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The extent to which economic agents consider domestic and foreign assets to be perfect substitutes is of fundamental importance to the conduct of monetary and fiscal policy. Under fixed exchange rates, it is well known that if assets are considered perfect substitutes, then monetary policy will be impotent for a small country, whereas for a larger country monetary policy will have (essentially) the same effect in all countries, regardless of which country initiates the change in policy. Comparably, under a flexible exchange rate regime, sterilized intervention to stabilize (or, simply, alter) exchange rates will be useless, since asset holders view the foreign and domestic assets as being equivalent. Furthermore, government deficits (due, for example, to real absorption by the government) will leave exchange rates (and forward rates) unaltered, provided none of the deficit is monetized.

In a world in which exchange rate movements are often large, and unpredictable, the assumption that agents consider assets denominated in different currencies to be perfect substitutes is a strong one. Under perfect foresight, and abstracting from transaction costs in switching between assets, it is highly plausible that the real return on assets should be equalized. However, under uncertainty, agents do not know, *ex ante*, the realized returns on these alternative assets; at best, they can only know the true probability distribution of these returns.

Standard portfolio theory implies that agents will attempt to diversify their portfolio, and hence implies that there is no ex ante reason for (expected) real returns to be equalized (unless agents are risk neutral). Furthermore, recent empirical studies (e.g., Hansen and Hodrick (1980)) indicate that the forward rate is not an unbiased predictor of the future spot rate, so that some risk premium seems to be embodied in the forward rate.

There have, of course, been other papers which analyze the determinants of the forward premium. Driskill and McCafferty (1980a, 1982) analyze the determination of spot and forward rates in a model in which ad hoc behavioral rules are postulated for the trade balance and for the speculative demand for foreign exchange. Frankel (1979a) analyzes the determinants of the forward premium in a portfolio setting in which agents possess quadratic utility functions (or returns are normally distributed). However, these models are essentially partial equilibrium ones. It is the purpose of this paper to present a general equilibrium model in which spot and forward rates (or interest rates) are simultaneously determined, and in which the behavioral rules are derived from individual optimizing behavior. Within this framework, we will analyze how the forward premium is determined, and present some comparative static results which show how changes in uncertainty, or asset stocks, alter the spot and forward rates.

The plan of the paper is as follows. In section II we present the basic model, and analyze the behavior of economic agents, given their expectations about current and future prices. In section III we specify the source of uncertainty, and show under what circumstances a rational expectations solution can be derived. Section IV presents some comparative static results concerning the rational expectations solution, while the last section contains our conclusions and suggestions for future research.

II) The Model

The purpose of our model is to allow us to derive asset demands from optimizing behavior, and to use these asset demands to determine how exchange rates - and real interest rate differentials - are determined under uncertainty. In specifying the model we have several objectives: (i) to allow a relatively tractable general equilibrium solution to emerge in the presence of uncertainty, and (ii) to specify a model so that the conventional results predicted by the Monetary Theory of Exchange Rate Determination will emerge if no uncertainty is present. Thus, we wish to focus on how uncertainty affects exchange rates and real interest rate differentials, not on how alternative specifications of money demands may alter the exchange rate equation.

In order to accomplish these goals we use a relatively simple consumption-loan model in which it is assumed that:

(i) there are two countries ($i = 1, 2$)

(ii) at any time, t, two generations are alive in each country; members of generation t are born at t and die at the end of $(t + 1)$.

(iii) each generation in country i consists of N^i individuals (a constant).

(iv) there is one homogeneous and perishable good "produced" in each country; aggregate output, $Q^i(t)$, and per capita output, $q^i(t)$, is exogenous, and belongs to the "young" at t.

(v) each country issues two nominal assets, currency $(M^i(t))$ and bonds $(B^i(t))$ denominated in its own currency unit.

(vi) all bonds are one period bonds that pay a market determined nominal interest rate $r^i(t)$.

(vii) country 1's currency is taken as numeraire; e(t), the market determined exchange rate, is the number of units of country 1's currency (dollars) needed to purchase one unit of 2's currency (marks).

These four assets are the only stores of value. If there is no specific transactions demand for money, country i money and bonds will be perfect substitutes ($r^i(t) = 0$). Even if there is a transactions demand for currency, the exchange rate may be indeterminate (Kareken and Wallace, 1981) if the currencies are considered perfect substitutes.[1] To avoid this problem, we postulate per capita money demands for each country as follows: [2,3]

(1) $m^i = \lambda^i P^i (q^i)^\eta (1 + r^i)^{-\gamma}$; $\eta, \gamma > 0$; $i = 1,2$

In (1), time subscripts are suppressed for simplicity; λ^i is a money demand parameter, P^i is the domestic currency price of the single good, q^i is per capita output, and m^i per capita money demand. Assuming commodity arbitrage:

(2) $P_t^2 = e_t P_t^1$

All agents in a country are identical, and only the young hold money. Hence, aggregate money demand is:

(3) $M_t^i = N^i \lambda_t^i P_t^i (q_t^i)^\eta (1 + r_t^i)^{-\gamma}$; $i = 1,2$

The exchange rate is given by:

(4) $e_t = (\tilde{M}_t^1/\tilde{M}_t^2)L_t[(1 + r_t^1)/(1 + r_t^2)]^{\gamma}$

(5) $L_t \equiv [\lambda_t^2(q_t^2)^{\eta}]/[\lambda_t^1(q_t^1)^{\eta}]$

In (4), $\tilde{M}_t^i$ is per capita money supply.

Under perfect foresight, interest arbitrage implies:

(6) $e_{t+1} = e_t[(1 + r_t^1)/(1 + r_t^2)]$

where e_{t+1} is also the one period forward rate at t. Using (4) and (6), and assuming $\lim_{\tau\to\infty} [(e(t + \tau))^{B^{\tau}}]\to 0$, $B \equiv [\gamma/(1 + \gamma)]$, the exchange rate is given by:

(7) $\ln e_t = [1/(1 + \gamma)][\sum_{i=0}^{\infty}\{(\ln[\tilde{M}_{t+i}^1 L_{t+i}/\tilde{M}_{t+i}^2])B^i\}]$

Thus, as usual, the current spot rate depends upon current, and time t expectations of future, variables. Under perfect capital mobility, real interest rates are equalized internationally, and bond stocks do not affect the exchange rate. To focus on the role of uncertainty, we use the *ad hoc* money demand functions (1) as a constraint on optimizing behavior.

Next, consider the individual agent's optimization problem. We assume:

(i) all agents have identical, homothetic preferences;

(ii) utility is an additive function of consumption in each period.

Assumption (i) is a standard trade theory assumption that allows one to ignore the income distribution, and (ii) is a common assumption in multiperiod models. Together, they imply:

(8) $U(C_t^i, S_{t+1}^i) = (1/\alpha)[(C_t^i)^\alpha + \phi(S_{t+1}^i)^\alpha]; \ \alpha < 1, \ \alpha \neq 0; \ \phi > 0$

where C_t^i,(S_{t+1}^i) is consumption by the young in t (old in (t + 1)), ϕ is the rate of time preference, and α is simultaneously a measure of relative risk aversion $(1 - \alpha)$ and intertemporal commodity substitutability $((1 - \alpha)^{-1})$. The assumption of constant relative risk aversion greatly simplifies the analysis since portfolio composition is independent of wealth.[4]

At t, the young choose current consumption and asset holdings to maximize expected utility. Current prices, and the "true" distribution of P_{t+1}^i are known, but the realized values of P_{t+1}^i are not known. Define:

(9) m_t^{ij} as the per capita demand by the young of country i for currency j;

(10) b_t^{ij} as the per capita demand by the young of country i for country j's bond.

By assumption, the m_t^{ij} are not choice variables, but are constraints imposed on the agent:

$$(11)\quad m_t^{ij} = (\delta_{ij})[P_t^j \lambda_t^j (q_t^j)^{\eta}(1 + r_t^j)^{-\gamma}];\ \delta_{ij} = 1,\ i = j;\ \delta_{ij} = 0,\ i \neq j$$

Each individual of country i receives an endowment (q_t^i) when young. In addition, he may pay taxes in the second period of life.[5] Let T_{t+1}^{ij} be nominal taxes that a resident of country i pays (when old) in country j's currency ($T^{ij} = 0$, $i \neq j$, if taxes are levied only in domestic currency). It is assumed the nominal (not real) value of taxes due is known <u>ex ante</u>.

The individual's budget constraints are:

$$(12)\quad q_t^i - C_t^i - [(m_t^{i1} + b_t^{i1})/P_t^1] - [e_t(m_t^{i2} + b_t^{i2})/P_t^1] \geq 0$$

$$(13)\quad [b_t^{i1}(1 + r_t^1) + m_t^{i1} - T_{t+1}^{i1}] + e_{t+1}[b_t^{i2}(1 + r_t^2) + m_t^{i2} - T_{t+1}^{i2}] - P_{t+1}^1 S_{t+1}^i \geq 0$$

Assuming non-satiation, (12) and (13) hold as equalities. Given $(q_t^i, P_t^1, e_t, r_t^i)$, and expectations concerning (P_{t+1}^1, e_{t+1}), the agent maximizes expected utility (from (8)), subject to constraints (11)-(13). Assuming an interior solution exists,[6] the first order conditions imply:

$$(14)\quad (C_t^i)^{\alpha-1} = \phi E[(S_{t+1}^i)^{\alpha-1}(1 + \rho_{t+1}^k)];\ k = 1,\ 2$$

$$(15)\ E[(S^i_{t+1})^{\alpha-1}([e_{t+1}(1 + r^2_t) - e_t(1 + r^1_t)]/P^1_{t+1})] =$$

$$E[(S^i_{t+1})^{\alpha-1}((1 + \rho^1_{t+1}) - (1 + \rho^2_{t+1}))] = 0$$

$$(16)\ (1 + \rho^k_{t+1}) \equiv [P^k_t(1 + r^k_t)/P^k_{t+1}];\ k = 1,\ 2$$

In (14) - (16), ρ^k_t is the realized real return on country k bonds, and $E(\rho^k_t)$ is the expected real return. Under risk neutrality (or perfect foresight) the (expected) real returns are equalized; for $\alpha < 1$, there is no such presumption.

Clearly, the introduction of a forward market will add nothing, provided there are no transaction costs. From covered interest arbitrage:

$$(17)\ f_t = [e_t(1 + r^1_t)/(1 + r^2_t)]$$

where f_t is the one period forward rate. Allowing forward positions would imply:

$$(18)\ E[(S^i_{t+1})^{\alpha-1}[(e_{t+1} - f_t)/P^1_{t+1}] = 0$$

which is equivalent to (15). As is well-known, the forward rate need not be an unbiased predictor of the future spot rate. As noted by Siebert (1982), the "risk premium" $\left(E(e_{t+1}) - f_t\right)$ depends on the degree of risk aversion and the covariance between (e_{t+1}, P^1_{t+1}). Even for risk neutrality, a "risk premium" can arise, although the expected real returns on bonds will be equalized.

Using the assumption of constant relative risk aversion, define

$$(19)\quad Y_t^i = e_t[b_t^{i2}(1 + r_t^2) + m_t^{i2} - T_{t+1}^{i2}]/[b_t^{i1}(1 + r_t^1) + m_t^{i1} - T_{t+1}^{i1}]$$

In (19), Y_t^i represents the ratio of next period's net wealth held in marks to dollars, evaluated at the current exchange rate. Using (19), (15) simplifies to:

$$(20)\quad E[(1 + (Y_t^i e_{t+1}/e_t))^{\alpha-1}(P_{t+1}^1)^{-\alpha}(e_{t+1}(1 + r_t^2) - e_t(1 + r_t^1))] = 0;$$

$$(20')\quad E[(1 + (Y_t^i e_{t+1}/e_t))^{\alpha-1}(P_{t+1}^1)^{1-\alpha}((1 + \rho_{t+1}^1) - (1 + \rho_{t+1}^2))] = 0$$

Since (20) (or(20')) is independent of agent specific variables, portfolio decisions (Y_t) are the same for all agents. Using the budget constraint and the first order conditions, the agent's behavioral rules may be simplified to:

$$(21)\quad C_t^i = [\tilde{q}_t^i/(1 + H_t)]$$

$$(22)\quad H_t = (\phi X_t)^{(1-\alpha)^{-1}}[(1 + r_t^1)^{-1} + Y_t(1 + r_t^2)^{-1}]$$

$$(23)\ X_t \equiv E[(1 + (Y_t e_{t+1}/e_t))^{\alpha-1}(P_t^1/P_{t+1}^1)^{\alpha}(1 + r_t^1)]$$

$$(24)\ \tilde{q}_t^i \equiv [q_t^i - ((r_t^1 m_t^{i1} + T_{t+1}^{i1})/(P_t^1(1 + r_t^1))) - e_t((r_t^2 m_t^{i2} + T_{t+1}^{i2})/(P_t^1(1 + r_t^2)))]$$

$$(25)\ (1 + r_t^1)b_t^{i1} = [P_t^1 \tilde{q}_t^i (\phi X_t)^{(1-\alpha)^{-1}}/(1 + H_t)] + T_{t+1}^{i1} - m_t^{i1}$$

$$(26)\ (1 + r_t^2)b_t^{i2} e_t = [Y_t P_t^1 \tilde{q}_t^i (\phi X_t)^{(1-\alpha)^{-1}}/(1 + H_t)] + e_t(T_{t+1}^{i2} - m_t^{i2})$$

Equations (22) - (24) are definitions, whereas (21), (25) and (26) are the commodity and asset demands (together with currency demands, (11)). In essence, $\tilde{q}_t^i$ is present discounted real income, net of tax liabilities and money holding costs. Also, X_t reflects the expected real return (in utility units) of current savings. Under risk neutrality, X_t is one plus the expected real return on bonds. Summing (25) and (26) across all agents yields aggregate demand for each bond. Equilibrium in the bond and currency markets yields current prices and interest rates, given expectations.[7]

To complete the model, we must specify government behavior, including how interest payments are financed. If there are no taxes and (part of) the interest payments are monetized, there is a direct link between bonds and exchange rates, even under perfect foresight. If all interest payments are financed through further bond issuance, no steady-state solution is possible, as nominal bonds (and the ratio of bonds to money) grow over time. Thus, we assume each government levies taxes (on the "old") that just cover the interest due on maturing bonds. Further, we assume each country issues bonds denominated only in its own currency units, so that the tax liability of country i residents depends on the stock of bonds denominated in that currency.[8] Total tax collections in country i ($\tilde{T}^i_{t+1}$) are:

$$(27)\quad \tilde{T}^i_{t+1} = (r^i_t B^i_t)$$

The government budget constraint is:

$$(28)\quad M^i_t + B^i_t = M^i_{t-1} + B^i_{t-1}(1 + r^i_{t-1}) - \tilde{T}^i_t + P^i_t G^i_t$$

$$= (M^i_{t-1} + B^i_{t-1}) + (P^i_t G^i_t)$$

where G^i_t is real government purchases. Hence, net debt (currency plus bonds) issuance corresponds to nominal government spending.[9] The per capita tax liability (assumed to be a lump sum tax) is:

$$(29)\quad T^{ij}_{t+1} = \delta_{ij}(r^i_t B^i_t / N^i);\ \delta_{ij} = 0,\ i \neq j;\ \delta_{ij} = 1,\ i = j.$$

Combining the government budget constraint with the aggregate consumer demands, and assuming all markets clear yields, after some simplification:

$$(30)\quad M^1_t = N^1 P^1_t \lambda^1_t (q^1_t)^\eta (1 + r^1_t)^{-\gamma}$$

$$(31)\quad e_t M^2_t = N^2 P^1_t (q^2_t)^\eta (1 + r^2_t)^{-\gamma} \lambda^2_t$$

$$(32)\quad (M_t^1 + B_t^1)Y_t = e_t(M_t^2 + B_t^2)$$

$$(33)\quad (M_t^1 + B_t^1)(1 + A_t + A_t Y_t) = A_t P_t^1 Q_t^T, \text{ where:}$$

$$(34)\quad A_t \equiv [\phi X_t]^{(1-\alpha)^{-1}};\ Q_t^T \equiv (N^1 q_t^1 + N^2 q_t^2)$$

Equation (32) is an aggregate bond market equation, and (33) is aggregate commodity demand; (34) is definitional. These four equations, plus (20) -which determines the portfolio rule -determine (P_t^1, e_t, r_t^1, r_t^2), given current asset stocks, output levels, and expectations. In the next section we specify how a rational expectations equilibrium is determined.

III) A Rational Expectations Solution

In order for the agent's expectations to be rational, these expectations must be derived from the same structural model used to derive the current period equilibrium. Unfortunately it is not generally possible to find a closed form solution.[10] The principal difficulty arises because it is not possible to obtain an analytic solution for interest rates, regardless of expectations. Thus, if P_{t+1}^i depends on (r_{t+1}^i), it is not possible to specify the true distribution of prices.

The problem is most severe when interest rates are non-stationary as would arise, for example, if one country grew more rapidly than the other, or the relative supply of bonds to money changed systematically over time. If we wish to find a closed-form solution, we must specify the model so that the probability distribution of interest rates is stationary. This entails assuming that the stock of nominal debt (currency plus bonds) denominated in each currency follows a first order Markov process (or, less plausibly, a stationary distribution), whereas all other disturbances are stationary. Define:

$$(35)\quad D_t^i = (M_t^i + B_t^i) \equiv M_t^i(1 + g_t^i); g_t^i \equiv (B_t^i/M_t^i)$$

$$(36)\quad K_t \equiv [L_t(1 + g_t^2)/(1 + g_t^1)] = [N^2\lambda_t^2(q_t^2)^\eta(1 + g_t^2)/N^1\lambda_t^1(q_t^1)^\eta(1 + g_t^1)]$$

In (35), D_t^i is the total stock of currency i denominated nominal debt, and g_t^i is the proportion of bonds to currency ($(1 + g_t^i)^{-1}$ is the fraction of debt that is monetized). K_t embodies other sources of uncertainty (plus g_t^i), specifically those which affect currency demands. Assume:

$$(37)\quad D_{t+1}^i = U_{t+1}^i D_t^i;\ i = 1,2$$

$$(38)\quad K_t = K^* Z_t$$

where (U_t^i, U_{t+j}^i) are assumed identically and independently distributed. Similarly, (Z_t, Z_{t+j}) have identical independent distributions, and U_t^i is distributed independently of U_t^j, or Z_t for any τ. K^* represents the stationary values (of q^i, λ^i, g^i) about which K_t may vary.11/

Under these assumptions, and using (30) - (32), the portfolio rule (20) is given by:

$$(39)\quad E[((1+g_{t+1}^1)\lambda_{t+1}^1(q_{t+1}^1)^{\eta}(1+r_{t+1}^1)^{-\gamma})^{\alpha}(V_{t+1}^1+Y_{t+1}V_{t+1}^2)^{\alpha-1}(Y_{t+1}V_{t+1}^2-W_tV_{t+1}^1)] = 0$$

$$(39a)\quad V_{t+1}^i \equiv [U_{t+1}^i]^{-1};\ W_t \equiv [Y_t^{(1+\gamma)}K_t^{-1}]^{(1/\gamma)}$$

The interest rate, an endogenous variable, still appears in (39); conceptually, it could be solved for in terms of the exogenous random variables, though this is not analytically possible. The expectation in (39) is taken over the exogenous variables (g^i, λ^i, q^i) and Y_{t+1} which, in turn, depends on K_{t+1} and the distribution of these variables.

Since the exogenous (t+1) random variables in (39) have stationary distributions, and since (r_{t+1}^1) depends only on (t+1) events, inspection of (39) indicates W_t must be constant for all time.

$$(40)\quad W_t = W^*\ \forall t;\ Y_t = [(W^*)^{\gamma}K_t]^{(1+\gamma)^{-1}}$$

$$(41)\quad e_t = (Y_tD_t^1/D_t^2) = [(W^*)^{\gamma}K_t]^{(1+\gamma)^{-1}}(D_t^1/D_t^2)$$

In general, W^* depends on K^* and the distributions of the random variables. Thus, one cannot infer from (41) how permanent changes affect the exchange rate, but it can be used to analyze transitory disturbances.

In addition, the forward rate, forward premium (f_t/e_t), and real interest differentials are:

$$(42)\ f_t = [e_t(1 + r_t^1)/(1 + r_t^2)] = (W^*)[D_t^1/D_t^2]$$

$$(43)\ (f_t/e_t) = [(1 + r_t^1)/(1 + r_t^2)] = [W^*/K_t]^{(1+\gamma)^{-1}}$$

$$(44)\ \Delta_t \equiv [(1 + \rho_t^1) - (1 + \rho_t^2)] = [(1 + r_t^2)P_t^2/D_t^2][(W^* D_{t+1}^1/P_{t+1}^1) - (D_{t+1}^2/P_{t+1}^2)]$$

In (44), Δ_t is the (distribution of) real interest rate differentials between assets. Due to our assumptions, P_{t+1}^i is linear homogeneous in D_t^i, and hence (D_t^i/P_{t+1}^i) is independent of time t events.

Given our assumptions, several results are immediately apparent.

Proposition I: The spot and forward rates are linear homogenous in total country i debt. A proportional expansion of both country i's bonds and money leads to an equiproportionate change in spot and forward rates, and P_t^1; all other variables are unaltered.

Thus, in general, the spot and forward exchange rates depend upon the total stock of country i debt, and its composition, not just on currency supplies. The other variables (except nominal prices) are independent of the stock of debt, but depend on its composition. The effect of transitory disturbances on the exchange rate can be seen from (41); for the perfect foresight case, the impact of transitory disturbances is seen from (7). A comparison indicates:

Proposition II: Transitory changes in money demand (λ^i, q^i) have the same proportionate effect on spot exchange rates under uncertainty and perfect foresight. In either case, the forward rate (and, by assumption, future spot rate) is unaltered.

The explanation for the equivalent effects is because expectations concerning future exchange rates are unchanged. Note, however, that the level of the exchange rate will not, in general, be the same for the two cases. The transitory disturbances will, in either case, affect the forward premium; under uncertainty, the expected real interest differential will only be affected if it is initially non-zero.

If asset demand disturbances are transitory, while asset supply disturbances (D_t^i) are assumed permanent, the spot rate will be more variable than the forward rate and the forward premium will be negatively correlated with (independent of) the spot (forward) rate. Finally, the forecast error, or "risk premium," embodied in the forward rate ($E[e_{t+1} - f_t]/f_t$) will have a stationary distribution. The magnitude of the risk premium cannot be ascertained without further assumptions.

As for transitory money demand disturbances, a transitory change in money supply due to an open market operation will have the same effect under uncertainty as under perfect foresight (for the latter case, it does not matter whether the change in money supply is due to open market operations or increased spending). From (41), holding D^i

constant, a temporary 1% change in M_t^1 will change e_t by $(1/(1 + \gamma))\%$, the same result as under perfect foresight. Again, the explanation for this rests on the constancy of e_{t+1} (and its distribution). As we shall see, a permanent change in the composition of the debt (given its time path) will not have equivalent effects.

Proposition III: An open market operation that is perceived to be transitory will have the same (percentage) effect on spot exchange rates under uncertainty and perfect foresight.

Next, consider how a (surprise) budgetary deficit affects the exchange rate. In order to concentrate on transitory changes, we assume expectations concerning next period exchange rates (and prices) are unaltered. Consistency requires that (the probability distribution of) future asset supplies, and their composition, are unaltered. Hence, we assume agents expect an offsetting surplus (lower deficit) next period, as well as unchanged debt composition.

Under perfect foresight, the (surprise) deficit will have no effect on spot and forward rates if it is bond financed, and will have the same effect as an open market purchase of bonds if it is financed by temporary money creation. For uncertainty the results differ. Since the debt does not follow a random walk, (39) does not hold. Instead, rewrite (20) as:

$$(45)\quad E[(1 + (D_t^2 e_{t+1}/D_t^1))^{\alpha-1}(P_{t+1}^1)^{-\alpha}([(D_t^2 e_{t+1})/(D_t^1 W_t)] - 1)] = 0$$

Since the distributions of (e_{t+1}, P_{t+1}^1) are assumed unaltered, W_t can-conceptually-be solved for in terms of (D_t^1/D_t^2). While no analytic solution is possible, comparative static results can be ascertained.

From differentiating (45), it can be shown that:

(46) $\hat{W}_t = -a_0(\hat{D}^1_t)$; $1 > a_0 > \alpha$ for $\alpha \geq 0$; (if $\alpha = 1$, $a_0 = 1$)

In (46), the notation $\hat{X}_t$ denotes the percentage change in $X_t(dX_t/X_t)$. Substituting into (32) and using the definition of Y_t (W_t) yields:

(47) $\hat{e}_t = [\hat{M}^1_t + \hat{D}^1_t\gamma(1 - a_0)]/(1 + \gamma)$

Using the expressions for the forward rate, and real interest differential, their changes (around $E(\Delta_t) = 0$) are given by:

(48) $\hat{f}_t = (1 - a_0)\hat{D}^1_t$

(49) $dE[\Delta_t] = [(1 + r^2_t)P^2_t/D^2_t][W_tD^1_t\ E(1/P^1_{t+1})](1 - a_0)\hat{D}^1_t$

For $a_0 = 1$ (risk neutrality), these are the same results as for perfect foresight. In general, for risk aversion the effect on the spot rate depends upon both money supply and bond supply changes, whereas the movement in the forward rate depends only upon the level of the deficit. From (47) and (48):

Proposition IV: Assuming agents are risk averse, a temporary increase in debt (temporary, and offset, deficit) leads to:

(i) A larger depreciation of the exchange rate under uncertainty than would occur under perfect foresight (assuming the same change in asset supplies)

(ii) A depreciation in the forward rate, even though expectations are unchanged.

Note that even if the deficit is fully financed by bonds, there is still an exchange rate depreciation, though a smaller one than if it is money financed. These results demonstrate that-unlike the perfect foresight case-money and bond supplies both affect spot rates. Further, they show that for given expectations, it is changes in the stock of debt (and not its composition) that affect the forward rate. This is plausible since the exchange risk is present for money and bonds; if agents are to be induced to hold, for example, more country i currency, they will wish to reduce the bonds (denominated in that currency) that they hold. Hence, the (real) rate on those bonds must rise, compared to other assets. Changes in the spot rate are dominated by money supply (or demand) movements, as differential interest rate movements enter indirectly; but the changes in the forward rate (or interest rates) are dominated by the total stock of debt (given expectations).

Further, note that there is no necessary correlation between the spot and forward rates (or forward premium). Tight monetary policy and a (temporary) deficit can lead to an appreciation of the spot rate, but a forward rate depreciation and increases in the real interest rate on domestic bonds (versus other bonds). A deficit and less restrictive monetary policy ($\hat{M}_t^1 > 0$) implies spot and forward depreciation, and higher (relative) real domestic interest rates. Since exchange expectations are given, relatively higher domestic real interest rates (than abroad) are compatible either with a currency that is expected to appreciate or depreciate. In sum, temporary budget deficits should lead to higher domestic interest rates (relative to abroad), but the movement in the spot rate will largely be dominated by monetary policy.

Finally, consider the effect of sterilized intervention that is assumed transitory. Clearly, the efficacy of the intervention depends on how future expectations are altered, as well as how current asset supplies change. In the spirit of this section, assume the intervention does not alter expectations concerning future spot rates (current supplies of assets to the private sector change, of course). To be consistent with the structural model, this means that expectations concerning future asset supplies are also unchanged.

If future asset supplies are unaltered, and if nominal government spending is also unchanged, then the profits (losses) of the intervention must be refunded to domestic residents. Thus, suppose the central bank of country 1 purchases $\tilde{B}_t^2$ units of country 2 bonds with $\tilde{B}_t^1 (= e_t \tilde{B}_t^2)$ units of its own bonds. At (t + 1) the position is liquidated; the net proceeds from the transaction $(e_{t+1}\tilde{B}_t^2(1+r_t^2) - e_t\tilde{B}_t^2(1+r_t^1))$ are refunded (lump-sum) to domestic taxpayers (the "old"). Clearly, if domestic residents (in the aggregate) increase their holdings of domestic bonds by $\tilde{B}_t^1 (= e_t \tilde{B}_t^2)$ and reduce their holdings of foreign bonds by $\tilde{B}_t^2$, they will remain in portfolio equilibrium (including the transfer/tax due to intervention). In essence, because of the transitory nature of the intervention, agents treat that part of the Central Bank's Balance Sheet as "inside" wealth (or debt), and the sterilized intervention is ineffective. It could be made effective by: (i) assuming the proceeds/losses affect nominal government spending; (ii) transferring the proceeds to the "young"; or (3) changing expectations, perhaps via a "permanent" intervention. Otherwise, the sterilized intervention will not have any effect.

Proposition V: Transitory sterilized intervention is ineffective if Central Bank profits/ losses are rebated to taxpayers.

That completes our analysis of transitory disturbances. In the next section we consider how "permanent" changes affect equilibrium values.

IV) Permanent Changes and Exchange-Rate Effects

The analysis of the previous section was facillitated by the assumption that future exchange rate and interest rate movements were exogenous. A permanent change, however, will lead to shifts in the distribution of these variables. Since an analytic solution for interest rates (in terms of exogenous variables) is not obtainable, it is not generally possible to derive comparative static results for permanent changes.

An examination of (39) shows there are two cases for which solutions can be found; (i) if r_{t+1} is constant, or (ii) if $\alpha = 0$[12]. The nominal interest rates will be constant over time if the only source of uncertainty concerns the rate of debt expansion. Hence, in the analysis that follows, we assume this to be the case. The qualitative results for our analysis and for the case in which $\alpha = 0$ and other disturbances (such as money demand, output levels, etc.) are present are identical, so we omit the latter case to save space.

Specifically, we assume: (i) the nominal debt in each country follows a first order Markov process, as in (37): and (ii) that agents assume current values of other variables (g^i, q^i, λ^i) will prevail forever. Given these assumptions, we inquire how permanent changes in these variables, or in the random rate of debt expansion, affect equilibrium values.[13] Under the above assumption, the portfolio rule (39) is:

$$(50) \quad E[(V_{t+1}^1 + Y_{t+1}V_{t+1}^2)^{\alpha-1}(Y_{t+1}V_{t+1}^2 - W_t V_{t+1}^1)] = 0.$$

Since the V_t^i are stationary, the solution to (50) implies W_t, and hence Y_t, (since K is constant) are constant. Dropping the time subscripts, (50) may be rewritten as an implicit function determining Y (in terms of K, and the distributions of V^i):

$$(51) \quad J(Y,K) = E[(V^2 - (Y/K)^{(1/\gamma)}V^1)(V^2Y + V^1)^{\alpha-1}] = 0.$$

Further, since $\lim_{Y\to 0}[J(Y,K)] > 0$, and $\lim_{Y\to\infty}[J(Y,K)Y^{1-\alpha}] < 0$, a solution $Y^*(K)$ exists. Finally, since $J_Y(Y, K) < 0$ at $J = 0$, the solution is unique.

While no analytic solution to (51) is possible, comparative static results can be derived. Totally differentiating (51) and simplifying yields (see Appendix for details):

$$(52) \quad \hat{Y} = a_1 \hat{K}; \; 1 > a_1 > [1 + \gamma(1 - \alpha)]^{-1} > 0, \; \alpha < 1$$

($\alpha = 1$ implies $a_1 = 1$). By definition:

$$(53) \quad \hat{K} = (\hat{\lambda}^2 - \hat{\lambda}^1) + \eta(\hat{q}^2 - \hat{q}^1) + [(dg^2/(1 + g^2)) - (dg^1/(1 + g^1))]$$

$$(54) \quad \hat{D}_t^i = \hat{M}_t^i + (dg^i/(1 + g^i)) = [(\hat{M}_t^i + g^i\hat{B}_t^i)/(1 + g^i)]$$

From (32), (42) and (52):

$$\begin{aligned}(55) \quad \hat{e}_t = \hat{Y} + \hat{D}_t^1 - \hat{D}_t^2 &= a_1[(\hat{\lambda}^2 - \hat{\lambda}^1) + \eta(\hat{q}^2 - \hat{q}^1)] \\ &+ ([\hat{M}_t^1(1 + a_1 g^1) + g^1(1 - a_1)\hat{B}_t^1]/(1 + g^1)) \\ &- ([\hat{M}_t^2(1 + a_1 g^2) + g^2(1 - a_1)\hat{B}_t^2]/(1 + g^2))\end{aligned}$$

$$(56) \quad \hat{f}_t = [((1 + \gamma)\hat{Y} - \hat{K})/\gamma] + \hat{D}_t^1 - \hat{D}_t^2$$

$$(57) \quad \hat{f}_t - \hat{e}_t = [(\hat{Y} - \hat{K})/\gamma]$$

where $(\hat{f}_t - \hat{e}_t)$ is the change in the forward premium.

Since g^i is assumed constant over time, prices in each country grow at the (random) rate of debt expansion, and nominal interest rates are constant. The difference in real returns on bonds is given by:

$$(58)\ \Delta_t = [(1 + r_t^1)(P_t^1/P_{t+1}^1) - (1 + r_t^2)(P_t^2/P_{t+1}^2)] = (1 + r^2)[(Y/K)^{1/\gamma}V_{t+1}^1 - V_{t+1}^2]$$

The change in $E(\Delta_t)$, around $E(\Delta_t) = 0$, is:

$$(59)\quad d[E(\Delta_t)] = (1 + r^2)(Y/K)^{1/\gamma}(E(V_{t+1}^1)((\hat{Y} - \hat{K})/\gamma) + d\ E(V_{t+1}^1))$$

The above comparative static results allow us to infer how changes in the exogenous variables affect equilibrium prices: however, the level of these prices cannot be inferred without specifying the distributions of (V^1, V^2). For now, assume that V^1 and V^2 have identical, independent distributions. Under this assumption, it is clear from (51) that (see Appendix):

$$(60)\quad Y \gtrless 1 \text{ as } K \gtrless 1;\ (Y/K) \lessgtr 1 \text{ as } K \gtrless 1.$$

The results embodied in (55) - (60) allow us to make the following inferences:

<u>Proposition V</u>: Under risk aversion, the expected real return on the two bonds will, in general, differ-even if disturbances are symmetric.

From (60), it is apparent the expected real returns (given identically distributed random debt growth) will be equal only if K = 1 -in essence, only if the two countries are identical. In general, the real return on country 1's bond will be higher if: (i) it is a larger country;

(ii) its agents have a relatively greater demand parameter (λ^1) for currency; or (iii) a larger fraction of its debt is issued in bonds. The effect of country size is comparable to recent results from international trade and capital movements (Grossman and Razin (1983)). Both effects (i) and (ii) occur because they imply country 1 agents wish to hold relatively more domestic currency which leads to an offsetting reduction in demand for domestic bonds; hence, the relative return on these bonds must rise. The third effect implies that relatively higher real returns are required to induce agents to hold the larger (relative) supply of country 1 bonds. Finally, note that the level of the debt does not affect the forward premium or real interest differential since all debt changes are assumed permanent, and interest rates are unaffected by (balanced) changes in debt.

Next, consider the relation between the spot rate under uncertainty and perfect foresight. Given the same (expected) rate of monetary (debt) expansion, the perfect foresight exchange rate is:

(61) $e_t^* = (M_t^1/M_t^2)K((1 + g^1)/(1 + g^2)) = K(D_t^1/D_t^2)$

Under uncertainty:

(62) $e_t = Y(D_t^1/D_t^2)$. Hence:

(63) $e_t \gtrless e_t^*$ as $Y \gtrless K$; i.e., as $K \lessgtr 1$.

Proposition VI: Given identically, independently distributed rates of monetary (debt) expansion, the efect of uncertainty is to lead to a depreciation of the currency for which real bond yields are larger.

The explanation is immediate--the higher interest rates (or forward rate) leads to a decreased demand for currency, and hence a depreciation of the spot rate. From (55) - (59) the effects of permanent changes on spot and forward rates can be deduced, and compared to their perfect foresight results. Unlike the case of transitory disturbances, permanent changes in money demand (via λ^i or q^i), or permanent changes in the composition of the debt (given its level) will have different quantitative effects under uncertainty than under perfect foresight. This is because these changes affect future spot exchange rates and hence, under risk aversion, have different affects on the forward rate change which is needed to induce agents to hold the existing asset stocks.

Comparing (61) to (62), it can be seen that the effect of uncertainty is to reduce the sensitivity of the spot (and forward) rates to shifts in money demand or supply (for given debt levels). Under perfect foresight, a 1% increase in demand for currency 1 (via increases in λ^1 or q^1) will lead to a 1% spot (and forward, due to stationarity) exchange rate appreciation, whereas under uncertainty the proportional spot (and forward) appreciations will be less. Comparably, a change in money supply (via a permanent open market operation) will have a smaller proportionate effect under uncertainty. Pure debt expansion, given the composition of the debt, will have the same proportionate effect under uncertainty and perfect foresight.

Furthermore, note that under uncertainty there is no direct correlation between movements in the spot rate and forward premium (or real interest differential) if debt levels also change. Given the stock of debt, the spot rate and forward premium move in opposite directions: an appreciation of the currency is associated with an increase in the forward premium and hence higher real returns on domestic (versus foreign) bonds. However, if debt stocks change, there is no such presumption. An increase in bond stocks (via a budgetary deficit) leads to a depreciation of the spot and forward rate and an increase in the forward premium and the relative real return on domestic bonds. Hence, the effect of a budgetary deficit on spot rates and the forward premium depends on how it is accomodated by monetary policy. A tight monetary policy (decrease in the proportion of debt that is monetized) will lead to higher relative real returns on domestic bonds; the spot rate may even appreciate if the monetary policy is sufficiently restrictive.

The preceeding analysis presumed that the (distribution of) rates of debt expansion were identical across countries. A natural sequel is to ask how changes in the rate-or variability- of debt expansion affects the spot and forward rates, and forward premium.[14] However, one must be careful in posing the question. A mean preserving spread of U^1 (the rate of country 1 debt expansion) will increase the expected real return (given nominal interest rates) on country 1 bonds because of Jensen's inequality $((P_t^1/P_{t+1}^1) = (U_{t+1}^1)^{-1})$. Thus, rather than considering changes in the distribution of debt expansion (U_{t+1}^i), we consider changes in the distribution of the intertemporal price level (or $(U_{t+1}^1)^{-1}$).[15]

First, consider a change in policy that leaves the relative variability of the intertemporal price ratio (compared to the rate of debt expansion) unchanged. Let:

(64) $V^1 = \Theta Z^1$; $U^1 = (\Theta Z^1)^{-1}$; $E(Z^1) > 0$

where Θ is a parameter, and Z^1 a random variable. An increase in Θ corresponds to an increase in the expected real return, and its variability, of country 1 bonds (given nominal interest rates); equivalently, it corresponds to a decrease in the rate, and variability, of nominal debt expansion.

Using (51), the impact of changes in Θ on Y (the portfolio rule) can be found. As shown in the Appendix:

(65) $Y = (-a_2)\Theta$; $\gamma > a_2 > (\alpha\gamma)(1 + \gamma\,(1 - \alpha))^{-1}$

Similarly, the "risk premium" on country 1 assets:

(66) $k = E[f_t/e_{t+1})] = (Y/K)^{1/\gamma}\ E[(V^1_{t+1}/V^2_{t+1})]$

(67) $\hat{k} = \hat{\Theta}[1 - (a_2/\gamma)]$

And the change in the expected real interest differential between country 1 and country 2 bonds is given by (59):

(68) $d[E(\Delta_t)] = (1 + r^2)\ E[V^1]\ \Theta(Y/K)^{1/\gamma}\ (1 - (a_2/\gamma))$

For risk neutrality ($\alpha = 1$), $a_2 = \gamma$, and there is no change in the risk premium or real interest diferential (which is zero). However, for $\alpha <$ 1, an increase in Θ increases the forward premium and the relative real return on country 1 bonds.

<u>Proposition VII</u> A decrease in the rate of country 1 debt expansion (given its relative variability) will, if agents are risk averse: (i) lead to a smaller percentage appreciation of country 1's currency than would occur under perfect foresight; and (ii) lead to an increase in the risk premium, or relative real interest rate, associated with country 1 assets.[16/]

Under perfect foresight, a 1% reduction in the rate of monetary (debt) expanion will lead to a (γ)% appreciation of the spot rate. From (55) and (65), the spot rate under uncertainty will appreciate by less than γ% (it may, conceivably depreciate if $\alpha < 0$). Hence, part (i) of the proposition follows, and reaffirms our earlier results that uncertainty reduces the sensitivity of the exchange rate to movements in exogenous parameters. Part (ii) of the proposition follows immediately from (67) and (68). The increase in the relative real return on domestic assets occurs because the variability of the real return on these assets is increased by an increase in Θ (decrease in the rate of debt expansion).

On the other hand, a mean preserving spread (MPS) of V^1 (i.e., an increase in the variability of the real return on country 1 bonds, for a given mean) will lead, as expected, to a spot depreciation of the exchange rate and an increase in the relative real return (and risk premium) on country 1 bonds. Let:

(69) $V^1 = a + \Theta Z^1$; a, scalar, $E(Z^1) = 0$, $E[(Z^1)^2] > 0$.

An increase in Θ corresponds to a MPS of (V^1), and hence of the real return on country 1 bonds (given nominal interest rates). As shown in the Appendix:

(70) $(dY/d\Theta) > 0$.

Consequently:

Proposition VIII: An increase in variability (a MPS) of the real return on country 1 bonds leads, under risk aversion, to: (i) a depreciation of 1's exchange rate, and (ii) to an increase in the risk premium, and relative real return, on country 1 assets.

However, care must be taken in interpreting this result; it does not imply that an increase in the variability of the rate of debt (monetary) expansion--that is, less predictability concerning future nominal deficits--will lead to an exchange rate depreciation and a higher "risk premium" for that country's currency. Due to Jensen's inequality, increased variability of U^1 (debt expansion) raises the expected real return on country 1 bonds (given nominal interest rates). Thus, as shown in the Appendix, if agents are not "too" risk averse ($\alpha > 0$) a MPS of U^1 will lead to an appreciation of country 1's currency. Consequently, in estimating how "variability" or "forecast errors" affect exchange rates and real interest rates, great care must be taker in specifying the source-and form-of the variability.

V) Conclusions

In this paper we have presented a general equilibrium model in order to investigate how exchange rates and real interest differentials are determined under uncertainty. Because the starting point of the analysis was the assumption of money demand functions typically employed in the Monetary Approach to Exchange Rate Determination, many of our results are qualitatively similar to those predicted by the Monetary Approach with perfect capital mobility.

Nevertheless, we have seen that the presence of uncertainty does lead to some modifications of these conventional results. As seen throughout the paper, the uncertainty and risk aversion reduces the sensitivity of exchange rate movements to changes in exogenous parameters, such as output levels, money demand disturbances, or expectations concerning future monetary (and debt) expansion. Furthermore, we have shown that it is the total stock of debt denominated in a particular currency, and not just currency supplies, that affect spot and forward exchange rates. For given money supply levels, an increase in the stock of bonds denominated in a particular currency will lead to a spot depreciation of that currency and to an increase in the "risk premium" associated with holding that currency.

Furthermore, we have been able to investigate the determination of the risk premium-or real interest differential-within the context of the mocel. Generally, we have seen that the real return on domestic bonds will be higher than on foreign bonds if (i) the domestic country is larger, (ii) a larger fraction of domestic debt is in terms of bonds; (iii) the expected rate of domestic monetary (debt) expansion is lower than abroad, or if (iv) the domestic intertemporal price level is more variable. Since the above factors can lead to appreciation or depreciation in the spot rate, there is no clear connection between movements in the spot rate and in the real interest differential between countries (or their bonds).

The current situation of large U.S. budgetary deficits, a strong (and appreciating) dollar, and high real interest rates in the U.S. versus its trading partners can be accounted for within the context of the model. If the current budgetary deficits are viewed as transitory, to be followed by lower deficits (than abroad), and if the deficit is largely financed through bond issuance, then the overall affect could be to lead to an appreciation of the spot rate. These factors--increases in the proportion of debt that is accounted for by bonds, and decreases in expected future deficits--will also lead to an increase in the expected real return on domestic (versus foreign) assets.

At the same time, much of importance has been omitted from the analysis. Since nominal assets are the only stores of value, one cannot ask how exchange rate variability--as caused by expectations concerning future asset demands or supplies--affects real investment, or compare the levels of real investment under fixed and flexible exchange rates. Similarly, since there are no wage contracts (denominated in nominal-or any-units), one cannot ask how exchange-rate variability affects employment, or compare the performance of fixed and flexible exchange rates with respect to stabilizing employment. However, a model of this type may be generalized to deal with these important issues.

Footnotes

1/ The Kareken-Wallace results imply one set of solutions is a constant, but indeterminate, exchange rate. Under uncertainty, other solutions may exist in which the exchange rate varies over time.

2/ Alternatively, the money demand functions could be "derived" by introducing currency i real money balances in the utility function of agents of country i. This procedure yields results that are comparable to those discussed in the paper. We use the money demand functions (1) to make our results comparable with the traditional monetary approach. Under certainty, and "perfect capital" mobility, the exchange rate will be independent of bond stocks.

3/ The use of a transactions technology is common in this type of model; for example, Helpman and Razin (1982a, 1982b) assume goods must be paid for in the seller's currency. However, since their demand functions are insensitive to interest rates, expectations do not affect the current spot rate. In order to emphasize the role of expectations, we assume that money demand is interest sensitive.

4/ Risk neutrality (α = 1) implies (C_t^i, S_{t+1}^i) are perfect substitutes and, in general, leads to corner solutions. An alternative specification: $U = (C_t^i)^{\lambda}(S_{t+1}^i)^{\alpha}$ allows for interior solutions and risk neutrality with respect to second period real income (α = 1). These two specifications yield equivalent portfolio demand rules.

5/ Allowing the young to pay taxes would not alter the analysis. The assumption that the nominal taxes due are known does simplify the analysis. Since these taxes will be identified later with interest due on the current debt, the assumption is consistent.

6/ Since short bond positions are allowed and since the Inada derivative conditions hold, an interior solution will exist.

7/ These four equations are independent since the total demand for wealth is endogenous. Alternatively, one bond market equation could be replaced by the commodity market equilibrium equation.

8/ If one country could issue bonds denominated in the other country's currency, it would become necessary to consider default risks. Clearly, economic agents do not consider dollar denominated bonds issued by Brazil to be perfect substitutes for those issued by the U.S.

9/ Although in our model bonds are outside wealth, an increase in the stock of bonds does not make agents feel richer because: (i) they are not used as transfers; and (ii) agents assume the interest liability on these bonds. Indeed, an increase in bonds (due to an increase in G) will cause agents to reduce their consumption demand by (rPG), just as they would if the spending were financed by taxes and agents attempted to equalize consumption over their infinite lives (or those of their offspring).

10/ As usual, a multiplicity of solutions can arise. From (20), one possible solution is: $e_{t+1} = [e_t(1 + r_t^1)/(1 + r_t^2)]$ for all (t + 1) realizations. Solving backwards, this yields:

$$\ln e_{t+1} = (1/\gamma)[\sum_{i=0}^{\infty}((\ln W_{t-i})((1 + \gamma)/\gamma)^i)],$$

$W_t \equiv [M_t^2/(M_t^1 L_t)]$, which is explosive. We rule out this solution, and seek one which only depends upon current variables and expectations.

11/ Since there are no transfers, the assumption on debt growth implies assuming nominal government puchases grow at a random (constant mean) rate, and expected real purchases are constant over time. By allowing taxes on the young, the (seemingly remote) possibility of budgetary surpluses could be handled as well.

12/ For $\alpha = 0$, the utility function in (8) is not defined. However, the utility function: $U = \ln C_t^1 + \phi \ln S_{t+1}^1$ yields behavioral rules identical to those derived in the text when α is set equal to zero. Even for $\alpha = 0$, the disturbances (other than debt expansion) must be stationary. If, for example, output levels follow a first order Markov process, we cannot derive a closed form solution.

13/ Within the context of the analysis, any change in (g^i, q^i, λ^i) must be viewed as a surprise. However, even if the change at t had been anticipated at (t - 1), this would not affect period t equilibrium, though the anticipation would affect, of course, the equilibrium at (t - 1). The affect of anticipated permanent changes can easily be derived by first showing how the actual change affects the equilibrium distributions, and then using (for t - 1) the transitory analysis of section III by viewing the (t - 1) variable value as a transitory change from its permanent level.

14/ Clearly, it is not appropriate to ask how changes in the variability of e_{t+1} affect spot rates since an increase in the variability of e_{t+1} (given its mean) will increase $E[(e_{t+1})^{-1}]$, and will also change the variability of $(e_{t+1}{}^{-1})$.

15/ Another reason for considering the reciprocal $(V^1 = (U^1)^{-1})$ is if it is real spending that is stochastic $P_{t+1}^1 G_{t+1}^1 = (U_{t+1}^1 - 1)\, D_t^1$; or $G_{t+1}^1 = (1 - V_{t+1}^1)[\lambda^1(q^1)^{\eta}(1 + g^1)(1 + r^1)^{-\gamma}]$. Hence, a mean preserving spread of real government spending corresponds to a MPS of V_{t+1}^1, not U_{t+1}^1 (given r^1).

16/ Essentially, this says that a positive risk premium is likely to be associated with an appreciating currency (i.e., a currency that is expected to appreciate). This result is consistent with "stylized facts," such as shown in Figure 1 of Wyplosz's article (1983, p.124).

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Appendix

Derivation of Comparative Static Results

(51) of the text can be rewritten as:

(1A) $J(Y, K) = E[(V^2(K/Y)^{1/\gamma} - V^1)(V^2Y + V^1)^{\alpha-1}] = 0$

For notational convenience, define:

(2A) $A \equiv [V^2(K/Y)^{1/\gamma} - V^1]$; $B \equiv (V^2Y + V^1)$

Let $J_z \equiv (\partial J/\partial z)$, $z = Y, K$. Then:

(3A) $[Y \cdot J_Y] = - E[((V^2/\gamma)(K/Y)^{1/\gamma}B + (1 - \alpha)YV^2 \cdot A)B^{\alpha - 2}] < 0$

since

(4A) $E[(YV^2/B)\ (A \cdot B^{\alpha - 1})] > 0$

since $(AB^{\alpha - 1}) \gtrless 0$ as $(V^2/V^1) \gtrless (Y/K)^{1/\gamma}$ and (YV^2/B) increases as (V^2/V^1) increases.

(5A) $[KJ_K] = (1/\gamma)E[V^2(K/Y)^{1/\gamma}B^{\alpha - 1}] > 0.$

Hence:

(6A) $(\partial Y/\partial K)(K/Y) = - [J_K K/(J_Y Y)] > 0.$

Let:

(7A) $\hat{Y} = a_1\hat{K}$; $(J_yY)\hat{Y} + (J_kK)\hat{K} = 0$ implies:

(8A) $E[V^2\{(K/Y)^{1/\gamma}((1-a_1)/\gamma)B - a_1(1 - \alpha)\ YA\}B^{\alpha - 2}] = 0$

For $a_1 = 0$, the LHS of (8A) is positive; for $a_1 = 1$, it is negative $(\alpha < 1)$. Thus, $a_1\ \varepsilon(0,1)$ for $\alpha < 1$.

Rewrite (8A):

(9A) $E[(YV^2)\ \{(K/Y)^{1/\gamma}(a_1(1 - \alpha) - ((1 - a_1)/\gamma))V^2$

$\quad - V^1(a_1(1 - \alpha)Y + ((1 - a_1)/\gamma)(K/Y)^{1/\gamma})\}B^{\alpha - 2}] = 0$

Since $0 < a_1 < 1$, the coefficient of V^2 must be positive:

(10A) $1 > a_1 > [1 + \gamma(1 - \alpha)]^{-1}$ which is the result of (52). This seems to be the closest bounds that can be placed on a_1.

If (V^1, V^2) have identical distributions:

(11A) $E[(V^2 - V^1) \cdot (V^2 + V^1)^{\alpha-1}] = 0$ by symmetry.

Thus, for $K = 1$, $Y = 1$ solves (1A). Since Y is unique (given distributions) and monotonically increasing in K; and since $0 < a_1 < 1$:

(12A) $Y \gtrless 1$ as $K \gtrless 1$; and $(Y/K) \lessgtr 1$ as $K \gtrless 1$,

which is the result asserted in (60).

For changes in the distribution of V^1:

(13A) $J(Y, K; \theta) = E[(V^2(K/Y)^{1/\gamma} - V^1)(V^2Y + V^1)^{\alpha-1}] = 0$

(14A) $V^1 = \theta Z^1$, θ, a scalar.

J_y is given by (3A). J_θ is:

(15A) $[\theta J_\theta] = E[(-V^1)(\alpha V^1 + (YV^2(1 + W)/W))B^{\alpha-2}] < 0$, $\alpha > 0$.

Hence, Y decreases as θ increases, for $\alpha > 0$; for $\alpha < 0$, we have not been able to sign (15A). Assume $\alpha > 0$ and define:

(16A) $\hat{Y} = -a_2\hat{\theta}$; $a_2 > 0$, $\alpha > 0$.

Since $[\hat{\theta}(\theta J_\theta) + \hat{Y}(YJ_y)] = 0$, simplification of (3A) and (15A) yields:

(17A) $E[(V^1)\{B(1 - (a_2/\gamma)) + (1 - \alpha)(1 + a_2)A\}B^{\alpha-2}] = 0.$

As in (4a), $E[(V^1/B)(AB^{\alpha-1})] < 0$. Hence, (17A) implies $(a_2 < \gamma)$. Rewriting (17A) as:

(18A) $E[(V^1/B)(C(K/Y)^{1/\gamma}V^2 - dV^1)B^{\alpha-1}] = 0$, where

(19A) $C = (1 - \alpha)(1 + a_2) + W(1 - (a_2/\gamma))$; $d = - \alpha + (1 - \alpha)a_2 + (a_2/\gamma)$

Since $C > 0$, (18A) implies $d > 0$, <u>i.e.</u>:

(20) $\gamma > a_2 > [(\alpha\gamma)/(1 + \gamma(1 - \alpha))] > 0$, $\alpha > 0$.

(Note (20) holds even for $\alpha < 0$; however, a_2 cannot be signed in this case.)

Next, consider a mean preserving spread of the reciprocal of the rate of debt expansion:

(20A) $V^1 = a + \theta Z^1$; $E(Z^1) = 0$, $E((Z^1)^2) > 0$; a - scalar.

As earlier, Y is determined by (13A), and $J_Y < 0$. Consider J_θ; from (13A):

(21A) $J_\theta = E[(-Z^1)\ \{B^{\alpha-2}[V^2Y(1 + ((1 - \alpha)/W)) + \alpha V^1]\}]$; let:

(22A) $M(V^1, V^2) = B^{\alpha-2}[V^2Y(1 + ((1 - \alpha)/W)) + \alpha V^1]$; and

(22'A) $\bar{M}(V^1) \equiv \underset{V_2}{E}[B^{\alpha-2}\{V^2Y(1 + ((1 - \alpha)/W)) + \alpha V^1\}]$

$(d\bar{M}/dV^1) < 0$ for $\alpha > 0$. Hence:

(23A) $\underset{(V^1)}{E_1}[(-Z^1)\bar{M}(V^1)] > E[(-Z^1)]E[\bar{M}(V^1)] = 0, \ \alpha > 0.$

Thus, for $\alpha > 0$, $J_\Theta > 0$ and $(\partial Y/\partial\Theta) > 0$, as in the text ($\alpha = 1$ implies $J_\Theta = 0$). For $\alpha < 0$, we have not been able to sign J_Θ.

Finally, consider a change in the variability of debt expansion (U^1):

(24A) $U^1 = a + \Theta Z^1$; $V^1 = (U^1)^{-1}$; a, scalar; $E(Z^1) = 0$.

(25A) $(dV^1/d\Theta) = -(V^1)^2 \cdot Z^1$.

As earlier, Y is determined by (13A); $J_Y < 0$.

From (13A);

(26A) $J_\Theta = E[(Z^1)\{(V^1)^2 \cdot M(Z^1, V^2)\}]$

where M is defined by (22A). For $\alpha > 0$:

(27A) $\frac{\partial}{\partial Z^1}[\underset{(V^2)}{E_2}\{(V^1)^2 \cdot M\}] < 0$

Hence:

(28A) $E[(Z^1)\{(V^1)^2M\}] < E(Z^1)E((V^1)^2M) = 0$

and $J_\Theta < 0$. Consequently, for $\alpha > 0$, $(dY/d\Theta) < 0$ - i.e., an increase in the variability of debt expansion causes an appreciation of the exchange rate for that country.