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GOLD MONETIZATION AND GOLD DISCIPLINE

by

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Gold and its price have emerged as frequent topics of both academic and government debate.[1/] Public attention focused on the gold market when gold's price surged briefly to $850/ounce by January, 1980; by July, 1981, the price collapsed to less than $400/ounce.

These price fluctuations have been coincident with the advent of three lines of thought concerning government gold market policy. First, Salant and Henderson (1978) and Salant (1981) have developed a partial equilibrium gold market model to analyze the interactions between rapid gold price rises, government gold auctions, and speculative attacks on government gold stocks. Second, Barro (1979) has constructed a model to study money and price dynamics under a version of the historical gold standard. Finally, other economists, led by Laffer (1979) and Lehrman (1980), (1981), have proposed a new monetary gold standard under which the government would maintain a fixed nominal gold price while retaining the freedom to issue money without fixed gold backing. Such a gold standard diverges from the historical gold standard because it does not require a fixed fractional gold backing of each currency unit issued. Rather, it bears a conceputal resemblence to the post-war fixed exchange rate regime: governments, which had some autonomy in conducting monetary policy, were constrained by the commitment to hold sufficient reserves to maintain fixed exchange rates.

The purpose of the present paper is to provide a framework suitable for analyzing various gold monetization policies. We integrate the work of Salant and Henderson with that of Barro; and, as an example of the usefulness of the conceptual framework, we analyze a transition to a gold standard of the Laffer-Lehrman type. More generally, the model developed here can be used to study a transition to any monetary system based on a storable commodity. In addition to analyzing a successful transition, we study the possibility of an unsuccessful transition to a gold standard, i.e. a government's implementation of a gold standard destined to collapse either immediately or after a finite interval following implementation. Our results on the collapse of a gold standard draw from the work of Salant and Henderson (S-H hereafter) and that of Krugman (1979).

We organize the paper in four sections. In Section I we present an informal discussion of what constitutes a complete policy to monetize gold. In section II we introduce the basic components of our model and study the paths of the relative gold price and the price level prior to a government's announcement that gold's nominal price will be fixed in the future. In section III we analyze the price and quantity effects of a current announcement to fix gold's price at a particular level in the future. In section IV we study policies which are inconsistent with maintaining a permanently fixed price of gold. Section IV is followed by some concluding remarks.

I. Fully Specified Monetization Policies and the Nature of Gold Monetization

In this section we will present an informal discussion of the nature of the gold monetization policies and the behavior of the public which we study in the rest of the paper. The aim here is to clarify the direction which our study follows by specifying explicitly a gold monetization policy and the monetary nature of gold. Since we abstract away from a banking system throughout the paper, money will be either government issued paper currency or, when gold is monetized, gold coins or bars held in private portfolios.

a. Completely Specified Monetization Policies

Recent proposals for government intervention in the gold market have suggested that the government should announce presently its intention to fix the nominal price of gold at the market price prevailing on a fixed future date.[2/] Since merely this announcement would not produce a deterministic future market price in the absence of an explicit monetary policy, we must expand on some of the proposals, which are incomplete as presently specified.[3/]

A complete monetization proposal is a specification from which the time of gold price fixing and the paths of gold's relative price, the nominal price level, and the stock of money can all be determined uniquely. Some of these components may be specified directly as exogenous policy variables; however, the others will be determined endogenously through the interaction among the exogenous variables, the structure of supply and demand in the gold and money markets, and expectations. Those components which are selected as exogenous will determine the nature of the analysis.

There are a number of ways to specify a complete policy. First, the government may designate the time for fixing the nominal gold price and the paths of its currency issue and of its gold reserve holdings. Such a specification will imply paths for the nominal price level and the relative price of gold, particularly the market value of gold at the time of fixing.[4/] Second, the government may specify the paths of its currency issue and gold stock holding together with the nominal gold price at which fixing will occur, given that the market price attains this level. The problem is then to solve for the time at which fixing takes place.[5/] Finally, the government may set the time of fixing and the price at which fixing will occur. Then we must seek the implied paths for the government's currency issue and gold stock holdings.[6/] All of these approaches are equivalent sets of policies since there is a unique relationship between price paths, money stock paths and the timing of gold price fixing.

Since the final approach produces solutions which are relatively easy to analyze, we will employ it in studying the monetization problem in the next two sections. Ostensibly, this approach appears different from the policy recommendations of those who wish to monetize gold by fixing its price at some market-determined level on some future date. However, since the goal of such a policy is not to monetize gold *per se* but to stabilize the nominal price level, the actual policy target is a given path of prices. Since merely fixing the gold price at some market determined level will not, in general, produce price level stability and may, indeed, be quite inflationary, it is reasonable to study gold monetization in the context of a policy specifying desired paths for the nominal price level. In this way we can readily determine a policy for currency and reserve gold holding which is consistent with price level stability. The choice of an appropriate price at which to fix gold depends on the goal the government is attempting to attain. Later in the paper we will provide calculations of the gold price which will attain nominal price level stability.

b. The Effect of Gold Monetization on the Money Stock

In this section we wish to discuss the changes in the money stock which will occur because of the nature of gold monetization. The policy of fixing the nominal price of gold monetizes gold because of its effect on the market for currency. Temporarily, we will ignore the aspect of the policy which pushes gold price fixing into the future and consider the effects of a permanently successful policy to fix gold's price immediately. The background for our discussions consists of an environment in which gold may be instantaneously exchanged between government reserves and private portfolio holding.

If a policy to fix gold's price is permanently successful, then either currency will dominate gold in private portfolios or portfolio gold and currency will be perfect substitutes. The first case hinges on currency's having a greater convenience than portfolio gold in making transactions. Since the two assets would be permanently fixed in price relative to one another, any purely speculative demand could be equally satisfied by currency balances. Since currency will therefore dominate portfolio gold after gold's price is fixed, agents will exchange all of their portfolio gold for currency at the fixed gold price. In the absence of sterilizing bond sales, the supply of currency will rise by the value of portfolio gold holdings, thus monetizing existing gold hoards.

Alternatively, if agents perceive that portfolio gold and currency are perfect substitutes, then both the amount of gold that the government must exchange for currency and the ratio of portfolio gold to currency held by the public are indeterminate. However, this indeterminacy is inconsequential; to the extent that portfolio gold is held after price fixing, it exactly fulfills agents' demands for money. Thus, the increase in both the supply of and the demand for nominal money at the time of gold's price fixing is identical in both cases.

In summary, when the government fixes the price of gold, the private sector monetizes portfolio gold by cashing in existing speculative hoards, by allowing hoards to fulfill money demands directly, or by a combination of the two. The nature of the government's commitment to exchange gold for currency at a fixed price produces an indeterminacy in the amount of gold exchanged between the government and the public.

So far the discussion has neglected some important aspects of gold monetization. First, the policies which we will study prescribe fixing gold's price in the future. How does a current announcement to fix gold's future nominal price immediately affect the gold market and the nominal price level? Second, we will consider policies which involve announcing currently the future fixed price for gold. How can we calculate the future gold price compatable with price level stability? Third, gold consumption imposes specific dynamics onto the gold market. Once gold is monetized, those dynamics become part of the money market. What are the money market implications of gold's dynamics? Fourth, our discussion above assumes that the policy to fix gold's price will be permanently successful. However, it is possible that the government may pursue policies inconsistent with permanently fixing gold's price. What is the nature of these policies and how will agents behave when they recognize that the government is unable to fix gold's price permanently? To study these questions we must turn to a formal analysis.

II. The World Prior to Gold Monetization

To obtain results concerning the effects of an announcement to fix gold's price we must first characterize the world prior to the announcement. The world we will study is very similar to that of S - H; however, there will be no government gold auctions, and we will modify their model in some aspects to suit our problem. Throughout the paper we employ a continuous-time model in which agents have perfect foresight. To minimize technical complexity, we present our ideas in the context of a linearized example.

We divide our economy into two sectors, a gold sector and a monetary sector. The real rate of interest, ρ, is fixed at a constant level exogenously to the gold and monetary sectors.[7/]

a. The Gold Sector

The operation of the gold market is described in equations (1) - (3):

$$I = D(t) + G(t) + R(t) \tag{1}$$

$$\dot{D}(t) = v[\overset{*}{D}(t) - D(t)],\ v > 0 \tag{2}$$

$$\dot{q}(t) = \rho q(t) \quad \text{for} \quad G(t) > 0. \tag{3}$$

I is a fixed total world stock of gold. D(t) is the quantity of gold at time t which has been put into consumption and industrial uses. G(t) is the quantity of gold held privately as ingots or coins in speculative hoards, and R(t) is the quantity of gold in government reserves. Until gold is monetized, we will assume R(t) to be the constant $\overline{R}$.[8/]

Equation (2) describes the law of motion for the total stock of gold in consumption and industrial use. Thus $\dot{D}(t)$ is equal to current consumption and industrial gold purchases. When $\dot{D}(t)$ is positive, speculative hoards are reduced to accomodate the flow of gold into consumption and industrial uses. $\overset{*}{D}(t)$ is a desired stock of gold used in industry and consumption. We assume[9/]

$$\overset{*}{D}(t) = \frac{\delta_0 + \delta_1[\dot{q}(t)/q(t)]}{q(t)}, \ \delta_0 > 0, \ \delta_1 > 0, \tag{4}$$

where q(t) is the relative price of gold in terms of other goods at time t. Hence, $\overset{*}{D}(t)$ depends positively on the rate of return to gold hoarding, $\dot{q}(t)/q(t)$, and negatively on the relative price of gold. Equation (2) is similar in form to a gold sector equation set up by Barro (1979), and the dependence of $\overset{*}{D}(t)$ on the real rate of return to gold is also consistent with Barro's model.

Equation (3) is the condition that gold's relative price rise at the real rate of interest while gold is held in speculative hoards. The intuition behind this condition is that speculators price gold presently such that its relative price can be expected to rise at the rate ρ. If relative price were expected to rise faster than ρ, then the current price would be bid up until $\dot{q}(t)/q(t)$ falls to ρ. If the price were expected to rise more slowly than ρ, then the current price would fall until $\dot{q}(t)/q(t) = \rho$.[10/]

While our gold market model is quite similar to that of S - H, it differs in some important aspects. S - H interpret the variable which, in the context of their paper, is analogous to our $\dot{D}(t)$ as a gold demand which depends only on the relative gold price. Since this gold demand always must be non-negative in their paper, S - H effectively treat the flow demand for gold as a demand for a commodity which disappears forever after use.

However, since the demand for gold stems from a demand for a continuing service flow, we include two elements in the $\dot{D}(t)$ equation which are not present in S - H.[11/] First, we assume that the flow demand for gold depends negatively on D(t), the existing gold stock already used in consumption, and industrial forms. This reflects diminishing marginal utility and productivity associated with additional consumption and industrial gold uses, respectively. Second, we assume that the flow demand for gold depends positively on the expected rate of change of gold's relative price since consumption and industrial demand for gold increase when expected capital gains rise.

Equations (1) - (4) form a system of differential equations in $D(t)$ and $q(t)$. In the remainder of this subsection we solve these equations for $D(t)$ and $q(t)$. Equation (3) yields

$$q(t) = q(0)e^{\rho t}, \quad \text{when } G(t) > 0. \tag{5}$$

Substituting from (3), (4), and (5) in (2), we obtain

$$\dot{D}(t) = -vD(t) + \frac{v(\delta_0 + \delta_1\rho)}{q(0)}e^{-\rho t}, \quad \text{when } G(t) > 0. \tag{6}$$

Equations (6) has the solution

$$D(t) = D(0)e^{-vt} + ve^{-vt}\int_0^t \frac{(\delta_0 + \delta_1\rho)}{q(0)}e^{(v-\rho)\tau}d\tau. \tag{7}$$

To complete our solution we require an initial condition for $D(0)$ and a condition on speculative gold holding to determine $q(0)$. First, we assume initially $D(0) = 0$.12/ Second, if T is the date when $\dot{D}(t) = 0$, then we must have $G(T) = 0$. Since hoards of speculative gold are held only in anticipation of future consumption and industrial gold uses, the portfolio gold stock must be exhausted when additional gold use in these areas ceases. Since $G(T) = 0$ and $\dot{D}(T) = 0$, equations (1) and (2) imply

$$I - \overline{R} = D(T) = \overset{*}{D}(T) \tag{8}$$

When $G(t) = 0$, as at $t = T$, gold is no longer held in speculative hoards; therefore, its price need not rise at the percentage rate ρ. In particular, $\dot{q}(T_+) = 0$, where T_+ indicates that the time derivatives is a right-handed one. With $\dot{q}(T_+) = 0$, we find from (4) that

$$\overset{*}{D}(T) = \frac{\delta_0}{q(T)} \tag{9}$$

Combining (9) with (8) yields

$$q(T) = \frac{\delta_0}{I - \overline{R}} \tag{10}$$

Equation (10) indicates the "choke price" for new consumption and industrial gold use, which is the terminal condition required to solve for $q(0)$.[13/] To find $q(0)$, we set

$$\frac{\delta_0}{I - \overline{R}} = q(0)e^{\rho T}, \tag{11}$$

or, solving for $q(0)$,

$$q(0) = \frac{\delta_0}{I - \overline{R}} e^{-\rho T}. \tag{12}$$

Our solution for the gold sector will be complete once we have determined the unknown "choke date", T. To find T we use (12), the condition that $D(0) = 0$, and the condition $D(T) = I - \overline{R}$ in (7) to obtain

$$D(T) = I - \overline{R} = ve^{-vT} \int_0^T \frac{(\delta_0 + \delta_1\rho)(I - \overline{R})e^{(v-\rho)\tau}}{\delta_0 e^{-\rho T}} d\tau. \tag{13}$$

Equation (13) may be solved for T, and yields

$$T = \frac{\log \left[\dfrac{\rho + (v\delta_1\rho/\delta_0)}{v + (v\delta_1\rho/\delta_0)}\right]}{\rho - v}, \quad \rho \neq v \tag{14}$$

We find the value of q(0) by substituting from (14) for T into equation (13). Thus,

$$q(0) = \frac{\delta_0}{I - \bar{R}} e^{\frac{-\rho \log\left[\frac{\rho + (v\delta_1\rho/\delta_0)}{v + (v\delta_1\rho/\delta_0)}\right]}{\rho - v}} \tag{15}$$

Equation (15) completes our solution of the gold sector. With q(0) determined by (15), equation (5) indicates the time path of q(t) from 0 to T; and q(t) = q(T) for $t \geq T$. Further, given the initial condition D(0) = 0, equation (7) determines the time path of D(t) from 0 to T with $D(t) = D(T) = I - \bar{R}$ for $t \geq T$. Thus, the gold sector alone determines gold's relative price path and the quantity of gold held in purely speculative hoards. To find the nominal price of gold, however, we must append a monetary sector to our model.

b. The Monetary Sector

The operation of the monetary sector has no real effects in the model prior to the monetization of gold. We introduce the monetary sector now however as a preliminary to the analysis of section III.

We assume that money market equilibrium is given by equations (16) and (17)

$$\frac{M(t)}{P(t)} = \beta - \alpha i(t) \tag{16}$$

$$i(t) = \rho + [\dot{P}(t)/P(t)], \tag{17}$$

where M(t) is the nominal quantity of money, P(t) is the aggregate price level. i(t) is the nominal rate of interest and $[\dot{P}(t)/P(t)]$ is the expected and actual rate of inflation. We view (16) as a linearization which is appropriate for $\beta - \alpha i(t) > 0$.

In this section we set $M(t) = \overline{C}$, where $\overline{C}$ is the constant quantity of currency in circulation. Since $M(t)$ is constant at $\overline{C}$, the price-level solution to (16) and (17) which is consistent with market fundamentals[14/] is that $P(t)$ be constant with

$$P(t) = \frac{\overline{C}}{\beta - \alpha\rho}.$$

We define the nominal price of gold as $Q(t)$, so $q(t) \equiv Q(t)/P(t)$.[15/] The path of the nominal price of gold in our model prior to the announcement of gold monetization is $Q(t) = q(t)P(t)$, with $q(t)$ given in the previous part of this section.

The model of this section is devoid of interesting interactions between the gold and monetary sectors. However, such interactions are introduced by a government announcement of gold price fixing, the topic of the next section.

III. The World After Gold is Monetized

In this section we will study the price-level, monetary, and gold-market implications of a government announcement to fix the price of gold. The plan of our presentation is to analyze first a world where gold's price is already fixed and then to investigate the transition from the world with a freely floating gold price to the world of a fixed gold price.

A. The Dynamics of the Gold Standard

The key element which integrates the monetary and gold sectors after monetization is the money supply. Once gold's price is fixed the money supply becomes

$$M(t) = \overline{C} + \overline{Q}[R(t) - \overline{R}] + \overline{Q}G(t), \tag{19}$$

where $\overline{C}$ is the fixed stock of currency which was outstanding at the time of monetization, $\overline{Q}$ is the fixed nominal price of gold, R(t) is the level of government gold reserves, $\overline{R}$ is government gold reserves at the time of price fixing, and G(t) is the quantity of gold used directly as money in items such as coins. The term $\overline{Q}[R(t) - \overline{R}]$ is the value of government gold purchases since the time of fixing. Gold purchases are open market operations which alter the quantity of money.

We expect G(t) to be quite small after monetization because gold will no longer earn the real rate of return. Gold's price will be fixed in terms of currency and gold's rate of return will be identical to that of currency. Thus, since currency is easier to store and transport than is gold, we expect that speculative gold hoards will be exchanged for currency. To the extent that they are not all cashed in, those hoards which remain must be, to agents, a perfect substitute for currency and thus part of the money supply. Since we cannot determine the split of monetary gold between government reserves and private holding it is useful to define the variable.

$$X(t) \equiv R(t) + G(t). \tag{20}$$

Our model will allow us to keep track of X(t) but not its components.

Given the definition in (20), we rewrite the money supply equation as

$$M(t) = \overline{C} - \overline{QR} + \overline{Q}X(t). \tag{21}$$

We use (21) in the money market to obtain money market equilibrium after the gold price fixing,

$$\frac{\overline{C} - \overline{QR} + \overline{Q}X(t)}{P(t)} = \beta - \alpha(\rho + [\dot{P}(t)/P(t)]). \tag{22}$$

The dynamics of X(t) can be found from the dynamics of D(t) since $X(t) = I - D(t)$. Substituting for D(t) and $\overset{*}{D}(t)$ in (2) yields

$$-\dot{X}(t) = v\left[\frac{\delta_0 + \delta_1(\dot{q}(t)/q(t))}{q(t)} - (I - X(t))\right]. \tag{23}$$

When gold's price is fixed, $\dot{q}(t)/q(t) = -\dot{P}(t)/P(t)$ because $\dot{Q}(t) = 0$. Substituting this result into (23) and rearranging (22) and (23), we find the pair of linear differential equations which determine the dynamics of P(t) and X(t) after gold monetization,

$$\dot{P}(t) = \left(\frac{\beta - \alpha\rho}{\alpha}\right)P(t) - \frac{\overline{Q}}{\alpha}X(t) - \frac{\overline{C} - \overline{QR}}{\alpha} \tag{24}$$

$$\begin{aligned}\dot{X}(t) = \frac{v}{\overline{Q}}\left[\frac{\delta_1(\beta - \alpha\rho)}{\alpha} - \delta_0\right]P(t) - v\left[1 + \frac{\delta_1}{\alpha}\right]X(t) \\ + v\left[I - \frac{\delta_1(\overline{C} - \overline{QR})}{\overline{Q}\alpha}\right].\end{aligned} \tag{25}$$

The system (24) and (25) exhibits saddle point stability since the determinant of the system, $-\frac{v}{\alpha}(\beta - \alpha\rho + \delta_0)$, is negative. Since the determinant is the product of the system's roots, the roots must be real and of opposite sign; so the system must have one stable (negative) root and one unstable (positive) root. The model's steady state is a saddle point, and there is a stable branch leading to the steady state. The variable $X(t) = I - D(t)$ is a "backward-looking" or predetermined variable, and P(t) is a "forward-looking" or currently determined variable. We assume that any (X(t), P(t)) pair will be located on the stable branch of the system; therefore, for any given value of X(t), we can uniquely determine P(t). This assumption requires the absence of speculative bubbles.

We solve (24) and (25) by first locating the steady state and then finding the stable path leading to it. To find the steady state, we set $\dot{P}(t) = \dot{X}(t) = 0$ in (24) and (25) and solve the resulting equations for $\hat{P}$ and $\hat{X}$, the values of P and X such that $\dot{P}(t) = \dot{X}(t) = 0$. These steady state values are

$$\hat{P} = \frac{\overline{Q}[I - \overline{R}] + \overline{C}}{\beta - \alpha\rho + \delta_0} \tag{26}$$

and

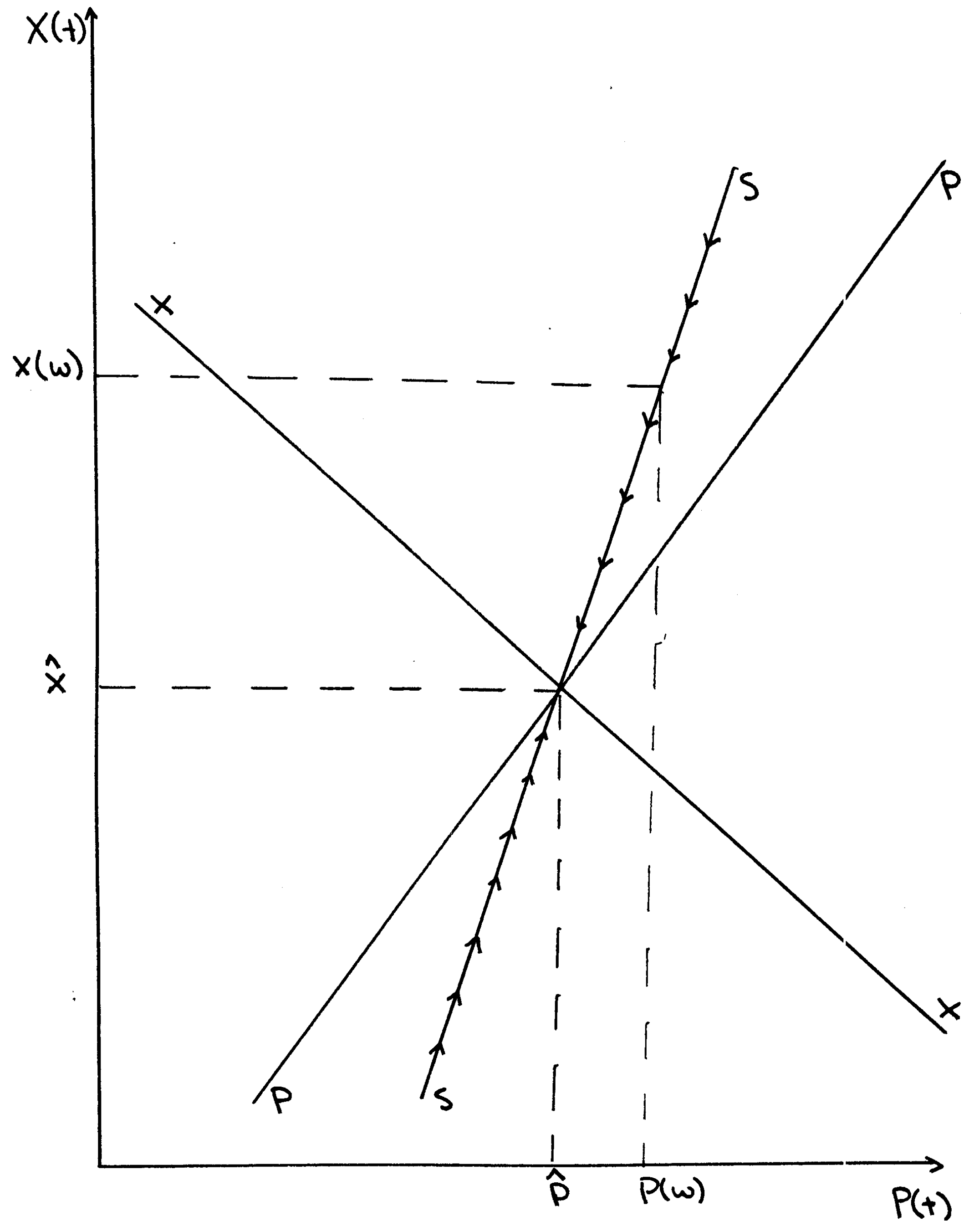
$$\hat{X} = \frac{(\beta - \alpha\rho)\overline{I} + \delta_0(\overline{R} - [\overline{C}/\overline{Q}])}{\beta - \alpha\rho + \delta_0}. \tag{27}$$

We may now use $\hat{P}$ and $\hat{X}$ to construct the equation of the stable branch,

$$P(t) = AX(t) + \hat{P} - A\hat{X}, \tag{28}$$

where $A \equiv \overline{Q}/(\beta - \alpha\rho - \alpha\lambda_1) > 0$ and λ_1 is the negative root of equation system (24) and (25).[16/]

Figure 1 depicts the dynamics of the model. The line SS is the stable branch given by equation (28); the line PP is the locus of points along which $\dot{P}(t) = 0$; and the line XX is the locus along which $\dot{X}(t) = 0$. These three lines intersect at the steady state whose co-ordinates are $\hat{P}$, $\hat{X}$. Given a value of X(t) such as X(w), P(t) is determined as that value P(w) located on the stable branch.



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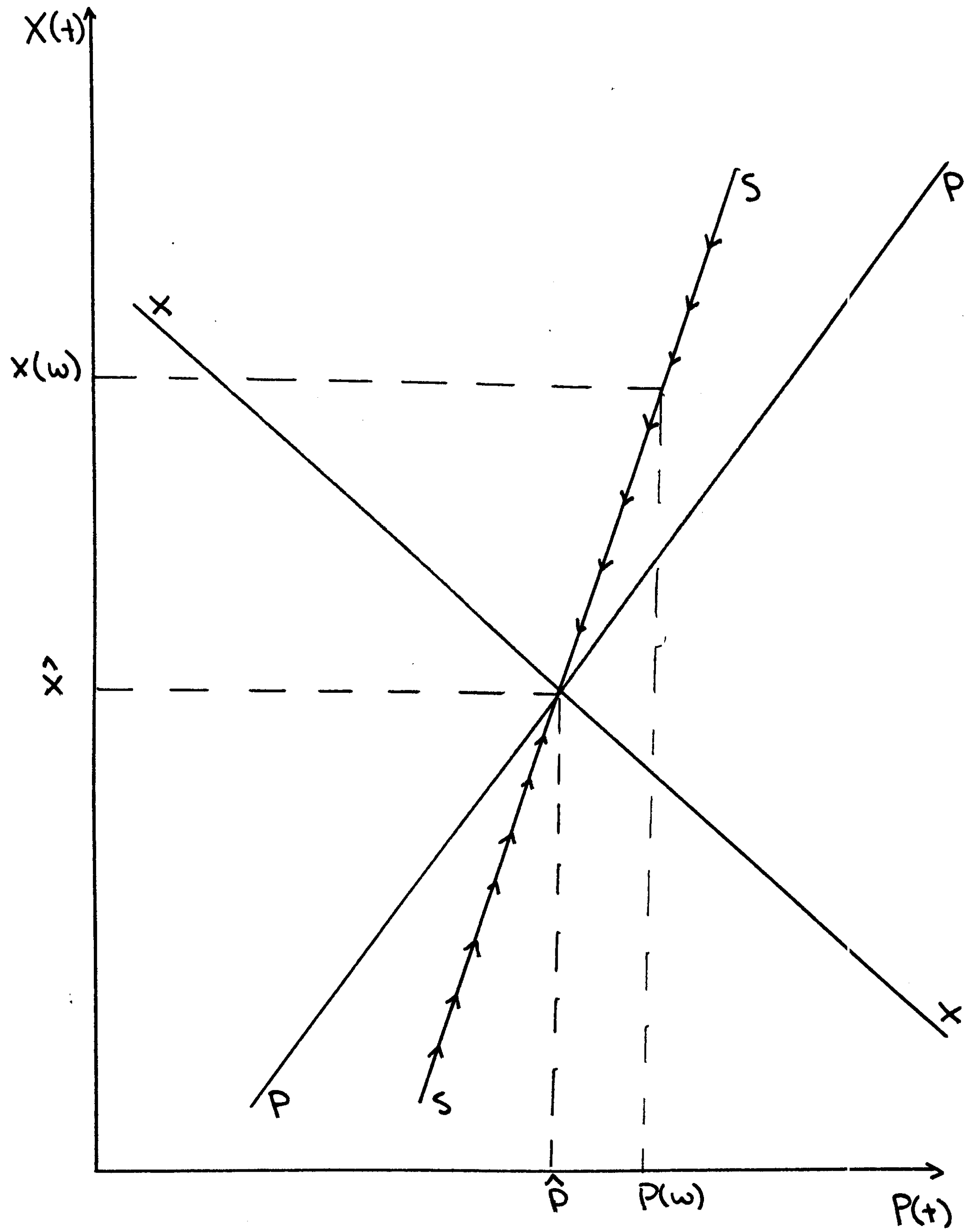

FIGURE 1

B. The Transition to the Gold Standard

Having derived the dynamics of $P(t)$ and $X(t)$ in the post-monetization environment, we are now ready to study the transition from the pre-monetization world of section II to the post-monetization environment. Crucial to our analysis is the determination of $\overline{Q}$, the fixed gold price. Since policies to monetize gold are intended as means of stabilizing the price level, we will determine the value of $\overline{Q}$ under a monetization policy which achieves such a goal. The policy will consist of a present announcement that gold's price will be fixed on a given future date. The value $\overline{Q}$ will be selected so that the aggregate price level $P(t)$ will remain constant after the fixing date without discretionary actions by the monetary authority. Since both markets in our model clear continuously, the value $\overline{Q}$ will be market determined at the time of the fixing, which is in the spirit of the current proposals. After first studying this policy with our framework, we will analyze some others.

Formally stated, the problem is to find the value $\overline{Q}$ which, if currently announced, will make $\dot{P}(t) = 0$ for all $t \geq w$, where w is the announced future date of price fixing. For all $t \geq w$, $P(t)$ is given by equation (28). Differentiating (28), we obtain

$$\dot{P}(t) = A\dot{X}(t),\ t \geq w. \tag{29}$$

If we are to find $\overline{Q}$ such that $\dot{P}(t) = 0$, $t \geq w$, then (29) implies that we must also have $\dot{X}(t) = 0$, $t \geq w$. Since this only occurs at the steady state of the system, the stable price level policy requires that the system reaches its steady state at the time of price fixing. For this to occur, the value of $X(t)$

at the time of fixing, $X(w)$, must equal the steady state value of $X(t)$, $\hat{X}$. Therefore, the policy requires that we find that value $\bar{Q}$ such that $X(w) = \hat{X}$.

Defining the instant that the monetization policy is announced as time zero, we use equation (7) to derive

$$X(w) = I - D(0)e^{-vw} - \frac{v(\delta_0 + \delta_1\rho)[1 - e^{(\rho-v)w}]}{q(w)(v - \rho)}. \tag{30}$$

Because speculators will price gold and other goods at levels such that no future price jumps are expected, we know $q(w) = \bar{Q}/\hat{P}$. Using equation (26) for $\hat{P}$ yields

$$q(w) = \bar{Q}/[\overline{Q}I + \bar{C} - \overline{QR})/(\beta - \alpha\rho + \delta_0)]. \tag{31}$$

The result in (31) may be substituted into (30) to yield a relationship between $X(w)$ and $\bar{Q}$. Also, equation (27) provides a relation between $\hat{X}$ and $\bar{Q}$. To find the value of $\bar{Q}$ which satisfies the policy, we equate $\hat{X}$ from (27) with $X(w)$ from (30) and solve the resulting equation for $\bar{Q}$. The result is a complicated formula, and its interpretation requires the introduction of issues which we do not treat until the next section. Therefore, we relegate the formula and its interpretation to the appendix.

The value of $\bar{Q}$ required to achieve $\dot{P}(t) = 0$ for $t \geq w$ depends on when the price will be fixed. A simple alternative to fixing $\bar{Q}$ in the future is to set $\bar{Q}$ at time zero such that $\dot{P}(t) = 0$ for $t \geq 0$. Then the time of the price fixing coincides with the time of the announcement. The $\bar{Q}$ that accomplishes this goal is that which equates the current value of $X(t)$ to the post-monetization steady state level, or

$$X(t) = \hat{X} = \frac{(\beta - \alpha\rho)I + \delta_0(\overline{R} - \overline{C}/\overline{Q})}{\beta - \alpha\rho + \delta_0} \tag{32}$$

Rearranging, we find that

$$\overline{Q} = \frac{\delta_0\overline{C}}{(\beta - \alpha\rho)I + \delta_0\overline{R} - (\beta - \alpha\rho + \delta_0)X(t)} \tag{33}$$

is the fixed nominal gold price which stabilizes the price level from the moment of the announcement.

C. Price Level Effects of Gold Monetization

The gold price fixing policy that we have studied is designed to produce a constant price level after price fixing, but it does not necessarily ensure a constant price level between announcement and implementation. To study the price-level effects of gold monetization, we compare the pre-announcement price level, $P(0) = \overline{C}/(\beta - \alpha\rho)$ with the post-fixing price level $P(w) = \hat{P} = [\overline{C} + Q(\hat{X} - \overline{R})]/(\beta - \alpha\rho)$. It is evident that the sign of $\hat{P} - P(0)$ depends on the sign of $\overline{Q}(\hat{X} - \overline{R})$, the value of gold which is monetized in the steady state.

Using the definition of X we may obtain

$$\hat{X} = X(w) = \overline{R} + G(w_-) = R(w_+) + G(w_+) \; . \tag{33}$$

In (33) we emphasise that while X is a continous variable, its components R and G are not. In general, at the time of fixing G will jump down ($G(w_-) > G(w_+)$) and R will jump up ($\overline{R} < R(w_+)$) as gold hoards are exchanged for cash. We use equation (33) to find

$$\hat{P} - P(o) = \frac{\overline{Q}G(w_-)}{\beta - \alpha\rho}, \tag{33a}$$

which may be further manipulated to yield

$$\frac{\hat{P} - P(o)}{P(o)} = \frac{\overline{Q}G(w_-)}{\overline{C}} \tag{33b}$$

Equation (33b) portrays an intuitive result of this monetization scheme: monetization will cause the price level to rise in percentage terms by the ratio of the money value of speculative gold holding at the instant prior to monetization to the quantity of currency at the instant prior to monetization.

The difference between P(o) and $\hat{P} = P(w)$ provides the information needed to derive the entire time path of P(t) between the announcement and price fixing. Agents will set prices now so thatforseen price jumps are eliminated. Thus, since $\hat{P} > P(o)$, on announcement P(t) will jump above P(o) but not all the way to $\hat{P}$. P(t) will then rise smoothly toward $\hat{P}$, reaching $\hat{P}$ at precisely the instant of gold price fixing. More formally, P(t) will follow

$$P(t) = [\hat{P} - P(o)]e^{(\frac{\beta - \alpha\rho}{\alpha})(t-w)} + P(o) \ ; \ t < w.$$

So far we have treated C as fixed at $\overline{C}$ with the government's only tool being $\overline{Q}$; and $\overline{Q}$ is set so that $\dot{P}(t) = 0$, $t \geq w$. However the government may also use open market operations in bond markets to alter C to ensure that monetization occurs without any price changes. These open market operations exactly sterilize any gold inflow or outflow at the instant of monetization. To find the size of the required intervention, we maintain a constant money supply both before and after monetization. This requires

$$\overline{C} = \overline{C} + \Delta\overline{C} + \overline{Q}[\hat{X} - \overline{R}], \tag{34}$$

where $\Delta\overline{C}$ is the size of the open market operations in the bond market. (34) implies that the required open market operation is $\Delta\overline{C} = -\overline{Q}[\hat{X} - \overline{R}]$. If C jumps at the instant of monetization then the expressions for $\hat{X}$ and $\hat{P}$ must be modified to reflect the jump. They become:

$$\hat{X} = \frac{(\beta - \alpha\rho)\overline{I} + \delta_0(\overline{R} - [(\overline{C} + \Delta\overline{C})/\overline{Q}])}{\beta - \alpha\rho + \delta_0} \tag{35a}$$

and

$$\hat{P} = \overline{C}/(\beta - \alpha\rho). \tag{35b}$$

Now, to solve for the value of $\overline{Q}$ such that $\dot{P}(t) = 0$, $t \geq w$ we must use (35a), (35b) and the condition $\Delta\overline{C} = -\overline{Q}[\hat{X} - \overline{R}]$. As before, finding the value of $\overline{Q}$ such that $\dot{P}(t) = 0$, $t \geq w$, yields a complicated expression which we report in the appendix.

IV. Crisis and Discipline of the Gold Standard

In this section we will analyze a gold standard which can exist only temporarily, succumbing eventually to a run on the government's gold reserves. We will study both the dynamics of prices and the money stock during the temporary gold regime and the timing of the crisis. Also, we will provide a formal definition of the often-invoked but elusive concept of "the discipline of the gold standard."

A. A Collapse without Discretionary Money Growth

Our analysis of gold price fixing has considered only the possibility of a permanently successful scheme. However, a government may attempt to peg too low a gold price; or it may adopt other policies inconsistent with maintaining gold's fixed price.

We introduce the problem of a gold standard's collapse using the model of Section III which allows no discretionary money supply movements. The total world quantity of gold is $I = D(t) + X(t)$. In the model's steady state $I = \hat{D} + \hat{X}$. As long as both $\hat{D}$ and $\hat{X}$ are positive, the gold standard is viable. Since $\hat{D} = \delta_0/\hat{q} > 0$, the potential of a breakdown arises when $\hat{X} < 0$, which implies that $\hat{D}$ exceeds I. Since this steady state cannot be attained, the dynamic system (24) and (25) must break down eventually.

In equation (28), we found that

$$\hat{X} = \frac{(\beta - \alpha\rho)I + \delta_0(\overline{R} - \overline{C}/\overline{Q})}{\beta - \alpha\rho + \delta_0};$$

so $\hat{X}$ can be negative if

$$\overline{C}/\overline{Q} > \frac{(\beta - \alpha\rho)I + \delta_0\overline{R}}{\delta_0}. \tag{36}$$

Equation (36) indicates that the eventual failure of a gold standard without discretionary monetary policy depends on the size of the gold value of currency outstanding, $\overline{C}/\overline{Q}$, relative to a linear combination of I and $\overline{R}$. Given I and $\overline{R}$, (36) may be fulfilled by a infinity of $\overline{C}$ and $\overline{Q}$ values.

We will now examine a collapse in detail. We suppose that a gold standard, introduced at time w, can function temporarily but that $\overline{Q}$ is low enough that (36) holds. When the system collapses at time z, $\overline{Q}X(z)$ is demonetized. At the instant of collapse, R(Z) will be withdrawn from the goverment in exchange for currency; and any gold in coins, G(Z), will be demonetized since it will now earn the real rate of return and will not circulate. Thus, the money supply will fall by $\overline{Q}X(z)$ since $X(z) \equiv R(z) + G(z)$. Hence, immediately following the collapse money market equilibrium is

$$\overline{C} - \overline{Q}R = (\beta - \alpha\rho)P(z_+) - \alpha\dot{P}(z_+). \tag{37}$$

Since we assume perfect foresight, the run on gold stocks is forseen by agents, whose behavior prevents prices from jumping discontinuously in response to future foreseen events. This implies that $P(z_+) = P(z_-) = P(z)$. Also, once the run takes place, agents expect a constant money supply, $\overline{C} - \overline{QR}$, in the future. Hence, the solution to (37) consistent with market fundamentals requires $\dot{P}(z_+) = 0$. The post-collapse price level solution is then

$$P(z) = (\overline{C} - \overline{QR})/(\beta - \alpha\rho). \tag{38}$$

With $P(z_-) = P(z_+) = P(z)$ and $\dot{P}(z_t) = 0$. Equation (28) gives a terminal condition which applies to the dynamic system (24), (25). In addition, that system also has an initial condition, that X at time w be the predetermined variable X(w).

Since the gold standard system will collapse in finite time, it is no longer appropriate to assume that the (X(t), P(t)) pair lies on the stable branch in Figure 1. However, the (X(t), P(t)) pair still satisfies the general solution to the model (24), (25) which is

$$P(t) = C_1Ae^{\lambda_1(t-w)} + C_2Be^{\lambda_2(t-w)} + \hat{P}, \qquad w \leq z, \tag{39a}$$

$$X(t) = C_1e^{\lambda_1(t-w)} + C_2e^{\lambda_2(t-w)} + \hat{X}, \qquad w \leq z, \tag{39b}$$

where $B \equiv \overline{Q}/(\beta - \alpha\rho - \alpha\lambda_2)$ and λ_2 is the positive root of the system.

C_1 and C_2 are constants which we will determine with initial and terminal conditions. At the start of the gold standard, time w, there is an initial endowment of X, $X(w)$. In addition, a terminal condition on the price level when the system collapses arises from the continuity of prices. As stated earlier, the time z price level is $P(z) = (\overline{C} - \overline{QR})/(\beta - \alpha\rho)$. From these conditions, we calculate

$$C_1 = \frac{[\{(\overline{C} - \overline{QR})/(\beta - \alpha\rho)\} - \hat{P} - (X(w) - \hat{X})Be^{\lambda_2 z}]}{Ae^{\lambda_1(z-w)} - Be^{\lambda_2(z-w)}} \tag{39c}$$

$$C_2 = \frac{[(X(w) - \hat{X})Ae^{\lambda_1(z-w)} - \{(\overline{C} - \overline{QR})/(\beta - \alpha\rho)\} + \hat{P}]}{Ae^{\lambda_1(z-w)} - Be^{\lambda_2(z-w)}}. \tag{39d}$$

When C_1 and C_2 are substituted into (39a) and (39b) those equations trace out the motion of $P(t)$ and $X(t)$ from time w to time z.

To find time z we use the general solution developed in (39a - d) along with our knowledge of conditions which must prevail in the gold market in the post-monetization era. At time z, when gold is demonetized, we know $q(z) = \overline{Q}/P(z) = \overline{Q}(\beta - \alpha\rho)/(\overline{C} - \overline{QR})$. By the continuity of prices we know that this fixed value of $q(z)$ is an initial condition for the post-monetization gold market. Furthermore, as in section II, we assume that a terminal condition is given by the choke price,

$$q(T) = \frac{\delta}{\overline{I}} \tag{40}$$

Equation (40) differs from equation (10) because government gold reserves are zero in the post-monetization era.

As in the pre-monetization era, while speculative gold hoards are held we must have

$$\dot{q}(t) = \rho q(t). \tag{41}$$

Combining the initial condition, q(z), the terminal condition, q(T), and the law of motion, (40), we find

$$T - z = \frac{1}{\rho}\ell n(\delta_0/q(z)I) \tag{42}$$

According to (42) gold will be held in speculative hoards for a period $\frac{1}{\rho}\ell n(\delta_0/q(z)I)$ during the post-monetization era. At the end of this period all gold will be absorbed into consumption and industrial use, i.e., $D(T) = I$. Condition (42) allows us to determine the size of $D(z) = \overline{I} - X(z)$ at the date of demonetization. D(z) must be of a size such that accumulation of D, which lasts a period $\frac{1}{\rho}\ell n(\delta_0/q(z)I)$ will exactly exhaust the stock of gold.

Equation (7) may be adapted to allow us to solve for D(z). In particular, the accumulation requirement is

$$I = D(z)e^{-v(T-z)} + e^{-vT}\int_z^T \frac{v(\delta_0 + \delta_1\rho)}{q(z)} e^{-\rho(\tau-z)} e^{v\tau} d\tau \tag{43a}$$

or

$$I = D(z)e^{-v\eta} + \frac{v(\delta_0 + \delta_1\rho)}{q(z)(v - \rho)}[e^{-\rho\eta} - e^{-v\eta}], \tag{43b}$$

where $\eta = \frac{1}{\rho}\ell n(\delta_0/q(z)I)$. Using the condition $I = D(z) + X(z)$, (43b) implies

$$X(z) = I - Ie^{v\eta} + \frac{v(\delta_0 + \delta_1\rho)}{q(z)(v - \rho)}[e^{(v-\rho)\eta} - 1]. \tag{44}$$

Equation (44) gives the final piece of information we need to determine **z**, the time of the run. To find z set X(z), from (44), equal to X(z) from (39). This results in the following equation in z

$$I(1 - e^{v\eta}) + \frac{v(\delta_0 + \delta_1\rho)[e^{(v-\rho)\eta} - 1]}{q(z)(v - \rho)} = C_1 e^{\lambda_1(z-W)} + C_2 e^{\lambda_1(z-W)} + \hat{X}, \quad (45)$$

where C_1 and C_2 are given in equations (39c) and (39d). Equation (45) is a single non-linear equation in the unknown collapse date **z**, which may be solved for **z** by numerical methods given numerical values for the parameters.

We have depicted the case of a negative $\hat{X}$ in Figure II. the $\dot{X}(t) = 0$ and $\dot{P} = 0$ loci are analogous to those in Figure 1, except that the steady state value $\hat{X}$ is negative. Since the consumption demand for gold under this gold standard eventually exceeds the total available, the system must collapse at some point. At the time of the collapse, currency will be exchanged for all the governments' reserves at the fixed rate $\overline{Q}$; and gold will cease to be money. Hence, the post-collapse money stock will be $\overline{C} - \overline{QR}$; and the price level will be $(\overline{C} - \overline{QR})/(\beta - \alpha\rho)$, represented by point A in Figure II.

In order to prevent the infinite profits associated with price level jumps, the price level must have attained $(\overline{C} - \overline{QR})/(\beta - \alpha\rho)$ at the moment of the collapse. Similarly, to prevent a jump in the nominal gold price Q, there must still be some gold in the hands of the public at the time of the collapse. The gold price then gradually rises, thereby reducing the slope of the $\dot{X}(t) = 0$ locus and rotating it counter-clockwise until it intersects the P-axis at A. The lines through the points FHA represent the path followed by the price level and X(t). The FH segment, located to the left of the stable branch SS, is a solution path for the system (24)-(25). Thus, we have forced a collapse of the gold standard even when the money stock and prices continually fall.

B. The Discipline of the Gold Standard

The analysis in the previous part of this section showed that even the seemingly innocuous policy of zero discretionary money growth can lead to the collapse of a gold standard. Hence it is of interest to determine which combinations of gold price and money growth are consistent with the permanent maintenance of a gold standard. In this part of this section we will derive a constraint on gold price and on the path of the money supply such that a gold standard will not collapse. We refer to this constraint as *the discipline of the gold standard*.

We now assume that a government maintains a gold standard at the gold price $\bar{Q}$ but uses discretionary monetary policy. The economy's money supply follows

$$M(t) = M(0)e^{\int_0^t g(h)dh} \qquad t < z \tag{46a}$$

$$M(t) = M(0)e^{\int_0^t g(h)dh} - K(z) \quad t = z \tag{46b}$$

$$M(t) = M(z)e^{\int_z^t g(h)dh} \qquad t \geq z, \tag{46c}$$

where K(z) is the amount of money destroyed during a collapse of the gold standard. It is of course quite possible that K(z) = 0, i.e., the gold standard may never collapse. Indeed, our methodology in deriving the discipline of the gold standard is to find the set of paths of money growth, g(h), such that K(z) = 0.

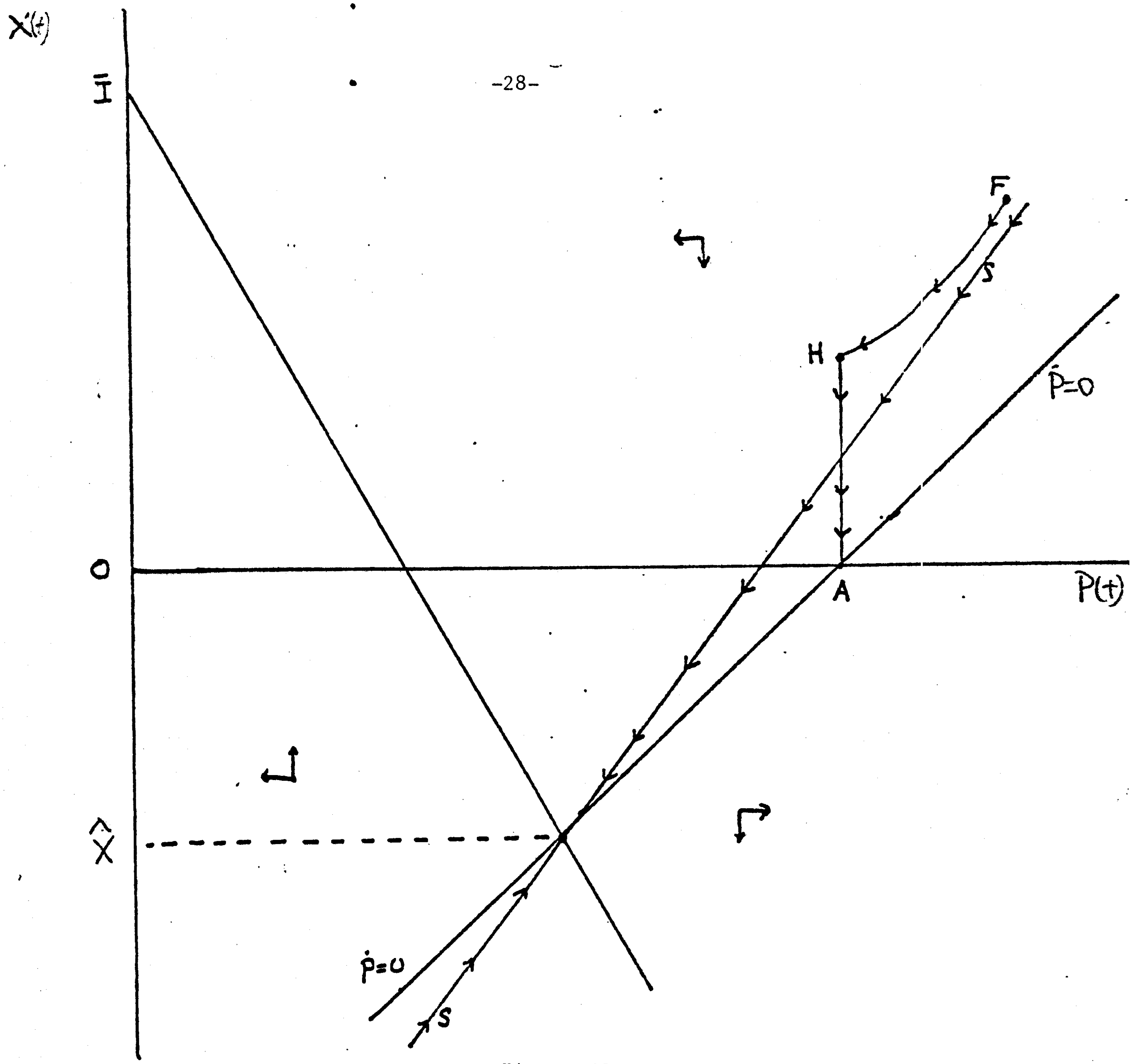

Figure II

As indicated in the previous part of this section our model requires $I \geq D(t)$, for all t. This is a real resource constraint on any gold standard. A gold standard, once imposed, will collapse if and only if agents forsee that they are on a path such that in the absence of a collapse there would be some finite t when $I = D(t)$, with further demands on government gold stocks expected.

We define $\tilde{D}(t)$ to be that value of D(t) constructed from our model setting $K(z) \equiv 0$. We emphasize that $\tilde{D}(t)$ need not be less than I. The $\tilde{D}(t)$ are a sequence of hypothetical values of D(t) constructed on the assumption that $K(z) = 0$. Actual values of D(t) must, of course, fulfill $D(t) \leq I$.

We define a gold-disciplined money growth-gold price policy as a policy for which there is no finite value of t such that $\tilde{D}(t) > I$.

We will now present two examples of policies which violate this definition and which will cause the gold standard eventually to collapse. First, we derive the price level solution for the system whose money supply process is given in (46a-c),

$$P(\tau) = e^{(\frac{\beta - \alpha\rho}{\alpha})\tau} \int_{\tau}^{\infty} \frac{M(j)}{\alpha} e^{-(\frac{\beta - \alpha\rho}{\alpha})j} dj. \tag{47}$$

Next, the solution for D(t), after gold price fixing is

$$D(t) = D(w)e^{-v(t-w)} + e^{-vt} \int_{w}^{t} \frac{v[\delta_0 P(\tau) - \delta_1 \dot{P}(\tau)]}{\overline{Q}} e^{v\tau} d\tau \tag{48}$$

We can use money market equilibrium to eliminate $\dot{P}(\tau)$ from (48); and we substitute from (47) into (48) with the condition that $K(z) \equiv 0$ to obtain

$$\tilde{D}(t) = D(w)e^{-v(t-w)} + e^{-vt}$$

$$\int_w^t \frac{v}{\bar{Q}}[\delta_0 - \delta_1\{(\frac{\beta - \alpha\rho}{\alpha})(e^{(\frac{\beta - \alpha\rho}{\alpha})\tau}\int_\tau^\infty \frac{M(0)e^{\int_0^{\tau-j} g(h)dh}}{\alpha} e^{-(\frac{\beta - \alpha\rho}{\alpha})j}dj) - \frac{M(\tau)}{\alpha}\}e^{v\tau}d\tau \quad (49)$$

A policy, consisting of a sequence of money growth rates, g(h), and a gold price, $\bar{Q}$ is gold disciplined if and only if there is no value of t such that $\tilde{D}(t)$, in (46), is greather than I.

The condition that (49) be greater than I is quite complicated. However, it must be satisfied for the policy studied in the first part of this section if $\overline{C}/\overline{Q}$ is too high since $\hat{X}$ is then negative. In this case $\hat{D} = \lim_{t\to\infty} \tilde{D}(t) > I$. Since $\tilde{D}(t)$ approaches $\hat{D}$ smoothly, there must be some finite t such that $\tilde{D}(t) > I$.

Similarly, a policy to increase the money supply at the constant, positive, finite rate g, where $0 < g < (\frac{\beta - \alpha\rho}{\alpha})$, will not be gold-disciplined.[17] If money is rising at the rate g, then prices must rise at the rate g. With $\bar{Q}$ fixed, $q(t) = \bar{Q}/P(t)$ must fall at the rate g. With q(t) falling, D^* will rise indefinitely; and D will eventually hit the barrier I. For a constant money growth rate we find

$$\tilde{D}(t) = D(0)e^{-vt} + \frac{v(\delta_0 - \delta_1 g)(e^{gt} - e^{-vt})}{\bar{Q}[(\frac{\beta - \alpha\rho}{\alpha}) - g](g + v)}. \quad (50)$$

Since we require $(\delta_0 - \delta_1[\dot{P}(t)/P(t)]) > 0$ and $\frac{\beta - \alpha\rho}{\alpha} > g$, $\tilde{D}(t)$ grows without bound as t becomes large.

V. Conclusion

There are many possible commodities that can serve as monetary standards. However, the sequence of items upon which the monetary system has been based in modern times has been so limited that it can readily be characterized as a "gold-paper-gold" cycle. Since the current "paper" phase of the cycle has allowed price level instability, there has been some agitation to return to the gold standard. Proponents of such a return presume that simply allowing the market to determine the price at which gold will be fixed is sufficient to produce an "appropriate" price. The basic point of this paper is that such policies will lead to price level stability only in the presence of very specific monetary policies. To determine the nature of such policies requires detailed knowledge of the structure of the gold and money markets, the nature of expectations, and the dynamics of the price level and of private gold accumulation. In the absence of such monetary control, specifying a policy to set the gold price according to a given day's market price may lead to a situation even more inflationary than that produced by the current system.

Nevertheless, given the results of the paper, it may be possible to compile this required knowledge by estimating the structural parameters of the money and gold markets. However, operational use of the model requires its extension to incorporate the technological nature of gold mining so that the model is specific to gold.

Footnotes

1/ The Reagan administration has decided to assess the role of gold in the domestic and international monetary systems through the appointment of a "Gold Commission".

2/ Laffer proposes to have the government announce that gold will be monetized at the market price prevailing on a certain future date. To prevent inflation in the interim, the government would follow an "austere" monetary policy and sell a large portion of its gold holdings.

Lehrman's proposal is somewhat different in that he requires that gold's price be fixed at its market determined level as long as wages are not required to fall.

3/ Lacking a specific policy on the money stock, the markets for gold and currency could not produce price solutions at the time of fixing. Otherwise stated, any market price for gold at the time of fixing is possible depending on which policy on government currency issue and gold reserves is selected. This includes the possibility of a market gold price which may be highly inflationary even after the price fixing.

4/ In an earlier version of this paper we studied this sort of policy. The government was assumed to announce the time of fixing, its currency issue, and its gold reserve holdings; and we computed the implied solutions for the price paths. Since we had to account for the post-fixing dynamics in the gold and money markets, the solution involved some very complicated algebra. Also, only for a single policy for currency issues and gold reserves did this scheme produce price level stability, even after the fixing of the nominal gold price.

5/ In Flood and Garber (1981), we studied this problem in the guise of a transition from a floating to a fixed exchange rate for foreign currency. However, since the model was stochastic, we solved for the probability density function over the time of fixing rather than for an exact time.

6/ The fixing of the price of gold at a given future time and price has a precedent in the Greenback period of the U.S. In 1875, Congress passed a law requiring a return to the gold standard in January, 1879 at the pre-Civil War parity. In this case gold circulated as money with a fluctuating greenback exchange rate. For details, see Friedman and Schwartz (1963), Ch. 2.

7/ Allowing for changes in the real rate of interest would greatly complicate our analysis; thus, the development of our models is an exercise in partial equilibrium analysis. In (S-H), ρ is given exogenously. In Barro, ρ is exogenously determined at a fixed value.

8/ These assumptions preclude any gold mining activity, a specification similar to that of S-H. An inclusion of gold mining would produce a richer dynamics than those studied here; because of the exhaustible resource nature of gold, marginal cost curves for extraction would shift upward with cumulative increases in previously mined gold and anticipated capital gains would affect gold extraction rates. Indeed, an inclusion of the technical nature of extraction is necessary to make the transition to monetization studied here specific to the gold commodity.

However, we have chosen to ignore the gold extraction aspect of the problem because it adds nothing to the qualitative results which we obtain while greatly increasing the technical complexity of the problem. On the other hand, a quantitative study, aimed at deriving explicit policy recommendations on money supply policies and on the price at which to fix gold, must account for the gold mining element since its parameters affect the steady state of the system.

9/ The specification in (4) is adopted purely on the basis of its convenience for analysis of the dynamic system which we create. We assume δ_0 to be large enough to make $\delta_0 + \delta_1[\dot{q}(t)/q(t)] > 0$ for all relevant values of $\dot{q}(t)/q(t)$.

10/The speculative pricing of such resources is discussed in more detail in S-H.

11/The elements which we add to the S-H specification were absent from their published paper, but some aspects were contained in a pre-publication version.

12/In later analysis we will relax the $D(0) = 0$ condition.

13/Notice that while $\dot{D}(T_-) = 0$, $\dot{D}(t)$ does not smoothly approach zero as $t \to T$ because $\overset{*}{D}(t)$ jumps downward at time t. We have

$$\dot{D}(T_-) = v\left(\frac{\delta_0 + \delta_1\rho}{q(T)} - D(T)\right).$$

Since $\dot{D}(T_+) = 0$ with $\overset{*}{D}(T_+) = D(T_+) = \delta_0/q(T)$, we find that

$$\dot{D}(T_-) = \frac{v\delta_1\rho}{q(T)},$$

which is the size of the final downward jump in $\dot{D}(T)$ to zero.

14/In Flood and Garber (1980b) we estimate a model similar to (16) and (17) and cannot reject the hypothesis that only market fundamentals govern price.

15/Since P(t) contains the rental price of gold in consumption form, our relative price $q(t) = Q(t)/P(t)$ is not equal to the price of gold divided by the price of goods other than gold. However, we assume the consumption rental price of gold to be proportional to gold's price. We also assume $P(t)$ to be Cobb-Douglas in gold's rental price and in the price of other goods. Hence, our $q(t)$ is proportional to the price of gold divided by the price of goods other than gold raised to the power $1 - \theta$, where θ is gold's share in the price index. In our analysis we assume that θ is sufficiently small that it may be ignored without consequence. However, if θ were large then our analysis would need to be altered in two ways: first equation (3) would become $\dot{q}(t) = \rho(1 - \theta)q(t)$, second, the rate of return term in $\dot{D}(t)$ would become $[\dot{q}(t)/q(t)]/(1 - \theta)$. Neither of these changes would alter our results substantively.

16/ The values of λ_1, the negative root, and λ_2, the positive root, are

$$\lambda_1 = \tfrac{1}{2}(- b - \sqrt{b^2 - 4c})$$

$$\lambda_1 = \tfrac{1}{2}(- b + \sqrt{b^2 - 4c})$$

with

$$b = - [\frac{\beta - \alpha\rho}{\alpha} - v(1 + \frac{\delta_1}{\alpha})]$$

and

$$c = - \frac{v}{\alpha}[\beta - \alpha\rho + \delta_0].$$

17/ That g be less than $\frac{\beta - \alpha\rho}{\alpha}$ is required for the convergence of the integral in (44). In Flood and Garber (1980a) we refer to this requirement as <u>process consistency</u>.

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Appendix

In this appendix we develop expressions for the setting of gold's price such that inflation will be zero after the date of price fixing. First we find that $\overline{Q}$ which makes $\dot{P}(t) = 0$, $t \geq w$ but does not ensure $\dot{P}(t) = 0$, $t < w$. Second, we find the value of $\overline{Q}$ which, when accompanied by an appropriate open market operation at the time of price fixing, yields $\dot{P}(t) = 0$, for all t.

With no open market operation at the time of fixing equations (31) and (32) yield

$$X(w) = I - D(0)e^{-vw} - \frac{v(\delta_0 + \delta_1\rho)[1 - e^{(\rho-v)w}][\overline{Q}I + \overline{C} - \overline{QR}]}{(v - \rho)\overline{Q}(\beta - \alpha\rho + \delta_0)}. \tag{A1}$$

The steady state value of X is

$$\hat{X} = \frac{(\beta - \alpha\rho)I + \delta_0[\overline{R} - (\overline{C}/\overline{Q})]}{(\beta - \alpha\rho \quad \delta_0)}. \tag{A2}$$

The value of $\overline{Q}$ which equates $X(w)$ and $\hat{X}$ is

$$\overline{Q} = \frac{[\delta_0 - \frac{v(\delta_0 + \delta_1\rho)}{v - r}(1 - e^{(\rho-v)w})]\overline{C}}{(\beta - \alpha\rho)I + \delta_0\overline{R} - (I - D(0)e^{-vw})(\beta - \alpha\rho + \delta_0) + \frac{v(\delta_0 + \delta_1\rho)[1 - e^{(\rho-v)w}](I - \overline{R})}{v - r}} \tag{A3}$$

If the government conducts an open market operation at time w of a magnitude $\Delta\overline{C} = -\overline{Q}[\hat{X} - \overline{R}]$ then the steady state will move to correspond to this open market operation. Substitute $\Delta\overline{C} = -\overline{Q}[\hat{X} - \overline{R}]$ into (36a) to find the new steady state X, which is

$$\hat{X} = I - \frac{\delta_0\overline{C}}{\overline{Q}(\beta - \alpha\rho)} \tag{A4}$$

Further, with $P(w) = \hat{P} = \overline{C}/(\beta - \alpha\rho)$ our equation determining $X(w)$ becomes

$$X(w) = I - D(0)e^{-vw} - \frac{v(\delta_0 + \delta_1)[1 - e^{(\rho-v)w}]\overline{C}}{(v - \rho)\overline{Q}(\beta - \alpha\rho)} \tag{A5}$$

Now, the value of $\overline{Q}$ which equates $\hat{X}$ and $X(w)$ is

$$\overline{Q} = \frac{[\delta_0 - \frac{v(\delta_0 + \delta_1\rho)}{(v - r)}[1 - e^{(\rho-v)w}]]\overline{C}}{D(0)e^{-vw}(\beta - \alpha\rho)}. \tag{A6}$$

If monetization is immediate $(w = 0)$ this becomes

$$\overline{Q} = \frac{\delta_0\overline{C}}{D(0)(\beta - \alpha\rho)} \tag{A7}$$

Notice that under our scheme $P(0) = \hat{P}$ and $\hat{D}(0) = \hat{D} = \hat{\overset{*}{D}}$. Thus, equation (A7) can be obtained quite directly from the condition $\hat{\overset{*}{D}} = \frac{\delta_0}{\hat{q}} = D(0) = \hat{D}$, and is a special case which provides some intuition for the complex formulas (A3) and (A6).