

International Finance Discussion Papers

Number 222

May 16, 1983

PURCHASING POWER PARITY AND REAL AFTER TAX INTEREST RATE ARBITRAGE

by

David H. Howard and Karen H. Johnson*

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment by a writer that he has had access to unpublished material) should be cleared with the author or authors.

Abstract

This paper examines the relationship between the goods market and capital market arbitrage conditions. It is shown that in a well integrated international capital market in which investors are concerned with real after tax rates of return, purchasing power parity will not necessarily hold even in the long run even if all shocks are monetary in nature. In such a world, the real exchange rate is not independent of relative inflation rates. Alternative tax regimes are investigated and some informal empirical evidence is presented.

The behavior of exchange rates during the floating rate regime of the 1970s and early 1980s has given rise to renewed interest in the ability of the purchasing power parity (PPP) doctrine to predict changes in nominal exchange rates. Jacob Frenkel (1981a, 1981b) points out that PPP predictions have not performed well during the recent floating rate period and contrasts this performance with that of the 1920s, when PPP predictions appear to have been much more reliable. Frenkel offers a number of reasons for the poor performance of PPP, including: shocks that require changes in relative prices and changes in equilibrium real exchange rates themselves (on this point, see also William Branson); transportation costs; commercial policies; and the inherent difference between exchange rates --asset prices -- and national price levels with regard to their short-run responsiveness to new information ("news"). This last reason pertains to short-run departures from PPP only, but the first three can mean that PPP need not ever hold, regardless of the length of the period involved. Indeed Frenkel points out that during the 1970s departures from PPP often appear to have been persistent and cumulative. Frenkel's analysis leads him to conclude that when real shocks require changes in relative prices, PPP may not hold in the long run and that the usefulness of PPP "is in providing a guide as to the general trend of exchange rates in particular in circumstances where the main shocks underlying the trend are of a monetary origin," (1981a, p. 162).

Another theme in the literature on exchange rates is concerned with the connection between changes in interest rates and

exchange rate movements. (See, for example, Frenkel, 1981b.) The present paper examines the relationship between the goods market (i.e., PPP) and capital market (i.e., interest rate) arbitrage conditions. It is shown that in a world in which there is a well integrated international capital market and where investors are concerned with their real after tax rates of return, one would not necessarily expect PPP to hold even in the long run even if all shocks are monetary in nature.^{1/} In such a world, the real exchange rate is not independent of relative inflation rates.

I. Interest Rate Arbitrage with Taxes

In order to examine the implications of a well integrated international capital market, the following notation is introduced:^{2/}

R_i = nominal rate of interest on an asset denominated in the currency of country i ,

π_i = expected rate of inflation in country i ,

P_i = price level in country i ,

r_i = expected real after tax rate of interest in country i ,

t_i = marginal rate of tax on income in country i ($t_i < 1$),

E = exchange rate (units of currency 1 per unit of currency 2),

e = expected rate of change of E .

It is specified that an individual investor equates the expected real after tax rates of return expressed in terms of a single currency, on similar assets (e.g., bonds) denominated in different currencies, at least up to a perhaps time-varying risk premium. If the bonds are perfect substitutes, arbitrage will ensure that:

$$(1) \quad r_1 = (1 - t_1) R_1 - \pi_1 = (1 - t_1) (R_2 + e) - \pi_1,$$

$$(2) \quad r_2 = (1 - t_2) (R_1 - e) - \pi_2 = (1 - t_2) R_2 - \pi_2,$$

which reduce to the familiar interest rate arbitrage condition: $R_1 = R_2 + e$. (Note that it is specified that taxes are levied on nominal interest income.) In Howard and Johnson, it is shown that if conditions (1) and (2) hold, a change in expected inflation in one country implies that either the real exchange rate must change, that is PPP will not hold, or there must be a differential impact on real after tax interest rates across countries, even if tax rates are equal. Some combination of these two effects is also possible.^{3/}

In the above analysis, no relationship between r_1 and r_2 is stated. Conditions (1) and (2) apply to investors who face a country-specific combination of tax rates and expected inflation. However, these investors can observe all such combinations, and some are in a position to change their location or set up a subsidiary in order to seek out the highest real after tax rate of return. A piece of evidence of this kind of behavior is the existence and use of tax havens. The fact that such havens are heavily utilized indicates that at least a significant portion of investors are concerned with after tax rates of return and are willing to move funds and operations across national boundaries in order to take advantage of various tax incentives. Such investors presumably move funds between assets denominated in different currencies and located in different countries until their expected after tax real rates of return are equal, at least in the simple case examined in this paper in which the assets are perfect substitutes and there are no barriers to such movements.

If expected after tax real rates of return are specified to be equal across countries,^{4/} it follows from conditions (1) and (2) that:

$$(3) \quad (1 - t_1)R_1 - \pi_1 = (1 - t_2)(R_1 - e) - \pi_2.$$

If condition (3) holds, then

$$e = R_1 + (\pi_1 - \pi_2)/(1 - t_2) - [(1 - t_1)R_1]/(1 - t_2),$$

or,

$$(4) \quad e \approx (\pi_1 - \pi_2)/(1 - t_2).$$

if $t_1 \approx t_2$. Condition (4) states that the expected rate of change of the exchange rate is systematically related to relative rates of inflation in a way that ensures a violation of PPP as long as the tax rate is not zero. (Note that PPP implies $e = \pi_1 - \pi_2$.) The expected change in the real exchange rate (x), that is, the exchange rate adjusted for price level changes, is defined to be:

$$(5) \quad x = e - \pi_1 + \pi_2.$$

Substituting for e in equation (5) using condition (4), one obtains:

$$(6) \quad x \approx [t_2/(1 - t_2)] (\pi_1 - \pi_2).$$

According to expression (6), investors expect the currencies of low inflation countries to appreciate in real terms and those of high inflation countries to depreciate in real terms. As long as the inflation differentials do not change, these movements in expected real exchange rates will continue, which may help explain Frenkel's observations that departures from PPP often appear to be persistent and cumulative.

It is also interesting to note that the preceding analysis implies a particular kind of "vicious" or "virtuous" circle with regard to exchange rates and inflation. The term vicious/virtuous circle is not well defined in the literature. As Henry Wallich and Jo Anna Gray (p. 50) point out, "virtually any succession of price-level increases (decreases) and exchange-rate depreciations (appreciations) experienced by a country may be labelled a vicious (virtuous) circle without doing violence to currently accepted uses of the term." A common theme in this literature is the observation that low inflation countries tend to have currencies that appreciate which in turn tends to slow inflation further and that the opposite is true for high inflation countries. The timing relationships involved in this process suggest a causal ordering, but, as Frenkel points out (1981a, p. 163), exchange rates and prices are endogenous variables that react to shocks with different speeds. Thus the observed timing relationship should not be interpreted as evidence of causality. Expression (8) has as an implication an empirical regularity -- a tendency for the currencies of high (low) inflation countries to depreciate (appreciate) in real terms -- that is a type of vicious (virtuous) circle. This empirical regularity does not depend on a causal relation between exchange rates and inflation rates.

II. Alternative Tax Regimes

In the preceding section, the tax regime considered is one in which each country's tax rates are approximately equal and in which taxes are levied against nominal interest payments. Alternative tax regimes include the more general case where tax rates can differ substantially across countries and regimes in which taxes are levied on inflation-adjusted, that is, real, interest payments.

Consider the regime in which taxes are levied on nominal interest payments but tax rates need not be approximately equal. In this case,

$$(7) \quad x = [t_2/(1 - t_2)](\pi_1 - \pi_2) + [(t_1 - t_2)/(1 - t_2)]R_1,$$

which, of course, is the complete version of expression (6). The second term on the right-hand-side of equation (7) indicates that if nominal interest rates are high enough, differential tax rates may reverse, or at least weaken, the tendency of currencies of low inflation countries to appreciate in real terms.

When tax rates apply to real interest payments, arbitrage conditions (1) and (2) become:

$$(8) \quad r_1^* = (1 - t_1)(R_1 - \pi_1) = (1 - t_1)(R_2 + e - \pi_1),$$

$$(9) \quad r_2^* = (1 - t_2)(R_1 - e - \pi_2) = (1 - t_2)(R_2 - \pi_2).$$

Equating these real after tax rates of return (r_1^* and r_2^*) and using equation (5), one obtains:

$$(10) \quad x = [(t_1 - t_2)/(1 - t_2)](R_1 - \pi_1).$$

Thus, as long as tax rates differ and the nominal interest rate (R_1) does not equal the inflation rate (π_1), the real exchange rate will change over time even in the absence of any shocks. In the particular case where R_1 exceeds π_1 , that is, where the real interest rate as conventionally defined is positive, the currencies of low tax rate countries will appreciate in real terms in this tax regime.

In an earlier paper (Howard and Johnson) it is shown that if taxes are levied on real interest income and no relationship is specified about real after tax interest rates across countries, the paths of real exchange rates and/or real after tax interest rates need not be affected by a change in expected inflation regardless of the tax

rates involved. Thus, the tax effect identified in that paper disappears if taxes on interest payments are adjusted for inflation. In the present paper, where a relationship between real after tax interest rates across countries is specified, it is not the case that the identified effect of taxes disappears when taxes are levied on real interest payments rather than nominal interest payments: the path of the real exchange rate can be affected by tax rates even in an inflation-adjusted tax environment.

The particular tax regime involved affects the path of the real exchange rate. Thus any serious empirical investigation of the propositions put forward in this paper must take cross-country differences in tax regimes into account. In the next section of the present paper some informal empirical evidence is presented in which it is specified, reasonably, that taxes are levied on nominal interest rates and, not so reasonably, that tax rates are equal across countries. Given the modest use made of the evidence, such an assumption is probably acceptable, but clearly there is scope for a more complete empirical investigation in which actual tax regimes are taken into account.

III. Behavior of Real Exchange Rates, 1975-1980

Expression (6) is not intended to be a theory of real exchange rate determination. The factors working against the PPP condition -- see the preceding discussion for a list of some of them -- as well as the goods arbitrage implicit in the PPP condition itself also help to determine the path of the exchange rate. Expression (6) merely is the logical implication of a well integrated international capital market in which investors are concerned with real after tax

rates of return. As such it represents just another force acting to prevent the PPP condition from holding, even in the long run. Nevertheless, it is tempting to investigate to what extent it is low inflation countries that tend to have currencies that have appreciated in real terms.

Figure 1 summarizes the inflation and bilateral real exchange rate experience of eight major European industrialized countries from 1975 to 1978. These eight countries are ones where it is not unreasonable to expect both capital and goods markets to be highly integrated, although, to be sure, some restrictions on capital movements and trade between these countries do exist and prevent the markets from being completely integrated. By 1975 the initial impact of the real oil price shock of 1973-1974 had already occurred and, to a large extent, the differences across countries in the 1975-1978 period reflect differences in policies (especially monetary policy) chosen to deal with the remaining price and income effects of the initial real shock. In the figure, countries are arranged in order of their consumer price inflation rates over the entire period, with Switzerland being the lowest. A plus sign indicates an appreciation of the real exchange rate (measured using consumer price indices) of the currency of the country listed vertically in terms of the currency of the country listed horizontally over the three year period. (Both prices and exchange rates are measured on an annual average basis.) A minus sign indicates depreciation of the real exchange rate. The implication of expression (6) is that such an ordering by inflation rate (if expected inflation roughly corresponded to actual inflation) should generate an array with pluses above the diagonal. For the period 1975-1978, this result is obtained in nearly all cases. Indeed the few

FIGURE 1

Real Bilateral Exchange Rate Changes: 1975-1978

	<u>Switzerland</u>	<u>Germany</u>	<u>Netherlands</u>	<u>Belgium</u>	<u>France</u>	<u>Sweden</u>	<u>U.K.</u>	<u>Italy</u>
Switzerland		+	+	+	+	+	+	+
Germany			-	-	+	+	+	+
Netherlands				-	+	+	+	+
Belgium					+	+	+	+
France						+	-	+
Sweden							-	+
United Kingdom								+
Italy								

Note: The countries are listed in order of (consumer price) inflation rate during the period, with Switzerland having the lowest rate. A plus (minus) sign signifies an appreciation (depreciation) of the real bilateral exchange rate of the currency of the country listed vertically with respect to that of the country listed horizontally. For example, the Swiss franc appreciated in real terms against the German mark.

minus signs in the diagram result mostly from cases where the real exchange rate change was in fact negligible and where the differences in inflation rates were quite small.

Continuous real appreciation of the sort pictured in Figure 1 cannot be expected to continue indefinitely. The arbitrage possibilities in goods markets implicit in a divergence from PPP plus other forces tending to affect the equilibrium real exchange rate, such as the oil price shock of 1979, would be expected to counteract the asset market arbitrage forces being highlighted in this paper. Indeed, in 1978-1980, a reversal of the conditions illustrated in Figure 1 did occur, as Figure 2 shows. Nevertheless, to the extent that 1975-1978 was a period during which, for these countries, monetary policy differences largely determined differences in expected and actual inflation and few other exogenous factors were present to alter the paths of real exchange rates, Figure 1 seems to corroborate the existence of the economic relationships summarized in expression (6).

IV. Conclusion

Tax considerations and the existence of a substantial number of investors in a well integrated international capital market who tend to equalize their expected real after tax rates of return introduce another potential reason for exchange rates to deviate from their purchasing power parities, even in the long run. Indeed, literal interpretation of expression (6) indicates that even if all shocks are monetary in nature, PPP will not hold as long as tax rates are nonzero. (Compare Frenkel, 1981a, p. 162.) There is a fundamental tension between the rates of return and goods arbitrage conditions involved;

FIGURE 2

Real Bilateral Exchange Rate Changes: 1978-1980

	<u>Switzerland</u>	<u>Germany</u>	<u>Netherlands</u>	<u>Belgium</u>	<u>Sweden</u>	<u>France</u>	<u>U.K.</u>	<u>Italy</u>
Switzerland	-	-	-	-	-	-	-	-
Germany			+	+	-	-	-	-
Netherlands				+	-	-	-	-
Belgium					-	-	-	-
Sweden						-	-	-
France							-	-
United Kingdom								+
Italy								

Note: see note to Figure 1.

both cannot hold when tax rates are nonzero and inflation rates are not identical. In this paper a bit of evidence is presented which is consistent with the arbitrage condition for real after tax interest rates prevailing (roughly) for a substantial period, but the main point of the paper is that neither condition can be expected to prevail consistently either over time or across currencies. Neither condition can substitute adequately for a general equilibrium model in which both the real exchange rate and inflation are determined as endogenous variables.

Literature Cited

- Blejer, Mario I., "Financial Market Taxation and International Capital Flows," IMF Departmental Memorandum, April 14, 1983.
- Branson, William H., "Comments: 'The Collapse of Purchasing Power Parities during the 1970's' by Frenkel," European Economic Review, 16, May 1981, 167-171.
- Cumby, Robert E. and Obstfeld, Maurice, "International Interest-Rate and Price-Level Linkages Under Flexible Exchange Rates: A Review of Recent Evidence," in Bilson, John F.O. and Marston, Richard C., eds., Exchange Rates: Theory and Practice, Chicago, 1983.
- Frenkel, Jacob A., "The Collapse of Purchasing Power Parities during the 1970's," European Economic Review, 16, May 1981a, 145-165.
- _____, "Flexible Exchange Rates, Prices, and the Role of the 'News': Lessons from the 1970s," Journal of Political Economy, 89, August 1981b, 665-705.
- Hodrick, Robert J., "Some International Evidence on the Equality of Expected Real Interest Rates," Working Paper, Graduate School of Industrial Administration, Carnegie-Mellon University, 1979.
- Howard, David H. and Johnson, Karen H., "Interest Rates, Inflation, and Taxes: The Foreign Connection," Economics Letters, 9, 1982, 181-184.
- Kane, Alex and Rosenthal, Leonard, "International Interest Rates and Inflationary Expectations," Journal of International Money and Finance, 1, April 1982, 97-110.

- Kimbrough, Kent P., "Price, Output, and Exchange Rate Movements in the Open Economy," Journal of Monetary Economics, 11, January 1983, 25-44.
- Mishkin, Frederic S., "Are Real Interest Rates Equal Across Countries? An Empirical Investigation of International Parity Conditions," NBER Working Paper No. 1048, December 1982.
- Wallich, Henry C. and Gray, Jo Anna, "Stabilization Policy and Vicious and Virtuous Circles," in Chipman, John S. and Kindleberger, Charles P., eds., Flexible Exchange Rates and the Balance of Payments: Essays in Memory of Egon Sohmen, New York, 1980.

Footnotes

- */ International Finance Division, Federal Reserve Board. This paper represents the views of the authors and should not be interpreted as reflecting the views of The Board of Governors of the Federal Reserve System or other members of its staff. We would like to thank Maureen Walsh for her assistance with the data used in this study and Jerry Caprio, Richard Freeman, Dale Henderson, Peter Isard, Raymond Lubitz, Larry Promisel, and Ralph Smith for their useful comments and suggestions.
- 1/ Kent Kimbrough presents a model in which unanticipated monetary shocks produce deviations from purchasing power parity. The conclusions of the present paper hold for anticipated shocks; unanticipated shocks are not analyzed specifically.
- 2/ The notation and analysis follows that presented in Howard and Johnson.
- 3/ Mario Blejer has extended the Howard-Johnson analysis by working out the implications for different tax regimes, including the important case in which foreign exchange gains and losses and interest payments are taxed at different rates.
- 4/ A recent paper by Alex Kane and Leonard Rosenthal presents some evidence that they contend indicates that real interest rates are equal across countries. Unfortunately Kane and Rosenthal do not take taxes into account. Their hypothesis is that $R_1 - \pi_1 = R_2 - \pi_2$, which follows immediately from substituting expectations of

purchasing power parity, $e = \pi_1 - \pi_2$, into the interest rate arbitrage condition, $R_1 = R_2 + e$. As mentioned in the text, the evidence presented by Frenkel indicates that purchasing power parity is not an attractive proposition about market expectations. The specification of PPP which is implicit in their analysis plus neglect of the role that taxes play in the portfolio allocation decisions of international investors may be important reasons why the evidence presented by Kane and Rosenthal, although perhaps suggestive, is not entirely convincing.

The Kane and Rosenthal evidence is not convincing chiefly for two reasons. First, Kane and Rosenthal interpret low t-ratios as supporting their hypothesis when, in fact, the t-ratios could just be reflecting the imprecision of their estimates. Second, even accepting the t-ratios as valid evidence, there is a large proportion of high t-ratios in the results reported. In Kane and Rosenthal's Table 4B, nearly half (31 out of 65) of the regressions contain at least one t-ratio greater than 1.73 (the critical value for 95 percent confidence level) in absolute value. The authors split their sample period (February 1974 - June 1979) in half and implicitly argue that the first half's experience is less relevant because it took time for agents to become familiar with a floating exchange rate regime. This division is, of course, arbitrary and one might argue that a 32 month learning period seems a bit long. Nevertheless even in the second half of Kane and Rosenthal's sample period there is a very large proportion of regressions with one or more t-ratios exceeding 1.73; in Table 4B, for months 33 through

65, one third of the reported regressions contain one or more t-ratios greater than 1.73.

Recent papers by Robert Cumby and Maurice Obstfeld and by Frederic Mishkin present evidence that real interest rates are not equal across countries. Like Kane and Rosenthal, Cumby, Obstfeld, and Mishkin do not take taxes into account in their empirical work. Indeed, the neglect of the role of taxes in these studies may be one reason for their negative findings. It is interesting to note that Cumby and Obstfeld regard their evidence against PPP as implying a rejection of real interest rate equality across countries; however, as shown in the present paper, when the role of taxes is taken into account such evidence is consistent with after tax real interest parity. Robert Hodrick reports mixed evidence on the equality of real interest rates across countries; Hodrick also omits the role of taxes in his empirical analysis.