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NASH-COURNOT EQUILIBRIUM FOR AN EXHAUSTIBLE RESOURCE LIKE OIL

by

Stephen W. Salant

ABSTRACT

of

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The theory of exhaustible resources is modified to take account of the industrial organization of the world oil industry. The oil cartel is viewed as a unified enterprise, engaging in a non-cooperative game against n firms in the non-cartel sector. Each of these n firms is assumed to own an equal share of the sector's oil; none, however, owns as much as the cartel. The Nash-Cournot equilibrium is examined. As the number of non-cartel firms increases, they behave like competitive price takers. The equilibrium converges to the standard "dominant firm model", adapted for exhaustible resources.

Several results in this paper have particular economic interest. Because its stock of oil is larger than that of any other single firm, the cartel has market power. It uses this power to continue selling after the other firms exhaust their supplies. Even if the competitive sector as a whole owns more oil, the cartel will out-last the other firms.

If the cartel were broken up, the competitive firms would lose money. In fact, the smaller firms gain more (in percentage terms) from the formation of the cartel than the members of the cartel themselves. These windfall profits of the smaller firms can be taxed away, however, without raising prices paid by consumers or affecting inter-temporal extraction rates. The tax revenue can be redistributed to the consumers.

The theoretical analysis abstracts from extraction costs and differences in the stock owned by each of the n "competitive" firms. However, such complications can be introduced into the framework for purposes of policy analysis.

Nash-Cournot Equilibrium for an Exhaustible Resource like Oil

by Stephen W. Salant*

I. Introduction

The current structure of the world's oil industry bears little resemblance to the structure assumed in the theoretical literature on exhaustible resources. There is neither a single cartel (or firm) which owns all the world's oil and thus has unchecked power to set prices over time; nor is there an abundance of measureless, "Mom-and-Pop" oil extractors dotting the globe. Instead, the industry contains one cartel with more power than any other individual extractor; but these other extractors do exist and have enough importance, individually and collectively, to restrain the full exercise of monopoly power.

In this paper, I modify the conventional theory of exhaustible resources so that it bears more resemblance to the current world oil industry. Since many extractive industries are "intermediate between monopoly and perfect competition"--as Hotelling[1/] observed in 1931--this modification may also be applied to other exhaustible resources.

The oil industry is assumed to be composed of firms in two sectors, each extracting oil at zero cost and selling it to consumers at the same price. One sector consists of the cartel, which acts in the coordinated manner of a single firm. The other sector consists of n firms. Each firm in the second sector is assumed to own an equal fraction of the sector's oil; none, however, owns as much as the cartel. The amounts owned by the two sectors are denoted, respectively, $\bar{I}^m$ and $\bar{I}^c$. The following diagram illustrates our assumptions:

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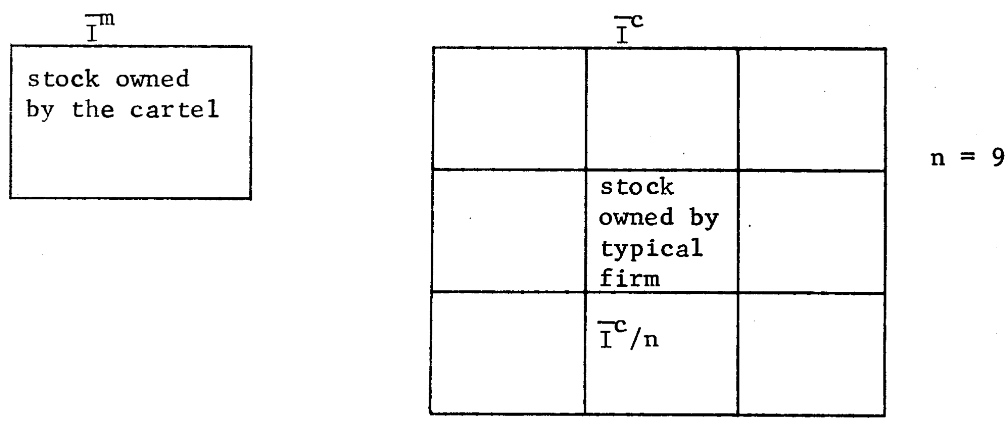
*/ This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

1/ Although Hotelling recognized the importance of oligopoly in extractive industries, he did not analyze the case in detail (JPE, April 1931, p. 171). Since his pioneering article, theoretical research on exhaustible resources has focussed exclusively on the rare extremes of monopoly and perfect competition.



$\bar{I}^c$ = stock owned by the non-cartel sector

$\bar{I}^c/n$ = stock owned by typical firm in that sector

$\bar{I}^c/n < \bar{I}^m$

Each firm is assumed to take the aggregate sales path of all the other firms as given. Each firm then selects its own sales path to maximize discounted profits. We consider the non-cooperative equilibrium where each firm's optimal strategy is what the others take as given. This model is analyzed in section IV. How extraction costs and asymmetries in the size of the n firms can be introduced is outlined in section V.

Throughout most of the paper, however, attention is focussed on the instructive, limiting case where the number of firms in the non-cartel sector is very large and, hence, their individual stocks of oil are very small. The small firms individually sell so little on each period that the revenue lost on inframarginal units from selling an additional unit is negligible when compared to the price received for it. Hence, these firms behave "competitively", taking the sequence of prices as given and choosing their own sales strategy to maximize discounted profits. The cartel, on the other hand, is large enough to consider the effect of its sales on price. It takes the sales of the competitive sector as given and chooses its own sales path to maximize discounted profits. The cartel, therefore, perceives that it is determining the price path.

Hence, for this limiting case, the non-cartel sector in effect takes the prices set by the cartel as given and chooses a sales path; and the cartel takes the sales of the competitive sector as given and chooses a price path. If the maximizing strategy of each sector is what the other took as given, neither sector has an incentive to alter its strategy. Because this solution concept was introduced by Cournot and refined by Nash, it is referred to as a Nash-Cournot equilibrium.

In section II, I describe a method to determine the non-cooperative equilibrium and then illustrate this technique with a specific example. Section III demonstrates diagrammatically various qualitative properties of the equilibria and concludes by discussing the relation of the dominant firm model to the extremes of pure competition and pure monopoly. Section IV considers the case where the non-cartel sector consists of a finite number of firms with an equal share of the sector's resource; it then shows how the previous results are the limiting case, when the number of non-cartel firms grows without bound.

II. Determining the Non-cooperative Equilibrium

Suppose the monopolist knows the stationary consumer demand curve, the sales path of the competitive sector and the stock of his own resource. The sales of the competitive sector include sales of competitive speculators who begin with nothing but are free to enter, purchase in any period, and sell a smaller or equal amount later. The monopolist's problem is to pick a path of prices, supported by his sales, to maximize his discounted revenues. Alternatively, he can be thought of as picking his sales path over time.

Since the monopolist takes the competitive sales path (sales of competitive extractors less speculative purchases) as given, he can deduct the given sales on each period from the consumer demand curve to obtain a sequence of excess demand curves. Each curve will indicate the amount of oil demanded of the monopolist, at a given price, in excess of what the competitors supply at that instant. Faced with these excess demand curves, the monopolist must choose a sales path which maximizes the sum of discounted revenues, but which does not exceed his initial inventory.

To accomplish this, the monopolist must compare the marginal revenue associated with selling another unit today to the discounted marginal revenue of selling it in the future. The marginal revenue associated with a sale by the monopolist of Q^m units is simply the price he would receive for selling another unit less the loss he would incur, because of the slight reduction in price, on the inframarginal units (Q^m) he sells:

$$M = P(Q^m + \overline{Q}^c) - Q^m \cdot a(P),$$

where

Q^m is the sales of the monopolist,

$\overline{Q}^c$ is the (given) sales of the competitive sector,

P is the price at which consumers would purchase $Q^m + \overline{Q}^c$ units,

M is the marginal revenue associated with the excess demand curve,

and $a(P)$ is the absolute value of the slope of the consumer demand curve (with respect to sales), expressed as a function of price.

Notice that this marginal revenue is derived from the <u>excess</u> demand curve faced by the monopolist.

If the strategy of the monopolist is optimal, it must generate marginal revenues of the same discounted value in all periods where the monopolist sells; once the monopolist stops selling, the discounted marginal revenue must never exceed the marginal revenue on any period of positive sales. Violation of either condition would provide opportunity for additional profits. Hence, along the optimal path, the marginal revenue grows at the rate of interest until sales cease; then it grows at a smaller (or equal) rate. Cumulated sales along the optimal path must, in addition, equal the monopolist's initial inventory; leaving some oil in the ground is never optimal.

Given the competitive sales path, the monopolist picks his sales path to maximize discounted profits. Alternatively, he can be viewed as selecting the optimal price path, supported by his sales. The price path he selects may have any shape, since the excess demand curves he faces in successive instants depend on the arbitrarily given competitive sales path. The price path may have regions where the price rises by more than the rate of interest or even falls absolutely. However, we must ask if all other agents in the model would react to that price path in the way assumed by the monopolist. If not, the price path does not result in non-cooperative equilibrium. To understand how the other agents will react, we turn to the competitor's problem.

The competitive sector takes the price path as given and knows its total stock. It chooses a sales strategy to maximize discounted profits without exceeding its initial inventory ($\overline{I}^c$). Since the competitive sector contains speculators who buy for re-sale--as well as extractors--we allow its "sales" to be negative (i.e., purchases) in some periods.

The competitor's optimization problem, like the monopolist's, is entirely conventional. The competitors will always exhaust their stock. As long as the price rises by the rate of interest, different ways of selling a given stock have the same discounted value. If the price rises by a greater amount between two periods, the competitive speculators would enter and attempt to make unlimited profits by buying in the first period and selling in the next. If the price begins to rise by less than the rate of interest, the competitive sector will find selling its inventory before this region is reached, to be optimal.

There are two ways to compute the non-cooperative equilibrium. The slow way involves trial and error. We pick a competitive sales path arbitrarily, figure out the price path the monopolist will set--given the arbitrary sales path--and then ask whether the competitors can make larger profits if they sell at the given prices different amounts over time than we arbitrarily assumed.[2/] If so, we have not found the equilibrium and must pick a (different) arbitrary sales path for the competitive sector.

A more sensible way to compute the non-cooperative equilibrium is to deduce its characteristics and eliminate all paths violating these conditions. Our two maximum problems give us enough information to characterize the equilibrium when it exists. Prices rise at the rate of interest as long as competitors hold stocks. Afterwards, they can rise at a smaller or equal rate. Prices can never rise by more than the rate of interest in equilibrium.

The marginal revenue derived from the excess demand curve must rise at the rate of interest, as long as the monopolist makes positive sales; when they are completed, the marginal revenue can rise at a smaller or equal rate. Each sector ultimately sells its entire stock.

To deduce the equilibrium price path, it is helpful to know which sector exhausts its supplies first.

<u>Proposition I</u>: As long as the monopolist has a positive initial inventory, he will conserve it so as to continue selling after the competitors exhaust their supplies.[3/]

2/ The Treasury Oil Study fails to ask this question. After arbitrarily assigning competitive sales over time, the Study shows that the monopolist should set a price path which declines over time. However, it is never observed that the competitors would react to this path in a different way than the monopolist assumed. Hence, the Study obtains the cheering, but incorrect result that a declining path of real prices can be an equilibrium. Even if the cartel <u>colluded</u> with the competitive sector, a declining price path would not result unless the consumer demand curve were non-stationary.

3/ On p. 27, this result is shown to hold for a small number of firms in the competitive sector, as long as each owns less stock than the cartel. The result also holds when marginal extraction costs are constant and equal for the two sectors.

<u>Proof</u>: Suppose the proposition were false and the monopolist completed his sales before the competitors. Then the price path would rise at the rate of interest while the two sectors coexisted and would continue to rise at that rate after the monopolist stopped selling.

Compare some early moment when the monopolist is selling to some later moment when his sales are zero. If his sales are positive, his marginal revenue will be less than the price; if his sales are zero, his marginal revenue will be equal to the price (since he has no inframarginal units on which to take losses). If the price grows at the rate of interest, the marginal revenue must grow by more; but this would give the monopolist an incentive to alter his strategy. Hence, in equilibrium, the competitors cannot continue selling after the monopolist drops out.

If both sectors begin with positive stocks, Proposition I implies that--for a period of time--they will both operate in the market simultaneously. When the price reaches some level, the competitors complete their sales and abandon the market to the monopolist.

Denote this "termination" price as P^*. Prior to the time P^* is reached, the two sectors coexist in the market; afterwards, the monopolist operates alone. This suggests a solution technique. We begin at P^* and the marginal revenue at that price associated with the consumer demand curve, $MR(P^*)$. Before P^* is reached, the price and marginal revenues grow at the interest rate; hence, u moments before termination of the first phase at P^*, the price $P(u,P^*)$ and marginal revenue $M(u,P^*)$ are:

$$P(u,P^*) = P^*e^{-ru}.$$

$$M(u,P^*) = MR(P^*)e^{-ru}.$$

The price and marginal revenue, u moments before termination at P^*, each depend on two variables: the sales of each sector at that time:

$$P(u,P^*) = P\{Q^m(u,P^*) + Q^c(u,P^*)\}.$$

$$M(u,P^*) = P\{Q^m(u,P^*) + Q^c(u,P^*)\} - Q^m(u,P^*) \cdot a(P(u,P^*)).$$

Substituting, we obtain:

$$P^* e^{-ru} = P\{Q^m(u,P^*) + Q^c(u,P^*)\} \tag{1}$$

$$MR(P^*)e^{-ru} = P^* e^{-ru} - Q^m(u,P^*) \cdot a(P^* e^{-ru}) \tag{2}$$

These two equations can be solved simultaneously to obtain the sales of each sector u moments before termination at P^*:

$$Q^c(u,P^*) \text{ and } Q^m(u,P^*).$$

Only these sales paths will permit price and marginal revenue to grow at the rate of interest until they reach, respectively, P^* and $MR(P^*)$.

When the price reaches P^*, the competitors drop out and the monopolist takes over the entire market. In this second phase, his marginal revenue must continue to grow at the rate of interest until the choke price is attained. The marginal revenue begins at $MR(P^*)$ and grows at the rate of interest until it reaches F. We denote the cumulated sales of the monopolist in the second phase as $\Delta(P^*)$.

The duration of the first phase (S) and the price (P^*) at which the competitors drop out are determined by two exhaustion equations:

$$\int_0^S Q^c(u,P^*)du = \bar{I}^c. \tag{3}$$

$$\int_0^S Q^m(u,P^*)du + \Delta(P^*) = \bar{I}^m. \tag{4}$$

The first states that P^* and S must be chosen so that the competitors sell their entire inventory during the first phase; the second states that P^* and S must be chosen so that the monopolist sells his entire inventory during the two phases.

By construction, any solution to the two exhaustion equations determines a path of prices, marginal revenues, and sales with the following properties:

(1) for $P < P^*$, price and marginal revenue grow at the rate of interest.

(2) for $P \geq P^*$, marginal revenue continues to grow at that rate until the monopolist's sales end; it then remains at the choke price (F).

(3) the competitors sell their entire inventory during the first phase.

(4) the monopolist sells part of his inventory in the first phase and the remainder in the second.

(5) the monopolist's sales are always positive.4/

Two other properties are necessary for the solution to be a non-cooperative equilibrium.

(6) In the second phase, price never grows faster than the rate of interest.

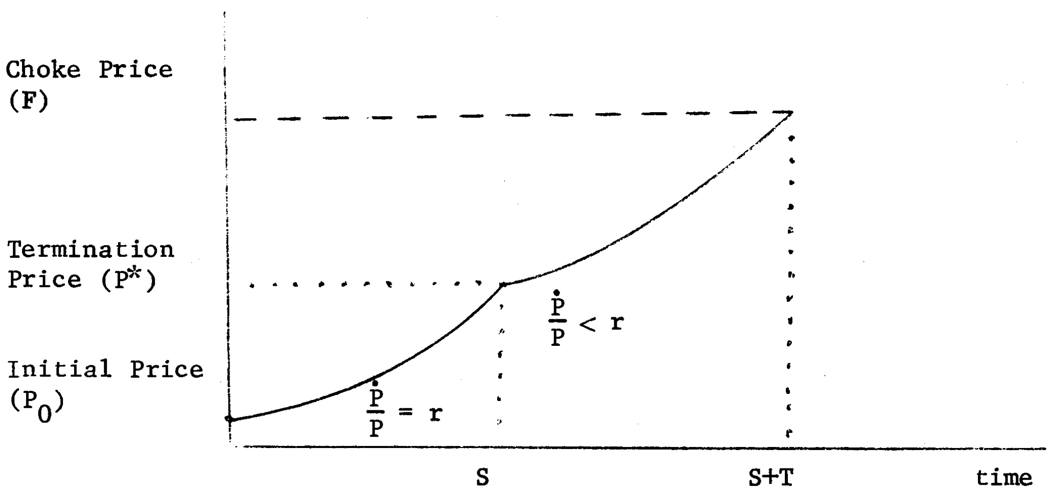
(7) The competitive sector (including the speculators) never sells so much that it has negative inventory in some periods.

Any solution with these seven properties constitutes a non-cooperative equilibrium. Given the price path of the monopolist, the competitors cannot make greater discounted profits by selecting a different, feasible sales path; given the sales path of the competitors, the monopolist cannot make greater profits by altering his price path.

4/ This follows easily from equations (1) and (2) and the fact that, on any downward sloping demand curve, price exceeds marginal revenue.

A sufficient condition for the existence of a non-cooperative equilibrium (with non-randomized strategies) is that the consumer demand curve have a point of unit elasticity ($\underline{P}$) and that elasticity along the curve increase strictly with price. The proof is relegated to the appendix. All linear or concave demand curves and many convex curves satisfy this sufficient condition.

Whenever an equilibrium exists, the price path will have the following appearance:



The price begins at some level, P_0, grows for S periods at the rate of interest, and then grows more gradually for T additional periods until the choke price is reached.

In the next section (p. 13-24), six qualitative propositions about the model are proved diagrammatically. Readers interested primarily in results may wish to skip the remainder of this section which provides an algebraic example of the determination of the equilibrium.

Suppose the consumer demand curve is linear:

$$P = F - aQ^{ag}.$$

Substituting this demand curve into equations (1) and (2) we obtain:

$$P^{*}e^{-ru} = F - aQ^{m}(u,P^{*}) - aQ^{c}(u,P^{*}). \qquad (1')$$

$$MR(P^{*})\cdot e^{-ru} = F - 2aQ^{m}(u,P^{*}) - aQ^{c}(u,P^{*}). \qquad (2')$$

These two equations may be solved to obtain the sales of each sector, u moments before termination of the first phase at P^{*}:

$$Q^{m}(u,P^{*}) = \frac{1}{a}\{F - P^{*}\}e^{-ru}.$$

$$Q^{c}(u,P^{*}) = \frac{F}{a}\{1 - e^{-ru}\}.$$

When the second phase begins, the marginal revenue the monopolist would receive for selling another unit will have grown to

$$MR(P^{*}) = 2P^{*} - F.$$

It must continue to grow at the rate of interest until the choke price is attained. For any price (P^{*}), the cumulated sales of the monopolist $(\Delta(P^{*}))$ during the second phase may be calculated.5/

5/

$$\Delta(P^{*}) = \int_0^T \frac{[F-(2P^{*}-F)e^{rx}]dx}{2a},$$

where the length of the second phase (T) is defined by $T = \frac{1}{r}\ln\frac{F}{(2P^{*}-F)}$.

This example illustrates the three general properties of the delta function:

a. $\Delta'(P^{*}) < 0.$

b. $\Delta(F) = 0.$

c. $\lim_{P^{*}\downarrow \underline{P}} \Delta(P^{*}) = \infty.$

Substituting our three derived functions into the exhaustion equations (3) and (4), we obtain two equations in the two unknowns, S and P^*:

$$\int_0^S \frac{F}{a}(1-e^{-ru})du = \bar{I}^c. \tag{3'}$$

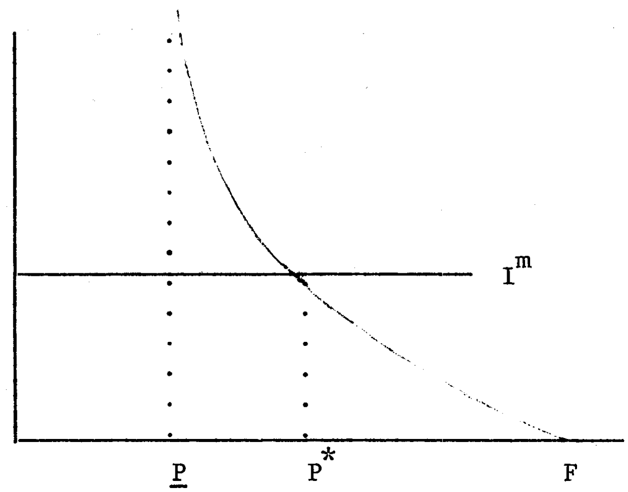
$$\int_0^S \frac{1}{a}\{F-P^*\}e^{-ru}du + \Delta(P^*) = \bar{I}^m. \tag{4'}$$

Since any linear demand curve satisfies the sufficient conditions discussed in the appendix, these two equations can be simultaneously satisfied. The solution, for this example, is unique.

Competitive sales u moments before the end of the first phase are positive and, in this example, are independent of the price (P^*) at which the monopolist takes over. Hence, the first exhaustion equation (3') uniquely determines the length of the first phase.

Substituting that value into the second equation (4') we can uniquely determine P^*. The right-hand side of equation (4') is a constant; the left-hand side is a continuous, monotonic function of P^*, which approaches infinity 6/ in the neighborhood of the unit elasticity point ($\underline{P}$) and zero at the choke price. Somewhere in between, it must reach the initial stock level $\bar{I}^m$. The diagram below plots the two sides of equation (4') and shows the graphical determination of the termination price (P^*):

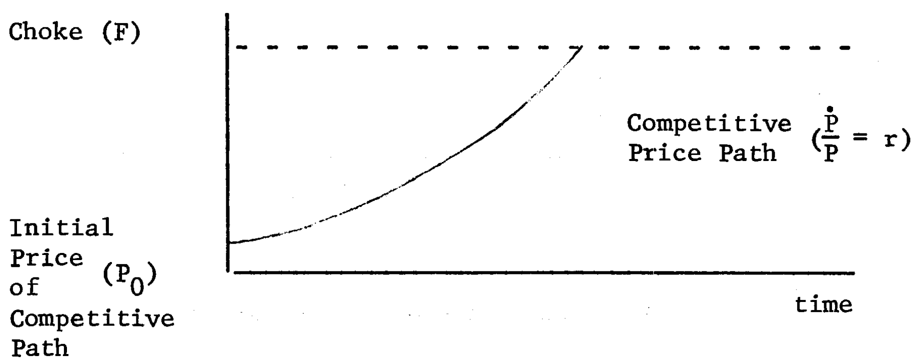
6/ This asymptotic behavior is a consequence of the last property of the delta function mentioned in footnote 5.



III. Implications of the Non-Cooperative Equilibrium

In this section, some important implications of the non-cooperative equilibrium are examined. We begin by considering the transition from a competitive world of many small extractors to a world where some of these extractors form a collusive cartel engaged in a non-cooperative game against the remaining small firms.

In the diagram below, the equilibrium price path prior to the formation of the cartel is portrayed. Price rises at the rate of interest until the world inventory ($\bar{I}^m + \bar{I}^c$) is exhausted; then the price remains at the choke level (F):

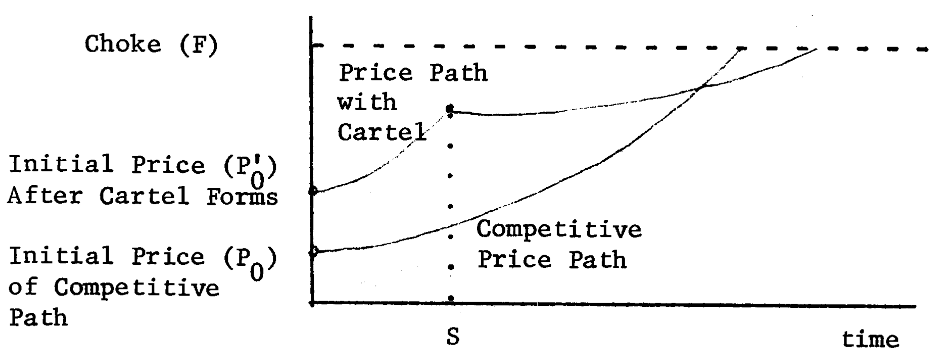


Since selling a barrel of oil in any period has the same discounted value, a firm receives a total discounted profit of P_0 multiplied by its initial stock.

If the firms owning $\overline{I}^m$ barrels decide to collude while the owners of the remaining $\overline{I}^c$ barrels continue to act competitively, the equilibrium price path changes. From the last section, we know the new path will rise at the rate of interest during the first phase and then grow at a slower rate during the second. Moreover, since world inventories are unchanged, the cumulative sales along each path must be identical.

Suppose the new price path began at P_0 or less. Then, it would lie below the competitive equilibrium price path somewhere and would never lie above it. But then cumulative demand along the proposed path would exceed the demand along the competitive equilibrium path and would, therefore, exceed the unchanged world stock.

Hence, the new price path must begin above P_0, rise at the rate of interest for S periods, and then grow at a slower rate thereafter. If cumulative sales along the two paths are equal, the new path must cut the competitive path. The following diagram illustrates the two equilibrium price paths:



We now consider who wins and loses from the formation of the cartel.

<u>Proposition II</u>: The formation of the cartel increases the total discounted profits of the collection of firms constituting the cartel and also those of each non-member; oil consumers are the only losers.

<u>Proof</u>: In either institutional setup, the discounted profit of each competitive firm is simply the initial price times its initial inventory of oil. Since the formation of the cartel raises the initial price, each competitor benefits.

The collection of firms in the cartel also benefits. Given the equilibrium sales path of the competitors, the cartel could have selected the competitive price path instead of the kinked path. The selection of the kinked path by the monopolist reveals it to be more profitable to the cartel than the competitive path.

In standard welfare analyses of extraction, it is shown that only the inter-temporal allocation of oil associated with the competitive price path is Pareto-optimal. Someone must lose in the movement away from the Pareto-optimal path. Since the extractors all gain, the consumers must lose. In principle, everyone could be made better off if some of the income (a constant marginal utility background good) of the consumers were redistributed to the extractors and the competitive allocation were restored.

That the formation of the cartel benefits the competitive non-members may seem surprising. After all, the cartel <u>is</u> their opponent; however, the game is not zero-sum. The cartel's power is directed not against the other oil producers, but against the consumers; the competitive firms ride on its coattails,[7/] benefitting from the higher prices.

7/ This analysis, of course, need not be confined to oil. When applied to gold, Proposition II implies that the small extractors in other countries benefit from South Africa's market power.

In fact, the competitive firms get the benefit of higher prices without, themselves, having to retard extraction in order to raise the prices. This suggests the following proposition:

Proposition III: The competitive firms benefit proportionately more from the formation of the cartel than does the cartel itself.[8/]

Proof: The profits of the competitive sector change from $P_0\overline{I}^c$ to $P_0'\overline{I}^c$ when the cartel forms. The profits of the firms which join the cartel are originally $P_0\overline{I}^m$. When they form the cartel, they sell part of their stock at the discounted value of P_0' per unit; however, they sell the remainder at a smaller discounted value. Hence, unlike the competitive firms, their collective profits rise by a smaller proportion than $\frac{P_0'}{P_0}$. The competitive firms are "free-riders".

The question naturally arises: can the profits of the firms in the competitive sector be taxed away? The imposition of a tax on oil stocks at less than 100% of their market value would be borne entirely by the extractors. By the familiar arguments of Henry George, the prices consumers pay would be completely unaffected. A tax sufficiently steep to offset the gain to the owners of the n firms resulting from the formation of the cartel would seem equitable to some (non-owners) and too small to others. Depending on its terms, such a tax need not even affect the private

8/ For an analogous result in static game theory, see Martin Shubik, Strategy and Market Structure (1959), p. 134-5.

search for additional oil. The tax might, for example, be imposed only on amounts owned on some earlier date.[9/]

We have not considered how the total profits of the cartel will be divided among its members. A subsequent paper will utilize cooperative game theory to discuss the division of the proceeds and the related question of the stability of the cartel.[10/] However, if each cartel member receives an equal profit per barrel, Proposition III has the following implication.

Proposition IV: A small firm within the cartel could make greater total profits from its oil sales if it defected from the cartel while the other members remained.

Proof: If the firm is small, its defection would influence the price path only trivially. Unfettered by the discipline of the cartel, the firm could sell all of its oil before time S, earning a discounted value of P_0' per barrel.

Such a calculation might lead the firm to demand compensation from the other members for staying in the cartel. However, each firm might make a similar demand

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9/ The transfer of this search activity from private industry to the government might, however, be desirable. In the past, other countries have concluded that the government can discover reserves more efficiently than the private sector. Moreover, serious ecological externalities often result in private scrambles for new resources. Finally, without government assitance, the private market is poorly situated to deal with the danger that the price may plummet if the cartel disintegrates or wishes to eradicate new entrants.

To solve this problem through the market mechanism, a permanent price floor has been proposed. This proposal seems excessively expensive. Any scheme guaranteeing firms the recovery of their set-up costs if the price falls should be adequate incentive for firms to enter and provide additional supplies.

10/ Using the analysis of the next section, we can compute the value of the non-cooperative game to the cartel and to each non-member, even if the excluded firms are large. With the game in "characteristic function form", the theories of Aumann-Maschler and others can be applied to answer questions about distribution and stability.

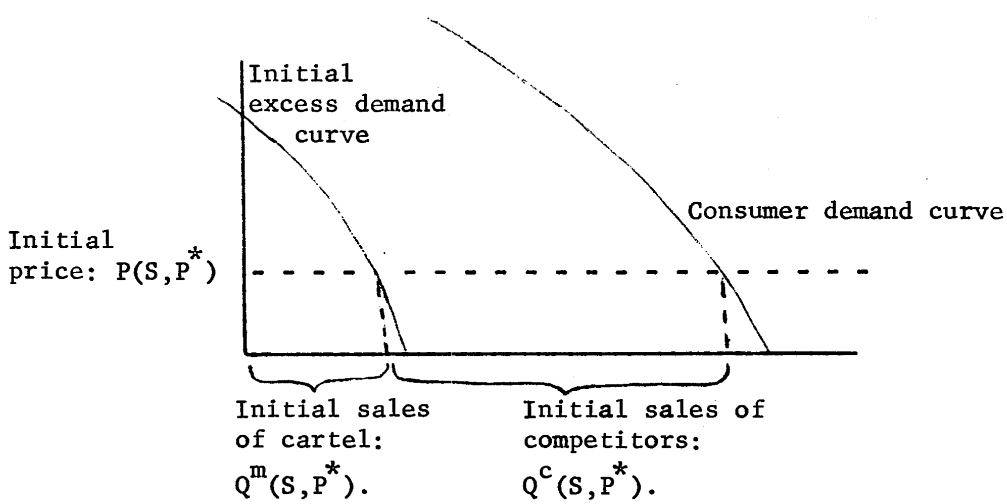
on all the others. Since the cartel earns less than $P_0'\overline{I}^m$, there is inadequate revenue to compensate each member.

On the other hand, each member of the cartel would be worse off defecting if his departure triggered the defection of the other members. For then, the price path would drop to the competitive path. The cartel is in their collective interest.

We have seen that the consumers lose surplus when the cartel forms and that the principal beneficiaries are firms outside the cartel. Their windfall profits, however, can be taxed away. Under these circumstances, the non-cartel firms may seek to persuade the public--whose representatives legislate taxes--that they are somehow aggrieved. The following proposition provides a pretext for such a claim.

Proposition V: Provided elasticity along the consumer demand curve rises with price, the discounted profit flow of the competitive sector declines as the coexistence phase progresses.

Proof: Since the discounted price is the same in every period of the coexistence phase, we must prove merely that the sales of the competitive sector decline during that phase. The following diagram shows the consumer demand curve, the excess demand curve, and the sales of each sector immediately after the cartel forms.



In the next period, price and the marginal revenue associated with the next excess demand curve must each grow at the rate of interest. Since

$$\frac{M}{P} = 1 - \frac{1}{\eta^{ex}},$$

marginal revenue and price can grow at the same rate only if the elasticity (η^{ex}) on each excess demand curve at the point chosen by the monopolist remains constant.

If elasticity rises as we move up the consumer demand curve, it increases with price along any excess demand curve. Since the price rises over time, the elasticity can remain constant during the coexistence phase only if the excess demand curve shifts to the right each period. This shift implies that the sales of the competitive sector decline as the first phase progresses. Hence, the profit flow of the competitive sector, although enormous at the beginning, does decline (eventually to zero).

Total discounted profit, nonetheless, is much larger than it would be without the cartel. The discounted profit flow of the monopolist, in contrast, may well increase during the initial phase.11/

11/ Since the discounted value of each price is the same during the coexistence phase, the discounted profit flow of the monopolist rises (falls) only if his sales rise (fall). Equations (1) and (2) imply that

$$\dot{Q}^m \gtrless 0 \text{ as } \frac{P}{a}\frac{\partial a}{\partial P} \lesseqgtr 1.$$

That is, sales of the monopolist rise if and only if a one-percent increase in the price raises the absolute value of the slope of the demand curve by less than one percent. This condition is satisfied by any linear or concave demand curve.

However, for a convex demand curve, the sales of the monopolist may remain constant or fall as the coexistence phase progresses. For example, suppose the demand curve were

$$Q^{ag} = \ell n \frac{F}{P}.$$

This demand curve is convex, has elasticity rising with price, and cuts the price axis. Since $\frac{P}{a}\frac{\partial a}{\partial P} = 1$, the sales of the monopolist do not change during the first phase. Hence, his discounted profit flow remains constant while that of the competitive sector declines. During the second phase, the discounted profit flow of the monopolist declines.

For the linear example of the previous section, it was proved that the first phase terminates at a price (P^*) above the unit elasticity point ($\underline{P}$). This is a general property of any non-cooperative equilibrium.[12/] Furthermore, during the first phase, the monopolist selects points on each excess demand curve which have the same elasticity. Since the last point he selects in that phase (P^*) is in the elastic region of the consumer demand curve, all previous points must also lie in that region. This proves the following proposition:

Proposition VI: The monopolist always operates in the elastic region of the demand curve he faces.

Although the elasticity of the excess demand curves exceeds unity at the points selected by the monopolist, the elasticity of aggregate demand may be smaller than unity early in the first phase. The two elasticities are related in the following way:

$$\eta^{ag} = \alpha\eta^{ex},$$

where α is the monopolist's share in total sales. As the first phase proceeds, the monopolist's share and the elasticity of aggregate demand grow at the same percentage rate. Ultimately, the share reaches 100% and the first phase terminates. At that time, the elasticity of aggregate demand will exceed unity. Before then, however, sales may occur in the inelastic portion of the aggregate demand curve.

12/ To prove this, note that a termination price in the inelastic portion of the demand curve implies a non-positive marginal revenue for the monopolist. A non-positive number can never grow by the interest rate until the choke price is reached. Hence, the monopolist would have to exhaust his supplies before reaching that price and then price would jump to the choke level. Such a situation is not an equilibrium.

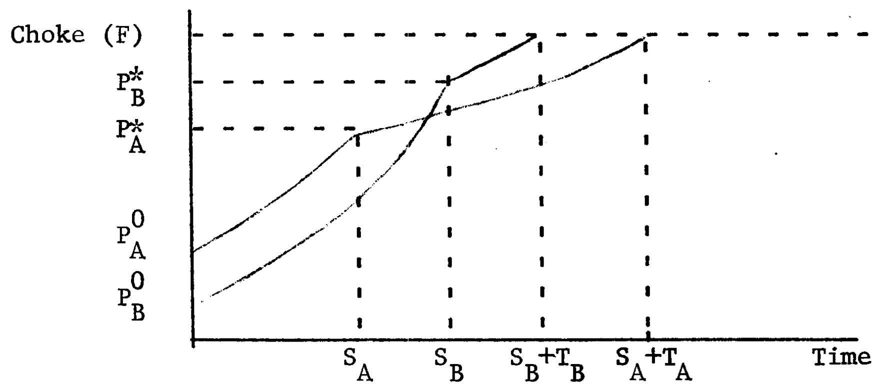
The foregoing remarks clarify an issue of apparent concern to econometricians studying the elasticity of demand for oil.[13/] There is no presumption that OPEC is behaving sub-optimally merely because the aggregate demand for oil is inelastic. This elasticity divided by OPEC's share of world supply is the relevant elasticity and it may well exceed unity.

As long as the equilibrium price path is uniquely determined by the initial inventories of the two sectors, it is meaningful to ask how the path will change if oil is transferred from the cartel to the competitive sector, holding the world total constant. Such a transfer would actually occur if some OPEC members defected from the cartel. Moreover, the comparison is of pedagogical value, since it reveals the sense in which the dominant firm model previously studied is intermediate between the conventional models of pure monopoly and pure competition. The changes in the equilibrium price path resulting from a redistribution of the fixed world stock are summarized in the following proposition.

Proposition VII: The greater is the share of the fixed world total stock of oil in competitive hands, the lower is the initial price (P_0), the longer is the first phase (S), the higher is its termination price (P^*), the shorter is the second phase (T), and the more quickly will the world supply be exhausted (S+T).

13/ A similar point may be made about the gold market. The measured elasticity of aggregate demand for gold appears to be less than 5. In 1970 and 1971, South Africa split its market, selling gold to the IMF and some Central Banks at $35 an ounce and to the free market at the higher market price of $40. Such behavior implies that South Africa feared lowering the private price and estimated the marginal revenue of selling another unit on the private market to be $35. This implies an estimated elasticity of excess demand of about 8, considerably above the estimates of econometric studies. However, when the elasticity of excess demand is multiplied by the share of South Africa in gold sales to consumers, the apparent discrepancy becomes sharply reduced. For further discussion, see "Analysis of the Gold Market" by S. Salant and D. Henderson (mimeo).

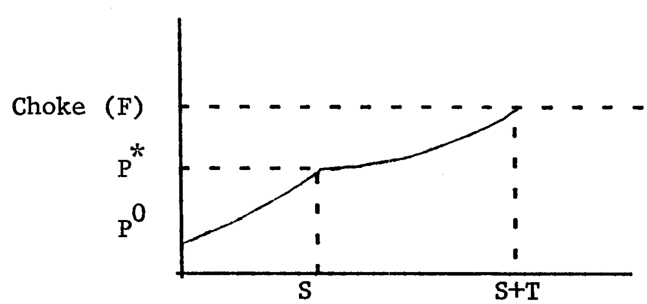
The following diagram illustrates Proposition VII:



Path B depicts the new equilibrium after some oil is transferred from the cartel to the competitive sector. If all the cartel's oil were transferred, the second phase of path B would disappear (P_B^* would coincide with F and the conventional competitive case would emerge). If, at the other extreme, the cartel acquired the entire world stock, the first phase would disappear (P_B^* would coincide with P_B^0, S_B would be zero and the standard monopoly case would emerge).

Proof: Consider any allocation of the fixed world stock between two sectors.

By assumption, the allocation results in a unique equilibrium path (sketched below):



Now consider the transfer of some oil to the competitive sector. The new path (not drawn) cannot coincide with the old path. If it did, its first phase would have the same duration (S) as the old path and would terminate at the same price (P^*). But, these variables determine the sales of the competitors at each moment of the first phase and, therefore, determine uniquely the cumulative sales of the competitors. If the new price path coincided with the old one, the competitors would sell the same total amount as before the transfer, although their initial stocks would be larger. Since the proposed (coincident) path would not permit the competitors to exhaust their supplies, it is not the new equilibrium price path.

The new equilibrium price path cannot lie entirely above (below) the old path. If it did, the total amount purchased by the consumers would be smaller (larger) than the unchanged world stock. For the same reason, the new path cannot partially coincide with the old path and partially lie above (below) it.

Hence, the new equilibrium path must cut the old path. The first phases of each cannot intersect, since the prices would then be equal and each would be rising at the same rate. For a similar reason, the second phases of each path cannot intersect. Hence, one phase of the new equilibrium price path must cut the other phase of the old path, as in our diagram of paths A and B.

In order to cut the higher path A in its second phase, the initially lower path B will require a longer first phase than path A. Path B must have a longer first phase and a higher termination price. When the second phase of path B commences (S_B), the price on path B (P_B^*) will be higher than the price is at the same time (S_B) on path A. Beyond S_B, the monopolist operates alone on each path. Since the price at S_B is larger on path B, the marginal revenue will then also be larger and, as long as it grows at the rate of interest, it must remain larger. Path B will, therefore, reach the choke price sooner.

We have not yet proved that the initially lower path B corresponds to the case where the competitive sector owns a larger fraction of the world's oil. However, it is well known that the path associated with pure competition begins below the path generated by pure monopoly. Beginning with the oil entirely in the hands of one sector, suppose we imagine transferring it until the other sector owns all the oil. The equilibrium price path would change with each reallocation. If the direction of this change ever reversed itself, then two (infinitesimally different) allocations of the world stock would generate the same price path. Since we proved above that the paths associated with different allocations never coincide, such a reversal cannot occur. Path B, therefore, does correspond to the case where the competitive sector owns a larger fraction of the world's oil.

Proposition VII indicates that the price path associated with the industrial organization being considered is intermediate between the extremes of pure monopoly and pure competition. The initial price lies between the high monopoly price and the low competitive one; and the world stock is exhausted more rapidly than under monopoly, but less quickly than under competition.

IV. N Extractors of Equal Size Face the Larger Cartel

In the previous sections, we assumed that one sector of the world's oil industry considers the effect of its sales on price and behaves monopolistically, while the other sector ignores its effect on price and behaves competitively. It may be argued that, in reality, few oil extractors currently in the industry behave competitively. However, if some do and some do not, the economic basis for this asymmetric behavior deserves examination.

In this section, the industry is assumed to consist of the cartel and n other firms which each own an equal share of the non-cartel oil stock. The number of firms dividing up the non-cartel stock is assumed to be sufficiently great so that none of these firms owns as much oil as the cartel. Only this difference in the initial stocks of oil distinguishes the cartel from any other firm.

Every firm in the oil industry (including the cartel) is assumed to take the aggregate sales path of the other n firms as given and, taking account of the effects of its sales on price, to devise its optimal, inter-temporal sales strategy. The price path is determined by the aggregate sales paths of the n+1 extractors. The equilibrium, where no firm has any incentive to change its sales path given the aggregate sales path of the others, can be studied.

There are several reasons for presenting this more general oligopoly model. First, it will enable us to show that--as the number (n) of firms sharing the non-cartel stock becomes very large--the equilibrium approaches (in the limit) the case we have examined in the previous sections. An economic basis for the asymmetry assumed in sections II and III is, therefore, provided. Second, the analysis can be extended to reveal the value--to any coalition owning a specified amount of oil--of playing a non-cooperative game against other coalitions of different sizes. This will permit, in future work, an examination of the distribution within each coalition and its stability. Neither the basis for the asymmetry nor the distribution question can be analyzed using the Stackelberg concept of equilibrium as opposed to the Nash-Cournot approach.

Following Cournot, assume that each of the n+1 extractors in the industry takes the aggregate sales path of the other n firms as given. Each firm computes the marginal to the excess demand curve it faces each period and chooses its sales path to maximize discounted total revenue. The characteristics of the optimal strategy for each firm (including the cartel) are that:

1) the marginal revenue rises at the rate of interest until the firm exhausts its supplies, and then rises at an equal or smaller rate; and

2) each firm exhausts its initial inventory.

Since each of the non-cartel firms is identical, only the behavior of a single representative need be considered. This representative small firm and the cartel behave in a similar manner. At any time when either firm operates, it equates its marginal revenue to the discounted marginal revenue it will receive on each future period until its supplies are exhausted. The marginal revenue of either firm is simply the price it would receive for selling an additional unit of oil less the revenue it would lose, because of the resulting slight reduction in price, on the inframarginal units it was selling. At any instant, the small firm and the cartel sell at the same price and an additional unit sold by either firm would reduce the price by the same amount; hence, <u>the firm selling the larger number of inframarginal units has the lower marginal revenue</u>.

By analogy with Proposition I, we begin by asking which of the two types of firms exhausts its supplies first. When one type of firm exhausts its supplies, its marginal revenue will then equal the price (since it has zero inframarginal units on which to take losses). The other type of firm will then be selling to the entire market and, therefore, its marginal revenue will be smaller.

At any moment of the coexistence phase, this same relationship must hold: the marginal revenue of the firm destined to drop out will exceed the marginal revenue of the firm which will take over. This difference in their marginal revenues indicates that, at every moment of the coexistence phase, the sales of the firm destined to drop out are smaller than the sales of the firm which will take over.

Which type of firm drops out first? If the cartel were the first to exhaust its supplies, it would sell less than each of the n identical firms at every moment of the coexistence phase. No matter how long that phase was, the cumulative sales of the cartel would be smaller than the cumulative sales of any other firm. But, if the phase were long enough to exhaust the cartel's supplies, it would over-exhaust each of the other firm's more modest supplies. The cartel cannot, therefore, drop out first.

The small firms must drop out first. Denote the termination price of the first phase by P^*. Then, since each type of firm equates its discounted marginal revenues across time:

$$M(u,P^*) = P^* \cdot e^{-ru}$$

$$M^b(u,P^*) = MR(P^*) \cdot e^{-ru}.$$

The first equation states that the marginal revenue of the representative small firm, u moments before it exhausts its supplies at P^*, is equal to its discounted marginal revenue at termination. The second equation states that the marginal revenue of the big cartel (denoted by superscript b), u moments before it takes over, will equal its discounted marginal revenue when n small firms depart. The marginal revenues for each type of firm are defined by:

$$M(u,P^*) = P\{nq(u,P^*) + Q^b(u,P^*)\} - q(u,P^*) \cdot a(P)$$

$$M^b(u,P^*) = P\{nq(u,P^*) + Q^b(u,P^*)\} - Q^b(u,P^*) \cdot a(P),$$

where $q(u,P^*)$ and $Q^b(u,P^*)$ are, respectively, the sales of the representative small firm and the cartel, u moments prior to the price (P^*) at which the cartel takes over.

Substituting, we obtain:

$$P^* e^{-ru} = P\{nq(u,P^*) + Q^b(u,P^*)\} - q(u,P^*) \cdot a(P). \tag{5}$$

$$MR(P^*) \cdot e^{-ru} = P\{nq(u,P^*) + Q^b(u,P^*)\} - Q^b(u,P^*) \cdot a(P) \cdot \tag{6}$$

To verify the results of our verbal reasoning, notice that since $P^* > MR(P^*)$, (5) and (6) imply (by subtraction):

$$q(u,P^*) < Q^b(u,P^*).$$

Each of the n firms sells less at each instant of the coexistence phase than the cartel.

Equations (5) and (6) can be solved for the two sales functions:

$$q(u,P^*) \text{ and } Q^b(u,P^*).$$

During the second phase, the marginal revenue of the cartel must continue to rise from $MR(P^*)$ by the rate of interest until the choke price is reached. The cumulative sales of the cartel along such a path will be the same as in our previous model: $\Delta(P^*)$.

These three functions can be substituted into two exhaustion equations to determine the length and termination price of the first phase:

$$\int_0^S q(u,P^*)du = \frac{\overline{I}^c}{n}. \tag{7}$$

$$\int_0^S Q^b(u,P^*)du + \Delta(P^*) = \overline{I}^m. \tag{8}$$

Our new system (5-8) is very similar to the model (1-4) studied in sections II and III. In fact, except for a difference in notation, (6-8) are exactly the same as (2-4). The only difference is between equations (1) and (5). The small firms no longer drive the current price down as far as the discounted value of the termination price, because they consider the losses on their inframarginal units. Hence, during the first phase, <u>price rises at less than the rate of interest in the oligopoly model</u>.

As the number of firms (n) dividing the non-cartel oil stock grows, however, the sales of an individual firm at each moment of the first phase become negligible. If the slope of the demand curve is bounded, the last term of equation (5) vanishes in the limit and our system (5-8) converges to the model (1-4) studied in previous sections.

To illustrate the limiting nature of our previous results, reconsider the case of the linear demand curve:

$$P = F - Q^{ag} = F - aQ^{b} - naq.$$

Substituting into equations (5) and (6), we obtain:

$$P^{*}\cdot e^{-ru} = F - aQ^{b}(u,P^{*}) - [n+1]\cdot aq(u,P^{*}).$$

$$MR(P^{*})\cdot e^{-ru} = F - 2aQ^{b}(u,P^{*}) - [n]\cdot aq(u,P^{*}).$$

Solving for our two sales functions, we obtain:

$$q(u,P^{*}) = \frac{F\{1-e^{-ru}\}}{a[n+2]}.$$

$$Q^{b}(u,P^{*}) = \frac{F+n(F-P^{*})e^{-ru} - (2P^{*}-F)e^{-ru}}{a(n+2)}.$$

From the demand curve, the price u moments before termination at P^{*} will be:

$$P(u,P^{*}) = F - aQ^{qg} = F - anq(u,P^{*}) - aQ^{b}(u,P^{*}).$$

If the number of identical non-cartel firms is very large, the sales functions of each sector converge to their previous form (p. 11):

$$\lim_{n\to\infty} nq(u,P^*) = Q^c(u,P^*) = \frac{F}{a}(1-e^{-ru}).$$

$$\lim_{n\to\infty} Q^b(u,P^*) = Q^m(u,P^*) = \frac{1}{a}(F-P^*)e^{-ru}.$$

Thus, the price path in the first phase will converge to its previous form:

$$\lim_{n\to\infty} P(u,P^*) = F-a\lim_{n\to\infty} nq(u,P^*) - a\lim_{n\to\infty} Q^m(u,P^*) = P^*e^{-ru}.$$

In the limit, the results of our earlier analysis (p. 11) emerge. The termination price and duration of the first phase for our n-firm case will then converge to their previous values.

V. A Comment About the Use of the Model for Policy Analysis

A principal short-coming of our model is the assumption of zero extraction costs. For the OPEC cartel, these costs are negligible (about 25 ¢ per barrel); but for the "competitive" sector they may be substantial.

Incorporating extraction costs in the model, however, seems relatively simple. If a firm's costs depend only on its current rate of extraction, its marginal profit (marginal revenue less marginal cost) must rise by the rate of interest and discounted profits must be non-negative. This entails a modification of equation (5).

If differences (in resource stock and/or cost structures) among the n "competitive" firms is desired, an exhaustion equation must be written for each different type of firm. The system of exhaustion equations will determine the duration of operation and the price at which exhaustion occurs for each different type of firm.

These details might best be introduced in a computerized version of the model. If the demand curve were for energy equivalents instead of "oil" and our firms produced "energy", the model could be used to evaluate different policies which have been proposed to ameliorate the current energy crisis.

The evaluation of any policy involves utility theory. Since we have made the "Marshallian" assumptions of a demand curve cutting the price axis and depending only on current price, a "constant-marginal-utility good" lurks in the background. This good is supplied by consumers to extractors in exchange for oil.

For the Marshallian case, the concept of consumer surplus can be rigorously defended. For any policy, we could evaluate the consumer surplus gained in each period. From the (discounted) sum of these surpluses, we would subtract the cost of the resources (measured in terms of the background good) used in extraction. The net benefit associated with the given policy could then be compared to that of any other policy.

VI. Conclusion

The model presented in this paper is stylized and abstract. In this section, its principal implications are summarized. The reader is cautioned that the results might change if the empirical details of the current situation were incorporated into the framework we have developed.

The formation of the cartel causes an immediate rise in the price of oil. In succeeding periods, the cartel raises the <u>real</u> price by the rate of interest until the competitors exhaust their supplies. Throughout this first phase, the elasticity of the <u>excess</u> demand curve faced by the cartel remains at a constant greater than unity. Initially, however, the elasticity of <u>aggregate</u> demand may be significantly smaller.

The formation of the cartel creates inter-temporal inefficiencies and redistributes income. Consumers lose surplus to owners of oil.

Members of the cartel benefit from the higher prices. Because they must restrict their sales, however, they cannot fully reap the rewards. This situation gives each member of the cartel an incentive to defect. However, maintenance of the cartel is in their collective interest.

When the cartel forms, the principal beneficiaries are owners of oil who are nonmembers. They can fully realize their capital gains by selling without constraint. That the flow of their discounted profits declines as the first phase progresses should not obscure their windfall gain in total, discounted profits.

When the cartel forms, each user of oil suffers a welfare loss as a consumer. The loss can be offset partially by a Henry George tax to eliminate the windfall capital gains of oil owners; the proceeds can then be returned to the consumers.[14/] Such a tax could be imposed in a way that would not raise prices or affect the search for new oil.

14/ Only a subset of the consumers and "competitive" extractors may fall within jurisdiction of a particular country. If the country consumes little relative to its own extraction (exporting the rest), a tax on the rents of local owners takes the surplus they capture from all consumers and redistributes it to the limited number of consumers under jurisdiction of the taxing authority.

APPENDIX

Proposition VIII: A sufficient condition for the existence of a non-cooperative equilibrium (of non-randomized strategies) is that the consumer demand curve have a point of unit elasticity ($\underline{P}$) and that elasticity of demand increase strictly with price.

Proof: To prove the proposition, it must be shown that--under the assumptions above--there will exist a solution satisfying all seven properties listed on p. 9.

For any S and P^*, the sales path of the monopolist is determined by equation (2); the sales path of the competitor can then be determined from equation (1). Since the monopolist's sales will always be positive (see footnote 4), property (5) is satisfied. In addition, the resulting price and marginal revenue paths satisfy property (1). Since elasticity is assumed to rise monotonically with price, property (2) and (6) hold for the second phase. Finally, we proved on p. 18 that the sales of the competitive sector will be positive if elasticity rises with price. This implies that, if property (3) holds so will property (7).

What requires proof is that there always exists a (P^*,S) combination that solves both exhaustion equations. If so, the remaining requirements for equilibrium (properties 3 and 4) will be fulfilled.

The two exhaustion equations insure that the sales of each sector match its initial inventory:

$$\int_0^S Q^c(u,P^*)du = \bar{I}^c \tag{3}$$

$$\int_0^S Q^m(u,P^*)du + \Delta(P^*) = \bar{I}^m. \tag{4}$$

For any termination price (P^*), we can find a unique duration (S) for the first phase satisfying equation (3). This follows from the fact that $Q^c(u,P^*)$ is positive for positive u. Hence, equation (3) implicitly determines a non-negative function $S(P^*)$.

Substituting that function into (4) changes it into a single equation in one unknown (P^*):

$$\int_0^{S(P^*)} Q^m(u,P^*)du + \Delta(P^*) = \bar{I}^m. \tag{4A}$$

From (2), it follows that

$$Q^m(u,P^*) \geq 0 \text{ for } P^* \leq F.$$

Furthermore, $\Delta(F) = 0$.

Thus, if the second phase were to begin at the choke price, cumulative sales of the monopolist would be zero in each phase. If, on the other hand, P^* were set below the choke price, cumulative sales of the monopolist would be positive in each phase.

In particular, if the termination price is set slightly above the point of unit elasticity, the cumulative sales of the monopolist would be positive in the first phase and would approach infinity in the second.

Since we can make the cumulative sales of the monopolist as small as zero or as large as any number by suitable choice of P^*, we can find (at least one) value for P^* which will equate the cumulative sales of the monopolist to his initial inventory. Once this termination price is found, the duration of the first phase may be obtained from (3). Since the solution satisfies all seven properties on p. 9, it is a Nash-Cournot equilibrium.

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