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MARKET ANTICIPATIONS, GOVERNMENT POLICY, AND THE PRICE OF GOLD

by

Stephen W. Salant and Dale W. Henderson

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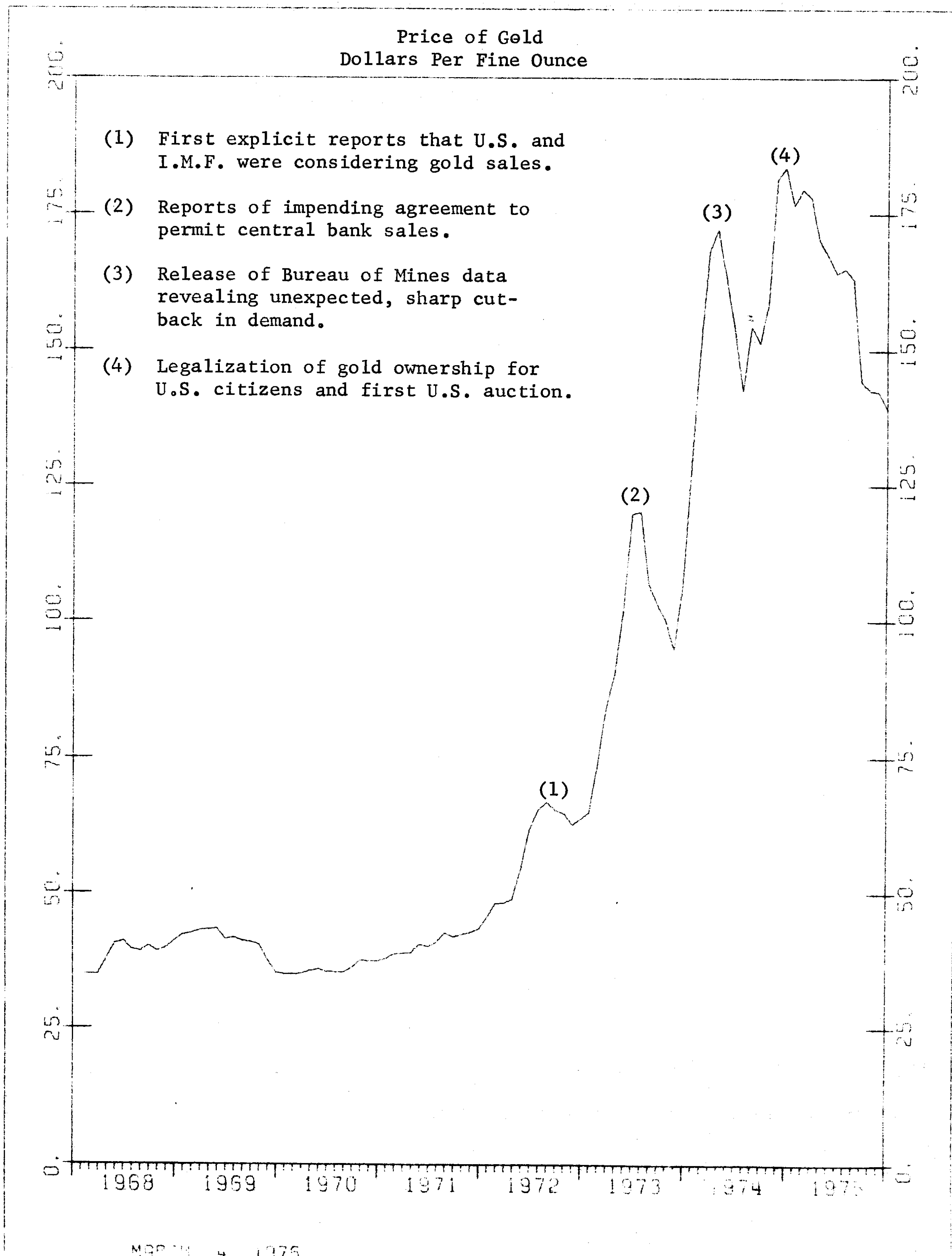
I. Introduction

Between March, 1968, when central banks stopped their pegging operations, and December 1974, when Secretary Simon announced the first U.S. government gold auction, the path of gold prices (shown in Figure 1) exhibited two striking features. First, the price rose at a rate much greater than the rate of interest for periods of as many as eight months at a time. Second, each upward surge was interrupted by a sharp setback.

Since gold, like any other ore, can be depleted (through use in dentistry and industry) but not created, it might be expected that the conventional theory of depletable resources would be useful in understanding this behavior.1/ However, it is argued in Section II that without some modification the standard theory cannot be used to explain how the price of a resource traded by competitive speculators could persistently rise by more than the rate of interest, nor can it be employed to account persuasively for the timing of the breaks in the price.

To be useful in explaining these phenomena, the standard theory must be modified to incorporate the influence on private sector behavior of the enormous stocks of gold controlled by world governments. These stocks total 25 times the peak rate of annual global extraction.2/ As the importance of gold in the international monetary system diminished, market participants recognized the substantial risk that government stocks would some day be sold on the private market. Section III is devoted to modifying the conventional theory of exhaustible resources to take account of anticipations of sales from government stockpiles.


Price of Gold
Dollars Per Fine Ounce
(1) First explicit reports that U.S. and I.M.F. were considering gold sales.
(2) Reports of impending agreement to permit central bank sales.
(3) Release of Bureau of Mines data revealing unexpected, sharp cut-back in demand.
(4) Legalization of gold ownership for U.S. citizens and first U.S. auction.
(1)
(2)
(3)
(4)
0.
25.
50.
75.
100.
125.
150.
175.
200.
1968
1969
1970
1971
1972
1973
1974
1975

It is, of course, widely understood that the risk of sales from government stockpiles depresses the gold price. However, since the lower price results in faster depletion, the stock overhang has intertemporal consequences which have traditionally been overlooked. Although the gold price in anticipation of government sales does begin lower, it rises more quickly and, unless government sales interrupt its ascent, must ultimately surpass the price that would have occurred in the absence of any stock overhang. In particular, it is shown that the stock overhang causes price to rise faster than the rate of interest and the percentage rate of increase, itself, to increase over time. Even if gold holders are risk neutral, they require this rate of increase as compensation for exposing themselves to the asymmetric risk of a price collapse which would result from a government auction. It is also shown that announcements which merely increase the perceived likelihood of such sales are enough to trigger attempts by the private sector to unload and, hence, result in a sudden drop in the price. If the risk of government sales remains higher, the price will, in the absence of the auction, begin a new upward surge at more than the rate of interest.

It is our belief that anticipations of such sales and announcements affecting their likelihood largely account for the surges and drops in the gold price in the period between March 1968 and the first U.S. auction. In Section IV, the timing of each sharp setback in the gold price during this interval is shown to be consistent with what the theory predicts would happen on the basis of the information that became available to market participants.

In the last year and a half, the United States has conducted two gold auctions. At the present time, planning for future IMF auctions is well advanced. Now that official sales are actually taking place, it seems particularly useful to analyze the effects of alternative sales policies. In Section V, the model is employed to predict the consequences of carrying out each of a variety of such policies, whether or not it was the one anticipated by the market. It is shown why any offer to sell gold at a fixed price must inevitably lead to a speculative attack of the kind experienced in March 1968.

II. The Inadequacy of the Conventional Theory of Exhaustible Resources as an Explanation of Gold Price Movements

Suppose the gold market conformed exactly to the most naive assumptions used in the conventional theory of exhaustible resources. Then the world's mine owners would extract gold costlessly and sell it competitively. In addition, there would be no uncertainty about the stock of gold available to the private market in the future. The world demand curve of "consumers" would be the same in every period, and there would be some high price, the "choke" price, at which no one would want to consume gold.[3/]

Under such assumptions, the theory predicts that as long as mine owners hold gold, the price of gold in any period must be one plus "the interest rate" times the price in the previous period.[4/] If price did not follow such a path, markets would not clear in all periods. One price would have a higher present discounted value than any other, and in order to maximize the present discounted value of their profits, mine owners would want to sell all their gold in the period with the highest present discounted price. However, since consumers would wish to purchase gold in other periods,[5/] there would be excess demand in all but one period. Thus, in order for the gold market to be in equilibrium in every period, the gold price must rise at a rate equal to the rate of interest. Given this set of prices, individual mine owners do not care when they sell since, whenever they sell, they make the same profits.

Knowing that prices grow geometrically by the rate of interest is not enough to determine their path completely. There are as many such paths as initial prices. In addition, sales along the path must exactly match the stock available. If, on the one hand, the initial price were too low, the mines would run out of gold before the price had risen enough to choke off demand. Consumers would then want ore they could not acquire. If, on the other hand, the initial price were too high, demand would be choked off before the supplies were exhausted. Hence, along the equilibrium path, price must rise by the interest factor and begin at a level such that when the gold is exhausted, demand is choked off.

The introduction of speculators to this model does not change the equilibrium price path. Extraction in a period where speculators buy (sell) exceeds (falls short of) gold consumption, the difference going into (coming from) speculative inventories. While competitive extractors make profits because of the rents on the scarce resource they own, competitive speculators make no profits.

An unexpected event which causes gold owners to believe that demand will be lower in future periods affects the gold price immediately. Mine owners and speculators foresee that if prices continued to rise along the original path, demand would be choked off before they could sell their supplies. To avoid the implied eventual loss, they sell. As a result, price drops and then follows a lower path which rises at the rate of interest.

This simple version of the theory of exhaustible resources yields poor predictions of the behavior of the gold price. It is inconsistent with any upward surges at greater than the rate of interest and cannot be used to explain either the existence or the timing of most drops in the price.[6/] However, since the assumptions of the naive model do not reflect several prominent features of the gold market, it may not be surprising that its predictions are poor. It is important to determine which particular characteristic of the gold market ruled out above was responsible for the observed qualitative behavior of the gold price.

Three characteristics of gold distinguish it from the exhaustible resource of the simple model. First, extraction costs in the gold industry are not negligible and have risen. Second, the gold market is not competitive, but is dominated by one seller, the South African Reserve Bank. The Reserve Bank acquires all South African production, three-quarters of the world total,[7/] and is the sole South African seller. This arrangement creates a presumption that the Reserve Bank knows its influence on the market price.[8/] Since the Bank sold part of its supply to the IMF for two years in order to avoid reducing the higher price it was getting on the free market, the inference that it knows its market power seems inescapable.[9/] Third, gold holders cannot know with certainty the size of the stock which will be available for private use since the possibility always exists that governments may sell to the private market some of their enormous holdings.[10/]

Of these three distinguishing features of the gold market, only the third could account for the basic characteristics of the observed gold price path. For, while the presence of soaring extraction costs or of monopolistic behavior could conceivably give rise to the observed pattern of prices in the absence of speculators,[11/] neither factor could explain the observed pattern in a world where competitive speculators can enter and do operate.[12/] Since competitive speculators have no extraction costs, they prevent the price from rising at a rate greater than the interest factor. For, if they foresee that the price will rise by more than the interest factor, they attempt to make unlimited profits by borrowing, buying gold in one period, and selling it in the next--thereby carrying cheaper ore into a period in which it would have been more expensive. Speculators' actions cause the price path to change until all opportunities for profitable arbitrage are removed.

III. An Alternative Model to Explain Gold Price Movements

Since competitive speculators would place an upper bound on the rate of increase of gold prices under the circumstances considered so far, a model which purports to explain movements in these prices must explain why speculators did not flood the gold market when faced with apparently unexploited profit opportunities. A necessary ingredient in such an explanation is that gold holders anticipated that (some of) the large stocks held by the world's governments would be sold on the private market at an unpredictable time. This possibility had been mentioned repeatedly in trade journals, official statements, and newspapers long before December 1974, when Secretary Simon announced the first official auction. In the words of the New York Times, "The 'sword of Damocles' over gold's high price is the huge dormant supply in the central banks."[13/] It is argued here that the threat that this sword would drop would have been sufficient to cause rapid increases in gold prices even in the absence of rising extraction costs and South African market power.

The conventional model of the market for an exhaustible resource can be modified to take account of the risk of government selling. In constructing the modified model, many of the basic assumptions of the simplest conventional model are retained. The consumers' demand curve for gold [$D(\cdot)$] is assumed to be stationary and downward sloping. There is a "choke" price (P_c) at which consumers' demand is zero. Mine owners have an initial stock of gold ($\bar{I}$) which they extract costlessly and sell competitively.

What makes the modified model different is the possibility that the government might auction off part of its gold. Mine owners and speculators believe that in each period there is some constant probability (α) that the government will auction a specified amount of gold ($\bar{G}$) in the next period given that it has not done so already.[14/] These private gold holders also believe that the auction of

$\overline{G}$ is the only one which might occur.[15/] They know that one price (P_t) will emerge at time t in the absence of an auction while another (f_t) will result if the auction occurs. Since they are risk neutral, they act to maximize discounted expected profits. The stock of gold in private hands at the beginning of period t in the absence of an auction is S_t.

Under these assumptions, five conditions determine the equilibrium paths of the variables in the model. In the initial period the private sector has what gold is in the mines:

$$S_0 = \overline{I}. \qquad \text{(Initial Condition)} \qquad (1)$$

Private gold holders sell all their gold before the endogenously determined terminal period (T) when price chokes off demand:

$$P_T = P_c, \qquad S_T = 0. \qquad \text{(Terminal Conditions)} \qquad (2)$$

If an auction occurred at time t, the private sector would then own $S_t + \overline{G}$ units of gold and would, by assumption, no longer operate under the threat of a future auction. Hence, price would grow over time at the rate of interest from the auction price (f_t) to the choke price. The potential auction price is determined by the condition that cumulative consumption along the price path exactly equals the stock in private hands:

$$\sum_{s=0}^{\infty} D\{f_t(1 + r)^s\} = \overline{G} + S_t. \qquad \text{(Equation Determining Potential Auction Price)} \qquad (3)$$

Since consumers demand gold in every period before T, mine owners and speculators must be willing both to sell gold and to carry some inventory of the metal. Risk neutral private gold holders will be indifferent between these two activities when they yield the same discounted expected profit. Opportunities for profitable arbitrage exist unless the price in period t is equal to the discounted value of the price then <u>expected</u> to prevail in period t + 1:[16/]

$$P_t = \frac{\alpha f_{t+1} + (1 - \alpha) P_{t+1}}{(1 + r)} . \qquad \text{(Arbitrage Equation)} \qquad (4)$$

The private stock is depleted between period t and period t+1 by the amount of non-recoverable consumption in period t:

$$S_{t+1} = S_t - D(P_t). \qquad \text{(Depletion Equation)} \qquad (5)$$

The model of equations (1) - (5) generates time paths for the price of gold (P_t) in anticipation of an auction, the potential auction price (f_t), demand [$D(P_t)$], and the private stock of gold (S_t). The number of periods (T) required to consume the initial private stock ($\bar{I}$) if an anticipated auction never occurs, is also determined.[17/]

To show that equations (1) - (5) determine these variables, a backward solution is utilized. At the end, the private stock (S_T) and the price (P_T), in the absence of an anticipated auction, are indicated by the terminal conditions. Using the private stock and equation (3), f_T, the price that would occur in the final period if there were an auction, can be calculated. Substituting P_T and f_T into equation (4) yields P_{T-1}, the price that must prevail at the beginning of the previous period in the absence of an auction. The stock in private hands in the previous period can then be determined from equation (5).

It has been shown that starting with the final values for the private stock (S_T) and price (P_T) in the absence of an auction, values for those same two variables one period earlier can be obtained. The process can be repeated to construct the time paths of the endogenous variables. Eventually, the private stock reaches the level specified by the initial condition, and the backward iteration is terminated. T is calculated by counting the number of backward steps required for termination.[18/] [19/]

The most striking part of the solution is the path of prices which would emerge in anticipation of an auction. This path rises at a rate greater than the interest rate, and the rate of increase grows as time passes.

Why would competitive extractors and speculators who are risk neutral require a path of prices which rises by more than the interest rate? According to the arbitrage condition, the expected price in period t+1 must have the same discounted value as the price in period t. Given the intuitive proposition that a government auction in any period would reduce the price,[20/] the price which would occur in period t+1 in the absence of government sales must have a larger discounted value than the price in period t.[21/] With a risk of government sales, no one would hold gold as an asset unless he received compensation for storing gold in the face of a possible capital loss.

The required compensation for storing gold must grow each period; that is, price must rise at an increasing rate. In order to establish this result, it must first be shown that the initial auction price rises at a rate less than the interest rate. If the government sold in period t-1, the stock in private hands at time t would be $S_{t-1} + \overline{G} - D(f_{t-1})$, and the price in period t would be $f_{t-1}(1+r)$. However, if the government did not sell in period t-1, but sold in period t instead, the stock in private hands at time t would be $S_{t-1} + \overline{G} - D(P_{t-1})$. Given that $P_{t-1} > f_{t-1}$ and that the demand curve slopes downward, the stock in private hands at time t would be larger if government sales did not occur until time t. Therefore, f_t must be less than $f_{t-1}(1+r)$ for all t. Since f_t is the (undiscounted) revenue per unit the government would receive for an auction at t, the longer the government waits to auction its gold, the smaller the present value of the proceeds.

The demonstration that price rises at an increasing rate can now be completed. Rearranging the arbitrage condition yields an expression for R_t, the percentage increase in price following period t:

$$R_t = \frac{P_{t+1} - P_t}{P_t} = \frac{\alpha}{1-\alpha}\{1 - \frac{f_{t+1}}{P_t}\} + \frac{r}{1-\alpha} \tag{6}$$

Since $f_{t+2} < f_{t+1}(1+r)$ and $P_{t+1} > P_t(1+r)$, $R_{t+1} > R_t$. The rate of increase in the price which would emerge in the absence of an auction must increase because the potential capital loss as a fraction of current price rises over time. From equation (6) it is evident that there is an upper bound to R_t:

$$R_t \leq \frac{\alpha + r}{1 - \alpha} \,. \tag{7}$$

It has been shown that market participants who are risk neutral require a path of prices which rises by more than the interest rate and accelerates as time passes to induce them to hold gold in the face of a possible capital loss. If holders of gold were <u>averse</u> to risk and therefore cared about properties of the discounted stream of profits besides its expected value, the return to gold holders would have to include a risk premium. Thus, the rise in gold prices would have to be even steeper, and R_t could exceed the upper bound given by (7).

The three gold price paths in Figure 2 illustrate what has been learned so far about the effects of the anticipation of government sales when gold holders are risk neutral. AA is the path of prices which would emerge with no possibility of government selling. Along this path, price rises at the rate of interest.

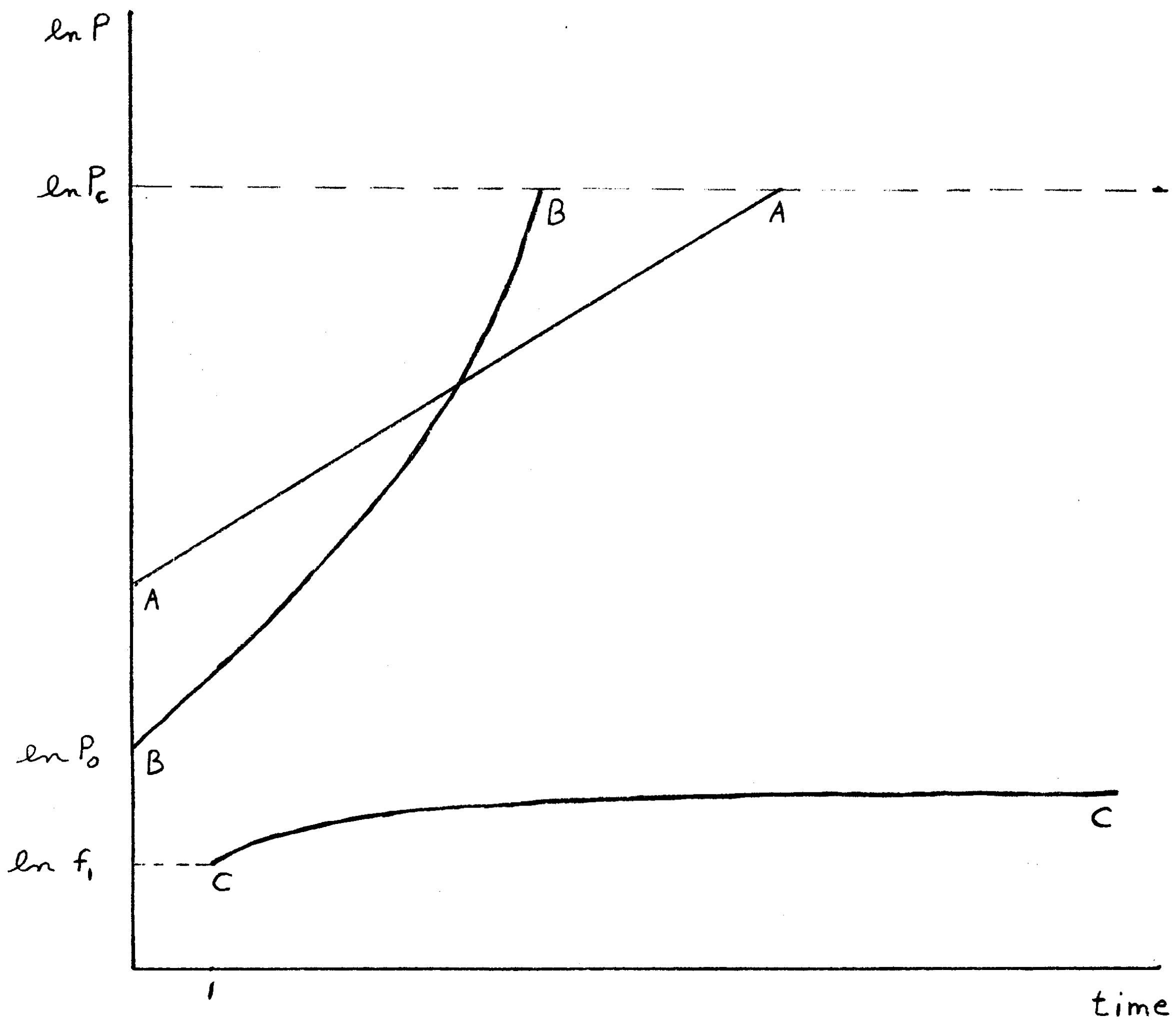


FIGURE 2

Since the natural logarithm of price is plotted against time, this path is represented by a straight line with a slope of $\ell n(1+r)$. BB is the path which would occur in the continued anticipation of sales, but prior to their occurrence. The slope of BB is always greater than the slope of AA, and the slope of BB rises as time passes. For completeness, CC shows the path of f_t, the price which would emerge each period if the government auctioned its stock in that period. CC rises at a rate which is always below the rate of interest.[22/]

Another important result is also reflected by paths AA and BB in Figure 2. The price path with risk begins lower, cuts the no-risk path, and chokes demand sooner. To understand why this is so, recall that the same stock must be depleted along both paths. Now suppose the two paths began together. Since prices on the path with risk rise at a faster rate, this path would remain above the other and would choke demand very early. However, since prices would always have been higher, less would have been consumed each period and, when demand is choked off, there would still be gold left. For the supply to be exhausted exactly when demand is choked, the price path with risk must begin lower. This implies that private gold stocks are exhausted more quickly than they would be without risk of government selling.

That the price path with a risk of government sales should start lower and rise more steeply is intuitively appealing. Suppose society had a fixed stock of some exhaustible resource and had some chance, each period, of finding a single additional, known amount. In that case, depleting more of the fixed stock in the early period than if there were no prospect of finding the additional amount would be sensible. However, if the additional stock were not "found", what remained after the early phase of rapid depletion should be rationed tightly. A group of competitive mine owners and speculators, acting in self-interest, would solve this "planning" problem by selling at low prices in early periods and very high prices later on.[23/] Of course, in the case of government sales of gold, the problem solved by the market is artificial and man-made; however, that makes the risk that additional supplies will topple the price no less real to private holders of gold.

As preparation for using the model to explain the historical path of the gold price in the next section, it is helpful to consider what the model predicts will happen to the path of prices when new information becomes available. Suppose an official statement is made about the sale of monetary gold. If this statement causes speculators and mine owners to raise their estimate of the probability of government sales, the price of gold falls and begins a new ascent; assuming that the intervention does not occur, the new path eventually rises above the old one and chokes off demand sooner. Figure 2 provides an illustration of this. When the odds of government selling increase (from zero to a positive fraction), the price immediately drops and follows the lower, steeper path. The reason for the drop, of course, is that holders of gold sell more quickly when faced with a higher probability of capital loss.

The discounted profit which a mine owner can expect from the sale of a given amount of gold is equal to its current market value. When the odds of government selling increase, the market value falls. This serves to remind us that while the better prospect of additional supplies is socially beneficial, the increased likelihood does injure owners of the existing stock.

Information causing an upward revision in the size of the anticipated government sale with no change in the probability that it will occur has a similar effect on the price path.[24/] The price path falls and, if anticipated auction does not occur, eventually rises faster, ultimately cutting the other path.

As in any exhaustible resource model, unanticipated events which cause market participants to revise their views about future demand affect the price immediately. In reality, market participants can only guess at the demand forthcoming at prices never before observed. If demand at those prices turns out to be substantially lower than expected, gold holders adjust their estimates of future demand downward. As a result, price jumps down and follows a new, lower path.

For simplicity, shocks causing daily fluctuations in the price of gold have been neglected. However. fluctuations caused by random shifts in demand or autonomous changes in sales by the Soviet Union can be easily incorporated into the analysis. Given shocks like these, the price path is no longer perfectly smooth. A low demand realization in one period, for example, reduces the price in that period. As in any "random walk" model, such a shock also reduces the discounted price then expected to prevail in all subsequent periods by the same amount. The reason is that the low demand realization leaves the private gold stock slightly larger than it otherwise would have been and this excess is expected to be sold off gradually over time.

IV. Gold Price Behavior in the Light of the Theory

During the period beginning in early 1970 and ending just after the first U.S. auction in early 1975, there were four sharp drops in the monthly average dollar price of gold (shown in Figure 1). Each of the first three price setbacks was followed by an upward surge in the price at a rate much greater than the rate of interest. The theory of Section III predicts that a price setback followed by a rapid advance will occur if market participants increase their estimate of the probability of subsequent government sales. In this section, market analyses and official statements are reviewed to determine whether or not gold holders had reason to revise upward their probability estimates at the time when each price drop occurred.

The first price decline began in August of 1972. Between August and November the price fell by 6%. Three events which occurred at the beginning of August might have led market participants to believe that official gold sales were more likely. First, there were reports in the first week of August that the U.S. Treasury might consider selling gold to domestic industrial users.[25/] Second, there were reports in the second week of August that the Fund would recommend selling gold from its stocks or from those of central banks to the private market.[26/] Third, in the first week of August a spokesman for the French Government denied rumors that France and Italy had agreed to urge a group of European countries to use an accounting gold price higher than the official price of $38 in dealings among themselves.[27/] This apparent failure to make progress toward a general agreement to use an accounting price in inter-central-bank transactions may have been interpreted as reducing the attractiveness of gold as reserve assets and, therefore, as raising the likelihood of government sales.

According to the theory, an increase in the probability of government sales is supposed to lead to an initial drop in price followed by a renewed rise. The rumors of government sales which had begun in August persisted, suggesting that market participants' estimates of the probability of government sales remained higher.[28/] Price began to rise again in November, 1972, and between that time and the break in July, 1973, it increased 91%. Although this acceleration is in keeping with the qualitative prediction of the theory, only aversion to risk could account for a rise of such magnitude.

The second major break in the gold price occurred in July, 1973. Between July and November, the price fell by 21%. In July, 1973, market participants had ample justification for revising upward their estimate of the probability of government sales. It was widely reported that the Committee of 20 Deputies were in favor of official sales of gold in the free market as part of the reformed international monetary system.[29/] In addition, some officials suggested that there was a possibility of such sales in the near future.[30/] These reports led to sales of gold from private holdings during the latter part of July. A noted market analyst observed that these reports "were instrumental in changing investors' attitudes to the gold market."[31/]

On November 13, an agreement permitting central banks to sell to the market was announced officially, but presumably because this information had already been discounted by the market, its effect was short-lived.

Soon after the official announcement was made, the price began to rise again in a way consistent with the theory. One gold broker, in concluding its 1973 review, left its readers with the warning that "the very large gold holdings of investors and central banks cannot be ignored."[32/] Prices rose quickly to a high of $172 in April, 1974, an increase of 82% over their level in November, 1973. The upward movement in prices was reinforced by suggestions by European monetary officials that European central banks should agree to exchange gold among themselves at a higher fixed price or at market related prices.[33/] These suggestions may have caused market participants to revise downward their estimates of the likelihood of government sales.

In April, 1974, the gold price began its third major decline. Between April and July the price fell by 17%. Although there were some renewed rumors of government sales during April,[34/] this dramatic price setback was probably largely due to a realization by market participants that they had misjudged the strength of demand at high prices never before observed. U.S. consumption data for the second half of 1973 reported by the Bureau of Mines on April 16, 1974, provided striking evidence that gold was being priced out of the U.S. market.[35/] While net purchases by U.S. industry had grown during each half year from 1970 through the first half of 1973, they fell by 23% between the first and second halves of 1973.[36/]

This decline was unexpected and pervasive. The annual report of one of London's major dealers, published only a month before the Bureau of Mines release, gave no indication that a decline in U.S. demand was anticipated. Indeed, the report expressed confidence in the continued buoyancy of U.S. demand.[37/] The actual change between 1972 and 1973, which could be calculated only after the Bureau of Mines release in April, was a drop of 8% (between 1971 and 1972, total demand increased by 5%); an advance in the first half was more than offset by the dramatic decline in the second.[38/] The release was sufficiently newsworthy for a leading commodity market newsletter to reprint it.[39/] Expecting a price decline, the editors advised their readers to sell gold short.

The reduction in net purchases by U.S. industry which took the market by surprise in April, 1974, has persisted, presumably in large part because of the higher prices which have continued to prevail. Net purchases by U.S. industry have remained far below previous levels since the second half-year of 1973, and world gold use has declined.[40/]

The rate of changes of gold prices up to April, 1974, may have been myopically correct in the sense that it provided the compensation gold holders required to offset the potential capital loss from government sales. However, the unexpected weakness of demand at high prices presumably revealed to market participants that the level of prices was too high and that a price collapse was inevitable in the future. This recognition would have led to large sales and a sharp decline in price.[41/]

In July, 1974, the gold price surged upward again and, except for a minor setback in September, continued to rise until December, 1974. By December price was 29% above the July level. The theory predicts that price should have begun to rise again once it had dropped to reflect market participants' reassessment of demand.

According to the theory, the announcement of an actual auction should bring to a halt a price surge such as the one which began in July, 1974. However, when Secretary Simon did announce on December 3, 1974, that the U.S. would auction an initial two million ounces of its gold stock in the following month, the price--after a momentary decline from $183 to $170.50--soon began to rise again, reaching an all time high of $197.50 on December 30.[42/] Only on December 31 did price begin the precipitous drop from which it has not yet rebounded.

The prompt recovery following the auction announcement and the ultimate collapse on December 31 can be understood only when account is taken of P.L. 93-373, which was enacted on August 14, 1974, after several months of consideration.[43/][44/] This law legalized ownership of gold bullion by U.S. citizens as of December 31 or, if the President so ordered, an earlier date. The belief by foreign speculators that U.S. citizens would desire to hold gold bullion in substantial quantities probably reinforced the tendency for the gold price to rise during the fall of 1974. A speech indicating that the U.S. government was considering a gold auction to offset this demand by U.S. citizens was delivered in October.[45/] As December approached, however, opposition to gold ownership for U.S. citizens began to mount in some official and Congressional circles. By unilaterally announcing the U.S. gold auction in his appearance before the Subcommittee on International Finance (House Committee on Banking and Currency), Secretary Simon effectively blocked further opposition to U.S. ownership. To foreign gold holders, Simon's announcement provided some good news along with the bad. This interpretation of possible effects of the announcement of the auction might explain why the price continued upward until December 30 with only a brief interruption. The failure of the anticipated U.S. demand to materialize on December 31 led to large sales and a collapse in the price.

That demand did not materialize is consistent with the simplified model. The legislation could not have affected U.S. demand for gold for industrial, artistic, or dental use, since licenses for such uses were freely available prior to its passage. Furthermore, given the risk neutrality assumption of the model, European speculators would already have driven the expected profits from speculation in gold bullion to zero, so there would have been no incentive for U.S. speculators to enter once they had the opportunity. Finally, it should be recalled that ownership of gold coins was legalized one year earlier.

The question which requires a response, then, is what form of U.S. demand could speculators in other countries reasonably have expected to materialize when ownership of gold bullion became legal on December 31. Clearly, it must have been some kind of demand for bullion which was unrelated to industrial, artistic, or dental use and which was not already satisfied by possession of gold coins. No matter what their attitudes toward risk, U.S. speculators should probably not have been expected to buy bullion since they already had access to coins which had virtually the same risk characteristics and sold for only a small premium (less than 10%) over bullion.[46/] It seems that the only unsatisfied demand which foreign speculators might have expected to materilaize was a demand for gold bullion to be held because of the services it would yield. There are, after all, people in the world who hoard gold bars because they believe that possessing gold in this form provides benefits unrelated to market price. Although it turned out that such people did not reside in large numbers in the U.S., market anticipation of their existence would not have been completely unreasonable, especially since possession of bullion had been a crime in the U.S. for forty years.

It must be admitted that the simplified model cannot account for the price movements related to the legalization of gold ownership. A model which could account for these price movements would have to include at least the possibility of a U.S. demand for the services of gold bullion.[47/] However, the price collapse on December 31 indicates that, if this demand existed at all, it was weaker than the market anticipated.

In summary, given the events which occurred, the theory predicts three of the four price drops in the period under consideration. The price rises which followed these drops are consistent with the qualitative predictions of the theory, but the presence of risk aversion is required to account for the magnitude of the rate of increase. While the theory does predict that the price should have begun to fall in December, 1974, it cannot without modification account for the price movements related to the legalization of U.S. gold ownership.

V. The Effects of Some Government Gold Policies

Up to now, attention has been focussed on the implications of market anticipations of a particular type of government gold policy. In this section, the effects of actually announcing and executing several alternative policies are considered in detail. So far, it has been assumed that any auction would be carried out the moment it was announced. Here, the case is considered in which a credible announcement is made that an auction will be held on a particular date in the future. In addition, two other government policies are analyzed: an announcement by the government that it will sell gold at any time at a specified price and an announcement by the government that it will both buy and sell gold at a specified price. For simplicity, it is assumed that only one government conducts the policy under consideration, or equivalently, that governments in all countries act in a coordinated fashion. It is also assumed that before a particular policy is announced the private sector anticipates in every period that the government might (with probability α) conduct an auction of a given size in the next period. This assumption is unimportant, however, since all the information about the path of prices prior to the announcement of a policy which is relevant for the analysis below is summarized in the size of the private stock available at the time the policy is announced.

Suppose that the gold price is moving along an accelerating path of the type described in Section III and that governments make a credible announcement at time k in Figure 3 that $\bar{G}$ units of gold will be auctioned off at some specified date in the future. Speculators and other holders of gold realize that on that future date the gold price will fall, leaving them with a capital loss. In the scramble to avoid that prospective loss, they sell gold they otherwise would have held in advance of the government auction.

The shape of the new price path depends on the length of time between the date when the auction is announced and the date when the auction actually occurs. If the auction occurs soon enough after the announcement, the equilibrium price path is path AA in Figure 3a. Path AA begins at f_k and rises at the rate of interest. This path is identical to the one which would emerge if government gold were actually sold on the date the auction is announced. However, if the auction occurs long enough after the announcement date, the equilibrium price path has two segments like BB and CC. The first segment lies above AA, the second below it, and both rise at the rate of interest. The exact position of each segment in this case depends on the date when the auction occurs.

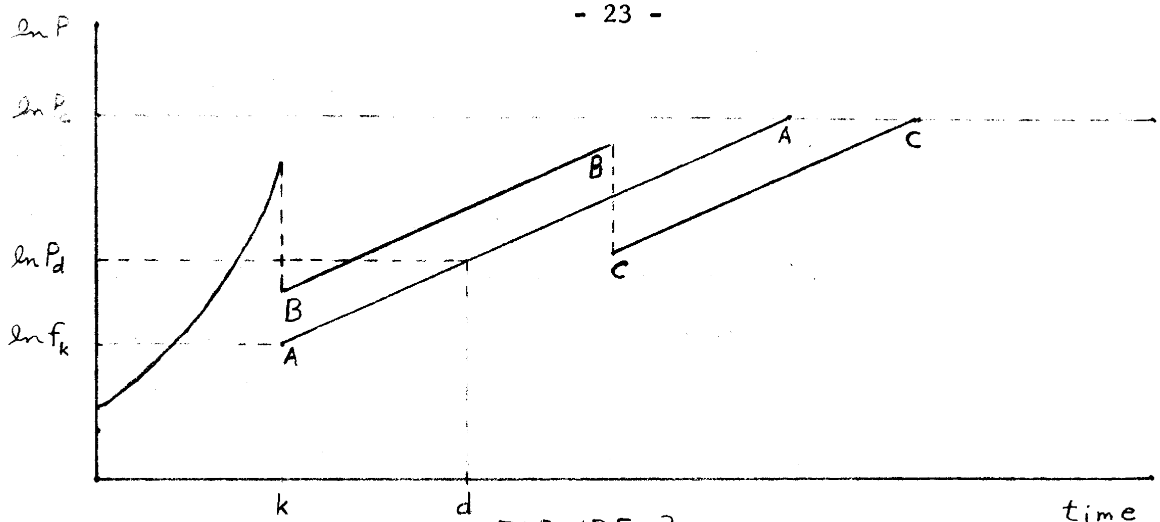


FIGURE 3a

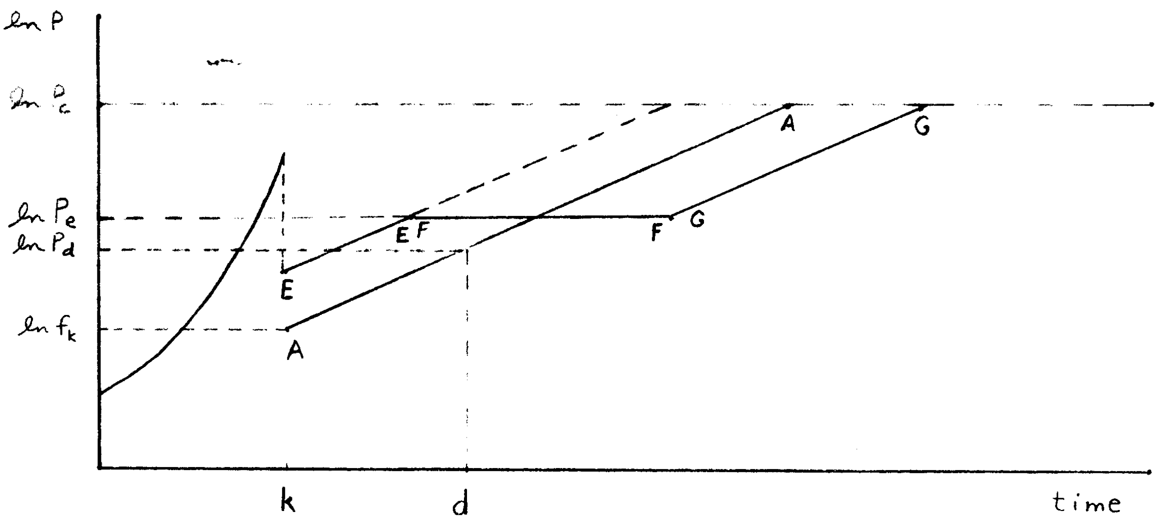


FIGURE 3b

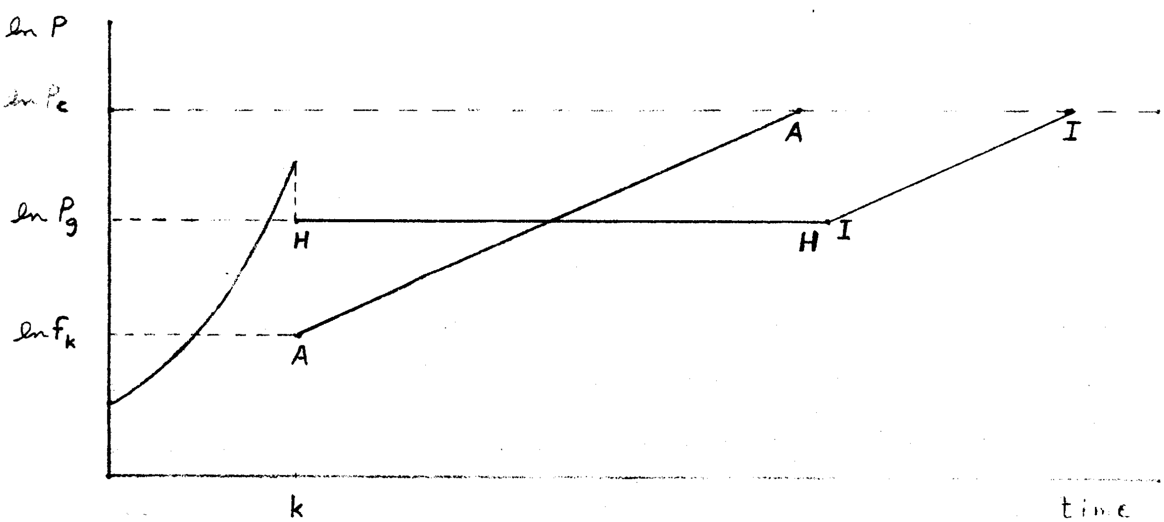


FIGURE 3c

Path AA results when the auction date is near enough to the announcement date. Along this entire path, the total world stock of gold available at time k is just equal to the cumulative demand. Moreover, at no intermediate point does the cumulative demand exceed the available stock. If the auction is to be conducted at some more distant date, say time d in Figure 3a, cumulative sales before d are just equal to the private stock available at time k, and cumulative sales after time d just exhaust the government stock.

If the auction is to be conducted at time a which is later than time d, AA could not be the equilibrium path. For along this path the private sector would run out of gold before it could acquire the government stock, and excess demand would occur between time d and time a. Instead, price first rises along path BB which is enough higher than AA to insure that cumulative sales before time a just exhaust the private stock available at time k. Then, at time a the auction occurs. Price must drop and then rise along CC which lies below AA since sales along CC must be equal to the entire government stock. Along this segmented price path consumers never want more gold than the private market has available. When faced with this segmented price path, holders of the existing stock want to deplete their inventories before the drop in price, and speculators are willing to purchase government gold at time a and sell it during the second phase. It is worth noting that the present discounted value of the revenue the government receives is smaller the further the auction date is beyond time d.[48]

The government could also announce that it will sell $\overline{G}$ units in a series of auctions of specified amounts scheduled for different dates in the future. It should be clear from the foregoing analysis that path AA can still be the equilibrium price path as long as cumulative demand at any point on this path is less than or equal to the private stock at time k plus the total amount the government will have auctioned by that time. Otherwise, the price path will consist of segments which rise at the rate of interest, and there will be price drops when auctions occur.

Consider the case in which the government announces at time k that it is willing to sell its gold at any time at some specified price. If this price is not too far above f_k, the equilibrium path is AA in Figure 3b. If the price is far enough above f_k, the equilibrium path is one like the path made up of the three segments EE, FF, and GG. In either case, speculators suddenly purchase the entire government gold stock at some time.

Path AA results when the specified price is below or equal to the dividing price, P_d. The dividing price is found by moving along path AA until cumulative sales just equal the private stock available at time k. It occurs at the dividing time, d. If the specified price is below or equal to P_d, speculators buy the government stock before or just when it is needed in order to satisfy demand along AA. The government stock is bought when the specified price is reached. Once again, the private sector behaves at time k as if it had the entire world stock at time k.

However, if the specified price is P_e, which exceeds P_d, AA is not the equilibrium path. The private sector would run out of gold too soon if price followed AA. Instead, price follows path EE which is enough higher than AA to insure that cumulative sales just match the private stock available at k by the time P_e is reached. Speculators are unwilling to buy the government stock when P_e is first reached because doing so would result in an immediate capital loss. The government stock is too large to be exhausted along the dashed continuation of EE. Demand is satisfied by government sales at price P_e, along segment FF, until the dwindling government stock is worth P_e to private gold holders. This happens when sales along a path beginning at P_e, and rising by the interest rate will just use up what gold is left. Speculators then "attack" and buy all the remaining gold from the government. This gold is sold along segment GG.

Both of the policies considered so far yield path AA unless the government auction is conducted too long after it is announced or the specified government selling price is too high. If governments offer at time k to buy as well as sell at a fixed price,[49/] path AA in Figure 3a will result only if the fixed price is below f_k. In that case, government gold is sold immediately at the fixed price. However, the market price falls only to f_k so those lucky enough to be able to buy from the government make an immediate capital gain. If the fixed price is set above f_k, say at P_g, the equilibrium price path has two segments like HH and II. The exact positions of HH and II depend on the level of P_g. The government can sustain P_g for a time. At first, it purchases the entire private stock from gold extractors who, under our assumption of zero extraction costs, extract and sell all their gold at the artificially high price.[50/] The government alone then supplies gold to consumers at the fixed price. Eventually, however, the augmented government stock is depleted to the point that it can be exhausted along a path like II which begins at P_g and rises by the interest factor. Then the remaining government gold is purchased in a swift speculative "attack".[51/] The events leading up to the dissolution of the central bank gold pool and the institution of a two tier market in March, 1968, suggest that this theoretical prediction cannot be taken lightly.

Appendix A

The purpose of this Appendix is to establish rigorously the intuitive proposition that a government auction in any period would reduce the gold price. Proof by contradiction is the technique employed. Suppose $P_t \le f_t$ for some t. Then it is shown that the same relationship would have to hold for all subsequent periods. However, the terminal conditions, equations (2) in the text, imply that $P_T > f_T$. Hence, the original supposition must be false and $P_t > f_t$ for all t.

If government sales occurred at t, equation (3) in the text implies that

$$\sum_{s=0}^{\infty} D[f_t(1 + r)^{s+1}] = S_t + \bar{G} - D(f_t). \tag{A1}$$

If, instead, the government auction occurred at t+1, equations (3) and (5) in the text imply that

$$\sum_{s=0}^{\infty} D[f_{t+1}(1 + r)^{s}] = S_t + \bar{G} - D(P_t). \tag{A2}$$

Now, suppose that $f_t \ge P_t$ for some t. If so, (A1) and (A2) imply that

$$\sum_{s=0}^{\infty} D[f_t(1 + r)(1 + r)^{s}] \ge \sum_{s=0}^{\infty} D[f_{t+1}(1 + r)^{s}]. \tag{A3}$$

Since the demand curve is downward sloping, $f_t(1 + r) \le f_{t+1}$. Hence,

$$P_t(1 + r) \le f_t(1 + r) \le f_{t+1}. \tag{A4}$$

Substituting this result into the arbitrage, equation (4) in the text yields

$$\alpha f_{t+1} + (1-\alpha)P_{t+1} \le f_t(1+r) \le f_{t+1}, \tag{A5}$$

or

$$P_{t+1} \le f_{t+1}. \tag{A6}$$

Thus, if $f_t \ge P_t$ for some t, the relationship must hold for all subsequent t. But, so long as $\overline{G} > 0$, it must be true that $P_T > f_T$. The supposition that $f_t \ge P_t$ for some t must, therefore, be false and instead, $P_t > f_t$ for all t.

Appendix B

In the text, we studied the effect on the private market of a single stochastic "arrival" of additional supplies of an exhaustible resource. It was shown that the price path is affected by the anticipations of owners of the existing stock even before the new arrival occurs. In the case of a stockpiled resource like gold, the arrival is the credible announcement of a government auction. However, the price path would be affected in the same way and for the same reasons if the stochastic arrival were instead a single new discovery resulting from exploration.

The results of Section III can be extended easily to situations of multiple arrivals. In the case of a stockpiled resource, many countries may decide independently when to auction specified amounts. Similarly, exploration may lead to new discoveries at different times. Since there should be considerable interest in incorporating new discoveries into the standard Hotelling model, we adopt the discovery terminology.

Suppose that there are n mines of known sizes left to be discovered, that the stock of "proved" reserves is X and that the rate of depletion $[D(\cdot)]$ depends on the current market price. Next period, one of the following 2^n mutually exclusive events must occur. No mines may be discovered, a particular one of the n mines may be located, a particular pair of the n mines may be found, etc. The probability that a specific one of the 2^n possible events will occur next period is assumed to depend only on the set of mines remaining to be located (or equivalently on the complementary set already located). The price resulting from each event will depend on the stock of proved reserves then available.

For simplicity, consider a case where n = 3. Denote by $\alpha_{AB\overline{C}}$ the probability that--with mines A, B, and C remaining to be discovered--A and B, but not C are located next period. Denote by $f(X|A,B,C)$ the price that would occur if only mines

A, B, and C remained to be located, and if the private stock were X. Our task is to determine the following 2^n (=8) equilibrium price functions:

$$f(X \mid \emptyset); \tag{B1}$$

$$f(X \mid A),\ f(X \mid B),\ f(X \mid C); \tag{B2}$$

$$f(X \mid A,B),\ f(X \mid A,C),\ f(X \mid B,C); \tag{B3}$$

$$f(X \mid A,B,C). \tag{B4}$$

These functions will enable us to compute the equilibrium price under every conceivable circumstance (i.e., state of the system).

The price that would emerge with no mines remaining undiscovered and X ounces in private hands $[f(X \mid \emptyset)]$ can be determined from equation (3) of the text.[52/] The price that would emerge in each of the three cases above where only one discovery remains and the private stock is X units $[f(X \mid A), f(X \mid B), f(X \mid C)]$ can be derived by solving the model of the text using each of the three different specifications for $\bar{G}$ (G_A, G_B, G_C). The remaining four functions listed above require further discussion.

With two particular mines left to be located (say A and B), four events can occur on the next period: both, a particular one, or none of the mines may be discovered. If S denotes the stock which the private market would then own in the absence of any new discovery, the four prices which might occur are $f(S \mid A,B)$, $f(S + G_A \mid B)$, $f(S + G_B \mid A)$, and $f(S + G_A + G_B \mid \emptyset)$. In equilibrium, the price this period will be driven equal to the discounted price now expected to occur next period:

$$f(S_t \mid A,B) = \frac{1}{1+r} [\alpha_{\bar{A}\bar{B}} f(S_{t+1} \mid A,B) + \alpha_{A\bar{B}}\, f(S_{t+1} + G_A \mid B)$$

$$+ \alpha_{\bar{A}B}\, f(S_{t+1} + G_B \mid A) + \alpha_{AB} f(S_{t+1} + G_A + G_B \mid \emptyset)], \tag{4''}$$

where

$$S_{t+1} = S_t - D[f(S_t \mid A,B)]. \tag{5''}$$

To compute the function $f(X \mid A,B)$, note that the probabilities and all the other functions in the arbitrage equation (4") above are known. Furthermore, the price which will clear the market should the private stock be exhausted before mines A and B are located is simply the choke price:

$$f(0 \mid A,B) = P_c. \tag{B5}$$

Using this terminal condition, the depletion equation and the enlarged arbitrage equation (4"), the remaining points[53/] of the function $f(X \mid A,B)$ can be computed by backward recursion. The same procedure can be used to compute the prices which would emerge if a different pair of mines remained to be discovered: $f(X \mid B,C)$ or $f(X \mid A,C)$.

An analogous procedure permits computation of the price if all <u>three</u> mines remain to be discovered ($f(X \mid A,B,C)$). The calculation would again involve the same depletion equation and terminal condition. However, the arbitrage equation would differ. With three mines remaining to be located, the expectation of next period's price must be taken over each of the <u>eight</u> possible events which might occur. This would require using each of the seven functions previously obtained.[54/] The eighth would be constructed by backward recursion. Once these 2^n functions are computed, we can use them to determine the price which would occur in any conceivable state as the stochastic process of discoveries or auctions unfolds.

In the case of multiple auctions or discoveries, price again rises smoothly by more than the rate of interest until a new supply arrives. Every time an auction or discovery occurs, the price drops. At any time, the price is again equal to the discounted value of the price then expected to occur in the future.

The rising discounted price which emerges in the absence of a new arrival is just enough to compensate risk-neutral gold owners for the asymmetric risk of a drop in price which would result from an additional supply.

Appendix C

In the model of the text it is assumed that gold bullion provides neither psychic nor rental income to its owners, so the only reason for holding gold is for expected capital gain. Since gold owners are assumed to be risk neutral, the expected capital gain must equal the real rate of interest. This model could, of course, be complicated in a variety of ways. For example, it might be assumed that gold bars can be irreversibly fabricated into jewelry which can be rented. But it is only if the bars themselves provide psychic or rental income that the expected rate of increase in their price can differ from the rate of interest.

In this Appendix it is assumed that people exist who benefit from the mere possession of gold bullion. Such people are willing either to rent bullion or own it when its expected rate of price increase is less than the rate of interest. The effect of these agents on the price path is explored, first in the absence and then in the presence of government stockpiling. As before, it is assumed that gold bullion transformed into jewelry, dental inlays, and industrial products is never recoverable so that the stock of gold bullion diminishes at the same rate as it is used as an input in these transformation processes.

1. The Effects of Stock Demand with No Government Stockpiling

To begin, assume that some demand for the services of the gold stock exists and that there is no risk of government sales. At the present time, the private gold stock is held in part by pure speculators and in part by those who enjoy its services. This pattern of gold holding implies that the stock of gold exceeds the amount which people enjoying its services would want at any positive rental. If gold were less abundant, those enjoying its services would presumably be willing to rent it from its owners or, alternatively, to own it themselves, even when its price ascent failed to keep up with the rate of interest. In this latter event, the foregone earnings would constitute an implicit rental. However, under current circumstances, the gold stock will only be held if its price rises at the rate of interest because the rate of price ascent attracts even people who do not enjoy its services and who receive no income from renting it.

In the future, however, industrial depletion will reduce the stock until it just matches the amount demanders are willing to hold at a zero rental. Thereafter, only those who enjoy the services of gold will physically possess it. They will either rent it from others or own their gold outright, foregoing the income that they could earn either from renting to others or from investing in alternative assets. Industrial use will continue, although at a diminishing rate, and the strength of stock demand will determine whether or not depletion ultimately ceases with some positive amount remaining.

These conclusions can be derived in a model which incorporates stock demand. Let $\hat{r}$ represent the force of interest; P, the purchase price per unit of gold; μP, the rental price per unit time for the services of a unit of gold; $D^S(\mu P)$, the stock demand; and $D^f(P)$, the flow demand for uses which deplete the stock.[55/]

In continuous time the dynamic system is

$$\mu\{S - D^S(\mu P)\} = 0, \tag{C1}$$

$$\dot{P}/P = \hat{r} - \mu, \tag{C2}$$

$$\dot{S} = - D^f(P). \tag{C3}$$

Equation (C1) says that the rental price induces people to hold the existing stock: either the market clears at a positive rental or, as is now the case, there is inadequate demand for the services of the stock at any positive rental and the rental is therefore zero. In this event, the excess supply is held by speculators who earn no rental income. Equation (C2) says that the percentage capital gain plus the percentage gain from renting the gold to someone else must compensate the rational gold owners for the interest he foregoes by not selling today and bank- the proceeds. Equation (C3) indicates the rate at which the stock is depleted through industrial use. For any price and stock level, the system determines the rate of change of the price and stock levels as well as the rental percentage required to induce people to hold the existing stock of gold. The entire evolution of the system is determined for any initial price and inventory.

The initial inventory ($\bar{I}$) is inherited. To see the consequences of different initial price assignments, a phase diagram (Figure 4) is helpful. Motion in the horizontal direction stops at all prices above the choke. If, at some price and stock level, gold holders are willing to pay rentals equaling the interest which gold owners forego by not selling their stocks ($\mu = r$), capital gains are zero and motion momentarily stops in the vertical direction. If the price at which this occurs is the choke, motion stops permanently. The locus of (P,S) combinations for which $\dot{P} = 0$ satisfies the following equation:

$$D^S(rP) = S. \tag{C4}$$

This locus is downward sloping and, if the current rental rate is assumed to be zero, cuts the horizontal axis to the left of $\bar{I}$. The locus cuts the vertical axis either below the choke price or above it, depending on whether $D^S(rP_c) \gtrless 0$. Assuming demand at that rental is positive, the locus intersects the $\dot{S} = 0$ line at the positive stock (S^*) solving $D^S(rP_c) = S^*$. Given the initial stock, all the initial price assignments except one result in eventual excess supply or demand. For markets to clear at every moment, the initial price must be set on the unique stable-arm going into the saddle point (P_c, S^*). With this initial assignment,

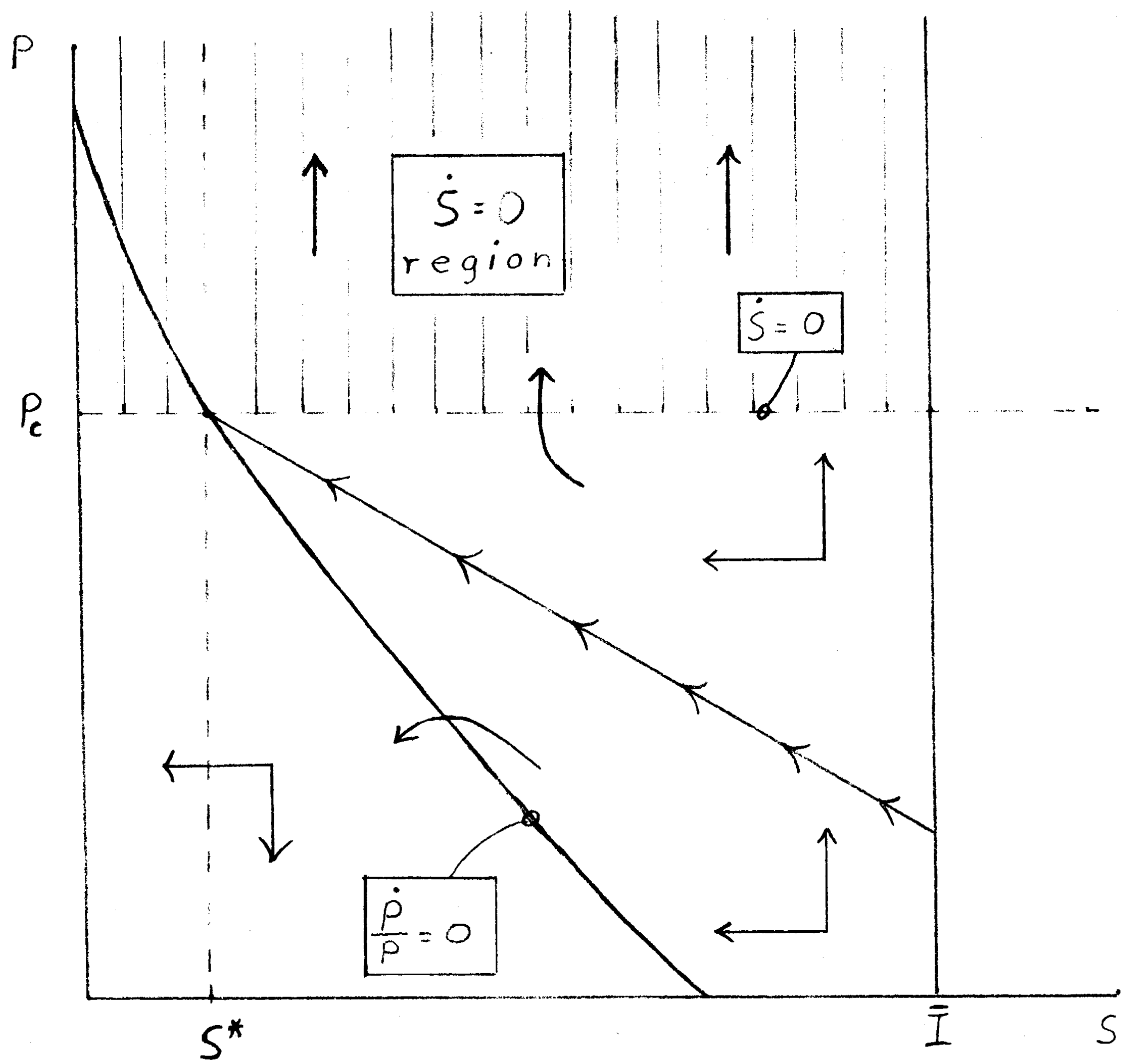


FIGURE 4

price begins to grow at the rate of interest and stock depletion proceeds as in the standard Hotelling model. However, rentals eventually become positive and the percentage increase in price then begins to decline. Finally, industrial use ceases (asymptotically) and eventually S* units of the stock remain to be either rented or owned outright by those who enjoy its services.[56/]

2. The Effect of Government Stockpiling

We now consider how risk of government sales alters this situation. Denote $\hat{\alpha}$ as the (instantaneous) probability of an auction, f as the price which would occur in that auction, and $\mu'f$ as the rental price which would prevail immediately after that auction. Then the following equations must hold at every moment:

$$\mu[S - D^S(\mu P)] = 0, \tag{C5}$$

$$\mu'[S + \bar{G} - D^S(\mu' f)] = 0, \tag{C6}$$

$$\frac{\dot{P}}{P} = \hat{r} - \mu + \hat{\alpha}(1 - f/P), \text{ [57/]} \tag{C7}$$

$$\dot{S} = - D^f(P), \tag{C8}$$

$$\frac{\dot{f}}{f} = [\hat{r} - \mu'] \frac{D(f)}{D(P)} . \text{ [58/]} \tag{C9}$$

Given any choice of S, f, and P, the system determines rates of change of each of these three variables as well as the rental percentages (μ, μ') required to induce people to hold the pertinent stocks. Once again, the initial inventory ($\bar{I}$) is inherited and any initial assignment of f and P determines the motion of the system forever after. Only one assignment, however, will satisfy the terminal conditions which must be chosen both to eliminate foreseeable profit opportunities, and to insure that markets clear at all moments in time whether or not the auction occurs.[59/]

If stock demand is identically zero, equations (C5)-(C9) reduce exactly to the continuous-time analogue of the model in the text.[60] But even if stock demand is positive, the motion of this complicated system coincides with that of the simpler case in the text, as long as rentals (μ, μ') continue to be zero. Price again grows at more than the rate of interest and the percentage rate itself increases for exactly the reasons previously discussed. Only in the future, when rentals become positive, will the price ascent be affected by the additional complications from which we abstracted.

Footnotes

*Economists, Board of Governors of the Federal Reserve System. Our colleague, Jeffrey Shafer, showed us that if there were some reason why price might fall to a constant floor, observed prices would accelerate away from that floor. For his initial idea and subsequent insights, we wish to acknowledge our debt. Helpful comments on Salant and Henderson (1974) and several memoranda which formed the basis for this paper were received from Ralph Bryant, George Henry, Walter Salant, Steven Salop, Charles Siegman, Edwin Truman, and Henry Wallich. No one but the authors is responsible for the paper's remaining shortcomings. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Federal Reserve System or other members of its staff.

1/ The basic theory of exhaustible resources was developed by Hotelling (1931). Solow (1974) provides a lucid exposition of this theory.

2/ Annual world gold extraction (excluding the U.S.S.R.) in metric tons for the years 1970 to 1974 was 1275, 1234, 1157, 1092, and 995, according to Samuel Montagu and Co., Ltd. (1974), p. 15, and (1975), p. 10. Gold production in 1970 was higher than in any post World War II year except 1966 when extraction was 1285, according to Samuel Montagu and Co., Ltd. (1973), p. 15. Gold reserves of the industrial countries at the ends of the years 1970 to 1974 in metric tons were 27677, 26965, 26460, 26457, and 26458 according to the International Monetary Fund (1975), p. 20. Gold reserves of the industrial countries were thus 22, 22, 23, 24, and 27 times annual extraction in the years 1970 to 1974. Gold reserves of all countries for the years 1970 to 1974 in metric tons were 32866, 31905, 31631, 31637, and 31593 according to ibid.

3/ Consumers, who purchase goods and services produced using gold, ultimately determine how much gold is bought by jewelry fabricators, dentists, and electronics firms. It is these derived demands which add up to the flow demand curve. Gold used for these purposes is assumed not to be used again, either because of the taboos of society, or because of prohibitive costs of recovery.

4/ The models investigated in this paper explain movements in the price of gold relative to the price of a constant-marginal-utility background good which cannot be stored. The relative price of gold must rise by the real rate of interest, r. r is defined by

$$(1 + r) = (1 + i^{\$})/(1 + \pi^{\$}),$$

where $i^{\$}$ is the nominal rate of return on dollar denominated assets and $\pi^{\$}$ is the rate of increase in the dollar price of the background good. Therefore, the nominal dollar price of gold must rise by the nominal rate of return on dollar assets. The nominal dollar price of gold is plotted in Figure 1, and the discussion in Section IV is conducted in terms of the nominal dollar price. Likewise, the pound price of gold, for example, must rise by the nominal interest rate on assets denominated in pounds, $i^{£}$, and

$$(1 + i^{£}) = (1 + i^{\$})/(1 + d),$$

where d is the rate of depreciation of the dollar. For simplicity, it is assumed in the models that the opportunity cost of storing gold can be represented by a single real interest rate. Of course, in attempting to check the theory against experience, one would have to construct some summary measure of the variety of real interest rates available to potential gold holders.

5/ They would not wish to consume if all the other prices exceed the choke price. But then, so would the price in the high period and sellers would never find buyers willing to purchase anything, let alone all gold in existence.

6/ It does suggest that gold extraction might fall as price rises, a phenomenon which has occurred, to the puzzlement of some analysts.

7/ South African extraction averaged 78.4% of annual world extraction (excluding the U.S.S.R.) during the period 1969-1973, according to figures from Samuel Montagu and Co., Ltd. (1974).

8/ Machlup (1969) suggests that South Africa would exercise monopoly power whenever it had the opportunity.

9/ In 1970 and 1971, South Africa split its market, selling gold to the IMF and some central banks at $35 an ounce and to the free market at the higher market price of $40. Such behavior implies that South Africa estimated the marginal revenue of selling another unit on the private market to be $35, or equivalently, estimated the elasticity of the (excess) demand facing it to be about 8.

To infer the elasticity of aggregate demand from this information requires an additional assumption about how South Africa perceived its dominant position in the gold market. If it did not take into consideration how other sellers would react to changes in its sales (the "Cournot" assumption), the elasticity of aggregate demand would simply equal the elasticity of excess demand, estimated above, multiplied by South Africa's market share at the time. If it considered that other extractors would sell more when it sold less (the "Stackelberg" assumption), the implied elasticity of aggregate demand would be lower.

10/ There may be a fourth characteristic of gold which distinguishes it from the exhaustible resource of the simple model. It has been suggested that some people may derive satisfaction from simply holding gold bullion no matter whether its price is rising or not. In Appendix C, the theory is modified to take account of stock demand for gold by individuals who derive services from holding bullion. In brief, it is shown there that, given a stock demand, gold will eventually become so scarce that it may be rented for the services it provides. For this reason, the level of the price path and the future rate of change of prices will be affected. However, since bullion is now held by some individuals purely as an investment, the rental is currently zero and the present rate of change of prices is not affected by this complication.

11/ Hotelling (1931) and Solow (1974) study competitive extractors who face positive marginal extraction costs. The behavior of a single monopolistic extractor is analyzed in Hotelling, Weinstein and Zeckhauser (1975), and Stiglitz (1976). What happens when a single dominant extractor coexists with a competitive fringe is described in Salant (1976) and Stiglitz (1976), using respectively a "Cournot" and a "Stackelberg" approach.

12/ The constraints imposed on a monopolist by competitive speculators are completely analogous to those considered by Smithes (1941). He considered the single producer of a commodity who wishes to sell it in spatially-separated markets at different prices, but who knows he cannot (because of arbitrageurs) let prices diverge between markets by more than transportation costs. In an intertemporal model the markets are separated in time, and the cost of transporting through time is the rate of interest.

The behavior of a monopolistic extractor constrained by the existence of competitive speculators has been analyzed by Stiglitz (1976).

13/ This observation was made by Sidney Rolfe in the New York Times of May 24, 1974.

14/ It might be assumed instead that gold holders are uncertain about the amount the government will sell if an auction occurs. The technique for solving the model when it contains this alternative assumption is explained in footnote 19.

15/ The behavior of the model when market participants believe that there may be multiple auctions is discussed in Appendix B.

16/ Equation (4) indicates that the discounted price expected in period t to prevail one period in the future is equal to the price in period t. It can be verified that a similar relation holds for the price in period t expected to prevail k periods in the future. That is,

$$\frac{E(X_{t+k} \mid P_t)}{(1+r)^k} = P_t,$$

where X_{t+k} is the random price in period t+k. X_{t+k} may take on the following k+1 values:

$$f_{t+i}(1+r)^{k-i} \quad \text{with probability} \quad \alpha(1-\alpha)^{i-1} \qquad i = 1,k$$

$$P_{t+k} \quad \text{with (complementary) probability} \quad (1-\alpha)^k.$$

Thus,

$$\frac{E(X_{t+k} \mid P_t)}{(1+r)^k} = \sum_{i=1}^{k} f_{t+i}(1+r)^{-i} \alpha(1-\alpha)^{i-1} + P_{t+k}\left[\frac{(1-\alpha)}{(1+r)}\right]^k .$$

That P_t is equal to this expression can be verified by starting with (4) and substituting successively for P_{t+j} values obtained by moving (4) forward j periods. After k-1 substitutions, the desired result is obtained.

17/ In continuous time, equations (3) - (5) become

$$\int_{s=0}^{\infty} D[f(t)e^{\hat{r}s}]ds = \overline{G} + S(t) \tag{3'}$$

$$\frac{\dot{P}(t)}{P(t)} = \hat{r} + \hat{\alpha}\left[1 - \frac{f(t)}{P(t)}\right] \tag{4'}$$

$$\dot{S}(t) = - D[P(t)] \tag{5'}$$

where $\hat{\alpha}$ and $\hat{r}$ are the continuous time analogues of α and r and

$$\hat{\alpha} = \ell n(1+\alpha),$$

$$\hat{r} = \ell n\ (1+r).$$

(4') may be derived in the following way. Holders of gold will force the current price to equal the discounted price now expected to prevail t moments in the future. This expected price is equal to the probability weighted average of the prices which would prevail at t if the auction occurred at different moments before t or if no auction occurred through t, so

$$P(0)e^{\hat{r}t} = P(t)e^{-\hat{\alpha}t} + \int_{s=0}^{t} f(s)e^{\hat{r}(t-s)} \cdot \hat{\alpha}e^{-\hat{\alpha}s}ds.$$

(4') is obtained by differentiating this equation and simplifying the result.

18/ The private stock may never build up exactly to the initial inventory since the analysis is in discrete, not continuous, time. In this event, the simplest computational trick is to replace the terminal price P_T with a number larger than the choke price but less than $[1/(1-\alpha)][1 + r - \alpha f_T/P_c]$ times as large.

19/ When it is assumed that gold holders are uncertain about the amount the government might sell, the solution technique is only slightly different. For simplicity, assume that the distribution of possible auction sizes is independent of the time of the auction and the state of the system. For every auction size (g), the initial price of the auction $[f_t(g)]$ can be obtained using the following equation:

$$\sum_{s=0}^{\infty} D[f_t(g) \cdot (1+r)^s] = g + S_t \ .$$

Using the probability distribution on $\tilde{g}$, the expected auction price $(\bar{f}_t)$ can be computed. This number replaces f_t in the backward solution described in the text.

20/ This proposition is established rigorously in Appendix A.

21/ If $f_t < P_t$, then from (4)

$$P_t(1+r) = \alpha f_{t+1} + (1-\alpha)P_{t+1} < \alpha P_{t+1} + (1-\alpha)P_{t+1}.$$

Hence,

$$\frac{P_{t+1} - P_t}{P_t} > r.$$

22/ It is instructive to rederive the result that the potential auction price $[f(t)]$ rises at less than the rate of interest using the continuous time formulation of the model. Equation (3') in footnote 17 implicitly defines the potential auction price $[f(t)]$ as a function of the private stock $[S(t)]$. The rate of depletion of the stock determines the rate of change of the auction price:

$$\dot{f}(t) = \frac{df(t)}{dS(t)} \dot{S}(t).$$

If an auction occurred, the resulting price would generate exactly enough depletion to make the price rise at the rate of interest:

$$\frac{\dot{f}(t)}{f(t)} = \frac{1}{f(t)} \frac{df(t)}{dS(t)} \{ - D[f(t)]\} = \hat{r}.$$

This relationship provides insight into the $f[S(t)]$ function. Specifically,

$$\frac{1}{f(t)} \frac{df(t)}{dS(t)} = - \frac{\hat{r}}{D[f(t)]} \cdot$$

In the absence of an auction, the actual price is higher and depletion $[\dot{S}(t)]$ is, therefore, lower. Hence, the auction price rises at less than the rate of interest:

$$\frac{\dot{f}(t)}{f(t)} = \frac{1}{f(t)} \frac{df(t)}{dS(t)} \dot{S}(t) = \hat{r} \frac{D[P(t)]}{D[f(t)]} \cdot$$

The rate of change of the potential auction price is thus equal to the force of interest multiplied by actual depletion expressed as a percentage of the depletion which would have occurred in the event of an auction, so the rate of change of the potential auction price is less than the force of interest.

The time derivative of the rate of change of the potential auction price can, of course, be obtained from the last of the expressions above. A sufficient condition for the time derivative to be negative (a property assumed in drawing Figure 2) is that the demand curve be "normal" in the sense that the absolute value of the elasticity of demand rises with price.

23/ If society can make the probability of discovering the additional stock in the next period unity without cost, such a policy is socially optimal.

24/ However, even if the hypothetical increase in α and the hypothetical increase in G are set so as to make the initial drop in price the same, the two price paths are not identical. For example, it can be seen from equation (7) that the limit on the rate of increase in prices would be different for the two paths since the α's would be different.

25/ These reports appeared in the New York Times, August 2, 1972, p. 47, and August 7, 1972, p. 39.

26/ News of Fund intentions was reported by the London Financial Times, August 4, 1972, p. 40, Reuters, August 4, 1972, and August 23, 1972, and the New York Times, August 7, 1972, p. 39.

27/ Rumors of a French-Italian agreement were reported by Reuters on August 1, 1972, and by Agence France Presse on August 2, 1972. Less specific reports that several European countries were considering entering into an agreement to use a price higher than the official price for inter-central-bank transactions appeared in the London Financial Times, July 31, 1972, p. 5, and August 1, 1972, p. 38, and in the Washington Post, August 1, 1972, p. D6. The French denial of the rumors regarding a French-Italian agreement was carried by Reuters on August 1, 1972, and by Agence France Presse on August 2, 1972.

28/ Samuel Montagu and Co., Ltd. (1973), p. 12, noted that "Serious consideration has therefore also been given to sales of gold by monetary authorities and these cannot be excluded in the future".

29/ Reports of the Committee of Twenty Deputies' views were carried by Reuters on July 13 and 16, 1973, Agence France Presse and United Press International on July 13, 1973, the Washington Star-News, July 14, 1973, p. A3, the Baltimore Sun, July 14, 1973, p. A4, and the London Financial Times, July 21, 1973, p. 14.

30/ An example is contained in the interview with Klassen in Business Week, July 14, 1973, p. 72.

31/ This quotation is from Samuel Montagu and Co., Ltd. (1974), p. 6.

32/ Ibid., p. 13.

33/ According to Samuel Montagu and Co., Ltd. (1975), p. 4, on January 24, 1974, the E.C. Commission Vice President appealed for a higher gold price in the E.C., and on February 11, 1974, the French Finance Minister suggested that E.C. countries should use gold in intra-community settlement at market-related prices.

34/ Reports of possible government sales were carried by Reuters and Agence France Presse in April 29, 1974.

35/ This data was reported in the Bureau of Mines (April 16, 1974), p. 6.

36/ Net purchases of gold by U.S. industry in thousands of fine ounces by half years for the period 1970I to 1973II were 2940, 3033, 3255, 3708, 3527, 3748, 3810, and 2919. The figures for 1970I to 1971II were obtained from the Office of Domestic Gold and Silver Operations, Department of the Treasury. The remaining figures were obtained from the Bureau of Mines, op. cit., p. 6.

37/ This view was expressed in Samuel Montagu and Co., Ltd. (1974), p. 10.

38/ According to the Bureau of Mines, op. cit., p. 6, annual net purchases of gold by U.S. industry for 1971 through 1973 in thousands of fine ounces were 6933, 7285, and 6729.

39/ The commodity market newsletter is Green's Commodity Market Comments, May 8, 1974, p. 4.

40/ Net purchases of gold by U.S. industry in thousands of fine ounces from 1974I to 1975I were 2219, 2432, and 1529 according to the Bureau of Mines (September 4, 1975), p. 11. World gold use in electronics, dentistry, and other industrial and decorative uses declined from 277.3 metric tons in 1973 to 208 metric tons in 1974 (one metric ton equals 32,150 fine ounces) according to Consolidated Gold Fields Limited (1975), pp. 22-24. The decline in gold use in 1974 and 1975 compared to previous periods can be explained only in part by the high gold price; the world economic slowdown reinforced the effect of the high gold price.

41/ At a meeting of E.C. Finance Ministers on April 23, there was apparently an agreement to propose to the U.S. and other countries that central banks should be permitted to carry out gold transactions among themselves at fixed prices. This agreement was reported by Le Figaro, April 24, 1974, p. 1, and by the London Financial Times, April 24, 1974, p. 20. The Journal of Commerce reported on April 25, 1974, p. 1, that the U.S. opposed any such arrangements, and on April 26, 1974, p. 1, that nothing had really been settled at the April 23 meeting. Apparently, market participants concurred, since Reuters, on April 24, cited disappointment with the results of the meeting as reason for a price decline that day.

42/ These price figures are from Samuel Montagu and Co., Ltd. (1975), p. 9.

43/ On May 29, 1974, the Senate passed a bill which included as an ammendment the repeal of the prohibition against the holding of gold by U.S. residents. The House passed a bill carrying an ammendment which contained a different version of the repeal on July 2, 1974. The final version of the repeal cleared both Houses on July 31, 1974. President Ford signed the repeal into law as P.L. 93-373 soon after his succession on August 1, 1974.

44/ Another event which might conceivably help to account for the prompt recovery in price was the agreement between the Presidents of France and the United States that countries could revalue their gold holdings at market related prices if they wished. This Martinique Agreement of December 16 may have convinced the market that gold would be more attractive as a reserve asset, so that further official sales would be less likely. However, that such news could offset the effect of the announcement of the U.S. auction seems improbable.

45/ The possibility of a U.S. auction was mentioned by Wolfe (October 1, 1974), p.2.

46/ This information appears in Simon (December 3, 1974), p. 3.

47/ In Appendix C, the simplified model is expanded to include demand for the services of gold bullion. The expanded model can be used to explain the price movements which occurred in response to the new legislation. An announcement that stock demand will be larger after some future date causes an immediate increase in the bullion price. If the increased demand does not materialize, however, price drops.

48/ None of the strategies considered in this Section maximize the discounted revenue the government receives by selling its gold stock. If the government sought to maximize its discounted revenue in a market with other sellers, it would behave like a dominant firm or an oligopolist. Salant (1976) and Stiglitz (1976) analyze such market structures for the case of an exhaustible resource.

49/ They could not buy without abrogating Article IV, section 2 of the IMF Articles of Agreement.

50/ With increasing marginal extraction costs, each mine extracts more gradually and sells to the government over time. Optimally, extraction proceeds at a declining rate so that marginal profit (the fixed government buying price less marginal cost) rises at the rate of interest. During this phase, consumers buy gold either directly from mines or else from the government. Eventually, however, the remaining government gold stock begins to be worth more than the price charged by the government. At that moment, the entire government stock is purchased in a speculative attack. It can be shown that speculators and extractors then coexist in the market for some period of time with both supplying bullion to consumers. Finally, however, the speculators exhaust their supplies and the mines alone supply consumers until demand is choked off.

51/ The analysis implies that pegging the price of gold relative to the constant-marginal-utility background good (see footnote 4) results in a speculative attack. Of course, pegging a nominal price in an inflationary world makes the inevitable occur more quickly.

52/ In discrete time each function must be approximated. In the continuous-time counterpart, this would be unnecessary. See footnotes 17 and 18.

53/ See the previous footnote.

54/ More generally, with k of n mines remaining to be discovered, 2^k possible events can occur on the next period. The arbitrage equation would involve the 2^k-1 functions previously obtained.

55/ The model would be further complicated if each demand curve depended both on the purchase price and the rental price as might be the case in industries where some fraction of gold inputs was recovered and the rest was depleted.

56/ If the $\dot{P} = 0$ locus cuts the vertical axis below the choke price $[D^S(rP_c) < 0]$, the phase diagram has no stationary point. In that case, the initial assignment of price is on the unique trajectory which leads to complete exhaustion.

57/ Equation (C6) may be derived from familiar arbitrage consideration. A person who owns gold could sell it and bank the proceeds, or he could rent someone the gold and sell it at the end of a short interval, h. For a short enough interval, the probability of a government sale and the interest accumulated are each directly proportional to its width. Therefore, the arbitrage condition is

$$P(t) + \hat{r}hP(t) = \mu(t)hP(t) + (1-\hat{\alpha}h)P(t+h) + \hat{\alpha}hf(t+h).$$

Dividing by hP(t) and collecting terms, yields

$$\frac{P(t+h) - P(t)}{hP(t)} = \hat{r} - \mu(t) + \frac{\hat{\alpha}P(t+h)}{P(t)} - \frac{\hat{\alpha}f(t+h)}{P(t)} .$$

As $h \to 0$, equation (C6) emerges in the limit.

58/ Equation (C8) may be derived from considerations discussed in footnote 22. If the auction had occurred, the price would drop to f and would begin growing at the percentage rate $\hat{r} - \mu'$. Depletion would then be occurring at rate $D^f(f)$. In the absence of the auction, however, the stock will be depleted at a rate only $\frac{D^f(P)}{D^f(f)}$ times as fast. Therefore, the ascent of the potential auction price is slower by that factor.

59/ In the absence of an auction, depletion will terminate at the price P_c. Denote by S** the stock which then remains. This stock will be combined with the government stock once the auction occurs. Therefore, its amount helps determine the price of that auction $[\bar{f}(S^{**} + \bar{G})]$. Both the price of that auction and the combined stock then determine the subsequent trajectory. The laws of motion of that subsequent trajectory conform to equations (C1)-(C3) in part 1, since no further risk is involved. Unless S** is chosen correctly, that trajectory will ultimately result in excess supply or demand. If markets are to clear at all subsequent moments after the auction, the stock (S**) in private hands prior to the auction but after depletion ceases must have a particular magnitude. The first phase can terminate with these prices and stock levels $(P_c, S^{**}, \bar{f})$ only if the current actual price and the potential auction price are set appropriately.

60/ See footnote 17.

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