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CAPITAL ACCUMULATION AND FOREIGN INVESTMENT TAXATION

by

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1. Introduction

This paper provides a general equilibrim model of capital accumulation in open economies. Capital movements between countries result from differences in endowments. The paths over time of wages, interest rates, and capital-labor ratios under autarky and laissez-faire are described, welfare levels under autarky and laissez-faire are compared, and the effect of restricting capital mobility by foreign investment taxation is discussed.

The major results are that if two countries differ only in initial capital abundance, then in autarky the initially more capital-abundant country will always be more capital abundant and will always have higher wages and lower interest rates than the initially less capital-abundant country. However, both countries converge to the same steady state, if a stable steady state exists. Neither an autarky nor a laissez-faire equilibrium is necessarily Pareto optimal. The set of efficient allocations is characterized and seen to depend on the relationship between the path of interest rates and population growth. Autarky and laissez-faire are shown to be Pareto non-comparable. A move from autarky to capital mobility makes the original holders of capital in the relatively capital-abundant country better off, the original holders of capital in the labor-abundant country worse off, later generations in the labor-abundant country better off and later generations in the capital-abundant country worse off. There will be no unanimity as to the optimal level of foreign investment taxation in either country. The optimal taxes from the point-of-view of the original capital holders are derived.

The old in the relatively capital- (labor-) abundant country will prefer a smaller (larger) tax than that which would maximize current national income. In at least one country, a social planner trying to maximize steady-state utility will choose to tax foreign investment earnings.

The above, unusual results follow from the assumptions of the model, which is dynamic, choice-theoretic, has agents who are heterogenous with respect to endowments, and countries which are homogenous except for the size of the initial capital stock.

It is important that a model be dynamic because a policy implemented in a given period will affect savings behavior in that period, and hence, capital stocks in all future periods. A well-known result is that a capital-exporting, or importing, country can act as a monopolist, or monopsonist, and increase its current national income by restricting capital flows.[1/] However, such a policy affects the growth of the capital stock, representing a secondary burden or benefit to later generations.

The agents in the model possess non-homogenous endownments. Some agents are purely capitalists, endowed with their country's capital stock. Others are laborers and capitalists, earning labor and rental wages dependent upon the size of the capital stock during their lifetime. Thus, there is disagreement among agents as to the optimal degree of capital mobility. For example, capital outflows tend to benefit current capitalists at the expense of future laborers.

Much of the trade literature has ignored this problem of competing groups by appealing to the Kaldor criterion of potential Pareto optimality. Under this criterion a policy is deemed to be optimal if the winners could bribe the losers into accepting a policy change. This notion may be vacuous here due to the difficulties in carrying out such a bribe. Transactions costs and difficulties with the moral hazard problem of effecting a transfer between two competing groups at one period of time may be formidable. Additional complications occur when competing groups appear at different points in time.

The model employed here is a two-country, overlapping generations (OLG) model with production. It is an international version of Diamond's (1964) model, which combines a one-sector Solow growth model with Samuelson's OLG model. An OLG model in this setup is one where in each period a given number of agents are born. The agents live two periods and then die. In their first period of life agents born in period t may trade only with agents born in t - 1 or t. In their second period of life they may trade only with agents born in t + 1 or t. The model begins at period one. At this time the young of generation one and the old of generation zero are alive. It will be seen that in the first period of life agents save and in the second period of life agents consume the return on their savings. Thus, the abstraction of a two-period life captures the essential feature of a life-cycle model: the agents begin life by saving, and at some point start to dissave.

The possible non-optimality of both the autarky and laissez-faire equilibria result from the structure of the OLG model. The double-infinity of agents and goods violates the assumptions of the First Theorem of Welfare Economics.[2/] The optimality theorem presented here extends the results for an OLG model with constant returns on storage to one with a variable rate of return.

In most of the international trade literature, unhindered capital flows lead to efficiency. However, foreign investment taxation by both capital importing and capital exporting countries represents a Nash equilibrium in policy space. An examination of the stylized facts reveals that neither the Pareto optimal nor the Nash solutions of standards trade models are in fact observed. There has not been a free international capital market since 1930, and policies with respect to foreign investment taxation are extremely inconsistent--not just across countries, but within a given country over time.[3/]

The model presented here is not inconsistent with the stylized facts. Allowing free trade in capital leads to factor-price equalization, but not necessarily to efficiency and certainly not to a regime which is Pareto superior to autarky. There will be no agreement within a country as to what constitutes an optimal tax on foreign investment. Thus, countries will not have an optimal level of foreign investment taxation.

2. Capital Accumulation in an Open Economy

2.1 Production

In each period $t > 0$ firms use capital and labor to produce output. The capital is completely used up in the production process and the output is purchased by consumers. Part is consumed in period t and the remainder is saved to be sold to the firms as capital for the period $t + 1$ production process. There is a neoclassical production technology which is assumed to be linearly homogenous. Thus, output per unit of labor is a function only of the capital-labor ratio. The allocation of factors is perfectly competitive; hence the wages of capital and labor are equal to their respective marginal products. Capital mobility ensures that factor wages are equated across countries. The stock of labor at time t is equal to the population born at time t. The population is assumed to grow at a constant geometric rate.

Notation

$K^i(t)$: = the capital stock owned by citizens of country i at time t; $i = H,F$.

$G^i(t)$: = the capital stock located in country i at time t; $i = H,F$.

$L(t)$: = the number of people born in each country at time t.

$k^i(t)$: = $K^i(t)/L(t)$; $i = H,F$.

$g^i(t)$: = $G^i(t)/L(t)$; $i = H,F$.

$r(t)$: = the rental price of capital at time t.

$w(t)$: = the wage rate of labor at time t.

It is assumed that $k^H(1) > k^F(1)$; hence the home country is relatively capital abundant.

Linear homogenity implies

$$(1)\quad F(G^i(t), L(t)) = L(t)f(g^i(t));\ i = H,F,$$

where $F(\cdot,\cdot)$ is the production function and $f(\cdot)$ is the average product of labor function.

The production function is assumed to satisfy the usual neoclassical assumptions

$$(2)\quad f \in C^3;\ f > 0,\ f' > 0,\ f'' < 0,\ \text{for every } g > 0$$

$f \to 0$ as $g \to 0$, $f' \to \infty$ as $g \to 0$, $f' \to 0$ as $g \to \infty$.

Equating wages to marginal products implies

(3) $f(g^i(t)) - g^i(t)f'(g'(t)) = w(t);\ i = H,F.$

$f'(g^i(t)) = r(t);\ i = H,F,$

where a prime denotes a derivative. System (3) is a set of four equations in four unknowns: $g^H(t)$, $g^F(t)$, $w(t)$, and $r(t)$. By (2), $g^H(t)$, $g^F(t)$, and $w(t)$ can be solved as unique functions of $r(t)$. Thus,

(4) $g^i(t) = g(r(t));\ i = H,F$

$w(t) = w(r(t)).$

The desired capital-labor ratios are equalized across countries. Totally differentiating (3) yields

(5) $w'(r(t)) = - g(r(t)) < 0$

$g'(r(t)) = 1/f''(g[r(t)]) < 0.$

Thus wages and the desired capital-labor ratio are decreasing in the rental price of capital.

It will also be assumed that

(6) $\varepsilon > 1$,

where $\varepsilon := -g'r/g$ is the elasticity of demand for capital. Equation (6) says that demand is elastic, and is true for all Cobb-Douglas functions.

Labor growth is given by

(7) $L(t) = L_0 n^t$,

where L_0 is the initial stock of labor in each country

2.2 The Consumers

2.2.1 The young agents

In period $t \geq 1$, $L(t)$ individuals are born at home and abroad. Individuals live for two periods. In the first period they are endowed with one unit of labor which they supply inelastically. They save part of their earnings in the form of capital, to be sold to firms, and consume part. In the second period they retire and consume the value of their capital earnings.

All individuals of generation $t \geq 1$ have identical tastes represented by a utility function which satisfies the usual regularity conditions, is homothetic and displays gross substitutability. Homotheticity is equivalent to assuming that the marginal rate of substitution is soley a function of the ratio of first to second period consumption. Gross substitutability ensures that an increase in the return to saving, with wages held constant, increases saving.

The life-time utility function is

$$U = U(c_1^i(t),\ c_2^i(t));\ t \geq 1,\ i = H,F,$$

where $c_j^i(t)$ is the consumption of an agent of generation t of country i in period $t - 1 + j$; $i = H,F$, $j = 1,2$.

U is homothetic; hence let

$$W(c_1(t)/c_2(t)) = U_1(c_1(t),\ c_2(t))/U_2(c_1(t),\ c_2(t))$$

be the marginal rate of substitution function. U is assumed to be such that

$$U \in C^3,\ U_j > 0,\quad (c_1(t),\ c_2(t)) \in \underset{\sim}{R}_{++};\ j = 1,2$$

(8) $\quad W' < 0,\quad c_1(t)/c_2(t) \in \underset{\sim}{R}_+,\ W \to \infty$ as $c_1(t)/c_2(t) \to 0$

$\quad W' \to -\infty$ as $c_1(t)/c_2(t) \to 0$, $W' \to 0$ as $c_1(t)/c(t) \to \infty$.

U satisfies gross substitutability; hence

(9) $\quad W + W'c_1(t)/c_2(t) \geq 0,\quad c_1(t)/c_2(t) \in \underset{\sim}{R}_+.$

Let $s^i(t+1)$ be the amount saved by the young of generation $t \geqslant 1$ in country i; i = H,F. Then the choice problem of a young agent in country i is

$$\max_{\{c_1^i(t),\ c_2^i(t),\ s^i(t+1)\}} U(c_1^i(t), c_2^i(t))$$

subject to

$$(10) \quad c_1^i(t) + s^i(t+1) \leqslant w(t)$$

$$(11) \quad c_2^i(t) \leqslant r(t+1)s^i(t+1).$$

$r(t+1)$ can be regarded as one plus the interest rate earned on savings because the capital is completely used up in the production process. Given the assumptions on $U(\cdot, \cdot)$, the solution to the choice problem is given by (10) and (11) with equality and

$$U_1(c_1^i(t), c_2^i(t)) - \lambda_1(t) = 0$$

$$(12) \quad U_2(c_1^i(t), c_2^i(t)) - \lambda_2(t) = 0$$

$$-\lambda_1(t) + r(t+1)\lambda_2(t) = 0,$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are non-negative multipliers associated with consraints (10) and (11), respectively.

By (8), W has an inverse function, V, such that

$$V(W(c_1(t)/c_2(t))) = c_1(t)/c_2(t).$$

By the properties of U, (10) - (12) can be solved to yield

$$(13)\quad c_1^i(t) = V(r(t+1))r(t+1)w(t)/[V(r(t+1))r(t+1)+1]$$

$$(14)\quad c_2^i(t) = r(t+1)w(t)/[V(r(t+1))r(t+1)+1]$$

$$(15)\quad s^i(t+1) = w(t)/[V(r(t+1))r(t+1)+1].$$

2.2.2 The old

The "old," or the agents of generation zero in each country are endowned with their country's initial capital stocks. These stocks are employed in the time $t = 1$ production process and the old consume the rental earnings. Thus,

$$(16)\quad c_2^i(0) = r(1)K_1^i/L_0;\ i = H,F,$$

where K_1^i is the original capital stock of country i.

2.3 Equilibrium

2.3.1 Definitions

In equilibrium the amount of capital supplied by individuals must be equal to the amount of capital demanded by firms. Thus,

$$(17)\quad L(t)s^H(t+1) + L(t)s^F(t+1) = L(t+1)g^H(t+1) + L(t+1)g^F(t+1);\quad t > 1.$$

Substituting equations (4), (6), and (15) into equation (17) yields

$$(18)\quad w(r(t)) = ng[r(t+1)]B[r(t+1)];\quad t > 1,$$

where $B(r(t+1)) := V[r(t+1)]r(t+1) + 1$.

In period 1

$$(19)\quad k_1 + k_1^* = 2g[r(1)].$$

It can be seen that all capital flows take place in period one. After period one owned capital-labor ratios, and thus neutral prices, are equalized; hence no capital flows take place. This result differs from Buiter (1981) and Ruffin (1979) where one country may run a countiual current account dificit. Buiter's result is driven by the assumption that the rate of time preference differs between countries. Ruffin's result depends on differing savings rates and technologies between countries and a depreciation rate between zero and one.

Def. 1 An equilibrium is a sequence, $\{r(t)\}_{t=1}^{\infty}$, which is bounded away from zero and infinity, such that (18) and (19) are satisfied.

Def. 2 A steady state is an $r \epsilon \underset{\sim}{R}_t$ such that

(20) $w(r) = ng(r)B(r)$.

2.3.2 Existence, uniqueness, and stability

As in Diamond's (1965) model of a closed overlapping generation economy with linearly homogenous production, it seems impossible to guarantee existence, uniqueness, and stability without strong assumptions on the relationships of parameters of the utility function to parameters of the production function. An example in Section 2.4 will be given of a utility and production function which yield a unique, stable steady state. This will ensure nonvacuousness.

By equation (4) the demand curve for capital at time $t \geq 1$ is unambiguously positive and downward sloping and it converges to both axes. Given a strictly positive, finite $r(t)$ there exists a strictly positive, finite $w(t)$. Then gross substitutability ensures that at time $t + 1$ the supply curve slopes upward. Thus, for any strictly positive, finite $r(t)$ there exists a strictly positive, finite $r(t + 1)$. Hence, there exists a sequence $\{r(t)\}$ which satisfies equations (18) and (19) and $r(t + 1)$ is a monotonic function of $r(t)$. If a unique steady state exists, rental rates converge monotonically to the steady state. The existence of a steady state, however, is not guaranteed. The difficulty is that $r(t)$ may approach zero or infinity as t goes to infinity.

If a unique, stable steady state equilibrium exists, it can be seen from equation (20) that the equilibrium depends only upon the growth rate, tastes, and technology. It does not depend upon initial endownments.

Proposition 1. If a unique, stable steady state equilibrium exists, an increase in the rate of population growth must be associated with an increase in the steady state rental rate.

Proof. Totally differentiating equation (20) gives

$$[w' - n(gB)']dr = gBdn.$$

Stability requires

$$\frac{dr(t + 1)}{dr(t)} < 1.$$

By (18), this implies $w' - n(gB)' > 0$. Thus $\frac{dr}{dn} > 0$.

The intuition is that as n increases, the aggregate demand for capital, which depends on the time t + 1 labor force, increases relative to the supply of capital by the time t young population.

2.3.3 Optimality

It is a well-known result for growth models that a social planner intending to maximize stationary welfare would set $f' = n$, the Golden Rule rate.[4/] In the fixed-coefficient technology overlapping generations model it is possible to focus on the stationary equilibrium. In that model setting $f' > n$ cannot be considered inefficient because a lower f' would require sacrifices from the current old. In this variable rate of return model it is not possible to focus on a stationary equilibrium because f' varies over time.

Proposition 2. An equilibrium is Pareto optimal if and only if $r(t) \geq n$ for all but a finite number of times.

Proof. See the appendix.

The result that a capital-stock steady state equilibrium in a model with a fixed-coefficient technology is Pareto optimal if and only if $r > n$ is the standard result in the overlapping generations literature.[5/] In this literature the nonoptimality of the Golden Rule rate follows from the fact that individuals in these models are endowed with goods rather than labor and r is the constant rate of return on storage. It is possible to dominate the equilibrium allocation with one resulting from no saving. If no savings occurred in the model presented here there would be no second period consumption.

2.3.4 The gains from trade

In this section it will be assumed that a unique, stable steady state equilibrium exists.

Under autarky the growth of rental rates in both economies is governed by equation (18). Initial rates are found by equating domestic endownments with domestic firms demand in the first period. Rental rates in both economies approach the same steady state level monotonically, with the path of home rental rates always below the path of foreign rental rates. Thus, capital-labor ratios and wages in both countries approach their steady state levels, which are identical across countries, monotonically with home capital-labor ratios and wages always above foreign levels. This is shown in propositions 3 and 4.

Proposition 3. In the absence of trade the home installed capital-labor ratio will always be as great as the foreign capital-labor ratio. In the steady state the two ratios will be the same.

Proof. Let $r^H(t)$ and $r^F(t)$ represent the autarky home and foreign rental prices at time t. $r^H(1)$ and $r^F(1)$ are determined by

(21) $K_1^H = L(1)g(r^H(1))$

$K_1^F = L(1)g(r^F(1))$.

$K_1^H > K_1^F$ by hypothesis; hence by (5), $r^H(1) < r^F(1)$. Convergence to the steady state is monotonic and both economies are governed by equation (18); hence $r^H(t) < r^F(t)$ for every $t \geq 1$. Therefore, by (5), $k^H(t) < k^F(t)$ for every $t \geq 1$.

Proposition 4. Capital mobility raises the wage rate in the capital importing country and lowers the wage rate in the capital exporting country.

Proof. This follows trivially from (5) and $r^H(t) < r^F(t)$ for every $t \geq 1$.

This is the same result as in MacDougall's (1964) and Kemp's (1960) static models, Ruffin's (1979) model in the steady state and Buiter's (1981) model.

Proposition 5. Capital mobility makes the old at home and the young abroad better off and the young at home and the old abroad worse off.

Proof. The old are purely capitalists whose consumption levels are equal to their earnings on capital. The opening of trade raises interst rates at home and lowers them abroad, thus it makes the old at home better off and the old abroad worse off.

By (13) and (14) the utility of a young agent of generation t is

(22) $U = U(\frac{[B(t + 1) - 1]w(t)}{B(t + 1)}, \frac{r(t + 1)w(t)}{B(t+1)})$.

Substituting (17) into (26) gives

$$(23)\quad U = U(n[B(t + 1) - 1]g(t + 1), nr(t + 1)g(t + 1)).$$

By (5), (6), and (9) $n[B(t + 1) - 1]g(t + 1)$ and $nr(t + 1)g(t + 1)$ are monotonically decreasing in $r(t + 1)$. An increase in rental rates lowers consumption in both periods, thus the young at home are made worse off and the young abroad are made better off.

It is seen from propositon 5 that all the gains from the opening of trade are captured by the original capitalists in the relatively capital-abundant country and the young agents in the relatively labor-abundant country.

This result differs from that obtained in Buiter (1981). In Buiter's model initial capital-labor ratios are identical and time t + 1 capital is installed at time t. The current old are not able to relocate their capital in response to interest rate changes and thus their welfare is unaffected by the opening of trade. This implicitly assumes that a move from autarky to trade cannot be anticipated. The effects of capital mobility on other generations is ambiguous in Buiter's model.

2.4 An Example

Suppose $U(c_1, c_2) = c_1^{\alpha}c_2^{\beta}$; $\alpha, \beta > 0$ and $F(K, L) = K^{\gamma}, L^{1-\gamma}$; $0 < \gamma < 1$. Then by (3)

$$(24)\quad g(t) = \left(\frac{r(t)}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$w(t) = (1 - \gamma)\left(\frac{r(t)}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$$

By definition

(25) $B(t) = \frac{\alpha + \beta}{\alpha}$.

By (18)

(26) $r(t + 1) = Cr(t)^{\gamma}$,

where $C: = \left[\frac{n(\alpha + \beta)\gamma}{\alpha(1 - \gamma)}\right]^{1-\gamma}$.

Equation (26) is a first-order difference equation with solution

(27) $r(t + 1) = C^{\frac{1-\gamma^t}{1-\gamma}} r(1)^{\gamma^t}$

where by (19), r(1) is given by

(28) $r(1) = \gamma\left(\frac{K_1^H + K_1^F}{2L(1)}\right)^{\gamma-1}$.

By (20) the steady rate is

(29) $r = C^{\frac{1}{1-\gamma}}$

By (27), $\lim_{t \to \infty} r(t + 1) = r$; hence the equilibrium exhibits global stability.

3. Foreign Investment Taxation

3.1 Taxes

It has been shown that laissez-faire is not Pareto-superior to capital mobility. Thus, restricting capital flows will raise the welfare of some generations in a country. It will be assumed that each country has one policy variable for curtailing foreign investment.

Country F may levy an ad valorem withholding tax τ^F, $0 \leq \tau^F \leq 1$, on the earnings of capital in country F owned by citizens in country H. This leaves a rate of return at time t of

$$(30) \quad r(t) = (1 - \tau^F)r^F(t)$$

for the home country, where $r^F(t)$ is the foreign rental price. $r(t)$ will be referred to as the "world" rental price.

The government of the home country may tax the after tax earnings of country H's capital exported to country F at the ad valorem rate τ^H, $0 \leq \tau^H \leq 1$. This leaves

$$(31) \quad (1 - \tau^H)(1 - \tau^F)r^F(t) = (1 - \tau^H)r(t)$$

available for investors in the home country. If $(1 - \tau^H)r(t) < r^H(t)$, where $r^H(t)$ is the home country rental rate, the taxes are prohibitive and no capital flows take place. If this is not the case, capital flows occur until owners of capital in the home country earn the same rate, net of taxes, at home and abroad. This is the case when

$$(32)\quad (1 - \tau^H)r(t) = r^H(t).$$

Let u^H: $= 1 - \tau^H$ and u^F: $1/(1 - \tau^F)$. Then (30) and (31) can be rewritten as

$$(33)\quad r^F(t) = u^F r(t) = u^F r^H(t)/u^H.$$

<u>Proposition 6</u>. There will always be non-negative capital flows from country H and country F.

<u>Proof</u>. By contradiction. Assume that at some time a strictly positive amount of country F capital is invested in country H. Let t be the first time at which this happens. Then investors in country F must be making the same return at home as abroad; hence $r^H(t) = r^F(t)$. For this to be true it must the case that $g^H(t) = g^F(t)$. If $t = 1$ this implies $K_1^H < K_1^F$ which is a contradiction. If $t > 1$ this implies $s^H(t) < S^F(t)$. For this to be true it must be the case that $w^H(t - 1) < w^F(t - 1)$, which implies $r^H(t - 1) < r^F(t - 1)$, which is a contradiction.

Total taxes collected by country i at time t are

$$(34)\quad T^i(t) = L(t)(u^i - 1)r(t)[k^i(t) - g^i(t)].$$

The governments are assumed to transfer tax revenue to the old of each generation on a lump sum basis.

3.2 The Producers and Consumers

If taxes are not prohibitive the solutions to the producers problems satisfy

$$(35) \quad f(g^i(t)) - g^i(t)f'(g^i(t)) = w^i(t); \; i = H, F$$

$$f'(g'(t)) = r^i(t); \; i = H, F.$$

There exist solutions

$$(36) \quad w^i(t) = w[u^i r(t)]; \; i = H, F$$

$$g^i(t) = g[u^i r(t)]; \; i = H, F$$

The consumption of the old of generation zero is equal to their rental earnings plus their transfers of tax revenues. Using (34) yields

$$(37) \quad c_2^i(0) = [r(1)u^i K_1 + T^i(1)]/L_0$$

$$= r(1)[K_1^i + (u^i - 1)G_1^i(1)]/L_0.$$

The choice problem of the young of generation t in country i is

$$\underset{\{c_1^i(t),\, c_1^i(t),\, s^i(t+1)\}}{\text{Max}} U[c_1^i(t), c_2^i(t)]$$

subject to

$$(38)\quad c_1^i(t) + s^i(t + 1) \leq w^i(t)$$

$$(39)\quad c_2^i(t) \leq u^i r(t + 1) + T^i(t + 1)/L(t).$$

The solutions are

$$(40)\quad c_1^i(t) = \frac{[B^i(t + 1) - 1][u^i r(t + 1)w^i(t) + T(t + 1)/L(t))}{u^i r(t + 1)B^i(t + 1)}$$

$$(41)\quad c_2^i(t) = \frac{u^i r(t + 1)w^i(t) + T(t + 1)/L(t)}{B^i(t + 1)}$$

$$(42)\quad s^i(t + 1) = \frac{u^i r(t + 1)w^i(t) - [B^i(t + 1) - 1]T^i(t + 1)}{u^i r(t + 1)B^i(t + 1)}$$

3.3 Equilibrium

Equilibrium requires that in country i, i = H,F, owned capital equals capital supplied by consumers. Thus,

$$(43)\quad s^i(t + 1) = nk^i(t + 1);\ i = H,\ F.$$

Substituting (43) into (42) yields

$$(44)\quad k^i(t + 1) - g^i(t + 1) = \frac{u^i[w^i(t) - nB^i(t + 1)g^i(t + 1)]}{n[B^i(t + 1) + u^i - 1]};\ i = H,\ F$$

In equilibrium capital exported by the home country must equal capital imported by the foreign country. Thus,

$$(45)\quad \sum_{i=H,F} \frac{u^i[w^i(t) - nB^i(t+1)g^i(t+1)]}{B^i(t+1) + u^i - 1} = 0$$

<u>Definition 3.</u> An <u>equilibrium with taxes</u> is a sequence $\{r(t)\}_{t>1}$, which is bounded away from zero and infinity such that (45), holds for every $t > 1$ and $r(1)$ is given by

$$(46)\quad g(u^H r(1)) + g(u^F r(1)) = K_1^H + K_1^F.$$

3.4 The Steady State Welfare Effects

<u>Proposition 7.</u> Suppose a unique, stable, steady state equilibrium exists. Then the implementation of small taxes in period one by country H causes the steady state values of r and r^F to rise and r^H to fall. The implementation of small taxes by country F causes the steady state values of r and r^H to fall and r^F to rise.

<u>Proof.</u> Totally differentiating (45) at the steady state and evaluating at $u^i = 1$, $i = H$, F yields

$$\left.\frac{\partial r}{\partial \tau^H}\right|_{u^H=u^F=1} = \frac{r}{2} > 0 \qquad \left.\frac{\partial r^H}{\partial \tau^i}\right|_{u^H=u^F=1} = -\frac{r}{2} < 0;\ i = H, F$$

$$\left.\frac{\partial r}{\partial \tau^F}\right|_{u^H=u^F=1} = \frac{r}{2} > 0 \qquad \left.\frac{\partial r^F}{\partial \tau^i}\right|_{u^H=u^F=1} = \frac{r}{2} > 0;\ i = H, F$$

Proposition 8. Suppose a unique, stable, steady state equilibrium exists. If $r > (<)\ n$ and a small tax is implemented by either country in period one then in the steady state, welfare at home is raised (lowered) and welfare in the foreign country is lowered (raised).

Proof. Substituting (34) into (41) and (42) gives steady state utility levels of

$$(47)\quad U^i = U\left(\frac{B^i - 1)[w^i - n(1 - u^i)g^i]}{(B^i + u^i - 1)}, \frac{ru[w^i - n(1 - u^i)g^i]}{(B^i + u^i - 1)}\right). \quad i = H, F$$

By (47), $\frac{dU^i}{\partial\tau^i} > 0$ if and only if $(n - r)g^iB^i\frac{dr}{du_i} > 0$.

Thus, $\frac{dU^H}{\partial\tau^F} > 0$ if and only if $n < r$ and $\frac{dU^F}{d\tau^F} > 0$ if and only if $n > r$.

By (47), $\frac{dU^i}{\partial\tau^i} > 0$ if and only if $(n - r)\ \frac{dr}{duj} > 0$

Thus, $\frac{dU^H}{\partial\tau^F} > 0$ if and only if $n < r$ and $\frac{dU^F}{\partial\tau^H} > 0$ if and only if $n > r$.

The imposition of a small tax by either country lowers r^H and raises r^F. Hence, if r^H is greater than the Golden Rule rate, n, a small tax will lower r^H, improving matters in the home country in the steady state. If r^H is greater than the Golden Rule rate a small tax will raise r^F, worsening matters in the foreign country in the steady state. If rental rates are lower than n, the reverse occurs.

The implication is that no small tax can make both countries better off in the steady state, but any small tax will make one country better off. Thus if $r \neq n$ it will be to the benefit of one and only one country to levy a small tax. There will be some tax which will be to the benefit of at least one country in the steady state.

Suppose a social planner in country i is attempting to maximize the steady-state utility in country i. If he can choose the tax rate and savings rate he will solve.

$$\max_{\{g^i, \tau^i, c_1^i, c_2^i\}} \text{ subject to}$$

$$c_1^i + c_2^i/n + ng^i \leq f(g^i) + r(k^i - g^i).$$

This amounts to simultaneously choosing τ^i and g^i so that $r = n$ and $f(g^i) + r(k - g^i)$, the national income, is maximized. Suppose however the economy is a market economy. Then the only policy variable is τ^i, and it may not be possible to simultaneously maximize the national product and achieve the optimal capital-labor ratio. Hence the optimal tax may be larger or smaller than the one which maximizes national product, depending on the effects of taxation on $r - n$.

3.4.2 The old

The traditional static literature posits that each country maximizes the value of national product plus the net after tax earnings of foreign investment. Thus countries solve

$$(48) \quad \max_{u_i} f[g^i(ru^i)] + r[k^i - g^i(ru^i)].$$

In equilibrium

(49) $\sum_{i=H,F}[k^i - g^i(ru^i)] = 0;$

hence,

(50) $\frac{\partial r}{\partial u^i} = \frac{-rg'(u^i r)}{\sum_{i=H,F} u^i g'(u^i r)}$; $i = H, F.$

Solving (48), using equation (50), yields

(51) $u^i = 1 - \frac{k^i - g(u^i r)}{\varepsilon^j g(u^i r)}$; $i,j = H,F$ $i \neq j,$

where $\varepsilon_i = - u^i rg'(u^i r)/g(u^i r)$, the elasticity of demand for capital in country i; $i = H,F$. Equation (51) is the Kemp-Mac Dougall solution.

Consider now the current old. Maximizing $c_2^i(0)$, as given by equation (37), with respect to u^i yields

(52) $[r(1) - (1 - u^i)\frac{\partial r(1)}{\partial ru^i}]L(1)g(u^i r(1) - (1 - u^i)r(1)$

$[r(1) + u^i\frac{\partial r(1)}{\partial u^i}]L(1)g'(u^i r(1)) + K_1\frac{\partial r(1)}{\partial u^i} \geq 0$ with equality if $\tau^i > 0.$

Substituting (50) evaluated at t = 1 into (52) gives

$$(53)\quad \dot{u}^i > \frac{\varepsilon^i}{\varepsilon^i - 1}\left[1 - \frac{k_1 - g(u^i r(1))}{\varepsilon^i g(u^i r(1))}\right] \quad i,j = H,F,j \neq i.$$

Therefore the home old favor no tax or a tax smaller than the Kemp-MacDougall optimal tax. The foreign old favor a prohibitive tax or a tax larger than the Kemp-MacDougall optimal tax.

3.4.3 The Young

The opening of unrestricted trade will cause the young of the home country to become identical to the young of the foreign country. All advantage arising from belonging to the capital-abundant country is lost with free rade and all benefits are captured by the old. All of the benefits from trading with the capital-abundant country are captured by the foreign young.

A tax imposed on foreign investment will have three effects. First, it will enable the setter to act as a monopolist or a monopsonist, as in the Kemp-MacDougall story. Second, it will help protect any advantage the home young might have from belonging to the capital abundant country and thirdly it will change the savings rate. Free trade makes the home young better off than in autarky. Thus it would seem that the advantages of acting as a monopolist are not as great for a young home agent as for an agent in a capital-abundant country in the Kemp-MacDougall world and the advantages of acting as a monopsonist are greater for a young foreign agent than for an agent in a labor abundant country in a Kemp-MacDougall world.

4. Conclusion

This paper presents a model which differs from the existing literature in that it is dynamic and choice-theoretic. It is shown that all of these attributes are important in a model of capital accumulation and foreign investment taxation. The implications of the model are not inconsistent with observed reality, as are the implications of the models in the existing literature. The model does not, however, allow prediction of the path of a countries taxes. The nonagreement between different groups as to what taxes should be suggest that a model with stronger positive results would contain a description of the political process by which taxes are set.

5. Appendix

5.1 The Proof of Proposition 5.

The strategy for the "only if" part is to construct a sequence $\{s(t)\}_{t=1}^{\infty}$ of feasible, Pareto improving transfers from the young of generation t to the old of generation t + 1.

For every $t \geq 1$, $W[c_1(t), c_2(t)] = r(t + 1)$. W is continuous; hence for every $\varepsilon > 0$, there exists $\delta > 0$,

$$\text{(A.1)} \quad U\{c_1(t) - \delta, c_2(t) + \delta[r(t + 1) + \varepsilon]\} > U[c_1(t), c_2(t)].$$

Let $s(t + 1) := s(t)[r(t + 1) + \varepsilon]$ for a given $s(1)$. Then for a given s(1) the sequence of individual transfers is

$$s(1), \{s(1)\Pi_{s-1}^{t-1}[r(t + 1) + \varepsilon/n^t]\}_{t-2}^{\infty}$$

The sequence is bounded if $r(t) < n$ for all but a finite number of times. Let

$$\sup_t\{\Pi_{t-1}^{t-1}[r(t + 1) + \varepsilon]/n^t\} = : M < \infty.$$

Choose $\{\delta(t)\}_{t=1}^{\infty}$ to satisfy (A.1) for every $t \geq 1$. Choose $s(1) \in (0, \inf \delta(t)/M)$. Then $\{s(t)\}_{t=1}^{\infty}$ makes the current old better off and the young at least as well off. Hence the sequence is Pareto improving.

The "if" part is shown by contradiction and borrows heavily from Wallace (1980). Let "o" over a variable denote a potentially Pareto superior allocation. Let "$\wedge$" over a variable denote an equilibrium allocation. Feasibility requires

(A.2) $L(t+1)[c_1(t+1) + k(t+2)] + L(t)c_2(t) \leq F(t+1)$

If the potentially Pareto superior allocation does not satisfy (A.2) with equality then it is not Pareto optimal. Thus, without loss of generality, assume the potentially Pareto superior allocation satisfies (A.2) with equality.

Case 1. $\{K^0(t+1)\} = \{\hat{K}(t+1)\}$ or at the first departure $K^0(t+1) > \hat{K}(t+1)$.

By (A.2) $c_2^0(t-1) < \hat{c}_2(t-1)$ or $\hat{c}_1(t) > c_1^0(t)$ or both.

Case 1a. $c_2^0(t-1) < \hat{c}_2(t-1)$.

$\{K^0(i)\} = \{\hat{K}(i)\}$ for every $i = 1, \ldots, t$; hence by (A.2), for every $i = 1, \ldots, t-1$

$$L(t-i)\hat{c}_1(t-i) + L(t-i-1)\hat{c}_2(t-i-1) = L(t-i)c_1^0(t-i) + L(t-i-1)c_2^0(t-i-1).$$

By Pareto superiority $c_1^0(t-1) > \hat{c}_1(t-1)$, and thus $c_2^0(t-2) <$

$\hat{c}_2(t - 2)$. Proceeding backward we get $c_2^0(0) < \hat{c}_2(0)$ which is a contradiction.

Case 1b.

Define the sequence $\{d(t)\}$ such that

$$\text{(A.3)}\quad d(t + 1) = \hat{c}_1(t + 1) + \hat{k}(t + 2) - [c_1^0(t + 1) + k^0(t + 2)]$$

It will be shown by induction that $\{d(t)\}$ is positive and unbounded. $\hat{c}_1(t + 1) + \hat{k}(t + 2)$ is bounded; hence this will rule out Case 1b. For the initial step we show $d(t + 1) > 0$. By (A.2)

$$L(t + 1)d(t + 1) = \hat{F}(t + 1) - F^0(t + 1) - L(t)\hat{c}_2(t) + L(t)c_2^0(t)$$

$$= L(t)[c_2^0(t) - \hat{F}_K(t + 1)k^0(t + 1)] + \hat{F}(t + 1) - F^0(t + 1) +$$

$$L(t + 1)[k^0(t + 1) - \hat{k}(t + 1)]\hat{F}_k(t + 1)$$

by Euler's rule and the first-order conditions of the consumers' problem, where indices have been dropped for convenience. The first term is strictly positive by strict quasiconcavity of the utility function and the Pareto superiority of the "o" allocation. The rest of the expression is strictly positive by the strict concavity of the production function.

For the induction step let $(\tilde{c}_1(t + 1), \tilde{c}_2(t + 1))$ solve

$$\min_{c_1(t+1),\, c_2(t+1)} c_2(t+1) \text{ subject to}$$

(A.4) $\hat{c}_1(t+1) + \hat{k}(t+2) - [c_1(t+1) + k^0(t+2)] \geq d(t+1)$

(A.5) $U[c_1(t+1), c_2(t+1)] \geq U[\hat{c}_1(t+1), \hat{c}_2(t+1)]$

$(\tilde{c}_1(t+1), \tilde{c}_2(t+1))$ exists because $(c_1^0(t+1), c_2^0(t+1))$ is feasible, and satisfies (A.4) and (A.5) with equality.

Let

(A.6) $\Delta(t+1) := k^0(t+2) - \hat{k}(t+2)$

Then by (A.4) with equality,

(A.7) $\hat{c}_1(t+1) - \tilde{c}_1(t+1) = d(t+1) + \Delta(t+1)$

Define $g: \underset{\sim}{R} \to \underset{\sim}{R}$ such that $U\{c_1(t), g[c_1(t)]\} = U[\hat{c}_1(t), \hat{c}_2(t)]$. g is continuously differentiable, with $g' = W$, the marginal rate of substitution function. By the Immediate Value Theorem

(A.8) $g(c_1) = g(\hat{c}_1) + (\hat{c}_1 - c_1)\{\phi(\hat{c}_1 - c_1) - W[\hat{c}_1, g(\hat{c}_1)]\}$,

where ϕ is a continuously differentiable, monotonically strictly increasing function and $\phi(0) = 0$.

By (A.5) with equality, (A.7) and (A.8)

$$\tilde{c}_2(t+1) = \hat{c}_2(t+1) + [d(t+1) + \Delta(t+1)]\{W[\hat{c}_1(t+1), \hat{c}_2(t+1)]$$

$$\text{(A.9)} \qquad + \phi[d(t+1) + \Delta(t+1)]\}.$$

By $c_2^0(t+1) > \tilde{c}_2(t+1)$ and (A.9),

$$c_2^0(t+1) - \hat{c}_2(t+1) > [d(t+1) + \Delta(t+1)]\{\hat{W}(t+1) +$$

$$\text{(A.10)} \qquad + \phi[d(t+1) + \Delta(t+1)]\}.$$

$$d(t+2) = \hat{c}_1(t+2) + \hat{k}(t+3) - [c_1^0(t+2) + k^0(t+3)].$$

By (A.2) this implies

$$L(t+2)d(t+2) = L(t+1)[c_2^0(t+1) - \hat{c}_2(t+1)] -$$

$$[F^0(t+2) - \hat{F}(t+2)]\Delta(t+1)/[k^0(t+2) - \hat{k}(t+2)].$$

This implies

$$d(t+2) > [d(t+1) + \Delta(t+1)]\{\hat{W}(t+1) + \phi[d(t+1) + \Delta(t+1)]\}/n$$

$$- r(t + 2)\Delta(t + 1)/n,$$

by (A.10), the first-order conditions of the firms' problem and strict concavity of the production function. This is equal to

$$d(t + 1)\hat{W}(t + 1)/n + \Delta(t + 1)[\hat{W}(t + 1) - r(t + 2)]/n +$$

$$[d(t + 1) + \Delta(t + 1)]\phi[d(t + 1) + \Delta(t + 1)]/n.$$

The second term is zero by the first-order conditions of the consumers' problem and the last term is positive by the properties of ϕ. Hence,

$$d(t + 1) > d(t + 1)\hat{W}(t + 1)/n = d(t + 1)r(t + 2)/n.$$

$r(t) > n$ for all but a finite number of times; hence $\{d(t)\}$ is unbounded.

Case 2. $\text{Min}\{t: K^o(t + 1) \neq \hat{K}(t + 1)\} = \text{Min}\{K^o(t + 1) < \hat{K}(t + 1)\}$

By (A.2), $\hat{c}_2(t) > c_2^o(t)$ or $c_1^o(t + 1) + k^o(t + 2) < \hat{c}_1(t + 1) + \hat{k}(t + 2)$ or both.

Case 2a. $c_2^o(t) < \hat{c}_2(t)$. Work backwards as in Case 1a to show $c_2^o(0) < \hat{c}_2(0)$.

Case 2b. $c_1^o(t + 1) + k^o(t + 2) < \hat{c}_1(t + 1) + \hat{k}(t + 2)$. The induction proof is repeated. Proceed up to the formulation of $d(t + 2)$. Here

$$\text{(A.11)} \quad L(t + 2)d(t + 2) = L(t + 1)[c_2^0(t + 1) - \hat{c}_2(t + 1)] + \hat{F}(t + 2) - F^0(t + 2)$$

Thus, by (A.10)

$$d(t + 2) > [d(t + 1) + \Delta(t + 1)]\{\hat{W}(t + 1) + \phi[d(t + 1) + \Delta(t + 1)]\}/n + [\hat{F}(t + 2) - F^0(t + 2)]/L(t + 2)$$

$$> d(t + 1)\hat{W}(t + 1)/n + \Delta(t + 1)[\hat{W}(t + 1) - \hat{F}_k(t + 2)]/n$$

by Euler's rule and the first-order conditions of the consumers' problem. Thus

$$d(t + 2) > d(t + 1)r(t + 2)/n.$$

Hence $\{d(t)\}$ is unbounded.

Footnotes

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1/ MacDougall (1960), Kemp (1964), Hamada (1966), Jones (1967), Pitchford (1970) and Manning (1974, 1975).

2/ For more on this issue see Shell (1977).

3/ Bergsten, Horst and Moran (1978), Chs. 1-6 and Kindleberger (1973), p.269.

4/ Phelps (1966).

5/ Wallace (1980).

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