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I. INTRODUCTION

Data on financial variables are available with essentially no lag. Today's nominal interest rate provides incomplete current information about aggregate disturbances in the money and goods markets. Employing a standard IS-LM model, Poole [1970] shows how the monetary authorities can best use this incomplete information to reduce, though not eliminate, the variance of output. Under his recommended "combination policy," the monetary authorities try to minimize the variance of the nominal interest rate when money market disturbances predominate and to amplify the variance of the nominal interest rate when goods market disturbances predominate. Of course, in Poole's static expectations model the variance of the nominal interest rate is the same as the variance of the expected real interest rate.

Viewed in the light of a decade of further research, Poole's analysis is open to an important criticism: it makes no allowance for the fact that private agents can, if they choose, make rational use of the same incomplete information to predict the price level and the inflation rate. In this paper, we characterize optimal monetary policy under alternative assumptions about which of three groups of agents--wage setters, investors, and the monetary authorities--"use" the current nominal interest rate. We establish that monetary policy is redundant when all the agents use the interest rate.[1/] However, we devote most of our effort to deriving monetary feedback rules that are optimal when either the monetary authorities or investors (or both) use the interest rate. Monetary policy is not redundant since we assume that wage setters do not use the interest rate, perhaps because the cost of "indexing" nominal wage contracts to the interest rate is too great.

When the monetary authorities use the current interest rate, the optimal contemporaneous feedback rule is the same whether or not investors do.[2/] However, the monetary authorities need not use the current interest rate when investors do. A lagged feedback rule based on last period's interest rate can be just as effective as a contemporaneous one.[3/] Neither feedback rule completely eliminates the variance of output because no agent has the complete information necessary to know exactly how much of the nominal interest rate innovation is due to the money market disturbance and how much is due to the goods market disturbance. However, both feedback rules reduce this variance to the level that would be reached if wage setters themselves used the interest rate. They save wage setters the cost of indexing.

Both lagged and contemporaneous feedback rules work by causing changes in investors' expected real interest rate that offset investors' expectation of the disturbance to the goods market. So, for example, when the variance of the disturbance to the goods market is relatively small, the monetary authorities seek to stabilize the real interest rate. Or, when the variance of the disturbance to the goods market is relatively large, the monetary authorities seek large offsetting changes in the real interest rate. Thus our rational expectations result is fundamentally the same as Poole's static expectations result.

However, lagged and contemporaneous feedback rules have strikingly different implications for the nominal interest rate. An optimal contemporaneous feedback rule implies a low variance for the nominal interest rate when money market disturbances predominate. But an optimal lagged feedback rule generates a low variance for the nominal interest rate when goods market disturbances predominate. This sharp difference arises because under the optimal contemporaneous feedback rule changes in the real interest rate are caused solely by changes in nominal interest rate, while under the optimal lagged feedback rule changes in the real interest rate are caused by changes in both the nominal interest rate and in investors' inflation expectations.

II. THE MODEL

The equations of the model are

$$\text{(1a)} \quad y_t = \overset{(+)}{\theta}(p_t - p_{t|t-1}),$$

$$\text{(1b)} \quad y_t = -\overset{(+)}{\Delta}[r_t - (p_{t+1|t-1} - p_{t|t-1})] + u_t,$$

$$\text{(1c)} \quad m_t - p_t = \eta y_t - \lambda r_t + v_t.$$

y, p, and m are (the deviations from trend values of the logarithms of) output, the price level, and the money supply.[4/] r is (the deviation from the trend value of) the nominal interest rate. u and v are independent, normally distributed random variables with zero means and variances σ_u^2 and σ_v^2.

There is an important difference between our model and the usual rational expectations IS-LM model. In making their "rational" predictions of p_t and p_{t+1}, some or all agents may use not only the information available at the end of period t - 1 but also the current interest rate, r_t.[5/] Adopting one standard notation, we denote predictions based on the lagged-only information set by $p_{t|t-1}$ and $p_{t+1|t-1}$ and predictions based on the current-interest-rate-augmented information set by $\overset{+}{p}_{t|t-1}$ and $\overset{+}{p}_{t+1|t-1}$. The introduction of some additional notation allows us to represent all the cases we want to consider with equations (1). We denote predictions based either on one information set or on the other by $\overset{(+)}{p}_{t|t-1}$ and $\overset{(+)}{p}_{t+1|t-1}$. Because none, some, or all three sets of agents (wage setters, investors, and the monetary authorities) may use the interest rate, equations (1) actually subsume eight different cases.

Equation (1a) is an aggregate supply function that incorporates the "natural rate" hypothesis. Output depends positively on the difference between the price level and (the deviation from the trend value of the logarithm of) the nominal wage, w:

$$(2) \quad y_t = \theta(p_t - w_t).$$

Wage setters set the nominal wage so that the expected value of output, based on their information set, is equal to the natural (trend) rate of output:

$$(3) \quad \overset{(+)}{y}_{t|t-1} = 0 = \theta(\overset{(+)}{p}_{t|t-1} - w_t).$$

Subtracting the right-hand equality in (3) from equation (2) yields the aggregate supply function (1a). Deviations of output from its natural rate are due to wage setters' price prediction errors.

Equation (1b) is the aggregate demand function; aggregate demand depends negatively on investors' expected real rate of interest. Some investors may also participate in the wage-setting decision. However, we assume that the investment decision may take place after the nominal wage is set. Thus, the price predictions that affect the investment decision may be based on the current interest rate when the price prediction that affects the wage decision is not.

Equation (1c) is the money market equilibrium condition. The demand for real balances depends positively on output and negatively on the nominal interest rate. As we explain in more detail below, the monetary authorities set the money supply on the basis of an information set which always includes all variables dated t - 1 and earlier and may in some cases include the current nominal interest rate as well. The monetary authorities have as their objective the minimization of the variance of output about its natural rate, which is equivalent to the minimization of the variance of wage setters' price prediction error.[6/], [7/]

III. THE PRICE PREDICTION ERROR OF AGENTS WHO USE THE INTEREST RATE

In this section we isolate the information content of the current nominal interest rate for those agents, private or official, who use the interest rate in predicting the current price level. We employ a general approach to calculating conditional price predictions in models of the type employed here.[8/] This approach highlights the information filtering problem faced by agents with incomplete current aggregate information. It also facilitates the discussion in later sections of how this information can be optimally exploited by wage setters or the monetary authorities in their attempts to minimize wage setters' price prediction error.

We begin by noting that agents who use the current interest rate in making their price predictions need predict only the aggregate demand disturbance u_t. This observation can be confirmed by inspecting the goods market equilibrium condition. Equating the right hand sides of equations (1a) and (1b) and rearranging yields an expression for the equilibrium price:

$$(4) \quad p_t = \overset{(+)}{p}_{t|t-1} - (\Delta/\theta)[r_t - (\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1})] + (1/\theta)u_t.$$

The point is that agents who use the current interest rate, r_t, know or can calculate everything on the right hand side of (4) except u_t. They have enough information to calculate wage setters' predictions and investors' predictions whether or not those predictions are based on the current interest rate. The price prediction of agents who use the interest rate is

$$(5) \quad \overset{+}{p}_{t|t-1} = \overset{(+)}{p}_{t|t-1} - (\Delta/\theta)[r_t - (\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1})] + (1/\theta)\overset{+}{u}_{t|t-1},$$

where $\overset{+}{u}_{t|t-1}$ is the expected value of u_t conditioned on lagged information and the current interest rate. Equation (5) is one of two key equations that are used repeatedly throughout this paper.

Next we determine what information about u_t agents can extract using their knowledge of r_t. Movements in the current interest rate are induced by disturbances in both the goods and money markets. In fact, since our model is linear, r_t depends upon a linear combination of the disturbances in the goods and money markets. The information content of r_t is this linear combination of u_t and v_t. Agents who use the interest rate can find this linear combination by using (1a) and (1b) to eliminate the unobservables y_t and p_t from (1c) and by rearranging to obtain

$$(6)\quad (1+\eta\theta)u_t + \theta v_t = \theta m_t + (\theta\lambda + \alpha)r_t - \alpha(\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1}) - \theta \overset{(+)}{p}_{t|t-1},$$

where $\alpha = \Delta(1 + \eta\theta)$. These agents can infer the linear combination of disturbances on the left-hand side of equation (6) because they either know or are capable of calculating everything on the right-hand side of the equation. In particular they can calculate the money supply m_t since by assumption all the information available to the monetary authorities is exploited by agents who use the interest rate.[9/] Equation (6) is the second of two key equations that are used repeatedly below.

And how is the information content of the interest rate used to predict u_t? The expected value of u_t conditional on $(1 + \eta\theta)u_t + \theta v_t$ is[10/]

$$(7)\quad \overset{+}{u}_{t|t-1} = \gamma[(1+\eta\theta)u_t + \theta v_t],$$

$$\gamma = \mathrm{cov}[u_t, (1+\eta\theta)u_t + \theta v_t]/\mathrm{var}[(1+\eta\theta)u_t + \theta v_t],$$

$$\gamma = (1+\eta\theta)\sigma_u^2/[(1+\eta\theta)^2\sigma_u^2 + \theta^2\sigma_v^2].$$

The price prediction error made by agents who use the interest rate is obtained by subtracting (5) from (4) to arrive at

$$(8)\quad p_t - \overset{+}{p}_{t|t-1} = (1/\theta)(u_t - \overset{+}{u}_{t|t-1}).$$

Since the prediction of u_t given by (7) minimizes the variance of agents' u_t prediction error, it also minimizes the agents' price prediction error. Given (7) the price prediction error (8) can be rewritten as

$$(9)\quad p_t - \overset{+}{p}_{t|t-1} = (1/\theta)\{u_t - \gamma[(1+\eta\theta)u_t + \theta v_t]\}.$$

The variance of this price prediction error simplifies to[11/]

$$(10) \quad \mathrm{var}(p_t - \overset{+}{p}_{t|t-1}) = \sigma_u^2\sigma_v^2/[(1 + \eta\theta)^2\sigma_u^2 + \theta^2\sigma_v^2].$$

Both the price prediction error and its variance are independent of the interest rate parameters Δ and λ. This result is interesting because the interest rate is the current variable that agents observe. The parameters which do affect the price prediction error and its variance are the price level and output parameters θ and η; the price level and output are the variables that the agents do not directly observe. It should be noted that expressions (9) and (10) are valid whether or not wage setters or investors use the interest rate.

One further result is very important in what follows. Agents who use the interest rate never have a price prediction error variance greater than agents who use only information available at the end of period t - 1:

$$(11) \quad \mathrm{var}(p_t - \overset{+}{p}_{t|t-1}) \leq \mathrm{var}(p_t - p_{t|t-1}).$$

Better informed agents can always achieve at least as low a price prediction error as less well informed agents since they are capable of calculating the price prediction of the less well informed agents.[12/]

IV. THE REDUNDANCY OF MONETARY POLICY WHEN WAGE SETTERS USE THE INTEREST RATE

If wage setters use the interest rate, then equations (1) become

$$(12a) \quad y_t = \theta(p_t - \overset{+}{p}_{t|t-1}),$$

$$(12b) \quad y_t = -\Delta[r_t - (\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1})] + u_t,$$

(12c) $m_t - p_t = \eta y_t - \lambda r_t + v_t.$

When wage setters are agents who use the interest rate, any monetary policy based on the interest rate is simply redundant.[13/] The wage setters' price prediction error and its variance are given by equations (9) and (10) of the last section. Both this price prediction error and its variance are independent of monetary policy because the information content of the interest rate, the linear combination of u_t and v_t given by equation (6), is independent of monetary policy based on the interest rate. Thus we have shown that monetary policy is redundant when wage setters and the monetary authorities use the same incomplete current aggregate information. This result is an extension of the Sargent and Wallace [1975] result that monetary policy is redundant when wage setters and the monetary authorities use the same lagged information.

Our result could be interpreted as implying that monetary policy is "ineffective" when wage setters and the monetary authorities use the interest rate.[14/] We prefer to use the word "redundant" because the result can also be interpreted as implying that monetary policy is quite important. If the monetary authorities exploit the information content of the interest rate, then the use of interest rate by wage setters is redundant.

In the next two sections we consider two different monetary policies designed to take advantage of the information conveyed by the interest rate. Both of these policies succeed in making $\overset{+}{p}_{t|t-1}$, the price prediction based on lagged information and the current interest rate, equal to $p_{t|t-1}$, the prediction based only on lagged information. Both reduce $\mathrm{var}(p_t - p_{t|t-1})$ to $\mathrm{var}(p_t - \overset{+}{p}_{t|t-1})$ thereby making it unnecessary for wage setters to use the interest rate. In one case the monetary authorities can fulfill this role even though they use information that is no more recent than the information used by wage setters and less recent than that used by investors.

V. OPTIMAL MONETARY POLICY WHEN THE MONETARY AUTHORITIES USE THE INTEREST RATE

In this section we show that if the monetary authorities use the interest rate, the optimal contemporaneous feedback rule is the same as the one derived by Poole [1970]. The monetary authorities' optimal rule is the same whether or not investors use the interest rate. Monetary policy is not redundant because we assume that wage setters do not use the interest rate.

If wage setters do not use the interest rate, then equations (1) become

$$\text{(13a)} \quad y_t = \theta(p_t - p_{t|t-1}),$$

$$\text{(13b)} \quad y_t = -\Delta[r_t - (\overset{(+)}{p_{t+1|t-1}} - \overset{(+)}{p_{t|t-1}})] + u_t,$$

$$\text{(13c)} \quad m - p_t = \eta y_t - \lambda r_t + v_t.$$

The symbol (+) has been retained in equation (13b) to indicate that investors may or may not use the interest rate. Both cases can be analyzed using equations (13).

Now, suppose the monetary authorities adjust the current money supply in response to the incomplete current information conveyed by the current interest rate. We establish that they should use a rule of the form

$$\text{(14)} \quad m_t = \beta(r_t - r_{t|t-1}),$$

and that β should be chosen according to the principles developed by Poole (1970). This monetary policy is optimal because it succeeds in lowering the price prediction error of wage setters down to the irreducible price prediction error of agents who use the interest rate.

It is convenient to begin by solving for some of the expectational variables. As a first step it is shown that given a contemporaneous feedback of the form (14),

$$(15) \qquad \overset{+}{p}_{t+1|t-1} = p_{t+1|t-1} = 0.$$

To derive these results it is necessary to obtain the difference equation that generates price predictions. Using (13a) and (13b) to eliminate y_t and r_t in (13c) leads to an expression for p_t:

$$(16) \qquad p_t = \lambda(\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1}) - (\eta + \lambda/\Delta)\theta(p_t - p_{t|t-1}) + m_t + \underset{(+)}{(\lambda/\Delta)}u_t - v_t.$$

Forwarding (16) by j periods, taking expected values conditioned on $|t-1$ information and rearranging produces the required difference equation:

$$(17) \qquad \overset{(+)}{p}_{t+j|t-1} = [\lambda/(1 + \lambda)]\overset{(+)}{p}_{t+j+1|t-1} + [1/(1 + \lambda)]\overset{+}{m}_{t+j|t-1}, \qquad j \geq 1.$$

The monetary policy rule (14) implies

$$(18) \qquad \overset{(+)}{m}_{t+j|t-1} = 0, \qquad j \geq 1.$$

After equation (18) is substituted into equation (17), the resulting difference equation can be solved by iterating forward starting at $j = 1$ to arrive at

$$(19) \qquad \overset{(+)}{p}_{t+1|t-1} = \lim_{T\to\infty}[\lambda/(1 + \lambda)]^{T-1}\overset{(+)}{p}_{t+T|t-1}.$$

Ruling out "speculative bubbles" by assuming that $\overset{(+)}{p}_{t+T|t-1}$ is finite, yields (15).[15]

As a second step, it is demonstrated that

$$(20) \quad p_{t|t-1} = r_{t|t-1} = 0.$$

To derive these results, first take the expectations of equations (13) conditioned on lagged information. Then, simplify the three resulting equations by noting that equation (14) implies that $m_{t|t-1}$ equals zero and that $p_{t+1|t-1}$ equals zero from (15). Finally, solve the three simplified equations for $y_{t|t-1}$, $p_{t|t-1}$, and $r_{t|t-1}$. It turns out that all three of these variables must be equal to zero.

We are now ready to derive an optimal monetary policy. To do so, we use versions of the key equations (5) and (6) that reflect the assumptions of this section. These equations are obtained by taking the (+) symbol off of wage setters' predictions:

$$(21) \quad \overset{+}{p}_{t|t-1} = p_{t|t-1} - (\Delta/\theta)[r_t - (\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1}) + (1/\theta)\overset{+}{u}_{t|t-1},$$

$$(22) \quad (1 + \eta\theta)u_t + \theta v_t = \theta m_t + (\theta\lambda + \alpha)r_t - \alpha(\overset{(+)}{p}_{t+1|t-1} - \overset{(+)}{p}_{t|t-1}) - \partial p_{t|t-1}.$$

The question is this: can the monetary authorities choose a contemporaneous feedback rule of the form posited in (14) which insures that the optimality condition $\overset{+}{p}_{t|t-1} = p_{t|t-1}$ is satisfied?[16] To see that they can, impose the optimality condition that $\overset{+}{p}_{t|t-1}$ be equal to $p_{t|t-1}$ which, in turn, equals zero from (20), and then eliminate $(1 + \eta\theta)u_t + \theta v_t$ using (7) and $\overset{(+)}{p}_{t+1|t-1}$ using (15). Equations (21) and (22) reduce to two optimality equations:

$$(23)\qquad 0 = -\Delta r_t + \overset{+}{u}_{t|t-1}.$$

$$(24)\qquad (1/\gamma)\overset{+}{u}_{t|t-1} = \theta m_t + (\theta\lambda + \alpha)r_t,$$

These optimality equations are satisfied if the monetary authorities set m_t according to the rule

$$(25)\qquad m_t = [-\lambda + (\Delta - \gamma\alpha)/\gamma\theta]r_t = \overset{*}{\beta} r_t,$$

$$\overset{*}{\beta} = -\lambda + [\Delta\theta/(1+\eta\theta)](\sigma_v^2/\sigma_u^2),$$

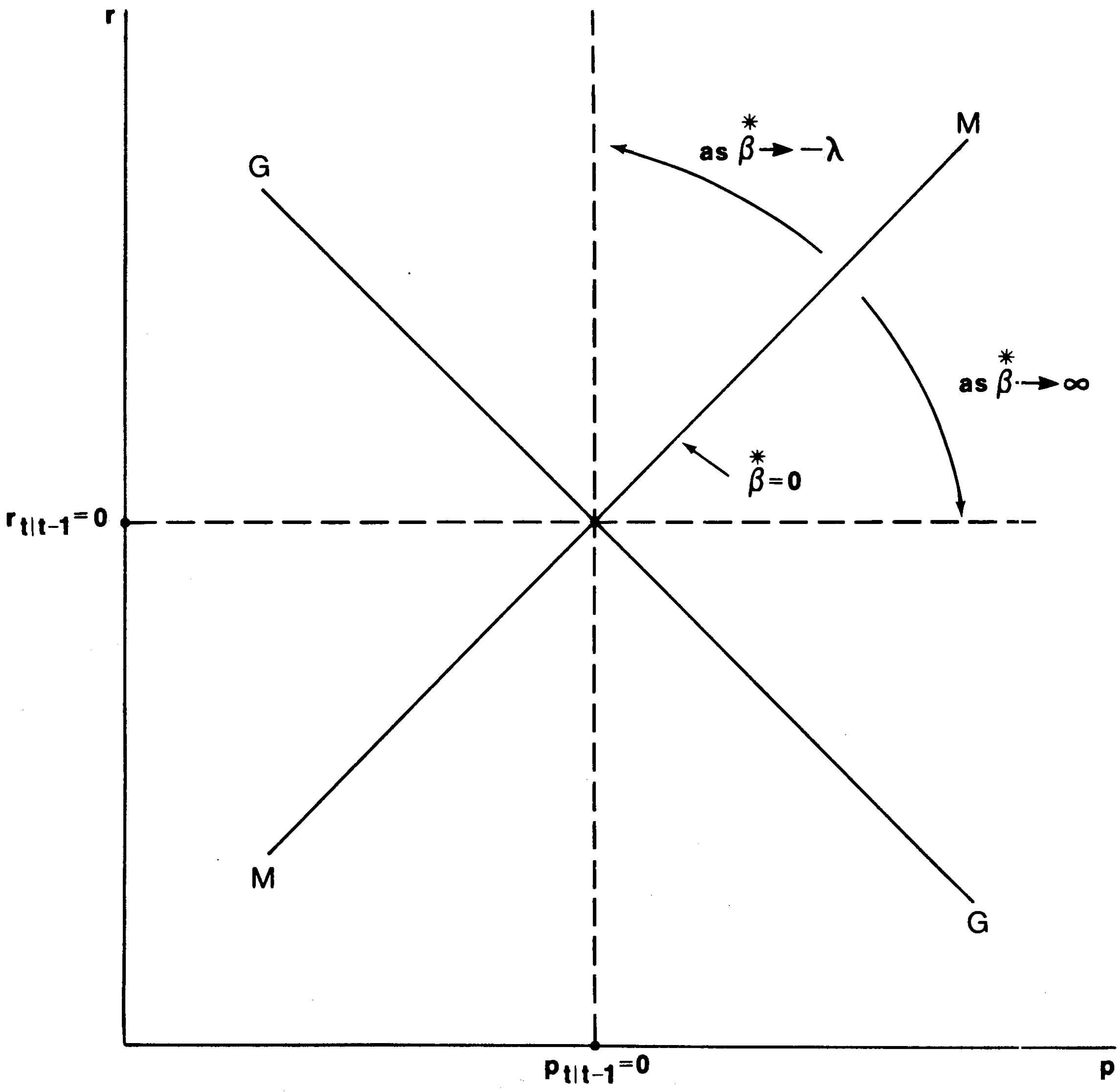
where use has been made of the definitions of α and γ given below equations (6) and (7) respectively. The monetary rule of equation (25) is indeed of the form posited in equation (14) since $r_{t|t-1} = 0$ from (20).

Now we show that our results are consistent with those derived by Poole [1970] from a standard IS-LM model. To do this we derive an optimal goods market equilibrium schedule and an optimal money market equilibrium schedule that together can be employed to represent the equilibrium of our model in nominal interest rate-price level space. The optimal goods market equilibrium condition is obtained by equating the right-hand sides of equations (13a) and (13b), recognizing that $\overset{(+)}{p}_{t+1|t-1} = 0$ from (15), and noting that $\overset{(+)}{p}_{t|t-1} = p_{t|t-1} = 0$ from the optimality condition and (20):

$$(26)\qquad \theta p_t = -\Delta r_t + u_t.$$

The optimal goods market equilibrium schedule, GG, in Figure 1 is derived using equation (26) with u_t set equal to zero. The "modified" optimal money market equilibrium condition is obtained by substituting the

FIGURE 1. Optimal contemporaneous feedback rule: Slope of MM schedule depends on optimal reaction parameter



right-hand side of equation (13a) for y_t in equation (13c), recognizing that $m_t = \overset{*}{\beta} r_t$, and recalling that $p_{t|t-1} = 0$ from (20):

$$(27) \qquad \overset{*}{\beta} r_t - p_t = \eta\theta p_t - \lambda r_t + v_t.$$

The optimal money market equilibrium schedule, MM, in Figure 1 is derived using equation (27) with v_t set equal to zero. While the slope of the optimal GG schedule is independent of $\overset{*}{\beta}$, the slope of the optimal MM schedule,

$$(28) \qquad (dr_t/dp_t)_{MM} = (1 + \eta\theta)/(\overset{*}{\beta} + \lambda),$$

depends on $\overset{*}{\beta}$ and, therefore, on the relative sizes of the disturbances to goods and money markets.[17/] It is easy to confirm that $\overset{*}{\beta} \to \infty$ as $\sigma_v^2/\sigma_u^2 \to \infty$ and that $\overset{*}{\beta} \to -\lambda$ as $\sigma_v^2/\sigma_u^2 \to 0$. Under our optimal contemporaneous feedback rule--just as under Poole's [1970] combination policy--if money market disturbances predominate, the optimal money market equilibrium schedule is almost horizontal, and if goods markets disturbances predominate, the optimal money market equilibrium schedule is almost vertical.

When goods market disturbances are relatively small, it is optimal to minimize the variance of the (investors' expected) real interest rate. Under the optimal contemporaneous feedback rule, the real interest rate is equal to the nominal interest rate:

$$(29) \qquad r_t - (\overset{(+)}{p_{t+1|t-1}} - \overset{(+)}{p_{t|t-1}}) = r_t,$$

as shown in the derivation of the optimal goods market equilibrium condition (26). Thus, the optimal contemporaneous feedback rule stabilizes the real interest rate by stabilizing the nominal interest rate. However, when goods market disturbances are relatively large, the optimal contemporaneous feedback rule induces offsetting changes in the real interest rate even larger than those that would occur if the money stock were held constant by causing changes in the nominal interest rate.

It is worth emphasizing that the optimal rule for monetary policy is the same whether or not investors use the interest rate in making their inflation predictions; the derivations in this section apply in either case. The explanation of this result is straightforward: the optimal contemporaneous feedback rule makes inflation predictions based on the current interest rate indentical to those that are based only on lagged information. Under any contemporaneous feedback rule of the form of equation (14), $\overset{+}{p}_{t+1|t-1} = p_{t+1|t-1}$, and under the optimal one the monetary authorities, in satisfying their objective of minimizing wage setters' price prediction errors, see to it that $\overset{+}{p}_{t|t-1} = p_{t|t-1}$ as well. So the optimal contemporaneous monetary feedback rule makes it unnecessary for investors as well as wage setters to use the interest rate; this is another redundancy result.

VI. OPTIMAL MONETARY POLICY WHEN ONLY INVESTORS USE THE INTEREST RATE

Suppose that only investors use the interest rate and that wage setters and the monetary authorities base their decisions solely on lagged information.[18/] We show that a lagged feedback rule for the monetary authorities works just as well as the contemporaneous feedback rule analyzed in the preceding section; it causes $\overset{+}{p}_{t|t-1}$ to equal $p_{t|t-1}$. Although this rule has the same implications for the expected real interest rate as the contemporaneous feedback rule, it has very different implications for the nominal interest rate.

If only investors use the interest rate, then equations (1) become

(30a) $y_t = \theta(p_t - p_{t|t-1})$,

(30b) $y_t = -\Delta[r_t - (\overset{+}{p}_{t+1|t-1} - \overset{+}{p}_{t|t-1})] + u_t$,

(30c) $m_t - p_t = \eta y_t - \lambda r_t + v_t$.

Now, suppose the monetary authorities use a lagged feedback rule of the form

(31) $m_t - m_{t-1} = \phi(r_{t-1} - r_{t-1|t-2})$.

When equation (31) is forwarded by one period to obtain

(32) $m_{t+1} - m_t = \phi(r_t - r_{t|t-1})$,

it becomes clear that this rule links the current growth rate of money directly to the current innovation in the interest rate. The lagged feedback rule can affect the variance of current output because it succeeds in relating investors' inflation prediction to the current interest rate. Under the optimal lagged feedback rule, it is the slope of the goods market equilibrium schedule rather than the slope of the money market equilibrium schedule that is made to depend on the relative sizes of the disturbances to the goods and money markets.

As in the preceding section we begin by solving for some of the expectational variables. As a first step it is shown that

$$(33) \quad p^{+}_{t+1|t-1} = m_t + \phi(r_t - r_{t|t-1}) = m_{t+1}.$$

for any value of ϕ. To do this we follow the procedure employed to derive equation (15). Using (30a) and (30b) to eliminate y_t and r_t in (30c) and rearranging leads to an equation for p_t:

$$(34) \quad p_t = \lambda(p^{+}_{t+1|t-1} - p^{+}_{t|t-1}) - (\eta + \lambda/\Delta)\theta(p_t - p_{t|t-1}) + m_t + (\lambda/\Delta)u_t - v_t.$$

Forwarding equation (34) by j periods, taking expected values conditioned on $^{+}|t-1$ information, and rearranging yields

$$(35) \quad p^{+}_{t+j|t-1} = [\lambda/(1+\lambda)]p^{+}_{t+j+1|t-1} + [1/(1+\lambda)]m^{+}_{t+j|t-1}, \quad j \geq 1.$$

The monetary policy rule (32) implies

$$(36) \quad m^{+}_{t+j|t-1} = m_t + \phi(r_t - r_{t|t-1}).$$

After equation (36) is substituted into equation (35), the resulting difference equation can be solved by iterating forward beginning with j = 1 to arrive at

$$(37) \quad p^{+}_{t+1|t-1} = \lim_{T\to\infty}[\lambda/(1+\lambda)]^{T-1}p^{+}_{t+T|t-1} + m_t + \phi(r_t - r_{t|t-1}).$$

Ruling out "speculative bubbles" yields the left-hand equality in (33). Of course, the right-hand equality follows from (32).

As a second step, it is noted that

$$(38) \quad p_{t|t-1} = m_t \text{ and } r_{t|t-1} = 0.$$

To confirm these results, take expectations conditioned on $|t-1$ information of equations (30). Then eliminate $p_{t+1|t-1}$ and $m_{t|t-1}$ using the relationships

$p_{t+1|t-1} = m_{t|t-1} = m_t$ implied by equations (33) and (31). The resulting equations can be solved for $y_{t|t-1}$, $p_{t|t-1}$, and $r_{t|t-1}$ in terms of m_t. The solutions for $p_{t|t-1}$ and $r_{t|t-1}$ are given by equations (38).

Now we are ready to prove that a lagged monetary feedback rule of the form (32) is optimal since it causes $\overset{+}{p}_{t|t-1}$ to equal $p_{t|t-1}$. We follow the procedure employed in the preceding section to show that a contemporaneous feedback rule of the form we posited was optimal. Specializing the key equations (5) and (6) to the case in which only investors use the interest rate yields

$$(39) \qquad \overset{+}{p}_{t|t-1} = p_{t|t-1} - (\Delta/\theta)[r_t - (\overset{+}{p}_{t+1|t-1} - \overset{+}{p}_{t|t-1})] + (1/\theta)\overset{+}{u}_{t|t-1},$$

$$(40) \qquad (1 + \eta\theta)u_t + \theta v_t = \theta m_t + (\theta\lambda + \alpha)r_t - \alpha(\overset{+}{p}_{t+1|t-1} - \overset{+}{p}_{t|t-1}) - \theta p_{t|t-1}.$$

Here the question is this: can the monetary authorities find a lagged feedback rule of the form posited in (32) which insures that the optimality condition $\overset{+}{p}_{t|t-1} = p_{t|t-1}$ is satisfied? To see that they can, impose the optimality condition that $\overset{+}{p}_{t|t-1}$ be equal to $p_{t|t-1}$ which, in turn, equals m_t from (38) and then eliminate $(1 + \eta\theta)u_t + \theta v_t$ using (7) and $\overset{+}{p}_{t+1|t-1}$ using (33). Equations (39) and (40) reduce to two optimality equations:

$$(41) \qquad 0 = - \Delta[r_t - (m_{t+1} - m_t)] + \overset{+}{u}_{t|t-1},$$

$$(42) \qquad (1/\gamma)\overset{+}{u}_{t|t-1} = (\theta\lambda + \alpha)r_t - \alpha(m_{t+1} - m_t).$$

These optimality equations are satisfied if the monetary authorities set the money supply according to the lagged feedback rule

(43) $\quad m_{t+1} - m_t = [1 + \theta\lambda\gamma/(\alpha\gamma - \Delta)]r_t = \overset{*}{\phi} r_t,$

$$\overset{*}{\phi} = 1 - [\lambda(1 + \eta\theta)/\Delta\theta](\sigma_u^2/\sigma_v^2),$$

where use has been made of the definitions of α and γ given below equations (6) and (7) respectively. The monetary rule of equation (43) is indeed consistent with the rule posited in (32) since $r_{t|t-1} = 0$ from (38).

Now we show that under the optimal lagged feedback rule the slope of the goods market equilibrium schedule depends on the relative sizes of the disturbances to the goods and money markets while the slope of the money market equilibrium schedule is independent of the relative sizes of these disturbances. The optimal goods market equilibrium condition is obtained by equating the right-hand sides of equations (30a) and (30b) while recognizing that $\overset{+}{p}_{t+1|t-1} = m_{t+1}$ from (33), that $\overset{+}{p}_{t|t-1} = p_{t|t-1} = m_t$ from the optimality condition and (38), and that $m_{t+1} - m_t = \overset{*}{\phi} r_t$:

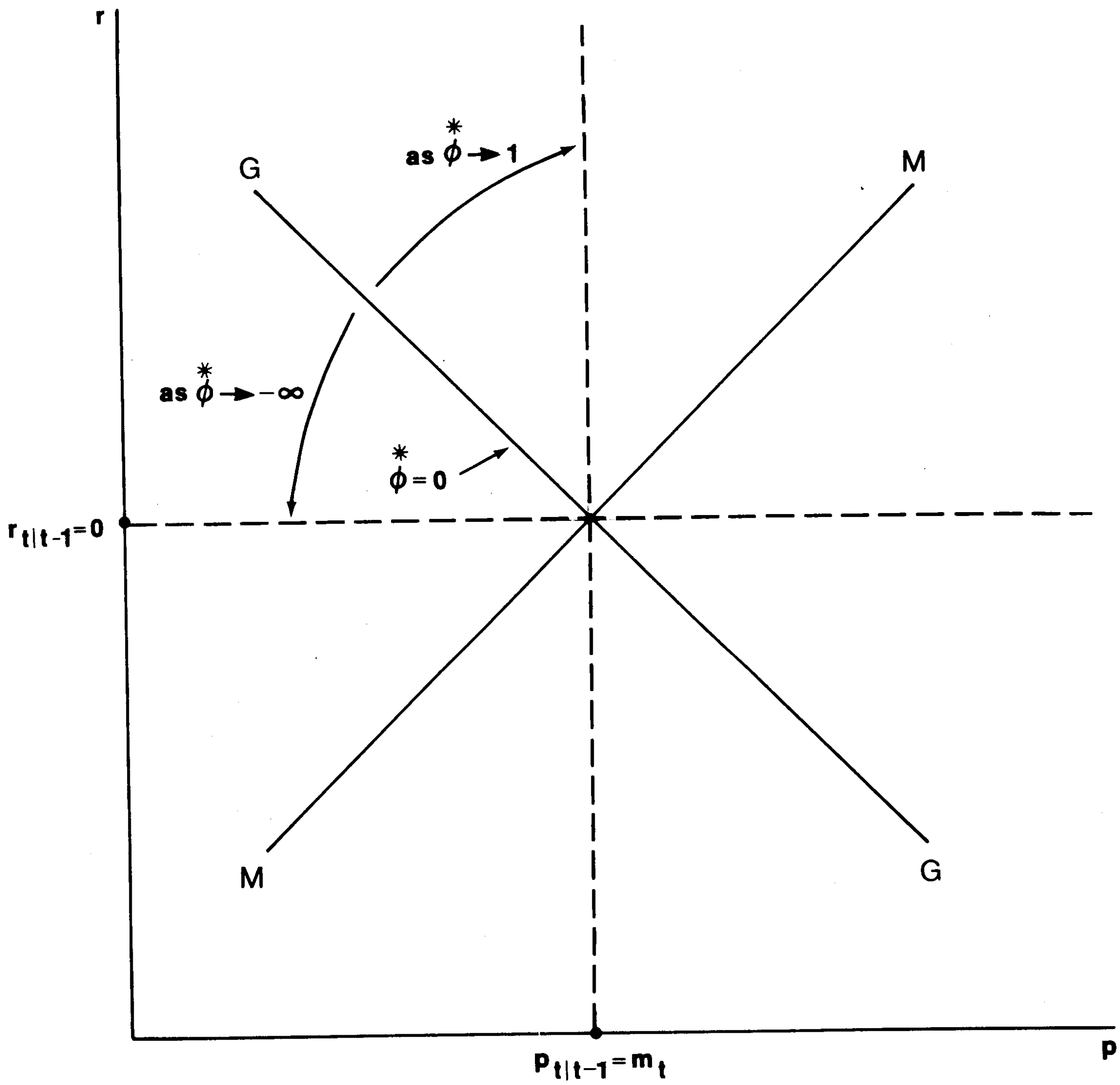
(44) $\quad \theta(p_t - m_t) = - \Delta(1 - \overset{*}{\phi})r_t + u_t.$

The optimal goods market equilibrium schedule, GG, in Figure 2 is derived using equation (44) with $u_t = 0$. The modified optimal money market equilibrium condition is obtained by substituting the right-hand side of (30a) for y_t in equation (30c) and recognizing that $p_{t|t-1} = m_t$ from (38):

(45) $\quad m_t - p_t = \eta\theta(p_t - m_t) - \lambda r_t + v_t.$

The optimal money market equilibrium schedule, MM, in Figure 2 is derived using equation (45) with $v_t = 0$. The slope of the optimal goods market equilibrium schedule is

FIGURE 2. Optimal lagged feedback rule: Slope of GG schedule depends on optimal reaction parameter



$$(46) \quad (dr_t/dp_t)_{GG} = -\theta/\Delta(1 - \overset{*}{\phi}),$$

and the slope of the optimal money market equilibrium schedule is independent of $\overset{*}{\phi}$. It is easy to confirm that $\overset{*}{\phi} \to 1$ as $\sigma_u^2/\sigma_v^2 \to 0$ and that $\overset{*}{\phi} \to -\infty$ as $\sigma_u^2/\sigma_v^2 \to \infty$. Thus, if money market disturbances predominate, the optimal GG schedule is almost vertical, and, if goods market disturbances predominate, the optimal GG schedule is almost horizontal.

The optimal lagged feedback rule has the same implications for the (investors' expected) real interest rate as the optimal contemporaneous feedback rule. Under the optimal lagged feedback rule, the real interest rate is given by

$$(47) \quad r_t - (\overset{+}{p}_{t+1|t-1} - \overset{+}{p}_{t|t-1}) = (1 - \overset{*}{\phi})r_t$$

as shown in the derivation of the optimal goods market equilibrium condition. When goods market disturbances are relatively small, the monetary authorities try to insure that the variance of real interest rates is small. They do this by setting $\overset{*}{\phi}$ near one so that nominal interest rate innovations induce nearly offsetting expected inflation rate innovations. When goods market disturbances are relatively large, the optimal lagged feedback rule generates large offsetting changes in the real interest rate by causing small nominal interest rate innovations to induce large reinforcing expected inflation rate innovations.

However, the optimal lagged feedback rule has implications for the nominal interest rate that are quite different from those of the optimal contemporaneous feedback rule. The reason for this difference is that the lagged feedback rule operates to change the real interest rate by inducing changes in both the expected inflation rate and the nominal interest rate while the contemporaneous feedback rule operates by inducing changes in the nominal interest rate alone. As a result, the lagged feedback rule yields a low (conditional) variance for the nominal interest rate when goods market disturbances predominate in contrast to the contemporaneous feedback rule which yields a low (conditional) variance for the nominal interest rate when money market shocks predominate.

An examination of the optimal modified money market equilibrium condition (45) suggests an explanation for why a low nominal interest rate variance makes it easier for wage setters to predict the price level when money market disturbances are small. Given a lagged money rule, wage setters know the current money supply before they set wages. Thus, if money market disturbances are small and if the monetary authorities can keep the variance of the nominal interest rate low, wage setters can use the money market equilibrium condition to make an accurate prediction of the current price level.

However, the optimal lagged feedback rule does not minimize nominal interest rate changes when money market disturbances are large. Under a lagged feedback rule the current nominal money supply is not allowed to change in response to a contemporaneous disturbance. Therefore, it is desirable to allow the nominal interest rate to change. Otherwise a large, unanticipated movement in the price level would be required to equilibrate the money market. The goods market remains insulated from nominal interest rate change because the lagged rule induces an offsetting movement in the expected inflation rate. Of course, it is optimal to have the entire change in expected inflation take place through a change in investors' prediction of next period's price level, so that investors' prediction of this period's price level coincides with the prediction of wage setters.

VII. CONCLUSIONS

Under rational expectations private agents, as well as the monetary authorities, may make use of the incomplete current information conveyed by the current interest rate. We show how private agents solve the information filtering problem they face when forming their price expectations. In this environment the monetary authorities can insure that wage setters benefit from superior information used by other agents, either the authorities themselves or investors, thereby saving wage setters the costs of indexing.

Viewed one way, our results confirm those of Poole [1970]. The underlying mechanism through which monetary policy works is the same. Optimal monetary policy stabilizes the real interest rate when goods market disturbances are relatively small and amplifies offsetting movements in the real interest rate when goods market disturbances are relatively large.

Viewed another way, our results imply that Poole's results must be qualified in an important way. If some private agents use more recent information than the monetary authorities, the practical policy prescription is just the opposite of Poole's. When using a lagged feedback rule, the authorities should act so as to stabilize the nominal interest rate when goods market disturbances predominate. Only if no one uses more recent information than the monetary authorities is the practical policy prescription the same as Poole's. When using a contemporaneous feedback rule, the authorities should stabilize the nominal interest rate when money market disturbances predominate.

FOOTNOTES

*/ Economists, Board of Governors of the Federal Reserve System. This paper is a revised version of a paper presented at the Federal Reserve System Committee on Business Analysis in June 1981. It represents a substantial extension of Canzoneri [1980]. We have benefited from comments by John Boschen, Patrick Corcoran, Robert Flood, Bennett McCallum, and Douglas Waldo. The paper represents the views of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

1/ Sargent and Wallace [1975] show that monetary policy is redundant when private agents and the monetary authorities make rational use of the same lagged information.

2/ Woglom [1979] derives a contemporaneous feedback rule under the assumption that wage setters do not use the interest rate. The question of whether investors use the interest rate to predict the inflation rate does not arise in Woglom's model because he assumes that investors base their decisions on the nominal rather than the real interest rate.

3/ Recently, Weiss [1980], King [forthcoming], and Waldo [1981] have demonstrated that a monetary policy based on lagged information can be effective when there are certain kinds of differences in information utilization among private agents. These authors find that lagged feedback rules can reduce the variance of output around its "natural rate" to zero. They obtain this result because they consider only one type of aggregate disturbance. Turnovsky [1980] analyzes a lagged feedback rule in a model with more than one aggregate disturbance. However, he too finds that the variance of output can be reduced to zero. He obtains this result even in the presence of more than one aggregate disturbance because he allows investors to have complete current information.

4/ The trend values of y_t, p_t, p_{t+1}, m_t, and r_t are the (not necessarily constant) expected values of those variables based on information available at the end of period $t - 1$.

5/ We assume that all agents, official and private, have indirect utility functions that are quadratic in the price prediction errors for the prices that are unknown to them. Under this assumption agents maximize their expected utility by setting their price predictions equal to the conditional expectations of the unknown prices whether or not the disturbance terms, and therefore prices, are normally distributed. However, assuming normality makes the calculation of conditional expectations straightforward as shown, for example, in Meditch [1969], chapter 3. We also assume that agents subjective expectations are identical to the mathematical expectations derived from the model.

If it is assumed instead that agents' utility functions are general concave functions of price prediction errors, then the normality assumption is sufficient to insure that agents maximize their expected utility by choosing price predictions equal to their conditional expectations.

6/ The equivalence obtains because we assume that there is no productivity disturbance (no disturbance term in the aggregate supply function). As shown by Gray [1976], the equivalence no longer obtains when this assumption is relaxed. This equivalence is not necessary for our results, but it does make it easier to derive them.

7/ The variance of output around its natural rate is a measure of the economic inefficiency that both the monetary authorities and wage setters are trying to eliminate. We use a "contracting" approach, like the one employed by Fischer [1977] and Gray [1976], in deriving the aggregate supply function (1a), but there are other ways to motivate this supply function; Taylor [1979] provides a good survey. The contracting approach has empirical foundation, but has been criticized as lacking the theoretical foundation of the "islands" approach. However, the two approaches may turn out to be very similar. As Lucas [1981] sees it,

> "[n]one of these [contracting] models offers an explanation as to why people should choose to bind themselves to contracts which seem to be in no one's self-interest, and my conjecture is that when reasons for this are found they will reduce to the kind of informational difficulties already stressed in my 1972 article, for example."

8/ Kareken, Muench, and Wallace [1973] and Leroy and Waud [1977] show how the monetary authorities should use this general approach to form conditional predictions of their target variable. In our model private agents as well as the monetary authorities follow this approach in calculating their conditional price predictions.

9/ In the text we implicitly assume that the monetary authorities set the money supply exactly. Suppose instead that the money supply process had a random component, e_t. Our analysis would be unaffected if m_t were regarded as the "systematic" part of monetary policy and e_t were incorporated into the money market disturbance v_t in equation (1c).

10/ See Meditch [1969], chapter 3.

11/ The model of equations (1) does not subsume the case in which the aggregate demand function is given by

$$(1b') \qquad y_t = - \Delta[r_t - (\overset{(+)}{p}_{t+1|t-1} - p_t)] + u_t.$$

In (1b') investors' expected rate of inflation depends on p_t rather than $\overset{(+)}{p}_{t|t-1}$. If (1b') is used instead of (1b), the results are very similar to those reported throughout the text. The variance of the price prediction error of an agent who uses the interest rate would be somewhat lower with (1b') than with (1b):

$$(10') \quad var(p_t - \overset{+}{p}_{t|t-1}) = \sigma_u^2\sigma_v^2/[(1 + \eta\theta)^2\sigma_u^2 + (\theta + \Delta)^2\sigma_v^2].$$

This variance is lower because a given increase in p_t causes a larger reduction in the excess demand for goods since it decreases demand as well as increasing supply. The variance of the price prediction error is lower even though $var(u_t - \overset{+}{u}_{t|t-1})$ is higher.

12/ A straightforward way to prove this result is to note that $p_t - p_{t|t-1}$ and $r_t - r_{t|t-1}$ have a joint normal distribution. This is true because the underlying shocks u_t and v_t are normally distributed, and the model is linear. The inequality (11) is a property of joint normal distributions. See De Groot [1975], p. 250.

13/ It should be noted that this result holds even if investors use the interest rate. In an otherwise very useful survey of rational expectations macroeconomic models, McCallum [1980] states incorrectly that monetary policy can be effective in this case.

14/ McCallum [1980] and others have referred to the Sargent and Wallace [1975] result as the "policy ineffectiveness result."

15/ In ruling out expected price (deviation) paths that increase or decrease without limit, we are following the suggestion of Sargent (1973). Note that the arguments used in the text to establish the results in equations (15) and (20) are clearly valid if β is finite. An expression for the optimal value of β, denoted by $\overset{*}{\beta}$, is given below equation (25). From that expression it is evident that the assumptions that σ_v^2 is finite and that $\sigma_u^2 > 0$ are necessary and sufficient to ensure that $\overset{*}{\beta}$ is finite. In the appendix we explain why the proofs of the results in equations (15) and (20) apply even in the extreme case in which $\overset{*}{\beta} \to \infty$ ($\sigma_v^2 \to \infty$ or $\sigma_u^2 = 0$).

16/ This rule must be of the form posited in (14) because we have imposed that form in deriving (15) and (20).

17/ It might be thought that if investors use the interest rate, then the optimal value of β would be different than if they did not, and the slope of the optimal goods market equilibrium schedule would depend on this different optimal value. It is true that for any value of β other than the $\overset{*}{\beta}$ of equation (25), investors' prediction of the current price level, $\overset{+}{p}_{t|t-1}$, is linked to the current interest rate, and the slope of the goods market equilibrium schedule depends on β. However, when the monetary authorities set β equal to the $\overset{*}{\beta}$ of equation (25), they fulfill the optimality condition that $\overset{+}{p}_{t|t-1}$ always be equal to $p_{t|t-1}$ (which has the same constant value whether or not investors use the interest rate.) Therefore, they make $\overset{+}{p}_{t|t-1}$

independent of r_t, and cause the slope of the goods market equilibrium condition to be the same as it would be if investors did not use the interest rate. It is worth noting that if the aggregate demand function were given by equation (1b') in footnote 11, the slope of the goods market equilibrium condition would be independent of β no matter what its value.

18/ We want to consider how monetary policy works when investors alone use superior current information. The way this objective can be achieved in the familiar model we employ is to assume that investors use the interest rate while the monetary authorities do not.

19/ McCallum (forthcoming) discusses this indeterminacy problem and specifies another set of circumstances under which it would not arise.

20/ Incidentally, the introduction of overlapping contracts would not rule out this indeterminacy. Agents forming $p_{t|t-k}$ would encounter the same difficulty as agents forming $p_{t|t-1}$.

21/ It would also be met by a tax schedule that changed in real terms when the price level changed.

APPENDIX

The model of equations (1) is expressed in terms of deviations of variables from their trend values. In this appendix we confirm that if the monetary authorities announce the trend path of the nominal interest rate but do not also announce at least one point on a mutually consistent trend money supply path, then the trend price path is indeterminate. Then we show that certain arguments used in the text remain valid, even in the extreme case in which r_t is fixed at $r_{t|t-1}$ ($\overset{*}{\beta} \to \infty$).

We use the following model:

(A1a) $\tilde{Y}_t = k,$

(A1b) $\tilde{Y}_t = q - \Delta[\tilde{R}_t - (\tilde{P}_{t+1} - \tilde{P}_t)],$

(A1c) $\tilde{M}_t - \tilde{P}_t = \eta\tilde{Y}_t - \lambda\tilde{R}_t.$

$\tilde{Y}_t$, $\tilde{P}_t$, and $\tilde{M}_t$ are the (logarithms of the) period t trend values of output, the price level, and the money supply respectively. $\tilde{R}_t$ is the trend value of the nominal interest rate. k is the exogenous natural rate of output; q is an intercept term in the aggregate demand function, and $k \geq q$. In order to simplify the analysis we assume that k and q are constants, but similar conclusions could be derived if they varied over time.

First we show why the model of equations (1) is indeterminate if the monetary authorities announce only a constant trend nominal interest rate. Eliminating $\tilde{Y}_t$ from equations (A1b) and (A1c) using equation (A1a) and fixing the trend interest rate at $\bar{\tilde{R}}$ reduces the model of equations (A1) to

(A2a) $k = q - \Delta[\bar{\tilde{R}} - (\tilde{P}_{t+1} - \tilde{P}_t)],$

(A2b) $\tilde{M}_t - \tilde{P}_t = \eta k - \lambda\bar{\tilde{R}}.$

Equation (A2a) is a first-order difference equation in the trend price level, $\tilde{P}_t$, with a unit root. This equation is satisfied by any constant-inflation price path. Equation (A2b) does not imply any further restriction on the trend price path since fixing the interest rate just sets trend real balances. Agents who try to calculate $\tilde{P}_t$ recognize that for any trend price level they choose, there is a corresponding trend money supply which is consistent with the fixed interest rate. Therefore, $\tilde{P}_t$ and $\tilde{M}_t$ are indeterminate for all t.

Next we show that the trend price path is determinate if the authorities announce one value on a trend money supply path that is consistent with the constant trend nominal interest rate. Suppose the authorities set the initial trend money supply, $\tilde{M}_0$, at a particular value. Then $\tilde{P}_0$ is determined by (A2b). Suppose further that the authorities announce that the future path of the trend money supply will be the one implied by $\tilde{M}_0$ and $\bar{\tilde{R}}$. Agents can determine this path using equation (A2a):

$$\text{(A3)} \qquad \tilde{M}_{t+1} - \tilde{M}_t = \tilde{P}_{t+1} - \tilde{P}_t = \bar{\tilde{R}} + (1/\Delta)(k - q), \qquad t \geq 0.$$

Thus all the trend values of both the money supply and the price level are determined. Announcing a constant trend interest rate along with a consistent trend money supply path is a well-specified trend monetary policy, but simply announcing a constant trend interest rate is not.

Finally, we explain why the proofs of the results in equations (15) and (20) of the text apply even if r_t is fixed at $r_{t|t-1}$ ($\beta \to \infty$, $\sigma_v^2 \to \infty$ or $\sigma_u^2 = 0$). The only steps of these proofs that are affected when $\overset{*}{\beta} \to \infty$ are the steps at which it is argued that $m^{(+)}_{t+j|t-1} = 0$, $j \geq 1$, and $m_{t|t-1} = 0$. Here we show that these steps and, therefore, the entire proofs are valid even if $\overset{*}{\beta} \to \infty$. We assume that the monetary authorities announce either (1) the trend path of the money supply or (2) the trend path of the nominal interest rate along with one value on a mutually consistent trend money supply path. The variables $\overset{(+)}{m}_{t+j|t-1}$, $j \geq 1$, and $m_{t|t-1}$ are expected deviations of actual money supplies (M_{t+j}, $j \geq 0$) from known trend money supplies ($\tilde{M}_{t+j}$, $j \geq 0$). Given a contemporaneous feedback rule and the form of equations (1) these expected deviations are equal to zero:

$$\text{(A4)} \qquad \overset{(+)}{m}_{t+j|t-1} = M_{t+j|t-1} - \tilde{M}_{t+j} = 0, \qquad j \geq 1.$$

$$\text{(A5)} \qquad m_{t|t-1} = M_{t|t-1} - \tilde{M}_t = 0.$$

Indeed, in this situation, both the expected deviation of any variable dated $t + j$, $j \geq 1$ (conditioned on $\overset{(+)}{|}t - 1$ information) from a known trend value and the expected deviation of any variable dated t (conditioned on $|t - 1$ information) from a known trend value are zero. Thus, the proofs of the results in equations (15) and (20) of the text apply even if $\overset{*}{\beta} \to \infty$.

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