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I. INTRODUCTION

Can a flexible exchange rate regime be unstable in the absence of destabilizing speculation? Several writers have suggested that portfolio composition alone can be a source of dynamic instability.[1/] When the residents of an open economy have net claims denominated in foreign currency, a depreciation of the home currency raises the home currency value of their wealth, thereby increasing their demand for home-currency assets. However, when they have net debts denominated in foreign currency, this conventional stabilizing valuation effect is reversed. A depreciation has the "perverse" effect of lowering the home currency value of wealth, thereby reducing demand for home-currency assets. Furthermore, even if home residents have a positive net foreign asset position, a depreciation of the home currency can have a destabilizing valuation effect if foreign residents have a negative net foreign asset position (that is, if they have net debts denominated in the currency of the home country).

The result that a flexible exchange rate regime may be unstable when home or foreign residents have negative net foreign asset positions is not merely a theoretical curiosum. It has been a source of serious concern in the empirical implementation of portfolio balance models for some countries. Martin and Masson (1979) are faced with the fact that Canadian residents have net debts denominated in U.S. dollars. Branson, Halttunen, and Masson (1979) and Obstfeld (1980) encounter the situation that foreign residents have net DM-denominated debts to West Germans.[2/]

Concern about the possibility of dynamic instability associated with negative net foreign asset positions has been tempered by the identification of two potentially stabilizing effects. The first is the valuation effect that exists if foreign (home) residents have a positive net foreign asset position even though home (foreign) residents have a negative net foreign asset position. The second is the expectations effect that arises if expectations are regressive. Obstfeld finds that these two effects are large enough empirically to stabilize his estimated model of the DM/dollar rate. Branson, Halttunen, and Masson and Martin and Masson do not reach such sanguine conclusions.

The negative net foreign asset case is not the only portfolio constellation that has led analysts to question the stability of open economy portfolio balance models. Enders (1977) and Masson (1980a) discuss the possibility that instability might arise when positive net foreign asset positions are "too large."

Here we ask whether either of these stability problems can arise in a model with rational expectations. If not, then they would be less worrisome. It would not be possible to identify portfolio composition as an independent source of dynamic instability in a world of flexible exchange rates.

We use a two-country, rational expectations portfolio balance model. It is distinguished by the absence of the usual simplifying assumptions that foreigners do not hold home-currency assets and that certain foreign variables are exogenous.[3/] The second assumption is often referred to as the "small country" assumption. While the stability issues studied here could also be addressed in the context of a small-country model, such an approach would leave open the question of whether the results would be affected by taking into account the endogenous behavior of the rest of the world. Thus, the results presented here should be regarded as being more decisive.

In Section II the model is described, and some properties of its long-run equilibrium are derived. Then, we analyze the stability of long-run equilibrium under static, regressive, and rational expectations in Sections III, IV, and V. Section VI is an extension of the stability analysis to the case of rational expectations with learning.

Section VII is of independent interest. There we obtain a small-country model as a limiting case of our symmetrically-specified, two-country model. The qualitative properties of this small-country model are similar to those of the usual small-country portfolio balance model, except that residents of both countries hold assets denominated in both currencies in equilibrium.[4/] We also consider another special case in which the popular assumption that the foreign price level is exogenous can be justified.

Section VIII contains our conclusions.

II. THE WORLD PORTFOLIO BALANCE MODEL AND ITS LONG-RUN EQUILIBRIUM

In this section we first describe the building blocks of a two-country world portfolio balance model and then derive some of its long-run equilibrium properties.

<u>The Asset Markets</u>. The model contains two assets, home money and foreign money. Residents of both countries hold both moneys. Home net wealth (W) and foreign net wealth ($\overset{*}{W}$), both measured in units of home currency, are given by

$$W = B + EF, \tag{1a}$$

$$\overset{*}{W} = \overset{*}{B} + E\overset{*}{F}. \tag{1b}$$

B ($\overset{*}{B}$) represents home (foreign) net private holdings of home money; F ($\overset{*}{F}$) represents home (foreign) net private holdings of foreign money. E is the exchange rate defined as the home currency price of foreign currency. Residents of each country always hold positive amounts of the money of their country ($B, \overset{*}{F} > 0$) but may have negative holdings of the money of the other country ($\overset{*}{B}, F \gtrless 0$). The outstanding stock of home money ($\tilde{B}$) less world central bank holdings (B^c) and the outstanding stock of foreign money ($\tilde{F}$) less world central bank holdings (F^c) are assumed to be positive:

$$\tilde{B} - B^c = B + \overset{*}{B} > 0, \tag{2a}$$

$$E(\tilde{F} - F^c) = E(F + \overset{*}{F}) > 0. \tag{2b}$$

Home (foreign) residents desire to hold a proportion $b(\cdot)$ [$\overset{*}{b}(\cdot)$] of their wealth in home money. These desired asset proportions depend on the expected rate of depreciation of the home currency (ε); they fall as the expected rate of depreciation rises ($b', \overset{*}{b}' < 0$).

Asset market equilibrium obtains when the stock of home money available to private agents ($\tilde{B} - B^c$) is willingly held in private portfolios:

$$\tilde{B} - B^c = b(\varepsilon)W + \overset{*}{b}(\varepsilon)\overset{*}{W}. \tag{3}$$

When the market for home money clears, the market for foreign money also clears by Walras' Law.[5/]

<u>The Goods Market</u>. The model contains one good. The law of one price holds:

$$P = E\overset{*}{P}.$$

P ($\overset{*}{P}$) is the home (foreign) currency price of the single good. In our model $\overset{*}{P}$ is an endogenous variable, while in many other monetary and portfolio balance models of open economies under flexible exchange rates it is taken to be exogenous.[6]

Private demand behavior in the home and foreign countries is characterized by Metzler-type, target-wealth savings functions, $S(\cdot)$ and $\overset{*}{S}(\cdot)$.[7] Both home and foreign residents save in order to gradually remove gaps between their target and actual real wealth levels. Real outputs in the home and foreign countries are equal to their fixed, full employment levels, $\tilde{Y}$ and $\tilde{\overset{*}{Y}}$. Target real wealth levels, $\alpha\tilde{Y}$ and $\alpha\tilde{\overset{*}{Y}}$, are proportional to incomes (outputs). Investment and, for the present, government spending and taxes are assumed to be zero in both countries. Equilibrium in the market for the world good obtains when world savings is zero:

$$S(\alpha\tilde{Y} - W/P) + \overset{*}{S}(\overset{*}{\alpha}\tilde{\overset{*}{Y}} - \overset{*}{W}/P) = 0, \tag{5}$$

where $S', \overset{*}{S}' > 0$.

<u>Asset Accumulation</u>. Home residents' total asset purchases equal home nominal saving:

$$\dot{B} + E\dot{F} = PS(\alpha\tilde{Y} - W/P). \tag{6}$$

$\dot{B}$ and $\dot{F}$ are the time derivatives of home residents' holdings of home and foreign currency respectively. When (2a), (2b), (5), and (6) hold, foreign residents' total asset purchases equal foreign nominal saving. Foreign residents' total asset purchases are the negative of home residents' total asset purchases, and foreign saving is the negative of home saving.[8]

In our model equation (6), which determines home residents' total asset purchases, conveys the same information as the balance of payments equation. The time derivative of (2a) implies that home residents' total asset purchases

($\dot{B}$ + E$\dot{F}$) equal the home capital account deficit ($-\dot{\overset{*}{B}}$ + E$\dot{F}$) since the home government prints no new money ($\dot{\tilde{B}} = 0$), and the world central bank does not intervene over time ($\dot{B}^c = 0$). Home saving equals the home current account surplus since investment, government spending, and taxes are zero.

The Long-Run Equilibrium. Except for one-time changes, the exogenous variables of the model are fixed. Thus, both nominal and real endogenous variables are stationary in long-run equilibrium. Although the model is highly nonlinear, long-run equilibrium is unique. This result implies that conclusions regarding the global stability properties of the model can be drawn from local stability analysis. Here the uniqueness result is proved. In addition, the long-run effects of two disturbances are derived.

In long-run equilibrium both the expected rate of depreciation of the home currency and home residents' total purchases of assets are zero:

$$\bar{\varepsilon} = \dot{\bar{B}} + \bar{E}\dot{\bar{F}} = 0. \tag{7}$$

A variable with a bar over it represents the long-run equilibrium value of that variable. Given restrictions (7), the long-run values of the endogenous variables are determined by the following equations:

$$\bar{W} + \bar{\overset{*}{W}} = \tilde{B} - B^c + \bar{E}(\tilde{F} - F^c), \tag{8a}$$

$$\bar{W}/\bar{P} = \bar{\omega}, \tag{8b}$$

$$\bar{\overset{*}{W}}/\bar{P} = \bar{\overset{*}{\omega}}, \tag{8c}$$

$$(\tilde{B} - B^c)/\bar{P} = b(0)\bar{\omega} + \overset{*}{b}(0)\bar{\overset{*}{\omega}}, \tag{8d}$$

$$S(\alpha\tilde{Y} - \bar{\omega}) + \overset{*}{S}(\overset{**}{\alpha\tilde{Y}} - \bar{\overset{*}{\omega}}) = 0, \tag{8e}$$

$$S(\alpha\tilde{Y} - \bar{\omega}) = 0. \tag{8f}$$

Equation (8a) is obtained by combining equations (1) and (2).[9/] Equations (8b) and (8c) define the long-run real wealths of home and foreign residents as $\bar{\omega}$ and $\bar{\overset{*}{\omega}}$ repectively. Equation (8d) is obtained by dividing equation (3) by $\bar{P}$ and applying the first restriction in (7). Equation (8e) is the same as equation (5), and equation (8f) results from applying the second restriction in (7) to (6).

Equations (8) imply a unique long-run equilibrium. Since $S(\cdot)$ is single-valued, (8f) determines a unique value of $\bar{\omega}$. Since $\overset{*}{S}(\cdot)$ is single-valued, (8e) then determines a unique value of $\bar{\overset{*}{\omega}}$. Since $b(\cdot)$ and $\overset{*}{b}(\cdot)$ are monotonic, $b(0)$ and $\overset{*}{b}(0)$ are unique. Thus, given $\tilde{B} - B^c$, (8d) determines $\bar{P}$. (8b) and (8c) then determine $\bar{W}$ and $\bar{\overset{*}{W}}$. Given $\tilde{F} - F^c$, (8a) then determines $\bar{E}$.

It is a corollary of the uniqueness result that a transfer of a combination of home and foreign money from foreign to home residents has no effects in the long run. Such a transfer can be accomplished by confiscating a combination of home and foreign money from foreign residents and distributing this combination to home residents. It is referred to below as a transfer of wealth. A transfer of wealth affects none of the exogenous variables in equations (8), so the long-run equilibrium is unaffected.

However, an intervention operation by the world central bank that increases the supply of home money and reduces the supply of foreign money ($dB^c < 0$, $\bar{E}dF^c = - dB^c$) does have long-run effects.[10/] While $\bar{\omega}$ and $\bar{\overset{*}{\omega}}$ are unaffected, $\bar{P}$ and, therefore, $\bar{W}$ and $\bar{\overset{*}{W}}$ rise by the same proportion as the rise in $\tilde{B} - B^c$. The long-run exchange rate also changes: the home currency depreciates proportionately more than the rise in $\tilde{B} - B^c$. This result can be confirmed by dividing (8a) by $\tilde{B} - B^c$ and noting that $(\bar{W} + \bar{\overset{*}{W}})/(\tilde{B} - B^c)$ must remain constant. Thus, the ratio of the home currency value of the private supply of foreign money to the private supply of home money, $\overline{E}(\tilde{F} - \overline{F}^c)/(\tilde{B} - \overline{B}^c)$, must remain constant. Since $\tilde{\overline{F}} - \overline{F}^c$ falls, $\overline{E}$ must rise proportionately more than $\tilde{B} - B^c$.

III. THE MODEL WITH STATIC EXPECTATIONS

Before the stability properties of the model can be analyzed it must be closed with an assumption about expectations formation. In this section expectations are assumed to be static, so that $\varepsilon = \overline{\varepsilon} = 0$.

Some Preliminaries. It is useful to lay some groundwork for the stability analysis. We first select a state variable for the system and then express the model in terms of deviations of the variables from their long-run values.

A convenient state variable is home residents' wealth valued at the long-run exchange rate (w), where

$$w = B + \overline{E}F. \tag{9}$$

The time derivative of w equals home residents' asset accumulation in the neighborhood of long-run equilibrium:

$$\dot{w} = \dot{B} + \overline{E}\dot{F}. \tag{10}$$

Foreign residents' wealth valued at the long-run exchange rate ($\overset{*}{w}$) is given by

$$\overset{*}{w} = \overset{*}{B} + \overline{E}\overset{*}{F}. \tag{11}$$

In deviation form the equilibrium conditions for the home money market, equation (3); the goods market, equation (5); and the expression for home residents' asset accumulation, equation (6), become

$$-dB^c = (b\overline{F} + \overset{*}{b}\overset{*}{\overline{F}})de + (b - \overset{*}{b})dw, \tag{12}$$

$$(S'\overline{W} + \overset{*}{S}'\overset{*}{\overline{W}})dp - (S'\overline{F} + \overset{*}{S}'\overset{*}{\overline{F}})de - (S' - \overset{*}{S}')dw = 0, \tag{13}$$

$$\dot{w} = S'\overline{W}dp - S'\overline{F}de - S'dw. \tag{14}$$

A variable with a d in front of it represents the deviation of that variable for its long-run value. p and e are the natural logarithms of P and E so that $dp = dP/\overline{P}$ and $de = dE/\overline{E}$. In deriving equations (12), (13), and (14) we have set $\overline{P}$ and $\overline{E}$ equal to one for convenience and have made use of the following relationships:

$$dW = dw + Fde, \tag{15a}$$

$$d\overset{*}{W} = d\overset{*}{w} + \overset{*}{F}de, \tag{15b}$$

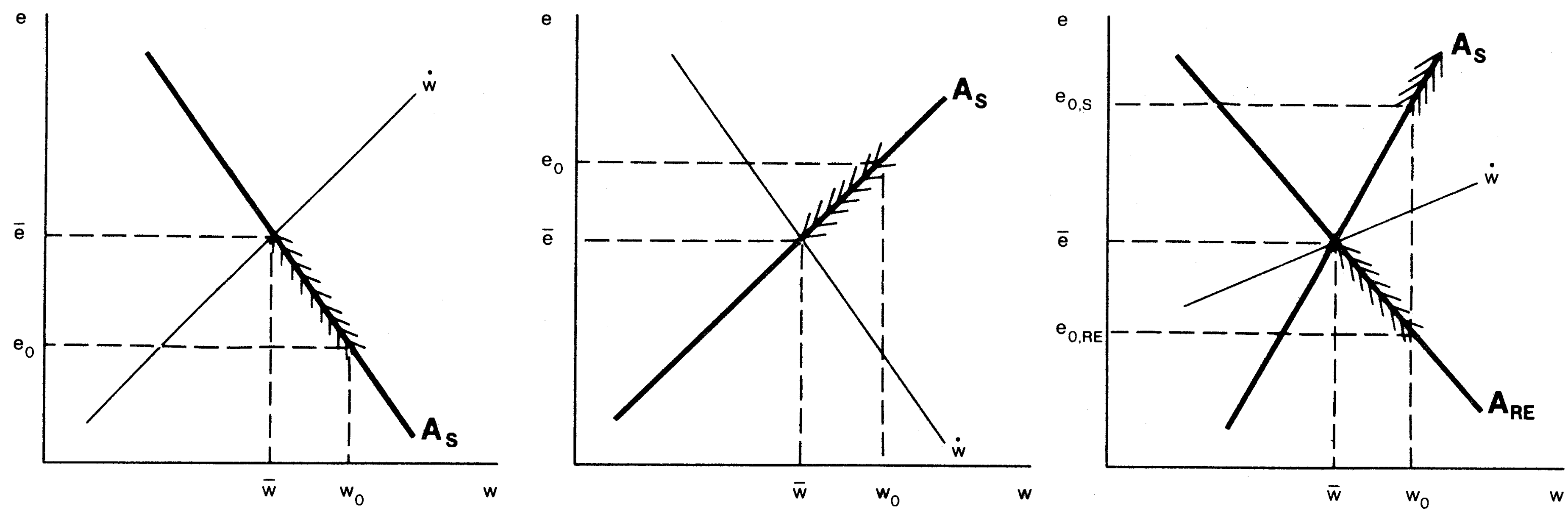
$$d\overset{*}{w} = d\overset{*}{B} + \overline{E}d\overset{*}{F} = - dB - \overline{E}dF = - dw. \tag{15c}$$

To derive equation (15c), sum equations (2) in deviation form to obtain

$$d\tilde{B} + \overline{E}d\tilde{F} - dB^c - \overline{E}dF^c = d\overset{*}{B} + \overline{E}d\overset{*}{F} + dB + \overline{E}dF. \tag{16}$$

(16) implies the equality of the middle two terms in (15c) because $d\tilde{B} = \overline{E}d\tilde{F} = 0$, and world central bank intervention is governed by $dB^c + \overline{E}dF^c = 0$.

<u>Stability Analysis Under Static Expectations</u>. We use two schedules in the analysis of stability under static expectations. Examples of these two schedules appear in Panel A of Figure 1. The $\dot{w}$ schedule shows the pairs of w and e that are compatible with both zero home saving and goods market equilibrium. The A_S schedule shows the pairs of w and e that are compatible with asset market equilibrum under static expectations.



Panel A. Static Expectations, Normal Case
$(b > \overset{*}{b},\ b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} > 0)$

Panel B. Static Expectations, Large Positive Net Foreign Asset Positions
$(b < \overset{*}{b},\ b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} > 0)$

Panel C. Static and Regressive Expectations, Negative Net Foreign Asset Positions
$(b > \overset{*}{b},\ b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} < 0)$

Figure 1. Wealth Transfer

The $\dot{w}$ schedule is referred to as the zero saving schedule. The first step in deriving the relationship between w and e represented by this schedule is to solve (13) for dp and substitute the result into (14) to obtain

$$\dot{w} = \psi(b - \overset{*}{b})\overline{W}\overline{\overset{*}{W}}de - \psi(\overline{W} + \overline{\overset{*}{W}})dw, \tag{17}$$

where

$$\psi = S'\overset{*}{S}'/(S'\overline{W} + \overset{*}{S}'\overline{\overset{*}{W}}) > 0, \tag{18}$$

and where use is made of the fact that

$$\overline{\overset{*}{F}B} - \overline{F\overset{*}{B}} = (1 - \overset{*}{b})b\overline{W\overset{*}{W}} - (1 - b)\overset{*}{b}\overline{\overset{*}{W}W} = (b - \overset{*}{b})\overline{W\overset{*}{W}}. \tag{19}$$

The second step is to set $\dot{w}$ equal to zero.

The zero saving schedule is upward sloping as in Panel A in the usual case in which home residents hold a larger proportion of their wealth in home money than do foreign residents ($b > \overset{*}{b}$). An increase in w represents a transfer of wealth from foreigners to home residents which reduces home savings.11/ If $b > \overset{*}{b}$, a depreciation of the home currency (a rise in e) is required to raise home savings back up to zero.12/ However, if $b < \overset{*}{b}$, an appreciation is required to raise home savings back up to zero, so the zero saving schedule is downward sloping as in Panel B.

The A_S schedule is referred to as the asset market equilibrium schedule under static expectations. The relationship between w and e represented by this schedule is derived by setting dB^c equal to zero in equation (12).

"Perverse" valuation effects that may be associated with negative net foreign asset positions have their impact through this relationship.

The asset market equilibrium schedule under static expectations is downward sloping as in Panel A in the "normal" case. In this case $b > \overset{*}{b}$ as is usual, and normal valuation effects dominate any perverse valuation effects $(b\overline{F} + \overset{**}{b\overline{F}} > 0)$. As shown by (12), if $b > \overset{*}{b}$, an increase in w raises the demand for home money. If valuation effects satisfy the restriction required for the normal case $(b\overline{F} + \overset{**}{b\overline{F}} > 0)$, an appreciation of the home currency (a fall in e) is required to reduce home money demand to its previous level.

Valuation effects always satisfy the restriction required for the normal case if there are no negative net foreign asset positions $(\overline{F}, \overset{*}{b} > 0)$. When $\overline{F}$ and $\overset{*}{b}$ are positive, an appreciation unambiguously lowers the demand for home money because it lowers both home and foreign wealth. Valuation effects may violate the restriction required for the normal case if either home residents or foreign residents have a negative net foreign asset position and definitely violate this restriction if both groups have negative net foreign asset positions. If home residents have a negative net foreign asset position $(\overline{F} < 0)$, an appreciation raises their wealth and, therefore, increases their demand for home money. If foreign residents have a negative net foreign asset position $(\overset{*}{b} < 0)$, an appreciation lowers their wealth. However, their demand for home money rises as their wealth falls.

In two perverse cases the asset market equilibrium schedule slopes upward. One case in which $b < \overset{*}{b}$ but normal valuation effects dominate any perverse valuation effects is shown in Panel B. The other case in which $b > \overset{*}{b}$ but perverse valuation effects associated with negative net foreign asset positions dominate any normal valuation effects $(b\overline{F} + \overset{**}{b}\overline{F} < 0)$ is shown in Panel C.

We investigate the stability of long-run equilibrium by analyzing an unanticipated transfer of wealth from foreign residents to home residents. This particular disturbance is convenient because, as has been demonstrated above, it leaves the long-run equilibrium values of the endogenous variables unchanged. Furthermore, recent commodity price increases have generated substantial interest in the effects of this kind of disturbance.

Stationary equilibrium is unambiguously stable under static expecations in the normal case. This result can be demonstrated with the use of Panel A. Suppose that there is a one-time wealth transfer to home residents from foreign residents, causing the initial value of w, w_0, to rise above $\overline{w}$. From equation (17) it follows that the direct effect of a positive dw is to cause $\dot{w}$ to become negative. The transfer also has an indirect effect on $\dot{w}$ through the change in the exchange rate required to clear the home money market. Since the home money market always clears, the initial value of the exchange rate must be the one implied by the A_S schedule when $w = w_0$. From equation (12) it follows that if $dw > 0$, the home currency must appreciate relative to its long-run equilibrium value $(de < 0)$ since the transfer increases the demand for home money. Thus, the indirect effect of the transfer through the induced appreciation of the home currency reinforces the direct effect in causing a negative $\dot{w}$. The economy proceeds along the A_S schedule until it reaches long-run equilibrium.

Long-run equilibrium is also definitely stable in one perverse case. Suppose that normal valuation effects dominate any perverse valuation effects ($b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} > 0$) but that foreigners hold a larger proportion of their wealth in home money than do home residents ($\overset{*}{b} > b$). This case in which net foreign asset holdings are "large" is analyzed using Panel B. As before, the direct effect of the transfer is to cause $\dot{w}$ to become negative. In contrast to the normal case, the home currency must depreciate ($de > 0$) since the transfer reduces the demand for home money. However, with $\overset{*}{b} > b$ a depreciation of the home currency reduces home savings, so once again the indirect effect of the transfer through the induced change in the exchange rate reinforces the direct effect in causing a negative $\dot{w}$. The "perverse" asset market effect is offset by the "perverse" effect of the exchange rate on home savings. Thus, the Enders (1977) problem of instability caused by large net foreign asset positions cannot arise in our model. The results for this case are intuitively appealing because the assumption that $\overset{*}{b} > b$ simply implies that the roles of home and foreign residents in the analysis are reversed; transferring wealth to home residents in this case is like transferring wealth to foreign residents in the normal case. Throughout the rest of this paper this case is ruled out by the assumption that $b > \overset{*}{b}$.

Stationary equilibrium is definitely unstable in the perverse case of primary concern here. Suppose that $b > \overset{*}{b}$ but that perverse valuation effects associated with negative net foreign asset positions of home residents or foreign residents, or both, dominate any normal valuation effects ($b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} < 0$).[13/] This case is shown in Panel C. Once again, the direct effect of the transfer is to cause $\dot{w}$ to become negative. Just as in the normal case, the transfer raises the demand for home money. However, in contrast to the normal case, a depreciation of the home currency is required to restore equilibrium in the home money market. Furthermore, the indirect effect of the transfer on home savings through the induced depreciation more than offsets the direct effect, so the net effect is a positive $\dot{w}$. In terms of Panel C, the A_S schedule is steeper than the $\dot{w}$ schedule.14/ The economy moves away from stationary equilibrium along A_S.

The stability properties of the model under static expectations can be summarized by setting $dB^c = 0$ in (12), solving (12) for de, and substituting the result into (17) to obtain15/

$$\dot{w} = -\psi[(\overline{B} + \overline{\overset{*}{B}})(\overline{F} + \overline{\overset{*}{F}})/(b\overline{F} + \overline{\overset{**}{bF}})]dw. \tag{20}$$

Stationary equilibrium is unstable ($d\dot{w}/dw > 0$) if and only if perverse valuation effects dominate any normal valuation effects ($b\overline{F} + \overline{\overset{**}{bF}} < 0$).

IV. THE MODEL WITH REGRESSIVE EXPECTATIONS

In this section the assumption that private agents have static expectations is replaced with the assumption that they have ad hoc regressive expectations:

$$\varepsilon = \theta(1 - E/\overline{E}),$$

where $\theta > 0$. Under regressive expectations it is more likely that a transfer of wealth to home residents will cause the home currency to appreciate initially and that stationary equilibrium will be stable.

Under regressive expectations there is a change in asset market equilibrium schedule. This schedule is again derived by setting $dB^c = 0$ in the deviation form of the home money market equilibrium condition (3) which becomes

$$-dB^c = [(1/\phi)\theta + (b\overline{F} + \overline{\overset{*}{b}\overset{*}{F}})]de + (b - \overset{*}{b})dw, \tag{21}$$

where

$$\phi = - 1/(b'\overline{W} + \overset{*}{b}'\overline{\overset{*}{W}}) > 0 \tag{22}$$

since b', $\overset{*}{b}' < 0$. Whatever its impact through valuation effects, a depreciation of the home currency relative to its long-run equilibrium level ($de > 0$) is more likely to increase demand for home money because of the expectations effect. A depreciation causes private agents to expect the home currency to appreciate and, therefore, to increase their demand for home money $[(1/\phi)\theta > 0]$. Consider the case represented in Panel C of Figure 1 ($b > \overset{*}{b}$ but $b\overline{F} + \overline{\overset{*}{b}\overset{*}{F}} < 0$). Under static expectations the asset market equilibrium schedule is upward-sloping and steeper than the $\dot{w}$ schedule. However, under regressive expectations if the expectations effect is strong enough $[(1/\phi)\theta + (b\overline{F} + \overline{\overset{*}{b}\overset{*}{F}}) > 0]$, the asset market equilibrium schedule represented by A_{RE} is downward-sloping.

Results obtained under static expectations may be reversed under regressive expectations. In the case of Panel C under static expectations a transfer of wealth to home residents causes the home currency to depreciate initially, and long-run equilibrium is unstable. However, when the effect of regressive expectations is strong enough that the A_{RE} schedule is downward-sloping, the home currency appreciates initially, and stationary equilibrium is stable.

The stability properties of the model under regressive expectations can be summarized by solving (21) for de and substituting the result into (17) to obtain[16/]

$$\dot{w} = -\psi\{[(\overline{B} + \overline{\overset{*}{B}})(\overline{F} + \overline{\overset{*}{F}}) + (1/\phi)\theta(\overline{W} + \overline{\overset{*}{W}})]/[(1/\phi)\theta + (b\overline{F} + \overset{**}{b}\overline{F})]\}dw. \quad (23)$$

If stationary equilibrium is stable under static expectations $(b\overline{F} + \overset{**}{b}\overline{F} > 0)$, it must also be stable under regressive expectations.[17/] However, even if stationary equilibrium is unstable under static expectations $(b\overline{F} + \overset{**}{b}\overline{F} < 0)$, it is stable under regressive expectations if the expectations effect is strong enough $[(1/\phi)\theta + (b\overline{F} + \overset{**}{b}\overline{F}) > 0]$.

V. THE MODEL UNDER RATIONAL EXPECTATIONS

In this section it is assumed that private agents have rational expectations so that $\varepsilon = \dot{e}$. Under rational expectations stationary equilibrium is always saddle-point stable. If speculation is stabilizing in a sense to be defined below, stationary equilibrium is definitely stable whether or not perverse valuation effects that arise from negative net foreign asset positions dominate any normal valuation effects. However, if speculation is destabilizing, stationary equilibrium is unstable. Thus, under rational expectations whether stationary equilibrium is stable depends on whether speculation is stabilizing.

Under rational expectations the approximation of the home money market equilibrium condition (3) becomes

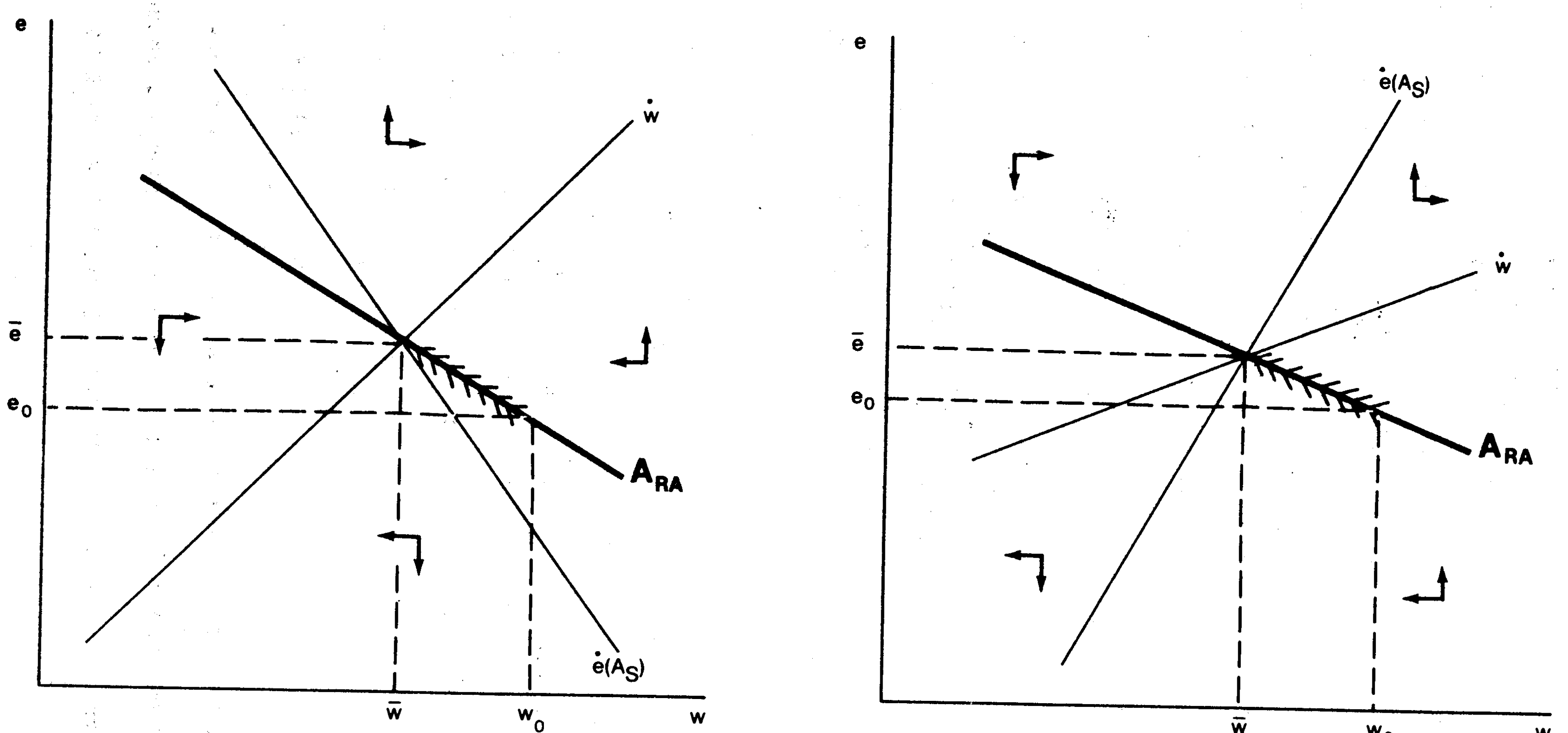
$$-dB^c = -(1/\phi)\dot{e} + (b\overline{F} + \overset{**}{b}\overline{F})de + (b - \overset{*}{b})dw. \tag{24}$$

Of course, it is impossible to obtain a unique relationship between w and e that is compatible with money market equilibrium without first solving for $\dot{e}$.

In order to determine $\dot{e}$, it is necessary to solve the system of two differential equations made up of equations (24) and (17). This system can be written in matrix form as

$$\begin{bmatrix} \dot{e} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \phi(b\overline{F} + \overset{**}{b}\overline{F}) & \phi(b - \overset{*}{b}) \\ \psi(b - \overset{*}{b})\overline{W}\overset{*}{\overline{W}} & -\psi(\overline{W} + \overset{*}{\overline{W}}) \end{bmatrix} \begin{bmatrix} de \\ dw \end{bmatrix} + \begin{bmatrix} \phi dB^c \\ 0 \end{bmatrix}. \tag{25}$$

Panel A of Figure 2 is a phase diagram for system (25). It is constructed under the assumptions that $b > \overset{*}{b}$ and $b\overline{F} + \overset{**}{b}\overline{F} > 0$. Under rational expectations the $\dot{w}$ schedule has the same interpretation as it has under static and regressive expectations. The $\dot{e}$ schedule shows the pairs of w and e that are compatible with home money market equilibrium when the rate of depreciation is zero ($\dot{e} = 0$). It coincides with the home money market equilibrium schedule under static expectations and is derived by setting $dB^c = \dot{e} = 0$ in (24). A value of e higher than the one on the $\dot{e}$ schedule for a given w leads to an excess demand for money, so $\dot{e}$ must be positive if the home money market is to be in equilibrium. Panel B of Figure 2 is a phase diagram for system (25) under the assumptions that $b > \overset{*}{b}$ and $b\overline{F} + \overset{**}{b}\overline{F} < 0$.



Panel A. Rational Expectations, Normal Case
$(b>\overset{*}{b},\ b\bar{F}+\overset{*}{b}\overset{*}{\bar{F}}>0)$

Panel B. Rational Expectations, Negative Net Foreign Asset Positions
$(b>\overset{*}{b},\ b\bar{F}+\overset{*}{b}\overset{*}{\bar{F}}>0)$

Figure 2. Wealth Transfer

As the arrows in Panel A and Panel B indicate, stationary equilibrium is a saddlepoint under rational expectations. Let A represent the matrix of coefficients of system (25). Since the determinant of A is negative,

$$\det A = -\psi\phi(\bar{B} + \overset{*}{\bar{B}})(\bar{F} + \overset{*}{\bar{F}}) < 0, \tag{26}$$

the two roots of the characteristic equation of system (25) are real, distinct, and of opposite sign, so stationary equilibrium is a saddlepoint. The negative and positive roots are given by λ_1 and λ_2 respectively:

$$\lambda_1 = C - D, \tag{27}$$

$$\lambda_2 = C + D, \tag{28}$$

where

$$C = \tfrac{1}{2}[\phi(b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}}) - \psi(\bar{W} + \overset{*}{\bar{W}})],$$

$$D = [C^2 + \phi\psi(\bar{B} + \overset{*}{\bar{B}})(\bar{F} + \overset{*}{\bar{F}})]^{\frac{1}{2}} = [H^2 + \phi\psi(b - \overset{*}{b})^2\bar{W}\overset{*}{\bar{W}}]^{\frac{1}{2}},$$

$$H = \tfrac{1}{2}[\phi(b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}}) + \psi(\bar{W} + \overset{*}{\bar{W}})].$$

The A_{RA} schedule in Panel A represents the stable arm of system (25) when $b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} > 0$, and the A_{RA} schedule in Panel B represents the stable arm when $b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} < 0$. The stable arm is the path the economy must follow after a disturbance if it is to reach stationary equilibrium. Let w_0 and e_0 represent the values of w and e immediately following a disturbance. Given w_0, stationary equilibrium is stable if and only if e_0 is chosen so that the solutions for e and w do not involve the positive root (λ_2), in which case

$$de_t = dw_0 J\exp(\lambda_1 t), \tag{29}$$

$$dw_t = dw_0 \exp(\lambda_1 t). \tag{30}$$

(Jk, k) is the characteristic vector corresponding to λ_1 where

$$J = \phi(b - \overset{*}{b})/[\lambda_1 - \phi(b\bar{F} + \overset{**}{b}\bar{F})], \tag{31}$$

and k is an arbitrary constant. The equation of the stable arm is the relationship between e and w implied by (29) and (30):

$$de = Jdw. \tag{32}$$

If $b > \overset{*}{b}$, the stable arm is always downward-sloping no matter what the sign of $b\bar{F} + \overset{**}{b}\bar{F}$ since

$$\lambda_1 - \phi(b\bar{F} + \overset{**}{b}\bar{F}) = -H - [H^2 + \psi\phi(b - \overset{*}{b})^2 \overline{W}\overset{*}{\overline{W}}]^{1/2} < 0. \tag{33}$$

The A_{RA} schedules in Panel A and Panel B can also be interpreted as the asset market equilibrium schedules under rational expectations when $\dot{e}$ is consistent with stability of long-run equilibrium. If the solution for e is given by (29), then

$$\dot{e} = \lambda_1 de \tag{34}$$

Substituting (34) into (24) with $dB^c = 0$ yields (32).

Now we define stabilizing and destabilizing speculation. Suppose that immediately after a transfer of wealth to home residents the value of w is w_0 in Panel A or Panel B. w can only adjust gradually over time through current account deficits or surpluses. However, under rational expectations the exchange rate "jumps" (that is, moves discretely at a point in time) to clear the home money market, just as it did under static and regressive expectations. If the bidding of market participants causes the exchange rate to jump to e_0, the exchange rate on the A_{RA} schedule corresponding to w_0, it will be said that speculation is stabilizing. When speculation is stabilizing the home currency appreciates, and stationary equilibrium is stable no matter what the sign of $b\overline{F} + \overset{*}{b}\overset{*}{\overline{F}}$. If the exchange rate remains unchanged at $\overline{e}$ or jumps to any value other than e_0, it will be said that speculation is destabilizing. When speculation is destabilizing, stationary equilibrium is unstable as indicated by the arrows in Panel A and Panel B. Under rational expectations instability can arise only because of destabilzing speculation and not because of perverse valuation effects associated with negative net foreign asset positions.

Destabilizing speculation is not ruled out by the assumptions made thus far. However, it is now conventional in rational expectations models to impose the additional assumption that speculation is stabilizing.[18] Assuming stabilizing speculation makes sense in our model since private agents in at least one country would probably want to avoid the situation that would result if the world economy followed any path other than the stable arm. It can be shown that along any path that began above e_0 there would come a time after which home real wealth would always be falling.[19] As home real wealth fell farther below target real wealth, home saving would increase. However, home consumption could never fall below zero.[20]

It can be shown that if home consumption reached zero in finite time, it would remain at this lower limit forever, and home real wealth would never again rise above the level it attained when consumption first reached its lower limit.[21] Thus, home residents would probably want to avoid paths that began above e_0. A similar line of argument leads to the conclusion that foreign residents would probably want to avoid paths that began below e_0.

VI. INTERVENTION OPERATIONS AND LEARNING

In this section we first analyze the effects of an unanticipated permanent intervention operation which private agents immediately recognize as being permanent. Then we sketch out what happens when private agents take time to discover that the intervention operation is permanent. This analysis indicates that if speculation is stabilizing, the model is definitely stable even if agents are not instantaneously fully informed.

Suppose that the world central bank makes an unanticipated offer to sell a given amount of home currency in exchange for foreign currency ($dB^c < 0$, $\overline{E}dF^c = - dB^c$).[22] Assume that private agents immediately recognize this intervention operation as being permanent. The response of the economy to this disturbance is shown in Figure 3. The pre-intervention $\dot{w}$, $\dot{e}$, and A_{RA} schedules (not shown) intersect at point a_0. If $b > \overset{*}{b}$, the post-intervention $\dot{w}$ and $\dot{e}$ schedules (not shown) and the post intervention A_{RA} schedule lie to the right of the pre-intervention schedules.

In the new long-run equilibrium, represented by point a_2, both e and w are higher. That e is higher has been established in Section II. It has also been shown there that the long-run values of both home and foreign real wealths are unchanged by an intervention operation. Since the ratio of real

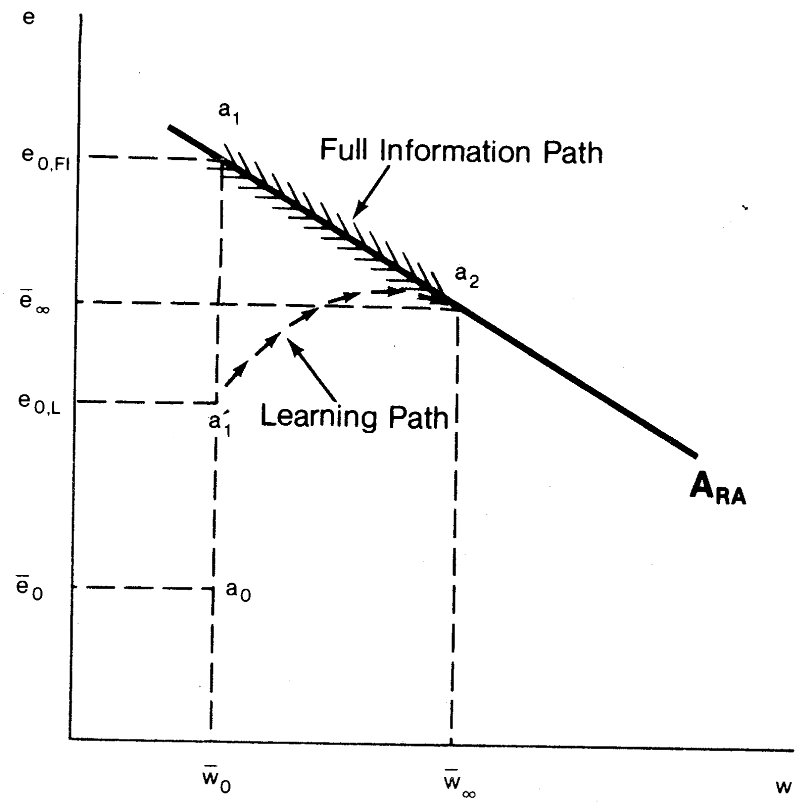


Figure 3. Permanent Intervention Operation, Rational Expectations, $(b > \overset{*}{b}, \, b\bar{F} + \overset{*}{b}\overset{*}{\bar{F}} \gtreqless 0)$

wealths remains constant, the ratio of nominal wealths must remain constant. If $b > \overset{*}{b}$, a rise in e increases home nominal wealth by a smaller proportion than it increases foreign nominal wealth. Thus, there must be a transfer of nominal wealth to home residents ($d\bar{w} > 0$), if nominal wealths in both countries are to rise by the same proportion.[23/]

At the time when the intervention operation occurs, the exchange rate depreciates to the level represented by point a_1. That is, it depreciates by more than the amount required to reestablish long-run equilibrium, so it must appreciate as the economy adjusts toward long-run equilibrium along the post-intervention A_{RA} schedule. Home residents' real wealth falls initially, and home residents restore their real wealth position by saving during the adjustment to long-run equilibrium.[24/]

Now suppose that agents live in a world of ongoing central bank intervention. Intervention operations may be permanent or transitory, and agents are able to distinguish between the two possibilities only through inference.[25/] The extent to which any intervention is regarded as permanent in such an environment depends on the relative variance of the two types of intervention operations.[26/] Even a truly permanent intervention operation, such as the one under consideration, is initially interpreted by private agents as being partly permanent and partly transitory. The A_{RA} schedule shifts up by an amount consistent with the part of the operation that private agents perceive as being permanent. Over time, agents will gradually realize that the whole operation was indeed permanent. The A_{RA} schedule will gradually shift upward as agents perceptions change. The dashed line in Figure 3 represents one type of path the economy might follow in the absence of further disturbances. Paths on which the exchange rate adjusts monotonically after its initial jump up are also possible both when a_1' lies below a_2 and when a_1' lies above a_2.

VII. SPECIAL CASES IN WHICH THE FOREIGN PRICE LEVEL IS EXOGENOUS

The most common portfolio balance model of an open economy is a one-country model in which the foreign price level is taken to be exogenous. In this section we show that such a model can be obtained as a special case of ours in two ways. One familiar way is to assume that the home country is "small" in the goods market but that it is "large" in the home money market. Another less familiar way is to assume that the foreign authorities use <u>fiscal</u> policy to peg the foreign currency price of goods.

As preparation for the investigation of these two special cases under rational expectations, it is useful to make a single modification in the model and to rewrite the model in a slightly different form. We assume that the foreign authorities engage in balanced-budget government spending and that foreign residents' desired wealth depends on their disposable income, so that their saving function becomes $\overset{*}{S}[\overset{*}{\alpha}(\tilde{\overset{*}{Y}} - \overset{*}{G}) - \overset{*}{W}/P]$. Dividing (34), the appropriately modified version of (13), and (14) by $\overline{W}$ and noting that $\overline{w} = \overline{W}$ and $d\overset{*}{p} = dp - de$ yields

$$-dB^c/\overline{W} = -(1/\hat{\phi})\dot{e} + [b(1 - b) + \overset{*}{b}(1 - \overset{*}{b})\overline{\overset{*}{W}}/\overline{W})]de + (b - \overset{*}{b})(dw/\overline{w}), \tag{35}$$

$$[S' + \overset{*}{S}'(\overline{\overset{*}{W}}/\overline{W})]d\overset{*}{p} + [S'b + \overset{*}{S}'\overset{*}{b}(\overline{\overset{*}{W}}/\overline{W})]de - (S' - \overset{*}{S}')(dw/\overline{w}) - \overset{*}{S}'(\overline{\overset{*}{W}}/\overline{W})\overset{*}{\alpha}(d\tilde{\overset{*}{G}}/\tilde{\overset{*}{W}}) = 0, \tag{36}$$

$$\dot{w}/\overline{w} = S'd\overset{*}{p} + S'bde - S'(dw/\overline{w}), \tag{37}$$

where

$$\hat{\phi} = -1/[b' + \overset{*}{b}'(\overline{\overset{*}{W}}/\overline{W})] > 0. \tag{38}$$

The state variable of the system is now the ratio of home wealth valued at the long-run equilibrium exchange rate to its long-run equilibrium value $(w/\bar{w})$.

Now suppose that the home country becomes small in the sense that the ratio of foreign wealth to home wealth rises without limit $(\bar{\overset{*}{W}}/\bar{W} \rightarrow \infty)$. Suppose also that the home country remains large in the home money market, equation (35), in the sense that as $\bar{\overset{*}{W}}/\bar{W}$ approaches infinity changes in $\dot{e}$ and e continue to lead to finite changes in the demand for home money $[\overset{*}{b}(\bar{\overset{*}{W}}/\bar{W}) \rightarrow m$, $\overset{*}{b}'(\bar{\overset{*}{W}}/\bar{W}) \rightarrow n$, where m and n are finite constants].[27] It follows that the home country becomes small in the goods market, equation (36), in the sense that $\overset{*}{p}$ is unaffected by changes in e and $w/\bar{w}$. Under the assumptions stated above, the coefficients on $d\overset{*}{p}$ and $d\overset{*}{\tilde{G}}/\bar{\overset{*}{W}}$ rise without limit while the coefficients on de and $dw/\bar{w}$ remain finite. Thus $\overset{*}{p}$ cannot change when e and $w/\bar{w}$ change and, therefore, can be taken to be exogenous. With $d\overset{*}{p}$ set equal to zero, equation (35) and equation (37) taken together constitute a version of the conventional small-country portfolio balance model. Setting $dB^c = 0$ in equation (35) yields the equation of the $\dot{e}$ schedule for this special case. Setting $\dot{w} = 0$ in equation (37) yields the equation of the $\dot{w}$ schedule. Of course, since this version of the small-country model is a special case of our two-country model, it is saddlepoint stable.

Now we consider the less familiar special case in which the foreign country is of roughly the same size as the home country $(\bar{\overset{*}{W}} \approx \bar{W})$ but in which the foreign authorities use fiscal policy to peg the foreign currency price of goods. Since $\bar{\overset{*}{W}}/\bar{W}$ is finite, the home country remains large in the home money market when $\overset{*}{b}$ and $\overset{*}{b}'$ are finite. If the foreign authorities vary $\tilde{\overset{*}{G}}/\overline{\overset{*}{W}}$ over time to exactly offset the effects of changes in e and $w/\overline{w}$ in the goods market, $\overset{*}{p}$ is unaffected by these changes and, therefore, can be taken to be exogenous. What is somewhat surprising is that continuous variation in foreign fiscal policy rather than foreign monetary policy is required to keep $\overset{*}{p}$ constant.[28/] Given the fiscal actions by the foreign authorities required to keep $d\overset{*}{p}$ equal to zero, equation (35) and equation (37) taken together constitute a saddlepoint-stable, one-country portfolio balance model. The qualitative properties of that model are indistinguishable from those of the small-country model described above.

VIII. CONCLUSIONS

In this paper a two-country portfolio balance model has been developed. This symmetrically-specified model has been employed to provide some perspective on two stability problems associated with portfolio composition: the problem arising from negative net foreign asset positions and the problem that some have suggested might arise from large and positive net foreign asset positions. The first of these problems has been a source of serious concern in both theoretical and empirical work on portfolio balance models. Using the rational expectations version of our model we have shown that portfolio composition does not constitute an independent source of instability in a floating exchange rate regime.

There is a strong connection between the instability problem associated with negative net foreign asset positions and a type of transfer problem. Our model is dynamically unstable if and only if a transfer of nominal wealth from foreigners to home residents leads to exchange rate and price level adjustments that cause home residents real wealth to end up lower than before the transfer. The joint assumptions of rational expectations and stabilizing speculation rule out this possibility. This result suggests that there is a type of transfer problem that arises only because of non-rational expectations or destabilizing speculation.

The stability analysis has yielded some useful by-products. For example, it has been shown that with rational expectations and stabilizing speculation, both the short-run and long-run qualitative effects of wealth transfers and intervention operations are the same with or without negative net foreign asset positions. Thus, there is no presumption that a purchase of Canadian dollars by the Canadian authorities will lead to a depreciation of the Canadian dollar just because Canadian private citizens have a negative net U.S. dollar asset position, even though such a depreciation is a possibility in a static expectations model.

The model presented here is of interest in its own right. It is a framework for analyzing the important case in which residents of both countries hold assets denominated in both currencies. Furthermore, as one special case it yields a version of the popular class of models in which a country is small in the goods market but large in the market for its own money.

FOOTNOTES

*/ Economists, Board of Governors of the Federal Reserve System. This paper is a revised version of a paper presented at the Meetings of the Econometric Society in September 1980, Cornell University, and Rice University. The authors have benefitted from discussions with Matthew Canzoneri, John Cuddington, Mark Gertler, Michael Mussa, Douglas Waldo, and Henry Wan. No one but the authors is responsible for the remaining shortcomings of the paper. The paper represents the views of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

1/ The instability problem associated with negative net foreign asset positions is a central issue in several recent papers: Branson, Halttunen, and Masson (1979), Martin and Masson (1979), Boyer (1977), and Obstfeld (1980). It is also discussed by Tobin and de Macedo (1981). Tobin (1980) summarizes a main conclusion reached in these papers:

> "Self-propelled, and at least temporarily self-justifying, speculation is not the only source of possible instabilities in the macroeconomic mechanism of national economies with distinct currencies, linked by trade and financial transactions. The wealth effects of exchange rate adjustments are stabilizing when countries have long positions in assets denominated in other currencies, but can be destabilizing when they have foreign currency debts."

In an appendix Kouri (1978) considers the implications of negative net foreign asset positions under rational expectations. Although Kouri's model is different from the one presented here, his conclusions can be reconciled with ours. Masson (1980) analyzes a model similar to Kouri's but reaches quite different conclusions.

2/ Data on the German case is readily available from the statistical supplements to the Monthly Report of the Deutsche Bundesbank. In particular, one can use the supplement Balance of Payments Statistics. Tables 7 and 8 of that publication contain data on both the domestic and foreign currency assets and liabilities of German banks and firms. At the end of 1979, foreigners had a net DM-denominated debt with respect to German banks and non-bank domestic enterprises of 50 billion DM.

3/ The usual formulation would also result if it were assumed that foreigners did hold domestic assets but that their holdings were totally unresponsive to changes in their wealth and relative rates of return.

4/ Kouri (1976), Calvo and Rodriguez (1977), and Flood (1979) analyze small-country portfolio balance models.

5/ That is,

$$E(\tilde{F} - F^c) = f(\varepsilon)W + \overset{*}{f}(\varepsilon)\overset{*}{W},$$

where $f(\varepsilon) \equiv 1 - b(\varepsilon)$ and $\overset{*}{f}(\varepsilon) \equiv 1 - \overset{*}{b}(\varepsilon)$ are the proportions of their wealths that home and foreign residents respectively desire to hold in foreign money. Adding this equation to (3) yields an identify since $W + \overset{*}{W} = \tilde{B} - B^c + E(\tilde{F} - F^c)$. This last equation can be obtained by adding (1a) and (1b), adding (2a) and (2b), and noting that the right hand sides of the resulting sums are equal.

6/ Since $\overset{*}{P}$ can be determined once P and E are known, $\overset{*}{P}$ and equation (4) are not referred to again except in Section VII.

7/ Our savings functions are identical to the ones used by Calvo and Rodriguez (1977), Dornbusch and Fischer (1980), and Henderson (1980).

8/ That is,

$$\dot{\overset{*}{B}} + E\dot{\overset{*}{F}} = P\overset{*}{S}(\alpha\tilde{\overset{*}{Y}} - \overset{*}{W}/P).$$

To confirm that $\dot{\overset{*}{B}} + E\dot{\overset{*}{F}} = -(\dot{B} + E\dot{F})$ add the time derivatives of (2a) and (2b), cancel the terms involving $\dot{E}$, and note that $\dot{\tilde{B}} = \dot{B}^c = \dot{\tilde{F}} = \dot{F}^c = 0$.

9/ See footnote 5.

10/ There is sometimes confusion about the exchange rate at which the central bank conducts its trade. The reason why it is permissible to view the trade as taking place at the initial long-run equilibrium exchange rate is presented in footnote 22.

11/ As shown by (14) the transfer causes home savings to decline directly. It also has an indirect effect on home savings though its effect on the price level required to clear the market for the world good. As is evident from (13) there are two cases to consider. If $S' < \overset{*}{S}'$ so that the decrease in home savings is smaller than the increase in foreign savings, then the price level must decline in order to maintain equilibrium in the market for the world good. As shown by (14), this decline further reduces home savings. If $S' > \overset{*}{S}'$, then the price level must rise. However, this price level rise is not large enough to raise savings back up to their pre-transfer level. The excess demand in the market for the world good caused by the transfer is smaller than the decrease in home savings because foreign savings increase. Price level increases work to eliminate this excess demand not only by raising home savings but also by raising foreign savings.

12/ There are three cases to consider. First suppose $\overline{F} > 0$ and $S'\overline{F} + \overset{*}{S}'\overline{\overset{*}{F}} > 0$. Then as shown by (14) a depreciation causes home savings to decline directly. However, since the depreciation also lowers foreign savings, the price level must rise in order to maintain equilibrium in the market for the world good as shown by (13). The increase in home savings induced indirectly by this rise in the price level more than offsets the direct decline. If the price level rose only by enough to keep home savings unchanged [$dp = (\overline{F}/\overline{W})de > 0$], there would still be an excess demand for the world good. Foreign saving would have fallen $\{[\overline{\overset{*}{W}}(\overline{F}/\overline{W}) - \overline{\overset{*}{F}}]de < 0\}$ because the proportion of foreign money in foreign portfolios $(\overline{\overset{*}{F}}/\overline{\overset{*}{W}} = 1 - \overset{*}{b})$ is larger than the proportion of foreign money in domestic portfolios $(\overline{F}/\overline{W} = 1 - b)$ when $b > \overset{*}{b}$. Second, suppose $\overline{F} < 0$ and $S'\overline{F} + \overset{*}{S}'\overline{\overset{*}{F}} > 0$. Then the depreciation causes home savings to rise both because of its direct effect and because of its indirect effect through the induced rise in the price level. Third, suppose $\overline{F} < 0$ and $S'\overline{F} + \overset{*}{S}'\overline{\overset{*}{F}} < 0$. Then the depreciation causes home savings to rise directly. In this case the price level must fall. However, the positive direct effect of the depreciation on home savings dominates the negative indirect effect of the induced price level decline. If the price level declined by enough to keep home savings unchanged, there would be an excess demand for the world good since foreign savings would have fallen.

13/ As shown below, if $b\overline{F} + \overset{*}{b}\overline{\overset{*}{F}} < 0$, the model is also definitely unstable when $\overset{*}{b} > b$.

14/ The slope of the A_S schedule minus the slope of the $\dot{w}$ schedule is equal to $-V/Z$ where

$$V = (b - \overset{*}{b})^2\overline{W}\overline{\overset{*}{W}} + (b\overline{F} + \overset{*}{b}\overline{\overset{*}{F}})(\overline{W} + \overline{\overset{*}{W}}),$$

$$Z = (b - \overset{*}{b})\overline{W}\overline{\overset{*}{W}}(b\overline{F} + \overset{*}{b}\overline{\overset{*}{F}})(\overline{W} + \overline{\overset{*}{W}}).$$

V is positive:

$$V = (b^2 - 2b\overset{*}{b} + \overset{*}{b}^2)\overline{W}\overline{\overset{*}{W}} + b(1 - b)(\overline{W}^2 + \overline{W}\overline{\overset{*}{W}}) + \overset{*}{b}(1 - \overset{*}{b})(\overline{W}\overline{\overset{*}{W}} + \overline{\overset{*}{W}}^2),$$

$$= b(1 - \overset{*}{b})\overline{W}\overline{\overset{*}{W}} + \overset{*}{b}(1 - b)\overline{W}\overline{\overset{*}{W}} + b(1 - b)\overline{W}^2 + \overset{*}{b}(1 - \overset{*}{b})\overline{\overset{*}{W}}^2,$$

$$= (\overline{B} + \overline{\overset{*}{B}})(\overline{F} + \overline{\overset{*}{F}}) > 0.$$

Therefore, sgn $(-V/Z) = -\,\text{sgn}(Z)$. If $b - \overset{*}{b} > 0$ and $b\overline{F} + \overset{**}{b\overline{F}} < 0$ so that both the A_S and w schedules have positive slopes as in Panel C, then $Z < 0$, so the A_S schedule is steeper.

15/ The numerator of the fraction in brackets in (20) is equal to the V of footnote 14.

16/ The first term in the numerator of the fraction in brackets in (23) is equal to the V of footnote 14.

17/ However, regressive expectations can cause an unstable positive root to become more positive. If $(1/\phi)\theta + (b\overline{F} + \overset{**}{b\overline{F}}) < 0$ so that the root of (23) is positive, increases in $(1/\phi)\theta$ make the root more positive.

18/ Sargent (1973) argues that it is sensible to impose this "no speculative bubbles" assumption. Kouri (1976) and Calvo and Rodriguez (1977) make it in models of the same general type as ours. Brock (1975) and Gray (1979) show that in models based on explicit utility maximization unstable paths can often be ruled out on the grounds that they are suboptimal.

19/ Proofs of the correctness of this assertion and the one made later in this paragraph are available from the authors on request.

20/ The lower limit on home consumption might be a subsistence level above zero.

21/ The discussion in the text is based on the assumption that home consumption would reach its lower limit when real wealth was still positive. This assumption is required because, as it stands, our model makes sense only when real wealth in both countries is positive. However, the model could be modified to make possible consideration of negative values of real wealth for one country or the other. In the modified model it would be possible to consider paths along which home consumption approached its lower bound assymptotically and home real wealth declined without limit.

22/ Of course, asset markets clear at the exchange rate that prevails immediately after the intervention operation, E_0, and the central bank also consummates its transaction at that rate. However,

$$dB^c + E_0dF^c = dB^c + dE_0dF^c + \overline{E}dF^c \approx dB^c + \overline{E}dF^c$$

where $\overline{E}$ is the initial long-run equilibrium exchange rate because the term dE_0dF^c, where $dE_0 = E_0 - \overline{E}$, is of the second order of smalls.

23/ The requirement that the ratio of real wealths remain constant implies

$$d(\overline{\omega}/\overline{\overset{*}{\omega}}) = [(\overline{W} + \overline{\overset{*}{W}})/\overline{\overset{*}{W}}]d\overline{w} + (\overline{W}/\overline{\overset{*}{W}})(\overset{*}{b} - b)d\overline{e} = 0.$$

If $b > \overset{*}{b}$, $d\overline{w} > 0$ since $d\overline{e} > 0$.

24/ For proof that a depreciation of the home currency is associated with an increase in home savings see footnote 12.

25/ Private agents would behave in an analagous way in the face of a disturbance to exogenous real income which they think might be either permanent or temporary.

26/ This section draws on results which have been derived when learning behavior is modeled as a Kalman filter problem. An early application of this approach in an open economy context is Mussa (1975). When learning behavior is modeled in this way, the economy reaches an ergodic state where the endogenous variables fluctuate around the full information path. While the short-run dynamics are quite different in the learning case, the long-run dynamic stability of the economy is unaffected.

27/ Usually is it assumed either that foreign residents do not hold the home money or that their holdings are invariant to changes in their wealth and changes in the expected depreciation of the home currency ($\overset{*}{b} = \overset{*}{b}' = 0$). Either of these conventional assumptions is sufficient but not necessary to insure that the home country remains large in the home money market. Under either conventional assumption the usual state variable F can be used instead of $w/\overline{w}$ since all changes in home residents' wealth must take the form of changes in their holdings of foreign currency.

28/ The foreign authorities could also use helicopter drops (and pickups) of the two moneys in order to adjust foreign nominal wealth so that foreign saving was always the negative of home savings at a fixed $\overset{*}{p}$. The foreign authorities would have to deliver both home and foreign money to their residents both as stock increases at the time of a disturbance and as flow increases during the ensuing adjustment period. Otherwise the exogenous increases in foreign nominal wealth would disturb home money market equilibrium. Only if $\overset{*}{b}$ were zero could they deliver only foreign money. Whether such a balanced-proportions helicopter drop policy should be regarded as fiscal policy or monetary policy may be a question of semantics. It is certainly distinct from the exchange of moneys discussed in Section VI. Among the policy actions possible in our model, this exchange of moneys is the action that most closely approximates the open market operations usually associated with monetary policy.

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