and $e_t^f c_t^f = 1$ is drawn in Figure I. The curve also serves as a means of inverting $E(e_t)$ and $E(c_t)$ to obtain $(E(e_t))^{-1}$ and $(E(c_t))^{-1}$. We know from the statistical theorem[1/] mentioned earlier that $E(c_t) > (E(e_t))^{-1}$ as shown in Figure I. The locations of the expected values and the distance between A and B depend upon the particular density function assumed. Roughly speaking, the smaller the variance the closer A and B are to one another.

1/ A discussion of this theorem with references to early proofs of it can be found in Joseph L. Fleiss, "A Note on the Expectation of the Reciprocal of a Random Variable," The American Statistician, XX (February 1966). The theorem also follows directly from Jensen's inequality.

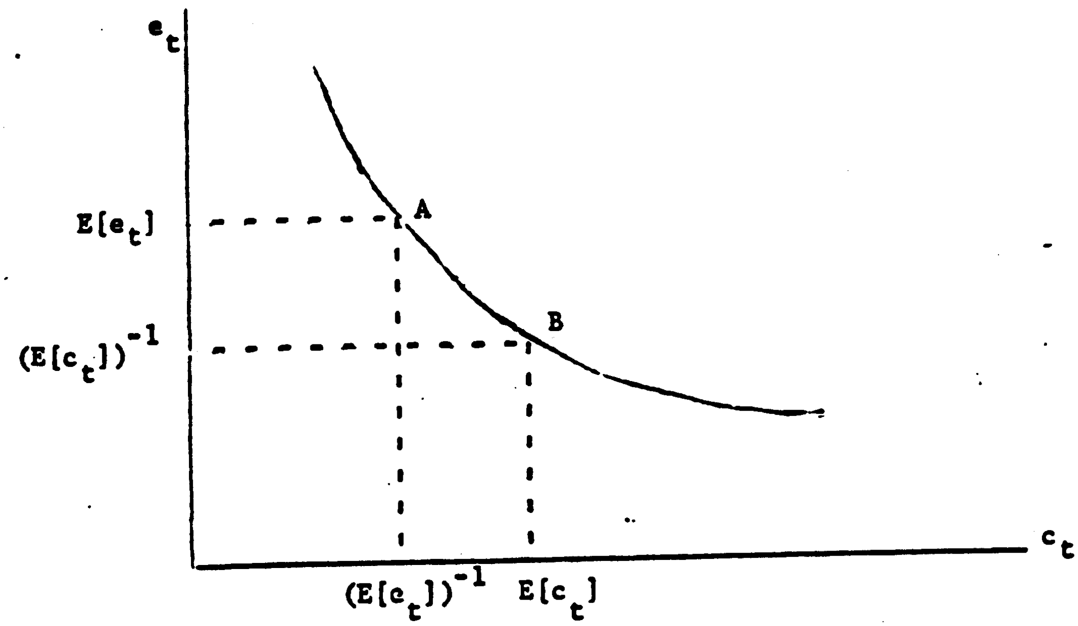

Figure I: Expected Value Relationships

Since A and B diverge, the forward rate cannot be equal to <u>both</u> $E(e_t)$ and $E(c_t)$. Consequently, the forward rate must be a biased estimate of the expected value of the anticipated future spot price of at least one currency. This conclusion is not an indication of any imperfections in the market and it is not based upon <u>any</u> assumptions about the behavior of market participants towards risk. It is the result of the fact that $E(e)$ and $E(\dot{c})$ are calculated in such a way that one is not the reciprocal of the other.

Although the forward rate must be a biased estimate of at least one of the expected values, it is useful to examine the economic behavior that would tend to make the forward rate converge to either of the expected values. This behavior can be identified by asking whether some market participants will employ one expected value rather than the other in their decision making. It will continue to be assumed that all market participants share the same subjective probability function concerning the anticipated future spot rate at time t and, following Siegel, the additional assumption of risk-neutrality will be imposed upon the participants.

Consider an investor who plans to convert D_t dollars into pounds at time t. He has the option to sell dollars forward or to wait until time t and sell them spot. His anticipated gain from selling dollars forward compared to selling them spot at time t is $(c_t^f - c_t)D_t$ and the expected value of this gain is

$$\left[\int (c_t^f - c_t) f(c_t) dc_t \right] D_t = \left[c_t^f - E(c_t) \right] D_t$$

where $f(c_t)$ is his subjective probability distribution and the integration is over the range of c_t. If the investor is neutral towards exchange rate risk, he will cover by selling dollars forward if the expected gain is positive and he will take an open position if $c_t^f - E(c_t) < 0$.

On the other hand, consider a foreign resident with pound liabilities, P_t, that he wants to service or retire at time t with dollars. The expected value of his loss from covering in the forward market compared with buying pounds spot in the future is

$$\left[\int (e_t^f - e_t) g(e_t) de_t \right] P_t = \left[e^f - E(e_t) \right] P_t$$

where $g(e_t) = f(c_t)|dc_t/de_t| = f(c_t)$. He will cover in the forward market if $e_t^f < E(e_t)$ and will buy pounds spot at time t if $e_t^f > E(e_t)$.

In summary, whether a person is converting pounds into dollars or vice versa, and whether a person holds an asset or a liability that he is considering covering, he will compare E(c) with the forward rate if the quantity of dollars is predetermined (i.e., independent of the value of the anticipated future spot rate) and he will use E(e) if the quantity of pounds is given. The market participants who compare $E(e_t)$ with e_t^f to determine whether or not the hedge provide pressure on e_t^f to stay in line with $E(e_t)$ on point A in Figure I.[1/] Similarly, those who use the criterion $c_t^f \gtrless E(c_t)$ put pressure on the forward rate to move toward $E(c_t)$ or point B.

As interesting as it may be to work out the implications of market participants using E(e) or E(c), these must be regarded as polar cases. In general, both the dollar and pound values of a transaction would be affected by the value of the anticipated future spot rate (in which case D_t and P_t would have to be included within the integration in the previous formulas). One could certainly find real world examples in which a person had, say, a liability fixed in terms of one currency such that the other-currency cost of retiring the liability would be calculated as a residual. However, in an abstract model free of money illusion, both the foreign and domestic currency values of a typical foreign exchange transaction would generally vary with the exchange rate.

1/ The interested reader can go through both the case in which a transactor is converting pounds into dollars and the case in which he is converting dollars into pounds to see that the decision to cover or not puts pressure on e_t^f to stay in line with $E(e_t)$.

Criticism of Siegel's Analysis

Siegel's result, that $E(c_t) > c_t^f$, is based upon an argument that can be usefully broken down (from a logical as opposed to a sequential standpoint) into two parts. In the first part he pointed out that E(e) is not equal to the reciprocal of E(c). The implications of this inequality have been explored above. The second part, which I think is incorrect, is an argument that the forward rate is equal to E(c).

Siegel's proof that $e_t^f = E(e_t)$ follows from two equations. Risk-neutral investors are willing to take upon positions such that they are "indifferent between the yields from holding either domestic or foreign securities" (304). Consequently, Siegel writes

(1) $\exp(r_t^d - r_t^f)t = E(c_o/c_t) = c_oE(1/c_t)$

where r_t^d and r_t^f represent the domestic and foreign interest rates with terms to maturity of time t. Siegel writes the arbitrage condition as

(2) $\exp(r_t^d - r_t^f)t = c_o/c_t^f$.

Equations (1) and (2) implies that $E(1/c_t) = 1/c_t^f$ or, in the notation of this paper, that $E(e_t) = e_t^f$.

However, equations (1) and (2) are incorrectly written. For instance, (2) says that the premium on domestic assets must equal the forward premium (rather than the forward discount) on pounds. (1) has a similar error.[1/] The mistake is easy to correct in (2) by inverting either the

1/ This same mistake is also present in his equations (6), (7), (7a), (8), (10), (13), and (14). The mistakes in equations (2) and (8) canceled out in his derivation equation (12) such that his interesting Figure II, which is derived from (12), is correct. He apparently corrected his arbitrage condition (2) before plotting it in his Figure I.

right on left hand side of the equation. But there are two corrected versions of (1), viz.,

(1a) $\exp(r_t^f - r_t^d)t = E(c_o/c_t) = c_oE(1/c_t)$ and

(1b) $\exp(r_t^d - r_t^f)t = E(c_t/c_o) = E(c_t)/c_o$.

Equations (1a) and (1b) depict the consequences of those market participants who make decisions to hedge or to remain open on the basis of E(e) and E(c), respectively.[1/]

In conjunction with (the corrected but unwritten version of) (2), (1a) implies that $E(e_t) = e_t^f$ and (1b) implies that $E(c_t) = c_t^f$. In terms of Figure I, (1a) holds if the forward rate is at point A and (1b) holds if the forward rate is at B. Both of these equations cannot be valid and it is not clear which equation Siegel meant to write. Since (1a) yields Siegel's conclusion that $c_t^f < E(c_t)$,[2/] let us proceed with the assumption that he meant to write (1a) and ask why it should hold rather than (1b).

1/ Equations (1a) and (1b) reflect the behavior of investors with portfolio allocations that include domestic and foreign assets. However, there are many market participants other than investors who need to decide whether to cover themselves or take an open position. Consequently, in the first part of this note the decision criteria for covering was developed in more general terms without reference to domestic and foreign interest rates.

2/ Siegel argues that his conclusion is contingent upon the way in which prices are quoted. In his words

> It should be noted that, if we quote prices as domestic currency in terms of foreign currency, this seeming paradox would fail to materialize since $E(e_t/e_o) = E(e_t)/e_o$ for $e = 1/c$. (304)

(In this quotation Siegel's notation "C" has been replaced with "e". The "seeming paradox" to which he refers is the surprising ease with which he obtains his strong result, $c_t^f < E(c_t)$.) When one carries out the substitution suggested in the quote, equation (1a) becomes

(1a') $\exp(r_t^f - r_t^d)t = E(e_t/e_o) = E(e_t)/e_o$.

It should be quite clear that (1a') differs from (1a) only in nomenclature. The use of e rather than c is for notational convenience only; it does not reflect different economic behavior.

The only assumption that differentiates Siegel's two regions and, therefore, might provide a possible reason why the forward rate should be forced toward E(e) rather than E(c) is that the domestic country is small relative to the rest of the world. Siegel did not suggest why this assumption might be crucial for his argument that $e_t^f = E(e_t)$. However, one line of reasoning might go as follows: _If_ foreign residents tend to dominate the pound-dollar market (and I don't think they would) and _if_ foreign residents used E(e) in their dicision rules, then (1a) might hold such that $E(e_t) = e_t^f$. A rationale behind the second "if" is that foreign residents make all their plans in terms of foreign currency such that they first decide on the dollar value of a foreign exchange transaction and the pound value comes out, as it were, as a residual. That Siegel had this sort of rationale in mind is suggested by his statement that "the natural numeriare is the unit of domestic currency" (304). However, since he was implicitly referring to domestic residents, this quote is relevant to (1b) rather than (1a). But, of course, (1b) implies that $c_t^f = E(c_t)$ which is contrary to Siegel's conclusion that $c_t^f < E(c_t)$.

But whether Siegel meant to write (1a) or (1b), the argument in the first part of this comment concluded that the use of E(e) or E(c) represented atypical polar cases in which the pound or the dollar value of the foreign exchange transaction was given independently of the exchange rate. And even in the polar cases, an example was given earlier in which the resident of the foreign country regarded the foreign currency value of a transaction as independent of the exchange rate. Consequently, residency should not be closely associated with equations (1a) and (1b) and, typically, neither equation should be expected to hold.

Conclusion

It has been argued that the decision to hedge or take an open position is influenced by the expected value of (the anticipated future spot price of) the domestic or the foreign currency if the domestic or foreign currency value of the transaction is determined independently of the exchange rate. Typically, however, both the domestic and foreign currency values of a foreign exchange transaction are influenced by the exchange rate such that neither expected value is solely applicable. The question "Is the forward rate an unbiased estimate of the anticipated future spot rate?" is ill conceived if it presupposes one expected value as the measure of the markets' anticipation of the exchange rate to prevail in the future.