#93

November 1976

# MODELING THE INTERNATIONAL INFLUENCES ON THE U.S. ECONOMY: A MULTI-COUNTRY APPROACH

by

Richard Berner, Peter Clark, Howard Howe, Sung Kwack and Guy Stevens

#23

# Modeling the International Influences on the U.S. Economy: A Multi-Country Approach*

Richard Berner, Peter Clark, Howard Howe, Sung Kwack and Guy Stevens
(Quantitative Studies Section)

## Introduction

Recent international events -- the U.S. devaluations, the move to flexible exchange rates, the quadrupling of the price of oil -- have dramatized the fact that the United States is significantly affected by external influences. Despite the increased awareness stimulated by these events, the development of the foreign sector of U.S. macro-economic models has lagged behind that of other sectors. This is probably in part a reflection of the view in the 1950's and 1960's that the United States was large enough to be modeled as a "closed economy," but it is also in part the result of the inherent difficulties in explaining U.S. international transactions.

There are two important obstacles to the successful development of models of the external sector of the U.S. economy; these are problems that should not be ignored, but they are not, we think, insurmountable. First, of course, is the recent change in the international

---

*The helpful comments on earlier drafts of this paper by many members of the Division of International Finance and by other interested economists are gratefully acknowledged. We are especially thankful to the following economists for sharing with us their detailed comments and suggestions: William Branson, Ralph Bryant, Dale Henderson, George Henry, Lawrence Lau, Patrick Minford, J. David Richardson, Jeffery Shafer, Jerome Stein, Edwin Truman and Janet Yellen.

monetary system from fixed (infrequently adjusted) to more flexible exchange rates. A more long-standing problem originates from the fact that many individual countries have a significant effect on the trade and capital flows of the United States; no single foreign country or bilateral exchange rate seems to be of such overwhelming importance as to merit exclusive attention. Thus the modeling of our international transactions requires, potentially, taking into account an enormous number of variables involving many countries.

This paper describes a model of the U.S. economy that emphasizes the impact of the foreign sector on key domestic variables, that allows for flexible exchange rates, and that strikes a reasonable compromise in the modeling of foreign country detail. We consider this a second-generation model, in that it will build upon and, we hope, improve upon efforts now completed or nearing completion -- notable the development of Kwack's balance-of-payments model and its linking with the FRB domestic model.[1] The present project will improve upon previous efforts, we hope, by better modeling the flexible exchange-rate system, by introducing recent developments in economic theory, and by incorporating country and policy detail that is of particular interest to the Division of International Finance.

---

[1] See the forthcoming paper by Sung Kwack entitled "Linkage of the Board Model with an International Transactions Model."

To be more specific, we see the goals of this project as the building of a model, for the purposes of forecasting and simulations, in which:

1. The international transactions and important exchange rates of the United States are endogenously determined.

2. The effects of international variables on the U.S. economy, particularly trade flows, capital flows and exchange rates can be specified and measured.

3. It is possible to analyze the effects of U.S. monetary policy on exchange rates, trade and capital flows.

4. It is possible to analyze the effects of exchange market intervention, both by the United States and foreign countries.

5. The most important effects of economic developments in the United States on foreign countries are measurable, and the feedbacks of these effects on the United States are specified.

To the greatest extent possible, consistent with considerations of cost and time to completion, the model should be constructed so that:

6. The impact of changes in foreign monetary policies on the U.S. economy can be analyzed.

7. The model can use as inputs detailed information on individual countries provided by the Division's World Payments Section and produce as outputs forecasts of important foreign variables that aid in analyzing these countries.

As noted above and in our previous paper,[1] to construct an adequate model, some difficult, even unique, problems must be solved. These deal primarily with the large number of countries that significantly affect U.S. international transactions and with the size of the United States relative to the rest of the world. In deciding how to attack these problems, important decisions have to be taken with respect to the modeling of the world outside the United States. These are discussed in Part I below, where we describe the scope of the overall model. To achieve the goals set out above, we deem it desirable to specify and estimate small structural models for the United States and four other countries important for U.S. trade and capital flows: Canada, Japan, the United Kingdom and West Germany. In so doing, as discussed below in Part I, we have rejected the alternative approach of using reduced-form equations for the important variables of these countries. In Parts II and IV of this paper we describe the structure of the prototypical country model. To facilitate linkages among them, the individual country models will have similar general structures, although they will of course differ in institutional detail. In building this basic structure we have mounted a sizable effort to develop a model that is empirically tractable and yet theoretically sophisticated enough to exhibit the significant features of industrial economies.

---

[1] R. Berner, P. Clark, H. Howe, S. Kwack and G. Stevens, "Simultaneous Determination of the U.S. Balance of Payments and Exchange Rates --An Exploratory Report." International Finance Discussion Paper No. 59, Feb. 3, 1975. (The reference to this paper is abbreviated in the text as IFDP #59.)

Part III describes how the component country models will be linked and how the overall system will be solved for its endogenous variables. In particular, we detail the process of exchange rate determination and the modeling of central bank intervention. We also discuss at length our use of ex ante balance-of-payments equations in the solution of the overall model.

In all four parts of this paper, the focus will be on the country sub-models and how they fit together into an overall world model. In order to maintain that focus and to avoid burying the reader in detail, in the present paper we shall only summarize both the reasoning that led to particular decisions and the arguments for and against alternative solutions. However, the details behind all our recommendations, arguments, and specifications are presented in a series of companion papers.[1]

## Part I: Scope of the Overall Model

In attacking the special problems inherent in the construction of a foreign sector for any U.S. model, we have found five choices or questions to be of primary importance:

---

[1] A list of these companion papers is as follows:
Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model." International Finance Discussion Paper (IFDP) No. 97
Peter Clark and Sung Kwack, "Asset Markets and Interest Rate Determination in the Multi-Country Model." IFDP No. 94
Howard Howe, "Price Determination in the Multi-Country Model." IFDP No. 98
Guy Stevens, "Alternatives for Modeling the World Outside the United States." IFDP No. 96
Guy Stevens, "Balance of Payments Equations and Exchange Rate Determination." IFDP No. 95.

1. Should the world outside the United States be modeled as a single undifferentiated region or should certain individual countries or areas be broken out?

2. Assuming that a few countries are broken out separately, what should be the treatment of those many other countries that cannot be so separated (i.e., rest of the world, ROW)?

3. Whatever the degree of country disaggregation in the world outside the United States, should the separate entities be modeled with structural models, with reduced-form equations for important variables, or with the important foreign variables treated exogenously?

4. If it is decided to use either structural or reduced-form models for important foreign countries, how should these be constructed?

5. Finally, to what sort of U.S. model should the chosen treatment of the world outside the United States be linked?

The present part of this paper concentrates on the answers to questions 1, 2, 3 and 5, i.e. on the more general aspects of our treatment of the world outside the United States and the U.S. sub-model. The specifics of the typical country model (question 4) are presented in Parts II and IV.

We start this part with a discussion of the fifth and third questions, so that the reader can appreciate at the outset what it means, in terms of time and effort, to recommend that one or several countries be modeled individually.

A. Proposed Treatment of the U.S. Sub-Model.

The centerpiece of the overall model will be of course a structural model of the United States.[1] But there are many choices left open once that is said. The fact that we are interested primarily in the United States implies that we should have a model disaggregated enough to capture the essential features of the United States economy and that we certainly should not sacrifice detail if that would threaten the realism of this focal point of our effort.

An important question was whether we could utilize an already existing model for the United States, the obvious choice being our own FRB model. For the present we have decided against that course; although the model we have specified for the United States will build on existing domestic models, particularly the FRB model, there are a number of reasons why we opt to build a new one. First, we would have to change substantially the foreign sector of any existing U.S. model, because no existing domestic model has an adequate foreign sector; this is particularly true regarding the treatment of capital flows. Moreover, other parts of existing domestic models would have to be restructured, in order to allow adequate linkages between the domestic U.S. economy and the international variables that are emphasized in this project. Second, because of the difficulties we expect to encounter in solving the system of simultaneous nonlinear equations implied by the endogenization of exchange

---

[1] Although we considered using reduced-form models for certain foreign countries, we never seriously considered this alternative for the U.S. model. The reasons are the same as those discussed in section B of this part, below.

rates, our first U.S. model should, we think, be on the small side; a large model like the FRB model might make it computationally infeasible to solve the overall system. Finally, by building our own U.S. model we are able to insure that certain key theoretical concepts are incorporated into its structure, and we maximize our ability to understand what will be happening when it comes time to simulate the overall model.

For this first step at modeling the international influences on the U.S. economy, the above considerations led to the following general framework. In fact, the basic structure, in terms of the markets to be modeled and the underlying behavioral theory, will be the same for the models of United States and for each foreign country to be modeled separately; however, the specific equations may differ amoung the country models because of differences in institutional structure, the availability of data, and the chosen degree of disaggregation. Five markets will be modeled for each country treated separately: markets for a composite of domestically-produced goods and services, labor services, high-powered money, and short and long term debt. The initial country sub-model will consist of approximately 30 behavioral equations, 14 of which are related to international transactions.

In specifying the structure of this basic model, we have attempted to link the behavioral equations to the accepted theory of microeconomic decision-making. We have also tried to incorporate frequently-observed macro-economic phenomena such as sticky prices and disequilibrium in selected markets. In the latter case, the models will take account of the spillover of the effects of disequilibrium into other markets, notably those related to international transactions and the exchange rate.

After the first stage of this project is completed, it would be possible to change the U.S. model in many ways. For example, it seems likely that it would be desirable to introduce more disaggregation into the trade sector. If desired, it would also be possible to restructure the U.S. model so that it would approximate or even be identical to the FRB model.

B. Foreign Country Sub-Models: Structural or Reduced Form?

As noted above we recommend that a small structural model be constructed for each of the foreign countries that will be treated separately. Given the decision to model a given country separately and to endogenize certain of its key variables such as its bilateral exchange rate with the United States, the only alternative to a structural model would be a reduced-form model. As discussed in our previous paper (IFDP #59) and in a companion paper by Stevens,[1] our conclusion is that a reduced-form model would be less rigorous, as difficult, and almost as time-consuming to construct as a small structural model. Moreover, reduced forms would sacrifice such goals as the estimation of feedback effects ( goal #5), the simulation of many kinds of intervention policies (#4) and, to a great extent, the endogenization of important country

---

[1]Guy Stevens, "Alternatives for Modeling the World outside the United States," section IV.

variables (#7) and the analysis of foreign monetary policies (#6). Given these deficiencies, we have decided against this alternative.

C. Degree of Country Disaggregation for the World Outside the United States: Two versus Multi-Region Models.

This is one of the most difficult and important questions to be answered in the shaping of this project. It is important because the number of countries that are modeled separately affects both the cost of the project and the ability of the final product to realize the goals set out above. The answer is difficult because there is no way we can demonstrate *ex ante* that our preferred degree of country disaggregation is conclusively superior to all others.

The major consideration determining the number and identities of the foreign countries to be modeled individually -- related of course to the goals set above -- is how many exchange rates, levels of income in foreign countries, foreign interest rates, and other foreign variables should be made endogenous. The answer to this question depends in turn on the degree of disaggregation necessary to achieve good forecasts and policy simulations under goals 1-4, and the desirability of country detail, goals 6 and 7.

The most immediate effect of a particular degree of country disaggregation is on the number of separate exchange rates that can be made endogenous and can appear in the equations for the U.S. trade and capital account. It seems obvious, and is in fact the case, that the number of separate regions or countries we need to model is equal to the number of endogenous exchange rates that appear in the overall model.[1] Below, we present evidence showing that the achievement of goals 1-4 is likely to be significantly hindered by a wholly aggregated treatment of the world outside the United States and the concomitant use of a single weighted-average exchange rate.

Probably the most obvious effect of a given choice of disaggregation in the world outside the United States will be on the overall model's ability to use and produce country detail. The greater one's interest in assessing the effects of U.S. activities on specific foreign countries (which includes the bilateral exchange rates between those countries and the United States), the greater will be the impetus for modeling individual countries; similarly, the greater the interest is measuring the effects of policy actions in certain foreign countries on the United States, the greater should be the push in this direction.

Concerning the question of the impact of the degree of country disaggregation on the forecasting and simulation accuracy of the overall model, there are a number of theoretical and practical considerations that can be brought to bear. Unfortunately, however, we will be forced to conclude below that none of these will conclusively point to one particular treatment of the world outside the United States.

Let us consider initially the *a priori* arguments based on aggregation theory. There are in fact two types of aggregation results that are relevant to this problem. First, as discussed at length in IFDP #59,

[1] For a more rigorous discussion of this and related points, see the companion paper by Stevens, "Balance of Payments Equations and Exchange Rate Determination."

are the conditions under which the bilateral exchange rates, which affect trade and capital flow equations, can be aggregated into a weighted-average or effective exchange rate. It was shown in IFDP #59 that the use of a single weighted-average exchange rate is theoretically defensible only if the effects of all bilateral exchange rates in one equation (i.e. on the dependent variable) are proportional to their effects in all other equations.[1] Since no one would claim that this stringent condition holds in reality, one is pushed to a reliance on an empirical or ad hoc justification for aggregation, or to the modeling of at least some individual foreign countries or regions.

Second, even if the above condition were present for aggregating bilateral exchange rates within a given equation, further conditions would have to be satisfied in order to aggregate across countries in the treatment of the world outside the United States. If no restrictions are put on the empirical variability of the endogenous variables in the system, this second aggregation condition requires that all equations in the separate country models be linear and, for corresponding equations, that the slopes for a given independent variable be equal.[2] This, too, is a condition that does not in fact hold.

---

[1] See Part II of IFDP #59, p. 8 ff. and H.A.J. Green, Aggregations in Economic Analysis (Princeton, 1964), Part II.

[2] H.A.J. Green, op. cit., Chapter 5.

Thus theory argues against aggregating countries in the world outside the United States. However, because of considerations of cost, data availability, and the large number of foreign countries that significantly affect U.S. trade and capital flows, a policy of disaggregating all significant foreign countries is impossible. The implication is that our initial stretegy for modeling the world outside the United States must be based a subjective weighting of the costs due to errors of aggregation, the multiplication of the number of individual country models, and the loss of country detail.

Given the infeasibility of the theoretically preferred strategy, it has been necessary to reconsider all alternatives, in particular the polar case of total aggregation -- treating the world outside the United States as a single, undifferentiated region. In weighing this alternative in the light of the broader set of criteria, mentioned above, we have still reached a negative conclusion concerning a wholly aggregated world outside the United States.

First, consider again the question of the errors in simulations and forecasts likely to be introduced by aggregation across countries. Above we outlined the theoretical case against aggregation. However, the magnitude of the error actually introduced by aggregation depends on empirical conditions: if the variables from different countries are highly correlated with each other[1] (e.g. exchange

[1]Of course if the components are perfectly correlated, then a weighted-average or any bilateral exchange rate can be used, because the system contains only one independent exchange rate.

rates), then the aggregation error may be small. The data for exchange rate movements indicate that the errors of aggregation are likely to be large. Table I shows that, for the most recent period 1974-75, the correlations between the exchange rates of our major trading partners -- Canada, Japan, Great Britain and West Germany -- ranged from a high of .76 to a low of -.33. The average correlation was only .37. The same picture appears for the first half of the period of flexible exchange rates, 1971-73. Table I also shows that the correlations changed substantially between the (arbitrarily) chosen sub-periods. To us this simple table undercuts the major empirical argument for total aggregation.

The weighing of other arguments leads, similarly, to the rejection of this polar case. The goal of endogenizing a number of foreign variables (including exchange rates) requires that we fit equations using foreign country data; if the world outside the United States were treated as a single region, composite equations would have to be estimated using data aggregated over many countries. Because of behavioral and institutional differences among countries, we would expect these composite equations to be very hard to defend. Finally, the Division's interest in bilateral exchange rates and individual country detail can only be realized by the modeling of some individual foreign countries.

**Table I:** Correlations Between Major U.S. Bilateral Exchange Rates, 1971-75 (Quarterly)*

1971 (2nd Quarter) to End of 1973

| | Can. | U.K. | W.G. | Jap. | W.ave. |
|---|---|---|---|---|---|
| Canada | 1.0 | -0.75 | .078 | .38 | .27 |
| U.K. | | 1.0 | -.067 | .022 | .034 |
| W. Germany | | | 1.0 | .91 | .97 |
| Japan | | | | 1.0 | .98 |
| Weighted Average** | | | | | 1.0 |

1974 - 75

| | Can. | U.K. | W.G. | Jap. | W.ave. |
|---|---|---|---|---|---|
| Canada | 1.0 | .76 | -.33 | .54 | .10 |
| U.K. | | 1.0 | .29 | .60 | .66 |
| W. Germany | | | 1.0 | .16 | .89 |
| Japan | | | | 1.0 | .43 |
| Weighted Average** | | | | | 1.0 |

*The quarterly observation for each exchange rate is measured by the quarterly average for Canada and the average of the end of month figures for W. Germany, Japan, the U.K., France, Italy, the Netherlands, Belgium, Sweden, and Switzerland.

**The average of the exchange rates of Canada (0.251), W. Germany (0.160), Japan (0.160), the U.K. (0.104), France (0.085), Italy (0.068), the Netherlands (0.061), Belgium (0.055), Sweden (0.028), and Switzerland (0.028) weighted by the average of each country's multilateral and bilateral trade weights in 1972 (computed by Bob Bradshaw in January 16, 1975 memo).

It seems, therefore, that both theoretical and empirical ce
point to the conclusion that some degree of disaggregation is ole
in constructing the world outside the United States. Neverth
we definitely believe that it would be desirable to compare t er-
ties of a highly aggregated two-region model (the United Sta
the rest of the world) with the more disaggregated model we
below. We believe that the linked Kwack model will produce what
is needed in this regard, despite the fact that the U.S. sub-
in the two exercises will not be identical. If more compara lua-
tion along these lines should eventually prove desirable, we he
best course would be to construct a new two-region alternati
our preferred disaggregated model is finished; at that time c-
ture of such a model could be tailored to the already existi -
ture of the completed country sub-models.

D. A choice of Countries for Individual Modeling

Since we reject a wholly aggregated treatment of the wo side
the United States, the only feasible route is somehow to sel o-
cedure for partially disaggregating a few important foreign es,
while at the same time leaving the large number of remainin
tries aggregated in a residual region, the rest of the world
The procedure that we propose below can be shown to be esse
independent of the treatment of ROW, so we shall postpone co -
tion of this latter part of the model until the next section

[1]See Stevens, "Alternatives for Modeling the World Out
United States", section III.

Two major criteria for country disaggregation have suggested themselves to us and readers of our earlier paper: (1) the degree to which the important variables of a country (exchange rates, etc.) are not highly correlated with those from other countries; (2) the "importance" of the country to the United States, based on the relative importance of its trade and capital flows with the United States and/or on special reasons for wanting to model that country's monetary and intervention policy.

We recommend that the following countries be modeled individually: Canada, Japan, the United Kingdom, and West Germany. This list was originally chosen primarily according to the second criterion; however Table I, above, shows that the exchange rates of these countries' currencies vis-a-vis the dollar are not highly correlated.

Table II shows that these four countries hold the first three places of importance for U.S. imports, exports, the stock of direct investment and the stock of portfolio claims on foreigners. A similar, though not quite so striking picture, can be observed for the stock of U. S. liabilities to foreigners and for all flows except the flow of total liabilities to foreigners; in the latter case, Switzerland and the Middle East oil exporters take the first two places.[1] Thus, in terms of totals, these four countries account for a large percentage of all trade and capital flows with the United States -- and a larger

---

[1]These data are for 1974, the most recent year for which all the data were available. The picture is similar for recent earlier years and for what we know about 1975.

TABLE II

TRADE AND INVESTMENT BY MAJOR COUNTRIES

(Millions of U.S. Dollars)

| | U.S. Trade (1974) | | U.S. Direct Investment (1974) | | U.S. Total Claims (Non-Direct) on Foreigners (1974) | | U.S. Total Liabilities (Non-Direct) to Foreigners (1974) | |
|---|---|---|---|---|---|---|---|---|
| | Imports | Exports | Stock | Flow (1974-1973) | Stock | Flow (1974-1973) | Stock | Flow (1974-1973) |
| W. GERMANY | 6428(3) | 4986(3) | 7998(3) | 348(12) | 1045(8) | 307(7) | 10429(4) | -3557(3) |
| JAPAN | 12455(2) | 10679(2) | 3337(8) | 666(6) | 14102(1) | 6438(1) | 14802(1) | 1406(5) |
| FRANCE | 2305(9) | 2942(9) | 4886(4) | 591(7) | 1347(6) | 199(11) | 4807(5) | 485(9) |
| UNITED KINGDOM | 4021(6) | 4574(5) | 12461(2) | 1421(2) | 5116(3) | 1676(2) | 10455(3) | 736(7) |
| CANADA | 22282(1) | 19932(1) | 28378(1) | 2837(1) | 5864(2) | 1330(3) | 4700(7) | -270(11) |
| ITALY | 2593(8) | 2752(10) | 2769(11) | 557(8) | 878(9) | 206(10) | 2869(11) | 1097(6) |
| NETHERLANDS | 1453(18) | 3976(6) | 3209(9) | 857(3) | 868(10) | 463(6) | 4086(8) | 535(8) |
| SWITZERLAND | 900(24) | 1150(19) | 4538(6) | 724(5) | 531 | 222(9) | 10862(2) | 5774(1) |
| MIDDLE EAST OIL EXPORTERS* | 4346(5) | 3215(7) | 2129(12) | -459(9) | 714(12) | NA | 4727(6) | 3981(2) |
| MEXICO | 3386(6) | 4855(4) | 2825(10) | 446(11) | 3112(4) | 961(5) | 1864(12) | 483(10) |
| VENEZUELA | 4676(4) | 1768(15) | 1772(13) | -279(13) | 1291(7) | 303(8) | 3494(9) | 1985(4) |
| BRAZIL | 1705(13) | 3089(8) | 3658(7) | 773(4) | 2503(5) | 966(4) | 1160(13) | 233(12) |
| 4 Country Total** | 45186 | 40171 | 52174 | 5272 | 26127 | 9751 | 40386 | -1685 |
| World Total | 100965 | 98524 | 118613 | 14938 | 61548 | 22577 | 112884 | 23504 |
| 4 Country % of World Total | 44.8% | 40.8% | 44% | 35.3% | 42.4% | 43.2% | 35.8% | 7.2% |

Source: U.S. Trade, Direction of Trade, Oct. 1975, p.2-4.
U.S. Direct Investment, Survey of Current Business, "U.S. Direct Investment Abroad", Oct. 1975, Tables 12, 13.
Total U.S. Claims on Foreigners, FRB Bulletin, Feb. 1975, A69, T.12, A75, T.23, A68, T.13, and Feb. 1976, A74, T.24, A73, T.23, A67, T.12 also Treasury Bulletin, May 1976, p.97, p.106, p.108.
Total U.S. Liabilities to Foreigners, FRB Bulletin, Feb. 1975, A66, T.8, A68, T.10, A75, T.23, and Feb. 1975, A64-A66, T.8-T.10, A69, T.15-T.16, A73, T.23 also Treasury Bulletin, May 1976, p.92, p.100, p.102, p.111-113.
*Middle East Oil Exporters are defined as in FRB Bulletin; Bahrain, Iran, Iraq, Kuwait, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates
[illegible] W. Germany, Japan, U.K. and Canada.

total than any other four-country breakdown.

Note, however, that Table II also demonstrate that <u>many</u> countries are important for the foreign sector of the United States: countries other than Canada, Japan, the United Kingdom, and West Germany still account for more than 50% of each of the columns in Table II.

Among those favoring disaggregation, no one has disputed the separate modeling of the above four countries; however, the importance of the rest of the world corresponding to that choice suggests even further disaggregation. Possibilities are France, Italy, Switzerland and/or the Middle East oil exporters. It might be potentially useful to model each of these additional countries or regions separately; nevertheless, as Table II shows, even if we were to specify separate sub-models for all of the above, the percentage of trade and capital flows left in the then-resulting rest of the world would still be very large. Given that a separate representation of more than four foreign countries would severely tax our ability to deliver a working product within a reasonable period of time, we present here an initial specification of the overall model that is restricted to six regions: the above four foreign countries, the United States, and an abbreviated ROW region. Given the way we propose to construct the trade and capital account sectors of the separate sub-models, it will be a relatively easy matter at some later time to add separate sub-models for more countries.

E. Treatment of Countries and Variables in the Residual, Rest-of-the World (ROW)

The above choice of four countries outside the United States to be represented by separate sub-models leaves a large number of alternatives with respect to the treatment of the multitude of countries that are left in the residual "rest of the world"(ROW). Our proposal is for a fairly limited ROW, at least at first. However, it might be useful to go over briefly some of the alternatives.

First let us consider the constraints on ROW imposed by data collection. Virtually no world aggregates exist; the only exception is for trade. Thus almost any ROW variable must be constructed by adding up the data for individual foreign countries. For most configurations of ROW this would involve considerable costs of data collection. Some help can come by using the individual country data banks for project LINK -- which we will use for some of our proposed country models; but most of the LINK country data, outside of the four we have chosen for individual modeling, are not quarterly and/or do not contain all the variables that we need. Thus any ROW that does not include the four countries already disaggregated involves some degree of data collection; this indicates why ROW variables in the past, for example in Kwack's balance-of-payments model, have been aggregates of a very limited number of countries. In Kwack's model, most ROW variables are aggregates of data for only four countries, those we have decided to treat separately.

Second is the question of the degree of endogenization of ROW variables and the type of model to be used for a given degree of endogenization. One option is to build a ROW that has exactly the same structure as our country sub-models; if one had already aggregated ROW data, this at least would be a feasible alternative. A somewhat similar strategy would be to endogenize ROW variables, but to use reduced forms for the variables rather than structural equations. Finally, to the extent that ROW is insignificant, uninteresting, and/or has little feedback onto U.S. variables, one can contemplate letting most or all ROW variables be exogenous.

The interplay of the considerations of data availability and the size of the rest of the world for already existing models of the U.S. foreign sector, has pushed us toward a minimal size for ROW, at least as a first cut at the problem. As noted above, in our proposed disaggregation we have already captured most of the country detail present in other models -- so even if a four-country breakdown does not look very impressive relative to the actual size of the world outside the United States, it compares favorably to what other models have done. Further, when we get beyond our four countries, we rapidly get to the point where it becomes difficult and costly to collect quarterly data; in fact most of the remaining countries in the world do not have readily available data on a quarterly basis for such variables as GNP, prices, money supply and so on. One could go on an annual ROW and do better in terms of data availability; this would be feasible, but would involve both considerable data-collection and programming complications, the latter in order to work the annual ROW into our quarterly framework.

In view of the above considerations, we feel justified in starting with a very limited ROW. Our view is that two variables in ROW should be endogenized and at least one enter the model exogenously. The latter would be the price of primary products; this variable, we feel, is important in the determination of import prices.

The variables we think should be endogenized are the Eurodollar rate and ROW exports. We discuss below in Part II why we feel it is a necessity to endogenize the Eurodollar rate. In fact, although the Eurodollar market will be outside all of our five country models, the rate really will not be a ROW variable, since it will be determined almost entirely by variables emanating from our five countries.

As we discuss in detail in explaining our framework for the determination of exports and imports, either ROW's exports or imports must be determined endogenously.[1] The specification of either of these permits the endogenous determination of exports and imports for each of our six regions. Normally we would prefer on theoretical grounds to determine ROW imports, as we do for all the other country sub-models. However, a ROW import function would require a good ROW activity or GNP variable to work well, and that may be hard to specify -- and harder

[1]See the companion paper by Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model," section II.C: "International Transactions."

to endogenize. Hence we will attempt to work with a ROW export function, which will be driven primarily by a GNP variable for our five disaggregated countries; this variable has the added desirable characteristic of being endogenous.

We discuss in Part III, below, the possibility of endogenizing a third ROW variable: either a weighted-average exchange rate between ROW and the United States or, if that exchange rate is assumed fixed, the change in international reserves for the countries in ROW. It is shown there that the U.S. balance-of-payments equation can be solved for either of these variables. However, we conclude that because we have data for so few of the important variables originating in ROW ( incomes, prices, interest rates, etc.), our overall model, for the present at least, will not be able to provide reasonable estimates of either the ROW exchange rates or the change in ROW reserves.

## Part II. The Structure and Basic Features of the Prototype Country Sub-Model

### Introduction

Part I has provided an overview of the issues and questions which we seek to address in our model-building efforts. In Part II we shall now describe the general characteristics of the typical country sub-model we have designed to answer these questions. We then explain in Part III how the five individual country models are linked together and solved simultaneously for the endogenous variables in the system. Finally, Part IV provides a listing and brief explanation of the specific equations that comprise our prototype country sub-model.

As pointed out in Part I, we have adopted an explicit multi-country approach in our modeling efforts. This does not mean, however, that we intend to build quite different sub-models for each country. On the contrary, we plan to use the same basic structure for each of the five countries. One reason for imposing a common specification on the sub-models is that we can then be sure of generating those variables that comprise the linkages among the countries in the overall model. Another consideration is that a uniform framework facilitates an understanding of how the entire model operates. We have therefore decided to portray a basic sub-model that will serve as the underlying structure for each country.

Although our point of departure will be the same for all countries, we do not intend to ignore all intercountry institutional differences.

On the contrary, we plan to incorporate the essential features of a country's economy insofar as they affect those variables (e.g., income, prices, interest rates) that are of particular interest to us. Differences in institutional structure will arise in the monetary sector, for example, and it will therefore be necessary to specify somewhat differently the instruments of monetary control from country to country. Nevertheless, we believe that it is useful to describe a common framework and to begin our empirical estimation of economic relationships on the same foundation. We shall modify the basic structure for each country on the basis of prior knowledge concerning features peculiar to an individual country and on the basis of our empirical results.

In our initial efforts we also plan to adopt approximately the same level of aggregation within each country sub-model. As our work progresses, however, we intend to focus more attention on the United States. Since we now break out only aggregate merchandise exports and imports, an important and necessary extension of the model obviously lies in the direction of disaggregating trade flows by type of commodity--at least for the United States. There are also many possibilities for further disaggregation in the domestic sector of the U.S. model. Since the specification of the domestic sector is not our primary focus, however, we have disaggregated only to the extent thought necessary to achieve the objectives outlined in Part I.

Part II is organized along the following lines. We first describe the agents in the country sub-model, the budget constraints within which they operate, their balance sheets and the flow of funds among the agents.

We show that by aggregating the budget constraints of all domestic agents one obtains the balance of payments, which is the budget constraint for the country as a whole. We then explain the major features of the important markets in the sub-model; these are for goods, labor and assets, where income, prices, the wage rate and interest rates are determined. Exchange rate determination is described in Part III in connection with the discussion of solving the overall model.

A. Agents and Flow of Funds

We differentiate four types of domestic decision-making agents in the prototype country sub-model: the central bank, the commercial banking sector, the government sector and the private non-bank sector. The salient features of each sector are described in turn. We then aggregate the four sectors to obtain the transactions of the entire economy with the rest of the world.

1. The Central Bank

We assume that the central bank has three domestic policy instruments: the discount rate, reserve requirements, and changes in holdings of domestic securities. In a world of managed floating the central bank can also use the purchase and sale of foreign assets as another instrument of monetary policy. In our initial modeling efforts we shall treat all four instruments of monetary control as exogenous variables. However, in those countries where domestic monetary policy has been significantly influenced by balance-of-payments considerations, we shall attempt to estimate a reaction function for the most important monetary policy variable at a later stage of our work. Also at a later stage we shall try to explain the foreign exchange intervention behavior of central banks since the advent of managed floating in 1973.

The typical central bank has the following balance sheet:

Central Bank Balance Sheet

| A | L |
|---|---|
| NFA | CUR |
| NGP | $\left.\begin{array}{l}\text{RR}\\\text{RX}\end{array}\right\}$ RT |
| RB | |
| OTH | NW |

where:

NFA = net foreign assets (international reserves)

NGP = net government position, i.e., claims on the government minus deposits held by the government at the central bank

RB = reserves borrowed by commercial banks from the central bank

OTH = other assets

CUR = currency

RT = total reserves of commercial banks at the central bank; broken down into:

a) RR = required reserves

b) RX = excess reserves

NW = net worth

The assets of the central bank constitute the "sources" of the monetary base; by increasing these claims the central bank generates an increase in its liabilities, which represent the "uses" of the monetary base. In particular, by increasing the total-reserves use of the monetary base, i.e., RT, the central bank can induce commercial banks to expand their holdings of earning assets, thereby depressing interest rates and encouraging additional expenditures.

The link between a sector's balance sheet and income and expenditure flows is its budget constraint, which states that the difference between income and expenditures, i.e., savings, is necessarily equal to the change in claims minus the change in liabilities, i.e., the change in net worth, where valuation effects, namely, capital gains and losses, are excluded from liabilities and assets. Taking the first difference of the items in the central bank's balance sheet, the budget constraint of this agent is given by:

(1) $$R_C - E_C \equiv \Delta NFA + \Delta NGP + \Delta RB + \Delta OTH - \Delta CUR - \Delta RR - \Delta RX$$

where:

R = receipts

E = expenditures

C = our mnemonic for central bank

2. The Commercial Banks

The balance sheet of the commercial banks is given by:

Commercial Bank Balance Sheet

| A | L |
|---|---|
| RR | RB |
| RX | DD |
| STS | TD |
| LTS | |
| FA | NW |

where:

STS = short-term securities

LTS = long-term securities

FA = foreign assets

DD = demand deposits

TD = time deposits

To simplify notation we have avoided the use of superscripts and subscripts, and we hope that our explanation of the balance sheets and budget constraints will make clear which are the assets and which are the liabilities of each sector. Thus the assets of commercial banks include required reserves (RR) and excess reserves (RX), which together equal total reserves (RT).[1] As we have seen above, these appear on the liability side of the central bank's balance sheet. We assume that there are two kinds of non-deposit financial assets: short-term securities (STS) and long-term securities (LTS), and that commercial banks are net holders of both types of assets. Finally, banks also hold claims on foreigners (FA). For our purpose at this point, it is not necessary to disaggregate these claims. The liabilities of banks include reserves borrowed from the central bank (RB), as well as demand deposits (DD) and time deposits (TD).

The budget constraint of the commercial banks, like that of the central bank, expresses the fact that the difference between receipts and expenditures is identically equal to the change in claims minus the change in liabilities (again, excluding capital gains):

(2) $$R_B - E_B \equiv \Delta RR + \Delta RX + \Delta STS + \Delta LTS + \Delta FA - \Delta RB - \Delta DD - \Delta TD,$$

where the letter "B" is our mnemonic for commercial banks. We have ignored real assets in both the balance sheets and the budget constraints of central and commercial banks because relative to their financial claims, their real assets are very small.

---

[1] Vault cash held by commercial banks is subsumed under RT, so that currency held by banks does not appear explicitly in their balance sheet.

3. The Private Non-Bank Sector

This sector contains two separate agents: firms and households. We distinguish between the behavior of these two agents when it comes to determining expenditures and prices. The theory of the firm is used to derive equations explaining investment and pricing decisions, whereas household behavior determines consumption expenditures. With regard to the supply and demand for assets, however, we do not separate firms from households, and therefore we deal here with the aggregate private non-bank sector (which, for short, we shall denote as the "private" sector).

The balance sheet of the private sector is given below.

Private Sector Balance Sheet

| A | L |
|---|---|
| CUR | STS |
| DD | LTS |
| TD | |
| FA | |
| KV | NW |

The private sector holds claims on the central bank in the form of currency (CUR), claims on commercial banks in the form of demand and time deposits (DD and TD), and claims on foreigners (FA).[1] A large fraction of the net worth of the private sector is in the form of real assets, which appear in the asset column of the balance sheet as the nominal value of the net capital stock (KV). The private sector issues both short- and long-term securities to other domestic sectors and to foreigners, and these appear (STS and LTS) in the liability column.

[1]As explained below, we break these claims down into two components: short-term and long-term portfolio claims, and long-term direct claims. At this point in our exposition, this disaggregation is not necessary.

Both STS and LTS are measured net of private-sector holdings of securities issued by the government, which would otherwise appear on the asset side of the balance sheet.

In the budget constraint of the private sector the change in the value of the net capital stock (ΔKV), i.e., net investment spending, is included along with all other expenditures. Therefore the change in the private sector's net claims on other domestic sectors and on foreigners is equal to total receipts (income net of taxes plus transfers) minus total (i.e., current and capital) expenditures:

(3) $$R_P - E_P \equiv \Delta CUR + \Delta DD + \Delta TD + \Delta FA - \Delta STS - \Delta LTS$$

4. The Government Sector

Because of the inherent difficulties in constructing a balance sheet for the government, we restrict ourselves here to a description of its budget constraint. Our treatment of this sector differs from the others in that we shall determine the government budget surplus or deficit as an endogenous variable. Our reason for doing this is that--as described in more detail below--the government deficit or surplus, when combined with the current account surplus or deficit, will yield an estimate of the change in the net worth (exclusive of capital gains or losses) of the combined non-government domestic sectors. There are three components of the government budget: government spending on goods and services (GV), which is assumed to be exogenous, and total government receipts (TV) and total government transfer payments (TRANV), both of which are explained within the model.

The surplus or deficit of the government is reflected in changes in its claims and liabilities. These are changes in government deposits at the central bank ($\Delta$BU), which are part of the net government position of the central bank ($\Delta$NGP), changes in demand and time deposits ($\Delta$DD and $\Delta$TD), issues of short- and long-term securities ($-\Delta$STS and $-\Delta$LTS), and changes in foreign assets held by the government ($\Delta$FA). The government budget constraint is therefore given by:

(4) $$\text{TV-TRANV-GV} \equiv \Delta\text{BU} + \Delta\text{DD} + \Delta\text{TD} + \Delta\text{FA} - \Delta\text{STS} - \Delta\text{LTS}$$

### 5. Flow of Funds and the Balance of Payments in the Country Sub-Model

By combining the budget constraints of all the domestic agents in a country sub-model we obtain a picture of the flow of funds among the domestic sectors, and by aggregating all these budget constraints together we can derive the flow of funds between a country and the rest of the world. To facilitate the exposition we make a few simplifications in certain components of individual budget constraints. First, we assume that changes in the net government position of the central bank ($\Delta$NGP) reflect only changes in its holdings of short- and long-term securities, so that changes in government deposits at the central bank are assumed to be zero, i.e., $\Delta\text{BU} = 0$. Second, we ignore changes in other assets of the central bank, so $\Delta\text{OTH} = 0$. Finally, since free reserves (RF) are equal to excess reserves (RX) minus borrowed reserves (RB), and since in each sub-model we shall explain RF rather than its components, we substitute $\Delta$RF for $\Delta$RX-$\Delta$RB in the central bank's and commercial banks' budget constraints. After incorporating these simplifications and substitutions, and after rearranging the terms in the budget constraints, we end up with Table 1.

## Table III. Flow of Funds, Budget Constraints and the Balance of Payments of the Country Sub-Model

| Real Side: Receipts - Expenditures | | | Financial Side: Change in Assets and Liabilities |
|---|---|---|---|
| 1. Central Bank: C | $R_C - E_C$ | $\equiv$ | $-\Delta CUR - \Delta RR - \Delta RF + \Delta STS + \Delta LTS + \Delta NFA$ |
| 2. Commercial Banks: B | $R_B - E_B$ | $\equiv$ | $+ \Delta RR + \Delta RF - \Delta DD - \Delta TD + \Delta STS + \Delta LTS + \Delta FA$ |
| 3. Private Non-bank: P | $R_P - E_P$ | $\equiv$ | $+\Delta CUR + \Delta DD + \Delta TD - \Delta STS - \Delta LTS + \Delta FA$ |
| 4. Government: G | TV-TRANV-GV | $\equiv$ | $+ \Delta DD + \Delta TD - \Delta STS - \Delta LTS + \Delta FA$ |
| 5. Total: T | $R_T - E_T$ | $\equiv$ | $0 \quad 0 \quad 0 - \Delta DD_T - \Delta TD_T - \Delta STS_T - \Delta LTS_T + \Delta FA_T$ |
| 6. $XGSV^S + XTRANV - MGSV - MTRANV$ | | $\equiv$ | $- \Delta DD^S - \Delta TD^S - \Delta STS^S - \Delta LTS^S + \Delta FA$ |
| 7. $XGSV^D + XTRANV - MGSV - MTRANV$ | | $=$ | $- \Delta DD^D - \Delta TD^D - \Delta STS^D - \Delta LTS^D + \Delta FA$ |

New variables: XGSV = receipts from exports of goods and services
XTRANV = receipts of transfers from abroad
MGSV = expenditures on foreign goods and services
MTRANV = expenditures in the form of transfer payments to foreigners

By aggregating the budget constraints of the four sectors in the sub-model we obtain the overall budget constraint for the country as a whole. This is shown in row 5, which is the sum of the first four rows. We assume that there is no foreign demand for currency, required reserves and free reserves, so that transactions in these assets net out to zero among the four sectors of the economy. The aggregate budget constraint for the country then states that the difference between total receipts and total expenditures ($R_T$-$E_T$) is necessarily equal to the change in aggregate liabilities to foreigners ($-\Delta DD_T-\Delta TD_T-\Delta STS_T-\Delta LTS_T$) plus the change in aggregate claims on foreigners ($\Delta FA_T$). Since row 5 is obtained by aggregating ex post identities, it is itself an ex post identity.

When we sum all the receipts and expenditures of the four domestic sectors, transactions between sectors net out in the same way that claims and liabilities between domestic residents cancel out. On the real side of the budget constraint we end up, as shown in row 6, with total receipts from foreigners minus total payments to foreigners, which is the current account of the balance of payments, namely, receipts from exports of goods, services and transfers minus payments for imports of goods, services and transfers. Similarly, the capital account of the balance of payments, namely, the change in claims on foreigners minus the change in liabilities to foreigners, is given by the financial side of row 6. Thus we have just demonstrated that the balance of payments, when regarded as the ex post equality of the current account and the capital account, can be derived from the overall budget constraint of the country. Since one can go just as easily in the opposite direction, we can assert that ex post a country's balance of payments and its overall budget constraint are equivalent or identical.

In somewhat similar fashion one can work on the ex ante rather than the ex post level.[1] Ex ante as well as ex post each agent has a budget or wealth constraint; this states the simple fact that at any given set of prices, income, etc., the total value of an agent's demands for goods and assets must equal the total value of what he supplies to the market (including labor and the services of his assets). Another way to say it is that ex ante his sources and uses of funds must be equal.

This ex ante budget constraint is a relationship among an agent's demand and supply functions; it is a constraint on the functions and since it must hold at all sets of prices, it is an identity. By adding these ex ante identities over all domestic agents we obtain, as we did for Table III above, an ex ante budget identity for the country as a whole. Again, as we showed above, this overall budget identity can be expressed equivalently as a balance-of-payments identity.

One can, furthermore, transform this ex ante identity into an ex ante balance-of-payments equation or equilibrium condition. Why this is desirable is explained below in section II.B.3 on asset markets, where the case is made for avoiding the use of the equilibrium condition for the short-term securities market and substituting in its place a balance-of-payments equilibrium condition.

---

[1] The points made in following paragraphs are developed in detail in Guy Stevens, "Balance of Payments Equations and Exchange Rate Determination."

There are many ways to convert the above identity into an equilibrium condition. The method we shall use is as follows.[1] Consider, for example, the entry for goods produced at home in the country's overall budget constraint; ex post this entry can be expressed either as the country's exports or the domestic excess supply of home goods; this is the first term on line 6 of Table III. Ex ante the entry would be the ex ante excess supply of the home good (i.e. the difference at the given set of prices between home demand and home supply), which is represented by $XGSV^S$, where "S" denotes domestic excess supply. At the assumed set of prices, this is the country's <u>potential</u> exports; but we do not know ex ante whether foreigners will purchase this supply at the assumed set of prices.

In <u>equilibrium</u>, however, we know that foreigner's demand for the home good will be equal to the above domestic excess supply. Similarly, in equilibrium foreign demand for domestic assets equals the domestic excess supply of these assets. Thus, for conditions of equilibrium, if we substitute foreign demand functions for the home excess supply functions in the balance-of-payments identity, the resulting expression will still equal zero. However, <u>out of equilibrium</u>, this new expression need not equal zero, since foreign demand will not generally equal home excess supply for all sets of prices.

This is the convention we will adopt for building an ex ante balance-of payments equation: take a given country's ex ante budget constraint and substitute <u>aggregate foreign demand</u> for <u>domestic excess supply</u> wherever the latter appears. In row 7 of Table III we have made

[1]See Stevens, <u>op</u>. <u>cit</u>., section III.A.

the appropriate substitutions, replacing the excess supply of home goods and services with foreign demand ($XGSV^D$, where "D" denotes foreign demand), and similarly, replacing equations describing the change in the excess supply of domestic assets with the change in foreign demand.

This substitution converts the balance of payments from an identity into an ex ante equation or equilibrium condition. The resulting expression equals zero in equilibrium, but can be non-zero for sets of prices (and other endogenous variables) that are not equilibrium values. It is proved by Stevens[1] that his balance-of-payments equation can be used as an equilibrium condition in our model and that it can be substituted for any of the other market clearing conditions.

Before turning to a description of the domestic markets for goods, labor and assets, we can use Table III to explain how we will obtain an endogenous measure of domestic private wealth. The change in net worth for the entire private sector (bank plus non-bank) is equal to net investment plus the change in net claims on other agents. We know from Table III that the change in net claims of the private sector is comprised of the change in net claims on foreigners and on the government, i.e., the current account plus the government deficit. Since both of these variables, along with net investment, will be determined endogenously in our model, we can calculate the change in private net worth by adding all three:

(5) $$\Delta NW = (IFV + IIV - CCAV) + (XGSV + XTRANV - MGSV - MTRANV) + (TRANV + GV - TV)$$

---

[1] Guy Stevens, op. cit., section III.B.

where:

$\Delta$NW = change in private net worth

IFV = gross fixed investment

IIV = inventory investment

CCAV = capital consumption allowances

The stock of net worth will be calculated by adding past changes to a benchmark figure. This is, of course, only a proxy for the true value of private net worth because capital gains and losses are ignored in this procedure.

B. The Markets for Domestic Output, Labor and Assets

1. Domestic Output and Price Determination

We make the simplifying assumption that each country produces a different commodity that is a composite of goods and services. The output of this commodity is determined by adding up the components of demand, i.e., GNP = C+I+G+X-M. Government spending is assumed to be exogenous, and equations explaining consumption, investment (broken down into plant and equipment and inventories), exports and imports (broken down into aggregate merchandise trade flows, investment income, and other services) are described in Part IV below.[1]

We assume that the quantity of domestic output demanded equals the quantity supplied at the current market price, that is, the market for domestic output clears. However, the price of output does not adjust to

---

[1]The derivation of these equations is explained in more detail in a separate paper by Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model."

equilibrate supply and demand each period; rather, the price is set as a markup over marginal cost, with the markup a function of capacity utilization.[1] Since there is some inertia in the price, inventories act as a buffer and absorb any discrepancy between current production and sales; it is assumed that there are no stockouts. Current production also adjusts somewhat in response to demand conditions. Consequently there is some adjustment in price, inventories and output so that supply equals demand each period.

The value of imports and exports are affected by exchange rate changes in two ways. First, the quantities of goods traded are affected by the prices of exports and imports and substitutes for these goods. Second, variation in these prices (which are endogenous variables in our model) directly affect the value of merchandise trade. The price of merchandise exports-- which will not be equal to the price of output sold domestically because this latter variable includes both goods and services --is also specified as a markup over marginal cost, where the markup is a function of foreign capacity utilization and the prices of competitive exports. Exchange rates convert these prices from foreign to domestic currency, and this link between export prices can be an important channel through which exchange rate changes affect the value of exports.

The price of imports also depends on exchange rates because they are used to convert the export prices of other countries, expressed in

---

[1] For a fuller description of the pricing behavior in the model, see the companion paper by Howard Howe, "Price Determination in the Multi-Country Framework."

foreign currency terms, into domestic currency units. Specifically, a country's import price is a weighted average of foreign countries' export prices. For each country we shall use the export prices of the four other countries in the overall model. A country's import price will also depend on the U.S. dollar price of primary products traded on world commodity markets. We take this latter price as exogenous, since it is beyond the scope of our model to determine primary product prices, which are subject to volatile swings caused by abrupt shifts in supply or demand. We have therefore decided to take as given a representative index of world-traded primary products, and then have this price influence the overall domestic price level through its effect on the price of imports. We therefore implicitly assume that the prices of domestically-produced primary products move in line with their world-traded counterparts.

2. The Labor Market and Wage Determination [1]

We adopt the standard assumption that the wage rate does not adjust to clear the labor market (union contracts, minimum wage legislation, etc.), so that there is disequilibrium in the form of excess labor supply. As pointed out by Clower, the implication of a fixed nominal wage is that consumption is no longer a function of the wage rate, but rather the income received by workers. Disequilibrium in the labor market thus gets reflected in the goods market and also other markets insofar as it influences the form of the consumption function.

---

[1] The labor market is described in more detail in the paper by Richard Berner, op. cit.

There are two basic equations in this market: one determines the unemployment rate and the other determines the percentage change in the wage rate. The former is a function of the demand and supply of labor. Labor demand is determined by lagged adjustment to desired labor input, which is given by the equality of the real wage with the marginal physical product of labor. The supply of labor is a function of population, the real wage, and migration. The rate of change in the nominal wage rate is determined primarily by the unemployment rate and the expected rate of change in the deflator for aggregate expenditures.

3. Asset Markets and Interest Rate Determination[1]

There are three asset markets in each sub-model; these are for money, short-term securities and long-term securities. We pay considerable attention to the money market because it is through this market that the instruments of monetary policy affect interest rates. As described in section II.A.5 above, one can use the balance-of-payments equation as an independent equilibrium condition. We shall use this equation to substitute for the equilibrium condition in the short-term securities market, and thereby avoid having to specify equations explaining the supply and demand for these securities. Finally, we take a simplified approach to the market for long-term securities: rather than equate the supply and demand for these instruments, we use a term-structure equation to determine the long-term interest rate. Thus we assume that in each sub-model domestic short-and long-term securities are very close if not perfect

[1]The asset markets are described in more detail in a separate paper by Peter Clark and Sung Kwack, "Asset Markets and Interest Rate Determination in the Multi-Country Model."

substitutes, and that arbitrage between the two markets makes the long-term rate behave as a weighted average of expected future short-term rates.

The basic building block of the monetary sector is the balance sheet of the central bank, which has been described above. Ignoring the net worth of the central bank, we have:

(6) $NFA + NGP + RB + OTH \equiv RT + CUR$

By varying its assets, the central bank controls the sources of the monetary base. Since borrowed reserves (RB) are to a large extent determined by the behavior of commercial banks, it is preferable to work with the unborrowed base, BU, because it is this quantity that is directly controlled - at least in a world of managed floating - by the central bank. Subtracting RB from both sides of (6), we obtain BU from the sources side:

(7) $BU \equiv NFA + NGP + OTH.$

Since $RT \equiv RR + RX \equiv aDD + bTD + RX$, where $\underline{a}$ and $\underline{b}$ are reserve requirements against demand and time deposits, respectively, and since free reserves are defined as $RF \equiv RX - RB$, the unborrowed base from the uses side is given by:

(8) $BU \equiv aDD + bTD + CUR + RF.$

Equation (8) can be used to determine one right-hand-side variable, since the unborrowed base from the sources side is given by equation (7).

Following the practice of the Federal Reserve econometric model, we have chosen demand deposits to be this variable. Rearranging (8) yields:

(9) $DD \equiv (BU - bTD - CUR - RF)/a$.

To complete the monetary sector of the sub-model we specify four asset demand functions to determine DD, TD, CUR, and RF. In making simulations and forecasts the equation for the demand for demand deposits in inverted so that the short-term interest rate (RS) becomes the left-hand-side variable. The complete monetary sector thus consists of six equations in six unknowns: BU, [determined by (7)], DD [determined by (9)], TD, CUR and RF (determined by asset-demand functions), and RS (determined by inverting the equation explaining the demand for DD).

It remains to describe our treatment of international financial transactions. If we were to adopt a full-fledged flow-of-funds approach to interest rate determination, we could dispense with standard capital flow equations. We would instead have to describe the foreign demand for specific domestic securities, and the interest rates on these securities would be determined by equating the domestic supply with the total, i.e., domestic and foreign, demand. As mentioned above, however, we are not pursuing this approach. We use term structure equations in our sub-models to explain long-term interest rates, rather than equating the supply and demand for long-term assets, and we drop the markets for short-term securities and substitute balance-of-payments equations. With the possible exception of the foreign demand for domestic money, we therefore do not need to identify and explain either the foreign demand for specific domestic assets or the domestic demand for particular foreign assets. We do, however, have to explain all international financial transactions because they enter the balance-of-payments equation. We have adopted a highly aggregated breakdown for these capital flows; we explain the change in total short-term and long-term portfolio claims (liabilities) and long-term direct claims (liabilities).

The possible exception mentioned above (the foreign demand for domestic money) arises because it relates to the one market where we specify explicitly the supplies and demands for an asset, namely, the money market. If foreign demand for domestic demand and time deposits (foreign holdings of domestic currency are presumably small enough that they can be ignored) have an important influence on domestic monetary conditions, then the foreign component of total demand should be modeled separately. The extent of this influence will vary from country to country, and therefore we have not incorporated it into the structure of our sub-model.

The demand functions for both domestic and foreign assets have basically the same form.[1] They are homogeneous in nominal magnitudes and include as explanatory variables private net worth, domestic and foreign interest rates, the expected change in exchange rates and a proxy variable for the volume of transactions. In the equations for demand and time deposits this latter variable is represented by nominal GNP, whereas in the equation for the

---

[1] The major exception is the equation for long-term foreign direct investment, which involves considerations regarding plant and equipment expenditures overseas and how best to finance these expenditures.

change in total short and long-term portfolio claims (liabilities), we use the value of exports (imports). The capital flow equations are derived from stock demand functions, that is, capital flows are a function of changes in the rate-of-return variables. Finally, our asset demand functions embody the assumption that economic agents allocate their portfolio during the current period not only on the basis of their wealth at the beginning of the period, but also on the basis of their current savings.

We do not plan to estimate a complete system of asset demand equations because we have only a proxy for private net worth, so that the sum of the assets held by domestic residents does not equal our net worth variable. Therefore we are not in a position to impose the usual constraints on the coefficients of a set of asset demand functions. Nevertheless, we shall attempt to measure the substitutability between domestic and foreign assets by including, as mentioned above, both foreign interest rates and expected exchange rate changes as explanatory variables in our demand and time deposit equations.

In the five countries for which we plan separate sub-models there is asset substitution not only between national financial markets but between the national and the Eurodollar market as well. For some countries, e.g., Germany, the most important foreign interest rate is the Eurodollar rate. We shall therefore make this interest rate an endogenous variable that will be explained by U.S. and foreign interest rates, expected exchange rate changes, U.S. and foreign wealth, the value of non-U.S. imports, and variables that influenced U.S. head office borrowing from their foreign branches.

## Part III: Combining the Country Sub-Models Into the Overall Model

In Part II we outlined the prototype sub-model that we shall use for each of the five countries that together comprise the overall model. It remains to describe how these five sub-models are linked together. These links appear explicitly in the international transactions of the five countries, and we explain first how the balances of payments of the five countries and the residual rest-of-the-world sector (ROW) are related. We then explain how the entire model will be solved for all endogenous variables, with attention focused on the determination of exchange rates. Finally, we describe how foreign exchange intervention by central banks can be handled by our model.

### A. Balance-of-Payments Relationships in the Overall Model

In Part II we showed that the balance of payments is the overall budget constraint of an economy, since it is obtained by aggregating the budget constraints of the individual sectors in a country. If we now aggregate the budget constraints ( = balance-of-payments identities) of the five countries and ROW, we end up with the balance-of-payments constraint for the world as a whole, which is identically equal to zero:

$$(10) \qquad \sum_{i=1}^{6} (XV_i - MV_i) \equiv \sum_{i=1}^{6} (\Delta FA_i - \Delta FL_i) \equiv 0$$

where all variables are expressed in the same currency and:

$XV_i$ = value of exports of goods, services and transfers

$MV_i$ = value of imports of goods, services and transfers

$\Delta FA_i$ = change in total claims on foreigners

$\Delta FL_i$ = change in total liabilities to foreigners

Equation (10) holds because:

$$(11) \quad \sum_{i=1}^{6} XV_i \equiv \sum_{i=1}^{6} MV_i, \text{ and}$$

$$(12) \quad \sum_{i=1}^{6} \Delta FA_i \equiv \sum_{i=1}^{6} \Delta FL_i.$$

Equation (11) states that for the world as a whole, the value of exports necessarily equals the value of imports (assuming that the data have been adjusted for f.o.b./c.i.f. differentials). Similarly, equation (12) asserts that the sum total of the claims of the five countries plus ROW on each other must equal their liabilities to each other.

Equation (10) is the world balance-of-payments constraint. If we wished, we could make use of it to compute the current account ( = capital account) of ROW, which is given by:

$$(13) \quad (XV - MV)_{ROW} \equiv - \sum_{i=1}^{5} (XV_i - MV_i) \equiv \Delta FA_{ROW} - \Delta FL_{ROW}$$

The world balance-of-payments identity thus implies that there are only n-1 independent balances of payments. Since we determine the components of the balances of five countries, we could derive the implied aggregated balance of payments for all other countries in the world combined, i.e., ROW. The balance of payments for ROW does not, however, serve any function in our model. Its only potential use would be as a check on our overall results, e.g., if in simulation experiments the five countries combined ran a huge current-account surplus (or deficit), this would not be reasonable in light of the implied deficit (or surplus) for ROW.

In Part II we also derived a balance-of-payments equation that will be used as an ex ante equilibrium condition in each country submodel, rather than as an ex post identity. This derivation was accomplished by substituting the foreign demand functions for the domestic supply functions of goods and assets. The balance of payments is then no longer an identity but an equilibrium condition that combines domestic demand for foreign goods and assets together with foreign demand for domestically-supplied goods and assets. It is nevertheless still true that the sum of the six balance-of-payments equations equals zero, so that only five of these equations are independent. As described above, the balance of payments for ROW can be derived residually as the negative of the sum of the equations of the five countries.[1]

We have constructed our model in such a way that we take account of the world trade and capital flow identities given by (11) and (12). In a complete world model one should specify the trade and capital flow equations so that the sum of each country's or region's imports equals (after suitable adjustment for f.o.b./c.i.f. differences and

[1]For further discussion of this point, see the paper by Guy Stevens, "Balance of Payments Equations and Exchange Rate Determination."

other valuation problems) the sum of each country's or region's exports. Our intention is not to build a complete model of world payments. Nevertheless, since we shall be constructing models for several countries, we are in a position to take account of the fact that an increase in the imports of these countries will affect the level of their exports, and similarly, an increase in their claims on foreigners will enlarge their liabilities to each other.

These relationships between transactions in a multi-country framework are embodied in our model in two ways.[1] First, the exports of each of the five countries are determined by the imports of these countries and the total exports of ROW. A trade-share matrix is used to convert these trade flows into the implied level of exports for each of the five countries. Actual exports of a country will not equal the figure computed in this manner because the data on imports and exports do not necessarily match and because the trade-share matrix for the previous quarter is used to calculate exports in the current quarter. Each country's total exports are therefore explained using a quasi-identity as a regression equation which includes computed exports and relative prices as explanatory variables. Nevertheless, when we solve the entire model our computation procedure has been designed to ensure that the value of total world exports generated by the model is equal to the sum of the imports of the five countries and

---

[1] In addition we take account of a third relationship that has already been alluded to in Section II.B.1, namely, a country's import price is a weighted average of other countries' export prices.

ROW[1].

Second, we intend to implement to some extent, at least, the relationship represented by equation (12), namely, that world claims equal world liabilities. In the same way that one country's import is someone else's export, so an increase in a country's claims on foreigners necessarily represents an increase in another country's liabilities to foreigners. It is clearly desirable in a multi-country model to ensure that the sum of liabilities to foreigners generated within the model matches the sum of claims on foreigners implied by the asset-demand functions. At this stage of our model-building efforts, however, it is not possible for us to achieve this degree of consistency, mainly because the data are not available to achieve this objective. For example, it is not possible to implement a solution similar to that outlined above for the import-export relationship because the data required to construct a matrix of claims and liabilities by country analogous to a trade-share matrix are lacking.

Nevertheless, it is possible to take some account of the asset-liability relationship in our model. This can be done by including in each country's liability equation the total claims on foreigners of the other four countries as an explanatory variable. These claims get allocated to the country in question depending on the values of the rate-of-return variables that are also included in the equations. This approach by no means guarantees that equation (12) holds because

---

[1]For further elaboration of this topic, see the paper by Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model."

the claims of ROW are not part of the total-claims variables.[1] Using this variable to explain a country's liabilities does, however, have the advantage that it ensures that an increase in the claims of the five countries in the model leads to an increase in their liabilities. One would expect this result in light of the close financial ties among these five countries. We plan to experiment with this approach as well as a more traditional specification of the liabilities equation that uses the net worth of foreign countries as an explanatory variable instead of their claims on foreigners.

## B. Exchange-Rate Determination and the Simultaneous Solution of All Country Sub-Models

In the preceding section we have described some of the relationships among the individual country sub-models. It now remains to show how the models fit together to determine exchange rates and other important variables in the system.

The basic structure of the multi-country system consists of five markets in each of the six regions--the five countries and ROW. The markets are for the output, labor, money, short-term securities and long-term securities that originate in each region. The output, labor,

---

[1]Another reason that equation(12) will not hold exactly in this formulation is that only part of a country's liabilities to foreigners (short-term and long-term portfolio liabilities) are specified as a function of other countries' claims; long-term direct investment liabilities do not depend on these claims.

money and securities of one region are imperfect substitutes for those in the other regions. Since the U.S. dollar can be regarded as the numeraire of the system with a price of unity, there are 29 separate prices in the basic structure: 6 commodity prices, 6 wage rates, 6 short-term interest rates, six long-term interest rates, and 5 independent bilateral exchange rates. From Walras' Law we know that in a system of n markets there are only n-1 independent market-clearing equations; therefore we have 29 independent equilibrium conditions for the markets in the basic structure of our model. For convenience we choose the redundant market to be that for short-term securities in ROW.

The 29 prices and interest rates could be determined by the 29 independent excess demand functions for the commodities, labor, money and securities. Under this solution procedure the budget constraint of each region, which is its balance of payments, could be used to compute as a residual the excess demand (excess supply in the case of ROW) for the short-term securities issued by ROW. Alternatively, as we described in Part II, one can transform the balance of payments from an ex post identity into an ex ante equilibrium condition by substituting foreign demand for the domestic supply of goods and securities. For reasons given below we have adopted this approach, and we have chosen to replace the equilibrium condition in the short-term securities market in each of the five countries with the balance-of-payments equation of that country. Thus we drop the market-clearing condition in the short-term securities market in all six regions, one through Walras' Law and the other five by substituting balance-of-payments equations. We still have 29 equations to determine our 29 unknowns.[1]

From an analytical point of view it makes no difference whether one uses the equilibrium condition for the short-term securities market or the balance of payments to close the model. The same information is contained in either case. Our reasons for choosing the latter approach are therefore primarily practical rather than theoretical. First, we must in any case explain each country's current account, and in certain cases part of its capital account, namely, foreign demand for domestic money. Since there is intrinsic interest in the overall capital accounts of these countries, it is convenient to combine the current and capital accounts together to complete the model. Second, we wish to explain the change in international reserves of the five countries in the fixed-rate period preceeding 1971, and we intend to explore the implications of going back to fixed rates using the complete model. Using the balance-of-payments equations to solve for the change in reserves under a fixed-rate system is considerably easier than trying to explain

---

[1]Recall from the discussion in Section III.A above that the balance of payments of ROW can be derived from the balance-of-payemnts equations of the five countries, and therefore it is not an independent equation. Also note that when we use the balance of payments (= budget constraint) as an equilibrium condition, it becomes necessary to specify a behavioral equation that explains the variable previously determined by the country's budget constraint, i.e., the demand for ROW's short-term securities, since this variable is one component of the balance of payments. In our actual equations we aggregate the demand for these securities together with the country's demand for all other foreign short-term and long-term securities.

the entire excess demand for securities. Third, there are difficulties in making the supply of short-term securities endogenous in a macro-economic model. Even if one restricts short-term securities to include only Treasury bills, it is a complicated matter to explain the quantity of bills made available to the public; this involves knowing the proportion of the government deficit financed with bills and the quantity of bills purchased by government agencies, including the central bank.

For these reasons we decided to adopt the traditional approach of using balance-of-payments equations in our model. This has the additional virtue of being more familiar, and therefore more readily understood, than the alternative approach involving exclusively domestic goods and asset market equilibrium conditions. Nevertheless, after the model has been developed we may try to implement this alternative procedure to see if it is feasible and to compare the results using it with those obtained using balance-of-payments equations.

The overall model we intend to implement departs in certain respects from the basic structure described above. In our initial modeling efforts we shall not attempt to explain variables (prices, interest rates, etc.) in ROW. This task is left to a later stage in our work. The only variables that are not generated by the country models are exports of ROW to the five countries, the Euro-dollar interest rate and a price index for world-traded primary products. The first two are endogenous and the last is exogenous. It is our view that these three variables, together with those generated by the five countries in the model, will be sufficient to explain the international transactions of these countries.

Furthermore, as described in Part II, our prototype model for each country is more disaggregated than the basic structure. This can be seen in the balance-of-payments equations of the five countries:

$$(14) \qquad (XGV_i - MGV_i) + [(XSYV_i + XSOV_i) - (MSYV_i + MSOV_i)] + (XTRANV_i - MTRANV_i)$$
$$- DPFC_i + DNPFL_i - DLTDC_i + DLTDL_i - NGKA_i - DNFA_i + DLO_i = 0$$
$$i = 1, \ldots, 5$$

where:

XGV = merchandise exports

MGV = merchandise imports

XSYV = investment income receipts

XSOV = receipts on service account except investment income

MSYV = investment income payments

MSOV = payments on service account except investment income

XTRANV = transfer receipts

MTRANV = transfer payments

DPFC = change in private financial claims on foreigners (i.e., private short-term and long-term portfolio claims)

DNPFL = change in financial liabilities to private foreigners net of liabilities to foreign central banks (i.e., short-term and long-term portfolio liabilities to private foreigners; includes liabilities of the government (e.g., Treasury securities) to private foreigners)

DLTDC = change in long-term direct claims (private)

DLTDL = change in long-term direct liabilities (private )

NGKA = net government capital account (excludes foreign private holdings of government securities, which are included in DNPFL)

DNFA = change in net foreign assets of the central bank; includes primarily changes in holdings of gold, SDRs (except for SDR allocations), reserve position in the IMF, and foreign currencies; also includes changes in the liabilities of the central bank to foreigners

DLO = change in liabilities to foreign official holders, mostly central banks; excludes liabilities of central bank to other central banks, which are included in DNFA

The overall model consists of the five country sub-models (see Part IV below for a listing of the prototype equations), and for each of these countries there is a balance-of-payments equation given by equation (14). There are numerous linkages among these sub-models. The links between imports and exports and between claims and liabilities have been described in Section III.A above. Changes in real trade and service flows affect each country's real income. Each country's import price is a function of other countries' export prices and exchange rates. Interest rates are directly connected to each other because the demand for money in each country is specified as a function of interest rates in other countries. Interest rates are also related through official intervention in the foreign exchange markets, unless the impact on the money supply of such intervention is sterilized by offsetting changes in the domestic component of the monetary base.

Exchange rates provide pervasive links among the five countries. Both import and export prices, as well as service account payments and receipts, are functions of the five exchange rates in the model. All capital flows (except the net government capital account, NGKA, which is exogenous) are a function of expected changes in exchange rates. The money demand equations in each country also include expected exchange-rate changes as explanatory variables. Thus variations in both actual and anticipated exchange rates affect all five countries through numerous channels.

Because exchange rates affect nearly all the variables in the entire system, we cannot say that these rates are determined exclusively in any particular market or equation. Ours is a general equilibrium model in which all markets are interrelated and all macroeconomic magnitudes (outputs, prices, wages, interest rates and exchange rates) are endogenous variables. In order to determine these variables we shall solve the entire system of five country sub-models. The solution procedure will involve searching for those values for exchange rates, interest rates, etc., that satisfy the equilibrium conditions in the model, i.e., that equate income and expenditure and the supply and demand for money in each country, and that satisfy the balance-of-payments-equals-zero constraints for each country. Thus our model is no different from any other system of simultaneous equations, and exchange rates are treated in essentially the same way as other endogenous variables.

We do, however, pay particular attention to exchange rate expectations, which are an important determinant of exchange rates in the current quarter. We shall assume that expectations are formed rationally, which is interpreted to mean that economic agents make their forecasts as if they had knowledge of the economic structure that generates exchange rates. Specifically, we assume that agents behave as if they used our model to predict exchange rates. This implies that values for exchange rates generated in the current period by the overall model will be assumed to be the exchange rates that are expected in the previous period to occur in the current period. When exchange rates are characterized by managed floating, the actual, observed values for exchange rates in period t will be used in the first stage of the estimation procedure as the initial estimates of exchange rates expected in period t-1 to occur in period t.[1] Following McCallum, we will use a consistent estimation technique in this first stage.[2] To obtain efficient estimates, as well as fitted values for exchange rates, we will use a technique developed by Brundy and Jorgenson in the second stage of estimation.[3] Finally, we shall solve the model for fitted values of

---

[1]This assumption is reasonable when exchange rates are floating because the hypothesis of rational expectations implies that the actual rate and the expected rate differ by no more than a random error term. For a discussion of this point, see Bennett McCallum, "The Role of Speculation in the Canadian Forward Exchange Market: Some Estimates Assuming Rational Expectations," unpublished manuscript. However, during episodes of fixed rates there have been numerous occasions when the rates maintained by the monetary authorities were clearly not the rates believed to be appropriate by market participants. Under such circumstances other initial estimates of expected exchange rates, e.g., forward rates, will be needed.

[2]Bennett McCallum, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates," Econometrica, Vol. 44, No. 1, January 1976.

[3]James Brundy and Dale Jorgenson, "Efficient Estimation of Simultaneous Equations by Instrumental Variables, "The Review of Economics and Statistics, Vol. 53, No. 3, August 1971.

exchange rates and utilize these estimates of expected exchange rates until the fitted and the expected exchange rates converge. In making simulations and forecasts with the model we shall use this same criterion in solving the model, namely, that the exchange rates expected to occur in period t and the values for exchange rates generated by the model in period t converge.

When we speak of exchange rates, we mean four bilateral rates and one weighted-average exchange rate for ROW, the weights for which reflect trade shares of ROW-countries with the United States. The basic data for these rates are bilateral dollar exchange rates, i.e., U.S. dollars per mark, pound, yen, Canadian dollar, and the currencies that comprise the weighted-average rate for ROW. Since the variables in the models other than the United States are expressed in local currency, we convert the bilateral dollar rates into bilateral rates appropriate for these countries (cross rates) using the currency arbitrage conditions:

(15) $$R_{ij} = R_{1j}/R_{1i}, \qquad i=2,\ldots,6, \quad j=2,\ldots,5, \quad i\neq j$$

where $R_{ij}$ denotes units of currency i per unit of currency j and "1" refers to the U.S. dollar.

The exchange rates will enter regression equations as explanatory variables either separately or in the form of a weighted average, where the weights will differ depending on the country and the nature of the dependent variable. We can construct different weighted averages in this way because except for the exchange rate for ROW (RROW), our model determines individual bilateral exchange rates rather than weighted averages.

Including RROW, we have a total of 25 exchange rates, 20 of which are determined by (15). The remaining 5--the four bilateral dollar rates and RROW--can in principle be determined along with all other variables in the model by the entire system of equations.

While our model provides the right number of equations to determine the endogenous variables, it does not contain sufficient information to solve for values of RROW that are economically meaningful. Exchange rates are determined basically by relative interest rates in the short run and relative prices in the long run. We specifically determine the interest rates and prices of five countries which provide the information necessary to explain four bilateral exchange rates. If we had in addition as part of our model proxies for the export price and interest rate-- which presumably would be weighted averages--of the countries whose currencies are included in RROW, then we would be in a position to provide meaningful estimates of RROW. We could treat these proxies as either exogenous or as endogenous variables.

Because of our desire to simplify the treatment of ROW in the first stage of the model, we have in fact omitted in our specification those variables emanating from ROW that are relevant in determining RROW. The only variable that is specific to ROW is its level of exports ($XGV_R$). The price of primary products, PP, is not the price of exports of ROW; rather, it is the price of homogenous internationally-traded commodities that are both imported and exported by the five countries. Both Canada and the U.S., for example, are large exporters of agricultural commodities. Similarly, the Eurodollar rate is not the interest rate that characterizes local money-market conditions in the ROW. Since our model lacks those variables that are the important domestic determinants of RROW, it will be treated as an exogenous variable.

With one less endogenous variable we can now either drop one equation or we can make another variable endogenous. The most obvious variable to make endogenous is foreign exchange intervention by ROW. During the period of managed floating since 1973 there has been some movement in RROW, but there has also been some foreign exchange intervention because certain countries in ROW have pegged their currencies to the dollar. Furthermore, when we make forecasts and simulation experiments with the model we shall assign specified values to RROW, and it seems reasonable to suppose that the average exchange rate of ROW is maintained by central banks purchasing or selling foreign exchange. Since many countries in ROW intervene in dollars, the U.S. balance-of-payments equation can be used to determine the changes in U.S. liabilities to foreign official holders in ROW, which becomes the new endogenous variable.

We cannot claim, however, that our model will be able to generate very accurate forecasts of this variable. The reason is that the same information, i.e., prices, interest rates, etc., in ROW needed to determine RROW is also required to explain intervention by ROW when its average exchange rate is assumed to be pegged to the dollar. Since this information is lacking, when we use the U.S. balance-of-payments equation to determine ROW intervention, we are making either of two implicit assumptions: that the behavior of prices, interest rates, etc., in ROW can be proxied by the average movement of these variables in Canada, Germany, Japan and the United Kingdom, or that these variables and/or economic policies in ROW adjust in such a way as to generate the intervention behavior that is computed as a residual from the U.S. balance-of-payments equation.

Finally, with regard to the general question of exchange-rate determination, we should point out the relationship of our model to what has been called the "asset-market approach" to exchange-rate determination. For example, Branson writes, "In the asset-market approach with flexible rates, the exchange rate is determined in the short run by requirements of asset-market equilibrium".[1] Branson, as well as those of a monetary persuasion, e.g., Kouri, imply that it is illegitimate to use the balance-of-payments equation as part of a system that determines an exchange rate.[2]

It should be clear that our model is completely consistent with the asset-market approach. It is a general equilibrium model that is characterized by both goods-market and asset-market equilibrium. In such a model one can either specify equilibrium conditions in <u>all</u> domestic markets, in which case the balance of payments is an ex post

---

[1]William H. Branson, "Comment" on a paper by Marina Whitman, "Global Monetarism and the Monetary Approach to the Balance of Payments," <u>Brookings Papers in Economic Activity</u>, No. 3, 1975.

[2]See Pentti Kouri, "The Exchange Rate and the Balance of Payments in the Short Run and the Long Run: A Monetary Approach," <u>Scandinavian Journal of Economics</u>, Vol. 78, No. 2, 1976.

identity, or one can drop a domestic market, treating it as redundant, and substitute a balance-of-payments equation in which functions explaining foreign demand for domestic goods and assets replace domestic excess supply functions. These are two formulations of the same model since both embody the same information. Because we have constructed the balance of payments as a legitimate ex ante equilibrium condition and have used it to replace another equilibrium condition, our model is immune from the criticisms that have been leveled against models that use the balance of payments to determine exchange rates.

## C. Central Bank Foreign Exchange Intervention in the Multi-Country Model

One use that we will make of the complete model is to calculate the effects of central bank foreign exchange intervention on exchange rates and on the price and income levels in the five countries. In particular, we want to examine the effects on these variables of specific intervention strategies. For example, if central banks wish to dampen significantly quarter-to-quarter changes in exchange rates, our model can be used to calculate the amount of intervention needed and the impact of such intervention on the international transactions and domestic economies of the countries involved.

To take account of official intervention it is necessary to distinguish between liabilities to private foreigners and those to official holders. This is done in our balance-of-payments equation [see equation (14) above], where we separate the change in liabilities to private foreigners, DNPFL and DLTDL, from the change in liabilities to foreign official holders, DLO. It is also necessary to recognize that intervention by one country affects other countries' reserves and/or liabilities to foreign official holders. This is simply another manifestation of the claims-liabilities relationship described in Section III.A above.

To incorporate this relationship into our model, define the official settlements balance of country i ($OSB_i$) as the sum of all components in equation (14) except $DNFA_i$ and $DLO_i$, i.e., the sum of the first thirteen terms in this equation. $OSB_i$ is equal to:

(16) $OSB_i = DNFA_i - DLO_i$

The change in net foreign claims of the central bank can be broken down into two components:

(17) $DNFA_i = DNOA_i + DCO_i$

where:

DNOA = change in net outside reserve assets; primarily changes in gold, SDRs (exclusive of those allocated during the current period), and in the country's IMF position

DCO = change in holdings of convertible foreign currencies.

The relationship between changes in official currency claims and changes in liabilities to official holders is given by:

(18) $DCO_i = \sum_{\substack{j=1 \\ j \neq i}}^{6} DLO_{ji}$

where $DLO_{ji}$ = liabilities of country j to country i.

This relationship is highlighted in Table IV, which presents a matrix of multi-country intervention. All variables have been converted to a common numeraire, and all columns sum to zero because of the world balance-of-payments constraint and equation (18). The point brought out by Table IV is that intervention by one central bank necessarily gets reflected in the balance of payments of one or more other countries. For example, purchases of dollars by Canada ($DCO_5>0$) get recorded not

Table IV. Multi-Country Intervention

| | | |
|---|---|---|
| \$ | 1. | $OSB_1 = DNOA_1+DCO_1-DLO_{12}-DLO_{13}-DLO_{14}-DLO_{15}-DLO_{16}$ |
| DM | 2. | $OSB_2 = DNOA_2-DLO_{21}+DCO_2-DLO_{23}-DLO_{24}-DLO_{25}-DLO_{26}$ |
| £ | 3. | $OSB_3 = DNOA_3-DLO_{31}-DLO_{32}+DCO_3-DLO_{34}-DLO_{35}-DLO_{36}$ |
| ¥ | 4. | $OSB_4 = DNOA_4-DLO_{41}-DLO_{42}-DLO_{43}+DCO_4-DLO_{45}-DLO_{46}$ |
| C\$ | 5. | $OSB_5 = DNOA_5-DLO_{51}-DLO_{52}-DLO_{53}-DLO_{54}+DCO_5-DLO_{56}$ |
| ROW | 6. | $OSB_6 = DNOA_6-DLO_{61}-DLO_{62}-DLO_{63}-DLO_{64}-DLO_{65}-DCO_6$. |

only in the Canadian balance of payments but also in the U.S. balance of payments, where it would appear as $DLO_{15}>0$.

Under a regime of managed floating there are two ways to treat intervention: as exogenous, or as explained endogenously by other variables within the model. In the first stage of estimating our model we shall treat the purchase and sale of foreign currency by the central banks of Canada, Germany, Japan, the United Kingdom and the United States as exogenous. As noted above, intervention by ROW in dollars, i.e., $DLO_{16}$, will be determined as a residual from the U.S. balance-of-payments equation. this implies that over the sample period of managed floating it will be necessary to separate $DLO_{16}$ from the change in U.S. official liabilities to Canada, Germany, Japan and the United Kingdom.

When we treat intervention as exogenous over the sample period of managed floating, we do not have to know which currencies were bought and sold. All we need to know is DNFA+DLO for each of the five countries (with adjustment for $DLO_{16}$ for the United States). Assuming there are no errors in the data, the aggregate figures for reserve changes will be consistent with each other because $DCO_i$ will be included in the jth country's balance of payments as $DLO_{ji}$. As can be seen in Table IV, the fact that intervention by one country affects another country's balance of payments is reflected in each country's data on aggregate reserve changes.

However, when we exogenously determine a country's intervention in making simulations and forecasts outside the sample period, we must specify the currency composition of this intervention. For example, suppose we wish to use the model to ascertain the effects of intervention by the Federal Reserve. If we change only $DCO_1$ and leave $DLO_{j1}=0$, $j=2,\ldots,5$, we are implicitly assuming that the intervention is done using ROW currencies, i.e., that $DCO_1 = DLO_{61}$. Table IV shows that if we do not wish to make this assumption, then we need to specify in which of the four currencies the Federal Reserve will do the intervention and we have to include this intervention in the balance-of-payments equations of these countries. The general point is that as long as the central banks of the five countries in the model do not buy and sell the currencies of ROW, intervention by one country must enter not only its balance-of-payments equation but also that of at least one of the four other countries as well.

After our basic modeling work is complete we shall try to explain the intervention behavior of the five central banks over the period of managed floating. If we are successful in finding some systematic behavior in central bank actions, we can use the estimated intervention functions in simulation and forecasting exercises. We also plan to simulate our model using alternative intervention rules, e.g., more or less smoothing of exchange rate changes, in order to observe the effects on exchange rates, international transactions, etc.[1]

[1]For further discussion of the specification and use of intervention functions, see the paper by Peter Clark and Sung Kwack, "Asset Markets and Interest Rate Determination in the Multi-Country Model."

# PART IV: Equations for the Prototype Country Sub-Model

## Notation Conventions

Insofar as possible, variables are assigned mnemonic names. The use of lower case letters and subscripts is minimized to permit the variable names to be machine-readable. This may lead to some additional complexity on the first reading, but in the long run will result in substantial simplification for the estimation and simulation stages.

Time subscripts are not used when variables are contemporaneous. When lagged values are specified the subscript -1 is used to indicate t-1. Distributed lags are specified with standard lag operator notation. Let the lag operator (L) be defined by

$$L^i x \equiv x_{t-i}.$$

Let a polynomial in L, A(L), be defined as

$$A(L) \equiv \sum_{i=0}^{T} a_i L^i.$$

Then A(L)x can be used to denote a polynomial distributed lag in x.

$$\begin{aligned} A(L)x &= a_0 L^0 x + a_1 L^1 x + a_2 L^2 x + \ldots \\ &= a_0 x_t + a_1 x_{t-1} + a_2 x_{t-2} + \ldots \\ &= \sum_{i=0}^{T} a_i x_{t-i} \end{aligned}$$

Polynomial notations A(L), B(L), C(L), D(L), H(L), Z(L), $\beta$(L) and $\theta$(L) are used in the equations that follow.

A key in the left hand margin indicates the type of the corresponding equation: behavioral (B), identity (I), and quasi-identity (QI). Quasi-identities (or bridge equations) are equations that would hold identically if exact data definitions could be used. Because of aggregation, differences in definitions, and data problems (e.g. FOB - CIF distinctions) the equations do not hold identically. To account for the discrepancy, the LHS variable is regressed on its determinants and the estimated coefficients permit bridging between the variables.

The letter V appended to a variable name indicates measurement in current value terms of national currency. When the V is absent, the variable is expressed in 1972 currency units.

# List of Variables

| Mnemonic | Description |
|---|---|
| BU | Unborrowed monetary base ≡ total base minus borrowed reserves |
| BARD | Variable representing the Bardepot |
| C | Consumption, CV ≡ C · P |
| CAPC | Variable representing capital controls |
| CCAV | Capital consumption allowances |
| CU | Capacity utilization rate, peak = 100. |
| CUR | Currency held by the non-bank public |
| DD | Demand deposits |
| DLO | Change in liabilities to foreign official holders |
| DLTDC | Change in long-term direct claims |
| DLTDL | Change in long-term direct liabilities |
| DNFA | Change in net foreign assets of the central bank |
| DNPFL | Change in short-term and long-term portfolio liabilities to non-official foreigners |
| DPFC | Change in short and long-term portfolio claims |
| DSV | Domestic sales ≡ GNPV + MGSV - XGSV |
| DT | Total deposits ≡ DD + TD |
| E | Employment |
| FCT | Total financial claims on foreigners |
| FGNP | Foreign GNP; a weighted average |
| FLT | Total financial liabilities to foreigners |
| FP | Foreign price; a weighted average |
| FR or $F_R$ | Weighted average of foreign exchange rates |
| FRL | Foreign long-term interest rate; may be a weighted average of national rates |

| Mnemonic | Description |
|---|---|
| FRS | Foreign short-term interest rate; may be a weighted average of national rates or the Eurodollar rate |
| G | Government expenditure, $GV \equiv G \cdot P$ |
| GNP, GNPV | Gross national product |
| GNPP | Potential GNP |
| IF | Fixed investment (plant and equipment + housing), $IFV \equiv IF \cdot P$ |
| II | Inventory investment, $IIV \equiv II \cdot P$ |
| ITR | Indirect tax rate |
| K | Capital stock |
| LF | Labor force |
| LFP | Potential labor force |
| LTDCT | Long-term direct claims on foreigners |
| LTDLT | Long-term direct liabilities to foreigners |
| MG, MGV | Imports of goods |
| MGS, MGSV | Imports of goods and services |
| MIG | Number of foreign workers |
| MSO, MSOV | All service account payments other than investment income |
| MSY, MSYV | Investment income payments |
| MTRANV | Transfer payments |
| NFA | Stock of net foreign assets of the central bank |
| NGKA | Net government capital account |
| NGP | Net claims of the central bank on the government |
| NW | Private net worth |
| OTH | All assets of the central bank other than NFA and NGP |

| Mnemonic | Description |
| --- | --- |
| P | Domestic absorption deflator $(P \equiv (CV+IFV+IIV+GV)/(C+IF+II+G))$ |
| PMG | Price of imported goods |
| POP | Population |
| PP | Price of primary products (expressed in dollars, determined exogenously) |
| PXG | Price of exported goods |
| R | Exchange rate; $R_{ij}$ = units of currency $i$ per unit of currency $j$ |
| RD | Discount rate |
| RE | Exchange rate expected next period |
| RED | Eurodollar interest rate (3 month) |
| REGM | Regulation M dummy variable |
| REGQ | Regulation Q dummy variable |
| RF | Free reserves (defined as excess reserves - borrowed reserves) |
| RL | Long-term interest rate |
| RR | Required reserves |
| RS | Short-term domestic interest rate |
| SV | Inventory stock |
| T | Time |
| TB | Special securities issued by the Treasury and Export-Import Bank to U.S. foreign bank branches |

| Mnemonic | Description |
| --- | --- |
| TD | Time deposits |
| TRANV | Government transfers |
| TV | Tax revenue |
| UC | User cost of capital |
| UN | Unemployment rate |
| UNMIN | Minimum unemployment rate, to determine frictional unemployment |
| USNW | U.S. net worth |
| USRS | U.S. short-term interest rate |
| W | Wage rate (computed as wage bill divided by manhours) |
| WMIN | Minimum wage rate |
| XG, XGV | Exports of goods |
| XGS, XGSV | Exports of goods and services |
| XSO, XSOV | All service account receipts other than investment income |
| XSY, XSYV | Investment income receipts |
| XTRANV | Transfer receipts |

## List of Equations

### A. The Market for Goods and Non-Labor Services

#### Expenditure Components

| Type | No. | Equation |
|---|---|---|
| I | 1. | $GNP \equiv C + IF + II + G + XGS - MGS$ |
| I | 2. | $GNPV \equiv P(C + IF + II + G) + XGSV - MGSV$ |
| B | 3. | $C = a_0 + a_1C_{-1} + a_2[V - RRL(a_0 + a_1C_{-1})]$ |

where $V \equiv NW_{-1}/P + (DYPV/P) \cdot \sum_{i=0}^{N-1} [(1 + g)/(1 + RL)]^i$,

$$RRL \equiv \sum_{i=0}^{N-1} (1 + RL)^{-i},$$

$$DYPV \equiv GNPV - TV + TRANV - CCAV,$$

and NW is private net worth (see Section IV. C below).

The consumption function is derived from the assumption that a representative consumer maximizes an intertemporal utility function subject to the budget constraint: net worth plus the present discounted value of expected future income (over his remaining life, N periods), or V, equals the present discounted value of future consumption. The utility function is specified so that a minimum consumption level (based on habit formation) is required each period. Disposable income is proxied by DYPV, which in fact includes corporate retained earnings.

| Type | No. | Equation |
|---|---|---|
| B | 4. | $TV = b_0 + (b_1 + TSL)(GNPV - CCAV)$, |

where TSL is generated by $(TV - \hat{b}_0 - \hat{b}_1[GNPV-CCAV])/(GNPV-CCAV)$.

TV is taxes of all forms, made a linear function of the aggregate

**Type** **No.** **Equation**

tax base, net national product. TSL is a slope adjustment, an exogenous variable, the data for which are computed from the estimated equation such that the residual error is zero. TSL serves as a marginal tax rate shift variable; the equation is estimated with TSL = 0.

B 5. $TRANV = c_0 + c_1 GNPV + c_2 UE$

Transfers from government include subsidies, other business transfers, social security and unemployment compensation. It is thus dependent on the level of activity and on the number of unemployed, where $UE \equiv (UN/100)\cdot(CU/100)\cdot LFP$.

B 6. $CCAV = d_0 + d_1 K_{-1} + d_2 P_{-1} + d_3[GNPV - TV - CV]$

Depreciation is both an accounting and a physical concept; the latter is proxied in this linear function by the lagged capital stock and price terms, and the former by retained earnings, here represented by GNPV - TV - CCAV - CV (CCAV is substituted out, since it is the dependent variable).

I 7. $K \equiv (1 - \delta)K_{-1} + IF$

Since $K \equiv \Delta K + K_{-1}$, $\Delta K \equiv IF - \delta K_{-1}$ or net investment can be substituted to obtain (7).

B 8. $IF = e_0 + E_1(L)\Delta GNPD + \delta K_{-1}$

where $GNPD \equiv [\hat{\alpha}_1(GNP - XG)\cdot PDS/(1-CU/100) + \hat{\beta}_1 XGV/(1-\Sigma z_{ij}\ CU_j/100)]/UC$

$$\ln PDS \equiv \ln P/\beta + (\beta - 1)/\beta \quad \ln PM,$$

**Type** **No.** **Equation**

and $PM \equiv MGSV/MGS$.

Fixed investment is derived from the Jorgensonian neoclassical theory as a factor demand (stock demand translated into changes), with domestic and foreign capacity utilization proxying for the demand elasticities for domestic and export output, and $\delta$ consistent with that in (7). E(L) is a lag operator.

I 9. $CU \equiv (GNP/GNPP) \cdot 100$

I 10. $UC \equiv P(RL + \delta)$

More complex versions of user cost may be used, depending on tax incentives for investment.

B 11. $\ln GNPP = \ln A + gT + (\hat{\alpha} \ln K + (1 - \hat{\alpha}) \ln EP)$,

where EP is potential employment (see labor market section below). Only ln A and g are estimated in this capacity output equation, which represents a constant returns Cobb-Douglas production frontier. The output elasticity is estimated from factor shares data, and the expression in parentheses is therefore subtracted fiom ln GNPP for estimation.

B 12. $$II = f_0 + f_1 MGU + f_2 MG + f_3 (C+IF+XG) + f_4 [\frac{1}{4} \sum_{i=1}^{4} (S/GNP)_{-i} - (S/GNP)_{-1}] + F_5(L)\ C_{-1} + F_6(L)(GNPV/UC)_{-1} + F_7(L)\ K_{-1}$$

The change in inventories embodies both the output decision rule and inventory speculation; it also serves as a buffer between production and sales of goods. Here, sales of goods are represented by three components, and partial adjustment to

| Type | No. | Equation |
|---|---|---|
| | | both desired output and a desired stock-sales ratio is the dynamic mechanism underlying (12). MGU is a proxy for unexpected imports, and is measured by the vector of residuals from the goods import equation (20) below. |
| I | 13. | $S \equiv S_{-1} + II$ |
| QI | 14. | $XGS = g_0 + g_1\,[XG + (XSYV + XSOV)/PX]$ <br> This equation bridges the gap between real exports of goods and services in the national income accounts, and real exports of goods and of services in the balance of payments accounts. <br> $PX \equiv XGSV/XGS$. |
| B | 15. | $XG = h_0 + h_1 XGVD/PXG + H(L)(PXG/PC)$ <br> where $XGVD_i \equiv \sum_{j=1}^{5} (a_{ij} - a_{iR}a_{Rj}/a_{RR})\, MGV_j \cdot R_{ij} + a_{iR} XGV_R \cdot R_i/a_{RR}$, <br> $i$ = countries in the model; <br> $PC_i \equiv \sum_k R_{ik} \cdot PXG_k a_{ki}$; <br> $a_{ij} = i,j^{th}$ element of trade matrix A (i=R=Rest of World) <br> Exports of goods for those countries in the model are derived from the behavioral interpretation of supply = demand in matrix form X = AM, where A is a trade matrix adjusted for f.o.b./c.i.f. differentials. To this "activity" variable is added a relative price term, in which the denominator is a competitors' price (a weighted average of others' export prices). The equation can be specified in logs, so that exchange rates could be broken out from both terms. |
| I | 16. | $XGV \equiv XG \cdot PXG$ |

**Type** **No.** **Equation**

B 17. $XGV_R = m_0 + m_1 FGNPV + m_2 FP + m_3 FR + m_4 \; PP/PC,$

where FGNPV, FP, FR and PC are weighted (with trade weights) averages of GNP, P, exchange rates and export prices of countries in the model. ROW exports is made a function of variables in the model in nominal terms; using this equation permits the deletion of $M_{ROW}$ from the system. PP is primary product price.

QI 18. $MGS = n_0 + n_1 [MG + (MSYV + MSOV)/FP]$

This is a bridge equation analogous to that for XGS.

I 19. $MG \equiv MGV/PMG$

B 20. $\frac{MGV}{DSV} = \alpha_M + \beta_M(L) \; \Delta ln \; PMGF + \beta_r(L) \; \Delta ln \; R + \beta_D(L) \; \Delta ln \; P$

$$+ \delta_M KV_{-1} + \lambda SV_{-1} + aSTR + bDSTR$$

where DSV is domestic sales, and the last two variables are dummies for domestic and dock strikes.

The functional form for the import demand equation is derived from aggregation of factor and consumer demands that come from the homogeneous indirect translog family of utility and cost functions. While the $\Delta ln$ price terms result from first differencing of stock demands, levels (in logs) terms may be added.

QI 21. $XGSV = s_0 + s_1 [XGV + XSYV + XSOV]$

QI 22. $MGSV = t_0 + t_1 [MGV + MSYV + MSOV]$

These are bridge equations analogous to (14) and (18).

| Type | No. | Equation |
|---|---|---|
| B | 23 | $XSYV = c_0 + C_1(L)(FRS \cdot FCT_{-1} + FRL \cdot LTDCT_{-1}) + C_2(L)[(\Delta\%R)(FCT_{-1} + LTDCT_{-1})$ |

Investment income receipts are explained by the product of interest rates and the stocks of claims on foreigners, as well as the percentage change in exchange rate(s) multiplied by the stock of claims on foreigners. Where we have the currency or country composition of a country's foreign claims, FRS will be a weighted average of foreign short-term interest rates, the weights reflecting the compositon of the foreign portfolio. Similarly, FRL will be a weighted average of foreign long-term rates. The exchange rate term is designed to take account of the fact that the domestic-currency value of investment income received in foreign currency is affected by exchange rate changes.

| Type | No. | Equation |
|---|---|---|
| B | 24 | $MSYV = d_0 + D_1(L)(RS \cdot FLT_{-1} + RL \cdot LTDLT_{-1}) + D_2(L)\ [(\Delta\%R)(FLT_{-1} + LTDLT_{-1})]$ |

Payments of investment income depend on the domestic short-term rate times total financial liabilities to foreigners and the long-term rate times long-term direct liabilities to foreigners. An exchange rate term has been added to capture the effect of translating investment income payments in foreign currencies into domestic currency.

| Type | No. | Equation |
|---|---|---|
| B | 25 | $\ln(XSOV/P) = b_0 + B_1(L) \ln FGNP + B_2(L) \ln(FP/P)$ |

All service account receipts other than investment income are explained as a function of foreign real GNP, reflecting the demand for foreign services, and the ratio of foreign to domestic prices, to capture substitution effects. Both FGNP and FP are weighted averages.

| | | |
|---|---|---|
| B | 26 | $\ln(MSOV/FP) = a_0 + A_1(L) \ln GNP + A_2(L) \ln(P/FP)$ |

All service account payments other than investment income depend on domestic GNP, the demand variable, and the ratio domestic to foreign prices. In both this and the preceeding equation the aggregate domestic price index, P, is used because it gives a heavy weight to domestic services.

| | | |
|---|---|---|
| B | 27 | $XTRANV = a_0 + a_1 FDYPV$ |

Transfer receipts are assumed to depend on a weighted average of foreign disposable income, FDYPV.

| | | |
|---|---|---|
| B | 28 | $MTRANV = b_0 + b_1 DYPV$ |

Transfer payments are a function of domestic disposable income. For some countries, e.g., Germany, this can be supplemented by another variable: the wage rate times the number of foreign workers.

## Price Determination

The marginal cost of domestic output, derived using a Cobb-Douglas production function, is expressed in terms of factor prices: user cost of capital (UC), wage rate (W), and price of imported intermediates which, in turn, is approximated as a function of the primary goods price (PP, exogenous) and a weighted average of foreign export prices (PXG). Domestic output is sold monopolistically in the domestic markets. Domestic and export prices consist of different (and variable) markups over marginal costs.

| Type | No. | Equation |
|---|---|---|
| B | 29 | $\ln P = c_0 - \beta \ln(1-ITR) + \rho_1 T + C_2(L)\ln CU + C_3(L)CU + C_4(L)\ln UC + C_5(L)\ln W + C_6(L)\ln PP + C_7(L)\ln R_{i1} + C_8(L)\sum_{j\neq i}^{N} w_{ij} \ln PXG_j + C_9(L)\sum_{j\neq i}^{N} w_{ij} \ln R_{ij}$ |

Although it is the price of domestic sales (PDS) that is determined behaviorally in the domestic market, we desire the domestic absorption deflator, $P \equiv (CV + IFV + IIV + GV)/(C + IF + II + G)$. The absorption deflator can be represented as a geometric mean of PDS and the price of final imports (PMF), $P = PDS^{\beta}\ PMF^{1-\beta}$; PMF is approximated by the price of imported goods (PMG).

The domestic markup is approximated as a linear function of capacity utilization (CU) and its log; the indirect tax rate (ITR) is the average rate obtained as the ratio of tax receipts to (CV + IV + GV); $\rho$ is the (uniform) rate of technical progress; and $w_{ij}$ are shares of goods imports from country i to country j as a fraction of total imports of country i.

| Type | No. | Equation |
|---|---|---|
| B | 30 | $\ln PXG = b_0 - \rho T + B_1(L)\ln \sum_{j\neq i}^{N} z_{ij}CU_j + B_2(L) \sum_{j\neq i}^{N} z_{ij}CU_j$ |

$$+ B_3(L)\ln UC + B_4(L)\ln W + B_5(L)\ln PP + B_6(L)\ln R_{i1}$$

$$+ B_7(L)( \sum_{j\neq i}^{N} w_j^* \ln PXG_j + \sum_{j\neq i}^{N} w_{ij}\ln PXG_j)$$

$$+ B_8(L)( \sum_{j\neq i}^{N} w_j^* \ln R_{ij} + \sum_{j\neq i}^{N} w_{ij}\ln R_{ij})$$

The export markup is approximated as a linear function of CU in other countries (using weights $z_{ij}$, the share of exports to country j as a fraction of total exports of country i) and competitor prices (using weights $w_j^*$, the share of country j's exports of goods in total world trade of goods).

In all other respects, the equation for the price of exported goods is the same as that for the domestic absorption deflator because the determinants of cost (factor prices) are identical.

| Type | No. | Equation |
|---|---|---|
| QI | 31 | $\ln PMG = a_0 + A_1(L)\ln PP + A_2(L)\ln R_{i1} + A_3(L)( \sum_{j\neq i}^{N} w_{ij}\ln PXG_j)$ |

$$+ A_4(L)( \sum_{j\neq i}^{N} w_{ij}\ln R_{ij})$$

The overall price of imported goods for a given country is approximated as a geometric mean of the primary import price (PP) and a weighted average of foreign export prices (PXG).

## B. The Labor Market

| Type | No. | Equation |
|---|---|---|
| B | 32 | $\ln(1-UN/100) = z_0 - \ln LFP + Z_1(L)\ \ln[(1-\hat{\alpha})GNP/(W/PV)] + Z_2(L)\ \ln[(W/P)^2] - Z_3(L)\ \ln C - Z_4(L)\ \ln(W/P) - Z_5(L)\ \ln MIG$ |

The unemployment rate is a reduced form equation derived from the fact that the log of the employment rate equals labor demand minus labor supply. Labor demand is specified from the first order condition on efficient factor usage from the Cobb-Douglas production function, and labor supply (labor force/potential labor force) depends on the real wage, the labor-leisure choice, and foreign workers. $PV \equiv GNPV/GNP$.

| Type | No. | Equation |
|---|---|---|
| B | 33 | $\ln LFP = a_0 + a_1 \ln POP + a_2 T$ |

The data for potential labor force are generated as a peak-to-peak Wharton-style index. The equation is specified as a trend and depending on population growth (captures the trend in the potential labor force participation)

| Type | No. | Equation |
|---|---|---|
| I | 34 | $EP \equiv LFP - UEF$, where $UEF \equiv LFP \cdot (CU/100) \cdot (UNMIN/100)$. |

Potential employment equals potential labor force minus frictional unemployment. UNMIN is specified as a constant. Obviously, $EP \equiv LFP(1-[CU/100] \cdot [UNMIN/100])$.

| Type | No. | Equation |
| --- | --- | --- |
| B | 35 | $\Delta\%WR = b_0 + B(L)/UN + b_2/\Delta\%UN + b_3\,\Delta\%P + b_4\,\Delta\%TV + b_5\,\Delta\%WRMIN + b_6\,\Delta\%MIG + b_7\,STR$ |

The wage rate is derived as a Phillips-Lipsey curve with allowance for natural-rate hypothesis testing. Variables to account for the way the wage rate is constructed (average wage) are proxied by TV (employer contributions to social security) and WRMIN. Strikes and foreign workers, cet. par. are expected to lead to higher and lower negotiated wage rates, respectively. Incomes policy dummies may also be added, and a distributed lag on 1/UN may allow for a vertical (or steeper) long-run curve.

## C. Asset Markets

### Interest Rate Determination

| Type | No. | Equation |
|---|---|---|
| I | 36 | $BU \equiv NFA + NGP + OTH$ |

This equation defines the unborrowed monetary base from the sources side as the sum of net foreign assets (NFA), net claims on the government (NGP), and other assets of the central bank (OTH).

| | | |
|---|---|---|
| I | 37 | $DD \equiv (BU - bTD - CUR - RF)/a$ |

This equation defines the supply of demand deposits, and is really a rearrangement of the identity defining the monetary base from the uses side: $BU \equiv aDD + bTD + CUR + RF$.

| | | |
|---|---|---|
| B | 38 | $DD = B_0(L)NW + B_1(L)(RS\cdot NW) + B_2(L)(FRS\cdot NW) + b_3(DRE \cdot NW) + B_5(L)GNPV.$ |

where $DRE \equiv (\dot{R}E - R)/R$

The demand for demand deposits, like all asset demand functions, is based on portfolio allocation decision-making. The size of the portfolio, private net worth, acts as a scale variable and multiplies all rate-of-return variables because the share of an asset in net worth is assumed to depend on interest rates,etc, and in some cases on the ratio of a transactions-demand variable to net worth. In eq. 40 this variable is nominal GNP. The demand for demand deposits depends negatively on domestic and

| Type | No. | Equation |
|---|---|---|
| | | foreign interest rates, and on expected exchange rate changes. Both FRS and DRE will be vectors of foreign interest rates and expected exchange rates, respectively, or weighted averages of these variables. In making simulations and forecasts this equation is inverted so that the domestic interest rate, RS, becomes the left-hand-side variable. |
| B | 39 | $TD = C_0(L)NW + C_1(L)(RS \cdot NW) + C_2(L)(FRS \cdot NW) + C_3(DRE \cdot NW) + C_5(L)GNPV$ |
| | | The demand for time deposits is assumed to be a function of the same set of variables that determine demand deposits. The equation does not include the time deposit rate as an explanatory variable because we assume either that the time deposit rate is fixed or that it moves closely enough with RS that its effect on TD is captured by RS. |
| B | 40 | $CUR = A_0(L)NW + A_1(L)(RS \cdot NW) + A_3(L)CV$ |
| | | Currency held by the non-bank public depends positively on net worth and on a proxy for transactions demand (nominal consumption) and negatively on the domestic interest rate. We have assumed that no substitution takes place between currency and foreign assets, so that neither FRS nor DRE appear as explanatory variables. |
| I | 41 | $NW \equiv NW_{-1} + \Delta NW$ |

| Type | No. | Equation |
|---|---|---|
| I | 42 | $\Delta NW \equiv (XGSV - MGSV + XTRANV - MTRANV) + [(G \cdot P) + TRANV - TV] + [(IF + II) \cdot P - CCAV]$ |

These two equations show how private net worth is generated. The change in private net worth is defined as the current account balance plus the government deficit plus net investment.

| | | |
|---|---|---|
| B | 43 | $RF = d_0\, NDD + d_1(RD - RS)NDD + d_2(RS \cdot NDD) + d_3\, \Delta RU + d_4\, \Delta RR$ |

The demand for free reserves, defined as excess reserves minus borrowed reserves, is homogeneous of degree one in nominal magnitudes and with respect to net demand deposits (aside from the short-run adjustment to the change in unborrowed reserves, ΔRU, and required reserves, ΔRR). The coefficient of (RD - RS), the discount rate minus the short-term rate, is negative, since an increase in RS raises the demand for borrowed reserves, whereas a rise in RD makes borrowing from the central bank less attractive. The short-term rate enters by itself because it represents the opportunity cost of excess reserves, and therefore it has a negative sign. The coefficient of ΔRU is expected to be positive because an increase in unborrowed reserves may not be fully allocated to earning assets and required reserves within the current quarter. Similarly, an increase in required reserves brought about by a change in reserve requirements may cause a short-run decline in free reserves.

| Type | No. | Equation |
|---|---|---|
| I | 44 | $NDD \equiv (1-a)DD$ |
| I | 45 | $\Delta RR \equiv \Delta a(DD_{-1}) + \Delta b(TD_{-1})$ |
| I | 46 | $RU \equiv BU - CUR$ |

The first equation defines net demand deposits, the second defines the change in required reserves due to a change in reserve requirements against demand deposits ($\Delta a$) and time deposits ($\Delta b$), and the third equation defines unborrowed reserves.

B 47 $RL = h_0 + h_1 RS + H_2(L)\ RS_{-1}$

The domestic long-term interest rate is expressed as a term-structure equation of the domestic short-term rate, where the current value of the short-term rate is entered separately.

## Changes in Private Claims on Foreigners and Liabilities to Foreigners

B 48
$$DPFC = A_0(L)\Delta\ NW + A_1(L)\ \Delta(RS \cdot NW) + A_2(L)\ \Delta(RL \cdot NW)$$
$$+ A_3\ \Delta XGV + A_4(L)\ \Delta FRS \cdot NW) + A_5(L)\Delta\ (FRL \cdot NW)$$
$$+ a_6\ \Delta(DRE \cdot NW) + a_7\ CAPC$$

This equation explains the change in demand for private financial claims (short-term plus long-term portfolio claims) on foreigners. It depends positively on current and lagged net worth, since full adjustment may not take

Type No. Equation

place within a quarter. Current and past changes in domestic and foreign short and long-term rates are scaled by NW, as in other asset demand equations. For several countries FRS may not be composed of national rates, but rather be restricted to the Eurodollar rate, which is described below. The change in private financial claims also depends on the current change in expected exchange rates. The change in the value of exports ($\Delta XGV$) is a proxy for transactions demand and trade credit. CAPC represents capital controls on the type of flow being explained.

B 49

$$DNPFL = B_0(L)\ \Delta FPFC + B_1(L)\ \Delta(RS \cdot FPFC) + B_2(L)\ \Delta(RL \cdot FPFC) + B_3(L)\ \Delta MGV + B_4(L)\ \Delta(FRS \cdot FPFC) + B_5\ \Delta(FRL \cdot FPFC) + B_6(L)\ \Delta(DRE \cdot FPFC) + b_7\ CAPC$$

where $FPFC_i = \sum_{\substack{j=i \\ j \neq i}}^{5} R_{ij}\ PFC_j$. This equation explains the change in short-and long-term portfolio liabilities to all foreigners except foreign official institutions. In this form of the equation the scale variable is the sum of the other four countries' financial claims on foreigners. As explained in Part III, we use this variable to reflect the fact that an increase in countries' claims is the aggregate will tend to raise their liabilities. We also plan to experiment with another form of this equation in which FPFC is replaced by

| Type | No. | Equation |
|---|---|---|
| | | FNW, which is the sum of the four other countries' net worth. This equation is similar to that above for DPFC, the main difference being that trade credit is generated by changes in the country's imports, MGV. |
| B | 50 | $DLTDC = \frac{a[(F^{e}_{R+1}) - F_R) - F_R(RL - FRL)]}{VAR(F_R)/F_R + b\ FRL} + c\ F_R\ PK\ [(1 - \frac{d\ VAR(F_R)/F_R}{VAR(F_R)/F_R + b\ FRL}] - e\ LTDC_{-1}$ |
| B | 51 | $DLTDL = \frac{a[F(1/R)^{e}_{+1} - F(1/R) - F(FRL - RL)]}{VAR[F(1/R)]/F(1/R) + b\ FRL} + c\ PK[1 \frac{d\ VAR[F(1/R)]/F(1/R)}{VAR[F(1/R)]/F(1/R) + b\ FRL} - e\ LTDC_{-1}$ |

Where: DLTDC and DLTDL are the flow of direct investment out of (abroad) and into the country in question. $F^{e}_{R}$, $F_R$ and $F(1/R)$ are weighted averages of expected and actual bilateral exchange rates, respectively; similarly $VAR(F_R)$ and $VAR[F(1/R)]$ are the (subjective) variances of future exchange rates. FRL is a given foreign long-term interest rate and RL is the domestic long-term rate; the term $F_R(RL - FRL)$ is a weighted average of the product of the bilateral exchange rate with a foreign

Type No. Equation

country (R) times the difference in interest rates between the domestic country and that foreign country; F(FRL - RL) is a similar weighted average of differences in rates. PK is the value of the capital stock in the domestic economy; $F_R$ PK is a weighted average of foreign capital stocks translated at the current exchange rate. The flow of direct investment is a financial capital flow equal to the net change in the ownership position of domestic firms in foreign firms. We hypothesize that the real asset decisions of the firm-notably plant and equipment expenditure-are determined prior to the financing decision, of which direct investment is one part. The decision to finance a given flow of asset changes by direct investment or by borrowing abroad is hypothesized to be determined by a balancing of considerations of relative costs (domestic vs. foreign interest rates) and the risk of capital losses due to exchange rate changes. Identical considerations determine direct investment in the domestic country.

We find, therefore, that the stock of direct investment is positively related to the stock of real capital owned abroad by domestic firms, and to the foreign interest rate and negatively related the expected depreciation of the domestic currency and the risk (variance) of exchange rate changes. In the above equation, for simplicity of exposition, we have

**Type** **No.** **Equation**

substituted the value of the foreign capital stock (PK) for the value of that capital stock owned by foreigners; in actual estimation of these equations the determinants of the capital stock or its change may be substituted in.

I 52 $FCT \equiv PFC_{-1} + DPFC + NFA_{-1} + DNFA$

I 53 $LTDCT \equiv LTDC_{-1} + DLTDC$

I 54 $FLT \equiv NPFL_{-1} + DNPFL + LO_{-1} + DLO$

I 55 $LTDLT \equiv LTDL_{-1} + DLTDL$

These four equations cumulate flows to generate stocks of claims and liabilities.

I 56
$$\begin{aligned} DNFA - DLO \equiv\ & XGV + XSYV + XSOV + XTRANV \\ & - MGV - MSYV - MSOV - MTRANV \\ & - DPFC + DNPFL - DLTDC + DLTDL \\ & - NGKA \end{aligned}$$

This equation is used to determine total balance-of-payments financing when exchange rates are fixed. The breakdown of this financing into DNFA and DLO is determined exogenously. When exchange rates are not fixed, DNFA-DLO will either be taken as exogenous or it will be explained by means of an intervention function. In this case equation (56) becomes an equilibrium condition.

| Type | No. | Equation |
|---|---|---|
| B | 57 | $RE$ = function of variables that determine $R_{+1}$ |
| B | 58 | $RED = a_0 + a_1 USRS + a_2 FRS + a_3 RE + a_4 FNW + a_5 FMV + a_6 USNW + a_7[(1-a_{US})DD_{US}] + a_8 CAPC_{US} + a_9 BARD + a_{10} REGQ + a_{11} REGM + a_{12} TB$ |

The Eurodollar interest rate (3 month) is explained by a reduced form equation, where the main explanatory variables are the U.S. and foreign interest rates (USRS and FRS), expected exchange rate changes, foreign and U.S. net worth, and the value of non-U.S. imports, which is a proxy for trade financing. There are also two capital control variables, one for the United States ($CAPC_{US}$) and one for Germany (BARD) which represents the Bardepot. The remaining variables reflect head office borrowing from their foreign branches and Treasury and Export-Import Bank issues of special securities to U.S. foreign branches, TB.

A. Policy Variables

| | | |
|---|---|---|
| 1. | G | Government spending on goods and services |
| 2. | ITR | Indirect tax rate |
| 3. | RD | Discount rate |
| 4. | NGP+OTH | Net claims on the government and other assets of the central bank |
| 5. | a | Reserve requirement on demand deposits |
| 6. | b | Reserve requirement on time deposits |

B. Other Exogenous Variables

| | | |
|---|---|---|
| 7. | NGKA | Net government capital account |
| 8. | MIG | Number of foreign workers |
| 9. | MIGP | Peak number of foreign workers |
| 10. | PP | Price of world-traded primary products |
| 11. | UNMIN | Minimum unemployment rate |
| 12. | WMIN | Minimum wage rate |
| 13. | T | Time |
| 14. | g | Future growth rate of disposable income |
| 15. | POP | Population |