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ASSET MARKETS AND INTEREST RATE DETERMINATION IN THE MULTI-COUNTRY MODEL

by

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In this paper we describe in more detail the asset markets and other financial aspects of the multi-country model project that is being undertaken by the Quantitative Studies Section.[1] The first section is devoted to an explanation of the monetary sector of the prototype country sub-model, which is where each country's short-term interest rate is determined. In this section we also describe how the long-term interest rate in each country is determined. This section is followed by another that specifies the financial linkages that connect the sub-models with each other. The next part of the paper deals with the determination of the three-month Eurodollar interest rate. We then discuss the considerations that are relevant in deciding whether the two sources of the monetary base -- foreign and domestic assets held by central banks -- should be treated as endogenous variables. Finally, we describe how the purchase and sale of foreign assets, i.e., foreign exchange intervention, by several central banks is accounted for in our model.

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[1]This multi-country model is described in a paper by Richard Berner, Peter Clark, Howard Howe, Sung Kwack and Guy Stevens entitled "Modeling the International Influences on the U.S. Economy: A Multi-Country Approach." This is referred to in the text of this paper as our "Summary Paper."

A. The Monetary Sector of the Country Sub-Model: Money Supply and Demand

The money supply process in our prototype country model is very similar to that used in the Federal Reserve's econometric model.[1] The supply of demand deposits is determined from the identity defining the unborrowed monetary base: that is, given the unborrowed base and given the uses of the base as reserves against time deposits, currency in the hands of the public and free reserves, the supply of demand deposits is determined as a residual. The demand for demand deposits is explained by a behavioral equation that includes the short-term interest rate as an explanatory variable. For simulation and forecasting purposes, this equation will be inverted so that the short-term rate becomes the left-hand-side variable.

Our interest in international financial linkages led us to adopt two modifications to this basic approach. First, we treat the unborrowed monetary base as an endogenous variable because it is affected by changes in international reserves. Second, the demand for domestic assets is made a function of foreign interest rates and expected changes in exchange rates (in addition to other variables) in order to allow for possible substitution between domestic and foreign assets.

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[1] We need to emphasize that what we describe in this paper is a <u>prototype</u> model, that is, it provides the basic structure and point of departure for the specification of the monetary sectors of the five countries in the overall model. It can be used with little or no modification for the United States, but it will have to be changed to take account of the different monetary institutions in the other four countries.

The basic building block of the monetary sector is the balance sheet of the central bank, which is described in Part II. A.1 of our Summary Paper. The assets of the central bank represent the monetary base from the sources side, and the liabilities constitute the uses of the base. The balance sheet is given by equation (1), where the net worth of the central bank is included (with a minus sign) in OTH:

(1) $$NFA + NGP + RB + OTH = RT + CUR$$

where:

NFA = net foreign assets (international reserves)

NGP = net claims on the government

RB = borrowed reserves of commercial banks

OTH = all other assets minus net worth

RT = total reserves of commercial banks

CUR = currency held by the non-bank public.

Since borrowed reserves, RB, are endogenous, it is preferable to work with the unborrowed base, BU, which is obtained by subtracting RB from both sides of (1). The reason for prefering BU is that at least in a world of managed floating, it is this variable which is directly controlled by the central bank. The unborrowed base from the sources side is defined as:

(2) $$BU \equiv NFA + NGP + OTH.$$

From the uses side we have:

(3) $$BU \equiv RU + CUR,$$

where RU = unborrowed reserves, which in turn can be defined as:

(4) $$RU \equiv RT - RB \equiv RR + RX - RB \equiv RR + RF,$$

where:

RR = required reserves RX = excess reserves

RF = free reserves $\equiv$ RX - RB

Required reserves are in turn equal to:

$$\text{(5)} \qquad RR \equiv aDD + bTD$$

where:

DD = demand deposits

TD = time deposits

a,b = reserve requirements against demand and time deposits, respectively.

Using equations (3)-(5), one ends up with the following equation for the uses of the unborrowed base:

$$\text{(6)} \qquad BU \equiv aDD + bTD + CUR + RF.$$

Since the monetary base from the sources side is determined by equation (2), equation (6) can be used to determine one right-hand-side variable. Following the practice of the Federal Reserve econometric model, we have chosen demand deposits as this variable. Rearranging equation (6) yields:[1]

$$\text{(7)} \qquad DD \equiv (BU - bTD - CUR - RF)/a.$$

Equation (7) states that given the unborrowed base from the sources side [equation (2)], and given the uses of reserves in the form of currency, free reserves and reserves against time deposits, the quantity of demand deposits is determined.

To complete the monetary sector of the sub-model we need to specify four asset demand functions to determine DD, TD, CUR and RF. The short-term interest rate can be thought of as equilibrating the demand for base money, as given by equation (6), with the supply of the monetary base, which is determined by equation (2). As mentioned above, in making

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[1] Equation (7) combines equations XV.6 and XV.7 in the Federal Reserve econometric model. See Quarterly Econometric Model Equations, January, 1975.

simulations and forecasts the equation for the demand for demand deposits is inverted so that the short-term interest rate becomes the left-hand-side variable. The monetary sector thus consists of six equations in six unknowns: BU, DD, TD, CUR, RF and RS, the short-term interest rate.

Asset demand functions in each country have the following general form:

(8) $A_i/NW = f_i(IR, T_i/NW)$

where:

A_i = nominal value of the i^{th} asset

NW = nominal value of private net worth in the country

IR = vector of rate-of-return variables

T_i = variable that generates a transactions demand for the i^{th} asset.

This general functional form embodies two basic properties: 1) it is homogeneous of degree one in nominal magnitudes, and 2) the transactions variable, T_i, is deflated by net worth so that in the long run--when T_i and NW move proportionately--the demand for A_i will be homogeneous in wealth. If one did not deflate the transactions variable by net worth, then the proportion of wealth devoted to money balances, for example, would grow over time. Following Tobin, we divide T_i by NW to avoid this anomalous result.[1]

[1] See James Tobin, "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit and Banking, Vol. 1, February, 1969, p. 20.

To emphasize the portfolio-allocation aspects of our asset demand functions we also follow Tobin in making the demand for money depend on net worth. This implies that for given interest rates a positive fraction of an increment in net worth will be allocated to cash balances. This assumption can be rationalized on the grounds that the additional money balances provide an implicit service in the form of fewer trips to the bank and fewer conversions between money and interest-earning assets.[1] In addition, one can argue that an increase in net worth generates an increase in the demand for money because of additional portfolio transactions.

In the monetary sector the rate-of-return variables are the domestic short-term interest rate, RS, a vector of foreign short-term rates, FRS, and a vector of expected exchange rate changes, DRE. We assume that market-determined short-term interest rates generally embody the current expected rate of inflation, and therefore higher inflation rates are assumed to reduce the demand for money-fixed claims because inflationary expectations will raise nominal interesr rates.

Equation (8) is assumed to be a linear function. Explicit functional forms for currency, demand deposits and time deposits for the i^{th} country are given below, where both sides of (8) have been multiplied by NW.

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[1] For a contrary view, i.e., that the demand for money is not a function of net worth, see Albert Ando and Franco Modigliani, "Some Reflections on Describing Structures of Financial Sectors," in Gary Fromm and Lawrence Klein, eds., The Brookings Model: Perspectives and Recent Developments, North-Holland, Amsterdam, 1975.

(9) $$CUR_i = A_{0i}(L)NW_i + A_{1i}(L)(RS_i \cdot NW_i) + A_{2i}(L)CV_i$$

(10) $$DD_i = B_{0i}(L)NW_i + B_{1i}(L)(RS_i \cdot NW_i) + B_{2i}(L)GNPV_i + \sum_{\substack{j=i \\ j \neq i}}^{6} B_{3ij}(L)(RS_j \cdot NW_i) + \sum_{\substack{j=i \\ j \neq i}}^{5} b_{4ij}(DRE_{ij} \cdot NW_i)$$

(11) $$TD_i = C_{0i}(L)NW_i + C_{1i}(L)(RS_i \cdot NW_i) + C_{2i}(L)GNPV_i + \sum_{\substack{j=1 \\ j \neq i}}^{6} C_{3ij}(L)(RS_j \cdot NW_i) + \sum_{\substack{j=1 \\ j \neq i}}^{5} c_{4ij}(DRE_{ij} \cdot NW_i)$$

where:

L = lag operator

$DRE_{ij} = (RE_{ij} - R_{ij})/R_{ij}$ = expected change in R_{ij}

RE_{ij} = expected R_{ij} next period

R_{ij} = units of currency i per unit of currency j.

There are several points worth mentioning about these equations.

1. The demand for currency is a function of nominal consumption rather than nominal GNP, as in (10) and (11), on the grounds that expenditures by consumers would appear to be more closely associated with the transactions demand for currency than income. An alternative approach --following the Tobin-Brainard lesson that asset demands are part of a system of equations that are restricted by a budget constraint--is to make each asset demand a function of all the variables that enter demand equations. We have chosen not to pursue this approach here because our net worth variable is not equal to the sum of the assets we are explaining,

and consequently the constraints on the coefficients across equations (sum of intercept terms equal to one, sum of rate-of-return coefficients equal to zero) do not hold.[1]

2. Foreign as well as domestic short-term assets are considered a substitute for domestic demand and time deposits, and therefore foreign interest rates, RS_j, as well as expected changes in the exchange rates, DRE_{ij}, enter as explanatory variables. If DRE_{ij} is positive, the exchange rate is expected to depreciate, so there is an incentive to move into assets denominated in foreign currencies. Consequently b_{4ii} and c_{4ij} are negative. Only the current value of DRE enters as an explanatory variable because it appears that asset demands adjust very quickly to expected exchange rate changes.

3. Uncovered foreign interest rates are used in (10) and (11). The reason is that the forward premium is determined mainly by the uncovered interest differential and the expected change in the exchange rate between the corresponding currencies. Since these variables appear in the asset-demand equations, the estimated parameters are really reduced form coefficients that take into account the induced response in the forward premium. If the premium is influenced in an important way by other variables, e.g., official forward intervention, then it will be necessary either to deal with the forward market explicitly, or to enter these other variables in the asset demand equations.

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[1] The manner in which net worth is computed is explained in the Summary Paper, Part II.A.5.

4. Equations (9)-(11) reflect the simplifying assumption that these assets are demanded only by domestic residents, since foreign net worth does not appear as an explanatory variable. If foreign holdings of these assets are significant and if equations (9)-(11) cannot adequately explain the behavior of these assets, then foreign demand can be taken into account by either:

a. adding foreign variables, e.g., foreign wealth, to these equations, or

b. separating out the foreign holdings of DD and TD and estimating equations for these components.

An example of a separate treatment of the foreign demand for domestic money is provided below for the case of Germany.

5. The asset demands are specified as functions of current and possibly lagged values of net worth. The rationale for including lagged values is that it may take more than a quarter for the demand for CUR, TD and TD to adjust to a change in wealth. More generally, it should be noted that generalized lag structures will be used in estimation, involving Almon or Shiller techniques, rather than the Koyck lag. There are two reasons for this: 1) the Koyck procedure involves using a lagged dependent variable as a regressor, which entails econometric problems, and 2) it imposes the same lag structure on all explanatory variables.[1]

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[1] For a recent comparison of alternative distributed-lag estimators—where it is shown that the Koyck procedure may give misleading results—see John Wilson, "Have Geometric Lag Hypotheses Outlived Their Time? Some Evidence in a Monte Carlo Framework," International Finance Discussion Papers, No. 82, April 1976, Board of Governors of the Federal Reserve System.

6. If the basic structure of the asset demand equation involves a disturbance term such that $A_i/NW = f_i(IR, T_i/NW) + \varepsilon$, where ε has a constant variance, then multiplying both sides by NW can make the disturbance heteroskedastic because NW has a strong time trend. Consequently we shall have to test our asset demand equations for heteroskedasticity, and if the errors have a non-constant variance, re-estimate the equations either using generalized least squares or in ratio form, i.e., CUR/NW, etc.

7. There are three different ways to measure foreign interest rates and exchange rates. One can:

a. use one interest rate, e.g., the Eurodollar rate, and the corresponding expected bilateral dollar exchange rate for countries other than the United States;

b. enter several interest rates and expected exchange rates separately, as is done in equations (10) and (11); or

c. form weighted averages of both variables.

We do not know a priori which procedure will yield the best empirical results, and these results will probably vary from country to country. We will proceed by using the least restrictive specification first, i.e., b, and then go on to c, or as a last resort use a, if the initial results are unsatisfactory.

8. We have two reasons for separating time from demand deposits, even though we specify the same equation for both variables. First, time and demand deposits are subject to different reserve requirements. Second, if both kinds of deposits were combined, it would be more difficult to estimate the interest-sensitivity of the demand for money. The reason

is that an increase in the market-determined short-term rate (the rate we shall be using in our country models) will tend to raise the rate paid on time deposits, which will lead to a shift from demand into time deposits. This shift within the aggregate used as the dependent variable will mask the interest-sensitivity of the demand-deposit component. By separating out time deposits we avoid this problem.

9. We have simplified the monetary sub-sector somewhat in that the time-deposit rate is not included in our country sub-models, even though it should appear as the own rate of return in the time deposit equation. Our reasons for making this simplification are that 1) at certain times there have been statutory ceilings on interest rates payable on time deposits, and 2) when not subject to government control, the time deposit rate tends to be correlated with the rate on short-term money-market instruments, RS. In this latter case a change in RS will have both a direct and an indirect effect (through the time deposit rate) on TD, so that the parameters in equation (11) are really reduced form coefficients.

To complete the specification of the monetary sector of the sub-model we need to determine free reserves, RF. We start with the balance sheet of the commercial banks, which is the same as that described in our Summary Paper, Part II.A.2:

A	L
RR	DD
RX	TD
STS	RB
LTS	
FA	NW

where:

STS = short-term domestic securities

LTS = long-term domestic securities

FA = foreign assets

NW = net worth of commercial banks

As with the private non-banking sector, the assets and liabilities of banks are influenced by their net worth, since this is fixed in the short run and constrains their net asset position. Unlike households and firms, however, banks lack control over one other item on their balance sheet, namely, the level of demand deposits, which is determined by the non-bank public.[1] Banks must invest what they receive as demand deposits net of required reserves.[2] The portfolio decision of banks is therefore constrained by their net worth plus their demand deposits net of required reserves:

(12) $NW + (1-a)DD = (RR-aDD) - TD + (RX-RB) + STS + LTS + FA$

(12') $NW + (1-a)DD = (b-1)TD + RF + STS + LTS + FA.$

Since we do not explain banks' net worth in our model, the demand for free reserves will be constrained only by net demand deposits, (1-a)DD, which play the same role here as the private net worth variable in the money-demand functions.

Since RF = RX - RB, we must look at the determinants of these two variables. The short-term interest rate represents the opportunity cost

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[1]If time deposits were subject to an interest rate ceiling, then they would also be exclusively demand determined. To the extent that banks can vary the rates they pay on time deposits, they can control the quantity of this particular liability.

[2]This is embodied in the model described by Tobin, op. cit.

of excess reserves, and therefore an increase in RS will reduce RX. The level of borrowed reserves depends on the difference between the discount rate (RD) and the short-term rate because an increase in RS, for a given RD, makes borrowing from the central bank more profitable, whereas a rise in RD discourages such borrowing. In addition, the demand for both RX and RB may reflect some short-run adjustment to changes in required reserves (ΔRR) caused by alterations in reserve requirements, and changes in unborrowed reserves (ΔRU). A rise in the latter will tend to be associated with a short-run increase in RX and a reduction in RB, since banks may not fully adjust their earning assets within one quarter. An increase in required reserves caused by a change in reserve requirements would tend to have opposite effects because it may be easier for banks to adjust excess reserves and borrowings than earning assets, especially loans, when faced with a reserve deficiency at unchanged deposit levels. The equation for free reserves for the i^{th} country therefore looks like:[1]

$$(13) \qquad RF_i = d_0 NDD_i + d_1(RD_i - RS_i)NDD_i + d_2 RS_i \cdot NDD_i + d_3 \Delta RU_i + d_5 \Delta RR_i$$

where:

$$NDD = (1-a)DD$$

$$\Delta RR = \Delta a(DD_{-1}) + \Delta b(TD_{-1})$$

$$RU = BU - CUR$$

Only contemporaneous values of the explanatory variables enter this equation

[1] This equation is very similar to the one used in the Federal Reserve econometric model. See equation XV.2 in Quarterly Econometric Model Equations, January, 1975.

because free reserves appear to adjust very quickly.[1] Equation (13) also embodies the assumption that the rate of return on domestic short-term securities represents the opportunity cost of borrowed reserves. If commercial banks in the countries in our model regard foreign assets, e.g., Euro-dollar deposits, as close substitutes for domestic securities, then the interest rate on these foreign assets should also be included in the equation explaining free reserves.

One final point should be made about the consolidated balance sheet of the commercial banks. Leaving aside net worth, we explain all bank assets and liabilities except non-deposit liabilities and holdings of domestic short- and long-term securities. (Banks' foreign assets are determined as a component of capital flows, which are described below.) We ignore the banks' demand for long-term securities because the long-term domestic interest rate is explained using a term-structure equation, rather than by equating the supply and demand for these securities. As described in our Summary Paper, Parts II.B.5 and III.B, the equilibrium condition in the market for domestic short-term securities is replaced by the balance-of-payments equilibrium condition, and therefore the demand for short-term securities by banks is regarded as being determined as a residual from their balance sheet constraint.

The term-structure equation explaining the long-term interest rate in each country is based on the assumption that domestic short- and

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[1] For evidence on this point, see Thomas Thomson, James Pierce and Robert Parry, "A monthly Money Market Model," Journal of Money, Credit and Banking, November, 1975.

long-term securities are very close if not perfect substitutes. If this assumption is valid, then arbitrage between the markets for short- and long-term domestic securities will insure that the long-term rate is a weighted average of expected future short-term rates. If one assumes that expectations regarding expected future short rates contain both extrapolative and regressive elements, then the long-term interest rate for the i^{th} country can be expressed as a function of a constant term (representing a risk factor) and current and lagged values of the short-term rate:[1]

(14) $$RL_i = a_0 + a_1RS_i + A_2(L)RS_{i-1}$$

This equation implies certain assumptions about the demand functions for domestic short- and long-term securities. Modifying equation (8) slightly, these demand functions can be written as follows:

(15) $$STS_i = f(RS_i, RL_i, FRS, FRL, DRE)NW_i$$

(16) $$LTS_i = g(RS_i, RL_i, FRS, FRL, DRE)NW_i$$

where FRS, FRL and DRE are vectors of foreign short rates, long rates and expected exchange-rate changes, respectively. The assumption of perfect substitutes implies that the partial derivatives with respect to RS_i and RL_i in (15) and (16) are very large relative to the partials of the other explanatory variables in the equations. The net effect of this assumption is that in an equation such as (14)—which is really a reduced-form equation for the long-term rate—the direct influence of FRS, FRL

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[1] For a derivation of this term-structure equation, see Franco Modigliani and Richard Sutch, "Innovations in Interest Rate Policy," American Economic Review, Papers and Proceedings, Vol. LVI, No. 2, May 1966, pp. 178-197.

and DRE on RL is effectively zero because the denominator in the reduced form [the coefficient of RS in (16)] swamps the structural coefficients of FRS, FRL and DRL in (16). Foreign interest rates and changes in expectations regarding future exchange rates will have an indirect effect on the domestic long-term rate, however, to the extent to which they influence the current and expected future domestic short-term interest rates.

B. International Financial Flows

If we were to adopt a full-fledged flow-of-funds approach to interest rate and exchange rate determination, we could then dispense with standard capital flow equations. We would instead have to describe the foreign demand for specific domestic securities, and the interest rates on these securities would be determined by equating the domestic supply with the total, i.e., domestic and foreign, demand. As explained in our Summary Paper, Part III.B, however, we are not pursuing this approach. We use term structure equations to explain long-term interest rates, rather than equating the supplies and demand for long-term assets, and we drop the markets for short-term securities and substitute balance of payments equations. With the possible exception of the foreign demand for domestic money, we have no need to identify and explain the foreign demand for specific domestic assets, and similarly, we are not forced to specify the domestic demand for particular foreign assets. Instead, our task is to explain the international financial transactions of the countries in our model without concern for the particular markets involved. This means that we can aggregate several of our demand functions for foreign assets.

The exception mentioned above relates to the one market where we specify explicitly the supplies and demands for assets, namely, the money market. If foreign demand for domestic demand and time deposits (foreign holdings of domestic currency are presumed small enough that they can be ignored) have an important influence on domestic monetary conditions, then one can make a case that the foreign component of total demand should be modeled separately. For example, there has in the past been heavy foreign demand for German deposits: at the end of 1968 foreigners held nearly eleven per cent of German demand deposits and eight and one-half per cent of the time deposits. At the end of 1974 these figures were eight per cent and under one-half percent, respectively. The effect of these shifts in and out of Germany on unborrowed reserves, and therefore on the German monetary sector, are taken into account in the German sub-model through the link between international reserve changes and capital flows. There is in addition a further effect that arises from the fact that in Germany differential reserve requirements are imposed on foreign deposits, so that changes in the mix of domestic and foreign deposits cause variations in the relationship between the unborrowed base and the total volume of deposits.

To take account of foreign deposits in Germany we plan to specify and estimate a separate equation for this variable. Required reserves now become:

(5a) $RR = aDD + bTD + cFD$

where FD = total foreign deposits (demand and time) and c = average reserve requirement against foreign deposits. The equation for the "supply" of demand deposits is given by.[1]

(7a) $DD \equiv (BU - bTD - cFD - CUR - RF)/a.$

Under fixed rates a change in FD has a two-fold effect on DD, since BU changes on-for-one with FD, and $dDD/dFD = (1-c)/a$. If, on the other hand, there are floating rates and no official intervention, then the effect comes only through a change in required reserves, so that $\partial DD/\partial FD = -c/a$.

Thus other than the foreign demand for domestic money, we do not have to isolate the demand by foreigners for particular domestic assets. This means that we can aggregate the international financial flows of the countries in our model. This aggregation simplifies the task of estimating our model because we need to explain <u>all</u> capital inflows and outflows in the five country sub-models, since we use the balance-of-payments equation in place of the equilibrium condition in the short-term securities market.

International financial transactions are typically broken down into three categories: short-term, long-term portfolio and long-term direct capital flows. There are several reasons for combining short- and long-term portfolio flows in our model. First, the same functional

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[1]Quotation marks are used for the word <u>supply</u> to warn the reader that equation (7a) is not a behavioral supply equation but a rearrangement of the reserve identity.

form, namely, equation (8), is used to describe both types of flows. On the other hand, a different functional form is employed to explain long-term direct investment; it is derived from considerations regarding plant and equipment expenditures overseas and factors affecting the financing of these expenditures.[1] Second, short-term flows are an aggregate that includes assets ranging in maturity from demand deposits to one-year securities. Since we have already assumed considerable substitution between domestic short- and long-term securities, it seems reasonable to suppose that such substitution also exists between foreign assets with different maturities. This implies that long-term rates should appear in the short-term capital flow equation, and short-term rates in the long-term equation. Finally, long-term flows include instruments with an original maturity of more than one year. When the time to maturity of such instruments drops below one year, however, they have all the characteristics of short-term assets.

The demand of the private sector for financial claims on foreigners (PFC) is therefore the sum of short-term claims (STC) and long-term portfolio claims (LTPC), and is a function of the same variables that explain the demand for domestic securities given by equations (15) and (16). We use in addition one other variable, namely, the value of the country's exports (XGV). This variable represents both the transactions demand for foreign currencies and the extension of trade credit on the part of domestic exporters to foreign buyers. To preserve long-run

[1] Equations explaining long-term direct claims and liabilities are described in the Summary Paper, Part IV, equations (50) and (51).

homegeneity in wealth, this variable is divided by domestic net worth. The general functional form for the i^{th} country's private financial claims on the j^{th} country is given by:

$$(17)\quad PFC_{ij} = f(RS_i, RL_i, FRS, FRL, DRE, \frac{XGV_i}{NW_i})\ NW_i$$

The bilateral claims represented in (17) are not only a function of rates of return in the two countries involved, but in principle depend also on interest rates in all other countries. This is why vectors of rates of return (FRS, FRL and DRE) are included in (17). If we now aggregate over countries to get total financial claims on foreigners, we obtain a function with the same form as (17), the only difference being that the coefficients of the explanatory variables are now the sum of the corresponding coefficients in the bilateral equations:

$$(18)\quad PFC_i \equiv \sum_{\substack{j=1\\ j\neq i}}^{6} PFC_{ij} = f'(RS_i, RL_i, FRS, FRL, DRE, \frac{XGV_i}{NW_i})\ NW_i \cdot$$

Equation (18) is a <u>stock</u> demand function. Since most of the data on international financial transactions are in flow form, we have to take the first difference of (18). Assuming that $f'(RS_i, \ldots)$ is linear, we have:

$$(19)\quad \Delta PFC_i = A_{0i}(L)\ \Delta NW_i + A_{1i}(L)\ \Delta(RS_i \cdot NW_i) + A_{2i}(L)\ \Delta(RL_i \cdot NW_i)$$
$$+ A_{3i}(L)\ \Delta XGV_i + \sum_{\substack{j=1\\ j\neq i}}^{6} A_{4ij}(L)\ \Delta(RS_j \cdot NW_i)$$
$$+ \sum_{\substack{j=1\\ j\neq i}}^{6} A_{5ij}(L)\ \Delta(RL_j \cdot NW_i) + \sum_{\substack{j=1\\ j\neq i}}^{5} a_{6ij}\ \Delta(DRE_{ij} \cdot NW_i)$$
$$+ a_7\ CAPC$$

where CAPC is a variable designed to capture the effects of capital controls. The change in financial claims on foreigners is assumed to adjust within one quarter to expected exchange rate changes, but there may be lagged adjustment to changes in interest rates, mainly because long-term portfolio claims appear to take more than one quarter to respond fully to variations in interest rates. The lagged adjustment to imports reflects the fact that the change in the stock of trade credit depends not only on the extension of credit due to an increase in exports in the current period, but also on the current repayment of trade credit extended in the past.

The specification embodied in (19) includes only one effect of exchange rates on capital flows, namely, the influence of expected exchange rate changes on the rate of return on foreign assets expressed in domestic currency. It does not take account of two effects of changes in the level of exchange rates on financial claims on foreigners. To the extent that these claims are denominated in foreign currencies, a change in exchange rates will affect the domestic-currency value of 1) claims on foreigners and therefore 2) private net worth. The former is typically called the "portfolio - rebalancing "effect and the latter the "wealth" effect of exchange rate changes.[1] The portfolio-rebalancing effect will outweigh the wealth effect, so that in the case of a devaluation, for example,

[1] For a discussion of these effects, see Lance Girton and Dale Henderson, "Central Bank Operations in Foreign and Domestic Assets Under Fixed and Flexible Exchange Rates," Dennis Logue and Thomas Willett, "The Effects of Exchange Rate Adjustment on International Investment," and Guy Stevens, "Comment on 'The Effects of Exchange Rate on International Investment'," all to appear in a forthcoming volume edited by P. Clark, D. Logue and R. Sweeney, The Effects of Exchange Rate Adjustments, U.S. Government Printing Office, Washington, D. C.

under ceteris paribus assumptions the exchange rate change will cause a reduction in desired claims on foreigners.

At a later stage of our modeling work we shall attempt to take account of these effects of exchange rate changes on capital flows. In the initial stage of model construction, however, these effects will be ignored, the primary reason being that information on the currency composition of the foreign claims of the five countries in our model is far from complete. An additional reason is that we wish to begin with a relatively simple model.

In our Summary Paper, Part III.A, we pointed out the following relationship between world claims and liabilities: when expressed in the same currency, the sum total of the claims of the five countries plus the rest of the world (ROW) on each other is necessarily equal to the sum of their liabilities to each other. We would like to be able to exploit this relationship between claims and liabilities in a manner similar to that by which we utilize the world imports ≡ world exports identity.[1] In a multi-country model it is desirable to ensure that the sum of liabilities to foreigners generated within the model matches the sum of claims on foreigners implied by the asset-demand functions. For example, if we had information both on bilateral claims and the claims of our aggregate ROW sector, and if we had asset-demand functions to explain these claims, we could then determine each country's liabilities to foreigners

[1] See the companion paper by Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model," for a description of the method by which our overall model takes account of the fact that world imports ≡ world exports.

as identically equal to the claims on that country. With regard to financial claims and liabilities, we would have:

$$(20)\quad NPFL_i \equiv \sum_{\substack{j=1 \\ j\neq i}}^{6} PFC_{ji}$$

where $NPFL_i$ = total financial liabilities of the i^{th} country <u>net</u> of those to foreign official monetary institutions. Unfortunately, the requisite data are not available which would enable us to implement this approach.

Nevertheless, the relationship between claims and liabilities can be embodied to some extent in our model. This can be done by explaining $NPFL_i$ as a function of the financial claims of the four other countries in the model, i.e., PFC_j, j = 1, ..., 5, j≠i, and by assuming that these claims are allocated to the i^{th} country depending on the values of the rate-of-return variables that appear in equation (18). If one follows this approach equation (20), which is an identity, can be replaced by the following equation:

$$(21)\qquad NPFL_i = \sum_{\substack{j=1 \\ j\neq i}}^{5} PFC_j \cdot g_j(RS_i, RL_i, FRS, FRL, DRE) + h_i(MGV_i)$$

In equation (21) the exports of the j^{th} country, j=1, ...,5, j≠i, have been replaced by the i^{th} country's imports (MGV) because an increase in its imports will generally be associated with an increase in trade-related borrowing abroad, not only from the four other countries in the model, but from ROW as well.

If we assume that the <u>g</u> and <u>h</u> functions are linear and take first differences, we obtain the following fully disaggregated equation explaining short- and long-term portfolio liabilities to foreigners:

$$(22)\quad \Delta NPFL_i = \sum_{\substack{j=1\\ j\neq i}}^{5} B_{oij}(L)\Delta PFC_j + \sum_{\substack{j=1\\ j\neq i}}^{5} B_{1ij}(L)\Delta(RS_i \cdot PFC_j)$$

$$+ \sum_{\substack{j=1\\ j\neq i}}^{5} B_{2ij}(L)\Delta(RL_i \cdot PFC_j)$$

$$+ \sum_{\substack{j=1\\ j\neq i}}^{6} \sum_{\substack{k=1\\ k=i}}^{5} B_{3ijk}(L)\Delta(RS_j \cdot PFC_k)$$

$$+ \sum_{\substack{j=1\\ j\neq i}}^{5} \sum_{\substack{k=1\\ k\neq i}}^{5} B_{4ijk}(L)\Delta(RL_j \cdot PFC_k)$$

$$+ \sum_{\substack{j=1\\ j\neq i}}^{5} \sum_{\substack{k=1\\ k\neq i}}^{5} b_{5ijk}\Delta(DRE_{ij} \cdot PFC_k)$$

$$+ B_{6i}(L)\ \Delta\ MGV_i + b_7\ CAPC$$

A comparison of (22) with (19) reveals that the former has many more terms than the latter. The reason, of course, is that the change in claims is determined by the behavior of residents in only one country, whereas the change in a country's liabilities reflects the decisions of residents in the four other countries in the model and the rest-of-the world sector. As it stands, (22) contains far too many explanatory variables to be estimated, so that it will be necessary to aggregate one or more of these variables. At a minimum, we will aggregate the private financial claims of the four countries.

It should be clear that equation (22) will provide only a rough approximation to the claims-liability relationship represented by equation (20), the major reason being that we are not in a position to explain the change in private financial claims of ROW. This is the reason why the index k in (22) runs only to five. However, the specification embodied in (22) does have the advantage that it ensures that an increase in the claims of the five countries in the model does lead to an increase in their liabilities. It seems reasonable to impose this specification because of the close financial ties among these five countries. We shall nevertheless also experiment with a more traditional specification of the liabilities equation that uses the net worth of foreign countries in place of their claims on foreigners. In this alternative specification ΔNW_j would replace ΔPFC_j in equation (22).

C. The Eurodollar Interest Rate

In our discussion of international financial transactions we have implicitly assumed that these capital flows are a function only of national interest rates. For the countries in our model, however, the short-term Eurodollar interest rate is an important determinant of financial flows. Since this interest rate is in turn a function of the financial flows among the countries in the overall model, it is necessary to treat it as an endogenous variable to be explained along with national interest rates.

For our purposes we can use a reduced form equation to explain the Eurodollar rate. We could derive this equation by aggregating the excess demand for Eurodollar deposits on the part of the five individual countries. We have found it more convenient, however, to specify the supply and demand for three-month Eurodollar deposits for only two regions:

the United States and the four other countries combined. We shall therefore deal with weighted averages of the relevant variables in these four countries. We have also found it simpler to describe separate supply and demand functions for short-term funds emanating from the two regions.

Starting with aggregated supply and demand functions for Canada, Germany, Japan and the United Kingdom (which we shall denote by NUS i.e., non-U.S.) we have:

$$(23)\qquad ED^{S}_{NUS} = a_0 + \overset{+}{a}_1 EDR + \overset{-}{a}_2 USRS + \overset{-}{a}_3 FRS + \overset{-}{a}_4 DRE + \overset{+}{a}_5 FNW$$

$$(24)\qquad ED^{D}_{NUS} = b_0 + \overset{-}{b}_1 EDR + \overset{+}{b}_2 USRS + \overset{+}{b}_3 FRS + \overset{+}{b}_4 DRE + \overset{+}{b}_5 FMGV + \overset{-}{b}_6 BARD$$

where:

ED^{S}_{NUS} = supply of three-month Eurodollar deposits by NUS (lending)

ED^{D}_{NUS} = demand for three-month Eurodollar funds by NUS (borrowing)

EDR = three-month Eurodollar interest rate

USRS = short-term U.S. interest rate

FRS = weighted average of the short-term interest rates in NUS

DRE = weighted average of the expected changes in the bilateral dollar exchange rates of NUS

FNW = weighted average of net worth in NUS

FMGV = weighted average of merchandise imports in NUS

BARD = dummy variable for the Bardepot, which was designed to reduce German firms' foreign borrowing.

Expected signs have been put above the coefficients. Exchange rates are defined as dollars per unit of foreign currency, so that an expected depreciation of the dollar reduces the supply of Eurodollar deposits ($a_4 < 0$) and increases the demand for borrowed dollars ($b_4 > 0$). Eurodollar borrowing is assumed to be a function of imports, i.e., it reflects trade financing (among other things), with a high weight probably going to Japan.

Supply and demand equations for the United States are:

$$(25)\quad ED^S_{US} = c_0 + \overset{+}{c}_1 EDR + \overset{-}{c}_2 USRS + \overset{-}{c}_3 FRS + \overset{-}{c}_4 DRE + \overset{-}{c}_5 CAPC + \overset{+}{c}_6 USNW$$

$$(26)\quad ED^D_{US} = d_0 + \overset{-}{d}_1 EDR + \overset{+}{d}_2 USRS + \overset{+}{d}_3 REGQ + \overset{-}{d}_4 REGM + \overset{+}{d}_5\, TB + \overset{-}{d}_6\, [(1-a)DD_{US}].$$

where:

ED^S_{US} = supply of three-month Eurodollar deposits by U.S. (lending).

ED^D_{US} = demand for three-month Eurodollar deposits by U.S. (borrowing).

CAPC = dummy variable(s) for capital controls restricting U.S. capital outflows

USNW = U.S. net worth

REGQ = difference between USRS and Regulation Q ceiling rate on time deposits if this difference is positive; otherwise, REGQ = 0.

REGM = dummy variable set equal to zero prior to 1969Q4; beginning 1969Q4 scaled to the size of the reserve requirement (Regulation M) against U.S. bank head office borrowing from foreign banks, mainly foreign branches of U.S. banks.

TB = average quarterly outstanding U.S. Treasury and Export-Import Bank securities especially issued to U.S. foreign branch banks. These securities were first issued in January, 1971, and were fully retired by the end of October , 1971.

DD_{US} = total demand deposits. The term (1-a)DD measures demand deposits net of reserve requirements (a) and represents a source of funds exogenously given to the banking system. An increase in (1-a)DD should reduce the need to borrow from foreign branches, and therefore $d_6 < 0$.

U.S. lending to the Eurodollar market was probably curtailed to some extent by capital controls, CAPC. U.S. use of this market has been dominated by head-office borrowing from their foreign branches, which explains the large number of institutional-type variables in the ED^D_{US} equation.

Setting $ED^S_{NUS} + ED^S_{US} = ED^D_{NUS} + ED^D_{US}$, we end up with the following reduced form equation for the Eurodollar interest rate:

$$(27)\quad EDR = \alpha_0 + \alpha_1 USRS + \alpha_2 FRS + \alpha_3 DRE + \alpha_4 FNW + \alpha_5 FMGV + \alpha_6 USNW$$
$$+ \alpha_7 [(1-a)DD_{US}] + \alpha_8 CAPC + \alpha_9 BARD + \alpha_{10} REGQ + \alpha_{11} REGM$$
$$+ \alpha_{12} TB + \alpha_{13} DQ1 + \alpha_{14} DQ2 + \alpha_{15} DQ4.$$

where:

$$\Delta = a_1 + c_1 - b_1 - d_1 > 0$$
$$\alpha_0 = (b_0 + d_0 - a_0 - c_0)/\Delta$$
$$\alpha_1 = (b_2 + d_2 - a_2 - c_2)/\Delta > 0$$

$$\alpha_2 = (b_3 - a_3 - c_3)/\Delta > 0$$

$$\alpha_3 = (b_4 - a_4 - c_4)/\Delta > 0$$

$$\alpha_4 = a_5/\Delta < 0$$

$$\alpha_5 = b_5/\Delta > 0$$

$$\alpha_6 = c_6/\Delta < 0$$

$$\alpha_7 = d_7/\Delta < 0$$

$$\alpha_8 = c_5/\Delta > 0$$

$$\alpha_9 = b_6/\Delta < 0$$

$$\alpha_{10} = d_3/\Delta > 0$$

$$\alpha_{11} = d_4/\Delta < 0$$

$$\alpha_{12} = d_6/\Delta > 0$$

Seasonal dummies have been added to the equation because it appears that "window-dressing" and other seasonal borrowing has had a short-run effect on the Eurodollar rate. Because available evidence suggests that there is rapid adjustment in this market, it is quite likely that only current values of explanatory variables need to be used. To avoid multicollinearity foreign and U.S. wealth could easily be combined to give one wealth variable.

Net excess demand for Eurodollars originating in the rest-of-the world sector could in principle have some effect on the Eurodollar rate. Empirical studies have shown, however, that nearly all the variation in this interest rate can be explained by the variables appearing in equation (27). We therefore believe that we can ignore changes in net Eurodollar demand on the part of countries other than the five explicitly included in the model.

D. Treatment of Monetary Policy Instruments

In our description of the monetary sector for each country sub-model we have included as part of our specification four instruments of monetary policy: intervention in the foreign exchange market, NFA; open market operations, NGP; reserve requirements, a and b; and the discount rate, RD. We have made these instruments explicit because one major function of our overall model will be to explore the effects of alternative monetary policies on the U.S. economy. The linkages between these policy instruments and ultimate targets - real income, the inflation rate and the unemployment rate - have been spelled out in the model, so that by means of simulation experiments we will be in a position to explore the implications for the economy of different assumptions about the time paths of the instruments of monetary control.

Policy instruments are typically treated as exogenous variables in macroeconomic models. As recently pointed out by Goldfeld and Blinder, however, two problems may be encountered if one makes this assumption.[1] These problems arise because policy instruments are generally manipulated to achieve the goal of stabilizing the economy, which implies that they will be adjusted in response to current economic conditions. If the instruments of monetary policy are systematically related to other variables in the model, then they should be viewed as endogenous variables.

[1]Stephen Goldfeld and Alan Blinder, "Some Implications of Endogenous Stabilization Policy," Brookings Papers on Economic Activity, No. 3, 1972.

The first problem identified by Goldfield and Blinder is an econometric one: when a policy instrument appears as an explanatory variable, estimation involving ordinary least squares will lead to the usual difficulties associated with using an endogenous variable as a regressor. Goldfeld and Blinder find that this problem is much less acute in structural models as compared with reduced form equations, the main reason being that policy instruments seldom appear as explanatory variables in behavioral equations. In our monetary sub-model, instruments appear in only one equation, that for free reserves. In addition, Goldfeld and Blinder report that their Monte Carlo experiments suggest that the bias in structural models is itself very small. It seems reasonable to conclude, then, that the estimation bias associated with using endogenous policy instruments in structural models is not severe.

On the other hand, they find that if one treats policy instruments as exogenous, then one can overstate the magnitude of policy multipliers. The effects of a change in a given policy instrument on the economy will not be offset if all other instruments are set exogenously, e.g., at constant levels. However, if the other instruments in fact respond endogenously to the impact on the economy generated by the change in the given policy instrument, then the magnitude of this impact will be reduced. If these induced policy responses are systematic and persist over time, then they should be included within a model that is designed to measure the impact on the economy of variation in specific policy instruments.

To measure policy multipliers more precisely it will be necessary to endogenise at least some of the instruments of monetary policy. If we can identify systematic relationships between instruments and other variables in the model, then this information can be used in simulation experiments when we exogenously change <u>one</u> policy instrument and compute the effects on the economy. Instead of setting other policy instruments exogenously, by estimating reaction functions we will obtain a more accurate picture of the response of the entire economy.

It is difficult, however, to specify reaction functions that hold across countries. The main reason is that the instruments of monetary policy differ among countries. Therefore this aspect of the construction of the overall model will be left to the specification of particular country models.

E. Endogenous Exchange Market Intervention in a Multi-Country Framework

One instrument of monetary policy that we shall treat as endogenous is intervention in the foreign exchange market by central banks. Such treatment will involve the specification and estimation of an intervention function designed to explain the purchase and sale of foreign exchange. Our objective here is not to describe the particular functional form and specific set of explanatory variables for such an intervention function; this will be done at a later stage of our work. Rather, we wish to explore some of the problems associated with constructing a set of consistent intervention functions.

In Section III.C of our Summary Paper we argued that exchange market intervention by a central bank will appear not only in its balance of payments but in another country's external account as well. This reflects the fact that the purchase (sale) of a foreign asset by one central bank necessarily gets recorded on the books of some other central bank. When intervention is treated as exogenous in making simulations and forecasts outside the sample period, one must specify the currency composition of this intervention: if, for example, the Federal Reserve intervenes in deutsche marks, one must make sure that this transaction is included in the German as well as the U.S. balance of payments. When, however, intervention is explained in terms of other variables within the model by means of functional relationships, it is necessary to specify these functions such that the purchase or sale of foreign exchange which they generate gets reflected in the balance of payments of two countries.

We shall demonstrate this point by means of a simple example. Our point of departure is the discussion of central bank intervention in Section III.C of our Summary Paper. There we noted that to take account of official intervention it is necessary to distinguish between liabilities to private holders and those to official holders, since the latter generally reflect intervention behavior.[1] This distinction between liabilities is incorporated into the definition of the official settlements balance (OSB),

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[1] We use the qualifier "generally" because the measured change in liabilities to official holders does not always represent intervention behavior as understood here, i.e., as the purchase and sale of foreign assets for the express purpose of influencing exchange rates. For example, the increase in U.S. liabilities to OPEC countries is not motivated by the desire to influence exchange rates.

which for country i is equal to:

(28) $OSB_i = DNFA_i - DLO_i$

where:

D = mnemonic for "Δ" or first difference

DNFA = change in net foreign assets of the central bank

DLO = change in liabilities to official foreign holders.

The change in net foreign claims of the central bank can be broken down into two components:

(29) $DNFA_i = DNOA_i + DCO_i$

where:

DNOA = change in net outside reserve assets; primarily change in gold, SDRs (exclusive of those allocated during the current period), and in the country's IMF position

DCO = change in holdings of convertible foreign currencies.

Combining equations (28) and (29) gives:

(30) $OSB_i = DNOA_i + DCO_i - DLO_i$

Since for the world as a whole the balance of payments (with no change in outside reserve assets) is identically zero, for our model involving five countries and the rest-of-the-world region we have:

(31) $\sum_{i=1}^{6} OSB_i \equiv 0 \equiv \sum_{i=1}^{6} (DNOA_i + DCO_i - DLO_i)$, where all magnitudes have been converted to a common numeraire currency. There are in addition separate identities relating changes in outside reserve assets and currency claims and liabilities:

(32) $\sum_{i=1}^{6} DNOA_i \equiv 0 \Rightarrow DNOA_i \equiv -\sum_{\substack{j=1 \\ j \neq i}}^{6} DNOA_j$

(33) $$\sum_{i=1}^{6} DCO_i \equiv \sum_{i=1}^{6} DLO_i \equiv 0 \Rightarrow DLO_i \equiv \sum_{\substack{j=1 \\ j \neq i}}^{6} DCO_j$$

These three identities highlight the fact that in a multi-country model reserve changes in one country are necessarily reflected in reserve changes in one or more other countries.

To simplify our discussion of intervention behavior we shall make two assumptions: 1) changes in net outside reserve assets (DNOA) are zero and 2) changes in official currency claims and liabilities of the rest of the world are excluded from the analysis. The first assumption reduces the complexity of the intervention functions without changing their essentials; the second reflects our decision to treat the exchange rate of the rest of the world as exogenous and to determine ROW intervention as a residual from the U.S. balance-of-payment equation.

To derive for purposes of illustration an intervention function that has the properties we desire, we assume that central banks pursue a smoothing strategy with respect to individual currencies. Specifically, we assume that a central bank purchases or sells a particular currency in proportion to the percentage change in the bilateral exchange rate of that currency:

(34) $$DCO_{ij} = \alpha_{ij} \ln[R_{ij} / R_{ij}(-1)] \qquad \alpha_{ij} < 0$$

where DCO_{ij} = change in currency claims by the i^{th} central bank on country j. If $[R_{ij} - R_{ij}(-1)] > 0$, the i^{th} currency is depreciating relative to the j^{th} currency, which will activate sales of the j^{th} currency and purchases of its own currency by the central bank in country i, i.e., $DCO_{ij} < 0$.

Equation (34) can be used to explain intervention by all central banks who purchase or sell the currency of country $\underline{i}$. This central bank intervention (CBI) is equal to:

$$(35) \quad CBI_i = DCO_i - DLO_i$$
$$= \sum_{\substack{j=1 \\ j \neq i}}^{5} DCO_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^{5} DCO_{ji}$$

If data on central bank intervention by currency are available, then in principle one could estimate the individual bilateral intervention functions given by (34). In each country's balance of payments there would appear a total of eight intervention functions: four describing the intervention behavior of the central bank of that country and the remaining four describing how the central banks of the other four countries intervene in the currency of that country. With intervention data by currency one can in fact use different functional forms for the intervention functions, since there is no need to aggregate these functions.

In either case, whether one uses the same or different functional form, the point is that the disaggregated intervention functions must enter two balance-of-payments equations. This ensures that a change in an argument of an intervention function —in the simple case of (34) the only argument is a bilateral exchange rate —will affect two balance-of-payments equations by an equal and opposite amount. This can be demonstrated by using (34) to substitute for DCO_{ij} in (35) and writing out CBI for countries $\underline{i}$ and $\underline{j}$:

$$(36) \quad CBI_i = \sum_{\substack{j=1 \\ j \neq i}}^{5} \alpha_{ij} \ln[R_{ij}/R_{ij}(-1)] - \sum_{\substack{j=1 \\ j \neq i}}^{5} \alpha_{ji} \ln[R_{ji}/R_{ji}(-1)]$$
$$= \sum_{\substack{j=1 \\ j \neq i}}^{5} \alpha_{ij} \ln[R_{ij}/R_{ij}(-1)] + \sum_{\substack{j=1 \\ j \neq i}}^{5} \alpha_{ji} \ln[R_{ij}/R_{ij}(-1)]$$

$$(37)\qquad CBI_j = \sum_{\substack{j=1\\ j\neq i}}^{5} \alpha_{ji}\ \ln[R_{ji}/R_{ji}(-1)] - \sum_{\substack{j=1\\ j\neq i}}^{5} \alpha_{ij}\ \ln[R_{ij}/R_{ij}(-1)]$$

$$= -\sum_{\substack{j=1\\ j\neq i}}^{5} \alpha_{ji}\ \ln[R_{ij}/R_{ij}(-1)] - \sum_{\substack{j=1\\ j\neq i}} \alpha_{ij}\ \ln[R_{ij}/R_{ij}(-1)]$$

From (36) and (37) it is clear that:

$$(38)\qquad \frac{\partial\ CBI_i}{\partial \ln[R_{ij}/R_{ij}(-1)]} = \alpha_{ij} + \alpha_{ji} = -\frac{\partial\ CBI_j}{\partial \ln[R_{ij}/R_{ij}(-1)]}\ .$$

On the other hand, if data by currency of intervention are not available, then it will not be possible to estimate bilateral intervention functions, and therefore such functions cannot be entered individually into balance-of-payments equations. We have, however, chosen an intervention function that can be easily aggregated. Total intervention by country i, DCO_i, and total intervention by other countries in the i^{th} currency, DLO_i, can be explained by aggregating the bilateral intervention functions, DCO_{ij}. To explain CBI_i we now need two equations rather than eight; but these two equations have exactly the same form as (36) and (37), since they are simply linear combinations of the same bilateral functions. It therefore follows that these aggregate intervention functions preserve the consistency property described by (38).

Given the specification of the underlying intervention function DCO_{ij}, one can go a step further and aggregate DCO_i and DLO_i. One then ends up with one equation explaining total intervention in currency i:

$$(39)\qquad CBI_i = DCO_i - DLO_i = \sum_{\substack{j=1\\ j\neq i}}^{5} (\alpha_{ij} + \alpha_{ji})\ \ln\ [R_{ij}/R_{ij}(-1)].$$

The equation for CBI_j is given by:

(40) $$CBI_j = DCO_j - DLO_j = -\sum_{\substack{j=1 \\ j\neq i}}^{5}(\alpha_{ji} + \alpha_{ij})\ \ln[R_{ij}/R_{ij}(-1)]$$

Equations (39) and (40) also have the property described by (38), namely, that a change in an exchange rate has an equal and opposite effect in two balance-of-payments equations, since CBI_i enters the balance-of-payments equation of country i and CBI_j enters the corresponding equation of country j.

In practice, the bilateral intervention functions that will form the basis for aggregate functions will probably not have the simple form of (34). There may, therefore be difficulties in aggregating these functions to obtain equations explaining both DCO_i and DLO_i. Such aggregation will most likely be necessary because bilateral intervention data is not very good.

In the event that we have different intervention functions for the countries in our model, it will nevertheless be desirable to ensure that the consistency property described by (38) holds. To do this it will be necessary to include as arguments in the function explaining the aggregate DLO_i those variables appearing in the DCO_j functions, $j\neq i$, that generate purchases or sales of the i^{th} currency. In actual estimation it may also be necessary to impose constraints on the coefficients of the regression equations.