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SDRs, INTEREST AND THE AID LINK: FURTHER ANALYSIS

by

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SDRs, Interest and the Aid Link: Further Analysis*

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In a recent article, John Williamson advocated a monetary reform package that includes both a development assistance link and a competitive interest rate on SDRs.[1/] In his view, the interest rate on SDRs should be raised to a level comparable to the interest rate on short-term dollar assets in order to make SDRs attractive substitutes for reserve currency holdings. He recognizes that charging a higher rate of interest on the net use of SDRs would reduce the development assistance content of an aid link. Link benefits would "not be eliminated entirely, because few less-developed countries could hope to borrow long-term on international capital markets at short-term dollar interest rates ...."[2/] Williamson is concerned, however, that a competitive interest rate would increase the danger that recipients of link aid would default on interest payments.[3/] To deal with this potential problem, Williamson proposes that the linked SDRs be allocated under a modification of the procedure currently used to distribute new SDRs. His modification involves paying directly out of new link allocations of SDRs the interest due to the developed countries on SDRs accumulated as a result of the prior use of link aid by less developed countries (LDCs).[4/]

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1/ John Williamson, "SDRs, Interest, and the Aid Link," Banca Nazionale del Lavoro Quarterly Review, June 1972, pp. 199-205.

2/ Ibid., p. 202.

3/ Ibid.

4/ Ibid., p. 204.

This note has three purposes. (1) We consider explicitly the manner in which the net transfer of resources for development would decline over time with a link under (a) the normal net use procedure for assessing interest and (b) our interpretation of the modified procedure that Williamson proposes. (2) We examine the implications of these two procedures. (3) We argue that interest payments by LDCs on their use of SDRs allocated under an aid link could be subsidized to increase the development assistance content of the link without impairing the relative attractiveness of the SDR as a reserve asset.

For our purposes, the LDCs can be treated as a unit. Suppose that S_t units of SDRs are allocated for link purposes in year t. These SDRs are available to the LDCs both to use in payment of interest (I_t) on their use of SDRs previously allocated under the link, and to spend for purposes of acquiring real resources (T_t). On the assumption that the LDCs do not use link aid to supplement their reserves[5/] and under the assumption that the real purchasing power of the SDR remains constant

$$T_t = S_t - I_t.$$

The major difference between the two schemes under comparison is the manner in which interest is calculated. Under the normal net use procedure for assessing interest, which we refer to

5/ The conslusions of our analysis are not affected by this assumption, which is made for expositional convenience. It is also unnecessary in our analyais to consider whether or not link aid is channeled to the LDCs through development finance institutions.

as the indirect interest scheme, the interest due in year t is

$$I_t = r\ (S_1 + S_2 + \ldots + S_{t-1})$$

where r is the constant annual interest rate charged to the users of link aid. The aid link under the indirect interest scheme provides in year t for a net transfer of real resources of

$$T_t = S_t - r \sum_{k=1}^{t-1} S_k . \qquad (1)$$

Under Williamson's modified procedure for assessing interest, which we refer to as the direct interest scheme, the developed countries would receive directly out of new link allocations the interest due on the prior use of link aid by the LDCs. We interpret the Williamson proposal to suggest that interest calculations would be based not upon the sum of all previous SDRs allocated for link purposes (the S_t), but rather upon the sum of the previous link allocations of SDRs actually used by LDCs to acquire real resources (the T_t)[6/]. On this basis, the interest payment in year t would be

$$I_t = r\ (T_1 + T_2 + \ldots + T_{t-1}).$$

Equivalently, $I_t = r \sum_{k=1}^{t-1} (S_k - I_k) = r \sum_{k=1}^{t-1} S_k - r \sum_{k=1}^{t-1} I_k .$

6/ This formulation adds a distinction between bases for the calculation of interest to Williamson's distinction between direct and indirect payment of interest to the developed countries. Williamson may not have had the former distinction in mind. Without the former distinction, however, Williamson's modification would only involve an administrative change which, as we show below, would only avoid the default problem for a limited number of years.

Thus, for a fixed time pattern of link allocations, the two schemes differ in that the direct interest scheme does not involve charging interest on interest, the last term in the expression above. The aid link under the direct interest scheme provides in year t for a net transfer of real resources of[7/]

$$T_t = S_t - r \sum_{k=1}^{t-1} (1-r)^{t-1-k} S_k. \qquad (2)$$

For the case in which link allocations of SDRs (S_t) take place at a constant rate (C) of, say, $1 billion in each year, expressions (1) and (2) simplify respectively to

$$T_t = C[1-r(t-1)] \qquad (1a)$$

and[8/]

$$T_t = C(1-r)^{t-1}. \qquad (2a)$$

Using the above analytical framework, we can note several implications of the alternative schemes. For the simple case of a constant rate of link allocations under the indirect interest scheme, the permitted net transfer of real resources to LDCs diminishes to zero in year $t_o = 1+1/r$. After that year, the interest due on previous link allocations exceeds the volume of new allocations.[9/] For the case in which link allocations increase over time, the permitted net transfer will, of course, decline less rapidly[10/]. As

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7/ The derivation of expression (2) can be found in the appendix.

8/ The derivation of expression (2a) can be found in the appendix.

9/ At a 5 percent interest rate, $t_o = 21$; at a 3 percent interest rate, $t_o = 34$: and at a 1-1/2 percent rate, $t_o = 68$.

10/ At a 5 percent interest rate and a 3 percent rate of growth of link allocations, the net transfer will turn negative after 32 years.

long as the interest rate exceeds, however, the rate of growth of link allocations, the net transfer will eventually turn negative. Such negative net transfers are likely to result with a competitive rate of interest on SDRs.

Consequently, an indirect interest scheme could lead to difficulties even greater than those explicitly recognized by Williamson. With a competitive interest rate, the LDCs would eventually have to transfer real resources to the developed countries in order to discharge the interest payments. This could be expected to lead to the familiar political problems associated with debt burdens.[11/] Simultaneously, it would accentuate the default problem about which Williamson is concerned.

In contrast, if link allocations were constant or increasing over time, a direct interest scheme not only would eliminate the default problem, but also would avoid requiring the LDCs to transfer real resources to the developed countries as a result of a link. For the simple case of constant link allocations, as illustrated in expression (2a), the permitted net transfer of real resources to LDCs approaches but never reaches zero.[12/] A further examination of the two interest schemes reveals that, for any time pattern of link

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11/ The need for such reverse transfers does not necessarily eliminate the overall economic benefits of link aid; see below.

12/ As illustrations: at a 5 percent interest rate, the net transfer shrinks to 5 percent of the first year's transfer after 59 years; at a 3 percent interest rate, this occurs after 99 years; at a 1-1/2 percent interest rate, this occurs after 200 years.

allocations, the direct interest scheme provides larger net transfers in every year than the indirect interest scheme.[13/]

Present discounted values provide a summary comparison of the benefits that the LDCs would receive under the alternative interest schemes. For the simple case in which link allocations are the same in each year, expressions (1a) and (2a) imply respective present discounted values of[14/]

$$PDV_{1a} = \frac{1+R}{R}\left(1-\frac{r}{R}\right)C \qquad (3)$$

and

$$PDV_{2a} = \frac{1+R}{R-r}C \qquad (4)$$

where the constant discount rate (R) can be viewed as the marginal opportunity cost of funds to the LDCs.

Under the indirect interest scheme the LDCs only benefit from a link if, in expression (3), the present discounted value of the link aid is positive. The present discounted value will be positive if the interest rate on SDRs is less than the marginal

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13/ The general expressions (1) and (2) show that the net transfer in year t under the direct scheme exceeds that under the indirect scheme by the amount $r \sum_{k=1}^{t-1} [1-(1-r)^{t-1-k}]S_k$, which is positive whenever the $(1-r)^{t-1-k}$ are less than one, or whenever $0 < r < 1$. In other words, the effective interest rate on link allocations in year t, defined as the ratio of interest (I_t) to the sum of all previous link allocations ($\sum_{k=1}^{t-1} S_k$), is lower under the direct interest scheme.

14/ The derivations of expressions (3) and (4) can be found in the appendix.

opportunity cost of funds to the LDCs. Under this condition, the link will benefit the LDCs, although after a certain number of years they will, as was mentioned earlier, have to transfer real resources to the developed countries to discharge their interest obligations. Under the direct interest scheme the LDCs always benefit from a link, as is revealed by the fact that the present discounted value in expression (4) is positive for any positive values of the interest rate on SDRs and the discount rate.15/

If the interest rate on SDRs is positive, the present discounted value of the link aid under the direct interest scheme will always exceed the present discounted value under the indirect interest scheme. The proportionate difference is given by the expression

$$\frac{PDV_{2a} - PDV_{1a}}{PDV_{1a}} = \frac{1}{(\frac{R}{r})^2 - 1} . \qquad (5)$$

For a marginal opportunity cost of funds to LDCs (R) of ten percent and an interest rate on SDRs (r) of five percent, the present discounted value of link aid under the direct interest scheme would exceed that under the indirect scheme by 33 percent. For an interest rate of three percent, the improvement would be ten percent; for an interest rate of one and one half percent, the improvement would be two percent.

15/ These conclusions also hold when link allocations increase over time.

It seems desirable to avoid the political problems that would arise under the indirect interest scheme when the net transfers of real resources to LDCs eventually became negative. This could be achieved without affecting the benefit, or present discounted value, of link aid, by substituting for the indirect scheme a direct interest scheme with an appropriately lower level of (constant) annual link allocations.[16/] For example with a discount rate of ten percent and an interest rate on SDRs of five percent, the appropriate link allocations under the direct interest scheme would be 75 percent of those under the indirect interest scheme.

If the LDCs were not required to pay interest on their use of link allocations, the two schemes would yield identical streams of net transfers. If a subsidized zero interest rate were charged on the use of link allocations, the LDCs would, of course, under either scheme receive a larger net benefit from link aid than if a competitive interest rate were charged. For a discount rate of ten percent, a reduction of the interest rate on SDRs to zero would increase the net benefit from link aid by 50 percent as compared with the net benefit under a direct interest scheme with an interest rate on SDRs of five percent. The increase would be 30 percent,

16/ In order to achieve equivalent benefits for individual LDCs, their discount rates must be identical. In order for equivalent benefits for the LDCs to imply equivalent costs to the developed countries, the discount rates for the two groups of countries must be identical.

if the reduction in the interest rate on SDRs were from three percent; and 15 percent, if the reduction were from one and one half percent.[17/]

These figures illustrate that the benefits of an aid link could be increased substantially by subsidizing the interest charged on LDC use of link allocations. Although Williamson argues convincingly that a competitive rate of interest on SDRs is desirable to reduce countries' incentives to shift their reserves from SDRs to reserve currencies, it is only the rate of interest on marginal changes in SDR holdings that is relevant to such incentives. If the rate of interest on SDRs were raised to a competitive level, the relative attractiveness of SDRs would not necessarily be impaired by forgiving the interest charges on LDC use of link allocations. For example, prespecified shares of link allocations could be assigned as norms to developed countries. Each developed country would be expected to earn its assigned share of linked SDRs, and would forego interest on this amount of its SDR accumulations. Those developed countries that failed to earn their assigned shares of linked SDRs would then have to pay the competitive interest rate on the shortfall to the countries that earned more than their shares.

This subsidization procedure would not require making a distinction between interest-bearing and non-interest-bearing SDRs. Under it, the default problem would, of course, not arise, although it would be necessary to insure that the link allocations were actually spent by the LDCs.

17/ The corresponding increases, as compared with the net benefit under an indirect interest scheme, would be 100 percent (for an original r = .05), 43 percent (for r = .03), and 18 percent (for r = .015).

In summary, we have examined two interest schemes that could be adopted under an aid link. Our focus has been on Williamson's criteria: avoidance of default and application of a competitive interest rate on SDRs. With a competitive interest rate on LDC use of link allocations the direct interest scheme involving the forgiving of interest on interest avoids the default problem, as long as link allocations are constant or increasing over time. Out of a given level of link allocations, it provides for a greater net transfer of real resources to LDCs than would the indirect interest scheme. Subsidization of the interest charged on LDC use of link allocations is also consistent with a competitive interest rate on the marginal use and accumulation of SDRs by all countries.

Appendix

Under the direct interest scheme, the interest subtracted from link allocations of SDRs in year t is

$$I_t = r \sum_{k=1}^{t-1} T_k .$$

Thus

$$T_t = S_t - r \sum_{k=1}^{t-1} T_k$$

$$= S_t - r\{ \sum_{k=1}^{t-2} T_k + S_{t-1} - r \sum_{k=1}^{t-2} T_k \}$$

$$= S_t - rS_{t-1} - r(1-r)\{ \sum_{k=1}^{t-3} T_k + S_{t-2} - r \sum_{k=1}^{t-3} T_k \},$$

which leads to

$$T_t = S_t - r \sum_{k=1}^{t-1} (1-r)^{t-1-k} S_k . \qquad (2)$$

Moreover, for the case in which $S_k = C$, a constant, in each year,[18/]

$$T_t = C - r \sum_{k=1}^{t-1} (1-r)^{t-1-k} C = C\{1 - r \sum_{j=o}^{t-2} (1-r)^j\}$$

$$= C\{1 - r[\frac{1-(1-r)^{t-1}}{1-(1-r)}]\},$$

which simplifies to

18/ For a review of the sums of the series that appear in this appendix, the reader is referred to George B. Thomas, Calculus and Analytic Geometry (Reading, Mass.: Addison-Wesley Publishing Company, Inc., Third Edition, 1960), Chapter 16.

$$T_t = C(1-r)^{t-1} \qquad (2a)$$

The present discounted value of the link allocations to the LDCs, measured in year 1, is defined as

$$PDV = \sum_{t=1}^{\infty} \frac{T_t}{(1+R)^{t-1}} .$$

For

$$T_t = C[1-r(t-1)] ,$$

$$PDV_{1a} = C \sum_{t=1}^{\infty} \frac{1-r(t-1)}{(1+R)^{t-1}} = C \left\{ \sum_{j=o}^{\infty} \frac{1}{(1+R)^j} - r \sum_{j=o}^{\infty} \frac{j}{(1+R)^j} \right\}$$

$$= C\left\{ \frac{1+r}{R} - r \left(\frac{1+R}{R^2}\right) \right\} ,$$

which simplifies to

$$PDV_{1a} = \frac{1+R}{R}\left(1 - \frac{r}{R}\right)C . \qquad (3)$$

Similarly, under (2a),

$$PDV_{2a} = C \sum_{t=1}^{\infty} \frac{(1-r)^{t-1}}{(1+R)^{t-1}} = C \sum_{j=o}^{\infty} \left(\frac{1-r}{1+R}\right)^j = \frac{C}{1 - \frac{1-r}{1+R}} ,$$

which simplifies to

$$PDV_{2a} = \frac{1+R}{R+r} C. \qquad (4)$$