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HIRSHLEIFER ON SPECULATION

by

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I. Introduction

In a recent article,[1/] Hirshleifer compares the views of speculation associated with the names of Hicks and Keynes to the alternative views of Holbrook Working. According to the Hicks-Keynes school, differences in aversion to risk motivate speculation. In contrast, Working is said to attribute speculation to differences among market participants in their estimates of the likelihood of events. In considering this dispute in a general equilibrium setting, where agents with risky endowments act optimally, Hirshleifer performs a valuable service. Before evaluating his conclusions, an overview must be briefly sketched of the environment and market situations which he considers.

Each agent knows that he will receive a fixed endowment of commodity "n" but different amounts of commodity "z" depending on which of the two states of nature is realized. In advance of the realization, agents are allowed to make feasible contracts (which will be binding after the realization) at market clearing prices selected by the Walrasian auctioneer. Institutions prevent state dependent contracts for commodity "n" but permit contingent claims on commodity "z".

Hirshleifer considers two situations. In the first, prior contracts are made, the realization occurs, and then the contracts are executed. By assumption, there

*/ The views expressed herein are solely those of the author and do not necessarily represent the views of the Federal Reserve System. Dale Henderson and Steve Salop, both of the Federal Reserve Board, as well as an anonymous referee, provided useful expository assistance on an earlier draft of this comment. In addition, I wish to express my indebtedness to Robert Townsend of Carnegie-Mellon, whose work on incomplete markets has clarified in my mind many of the issues discussed in this comment.

1/ J. Hirshleifer, "Speculation and Equilibrium: Information, Risk, and Markets," Quarterly Journal of Economics, LXXXIX, (November 1975), pp. 519-542.

is no opportunity for a subsequent round of trading. Each agent chooses among lotteries with the consumption of the two goods in the two states as prizes. However, since individual endowments of commodity "n" are the same in either state and since prior contracts on that commodity are required to be independent of state, all the lotteries from which an agent can choose provide equal amounts of commodity "n" in the two states. His consumption of the socially risky commodity "Z" may, however, be state dependent.

Hirshleifer contrasts this situation to a second one where a subsequent round of trading is permitted. Prior contracts are made, agents are then informed accurately about which state will occur; they are then given a new opportunity to trade, the realization occurs, and the initial contracts are executed. As long as the information the agents receive is conclusive, this second situation can be compressed to something more familiar: agents make prior contracts knowing they will be allowed to retrade, the realization actually occurs, and they retrade in the spot market. This second situation envisaged by Hirshleifer provides a different set of lotteries within which the agents can choose. The agents are no longer constrained to consume the same amount of commodity "n" in the two states, although they may find that choice optimal.

Agents who utilize the second (informational) situation to alter their choice of consumption lottery from what it would have been in the first situation are defined by Hirshleifer to be speculators. Anyone choosing the same lottery in the two situations is a non-speculator.

Hirshleifer derives two central results:

1. A person differing from the mass of identical agents will speculate if and only if he differs from them in his views about probabilities; if he differs from them in his endowments or utility function, he will not speculate.

2. In the second situation where retrades are allowed in the spot market, the implicit future price of commodity Z (relative to n) in the prior market is

equal to the price of Z (relative to n) then expected to prevail in the subsequent spot market (provided all agents have concordant beliefs).

With the first result, Hirshleifer claims to refute the theories of Hicks and Keynes where utility deviance motivates speculation:

> The crucial result attained in the analysis above can be stated: Only those individuals deviating from representative beliefs in the market will hedge or speculate. In particular, contra the Keynes-Hicks or "risk transfer" theory, differences in degree of risk aversion alone will not lead to hedging or speculative behavior.2/ [His emphasis]

In the second result, Hirshleifer claims to have established, rigorously, a martingale theorem.

Neither result, however, is a logical consequence of the valuable and general framework Hirshleifer provides to study speculation. Instead, each of these results arises from a special assumption about the von Neumann-Morgenstern utility functions of all agents--the requirement that each exhibit "zero complementarity" ($U_{12} = 0$). No one with such a utility function would, for example,3/ have a preference between (1) a lottery promising a pair of shoes or no shoes at even odds and (2) a lottery which promises a left shoe but no right shoe or a right shoe but no left shoe at even odds. An assumption implying such behavior is not credible to many economists and its importance to specific results deserves more emphasis than Hirshleifer

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2/ p. 539.

3/ Let n = the number of left shoes and Z = the number of right shoes. Then, if $U(n,Z) = f(n) + g(Z)$, the two lotteries of the text will each have the same expected utility: Lottery 1 has expected utility $1/2\{f(1) + g(1)\} + 1/2\{f(0) + g(0)\}$. Lottery 2 has expected utility $1/2\{f(1) + g(0)\} + 1/2\{f(0) + g(1)\}$. Lottery 1 might, to take a different example, provide an expenses-paid vacation for a man and his wife or nothing, while lottery 2 might offer a trip for either one, but not the other. A third example might involve as prizes the consumption of different grades of wheat.

provides.[4/] As will be shown, results quite different from those listed above emerge from his framework when this assumption is altered.

In the next section, it will be shown that--in agreement with proponents of risk-transfer theory--a utility-deviant agent will speculate within the context of Hirshleifer's model as long as his utility function, unlike those of other agents, has a non-zero cross-partial ($U_{12} \neq 0$). Hirshleifer, in deducing his opposite conclusion, gives his deviant a utility function which can only differ from the masses in other ways.[5/] Since, however, the lone individual is assumed to be utility-deviant, it would seem fair to give him any utility function different from that of the masses. In the third section of this comment, Hirshleifer's martingale result is shown to depend not only on concordant beliefs but again on the peculiar assumption of zero complementarity. I conclude with a comment about speculation in a world of imperfect information.

II. Do Utility-Deviants Speculate?

Following Hirshleifer, consider an economy where all agents have the same beliefs about the probability that state "a" will occur. Assume that the mass of agents who determine prices has utility functions with $U_{12} = 0$. A lone individual, however, exists who has a utility function with $U_{12} \neq 0$. Will this utility deviant speculate? We will see that he will use the subsequent spot market to consume different amounts of "n" in the two states. Since he is unable to accomplish this without the opportunity to retrade, he is using the

4/ The assumption is introduced in the second section of the article (p. 526) as a mere technical simplification. Although it is given more stress in the last section (p. 540-1), it is left unclear how particular results would be affected if the assumption were altered.

5/ This is implicit in the step of his proof on p. 537 immediately following reference in the text to footnote 21, where he infers from

$$\frac{\partial u}{\partial n'} = \frac{\partial u}{\partial n''}$$

that $n' = n''$ for the deviant.

second market to advantage: this sufficies to make him a speculator by Hirshleifer's definition.

Denote: n, Z^A, Z^B as prior claims on goods n and Z contingent on states a and b.

P_Z^A, P_Z^B as prices of these prior claims relative to a certain claim on n.

π as the belief that state "a" will occur, shared by all agents.

q_a, q_b as the subsequent spot price of good Z (relative to good n) in the two states.

C_n^a, C_n^b, C_Z^a, C_Z^b as the consumption lottery chosen by the individual.

$\bar{n}$, $\bar{Z}^a$, $\bar{Z}^b$ as endowments.

Define: $C_n^a \neq C_n^b$ as a sufficient criterion for speculation.

Then each consumer wishes to maximize (with respect to C_n^a, C_n^b, C_Z^a, C_Z^b, n, Z^a, Z^b)

$$\pi U(C_n^a, C_Z^a) + (1-\pi)U(C_n^b, C_Z^b)$$

subject to: $n + P_Z^a Z^a + P_Z^b Z^b \leq 0.$

$$C_n^a + q_a C_Z^a \leq n + \bar{n} + q_a(\bar{Z}^a + Z^a).$$

$$C_n^b + q_b C_Z^b \leq n + \bar{n} + q_b(\bar{Z}^b + Z^b).$$

(For simplicity, I have omitted subscripts to denote each individual.) The correspondence of this notation to that used by Hirshleifer in his equations (7b) is explained in the table at the end of this comment.

The first equation restricts his acquisition of claims to the value of the claims he issues and assumes no state dependent claims on "n" are allowed; the second and third restrict the value of his expenditure in the spot market in each state to the value of his endowments augmented by his net claims on the endowments of other people. Eliminating n, we can associate λ_a, λ_b respectively with the second and third constraints.

The first order conditions are:

1. $\pi U_1(C_n^a, C_z^a) - \lambda_a = 0.$

2. $(1-\pi)U_1(C_n^b, C_z^b) - \lambda_b = 0.$

3. $\pi U_2(C_n^a, C_z^a) - \lambda_a q_a = 0.$

4. $(1-\pi)U_2(C_n^b, C_z^b) - \lambda_b q_b = 0.$

5. $\lambda_a(q_a - P_z^a) - \lambda_b P_z^a = 0.$

6. $- P_z^b \lambda_a + \lambda_b(q_b - P_z^b) = 0.$

In addition, we have the two budget constraints. The optimal choice of the four consumption goods and two prior claims on "z" must satisfy these necessary conditions.

We must begin by considering prices in general equilibrium. Each agent takes the prices as given. The auctioneer sets the prices to clear the markets. Since social endowments of commodity "n" are independent of state, prices must adjust so that aggregate demand for "n" is independent of state. Assuming that the mass of agents who determine prices have utility functions with zero complementarity ($U_{12} = 0$), we can deduce that the market clearing prices must become:

$$\frac{q_a}{P_z^a} = \frac{1}{\pi}\text{, and}$$

$$\frac{q_b}{P_z^b} = \frac{1}{1-\pi}$$

To prove this, we use equations (1), (2), (5), and (6). Combining (1) and (2), we obtain:

$$7. \quad \frac{\lambda_b}{\lambda_a} = \frac{1-\pi}{\pi} \frac{U_1(C_n^b, C_z^b)}{U_1(C_n^a, C_z^a)} \quad .$$

From (5), we get:

$$\frac{\lambda_b}{\lambda_a} = \frac{q_a}{p^a_{\cancel{z}}} - 1.$$

Hence,

8. $$\frac{1-\pi}{\pi} \frac{U_1(C^b_n, C^b_{\cancel{z}})}{U_1(C^a_n, C^a_{\cancel{z}})} = \frac{q_a}{p^a_{\cancel{z}}} - 1.$$

Every agent in the economy faces the same prices and has the same beliefs. Moreover, each agent among the mass who determine prices has a utility function with zero complementarity ($U_{12} = 0$). Thus, for each agent of the mass, the marginal utility from consuming good "n" depends on the amount of that commodity consumed and not on the amount of the other commodity ($\cancel{z}$).

Suppose prices such that $\frac{q_a}{p^a_{\cancel{z}}} - 1 > \frac{1-\pi}{\pi}$ were proposed by the auctioneer. Each agent of the mass would, from (8), set his own $C^a_n > C^b_n$. Aggregate demand for commodity "n" would then be larger in state a than in state b. But, aggregate endowments of commodity "n" are identical in the two states. Hence, the proposed prices could not clear the markets. By similar argument, if the auctioneer called out prices such that $\frac{q_a}{p^a_{\cancel{z}}} - 1 < \frac{1-\pi}{\pi}$, disequilibrium would result. For markets to clear, prices must adjust so that:

$$\frac{q_a}{p^a_{\cancel{z}}} - 1 = \frac{1-\pi}{\pi}$$

or, equivalently,

9. $$\frac{1}{\pi} = \frac{q_a}{p^a_{\cancel{z}}}.$$

In equilibrium, (6) - (9) must hold; together they imply that

$$10. \quad \frac{1}{1-\pi} = \frac{q_b}{p_z^b} .$$

These equilibrium relative prices result from the assumption that the utility function of the mass who determine prices exhibits zero complementarity. The members of this group could differ in endowments or in other characteristics of their utility functions without altering (9) and (10).

We now know enough about equilibrium prices. Let us focus on the consumption decision of a lone agent with the same beliefs, faced with the same prices, but with a deviant utility function which exhibits complementarity ($U_{12} \neq 0$).

From equation (5) and our knowledge about market-clearing prices, we learn that $\frac{\lambda_b}{\lambda_a} = \frac{1-\pi}{\pi}$. Dividing equations (2) by (1) and (4) by (3), we obtain:

$$U_1(C_n^a, C_z^a) = U_1(C_n^b, C_z^b)$$

$$U_2(C_n^a, C_z^a) = \frac{q_a}{q_b} U_2(C_n^b, C_z^b)$$

The optimal choice of our utility deviant must simultaneously satisfy these two equations, among others.

If our deviant were not a speculator, as Hirshleifer contends, he would find setting $C_n^a = C_n^b$ optimal. Let us assume $C_n^a = C_n^b$ and ask whether these two necessary conditions could be satisfied. By our assumption, the deviant's utility function exhibits complementarity. Hence, the function $U_1(C_n, C_z)$ is monotonic in the second argument. For the first equation to be satisfied, we must set $C_z^a = C_z^b$. But then, the second equation would be violated (since $\frac{q_a}{q_b} \neq 1$). The optimal choice for our lone agent must, therefore, be one where $C_n^a \neq C_n^b$. Hence, our utility deviant does speculate in accord with the views of Hicks and Keynes.

III. Do Concordant Beliefs Imply the Martingale?

From (9) and (10), we can obtain an interesting relation between the prior prices and subsequent spot prices:

$$P^a_{ƶ} + P^b_{ƶ} = \pi\, q_a + (1-\pi)\, q_b .$$

In general equilibrium, the future price of "ƶ" in the prior market is equal to the spot price of "ƶ" then expected in the subsequent market. Hirshleifer refers to this relation as a martingale. We derived it by assuming the mass of agents who determine price had concordant beliefs <u>and</u> zero complementarity in their utility functions.

The martingale result depends crucially on the assumption of zero complementarity. To see this, let us return to equation (8), where equilibrium prices were determined. Assume now that all agents in the economy have identical beliefs, endowments, and utility functions. Then prices must adjust so that each agent wishes to consume his endowments:

$$C^a_n = C^b_n = \bar{n}.$$

$$C^a_{ƶ} = \bar{ƶ}^a \neq \bar{ƶ}^b = C^b_{ƶ} .$$

If we now assume that the utility function for all agents exhibits complementarity, equation (8) implies that:

$$9'. \quad \frac{1}{\pi} \neq \frac{q_a}{P^a_{ƶ}} .$$

(6-8 and 9') imply that

$$10' \quad \frac{1}{1-\pi} \neq \frac{q_b}{P^b_{ƶ}} .$$

From this it follows that the martingale result need not hold, despite the concordant beliefs of all agents.

Hence, the two key results of Hirshleifer's analysis disappear if his assumption of zero complementarity is altered. For then we find that utility deviants may speculate and that prices may not follow a martingale.

IV. The Regime of Markets

In the final section of his article,[6/] Hirshleifer characterizes the requirement that prior contracts in the riskless commodity be independent of state as "inessential" for his analysis.[7/] I cannot agree. If contracts contingent on each state were allowed for each commodity, speculation (by Hirshleifer's definition) would never occur.

Arrow, among others, has shown that no one could use the spot market to advantage if he had been able to make complete contingent contracts. It follows that no one allowed to make such contracts would alter his consumption decision (speculate) in the informative situation. Hirshleifer's use of constrained-contingent markets is not an "inessential simplification" but a necessary condition for speculation by his definition.[8/]

Indeed, with complete markets, no one would speculate even if the injection of information were inconclusive. Suppose the two possible states were rain and shine and the two possible announcements were "it will rain" and "it will shine". Then there would be four exhaustive, exclusive events which could occur. Full contingent markets would allow a person to purchase a claim to X, binding if the forecaster said rain and it shined, etc. There would be four claims for each of

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6/ p. 541.

7/ In his footnote 13, Hirshleifer shows that--when retrades are not allowed--only belief deviants (and people with complementarity in their preferences) would use fully complete markets differently than they would use semi-complete markets. However, he does not go on to say that, with complete markets, even these deviants would not speculate. For they would then have sufficient market opportunities to make retrading after the information emerges unnecessary.

8/ See Robert Townsend's "Incomplete Forward Markets in a Pure Exchange Economy with Stochastic Endowments", University of Minnesota Discussion Paper 74-47 (November 1974), Section IV.

the two goods. With such a contingent market, no one would alter his desired consumption when given access to a subsequent spot market. Hence, no one would speculate.

Correspondence of Notation

	Salant	Hirshleifer
Consumption (State a)	C_n^a, C_z^a	n', z'_a
Consumption (State b)	C_n^b, C_z^b	n'', z''_b
Endowments	$\bar{n}$, $\bar{z}^a$, $\bar{z}^b$	n^e, z_a^e, z_b^e
Prior Contracts	n, z^a, z^b	$n^t - n^e$, $z_a^t - z_a^e$, $z_b^t - z_b^e$
Prior Relative Prices	P_z^a, P_z^b	P_{za}^0, P_{zb}^0
Posterior Relative Prices	q_a, q_b	$\bar{P}'_{za}$, $\bar{P}''_{zb}$