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TOTAL IMPORT AND GROSS OUTPUT DEMANDS IN THE CONTEXT OF A MULTISECTOR GENERAL EQUILIBRIUM MODEL

by

Richard Berner

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I. Introduction

In many studies of the determinants of international trade in general, or of import demand in particular, no consideration is given to the fact that imports consist of both final and intermediate goods.[1] Specification of "a" demand for imports, exports, or domestic output is theoretically incorrect, given the existence of intermediate goods that are traded and produced at home. The correct specification involves aggregation of demands over different economic agents for both final and intermediate uses. A "good" belonging to a certain commodity classification is not exclusively either a final or an intermediate good.[2] This is particularly true of homogeneous commodities, such as chemicals and rubber. Aggregation over commodities compound the problem.

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[1] See, for example, for the import equations for the countries in the LINK model, Basevi (1973); the models reviewed in Magee (1975); and the disaggregated studies of Barker (1969) and Kwack (1972).

[2] Cf. Rhomberg (1973).

This paper reports estimates of demands for gross output and imports that are part of a larger study, in which a complete multi-country, multisector general equilibrium model is specified, estimated and simulated.[1] The major goal in constructing the model was to examine commercial policy in the Common Market. The five countries in the model are Belgium, Netherlands, W. Germany, Italy and France -- the EEC "Six" of pre-1973, not counting Luxembourg. In the larger study, the supply of output, demand for primary factors, for exports, and prices corresponding to all these variables are made explicit for each of twelve industrial sectors in each of the five countries. The specification of the demands and supplies mentioned above is grounded in the micro-economic theory of production. It is assumed that a production function of all inputs is defined. In general, one could start with

(1) $X_j = X_j(L_j, K_j, x_{ij}^k)$, $j, i=1,\ldots, n;\ k=1,\ldots, m.$

where X_j is real gross output of the j^{th} industry, in one country,

L_j are labor inputs to industry j,

K_j are capital inputs to industry j,

x_{ij}^k are intermediate inputs from industry i to industry j from country k,

and m is the number of countries.

The absence of a country superscript on X_j, L_j, and K_j indicate that they all refer to the one country in question.

[1]Berner (1976).

Demands for factors and supply of output can be derived by assuming that producers maximize profits subject to (1) or that they minimize costs required to produce a given output, which also may involve (1). In either case, specification involves the choice of a specific functional form for (1). Since a model of five countries with twelve sectors in each country might entail thousands of intermediate input demand equations, a feasible approach, in which these demands are taken into account, yet not all specified explicitly, is necessary.

Questions that may be legitimately raised from consideration of (1) are: Why pay so much attention to demand for intermediate goods? and why attempt to separate imported intermediates from domestically supplied inputs? Why not simply treat output as a function of two primary inputs, and not worry about intermediate inputs? There are four reasons.

First, the effects of a devaluation on a country which imports many intermediates may differ significantly from those on a country which imports only final goods. It has been argued that "the price increase of imported intermediate products tends to reduce and perhaps nullify the advantages that devaluation grants to export industries and to import competing industries."[1] Estimates of parameter values are needed to establish what the implications of traded intermediates are for exchange rate policy.

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[1]Basevi (1969), p.1. Contemporary examples are food and oil.

This argument is obviously of crucial importance to current policy. If revaluations are ineffective in reducing European trade surpluses because cheaper intermediates mean cheaper exports and import competing goods, then exchange rate changes to which both monetary and real phenomena have led may be insufficient to bring about current account equilibrium.

Second, if intermediate imports are sufficiently important as inputs to current production, a priori expectations of qualitative results in general equilibrium models in which intermediates appear may be unfulfilled, and perverse results may arise.[1] Such perverse results as downward sloping supply curves may be ruled out by restricting the magnitudes of elasticities of substitution among the various inputs. In turn, however, these magnitudes are in general not independent of factor shares; i.e., the importance of imported intermediates as inputs.

Third, a model that seeks to examine the effects of commercial policy on domestic economic activity as well as on the pattern of trade, as does the one of which this study is a part, can hardly ignore the influence of the effective protection concept.[2] While the concept itself is somewhat paradoxical, in that it seeks to answer questions about domestic resource allocation in a general multi-commodity model using partial equilibrium analysis, its implication is clear. If intermediate inputs play an important role in the production process, changing their prices via tariffs on imports may produce allocations of resources that differ from those obtained in a model in which intermediate imports are unimportant.

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[1] See Batra (1973), especially Chapters 7 and 8. The perversity arises because labor might be a closer substitute for the intermediate than it is for capital, or vice-versa (read capital for labor and conversely). It must be admitted that the assumptions made here prejudge this issue, however,

[2] See Grubel and Johnson, eds. (1971) and Corden (1971) for comprehensive bibliographies.

Fourth, a model seeking to determine the demands for imports that is theoretically based solely on the behavior of one kind of economic agent is in error if imported intermediates are important. Typically, the specification of import demands is based on the theory of consumer behavior.[1] However, it is well known that demands for factors by producers as inputs differ in a fundamental way from the demand for goods by consumers. In particular, "in factor demand theory there is no such thing as the income effect of price changes. Income is replaced by output, which is expressed in volume terms, not money units."[2] To base demands for imports on either kind of behavior exclusively is justifiable only if imports are nearly all consumer goods or nearly all intermediate inputs.

Since these four reasons are contingent on the importance of intermediates, that evidence must be examined for the five countries of the pre-1973 EEC.

For the countries considered, intermediates account for a large fraction of total imports. In 1965, this fraction for imports varied between 69 and 78 percent, for the five (excluding Luxembourg) EEC countries. Table 1 summarizes the distribution of the totals.

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[1] "The basic explanatory variables [in an import demand function] are suggested by the theory of demand, according to which the consumer allocates his income among consumable commodities in an effort to achieve maximum satisfaction." Leamer and Stern (1970), p.9.

[2] Theil (1967), p.306. In Jorgenson's (1963) investment qua factor demand theory, the equation is transformed to be uncompensated.

Table 1: Distribution of Imports Among Uses, 1965, percent of value

	Intermediates	Consumption	Investment	Change in Inventories	Re-Exports
Belgium	68.6	16.9	9.6	0.4	4.5
France	71.1	15.8	12.9	0.0	0.1
W. Germany	73.5	19.0	5.9	0.6	1.0
Italy	77.5	13.6	8.1	0.7	0.1
Netherlands	69.0	15.1	12.0	1.8	2.0

Source: OSCE, Tableaux Entrées-Sorties 1965, 6 vols.

Table 2: Percent of Total Intermediate Purchases Represented by Imports, 1965, Values

Belgium	34.6
France	12.4
W. Germany	13.2
Italy	15.0
Netherlands	34.2

Source: Ibid.

Table 2 summarizes the degree to which imports are important in intermediate input demand for each country in 1965. From these two tables, one may safely infer that imports play an important role in intermediate input demand, yet all intermediates cannot be considered to be imported. Similarly, intermediates account for over two-thirds of import demand, yet one cannot consider all imports to be intermediates. Some feasible approach that takes into account some of all of these problems is necessary, given the demonstration that intermediate imports are important.

II. Specification of the Demands

As stated above, it is theoretically possible to derive the demands for imports that are intermediates as factor demands using the production function (1). The first problem with such an approach is that the interindustry flow data do not exist in time-series form.

The non-existence of a time series of interindustry flows is a problem with which economists have wrestled for some time.[1] Arrow and Hoffenberg (1959) attempted to estimate changes in coefficients econometrically and with linear programming techniques. Given marginal (row and column) totals, Stone (1963), Bacharach (1970), and Preston (1973) have constructed time series of interindustry flows or transformations thereof using the RAS technique. Theil (1967) uses a minimum squared error technique.

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[1]The Netherlands (from 1949), France (from 1959) and all other EEC countries (from 1970) do have time series of input-output tables, but for total flows only, not disaggregated by country of origin.

However, even if such time series data could be constructed, it would be unwieldy to estimate each of the cells (assuming all are non-zero) of a 36x12[1] transactions table for each country -- over 2000 equations -- as factor demands. Interest in this study focuses primary on aggregate output, import demand classified by industry and origin, prices, and the way in which imports enter as intermediate demands, rather than on the particular demand by industry j for good i produced in country k.

One solution to this problem, used by Burgess (1974a), (1974b), is the following. If one considers the demand for imports to be wholly an input demand in the production of net output (final demand) a joint cost function[2] may be used to derive imports as factor demands as well as supply of net output. While Burgess considers only total imports together with two primary factors, clearly the approach could be generalized (up to the point where degrees of freedom run low) to several inputs, some of which are imported. An objection to this approach is one raised above: it blurs the direct demands by final demanders for imported goods, and could be considered a misspecification; see footnote 2, p.5, above.

Figure 1 illustrates the input-output accounting framework used in the model in this study. In the top panel, domestic and imported intermediate and final demands are distinguished. Domestic gross outputs by sector are the row sums of the flows in the top row of

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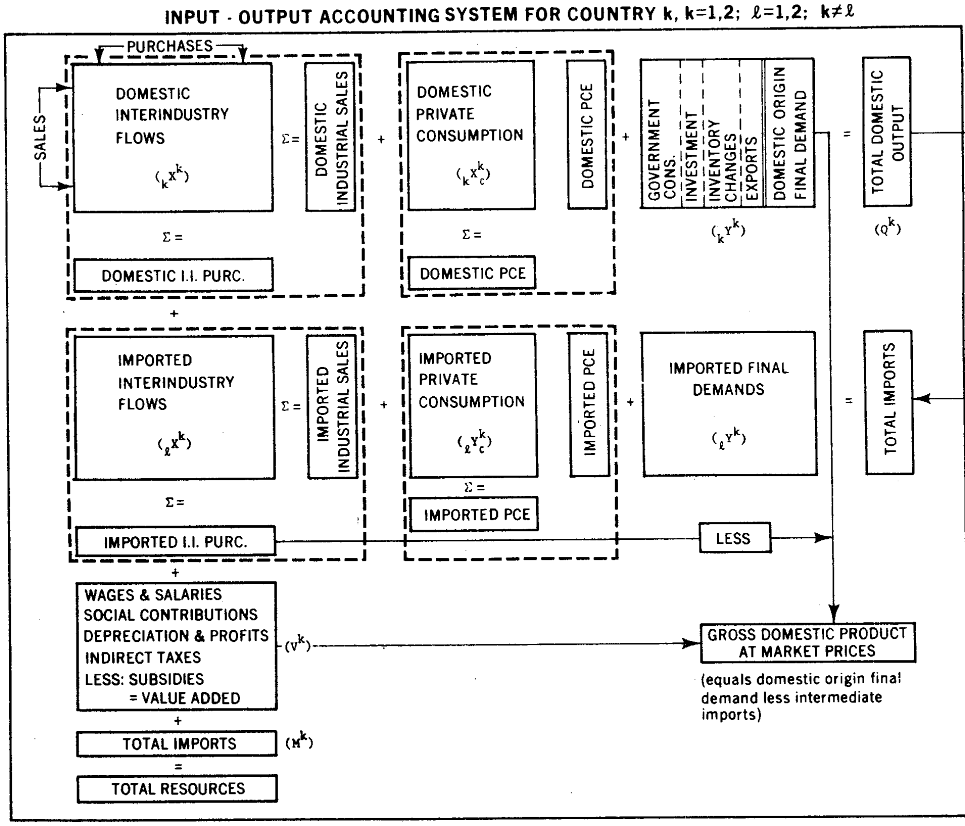
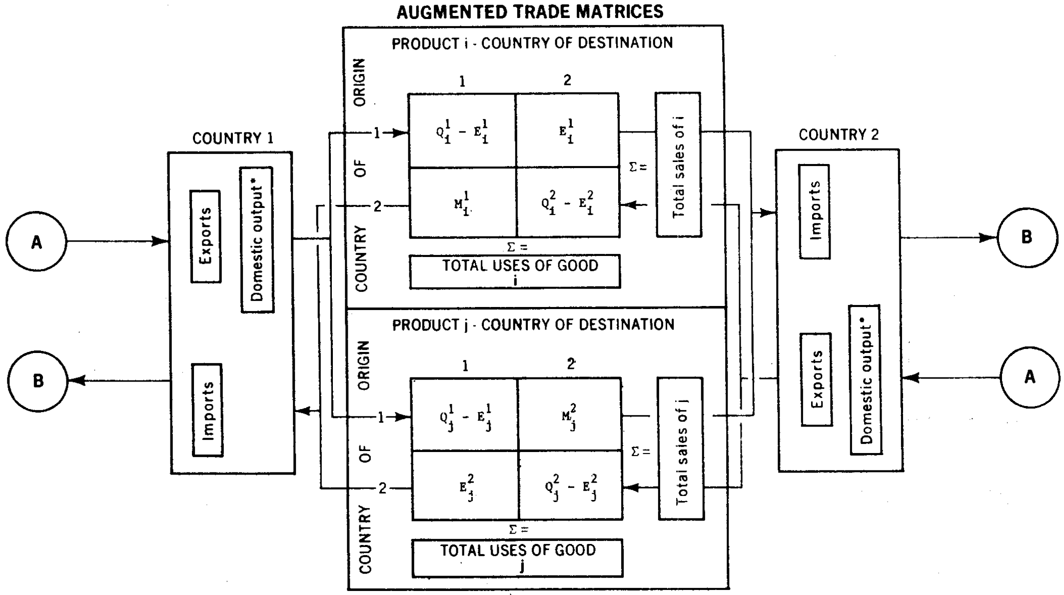
[1]Discussion in the text will clarify these dimensions.

[2]See, for example, Hall (1973) for definitions.

INPUT - OUTPUT ACCOUNTING SYSTEM FOR COUNTRY k, k=1,2; ℓ=1,2; k≠ℓ



AUGMENTED TRADE MATRICES



*Domestic output equals exports plus domestically produced total uses of good k.

FIGURE 1 INPUT - OUTPUT ACCOUNTING SYSTEM AND RELATION TO TRADE MATRICES boxes, and imports are equal to the sums of the flows in the second row of boxes - intermediates plus final demands equal total demand. The sum of imports and domestic output is equal to the sum of all inputs plus imports. This accounting identity is represented by summing down the columns of the left-most set of matrices, and means that the value of domestic output exhausts the value of its inputs for each sector. In this model, the row identities are used as the basis for demands for output and imports, and the column relations, in production function form, are used as the basis for the supply of output. Figure 1 depicts imports from one origin only. In this model, imports come from two origins: EEC partners and the rest of the world (ROW). Hence, twelve industrial sectors result in a 36x12 (=3·12x12) coefficient matrix for one country.

The second panel in Figure 1 represents the links from the domestic economy to the trade block of the model. In a complete model of world trade, the "centerpiece" is the matrix of trade shares, A.[1] If the share matrix (exports per unit of imports from country j) is explained, and an n-vector of goods import demands (M) is given from elsewhere in the model, assuming the trade matrix is adjusted for f.o.b./c.i.f. differentials, the n-vector of exports (E) follows from the matrix identity

(2) $E \equiv AM.$

[1] See, for example, Hickman (1973).

Whether or not the elements of the share matrix are explained individually, the A matrix is central to such a model. The trade matrices in the lower panel of Figure 1, for goods i and j, differ from the norm in that their "domestic" or diagonal elements represent domestic activity, so that row sums yield total sales including domestic, and column sums represent total uses, including the domestic sources.[1] Total outputs (circle A) are used for domestic and export sales; "imports" (circle B) are from both "domestic" and foreign sources.

In this study, equilibrium conditions are used to derive the demands for both gross output and total imports. By gross output is meant gross domestic output; i.e., the sum of all sales from a given industrial sector plus changes in stocks. Total imports means the sum of purchases by all users, intermediate and final, of imported goods or services from a given geographic origin that are classified with a single industrial sector. The equilibrium conditions are similar to those used by Saito (1972) and Allingham (1973).

In the context of this model, the input-output accounting system is different from that used by either Saito for Japan or Allingham for the U.K. In Saito's system, imports are allocated as purchases by the industry in which they might be "competitive"; "noncompetitive" imports are allocated in a special row like a primary factor. By contrast, in the present system, the accounting system allocates as purchases by each industry <u>only</u> those intermediate purchases from

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[1]Thus, "goods" are from all sources; "products" differentiate goods by geographic origin. This terminology, as well as the idea of a domestic, diagonal element for a trade matrix, is due to Armington (1969).

abroad used as inputs, and the allocation is exhaustive. Final demands are allocated similarly.

Saito's equilibrium conditions take the form of excess supply functions for each industry's output, as follows:

$$(3) \qquad E_i = X_i + M_i - \sum_{j=1}^{n} x_{ij} - Y_i, \qquad i=1,\ldots,n,$$

where E_i is excess supply of output i,

M_i is imports of goods classified under industry i,

Y_i are final demands.

Saito differentiates equations (3) with respect to each of the exogenous variables in his model to obtain the Jacobian of these excess supply functions. Being linear and nonsingular, the Jacobian can be inverted to solve for price changes in response to change in exogenous variables.

Allingham (1973) uses equilibrium conditions not unlike (3) to obtain predictions of the demand for each industry's output. Assuming that the supply of that output equals demand, the supply function can be renormalized to solve for one of its arguments (assuming it is invertible). The form of the demand equation is similar to (3), except that the coefficients are not assumed to be unity; i.e., the identity, given constant input-output coefficients, is estimated, as described below.

It is evident that one can obtain solutions for output and imports necessary to satisfy final demands in the context of an input-output system such as the one depicted in Figure 1.[1] To obtain predictions of output and imports consistent with final demand, one could modify the Kresge-Klein-Fromm (1969), Chapters 4 and 11, or Preston and Evans (1969), techniques of "output conversion," which update the static I-O predictions with autoregressive trends. It was judged preferable to separate the final demand predictions from those for intermediate demands in the present study. Allingham's device is then used to combine these two predictions into predictions for each country of the three demand vectors of interest: gross domestic output, imports from EEC partners, and imports from the rest of the world.

This separation of final demands from intermediates demands involves two steps:

- compute the input-output prediction of each demand in value terms.

- estimate the coefficients of the pseudo identity that results from adding the two demands to obtain the total, so that each component has its own coefficient.

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[1] See the Appendix.

That is, for gross output,

$$P_iX_i = b_0 + b_1 \sum_j P_i x_{ij} + b_2 \sum_k P_i H_{ik} \qquad i=1,\ldots,n, \tag{4}$$

where $x_{ij} \equiv a_{ij}X_j$,

$H_{ik} \equiv h^d_{ik}G_k$,

G_k = k^{th} final demand component,

h^d_{ik} = i, k^{th} element of the "bridge table" in coefficient form that represents sales from domestic industry i to final demand k, given in the input-output tables,

a_{ij} = input-output (value) coefficients for domestic origin purchases, given in the input-output tables.

Similarly, for imports classified by industrial sector,

$$\rho_r P^m_{ir} M_{ir} = c_{0r} + c_{1r} \sum_j P^m_{ir} T_{ir} \rho^*_r x^r_{ij} + c_{2r} \sum_k P^m_{ir} T_{ir} \rho^*_r H^r_{ik}, \tag{5}$$

$$i=1,\ldots,n; \quad r = \text{EEC, ROW},$$

where P^m_{ir} = import price for product i from origin r, in dollars,

ρ_r = units of local currency/dollar (star denotes index, base = 1),

T_{ir} = index of tariffs for product i from origin r of the form $\frac{(1+t)}{(1+t_0)}$,

M_{ir} = volume of imports classified under industry i from origin r (constant dollars),

$x^r_{ij} = a^r_{ij}X_j$,

$H^r_{ik} \equiv h^r_{ik}G_k$,

a^r_{ij} = input-output (value) coefficients for origin r purchases by domestic producers, given in the I-0 table,

h^r_{ik} = bridge table coefficients for origin r purchases by final demander k, given in the I-0 table.

In (4) and (5), the b_i and c_{ir} are coefficients to be estimated.

These quasi-identities are in value terms; the implied own-price elasticities are therefore $-1(P_i/X_i)$ and $-1(P^m_{ir}/M_{ir})$, respectively. A log-linear specification of (4) and (5) would yield constant elasticities. However, even without the addition of a relative price term, more price responsiveness, both to the own price and those of substituties, is built into (4) and (5) than is immediately apparent. The two explanatory variables in each equation, being in value terms, change when prices change, but in addition, both the G_k and X_j (volumes or quantities) are themselves functions of prices. The model includes consumer demand systems for imported and domestic goods that are functions of both domestic and import prices, see Berner (1975). Investment and inventory demands are sensitive to the overall final demands prices corresponding to them; see Berner (1976), Chapter VII. In turn, these prices will change when any of their domestic or imported components (or the exchange rate or tariffs) change; see Berner (1976), Chapter VI. Exports are sensitive to total world trade price changes, as well as to those of own-prices.

As for the supply of gross output, it is assumed that all prices enter in a unit elastic manner, so that the price terms in (4) involve P and ℓn P multiplicatively, see Berner (1976), Chapter III. The prices of gross output are functions of all domestic and imports prices, so that $X_j \equiv X^*_j/P_j$ depends on prices in the other four countries as well; see Berner (1976), Chapter VI.

The implication is that demand elasticities are not -1; the model must be partially simulated to assess their magnitudes. In spite of this fact, a relative price term is added to some equations to enhance price sensitivity.

Furthermore, the dependent variables of (4) and (5) are designed to represent "desired" gross output and "desired" imports. A partial adjustment to desired levels is postulated and implemented in the estimates in Section III below.

In (4) and (5), the time subscript has been dropped; the X_j and G_k as well as dependent variables and prices are time series data, but the coefficients a and h are time-invariant (1965 is the year used). Since they are estimated in value terms, (4) and (5) imply unit price elasticities as discussed above.

Personal consumption expenditures are data constructed by "country" of origin (the same three origins that appear here) for use in the demand systems estimated in Berner (1975). In these systems, fifteen products are identified: there are five goods categories for each of three country origins. This country origin distinction is not made for other final demands (investment, etc.), so that not all columns of the h matrices are treated alike. In particular, for each country origin, r,

(6) $\sum_i h^r_{ik} \equiv 1, \qquad k=1,\ldots,s, \qquad r\varepsilon D,E,R,$

where s is the number of personal consumption expenditure final demand categories (food, etc.),

and D,E,R, are the three geographic origins: domestic, EEC partners, and rest-of-world.

However, the identity (6) does not hold for the other columns in the h matrices, which consist of fixed coefficients. Instead,

(7) $\sum_r\sum_i h^r_{ik} \equiv 1, \qquad k=s+1,\ldots,m,$

where m is the total number of final demands (s personal consumption categories plus m-s other final demands).

An alternative way of looking at this nonuniform treatment of the components of final demand is simply to consider that there are 3·s personal consumption categories and (m^*-3·s) other final demands, where m^* now exceeds m of the previous paragraph by 2·s. This means that the column sums of the h matrices will all equal unity, when summed over all rows: (7) applies for k=1,...,m^*. This is due to the fact that, under this alternative,

(8) $$\sum_i h^r_{ik} \equiv 0,$$

for the following pairs of indices:

- rεE,R; k=1,...,s;
- rεD,R; k=s+1,...,2·s;
- rεD,E; k=2·s+1,...,3·s.

The Allingham system of determining demands for outputs, and, as extended here, for imports, might be viewed as a special case of a more general model in a slightly different context. Hickman and Lau (1973) have shown that a linarized export demand function derived from utility maximizing behavior, and assuming a CES utility function à la Armington (1969), takes the following form (deleting their time trend term):[1]

(9) $$x_{ij} = \alpha^0_{ij} m_j - \alpha_j x^0_{ij} (p^x_{ij} - \sum_{k=1}^{n} \alpha^0_{kj} p^x_j),$$

x_{ij} = constant dollar quantity of exports from i to j,

α^0_{ji} = base year share of i^{th} country's export in j's imports,

[1]This comparison with Hickman-Lau serves to introduce the relative price term; it is not claimed that a utility function of total imports is justifiable. Cf. Berner (1975). The use of x_{ij} here for exports is an effort to use H-L notation; it should not be confused with intermediate flows.

m_j = imports (constant dollars) of $j = \sum_{i=1}^{n} x_{ij}$,

σ_j = elasticity of substitution between imports from any two countries in j,

p_{ij}^{x} = price index of exports from i to j.

Summing x_{ij}'s over j, the export demand from i to all markets is written as:

$$(10) \qquad x_i = \sum_{j=1}^{n} \alpha_{ij}^0 m_j - \bar{\sigma}_i x_i^0 (p_i^x - p_i^{xc}),$$

where $\bar{\sigma}_i$ is now a weighted average of elasticities in all markets (j=1,...,n): $\bar{\sigma}_i = \sum_{j=1}^{n} \sigma_j x_{ij}^0 / x_i^0$, and $p_i^{xc} = \sum_{j=1}^{n} (\sigma_j / \bar{\sigma}_i)(x_{ij}^0 / x_i^0) \sum_{k=1}^{n} \alpha_{kj}^0 p_{kj}^x$, an export "competitive price index" of country i. p_{ij}^x is assumed to be equal to p_i^x, for all i and j; i.e., transport costs are equal and there is no price discrimination.

The export shares, α_{ij}^0, are partly analogous to the input-output coefficients used in (4) and (5), and these equations are analogous to (10), except that the latter are expressed in value, not volume, terms. As a result, on the assumption that the deflator used in (4) and (5) is the analogoue to p_i^x (that is, a deflator with a similar weighting scheme as that used for p_i^x), and assuming that the effects of $\bar{\sigma}_i x_i p_i^{xc}$ are captured in the constant terms, b_0 or c_{0r}, the import (and gross output) demand equations used here are thoroughly analogous to the Hickman-Lau-Armington (HLA) export demand equations. In fact, as already noted above, Preston (1973) (see p.7, above) uses the HLA system to determine values added as a function of share-weighted averages of final demands and appropriately defined relative prices.

In the present case, the unit elasticity of substitution assumption implicit in (4) and (5) can be relaxed by adding a linearized relative price term analogous to the right hand member of (10). The form of this price term will be, for (4):

$$(11) \quad RP_i = (1 - \bar{\sigma}_i)X_i^0(P_i - P_i^c), \qquad i=1,\ldots,n,$$

and for (5)

$$(12) \quad RP_{ir}^m = (1 - \bar{\gamma}_{ir})M_{ir}^0(P_{ir}^{m*} - P_i^c),$$

where $P_i^c \equiv \sum_j d_{ji}P_j + \sum_s \rho_s^* \sum_j e_{ji}^s P_{js}^m T_{js}$

d_{ji} = j, ith element of the first price conversion matrix $B^{-1}(I-A_D)^{-1}$, (see Appendix),

e_{ji}^s = j, ith element of the second and third price conversion matrices, $A_s(I-A_D)^{-1}$, $s \varepsilon E,R$,

$1 \equiv \sum_j (d_{ji} + \sum_s e_{ji}^s)$, $\qquad$ i=1,...,n,

$\bar{\gamma}_{ir}$ = substitution elasticity for import demands,

$P_{ir}^{m*} \equiv \rho_r^* P_{ir} T_{ir}$.

The assumption that the price of a product is the same in all local markets (for both industrial users and final demanders) permits the use of a single price in both terms in the second parentheses of both (11) and (12). The expressions in the first parentheses in these equations (e.g., $1-\bar{\sigma}_i$) derive from the fact that (4) and (5) are expressed in value, not volume terms. Hence, the estimated relative price coefficients are of the form in these first parentheses, and they must be less than positive 1 in order that goods be substitutes.

Estimates of the parameters of (4) and (5), with and without the terms RP_i and RP^m_{ir} from (11) and (12), respectively, are presented in Section III which follows.

III. Estimation Results

Estimation Techniques

The parameter estimates for the gross output and imports demands are presented in Tables 3-12 below. To obtain these estimates, the biased estimation technique known as ridge regression was employed.[1] This estimator and the reasons for using it are described briefly here.

Consider the general linear model

$$y = X\beta + u, \tag{13}$$

where y is Tx1, X is Txk, β is kx1 and u is Tx1, $E(u) = 0$, $E(uu') = \sigma^2 I$, the u_t being independent and identically distributed random variables. Consider an estimator of β, $\tilde{\beta}$, such that $\tilde{\beta}'\tilde{\beta} \leq r^2$, where r is the radius of a hypersphere constraining the vector β.

If $r \to \infty$, there is no constraint, and the most efficient estimator of β is the OLS estimator. However, suppose the analyst knows a priori that $r < \infty$. Then it can be shown that the constrained estimator has a smaller mean squared error than OLS, provided that a parameter representing the constraint is chosen properly. More efficient estimates mean smaller standard errors, or more precise estimates, which deals (in part) with the multicollinearity problem.

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[1] See Hoerl and Kennard (1970).

Minimization of $\frac{u'u}{\sigma^2}$ subject to $\tilde{\beta}'\tilde{\beta} \leq r^2$ yields the ridge estimator:

$$(14) \qquad \tilde{\beta} = \left(\frac{X'X}{\sigma^2} + \mu I\right)^{-1} \frac{X'y}{\sigma^2} ,$$

where μ is the Lagrange multiplier associated with the side constraint.[1] Obviously, if $\mu = 0$ then the constraint is not binding and (14) reduces to OLS. $\tilde{\beta}$ is a biased estimator, but it is more efficient than OLS if $\mu < 2/\beta'\beta$.[2] The estimator (14) is a special case of the general Bayesian estimator proposed by Chipman (1964) to deal with singularity or near-singularity (resulting from multicollinearity) of the moment matrix X'X.[3]

In the present case, multicollinearity among the regressors of (4) and (5) resulted in large standard errors for some variables, using the Gram-Schmidt orthonormalization algorithm to solve the OLS normal equations, indicating near-singularity of X'X. Ridge regression was employed, and for each equation, inspection of the <u>ridge trace</u> (the values of the coefficients given a grid over μ from $\mu = 0$ to 1) revealed those variables causing the multicollinearity problem. Most frequently, $\hat{G}^*$ and $\hat{X}^*$ were highly collinear, but the relative price term was also collinear with both of these. As explained below, the variables causing the problem become insignificant as μ was increased from zero. The above criterion for efficiency was almost always satisfied; since most coefficients were less than one in absolute value, any $\mu < 1$ would certainly

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[1] See Rappoport and Swamy (1974), and Meeter (1966), Theorem 1a, p. 1177.

[2] See Rappoport and Swamy, *op. cit.*, and Theobald (1974).

[3] See Bacon and Hausman (1974), and Swamy and Mehta (1974).

give a more efficient estimator than OLS. In general, μ was chosen to be as small as possible subject to the constraint that all explanatory variables be significant at least at the 10% level. Incidentally, minimizing μ also minimizes the bias of the ridge estimator, since if $\mu = 0$, $\tilde{\beta}$ is the OLS estimator, which is unbiased.

In one case, a distributed lag was specified for one of the explanatory variables. A technique used extensively elsewhere in the model, that of Shiller (1973), was employed to obtain a flexible lag pattern. With annual data, this is always a problem, since lags are rarely larger than 3-4 periods. In most cases, one-period lags were sufficient.

Following is a list of the industrial sectors and their acronyms used in the tables:

1. Agriculture, forestry, fishing - AGF
2. Mining and crude petroleum - MIN
3. Food, beverages and tobacco - FBT
4. Textiles, clothing, and leather - TCL
5. Chemicals, rubber, wood and paper - CRW
6. Stone, clay and glass - SCG
7. Basic metals - MET
8. Machinery and transport equipment - MTE
9. Construction - CON
10. Transport and communications - TRC
11. Electricity, gas and water - UT
12. Services, government (except France) - SVC

Demands for Gross Output and Imports

1. Gross Output

The quasi-identity (4) proves to be highly successful in providing regressors for the explanation of gross output in value terms for the five countries considered. The estimates, presented in Tables 3-7, are derived using the ridge estimator.[1] They are largely self-explanatory, but some explanation of particular features is worthwhile.

First, some of the coefficients for the final demand proxy, $\hat{G}*$, and for the intermediate demand proxy, $\hat{X}*$, were initially negative. These negative coefficients for $\hat{G}*$ were due to the dominance, for a particular industry, of the inventory change component of final demand, which is frequently negative. The explanation of a negative coefficient on $\hat{X}*$ is more difficult, and it must be admitted that it probably resulted from coefficient change in the input-output tables not explainable by Cobb-Douglas (unit price elastic) factor demands or time trends, also included in many equations. Both were for the construction industry, for France and the Netherlands. When a negative coefficient was obtained on either X* or G*, the two components were added together, to impose a single coefficient on both. This was done for twenty of the sixty equations in the tables.

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[1]A special case is OLS, when $\mu = 0$.

Table 3
Belgium Value of Gross Output, 1954-1970

	$\hat{G}^*$	$\hat{X}^*$	TIME	X*(-1)	Constant	$\bar{R}^2$	SE	DW/H[1]
BXAGF*[2]	.266			.474	22.1	.890	2.91	1.82
	(2.64)			(2.18)	(2.04)			.84
BXMIN*[3]	-.104	1.45		.263	-.075	.836	1.00	1.87
	(4.61)	(6.41)		(2.06)	(.199)			.32
BXFBT*	.195	2.75		-.225	48.8	.994	1.36	1.83
	(2.27)	(7.64)		(2.07)	(8.58)			.39
BXTCL*[2]	.146			.679	16.2	.927	3.61	1.89
	(1.52)			(3.42)	(1.82)			.39
BXCRW*	.220	.441				.980	1.34	1.59
	(6.94)	(9.41)						
BXSCG*	.569	3.02			50.5	.961	3.46	1.86
	(1.42)	(7.02)			(9.14)			
BXMET*	.080	2.17	-.887			.997	1.70	1.57
	(2.10)	(41.2)	(3.38)					
BXMTE*	.091	2.95				.989	3.27	1.15
	(2.40)	(23.7)						
BXCON*[2]	.386			.353	28.3	.883	9.15	2.13
	(2.98)			(1.70)	(2.80)			.52
BXTRC*	1.31	3.96	-2.60		-69.7	.969	5.95	2.08
	(3.40)	(3.14)	(2.30)		(5.10)			
BXUT*	1.10	1.38			-11.6	.990	.939	1.57
	(2.46)	(5.25)			(8.02)			
BXSVC*	.164	5.63			-52.3	.997	8.42	1.23
	(2.16)	(12.8)			(6.45)			

[1] H is Durbin's (1970) statistic to test for serial correlation in the presence of a lagged dependent variable.

[2] $\hat{G}^*$ in these equations is $\hat{G}^* + \hat{X}^*$.

[3] This equation is in first difference form. $\hat{G}^*$ in this equation is a one period lag of BXTRC*; $\hat{X}^*$ is BVMN.

Table 4
France Value of Gross Output, 1954-1970

	TIME	$\hat{X}$*	$\hat{G}$*	X*-1	Constant	$\bar{R}^2$	SE	DW/H[3]
FXMIN01*	1.49 (3.46)	.728 (5.96)			6.30 (5.89)	.994	1.62	1.48
FXMMIN*[1]		1.65 (64.2)			4.18 (8.46)	.996	.945	.798
FXMIN02*		2.20 (3.83)	.493 (2.86)	.249 (2.77)	1.77 (1.16)	.999	.567	2.21 .47
FXMIN11*[1]		1.00 (50.3)			5.71 (6.49)	.993	1.45	1.82
FXMCRW*		1.43 (13.8)	.670 (4.01)		-4.60 (9.87)	.999	.705	.880
FXMIN06*	.126 (2.04)	.994 (19.6)			.175 (.483)	.998	.302	1.65
FXMMET*		1.62 (33.9)			5.76 (4.25)	.986	2.53	.793
FXMIN09*[1]		.929 (139.)			-2.45 (3.15)	.999	1.60	1.20
FXMIN13*[2]		1.01 (41.2)	.655 (3.58)		-3.41 (1.41)	.998	1.68	1.73
FXMIN14*		.872 (3.94)	.989 (2.91)		1.58 (4.36)	.998	.656	.947
FXMIN04*		.504 (4.29)	1.07 (4.23)		.020 (.098)	.997	.272	1.05
FXMSVC*		1.15 (2.49)	.615 (3.69)		-9.52 (7.64)	.999	1.83	.671

[1] In these equations, $\hat{X}$* represents $\hat{X}$* + $\hat{G}$*.

[2] In this equation, $\hat{X}$* represents $\hat{X}$* + $\hat{G}$*, and $\hat{G}$* represents the 1st order serial correlation coefficient.

[3] H is Durbin's (1970) statistic to test for serial correlation in the presence of a lagged dependent variable.

Table 5

Germany Value of Gross Output, 1954-1970

	$\hat{G}^*$	$\hat{X}^*$	TIME	CONSTANT	$\bar{R}^2$	SE	DW /H[3]
GPRAG*[1]	.601 (38.6)			6.09 (9.58)	.989	.685	1.45
GPRMN*[2]	.289 (3.07)	.770 (4.08)	-.463 (2.23)	2.64 (1.71)	.884	.658	2.14 .46
GPRFBT*	.960 (41.3)			7.07 (5.78)	.991	1.35	1.79
GPRTCL*[2]	.220 (2.48)	.415 (2.14)		13.3 (3.42)	.899	1.67	1.89 .38
GPRCRW*		3.14 (8.35)	-5.31 (2.71)	-14.9 (1.94)	.977	8.61	.931
GPRSCG*[2]	.350 (3.76)	.499 (3.59)		2.48 (3.71)	.982	.613	1.51 1.23
GPRMET*		.617 (19.8)		23.1 (13.3)	.961	2.53	1.73
GPRMTE*[1]	1.06 (28.0)			-21.3 (3.79)	.980	.924	1.22
GPRCN*	.695 (9.01)	1.09 (2.87)		-2.66 (3.15)	.998	.942	1.36
GPRTC*	.787 (3.18)	.748 (7.18)	.621 (5.74)		.999	.384	1.99
GPRE*	.340 (1.95)	.730 (8.19)	.258 (2.53)		.997	.382	2.30
GPRSVC*	.157 (2.65)	3.72 (26.6)	4.17 (8.51)		.999	1.83	1.91

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[1] In these equations, $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$.

[2] In these equations, $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$, and $\hat{X}^*$ represents the lagged dependent variable.

[3] H is Durbin's (1970) statistic to test for serial correlation in the presence of a lagged dependent variable.

Table 6

Italy Value of Gross Output, 1954-1970

	TIME	$\hat{G}^*$	$\hat{X}^*$	X*(-1)	RHO	CON- STANT	$\bar{R}^2$	SE	DW/H[2]
IPRAGF*[1]		.464 (19.5)				3.34 (21.7)	.959	.177	1.14
IPRMIN*			.388 (8.12)		.662 (3.53)	.638 (8.87)	.976	.034	1.93
IPRFBT*[1]		.266 (2.34)		.570 (2.70)		.799 (2.02)	.983	.159	2.02 .08
IPRTCL*[1]	-.479 (2.30)	1.25 (2.88)			.876 (7.25)	7.65 (3.26)	.916	.177	1.42
IPRCRW*[1]		1.14 (35.5)				-.239 (1.46)	.987	.279	.936
IPRSCG*	.063 (7.32)		.272 (2.05)				.977	.064	1.09
IPRMET*	-.048 (2.23)		1.36 (11.5)			.096 (1.37)	.995	.074	1.20
IPRMTE*[1]		.611 (26.2)				1.13 (8.62)	.977	.239	.430
IPRCON*	-.069 (2.36)	.481 (5.90)	7.58 (7.30)			-.276 (2.22)	.996	.117	1.57
IPRTRC*		.475 (4.49)	1.85 (13.6)				.997	.076	1.48
IPRUT*		.403 (4.86)	.877 (17.5)			.175 (14.0)	.998	.018	.865
IPRSVC*		.690 (6.65)	.909 (2.81)	.341 (6.17)		-1.00 (8.08)	.999	.092	2.70 1.48

[1] In these equations, $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$.

[2] H is Durbin's (1970) statistic to test for serial correlation in the presence of a lagged dependent variable.

Table 7

Netherlands Value of Gross Output, 1954-1970

	TIME	$\hat{G}^*$	$\hat{X}^*$	RHO	CONSTANT	$\bar{R}^2$	SE	DW/H[3]
NXIO01*		1.31 (4.99)	.347 (2.81)		1.72 (6.74)	.998	.124	2.83
NXMIN*[1]	-.021 (3.15)	.753 (4.56)	.378 (1.72)		.269 (2.15)	.983	.045	1.45 2.68
NXFBT*[2]		1.11 (64.9)			1.36 (5.91)	.996	.318	1.63
NXTCL*[2]	.129 (2.54)	.317 (2.40)			2.49 (11.9)	.957	.265	1.56
NXCRW*		.362 (2.49)	2.53 (14.2)		-.858 (5.28)	.999	.186	1.18
NXIO14*	.022 (2.65)		.452 (12.2)		-.162 (4.36)	.995	.043	1.18
NXIO15*	-.146 (5.42)		2.32 (25.1)	.741 (4.42)	.839 (2.89)	.998	.068	2.40
NXMTE*	.257 (5.06)	.979 (18.1)			.155 (.573)	.997	.325	1.69
NXIO20*[2]		.956 (15.3)		.683 (3.74)	.411 (.506)	.991	.448	1.33
NXTC*		.461 (2.90)	1.69 (6.35)		1.33 (14.3)	.997	.152	1.46
NXIO21*	-.112 (5.18)	.854 (1.72)	2.94 (5.09)	.525 (2.47)		.993	.085	1.49
NXSVC*		.171 (2.57)	3.06 (9.83)		-1.77 (4.84)	.999	.281	1.52

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[1]In this equation, $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$, and $\hat{X}^*$ represents the lagged dependent variable.

[2]In these equations, $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$.

[3]H is Durbin's (1970) statistic to test for serial correlation in the presence of a lagged dependent variable.

Relative price terms do not appear in any of the results reported here because they were attempted for several of the industries in each country without success. The $\hat{G}^*$ and $\hat{X}^*$ variables dominated the relative price terms, so that the latter were insignificant, even when ridge regression was used. It is important to recall that both $\hat{G}^*$ and $\hat{X}^*$ are themselves functions of all (domestic and imported) prices, however.

Third, it is occasionally true that one or the other of $\hat{G}^*$ or $\hat{X}^*$ is insignificant in an equation. This would support the view that goods classified by industrial sector can be assigned to either final demand or intermediate demand. That this is not so in the majority of cases is evidenced by the results presented here, which therefore supports the use of the accounting framework and of input-output proxy regressors.

Fourth, twelve of sixty equations made use of the partial adjustment to desired levels hypothesis, especially those for Belgium. In all but one case, the Durbin h statistic indicates that serial correlation is not present in the residuals. Six equations, one for France, two for Italy, and three for the Netherlands, were corrected for serial correlation; the coefficients are presented in Tables 6 and 7 under RHO, and in Table 4 under $\hat{G}^*$.

Finally, the dependent variable names that appear on the left-hand side of each table are those acronyms that are used in the databank system for this model. They all refer to the value of gross output for the industries listed in order in, e.g., Table 3. It should also be mentioned at this point that many of the time series on the value of gross output were constructed using the 1965 benchmark value from the input-output tables to scale industrial production indices.[1] Therefore, they do not reflect all of the price movements that would be captured in a true value figure.

2. Imports

Tables 8 - 12 present the estimates for import demands, estimated in value terms in local currency, for the nine traded goods (and services) industries, for each of two origins, as formulated in (5) in quasi-identity terms. The quality of these results is not as uniformly high as those for gross output, although the introduction of the relative price term as expressed in (12) adds significant and frequently large (greater than unity) linearized relative price elasticities. Recall that in (12) the elasticity is the estimated coefficient minus 1. Thus, some of the elasticities have the "wrong" sign, indicating that these goods are slightly complementary to the bundle of all other industrial outputs, domestic and imported. This is not surprising if a relative price increase of a domestic industry's output increases supply, which in turn calls forth more energy needs (MIN), or the construction industry's increased output as a supply response evokes higher demand for construction materials (SCG). These are in fact two of the cases for which the price coefficient is positive in Table 10 for Germany.

The partial adjustment to desired imports plays a substantial role in several of the equations presented here, while the time trend, to account for non-price induced coefficient change, plays a minor role.

[1]This is true for Belgium and Italy, and all German manufacturing. The deflators for gross output were calculated using the price conversion techniques described in Berner (1976), Chapter VI.

Table 8

Belgium - Imports: EEC, ROW; 1954-1970

	RP	$\hat{G}$*	$\hat{X}$*	TIME	M*(-1)	CON-STANT	$\bar{R}^2$	SE	DW/H
EEC									
BMCAGF*[1]	.047	3.95	-3.41			18.4	.901	1.00	1.68
	(1.76)	(7.23)	(3.92)			(5.20)			
BMCMIN*		2.63	.830				.928	1.32	2.11
		(7.41)	(5.50)						
BMCFBT*	-.466	2.30				-4.91	.965	1.26	1.75
	(2.71)	(21.6)				(6.29)			
BMCTCL*		1.57	1.89			-13.8	.965	1.69	1.55
		(4.56)	(3.09)			(3.05)			
BMCCRW*	-.358	4.64				2.10	.960	2.80	2.11
	(1.68)	(7.74)				(.745)			
BMCSCG*		7.86	-1.51		.661	8.63	.365	1.68	2.35
		(1.70)	(1.50)		(3.03)	(1.81)			1.67
BMCMET*	-.395		.821			-1.18	.493	1.98	1.85
	(1.92)		(3.52)			(.263)			
BMCMTE*	-.192	2.26				-19.2	.872	10.8	3.02
	(1.48)	(8.78)				(2.80)			
BMCSVC*		28.9	-.889				.898	.510	1.49
		(9.60)	(6.91)						
ROW									
BMRAGF*			.513			14.6	.337	2.53	.870
			(3.02)			(3.40)			
BMRMIN*		-22.9	5.70	3.70		-118.	.354	4.29	2.18
		(2.61)	(3.29)	(3.27)		(3.04)			
BMRFBT*		4.30	-2.29	.340			.910	1.52	1.42
		(6.58)	(4.45)	(2.31)					
BMRTCL*	-.171	1.73	1.08				.919	.586	2.04
	(3.29)	(4.53)	(5.61)						
BMRCRW*	-.154	6.37				6.97	.710	4.24	1.58
	(1.72)	(6.02)				(2.87)			
BMRSCG*			2.81	-.431		-.091	.139	1.15	1.80
			(2.21)	(2.37)		(.037)			
BMRMET*	1.30	6.80	.283			3.21	.629	1.73	1.99
	(4.28)	(2.22)	(1.86)			(1.04)			
BMRMTE*		2.31				-13.5	.883	6.24	1.92
		(11.4)				(3.52)			
BMRSVC*		18.4	-.738				.839	.561	1.88
		(7.53)	(5.65)						

[1] In this equation, the coefficient under RP represents RP_{-1}.

Table 9

France - Imports: EEC, ROW; 1954-1970

	RP	$\hat{G}*$	$\hat{X}*$	TIME	M*(-1)	CONSTANT	$\bar{R}^2$	SE	DW/H	$\hat{X}*(-1)$	$\hat{X}*(-2)$	$\hat{X}*(-3)$
EEC												
FMCAGF*			1.39			-.123	.844	.189	1.13			
			(9.35)			(.959)						
FMCMIN*		1.43	.407			-.048	.908	.127	1.92			
		(9.44)	(10.1)			(.479)						
FMCFBT*[1]		1.93	.481	-.061		-.040	.993	.087	1.54			
		(15.0)	(2.26)	(3.00)		(.205)						
FMCTCL*		1.63	.685			-.305	.996	.087	1.86			
		(16.8)	(3.52)			(2.57)						
FMCCRW*		5.10				-.022	.972	.507	1.61			
		(23.4)				(.125)						
FMCSCG*[2]	-1.27	-.389	.518			.093	.681	.149	2.21			
	(4.02)	(1.69)	(4.02)			(1.16)						
FMCMET*			.809			-.368	.924	.568	1.67			
			(14.0)			(1.19)						
FMCMTE*[3]	.491	2.50	-3.89	-.863		1.03	.998	.347	1.57			
	(2.26)	(21.9)	(6.51)	(8.88)		(1.63)						
FMCSVC*			.056			-.005	.566	.026	2.33			
			(4.82)			(.357)						
ROW												
FMRAGF*					.536	3.69	.283	1.21	2.29			
					(2.70)	(2.39)			1.05			
FMRMIN*			.549			-.936	.979	.418	1.51			
			(27.5)			(3.57)						
FMRFBT*[4]		1.87		-.316	.468	-.763	.901	.571	1.39			
		(3.00)		(2.02)	(2.56)	(1.77)			1.92			
FMRTCL*			.579		.425	-.178	.963	.084	1.47			
			(4.98)		(3.46)	(2.68)			1.26			
FMRCRW*		6.03	.483				.983	.251	1.58			
		(3.70)	(7.19)									
FMRSCG*	.155		1.81			.800	.250	.128	1.55	.478	-.852	-2.18
	(1.50)		(2.27)			(8.43)				(2.07)	(2.27)	(2.29)
FMRMET*			1.01	-.207		1.25	.968	.216	1.66			
			(10.9)	(5.25)		(6.31)						
FMRMTE*[5]	.509	1.84	-2.75	-.476		-.143	.994	.365	1.41			
	(2.44)	(11.1)	(8.10)	(3.34)		(.141)						
FMRSVC*			.049			-.003	.614	.023	2.22			
			(4.82)			(.219)						

Notes for Table 9

1. The coefficient under $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$; the coefficient under $\hat{X}^*$ is the first-order autoregressive coefficient.
2. The coefficient under RP represents a one period lag of RP; the coefficient under $\hat{G}^*$ is the first-order autoregressive coefficient
3. The coefficient under RP represents a two period lag of RP; the coefficient under $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$; the coefficient under $\hat{X}^*$ represents a dummy variable whose value is 1 in 1968, 0 elsewhere.
4. The coefficient under $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$.
5. The coefficient under RP is the first-order autoregressive coefficient; the coefficient under $\hat{G}^*$ represents $\hat{G}^* + \hat{X}^*$; the coefficient $\hat{X}^*$ represents a dummy variable whose value is 1 in 1968, 0 elsewhere.

Table 10

Germany - Imports: EEC, ROW, 1954 - 1970

	RP	$\hat{G}$*	$\hat{X}$*	$\hat{G}$*+$\hat{X}$*	TIME	M*(-1)	DUMMY 1	DUMMY 2	CONSTANT	$\bar{R}^2$	SE	DW/H
EEC												
GMCAGF*		.154 (7.49)	.167 (2.04)		-.017 (1.88)					.908	.029	1.27
GMCMIN*				1.93 (7.36)	-.151 (4.79)				-.015 (.147)	.912	.149	2.12
GMCFBT*[1]				2.83 (29.8)	-.069 (5.98)		.389 (1.74)		-.415 (4.66)	.998	.058	1.82
GMCTCL*		3.44 (12.7)	2.13 (3.83)		-.126 (2.40)					.988	.240	2.12
GMCCRW*	-1.87 (3.92)		1.42 (9.91)						-1.95 (6.00)	.974	.403	1.34
GMCSCG*	.441 (2.92)		1.48 (5.66)						-.295 (1.87)	.726	.102	1.12
GMCMET*[2]				2.29 (5.80)	-.322 (4.06)		-65.1 (2.53)		13.7 (2.38)	.797	.434	2.38
GMCMTE*[1]	.304 (1.87)			1.39 (7.82)	-.499 (4.22)		.301 (1.30)		-1.04 (1.90)	.963	.401	1.94
GMCSVC*[7]			.049 (3.18)			.255 (4.05)	.250 (12.2)		-.027 (1.62)	.991	.018	1.27 1.55
ROW												
GMRAGF*			.186 (2.00)			.605 (3.05)			-.475 (.796)	.702	.596	1.80 .719
GMRMIN*[3]	1.60 (7.36)			1.06 (3.26)			11.7 (7.82)	3.29 (3.07)	2.55 (1.88)	.894	1.45	1.71
GMRFBT*[4]	-.857 (2.03)			3.64 (20.2)	-.505 (7.60)				-.484 (.683)	.980	.572	1.77
GMRTCL*		4.01 (9.25)	1.29 (2.17)		-.133 (1.91)					.962	.372	2.09
GMRCRW*		11.7 (2.30)	.511 (2.29)						-.147 (.354)	.939	.756	2.00
GMRSCG*[5]	.770 (3.74)	10.1 (4.96)	3.34 (2.15)	-3.44 (1.30)	-.059 (3.34)	-.030 (2.79)	.136 (2.45)	.036 (1.99)	3.63 (3.60)	.794	.075	2.56
GMRMET*[6]				1.38 (12.3)		.133 (1.79)	-4.97 (8.09)	-4.15 (10.7)	-2.96 (5.32)	.965	.573	1.78 .466
GMRMTE*	.310 (2.05)	3.24 (12.1)							-4.47 (5.47)	.912	1.32	1.80
GMRSVC*[7]	.775 (4.19)	6.41 (2.13)					.261 (5.47)		-.015 (.420)	.950	.060	1.92

Notes for Table 10

1. The coefficient under Dummy 1 is the first-order autoregressive coefficient.
2. The coefficient under Dummy 1 represents the \$/DM spot rate.
3. Dummy 1 is 1. in 1967, 0. elsewhere; Dummy 2 is 1. in 1964-1965, 0. elsewhere.
4. The coefficient under RP in this equation represents a one period lag of RP.
5. The coefficients under $\hat{G}*$, $\hat{X}*$, and $\hat{G}* + \hat{X}*$ represent the current, first period lag, and second period lag, respectively, of $\hat{G}*$; the coefficient under M*(-1) represents the capacity utilization rate for chemicals, CPCHM; the coefficient under Dummy 2 represents the percentage change in the \$/DM spot rate; Dummy 1 is 1. in 1964-65, 0. elsewhere.
6. Dummy 1 is 1. in 1967, 0. elsewhere; Dummy 2 is 1. in 1962-1964, 0. elsewhere.
7. Dummy 1 is 1. in 1965-70, 0. elsewhere.

Table 11

Italy - Imports: EEC, ROW; 1954-1970

	RP	$\hat{G}$*	$\hat{X}$*	TIME	SR	DUMMY 1	DUMMY 2	M*(-1)	CONSTANT	$\bar{R}^2$	SE	DW/H
EEC												
IMCAGF*[1]	.312		1.04			.064		.632	-.076	.944	.015	2.27
	(2.52)		(2.70)			(3.70)		(3.97)	(2.61)			.733
IMCMIN*	.868		9.16	-.014					.052	.532	.008	1.79
	(2.21)		(3.82)	(4.09)					(8.60)			
IMCFBT*[2]			2.15	-.806	.007	.045		.722	-4.59	.972	.022	2.46
			(2.24)	(5.44)	(2.17)	(2.18)		(8.65)	(2.18)			1.01
IMCTCL*		$151.\times 10^3$	1.22						-.047	.971	.007	2.37
		(10.0)	(4.29)						(3.23)			
IMCCRW*[3]		.776			.007	-.071	.115	.754	-4.28	.994	.017	2.02
		(4.44)			(2.28)	(3.77)	(5.77)	(8.33)	(2.31)			.044
IMCSCG*	.575	2.18							.040	.261	.014	1.62
	(2.73)	(1.75)							(3.22)			
IMCMET*[4]			2.28	.233	-.182	-.180	-.189		-.011	.842	.047	1.81
			(4.58)	(4.05)	(3.38)	(3.10)	(2.19)		(.216)			
IMCMTE*[5]		6.65		-.190	-.062	-.390			38.8	.945	.074	2.28
		(7.57)		(5.84)	(4.33)	(4.74)			(4.31)			
IMCSVC*[6]			.145		$-.3\times 10^{-3}$	-.006			.190	.494	.002	1.97
			(4.14)		(1.50)	(2.90)			(1.51)			
ROW												
IMRAGF*			1.13					.412	-.431	.891	.118	1.82
			(2.65)					(1.88)	(2.05)			.877
IMRMIN*			.818					.508	-.248	.901	.094	1.92
			(1.92)					(1.76)	(1.57)			
IMRFBT*[7]	-1.65		.988			-.072	4.50		-.018	.956	.044	2.20
	(2.32)		(2.25)			(1.83)	(5.82)		(.258)			
IMRTCL*[8]		$163.\times 10^3$	2.88			-.064		.161	-.133	.966	.011	1.96
		(4.79)	(3.06)			(8.16)		(1.89)	(3.15)			.095
IMRCRW*	-.429		.833						.074	.951	.045	1.64
	(3.33)		(8.24)						(1.79)			
IMRSCG*[9]												
IMRMET*[10]	.433			.543		.291			.211	.606	.065	1.97
	(3.41)			(2.66)		(4.86)			(5.41)			
IMRMTE*[11]		3.15		-.049		.199	-.299		-.108	.930	.063	1.48
		(5.99)		(3.24)		(3.02)	(4.39)		(2.34)			
IMRSVC*[9]												

Notes for Table 11

1. DUMMY 1 is a dummy variable that is 1. in 1969, 0. elsewhere; RP represents RP_{-1}.

2. SR represents SR_{-1}; the coefficient under TIME represents the first order serial correlation coefficient; DUMMY 1 is a dummy variable whose value is 1. from 1965 through 1970, 0. elsewhere.

3. $\hat{G}*$ represents $\hat{G}* + \hat{X}*$; SR represents SR_{-1}; DUMMY 1 is a dummy variable that is 1. in 1964, 0. elsewhere; DUMMY 2 is 1. in 1969, 0. elsewhere.

4. TIME represents a dummy variable whose value is 1. in 1963, 0. elsewhere; SR is a dummy variable that is 1. from 1964 through 1967, 0. elsewhere; DUMMY 1 is 1. in 1968, 0. elsewhere; DUMMY 2 is 1. from 1968 through 1970, 0. elsewhere.

5. $\hat{G}*$ represents $\hat{G}*_{-1} + \hat{X}*_{-1}$; DUMMY 1 is a dummy variable whose value is 1. in 1968, 0. elsewhere.

6. DUMMY 1 is a dummy variable that is 1. in 1964, 0. elsewhere; SR represents SR_{-1}.

7. DUMMY 1 is a dummy variable that is 1. in 1962 and 1963, 0. elsewhere; DUMMY 2 is 1. from 1965 through 1970, 0. elsewhere.

8. DUMMY 1 is a dummy variable whose value is 1. from 1964 through 1970, 0. elsewhere.

9. Variable exogenized.

10. The coefficient under TIME represents the first order serial correlation coefficient; DUMMY 1 is a dummy variable that is 1. in 1963, 0. elsewhere.

11. $\hat{G}*$ represents $\hat{G}* + \hat{X}*$; DUMMY 1 is a dummy variable that is 1. in 1964, 0. elsewhere; DUMMY 2 is 1. in 1968, 0. elsewhere.

Table 12

Netherlands - Imports: EEC, ROW; 1954-1970

	Time	$\hat{G}*$	$\hat{X}*$	$\hat{G}*+\hat{X}*$	RP	Dummy 1	Dummy 2	Constant	ρ	$\bar{R}^2$	SE	DW
EEC												
NMCAGF*[1]				.848 (4.85)		-.179 (3.04)		.069 (.732)	.548 (2.71)	.836	.066	1.51
NMCMIN*	.036 (8.03)	.692 (2.33)	-1.44 (5.21)					.276 (9.27)		.876	.035	2.29
NMCFBT*[2]				1.69 (25.9)		-10.5 (4.12)		2.43 (3.65)		.987	.046	1.74
NMCTCL*		1.43 (14.6)	.733 (3.66)					-.471 (2.47)		.994	.077	2.35
NMCCRW*[3]	-.096 (4.00)	1.50 (1.91)	2.31 (8.46)			-.530 (4.31)		-1.07 (12.0)		.990	.117	1.97
NMCSCG*[4]				.571 (3.71)	1.24 (2.08)	-.305 (1.74)	-.526 (4.63)	.625 (6.63)		.569	.162	2.35
NMCMET*[5]	-.186 (3.17)		2.01 (4.90)			.457 (2.64)		1.62 (4.76)	.464 (2.16)	.848	.716	2.47
NMCMTE*[1]				1.02 (18.8)	-.245 (2.46)	-3.37 (8.92)		-2.11 (6.01)	.366 (1.62)	.977	.392	1.83
NMCSVC*[9]		-2.34 (2.26)	5.95 (2.29)		-.799 (2.46)	.124 (7.45)	.101 (2.32)	.607 (2.32)		.963	.016	1.82
ROW												
NMRAGF*[3]				.489 (12.0)		.795 (4.45)		1.24 (9.56)		.910	.173	1.63
NMRMIN*		6.06 (3.97)	19.1 (18.9)		.197 (1.94)			-.326 (2.17)		.958	.179	1.79
NMRFBT*[6]	-.078 (2.92)		2.39 (5.04)		-.716 (2.27)	-.327 (2.87)		.026 (.309)		.910	.103	1.78
NMRTCL*		.952 (2.97)	1.43 (2.95)					-.193 (1.77)		.976	.036	1.76
NMRCRW*			.977 (32.5)					-.203 (3.23)		.985	.101	1.43
NMRSCG*[7]	-.546 (5.02)			5.74 (4.82)				7.51 (4.35)		.616	.213	1.57
NMRMET*[3]	-.081 (3.55)			1.13 (4.02)		.310 (2.39)		1.22 (8.60)	.256 (1.09)	.615	.131	1.96
NMRMTE*[1]				1.25 (16.6)		-1.37 (7.54)		-.922 (3.78)	.546 (2.69)	.981	.206	1.50
NMRSVC*[8]			.027 (8.76)			-.020 (1.74)	-1.05 (2.23)	.254 (2.08)	-.401 (1.80)	.825	.011	2.13

Notes for Table 12

1. Dummy 1 is 1. in 1968, 0. elsewhere.

2. The coefficient under Dummy 1 represents a one period lag on the $/Guilder spot rate.

3. Dummy 1 is 1. in 1964, 0. elsewhere.

4. Dummy 1 is 1. in 1958, 0. elsewhere; Dummy 2 is 1. from 1961-70, 0. elsewhere.

5. Dummy 1 is 1. in 1963 and 1964, 0. elsewhere.

6. Dummy 1 is 1. in 1963, 0. elsewhere.

7. This equation is in log-linear form.

8. Dummy 1 is 1. in 1968, 0. elsewhere; the coefficient under Dummy 2 represents the $/Guilder spot rate.

9. Dummy 1 is 1. in 1959, 0. elsewhere; Dummy 2 is 1. from 1965-1970, 0. elsewhere; the coefficient under $\hat{G}*$ represents the $/Guilder spot rate.

Many more of the equations than was the case for gross output had estimated coefficients for the intermediate demand proxy $\hat{X}*$ that were negative. Again, the two demand proxies were frequently added together when this was the case. This was not necessary for France and the Netherlands for gross outputs, but it was frequently done for imports. There are many shifts in series for the Netherlands, necessitating several dummies. The true import coefficients may be highly changeable, whereas the demand proxy used here assumes fixed (value) coefficients (see equation (5)).

It is significant that, in both the gross output and imports equations estimates, a gain is realized by hypothesizing, à la Allingham (1973), that the coefficients on the final demand proxy differ from those of the intermediate demand proxy. In fact, Allingham's original treatment involved actually estimating the individual coefficients on the final demand bridge tables. Due to the multicollinarity between the two regressors encountered here Allingham's approach was not attempted, even with ridge regression.

Industries that caused difficulty in import demands were, notably, SCG as well as MET and AGF for France and Belgium. SCG is always an extremely small import category. The results are generally better for EEC-origin imports than for ROW imports, suggesting that ROW imports in the input-output tables might have been treated as the residual item (between domestic production plus EEC imports and total output) in constructing the flow tables. Too, prices are more sharply defined for EEC partner imports, since the ROW price was constructed as a residual by the author (see Berner (1976), Data Appendix), and since changes in unit value composition are more likely among a larger and more diverse group of countries such as ROW, than they are among EEC partners.

In general, the performance of this type of industrial sector gross output or import equation, linked to disaggregated demand proxies (or import content variables) is judged satisfactory. The linearity of this functional form for imports could easily be modified by using the same regressors, changing the dependent variable to the ratio of imports to gross output of an industry, as in Barker (1970). This would yield interaction terms derived from the separate regressors used in (5).

Finally, these equations performed well in a full system (all five countries) simulation, yielding root mean squared percentage errors (in the aggregate) of 2-4% for gross output and 4-6% for imports for a within-sample ten year dynamic simulation.[1]

Individual equations showed a tendency to wander only as a result of explanatory variables wandering. It must be pointed out that the trade data especially were highly volatile. This resulted partly from the fact that they were collected from at least three sources with varying degrees of coverages (OECD Series IV, OECD Series C, and NIMEXE (EEC) tapes), explaining the breaks in the series. Partly, however, the sector imports data for some sectors simply had high variances. The result is a set of gross output and import demands, that, together with the consumer import demand equations of Berner (1975), represents a theoretically correct econometric specification of these important variables for a multisector model.

[1]See Berner (1976), Chapter VIII.

Appendix

The purpose of this appendix is to derive the output expressions referred to on page 11 and the price conversion matrices used for the relative price terms in equations (11) and (12) in the text.

For a three-origin input-output accounting system such as the one described in the text, the balance equations, in matrix notation, are

$$\text{(A.1)} \qquad \begin{bmatrix} Q \\ \hdashline M_E \\ \hdashline M_R \end{bmatrix} = \begin{bmatrix} (I-A_D) & 0 & 0 \\ -A_E & I & 0 \\ -A_R & & I \end{bmatrix}^{-1} \begin{bmatrix} Y_D \\ \hdashline M_E^f \\ \hdashline M_R^f \end{bmatrix},$$

where Q, Y_D, M_i, M_i^f denote respectively vectors of domestic outputs, final demands of domestic origin, total imports from origin i ($i \varepsilon$ E, R) and final demands from imported origin i. A_i denotes submatrices of input-output coefficients relating to domestically produced and imported inputs from origin i, and I is the identity matrix. Solving for Q from the first row of (A.1) yields

$$\text{(A.2)} \qquad Q = (I-A_D)^{-1}\, Y_D$$
$$= A_D Q + Y_D,$$

noting that exports constitute one of the columns of Y_D. From the second and third row of (A.1),

$$\text{(A.3)} \qquad M_i = A_i(I-A_D)^{-1}\, Y_D + M_i^f, \quad i\varepsilon \text{ E, R},$$
$$= A_i Q + M_i^f$$

in which total imports are the sum of intermediate demands used in the production of output for domestic final demand and of final demands of imported origin.

In the quasi-identities (4) and (5) in the text, the form of (A.2) and (A.3) that is used is that of the second line in each equation, not involving the $(I-A_D)^{-1}$ matrix. Use of that matrix solves out for Q and gives only one regressor in (A.2).

The $(I-A_D)^{-1}$ matrix is useful in price conversion, however. Suppose that

(A.4) $Q = BV,$

where V is a vector of values-added, and B is a diagonal matrix such that $b_{jj} = 1/(1-\sum_i a_{ij})$. Substituting (A.4) into (A.2) yields

(A.5) $V = B^{-1}(I-A_D)^{-1}\ Y_D.$

Now define C such that

(A.6) $C \equiv B^{-1}(I-A_D)^{-1},$

and assume that the columns of C in current prices sum to unity; i.e.,

(A.7) $\iota'\ \hat{P}^V\ C\hat{P}^{-1} = \iota'$

where ι is the unit vector, P^V is a vector of value added prices, and P is a vector of output prices. $\hat{P}^V\ C\hat{P}^{-1}$ is the current price C-matrix. (A.7) implies, for P,

(A.8) $P' = P^{V'}\ C.$

This holds for a constant - price C - matrix. Similarly,

(A.9) $\iota'\ \hat{P}^E\ A_E\ \hat{P}^{-1}(I - \hat{P}\ A_D\ \hat{P}^{-1})^{-1} + \iota'\ \hat{P}^R\ A_R\ \hat{P}^{-1}(I - \hat{P}\ A_D\ \hat{P}^{-1})^{-1}$

$+ \iota'(\hat{P}\ B\ \hat{P}^{V-1})^{-1}(I - \hat{P}\ A_D\ \hat{P}^{-1})^{-1} = \iota'$

implies

(A.10) $P' = P^{E'} A_E (I-A_D)^{-1} + P^{R'} A_R (I-A_D)^{-1} + P^{v'} B^{-1} (I-A_D)^{-1}$,

which is the basis for the relative price terms used in (11) and (12) in the text.

These are only approximations because of two facts. First $P^{v'}$ appears in the last member of A.10, and P is used in the text; the B matrix was used so that the weights summed to unity. Second, the matrices are really in current prices, so the price indices should be harmonic, not arithmetic means. For example, if C^* is the current price C matrix, then

(A.11) $C = \hat{P}^{v-1} C^* \hat{P}$,

and if the columns of C in constant prices sum to unity,

(A.12) $\iota' \hat{P}^{v-1} C^* \hat{P} = \iota'$,

or

(A.12') $\iota \hat{P}^{v-1} C^* = \iota' \hat{P}^{-1}$.

Hence,

(A.13) $P_j = 1/\sum_i C^*_{ij}/P^v_i$, $j = 1,\ldots,n$,

a harmonic mean of the P^v's.

See Berner (1976), Chapters II and VI for further details.

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