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ESTIMATING CONSUMER IMPORT DEMAND EQUATIONS

by

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This paper examines alternative functional forms for consumer import demand functions for multi-country trade models. Two variants are estimated and compared.

The point of departure is the use of a system of demand equations, based on two assumptions: imports are not perfect substitutes for domestic goods, and consumers' demands for imports are distinct from the demand for imports by other agents in the economy. While this approach has much in common with that of Armington (1969), and Hickman and Lau (1973), on the one hand, and Burgess (1974a), (1974b), on the other, it will be shown to be distinct from each of those.

First, a general discussion relates the demand system approach to the problems involved in constructing multi-country trade models. Two functional forms are considered, estimation problems and results are discussed for the two systems: the S-Branch and Rotterdam models. The models are implemented for annual data over the period 1954-1970 for Belgium, Netherlands, France, Italy, and W. Germany.

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I. Why Consumers' Demands for Imports? Why Demands by Country of Origin?

Consumer demand systems that distinguish goods by country of origin were motivated by the construction of a multi-country trade/macro model designed to examine the multisectoral impact of discriminatory (as in a customs union) tariff changes and of exchange rate changes. The multi-sector input-output accounting system used in this model involves a two-way classification of all flows: by origin (industrial sector and geographic) and destination (to interindustry purchases or final demanders), as in the upper panel of Figure 1. Total purchases from all origins by any demander are obtained by summing the appropriate column, and total uses from any origin are the sum of the corresponding row.

Thus, total imports classified under a particular industrial sector are the sum of all intermediate sales from that sector, the sum of imported plus private consumption over several categories (food, clothing, etc.) originating in that industry, plus other imported final demands (see the row of matrices aligned with B in the upper panel). Total private consumption expenditures for say, food, are equal to the sum of all industrial sectors' contributions from the various geographic origins (middle set of matrices in the upper panel). Of course, the sum of total food, clothing and other consumption expenditures equals total private consumption.

This accounting system was used for the model mentioned above in order to empirically capture the implications of imported intermediate and final goods that are close, but not perfect, substitutes for similarly classified domestically originating goods. Intermediate import

Figure 1

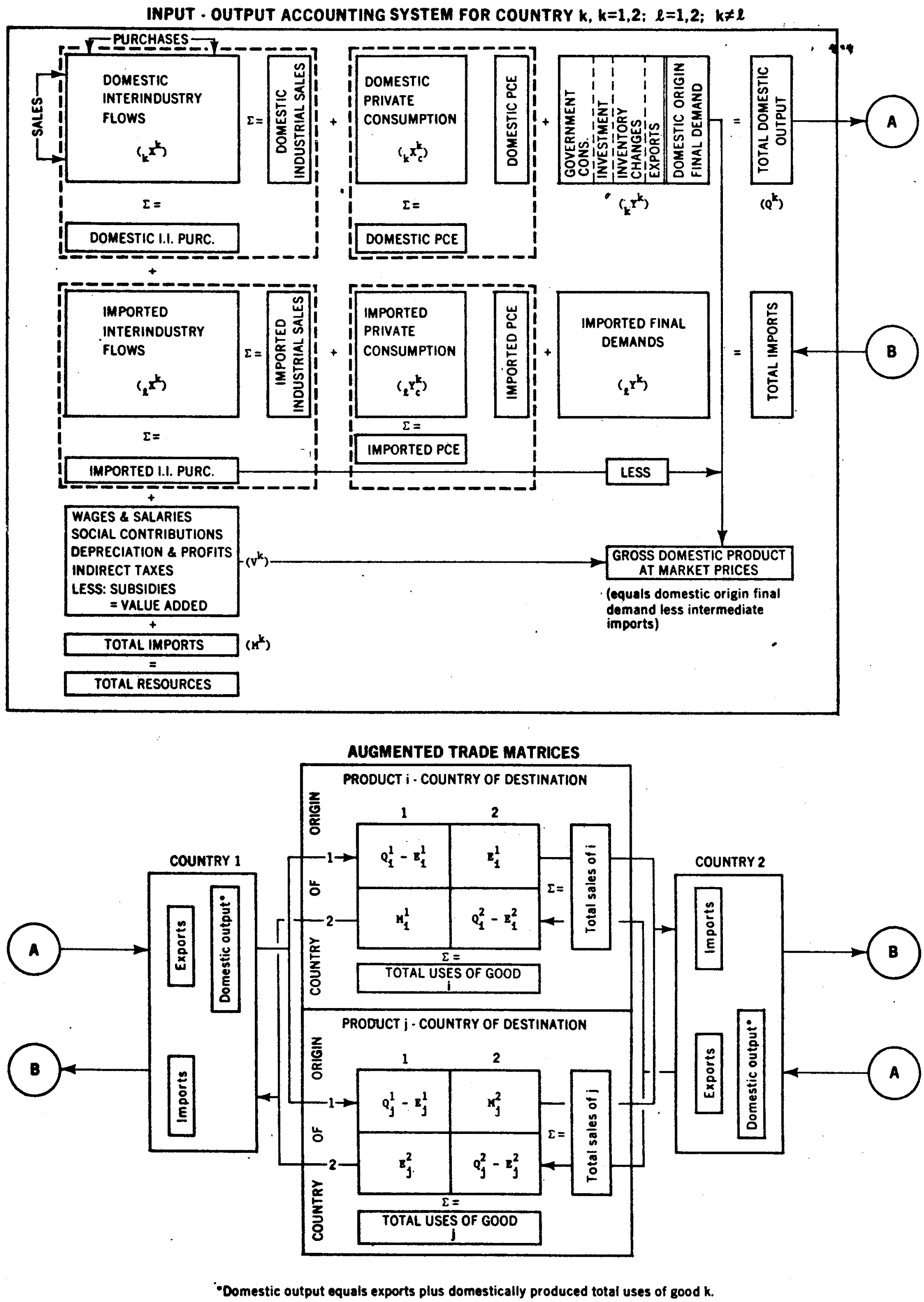

FIGURE 1 INPUT - OUTPUT ACCOUNTING SYSTEM AND RELATION TO TRADE MATRICES

demands are considered to be factor demands, and therefore have different functional forms from consumer demands. When prices of imports rise because of a tariff change, for example, and intermediate inputs become more expensive, the increase in the cost of producing domestic goods may partially offset the price advantage conferred on them by the tariff change--a result well known from the effective protection literature. The same is true for a devaluation. In fact, intermediates comprise as much as 75% of total imports. Thus, it was judged necessary to distinguish consumers' demands for both imported and domestic goods from those of producers.

To distinguish goods by geographic origin was judged necessary since price as between imports and domestic goods differed. A distinction between imports from EEC partners and the rest of the world (ROW) is the minimum necessary in the presence of tariff changes that discriminate in favor of the former.

Armington (1969) proposed and Hickman and Lau (1973) (HL) estimated demand systems that follow this goods-by-origin principle. In implementation, HL delete domestic origin goods, while the present system retains them. No distinction is made in either case between intermediate and final demands for imports. Burgess (1974a, 1974b) treats imports as inputs in the production of final demand, finessing the intermediate-final distinction entirely. No country-of-origin distinction is made in his model. An advantage of Burgess' approach and the one used in this paper is that prices of imports enter the demands for domestically originating goods, and vice-versa. This ensures that the shifts in demand from imports to domestic origin "importables" accords with that hypothesized in the theory of tariff and exchange rate changes.

II. Functional Forms

Two functional forms are employed: the S-Branch system of Brown and Heien (1972), and a block-additive relative price version of the Rotterdam model (see, for example, Theil (1975)). The S-Branch system is also block-additive. Five categories of goods are distinguished: food, clothing, shelter, durables, and other. Given the three origins for each good (domestic, EEC, ROW), a system of fifteen equations must be estimated.

Block-additivity (or strong separability) is imposed *a priori* to restrict the number of parameters to estimate. This restriction is based on a two level budgeting or utility tree view of the decision-making process: total expenditure is first allocated among goods, then expenditure for each good is allocated among products. This assumes that products from a given origin are homogeneous. For example, given the three origins, a Frenchman is assumed to distinguish a Renault from a VW and each from an Austin. Yet Mercedes, VW, Ferrari and Fiat are part of the same homogeneous product.

It has been argued that it would be more useful to distinguish products by quality class, following Lancaster (1966), rather than by country of origin. The approach taken here implies that these two classifications are related very simply; i.e., 1:1. Unfortunately, the characteristics or hedonic approach is impossible to implement for a complete macromodel, and more important, the phenomenon of interest is that prices differ and are changed differentially according to geographic origin.

The S-Branch demand functions to be estimated are

$$(1) \qquad q_{si} = \delta_{si} q_{si,-1} + \left(\frac{b_{si}}{p_{si}}\right)^{\sigma_s} [\alpha_s{}^{\sigma} X_s{}^{(\frac{\sigma-1}{\sigma_s-1})^{-1}}]$$

$$\cdot [\sum_{r=1}^{s} \alpha_r{}^{\sigma} X_r{}^{(\frac{\sigma-1}{\sigma_s-1})}]^{-1} [m - \sum_{r=1}^{s} \sum_{j\varepsilon r}^{n} p_{rj}\gamma_{rj}]$$

$$+ u_{si}, \quad i\varepsilon n_s, \quad s=1,\ldots,S$$

where $X_s = \sum_{j=s}^{n_s} (b_{sj}/p_{sj})^{\sigma_s} p_{sj}$,

n_s = the number of products in a goods class (here = 3),

S = the number of goods classes or branches (here = 5),

q_{si} = quantity demanded of product i in branch s,

p_{si} = corresponding price,

m = total expenditure

b_{si}, δ_{si}, α_s, σ_s and σ are parameters to be estimated, and u_{si} is a random error term.

As detailed by Brown and Heien and Deaton (1974), this system allows within-block complements, unlike its progenitor, the linear expenditure system (LES), but only if all products within a block are complements to each other.

The block-additive Rotterdam model estimating equations are

$$(2) \qquad w^*_{it} Dq_{it} = \mu_i Dq_t + \phi A_{it}(\mu) + \sum_{\substack{j\neq 1 \\ j\varepsilon n_s}} \nu_{ij} (Dp_{jt} - Dp_{it}) + u_{it},$$

where

$$(3) \qquad A_{it}(\mu) = \mu_i[(Dp_{it} - Dp_{nt}) - \sum_{k=1}^{n-1} \mu_k (Dp_{kt} - Dp_{nt})],$$

(4) $w^*_{it} = (w_{it} + w_{it-1})/2,$

(5) $Dq_t = \sum_k w^*_{kt} Dq_{kt},$

(6) $w_{it} = p_{it}q_{it}/m_t,$

(7) $Dx_t = \ln x_t - \ln x_{t-1}$ for any x,

(8) $\nu_{ii} = \phi\mu_i - \sum_{\substack{j\neq i \\ j\varepsilon n_s}} \nu_{ij}$

(9) $\Sigma\, \mu_i = 1$

(10) $\nu_{ij} = \nu_{ji}$

and $n = \sum_s n_s.$

As explained in Theil (1975), this version of the Rotterdam model permits either (specific) substitution or complementarity between pairs of goods within a block. As with the S-Branch model, substitution between goods is represented by a single parameter: σ in the S-Branch model and ϕ in the Rotterdam model.

III. Estimation Results and Within-Sample Comparisons

The S-Branch estimates (done by FIML) almost always yielded elasticities of substitution σ_s and σ significantly different from both 0 and 1 at the 5% level, implying the inadequacy of the LES for these data.[1] However, supernumerary expenditure (the quantity in the last bracket of the second line in (1)) was not infrequently negative at the sample

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[1]Thanks are due Murray Brown and Dale Heien for their version of the estimation program.

means, which violates the assumptions of the utility function from which the system is derived, and gives rise to negative income elasticities, as will be seen. The results are available from the author, but not presented here.

Space limitations prevent the inclusion of all but a sample of the Rotterdam model parameters, which were estimated using both iterated GLS and mixed estimation.[1] Given the converged GLS sample estimates, priors and standard errors are assigned to the marginal budget shares in the form of income elasticities. These are generally less than unity for domestic origin products, and greater than one for imported products.[2]

The sample and mixed estimates for Germany are presented in Table 1. The introduction of prior information on the marginal budget shares, μ_i, does not drastically change their magnitudes, but it does substantially reduce their standard errors. As a result, the standard errors of the derived parameters, ν_{ii}, the own price coefficients, are reduced as well, in view of (8). The compatibility test statistic is too low to reject compatibility between the simple and prior information (see bottom of Table 1).[3] The share of the precision of the mixed estimate attributable to the prior estimates is only .14.[4]

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[1]Thanks are due John Paulus for providing his estimation program, modified by the author for this study.

[2]See Chapter IV of Berner (1975) for details on the priors. See Paulus (1975) for details on the estimation procedures.

[3]See Theil (1963) for the derivation of this statistic, together with the derivation of prior and sample shares.

[4]Ibid.

TABLE 1

Rotterdam Model - W. GERMANY, 1954-1970

Income and Price Coefficients - Sample and Mixed Estimates

Product	Marginal Shares (μ_i)		Price Coefficients (ν_{ij})					
			Sample			Mixed		
	Sample (1)	Mixed (2)	Domestic (3)	EEC (4)	ROW (5)	Domestic (6)	EEC (7)	ROW (8)
FOOD								
1. Domestic	.2740	.2573	-7.74	.421	-.149	-7.28	.421	-.148
	(.0288)	(.0152)	(1.23)	($.2x10^{-3}$)	(.004)	(.827)	($.2x10^{-3}$)	(.004)
2. EEC	.0068	.0081		-.678	.072		-.714	.072
	(.0039)	(.0025)		(.327)	($.2x10^{-3}$)		(.177)	($.2x10^{-3}$)
3. ROW	.0011	.0220			-.233			-.523
	(.0239)	(.0102)			(1.05)			(.885)
CLOTHING								
4. Domestic	.1100	.1055	-3.12	.112	.013	-3.00	.112	.013
	(.0081)	(.0067)	(.935)	($.2x10^{-3}$)	($.1x10^{-3}$)	(.775)	($.2x10^{-3}$)	($.1x10^{-3}$)
5. EEC	.0054	.0066		-.323	.065		-.356	.065
	(.0028)	(.0014)		(.319)	($.1x10^{-3}$)		(.116)	($.1x10^{-3}$)
6. ROW	.0057	.0075			-.234			-.284
	(.0025)	(.0015)			(1.08)			(.804)
SHELTER								
7. Domestic	.1249	.1250	-3.49	.080	.010	-3.49	.080	.010
	(.0106)	(.0090)	(.658)	($.4x10^{-4}$)	($.1x10^{-3}$)	(.572)	($.4x10^{-4}$)	($.1x10^{-3}$)
8. EEC	.0004	.0007		-.075	-.016		-.084	-.016
	(.0005)	(.0003)		(.279)	($.4x10^{-4}$)		(.129)	($.4x10^{-4}$)
9. ROW	.0010	.0007			-.021			-.013
	(.0016)	(.0004)			(.577)			(.484)
DURABLES								
10. Domestic	.2658	.2691	-7.81	.251	.319	-7.90	.251	.319
	(.0103)	(.0077)	(.280)	($.1x10^{-3}$)	($.1x10^{-3}$)	(.209)	($.1x10^{-3}$)	($.1x10^{-3}$)
11. EEC	.0040	.0054		-.309	-.050		-.348	-.050
	(.0016)	(.0008)		(.046)	($.4x10^{-4}$)		(.025)	($.3x10^{-4}$)
12. ROW	.0039	.0036			-.376			-.366
	(.0026)	(.0012)			(.070)			(.033)
OTHER								
13. Domestic	.1842	.1853	-5.21	.045	.149	-5.24	.045	.149
	(.0060)	(.0048)	(.163)	($.3x10^{-4}$)	($.4x10^{-4}$)	(.131)	($.3x10^{-4}$)	($.2x10^{-4}$)
14. EEC	.0008	.0011		-.060	-.008		-.066	-.008
	(.0004)	(.0002)		(.009)	($.3x10^{-4}$)		(.005)	($.3x10^{-4}$)
15. ROW	.0019	.0021			-.191			-.198
	(.0010)	(.0007)			(3.26)			(1.39)
ϕ	-.2724	-.2724						
	($.98x10^{-4}$)	($.98x10^{-4}$)						

Notes: Figures in Columns 3.8 are to be divided by 10^2.
Figures in parentheses are standard errors.
Compatibility test statistic (χ^2_{14}): 10.17

Sample share: .86 **Prior Share:** .14

In two instances for two other countries, not presented here, sample marginal budget share estimates were negative, and the imposition of prior μ_i that are positive results in positive mixed estimates of these parameters.[1] Paulus (1975) has evidence at the goods level of strong specific substitution between durables and shelter and of strong specific complementarity between clothing and other for the Netherlands, one of the problem countries. Hence, the block-additive model must be a misspecification for the Netherlands. However, an advantage of the Rotterdam model is that off-diagonal blocks of coefficients may be added to the model without major surgery. Each would involve nine additional parameters in the present case, assuming symmetry, for a total of eighteen additional parameters. This will be done in the near future.

As seen in Table 2, the Rotterdam model wins the performance race based on information inaccuracy[2] (column I in Table 2) over both the S-Branch model and a naive model of the following form:

$$(11) \qquad \hat{w}_{it} = w_{it-1}$$

where the hat denotes predicted. Equation (11) corresponds to an autoregressive version of the linear Engel curve through the origin model:

$$(12) \qquad \hat{q}_{it} = w_{it-1} \cdot m_t/p_{it}.$$

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[1]This occurred for France and the Netherlands for imported shelter, which is largely fuels and electricity, where the budget shares (sample mean average) are extremely small--even smaller than those marginal shares for Germany in Table 1.

[2]See Theil (1967), Chapter 7.

TABLE 2

GERMANY MODEL COMPARISONS: INFORMATION INACCURACIES AND R^2s

MODEL	NAIVE		S - BRANCH		ROTTERDAM - SAMPLE		ROTTERDAM - MIXED	
CRITERION	I	R	I	R	I	R	I	R
FOOD								
1. Domestic	311	.99	660	.98	215	.99	215	.99
2. EEC	261	.94	869	.87	293	.94	299	.93
3. ROW	2217	.33	46418	.62	2356	.29	2273	.29
CLOTHING								
4. Domestic	39	.99	123231	.99	35	.99	35	.99
5. EEC	240	.96	1075	.94	2697	.47	6804	.10
6. ROW	117	.97	1137	.96	297	.93	494	.92
SHELTER								
7. Domestic	107	.99	7292	.99	64	.93	62	.93
8. EEC	14	.92	1311	.91	31	.77	42	.75
9. ROW	95	.64	592	.68	103	.51	102	.51
DURABLES								
10. Domestic	64	.99	108301	.99	37	.99	38	.99
11. EEC	55	.98	3728	.97	98	.96	137	.96
12. ROW	114	.92	40680	.96	124	.91	121	.91
OTHER								
13. Domestic	22	.99	21173	.98	15	.97	15	.97
14. EEC	19	.98	87977	.94	31	.97	41	.97
15. ROW	30	.94	96610	.96	42	.11	43	.29
AVERAGE	3534	.99	506805	.98	3100	.97	3043	.97

NOTES: I is the information inaccuracy for budget shares, and R is the uncorrected R^2 on quantity level predictions. Information inaccuracies are to be multiplied by 10^{-6}.

The Rotterdam model loses, however, based on an R^2 on levels criterion. The habit formation specification of the S-Branch model (see (1)) and the changing budget share of the naive model account for this phenomenon. This says nothing about the predictive ability of these models outside the sample--a comparison that will be made in the context of the abovementioned macro/trade model.

It should be mentioned that all criteria are adjusted for degrees of freedom according to

(13) $$k = (n - 1)T/[(n - 1)T - m]$$

where n = the number of equations (15)

T = the number of observations (17)

m = the number of estimated parameters.

Since m = 0 for the naive model, k = 1, and the naive model has an immediate advantage. For the S-Branch model, m = 40 and for the sample estimates of the Rotterdam model, m = 30.[1] Following Paulus[2], m is redefined to be $\alpha_s m$ in the mixed case, where α_s is the sample precision share. The new m "plays the role of the number of unconstrained parameters

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[1]For the S-Branch model, there are 15 b_{si}, 15 δ_{si}, 4 α_s, 5 σ_s, and one σ. This differs slightly from the original Brown-Heien specification, in which a b was dropped. Dale Heien has convinced me that an α should be dropped instead, as in the present formulation. In the Rotterdam model, there are 14 μ_i, one ϕ, and 15 ν_{ij} (3 ν_{ij} in each of five blocks, assuming symmetry); see equations 8-10 above.

[2](1975), pp. 128-130.

in the correction for loss of degrees of freedom after stochastic prior information is introduced."[1] For the German data, $\alpha_s m = 25.8$. Thus, without this correction, the sample estimates would win the performance race using the information inaccuracy criterion. It should be obvious that some experimentation with off-diagonal price coefficients is in order. Given the same dependent variables for two versions of the Rotterdam model, F-tests become appropriate for measurement of the significance of additional price terms.

To conclude, Table 3 presents a comparison across countries of income and own price elasticities or sample means for the Rotterdam model (mixed estimates) and for the S-Branch model for Italy (under Italy-S). These appear reasonable, and illuminate some substantial differences between elasticities for the two import origins, marking them as distinct products. While the income elasticities reflect the prior estimates, the own price elasticities reflect these only partially, in view of (8).[2] In the case of EEC shelter for the Netherlands, both elasticities are dominated by the introduction of a rather high prior income elasticity. However, the marginal budget share here is miniscule. Again, experimentation with non-block additive models is the next order of business.

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[1] Ibid.

[2] Own price elasticities are $\eta_i = (\nu_{ii}(1 - \mu_i) - \mu_i w_i)/w_i = [(\phi\mu_i - \sum_{j \epsilon n_s} \nu_{ij})(1 - \mu_i) - \mu_i w_i]/w_i$.

Table 3

INCOME AND OWN PRICE ELASTICITIES
ROTTERDAM MODEL - MIXED ESTIMATES

Product	Belgium		France		W. Germany		Italy		Netherlands		Italy-S	
	Income	Price	Income	Price	Income	Price	Income	Price	Income	Price	Income	Price
FOOD												
1. Domestic	.65	-.43	.77	-.34	.75	-.41	.84	-.52	.46	-.29	.28	-.02
2. EEC	1.5	-.98	2.7	-.71	1.9	-1.6	3.4	-1.4	2.3	-2.3	6.4	-.12
3. ROW	.64	-.84	.43	-.81	1.9	-.46	1.1	-1.3	.61	-1.1	6.4	-.34
CLOTHING												
4. Domestic	.77	-.36	.98	-.19	.87	-.33	1.1	-.31	.52	-.11	5.2	-.79
5. EEC	2.0	-1.3	4.6	-2.1	3.7	-2.0	3.1	-.32	1.2	-.001	6.4	-.70
6. ROW	.73	.75	2.2	-1.3	3.8	-1.4	1.6	-.56	1.1	-.75	6.4	-.70
SHELTER												
7. Domestic	.60	-.33	.99	-.15	1.1	-.39	.81	-.25	.21	-.97	-2.8	.43
8. EEC	1.8	-1.6	.02	.36	1.6	-1.9	.96	-.75	16.	-39.	6.4	-.70
9. ROW	.60	-.50	.25	.90	.72	-.14	.92	-.94	.05	1.3	1.3	-.14
DURABLES												
10. Domestic	1.4	-.59	1.2	-.31	1.2	-.53	1.7	-.49	.33	-.15	5.2	-.79
11. EEC	3.0	-1.3	3.7	-1.9	2.5	-1.6	1.6	-1.7	.94	-.76	6.4	-.61
12. ROW	1.8	.37	1.5	-.60	1.2	-1.3	1.2	-.04	1.4	-.91	6.4	-.61
OTHER												
13. Domestic	1.4	-.63	1.1	-.34	1.1	-.44	.89	-.28	2.6	-.75	-.86	.13
14. EEC	2.6	-1.8	4.2	-1.7	2.4	-1.5	3.0	-1.3	2.2	-.80	6.4	-1.1
15. ROW	.85	-.93	2.4	-.82	1.5	-1.4	1.7	-.98	2.7	-.88	-.68	.12

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