

International Finance Discussion Papers

Number 271

December 1985

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The authors have benefited from discussions with Rao Aiyagari as well as from seminars at Harvard and Indiana Universities. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Board of Governors of the Federal Reserve System or any of its staff. Financial support from the Alfred P. Sloan Foundation is gratefully acknowledged.

Elections and Macroeconomic Policy Cycles

ABSTRACT

There is an extensive empirical literature on political business cycles, but its theoretical foundations are grounded in pre-rational expectations macroeconomic theory. Here we show that electoral cycles in taxes, government spending and money growth can be modeled as an equilibrium signaling process. The cycle is driven by temporary information asymmetries which can arise if, for example, the government has more current information on its performance in providing for national defense. Incumbents cheat least when their private information is either extremely favorable or extremely unfavorable. An exogenous increase in the incumbent party's popularity does not necessarily imply a damped policy cycle.

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I. Introduction

At one time, research on political business cycles received a great deal of attention. Nordhaus (1975) and McRae (1977) provide important examples.¹ Their models suggest that politicians will inflate during election years in order to exploit a Phillips curve tradeoff which is more favorable in the short-run than in the long-run. Interest in these adaptive expectations models waned, however, after the rational expectations revolution of the seventies. As long as private agents (such as wage setters) understand the government's incentives, one would not expect to observe any systematic rise in employment prior to elections.² But the objections to conventional political business cycle models go beyond their Phillips curve formulation, and apply to any model in which the government takes artificial measures to make itself look good. Suppose the government tries to please the public before elections by raising transfers or by lowering taxes; according to Tufte (1978), this is the most robust empirical characteristic of the electoral cycle. Why should voters prefer a candidate who is suboptimally distributing tax distortions over time? Moreover, why should such actions suggest that the incumbent will do a good job over the course of the coming term?

Here we argue that electoral cycles in certain macroeconomic policy variables -- such as taxes, government spending, deficits and money growth -- derive from temporal information asymmetries.³ We assume that the government observes an indicator of its performance (e.g., in providing national defense efficiently) before the representative voter does. Administrative performance is correlated over time; hence prior to election periods the incumbent has an incentive to try to "signal" that it is doing well. This gives rise to an electoral cycle in macroeconomic policy. (It is important to stress that our model does not directly provide a rationale for an electoral cycle in unemployment.)⁴

In section II we present the basic model. Governments in this setup are differentiated in part by their level of "competency". Although the analysis could be extended to encompass other aspects of a government's performance, the notion of competency that we use is as follows: The more competent that a government is, the less revenue it needs to provide a given level of government services. This particular measure of competency stresses the administrative abilities of the policymaker. Naturally, other things being equal, voters prefer more competent governments. The government obtains information about its (serially correlated) competency more quickly than voters can. However, at the beginning of each period voters receive a signal; they learn the level at which the government is going to set income or poll taxes. Because taxes are set at the beginning of the period, any (intentional or unintentional) error the government makes must be made up through issuing bonds or drawing seignorage. (In another variant of the model, government spending can also be adjusted.) Thus the public observes the government's competency directly, but with a lag. During an election year, a vote-conscious incumbent party has incentive to make its most recent "competency shock" appear large. The incumbent party's incentive to "cheat" (rely on excessive seignorage or bond financing) is tempered only by the fact that it places some weight on social welfare.

There is only one really important difference between the incumbent party and the opposition party in our model. The opposition party has no credible way to "signal" the effectiveness of its policies. Obviously, there are many factors other than "competency" which influence the outcome of an election, but our model treats these as exogenous. Nevertheless, the framework developed here is sufficient to capture the essential elements of a political business cycle.

In section III, we consider equilibria in which voters' expectations depend only on known characteristics of the two parties, past observations on the competency shocks, and the government's most recent tax bill. In particular, expectations do not depend on how much a party might have "cheated" (set taxes suboptimally low) in previous elections. The equilibria in section III are the only possible equilibria when the political parties have finite horizons, given the assumed information structure. Using only mild restrictions on the utility functions, we are able to prove the existence of a unique separating equilibrium. In the separating equilibrium, voters are able to exactly infer the incumbent's information from the tax bill. Because the competency shock is directly observed by the public with a lag, there is never any "cheating" (suboptimal use of seignorage or bond financing) during off-election periods.

During election periods, the equilibrium has the following characteristics: If the incumbent party knows that its competency shock is the lowest possible, then it will not cheat at all. This result obtains even if the incumbent's temptation to improve its image is greatest when its competency is lowest. Cheating is increasing over a range of realizations of the unobservable competency shock, but is declining at higher levels of competency. Incumbents of intermediate ability cheat the most. (This conclusion must be amended if there is an upper bound to the level at which taxes can be set.) The model also yields some interesting conclusions with respect to changes in observable factors. For example, the conventional wisdom is that the incumbent party will inflate less if it becomes more popular for noneconomic reasons. We show, however, that although an increase in popularity may cause the incumbent to cheat less if its unobservable competency shock is high, it will cheat more if its competency is low.

In section IV, we examine other equilibria which can arise if the two political parties have infinite horizons. If their discount rates are low enough, then there will exist an equilibrium in which there is no macroeconomic policy cycle. "Enforcement" of the cooperative equilibrium can come through a number of related channels, including both voters' expectations and the strategic interactions of the two political parties. If the parties have sufficiently high discount rates, there will always be a cycle, but it may be damped. We illustrate some properties of sustainable equilibria for this case. Finally, in section V, we briefly consider what happens if the incumbent party cannot perfectly project its revenue needs; i.e., has imperfect information about its competency shock. The analysis of section III generalizes even if the forecast error is strictly private information. In the conclusions, we discuss some empirical implications of our model, and some possible extensions.

II. The Model

Every other period, atomistic voters choose between the two political parties, "R" and "D". A major factor in the election is voters' perception of the relative "competencies" of the two parties. A party's competency is defined as follows: All governments are required to provide a fixed level of government services, G . (The operative assumption is that G is observable, not that it is fixed.) The more competent the government, the less revenues it requires to deliver G :

$$(1) \quad G = \epsilon + \tau + \Delta ,$$

where ϵ is the government's competency, τ is direct taxes (or transfers if negative), and Δ represents seignorage or bond-financing. "Competency" is a broad index which captures the administrative abilities of the incumbent party, and the success of its policies in providing necessary government services efficiently. For example, a highly competent government will use well-designed bidding procedures on government contracts, and will make good choices about which weapons systems to purchase. (Although the analysis below focuses on the definition of competency embodied in (1), it would be possible to apply our framework to other measures of a government's performance).

The underlying macroeconomic model is rudimentary. In each period t , every voter receives a fixed, known amount of the perishable good. Agents pay a total of τ_t in real taxes. Money is required to conduct transactions, and the transactions technology is such that the equilibrium price level, P_t , is increasing in the money supply, M_t . We assume that there is no bond financing and that seignorage revenues, $\Delta_t = (M_t - M_{t-1})/P_t$, are strictly increasing in M_t over the relevant range. Our analysis applies equally well to the case of

bond financing if taxes are distortionary. In this case there generally exists an optimal distribution of tax distortions over time.⁵

Each of the identical voters has a time-separable indirect utility function, which depends on the incumbent's competency, ϵ , the level of distortions arising from seignorage or suboptimal deficit financing, Δ , and exogenous noneconomic factors which depend on which party is in power, η . (It is straightforward to extend the model to allow for different voters to have different preferences with respect to exogenous factors.) Social welfare in period t is given by

$$(2) \quad \Omega_t = -W(\Delta_t) + \epsilon_t + \eta_t,$$

where $W:]-\bar{\Delta}, \bar{\Delta}[\rightarrow \mathbb{R}$ is twice-continuously differentiable, except possibly at zero. $W'(<)0$ for $\Delta >(<)0$, and $W'' > 0$. Thus, W is minimized at $\Delta = 0$, and $W(\Delta) \rightarrow \infty$ as $\Delta \rightarrow \bar{\Delta}$.⁶

Each party's competency shock is serially correlated so that for party $j = D, R$

$$(3) \quad \epsilon_t^j = \alpha_t^j + \alpha_{t-1}^j,$$

where (α_t^j) is an i.i.d. stochastic process on $A = [0, \bar{\alpha}[$, where $\bar{\alpha}$ may be infinite. It is assumed that α^D and α^R are independent $\forall s, t$, that they have the same twice-continuously differentiable distribution function, and that $E_{t-1}(\alpha_t^j) = \hat{\alpha}$. It is crucial that (ϵ_t^j) display some serial correlation, or there would be no reason to vote for a party just because it appears more competent today. The assumption that the competency shocks follow MA(1) processes simplifies the analysis by making it possible for elections to be independent. However, we present results in section III indicating some qualitative features

of the more general case. The fact that the shocks vary over time may be justified by noting that the leaders of a political party change, and that policy prescriptions suited for one historical episode may be inappropriate in other circumstances.

We assume that exogenous, party-specific, non-economic preference shocks also follow a MA(1) process so that

$$(4) \quad \eta_t^R - \eta_t^D = q_t + q_{t-1} ,$$

where (q_t) is an i.i.d. stochastic process on \mathbb{R} . Furthermore, the density function of q is unimodal, continuously differentiable, and symmetrically distributed around zero.

Any promises the two parties might make before an election have no impact on the voters in our model. (All the voters are from Missouri.) What does make an impression on voters is their observations on the incumbent party's performance, from which they can infer something about its most recent competency shock. (We will describe the information structure shortly.) The macroeconomic policy cycle will arise because the incumbent party has an incentive to try to signal that its most recent competency shock is high. The opposition party can make promises, but it lacks an effective way to reveal how well it would have performed if it were currently in office. In fact, all the public knows about the opposition party is the probability distribution of its competency factor, ϵ . Because ϵ is an MA(1) process, the fact that the opposition party may once have been in power is not relevant.⁷

It would be pointless for the incumbent party to try to deceive the public unless it has an information advantage. Our assumptions about the information structure and the timing of elections are as follows. The incumbent party has

contemporaneous information about its most recent competency shock, α_t . (In sections III and IV, the information is perfect; in section V it is not.) Citizens observe α_t only with a one-period lag. However, at the beginning of each period, voters receive a signal from the incumbent in the form of a period- t tax bill. (When setting taxes, the incumbent party does not yet know q_t , the most recent exogenous shock to voters' preferences.) After receiving their tax bill and observing q_t , citizens vote if it is an election period. At the end of the period, markets meet and the price level is determined. If the government set taxes too low, it will be forced to use the inflation tax to balance its budget. (Again, our results extend to bond financing if taxes are distortionary.) Citizens can infer Δ_t and ϵ_t at this point, though we make the stronger assumption that citizens learn enough by $t+1$ to observe α_t directly. The above scenario is consistent with the lag between conception and implementation of fiscal policy.

At time t , voters will prefer party R to party D if their expected utility from having party R in office during periods $t+1$ and $t+2$ is greater than that from having party D in office. Thus, party R will win if

$$(5) \quad E_t^P[\Omega_{t+1}^R + \Omega_{t+2}^R - (\Omega_{t+1}^D + \Omega_{t+2}^D)] \geq 0,$$

where E_t^P is the expectations operator conditioned on time- t public information, which includes α_{t-1} , τ_t , G_t , q_{t-1} and q_t . We will temporarily assume that voters' expectations about Δ_{t+1} and Δ_{t+2} (suboptimal use of seignorage) do not depend on which party wins. Thus

$$(6) \quad E_t^P[(W_{t+1}^R + W_{t+2}^R) - (W_{t+1}^D + W_{t+2}^D)] = 0.$$

This assumption will turn out to be correct in equilibrium because:

- (a) no party ever chooses to inflate in the off-election year $t+1$, and
- (b) conditional on time t information, α_{t+2}^D and α_{t+2}^R have the same distribution. Thus, despite the fact that Δ_{t+2} will turn out to be a function of α_{t+2} , voters have no information at time t to help predict which party will set Δ_{t+2} higher. Assumption (6) must be relaxed when ϵ and q follow more general stochastic processes.

As the opposition party D has no way to signal its most recent competency shock, then by (3),

$$(7) \quad E_t^P(\epsilon_{t+1}^R + \epsilon_{t+2}^R - \epsilon_{t+1}^D - \epsilon_{t+2}^D) = E_t^P(\alpha_t^R) - \hat{\alpha}.$$

By (4),

$$(8) \quad E_t^P(\eta_{t+1}^R + \eta_{t+2}^R - \eta_{t+1}^D - \eta_{t+2}^D) = q_t.$$

Thus, by (2) and (5) - (8), when party R is the incumbent it will win if⁸

$$(9) \quad E_t^P(\alpha_t^R) - \hat{\alpha} + q_t \geq 0.$$

The incumbent party, R, does not observe the disturbance to voters' preferences, q_t , at the time it sets taxes. Therefore its estimate of the probability it will win the election is⁹

$$(10) \quad U_t^R = U[E_t^P(\alpha_t^R)] = c \cdot \text{Prob}[E_t^P(\alpha_t^R) - \hat{\alpha} + q_t \geq 0], \quad c > 0.$$

Given our assumptions about the distribution of q (see eq. (4)), we can infer that U is twice-continuously differentiable, strictly increasing in $E_t^P(\alpha_t^R)$, $\lim_{E(\alpha) \rightarrow -\infty} U'[E_t^P(\alpha_t^R)] = 0$, and that $U''[E_t^P(\alpha_t^R)] > (<) 0$ as $E_t^P(\alpha_t^R) < (>) \hat{\alpha}$.

We now specify the objective functions of the two political parties. Each party aims to maximize a present-discounted-value functional which depends on (a) their probability of being in office, and (b) the social welfare losses due to suboptimal use of seignorage or bond-financing.¹⁰ The R party's objective function is

$$(11) \quad \Psi_t^R = E_t \left[x \sum_{k \in S} \beta^{k-t} U_k^R - (1-x) \sum_{k=t}^T \beta^{k-t} W(\Delta_k) \right],$$

where S is the set of even-numbered (election) periods, T is the (possibly) infinite time horizon, and x is the weight the party places on being elected; x is contained in the open interval $]0,1[$. Party D's utility function is identical.¹¹

Because the assumption that the policymaker has a temporary information advantage over the public (at least, off the equilibrium path) drives the results below, we need to discuss the reasonableness of our specification: The representative voter understands the model. However, it is not worth it for him as an individual to monitor the government closely enough to have complete contemporaneous information on how effectively the government is spending his tax dollars. It is certainly reasonable to assume that a voter does not engage in costly information-gathering activities solely to decide his own vote, which has infinitesimal weight. Implicitly, we are assuming that other information which the voter does gather (because it is worthwhile in his production or consumption activities) does not allow him to directly observe α_t (until $t+1$). We are, of course, also assuming that statements from the opposition party cannot be trusted, and that there is no public watch group which can provide free, complete and unbiased information.

Another variant of our model involves interpreting $G + \epsilon$ as the effective level of government services. The public only observes expenditures, G . This seems quite realistic if one views $G + \epsilon$ as national defense, given the secrecy of military documents. (The country would not necessarily want to make these documents public just to mitigate the political business cycle.) During an election year, the incumbent might have an incentive to shave G and τ , claiming that military preparedness is adequate because the funds are being used efficiently (high ϵ). More generally, our model will suggest why the incumbent is likely to cut back on expenditures which have low short-run visibility, and focus on expenditures (and tax reductions) which have high immediate visibility.

III. Finite-Horizon Equilibrium

In this section, we analyze the case where the two political parties have finite time horizons. Because the information asymmetries are temporary, and because the random disturbances are MA(1) processes, each election cycle turns out to be independent of previous election cycles. We are able to show that there exists a unique (perfect) separating equilibrium, in which the incumbent party's action (tax bill) fully reveals its information (competency shock).¹² We also demonstrate the qualitative features of the model described in the introduction.

If there were full information, so that voters knew the competency shock α_t at election time, then the incumbent party would have no incentive to cheat. For then it could not possibly influence voters' perceptions of its competency. Moreover, by cheating, it would only lower social welfare by increasing seignorage distortions. With asymmetric information, however, the incumbent party may have an incentive to lower taxes in election years to try to exaggerate its competency. We will temporarily posit that voters recognize this incentive and believe that the level of inflation (cheating) depends on the government's competency shock, α_t . Later, we will verify that if voters have rational expectations, then this supposition is correct. Denote the voters' conjecture of inflation as a function of α_t by

$$(12) \quad \Delta_t^* = \Delta^*(\alpha_t),$$

where the superscript referring to the political party has been dropped for notational convenience.¹³ Then by equation (1), voters believe that the tax bill as a function of α_t is

$$(13) \quad \tau^*(\alpha_t) = g_t - \alpha_t - \Delta^*(\alpha_t),$$

where $g_t = G - \alpha_{t-1}$. We temporarily assume that τ^* is continuous and strictly decreasing. Then τ^* has an inverse function, τ^{*-1} and the public's time- t expectation of α_t is

$$(14) \quad E_t^P(\alpha_t) = \tau^{*-1}(g_t - \alpha_t - \Delta_t).$$

Substituting (14) into (10), and the result into (11) yields the incumbent party's maximization problem

$$(15) \quad \max_{\Delta} \{xU[\tau^{*-1}(g-\alpha-\Delta)] - (1-x)W(\Delta)\}, \quad \forall \alpha,$$

where the subscript t has been dropped. Given the public's beliefs (12), the incumbent party's choice of Δ affects only current-period elements of its objective function, (11).

The first- and second-order conditions for an interior solution to (15) are

$$(16) \quad -xU'[\tau^{*-1}(g-\alpha-\Delta)]\tau^{*-1'}(g-\alpha-\Delta) - (1-x)W'(\Delta) = 0,$$

$$(17) \quad xU''[\tau^{*-1}(g-\alpha-\Delta)][\tau^{*-1'}(g-\alpha-\Delta)]^2 + xU'[\tau^{*-1}(g-\alpha-\Delta)]\tau^{*-1''}(g-\alpha-\Delta) - (1-x)W''(\Delta) < 0.$$

In a separating equilibrium, voters' conjectures must be consistent, and hence $\Delta^* = \Delta \forall \alpha$. Thus, equations (16) and (17) can be rewritten as the interior equilibrium conditions:

$$(18) \quad \Delta'(\alpha) = \frac{xU'(\alpha)}{(1-x)W'[\Delta(\alpha)]} - 1,$$

$$(19) \quad \frac{xU''(\alpha)}{[1+\Delta'(\alpha)]^2} - \frac{xU'(\alpha)\Delta''(\alpha)}{[1+\Delta'(\alpha)]^3} - (1-x)W''[\Delta(\alpha)] < 0,$$

where we have made use of the fact that $f^{-1}[f(x)] = 1/f'(x)$, and $f^{-1}''[f(x)] = -f''(x)/[f'(x)]^3$. Inspection of (18) confirms our assumption that τ^* is continuous and strictly monotonic, since $\tau' = -(1+\Delta')$.

The second-order condition (19) allows us to rule out equilibria involving negative Δ :

Proposition 1. If Δ maximizes (15) and if voters' conjectures are consistent, then $\Delta(\alpha)$ cannot be strictly negative for any α .

Proof. By (13), $\tau' = -(1 + \Delta')$. By (18), $\tau' > (<) 0$ for every α if and only if $\Delta < (>) 0$ for every α . Differentiating both sides of the equilibrium condition (18) with respect to α gives

$$(20) \quad \Delta'' = \frac{(1+\Delta')^2}{xU'} \left[\frac{xU''}{1+\Delta'} - (1-x)W''\Delta' \right].$$

Substituting (20) into (19) gives the result that the second-order condition holds as long as $W''/(1+\Delta') > 0$.

Note that the proof of proposition 1 also established that (19) holds for every $\Delta > 0$ which solves (18). The proof required $W'' > 0$, but did not need any restrictions on the sign of U'' . The intuition behind this is that in equilibrium, U is not a function of the choice variable.

Equation (18) is a first-order differential equation with no apparent initial condition. The next proposition provides a boundary condition.

Proposition 2. A perfect equilibrium requires $\Delta(0) = 0$.

Proof. By proposition 1, $\Delta(0) \geq 0$. Assume that voters' expectations $\Delta^*(\alpha)$ are governed by (18) with initial condition $\Delta^*(0) = \delta > 0$. Define

$$D(\alpha) = xU(0) - (1-x)W(0) - xU(\alpha) + (1-x)W[\Delta^*(\alpha)].$$

Since $U(0)$ is the lower bound on U , $D(\alpha)$ represents the minimum gain to a type α who defects and sets $\Delta = 0$ instead of equal to $\Delta^*(\alpha) > 0$. Clearly $D(0) > 0$ for $\Delta^*(0) = \delta$. Furthermore, since D is continuous in α , there exists a neighborhood of zero in R_+ such that $D(\alpha) > 0$. Hence defection is not a probability measure zero event, since all α within a neighborhood of zero will defect.

Comment. It might seem that the "natural" boundary condition would be $\Delta(\bar{\alpha}) = 0$ rather than $\Delta(0) = 0$. The best type cannot gain by posing as a better type. Proposition 2 tells us, however, that it is $\alpha = 0$ who gains nothing by cheating in equilibrium. As long as other agents are cheating enough so that, by (18), it is not worthwhile for $\alpha = 0$ to raise Δ above $\Delta^*(0)$, then he might as well not cheat at all since he will be recognized as a zero in equilibrium anyway.

The proof of proposition 2 relied on the fact that $\tau = g$ is a feasible level of taxes. Suppose there exists some $\tau^{\max} < g$ such that taxes cannot exceed τ^{\max} . (It would be plausible to posit that taxes cannot be raised above τ^{\max} without fundamental changes in the tax system, and that these changes would take several periods to implement.) Then $\Delta^*(0) = g - \tau^{\max}$ is the only consistent conjecture for voters. The proof is analogous to the proof of proposition 2.¹⁴

The incumbent party's maximization problem (15) does not always have an interior solution on all of A . This issue definitely arises if $W'(0) = \lim_{\Delta \rightarrow 0^+} W'(\Delta) > 0$. Suppose for example that

$$(21) \quad Q(\alpha) = xU'(\alpha) - (1-x)W'(0),$$

is strictly negative for $\alpha = 0$. Then (18) implies that $\Delta'(0)$ is negative, but this possibility is ruled out by propositions 1 and 2. Since $U'' > 0$ for $\alpha < \hat{\alpha}$,

it is possible that $Q(\alpha) > 0$ for some $\alpha > 0$. Denote the smallest α such that $Q(\alpha) \geq 0$ as α_L . (If $Q(\alpha) < 0 \forall \alpha$, let $\alpha_L = \bar{\alpha}$.) Then consider a solution path to (18) initiating at $\Delta(\alpha_L) = 0$. It is easy to show that Δ will initially rise from zero, but eventually will decline and cross the α axis from above at some $\alpha_M < \infty$. (See Appendix A. If $W'(0) = 0$, the curve may asymptote to the α axis rather than intersecting it.) Furthermore, it must be true that $Q(\alpha)$ is strictly negative for all $\alpha \geq \alpha_M$ (since $U'' < 0$ for $\alpha > \hat{\alpha}$). This leads us to proposition 3.

Proposition 3. $\Delta = 0$ for $\alpha \geq \alpha_M$ and for $\alpha \leq \alpha_L$.

Proof. Suppose voters believe $\Delta^*(\alpha) = 0 \forall \alpha \geq \alpha_M$, so that

$$(22) \quad \tau^{*-1}(g - \alpha - \Delta) = g - (g - \alpha - \Delta) = \alpha + \Delta \quad \forall \alpha \geq \alpha_M.$$

Then since $Q(\alpha) < 0 \forall \alpha \geq \alpha_M$, $\Delta = 0$ is optimal for $\alpha \geq \alpha_M$. The proof of the second half of proposition 3 is similar and is subsumed in the proof of Theorem 1 in Appendix A.

Definition 1. A separating equilibrium is a continuous function $\Delta^S: A \rightarrow [0, \Delta]$ such that

- (i) $\Delta^S = 0$ if $\alpha \leq \alpha_L$.
- (ii) Δ^S satisfies (18) if $\alpha_L \leq \alpha < \alpha_M$.
- (iii) $\Delta^S = 0$ if $\alpha \geq \alpha_M$.

Theorem 1. A unique separating equilibrium exists.

Proof. See Appendix A.

The solid line in figure 1 is a graph of the equilibrium Δ^S function. (To insure a unique turning point, we require $U''' < 0$ for $\alpha < \hat{\alpha}$.) Figure 1 is drawn for

the case where $W'(0) > 0$, so the solution to (18) intersects the α axis from above instead of asymptoting to it. To understand the figure, it is helpful to consider two closely neighboring realizations of the competency shock, α_2 and α_1 , $\alpha_2 > \alpha_1$. Suppose both α types are thought to cheat by the same amount, $\Delta^*(\alpha_2) = \Delta^*(\alpha_1)$. Then when the incumbent party draws α_1 , it would have to set $\Delta = \Delta^*(\alpha_1) + \alpha_2 - \alpha_1$ in order to convince the public it had drawn α_2 . In deciding whether to take this action, the incumbent would compare the increased expectation of winning with the marginal social welfare cost of distortions. If U' is high, and if Δ and hence W' is low, then the temptation to cheat is great. To discourage an α_1 type incumbent from defecting, it is necessary to force it to cheat more to gain any given increase in votes. This implies $\Delta^S(\alpha_2) > \Delta^S(\alpha_1)$. Good types must "run away" from bad types. As Δ rises, W' rises and the temptation to cheat falls. Since U' begins to fall at $\hat{\alpha}$, Δ^S must eventually begin to fall. The dotted line in figure 1 is a graph of Δ^S for the case where type zero must use seignorage since $g - \tau^{\max} > 0$. In this case, cheating may be strictly decreasing in α . It is simple to prove that $\Delta^S(\alpha)$ is nondecreasing in $g - \tau^{\max}$.¹⁵

We have shown that a unique separating equilibrium exists.¹⁶ We now discuss pooling equilibria. First, note that a pooling equilibrium would have to involve a range of α types setting the same level of τ (taxes), not the same level of Δ (seignorage). Recall that voters observe $\tau = g - \alpha - \Delta(\alpha)$, not $\Delta(\alpha)$. Second, there cannot be a pooling equilibrium where voters ignore taxes completely, even though politicians would then have no incentive to cheat. In this case, the incumbent party's tax bill would fully reveal its type, and it would not be rational for voters to ignore this information. So a pooling equilibrium must involve having all α on some interval $[\alpha_1, \alpha_2]$ set the same level of taxes. But this cannot be an equilibrium if voters' mapping

from taxes to competency is monotonic. Suppose all types $\alpha \in [\alpha_1, \alpha_2]$ set $\tau = \tau^a$. Then $E^P[\alpha | \tau = \tau^a] = \hat{\alpha}^a$, where $\hat{\alpha}^a$ is the mean value of α on $[\alpha_1, \alpha_2]$. But faced with these expectations, type α_2 has an incentive to set taxes slightly lower. He thereby gains a discrete increase in votes, $\alpha_2 - \hat{\alpha}^a$, at the cost of an infinitesimal rise in seignorage distortions. The only pooling equilibria that might exist in our model would have to involve perverse (and nonmonotonic) expectations. For example, the following type of pooling equilibrium might exist: Voters believe $\alpha = \hat{\alpha}$ if $\tau = \bar{\tau}$, and believe $\alpha = 0$ otherwise. It would seem very reasonable to rule out such equilibria.

We now turn to establishing some comparative statics properties of the model. Define $\Delta(\alpha, x)$ as an equilibrium path for a given value of x , the weight the two parties place on votes. (For the remainder of this section, we omit the "s" superscript.)

Proposition 4. $\partial\Delta(\alpha, x)/\partial x \geq 0 \forall \alpha$, with strict inequality if $\Delta > 0$ and $\alpha > 0$.

Proof. If $\alpha_L > 0$, then $xU'(\alpha_L) = (1-x)W'(0)$, and $U''(\alpha_L) > 0$. Thus $d\alpha_L/dx \leq 0$, with strict inequality if $\alpha_L > 0$. Consider first the case $\alpha_L = 0$. Then by (18), for $\alpha \leq \alpha_M$,

$$(23) \quad \frac{\partial\Delta'(\alpha, x)}{\partial x} = \frac{U'(\alpha)}{(1-x)^2 W'[\Delta(\alpha, x)]} - \frac{xU'(\alpha)W''[\Delta(\alpha, x)]}{(1-x)W'[\Delta(\alpha, x)]^2} \cdot \frac{\partial\Delta(\alpha, x)}{\partial x}.$$

Evaluating (23) at $\alpha = 0$ we get

$$\frac{\partial\Delta'(0, x)}{\partial x} = \frac{U'(0)}{(1-x)^2 W'(0)} > 0$$

because $\Delta(0, x) = 0 \forall x$ implies $\frac{\partial\Delta(0, x)}{\partial x} = 0$. The function $\partial\Delta/\partial x$ is continuous in α ; hence there exists a deleted RHS-neighborhood N of zero where $\partial\Delta(\alpha, x)/\partial x > 0, \forall \alpha \in N$. Now suppose there exists an $\alpha > 0$ such that

$\partial\Delta(\alpha, x)/\partial x < 0$. Then it must be the case that there exists an $\alpha^* > 0$ such that $\partial\Delta(\alpha^*, x)/\partial x = 0$, and $\partial\Delta'(\alpha, x)/\partial x < 0$, within a RHS-neighborhood of α^* . But by (23),

$$\frac{\partial\Delta'(\alpha^*, x)}{\partial x} = \frac{U'(\alpha^*)}{(1-x)^2 W'[\Delta(\alpha^*, x)]} > 0.$$

Hence $\frac{\partial\Delta'(\alpha^*, x)}{\partial x} > 0$ within a RHS-neighborhood of α^* . And this is a contradiction.

If $\alpha_L = \tilde{\alpha} > 0$, then clearly $d\Delta(\tilde{\alpha})/dx > 0$ and otherwise the proof is the same as when $\alpha_L = 0$.

As x rises, the incentive to cheat rises. Equilibrium requires that each type must cut taxes by more to pose as the next highest type. Thus "good" types must cheat more relative to "bad" types. Since by proposition 2, $\Delta(0) = 0$, then Δ must rise for all $\alpha > 0$. The same result obtains if $\Delta(0) = g - \tau^{\max}$. In figure 2, we illustrate proposition 4.

Now suppose that voters like the incumbent party for observable "noneconomic" reasons, indexed by the parameter ν . In particular, suppose we modify equation (9) so that the incumbent party wins if

$$(24) \quad \nu + E_t^P(\alpha_t) - \hat{\alpha} + q > 0.$$

In order to maintain our assumption that $E_t^P(\Delta_{t+2})$ is the same for both parties, we will assume that the popularity disturbance is transitory, affecting only the current election. Then the first-order condition, (18), can be rewritten as

$$(25) \quad \Delta'(\alpha) = \frac{xU'(\alpha+\nu)}{(1-x)W'(\Delta)} - 1.$$

Define $\Delta(\alpha, \nu)$ as an equilibrium Δ function for a given value of ν .

Proposition 5 $\partial\Delta(\alpha, \nu)/\partial\nu \geq 0$ for $\alpha + \nu < \hat{\alpha}$, with strict inequality if $\alpha > \alpha_L$.

Proof. The proof is analogous to the proof of Proposition 4.

Proposition 5 contradicts the conventional notion that more popular incumbents are less likely to engage in a political business cycle.¹⁷ If the incumbent party draws a low α , then a rise in its popularity increases its temptation to cheat. As $\alpha + \nu$ rises from zero to $\hat{\alpha}$, a small amount of cheating yields larger and larger benefits in terms of increased probability of election. Only for $\alpha + \nu > \hat{\alpha}$, so that $U'' < 0$, can an increase in popularity lead to a lower level of cheating.

Proposition 5 deals with a small change in popularity, ν . We conjecture that a large increase in ν will move most types into the range where U' is declining, and whereas some types will cheat more, most will cheat less. Proposition 5 has a second important interpretation. The shift parameter ν may be viewed as an observable component of the incumbent's competency shock. We illustrate proposition 5 in figure 3.

Thus far, we have only analyzed the government's behavior during even (election) periods. But clearly, there will be no gain to cheating in off-election years, since the public will have observed α_{t+1} by period $t+2$. This supports our assumption in section II that expected inflation is the same regardless of which party wins the election. If the ϵ 's followed a higher-order MA process, then expected inflation in $t+2$ (conditional on t information) would not necessarily be the same for both parties (by proposition 5).

However, proposition 5 does not allow us to unambiguously state whether expected $t+2$ inflation is higher or lower for the high ϵ party. Also, we have only analyzed the case where the "R" party is the incumbent, but obviously the analysis is the same when the "D" party is the incumbent. The fact that the parties tend to move in and out of power (depending on competency shocks and voter preference shifts) does not complicate our analysis because we assume that the public directly observes α_t in period $t+1$. If they were only able to observe $\epsilon_t = \alpha_t + \alpha_{t-1}$, then there would be a "start-up" problem whenever a new party moves into office. (The public doesn't know α_{t-1} and therefore can't sort out α_{t-1} and α_t .) This case is much more complicated.

It should be emphasized that elections are not necessarily a bad thing, just because they result in excessive inflation or a suboptimal distribution of tax distortions over time. By holding elections, the public gets a more competent government, on average. It is possible in principle to explicitly analyze the tradeoff in a model such as the present one. But the exercise would not be interesting without allowing for a richer stochastic structure.

IV. Infinite-Horizon Equilibria

If the time horizons of the two political parties extend infinitely far into the future, then equilibria which Pareto-dominate the "noncooperative" equilibrium of section III will generally be attainable.¹⁸ There are three related mechanisms for enforcing a more "cooperative" equilibrium. First, the public can assume that a party which has defected in the recent past will not cooperate during its next term(s) in office. The incumbent party must then recognize that a defection will lower expected future social welfare and thereby its own, since the social costs of distortions are an element of its utility function. Second, if the public assumes there will be a reversion to the noncooperative equilibrium next time the defector wins an election (and hence higher expected distortions), then it will be less likely to vote for the defector. The third mechanism for enforcing cooperation involves strategic interactions between the two political parties. If the incumbent party cheats, the opposition party can respond by cheating more the next time it is in office. Since both parties care about social welfare, each may be willing to restrain itself to keep the other party in line.¹⁹

When politicians have high discount rates, even the best attainable equilibrium will involve some cheating. We illustrate why in this case, any sustainable equilibrium will still have the characteristic that higher α types cheat to "run away" from low α types, just as in the finite horizon case. However, to enforce any cooperative equilibrium, it is typically not necessary (to threaten) to punish very high α types as severely. Here punishments are never actually meted out along an equilibrium path, but the issue is relevant in the strictly private information setting of section V.

There are a rich variety of supergame equilibria. However, it is possible to exposit some of the general features of the infinite horizon case if we consider a special class of equilibria in which only the first and third enforcement mechanisms discussed above are relevant. Let $\Delta^T(\alpha) \geq 0 \forall \alpha$ be a candidate cooperative equilibrium, and suppose the public adopts the following trigger-strategy expectations:²⁰

Definition 2. Voters have a $\Delta^T(\alpha)$ trigger strategy for $\Delta^*(\alpha)$ if:

(a) When t is odd, $\Delta_t^*(\alpha) = 0 \quad \forall \alpha$.

When t is even, then

(b) $\Delta_t^*(\alpha) = \Delta_t^T(\alpha) \quad \forall \alpha$ if $\Delta_{t-2}(\alpha) \leq \Delta_{t-2}^T(\alpha)$, or if $\Delta_{t-2}^*(\alpha) = \Delta_{t-2}^S(\alpha)$.

(c) $\Delta_t^*(\alpha) = \Delta_t^S(\alpha)$ if $\Delta_{t-2}^*(\alpha) = \Delta_{t-2}^T(\alpha)$ and $\Delta_{t-2}(\alpha) > \Delta_{t-2}^T(\alpha)$.

(d) $\Delta_0^*(\alpha) = \Delta_0^T$, where zero is the initial period.

The trigger strategy expectations described above have the property that whenever the incumbent party defects and sets Δ_t too high, then there will be a temporary reversion to the noncooperative equilibrium, Δ^S , at the next election in $t+2$. This reversion takes place regardless of which party is in office, so expected inflation is the same for both parties and assumption (6) is intact. Note that there is no punishment for cheating too little; i.e., if $\Delta_t(\alpha) \leq \Delta_t^T(\alpha)$.

Let us first consider conditions under which $\Delta^T(\alpha) = 0 \forall \alpha$ is a trigger-strategy equilibrium. For the trigger strategies described in definition 2 the expected cost to the incumbent of defecting, P , is solely due to a higher

expected inflation rate (level of cheating) during the punishment period:

$$(26) \quad P = \beta^2 \left(\int_0^{\bar{\alpha}} W[\Delta^S(\alpha)] dF(\alpha) - W(0) \right),$$

where $dF(\alpha)$ is the continuous probability density of α . The expected cost P will be the same for any $\Delta_t > 0$.

In deciding whether or not to defect, the incumbent party has to compare its expected utility from defecting with its expected utility from cooperating. If in period t voters believe the incumbent party will not cheat, then by (13), $\tau^* = g_t - \alpha_t$, and

$$(27) \quad E^P(\alpha|\tau) = g - \tau = \alpha + \Delta,$$

where time subscripts are omitted. If the party does not cheat, it attains

$$(28) \quad L^C(\alpha) = xU(\alpha) + (1-x)W(0),$$

where L^C stands for the party's utility level if it cooperates. If, on the other hand, the party defects, it will attain

$$(29) \quad L^D(\alpha) = \max_{\Delta} \{xU(\alpha+\Delta) - (1-x)W(\Delta)\} - P.$$

Proposition 6. If $L^C(\alpha) - L^D(\alpha) \geq 0 \quad \forall \alpha$, then $\Delta^T = 0 \quad \forall \alpha$ is a Δ^T trigger strategy equilibrium.

The particular trigger strategy described in (a)-(d) involves only a one-election punishment period. If P is insufficient to prevent all α types from cheating, then it still may be possible to attain the zero-cheating equilibrium by extending the (threatened) punishment interval. In general, the threatened

punishment interval may be shorter when the incumbent draws a high α , since then the potential gains from cheating are very small. (Since α is observed with a one-period lag, conditional punishment strategies are feasible.)

If the incumbent's discount rate is very high, there may not exist any punishment interval, or more general punishment strategy, sufficient to discourage all α types from defecting. However, it may still be possible to sustain an equilibrium which Pareto dominates the "noncooperative" equilibrium.²¹ Although we do not provide a general analysis of this intermediate case, we do provide an example which illustrates some of the special features of our model. We construct our example in a way such that the maximum punishment, \bar{P} , is given. In equation (11) of section II, the two political parties are assumed to have a constant discount rate β . Here we assume that β is time varying such that $\beta(t)$ is much less than $\beta(s)$ for all $s > t$. We assume that after period t , β is large enough so that a zero-cheating equilibrium can be achieved in all future election periods. But $\beta(t)$ is so small that the zero-cheating equilibrium cannot be sustained in election period t . (It is always possible to choose (β) to satisfy these assumptions.) To establish an enforceable period- t trigger-strategy equilibrium, it is useful to ask what would happen if voters think that no one cheats, so $\Delta^* = 0$.

Consider an agent of type α . Let

$$(30) \quad G(\alpha, \Delta) = xU(\alpha + \Delta) - (1-x)W(\Delta) - \bar{P} - [xU(\alpha) - (1-x)W(0)]$$

be the gain from cheating by Δ rather than cooperating when $\Delta^* = 0$. By hypothesis, there is some $\alpha \in A$ such that there is a $\Delta > 0$ where $G(\alpha, \Delta) \geq 0$. Let α_0 be the minimum such α and let Δ_0 be the minimum Δ such that $G(\alpha_0, \Delta_0) \geq 0$. Then $\alpha_0 + \Delta_0$ is the minimum α type any other type α would ever be willing to

"impersonate". Let A_0 be the subset of A where the differential equation given by (18) with initial condition $\Delta(\alpha_0 + \Delta_0) = 0$ has a strictly positive solution, Δ^Z . Let

$$(31) \quad \Delta^B(\alpha) = \begin{cases} \Delta^Z(\alpha) & \alpha \in A_0 \\ 0 & \alpha \in A/A_0. \end{cases}$$

Clearly $\Delta^B \leq \Delta_t^S \quad \forall \alpha \in A$. See figure 4. We now show

Proposition 7. $\Delta_t^* = \Delta_t^B$ is the time- t part of a trigger-strategy equilibrium.

Proof. Clearly a party with $\alpha > \alpha_0 + \Delta_0$ will not defect. If a party with $\alpha < \alpha_0 + \Delta_0$ wants to defect, then by construction it must want to pose as $\alpha_1 > \alpha_0 + \Delta_0$. But, since $W'' > 0$, then $\forall \alpha < \alpha_0 + \Delta_0$ and $\forall \alpha_1 \in A_0$,

$$(32) \quad \frac{xU'(\alpha^1)}{1+\Delta^B'(\alpha_1)} - (1-x)W'[\Delta^B(\alpha_1)+\alpha_1-\alpha] < \frac{xU'(\alpha^1)}{1+\Delta^B'(\alpha_1)} - (1-x)W'[\Delta^B(\alpha_1)] = 0$$

by (18); hence this is impossible.

Δ^B is not necessarily the best sustainable trigger-strategy equilibrium. However, it seems clear that the optimal Δ function must still begin rising from zero at $\alpha_0 + \Delta_0$, though possibly more slowly than by (18). Thus the optimal equilibrium should have similar qualitative features to figure 4. Although we have structured our example so that the punishment \bar{P} is constant, the equilibrium will have the general shape of figure 4 even if P is determined jointly with $\alpha_0 + \Delta_0$, the "start-up" point.

V. Equilibrium When Cheating is Never Directly Observed

Until now, we have assumed that the incumbent government's information set becomes common knowledge after a one-period lag. Thus in period $t+1$, the public can directly confirm its inference about how much the government was cheating in period t . Here we modify the information structure so that electorate can never know for certain how much the incumbent cheated.²² Our main purpose is to show that the results of section III generalize.

We assume as before that in period $t+1$, all agents observe the lagged competency shock, α_t , and lagged seignorage (or bond financing), Δ_t . However, seignorage and cheating are not equivalent if the incumbent party cannot perfectly predict its revenue needs when setting τ_t . In particular, suppose it only observes the noisy signal $\theta_t = \alpha_t \phi_t$, where ϕ_t is an i.i.d. stochastic process on $[0,1]$, and is independent of α_t .

The analysis can be simplified somewhat with the assumption that the social-loss function takes the special form:

$$(33) \quad W = \Delta^2/2.$$

Since the quadratic form of W implies certainty equivalence, it would be socially optimal for the government to set

$$(34) \quad \tau^H[\Delta(\theta)] = g - \Delta(\theta),$$

where $\Delta(\theta) \equiv E(\alpha|\theta)$.

Since the government has incentives to cheat as before, the public believes that taxes are actually set according to

$$(35) \quad \tau^*[\Delta(\theta)] = g - \Delta(\theta) - \kappa^*[\Delta(\theta)],$$

where we now refer to κ rather than Δ as the level of cheating. κ_t^* denotes voters' expectations of κ_t , given their observation on τ_t . Ex-post, seignorage may be high in part because the incumbent party made a mistake in predicting α_t , and in part because it was cheating:

$$(36) \quad \Delta = \hat{\alpha}(\theta) + \kappa - \alpha.$$

Despite the fact that the public never directly observes κ , there still exists a separating equilibrium analogous to that of section III. The steps involved in deriving the equilibrium are as before. Equation (37) is the same as the first-order condition (18), except that α and Δ are replaced everywhere by $\hat{\alpha}(\theta)$ and κ :

$$(37) \quad \kappa'[\hat{\alpha}(\theta)] = \frac{xU'[\hat{\alpha}(\theta)]}{(1-x)\kappa[\hat{\alpha}(\theta)]} - 1.$$

Again, the separating equilibrium reveals the incumbent's information $\hat{\alpha}(\theta)$. As long as α_{t+1} is observed in period $t+2$, the incumbent still has no incentive to cheat in off-election periods. As in section III, this is the only equilibrium in the finite-horizon case, and is an equilibrium in the infinite-horizon case.

The infinite-horizon trigger-strategy equilibria of section IV do not extend directly to the case of imperfect monitoring. The observation that the incumbent party needed to rely on seignorage or suboptimal bond financing no longer implies that it has cheated. An interesting topic for future research would be to find the optimal equilibria for this case, drawing on the literature on repeated principal agent problems [see, for example, Radner (1985)].

Conclusions

Our analysis illustrates the essential role of temporary information asymmetries in explaining electoral cycles in macroeconomic policy variables. Much of the extant empirical evidence on political business cycles deals with national elections in the United States and Germany. Our model is broadly consistent with this evidence. However, the general framework we develop can also be used to examine other types of electoral policy cycles, such as those associated with state and local elections. (In this case, of course, seignorage will not be a factor, but bond financing still is.) A limitation in applying our model to some other countries is that it does not allow for endogenous timing of elections. Nevertheless, if there is a sufficient lag between the time when elections are called and the time when they are held, the general notion that the incumbent will try to look good remains relevant.

The analysis of the text focused on an electoral cycle in taxes, inflation and deficits. But it is straightforward to extend the model to allow for variable government spending. Consider again the highly plausible example where the government has private information about the effective level of national defense. The government might claim that it is efficiently obtaining a lot of defense for any given input of expenditures. Therefore, it might signal its competency by cutting defense appropriations. More generally, one would also expect election-year shifts in the composition of government spending, towards services with high immediate visibility (potholes), and away from expenditures which the average voter only observes with a lag (contributions to government pension funds).

It would be of interest to extend this framework to address other aspects of political economy, and to allow for more differences between the two political

parties. A related extension would be to allow for imperfect information about the incumbent's preferences. But the rationale presented here for an electoral policy cycle should carry over to these more general frameworks.

Appendix A

Proof of Theorem 1: Suppose $xU'(0) - (1-x)W'(0) > 0$. Then an equilibrium is given by

$$(A.1) \quad \Delta'(\alpha) = \begin{cases} \frac{xU'(\alpha)}{(1-x)W'[\Delta(\alpha)]} - 1 & \text{if } \Delta(\alpha) > c \text{ or if } \alpha = 0, \\ 0 & \text{otherwise.} \end{cases}$$

$$\Delta(0) = c,$$

where $c = 0$. Note that we have not ruled out $W'(0) = 0$ and A is not a domain; hence we cannot immediately apply the standard existence and uniqueness theorem for differential equations. For $c \in]0, \bar{\Delta}[$, it is useful to construct

$$\tilde{W}(\Delta) = \begin{cases} W(\Delta) & \text{if } \Delta \in [c, \bar{\Delta}[, \\ W(c) - (1 - e^{\Delta-c})W'(c) & \text{if } \Delta \in]-\bar{\Delta}, c[. \end{cases}$$

$$Y(\alpha) = \begin{cases} U'(\alpha) & \text{if } \alpha \geq 0, \\ U'(-\alpha) & \text{if } \alpha < 0. \end{cases}$$

Let $f(\alpha, \Delta) = \frac{xY(\alpha)}{(1-x)\tilde{W}'(\Delta)} - 1$. The strategy is to show that there exists a unique solution, $\tilde{\Delta}_c(\alpha)$, to

$$(A.2) \quad \frac{d\Delta}{d\alpha} = f(\alpha, \Delta); \quad \Delta(0) = c \in]0, \bar{\Delta}[,$$

and hence to (A.1) for $c > 0$. We then show that as $c \rightarrow 0$, the limit of the solutions to (A.1) exists and satisfies (A.1) for $c = 0$. In order to prove the existence of a solution to (A.2), we need the following lemma.

Lemma If a solution to (A.2) exists, then there exists a $\Delta_U < \bar{\Delta}$ such that $\tilde{\Delta}_c(\alpha) < \Delta_U$, $\forall \alpha \in A$, if $c < \Delta_U$.

Proof. Suppose $\tilde{\Delta}'_c(\alpha) \geq 0$. Then

$$\tilde{W}'[\Delta(\alpha)] \leq \frac{x}{1-x} Y(\alpha) < \infty.$$

$\tilde{W}' \rightarrow \infty$ as $\Delta \rightarrow \bar{\Delta}$; hence $\tilde{\Delta}_c(\alpha)$ must have an upper bound which is strictly less than $\bar{\Delta}$.

f is a continuous function on the domain $D \equiv \mathbb{R} \times]-\bar{\Delta}, \bar{\Delta}[$ and $(0, c)$ is an interior point of D . Thus by theorem 10.1 in Ross (1965), (A.2) has a solution $\tilde{\Delta}_c(\alpha)$, on $[-h, h]$, where $h \equiv (\bar{\Delta} - \Delta_U) / \max|f| > 0$. Likewise, (A.2) with initial condition $\Delta(h) = \tilde{\Delta}_c(h)$, has a solution on $[0, 2h]$. Proceeding in this manner we can show (A.2) has a solution on \mathbb{R} . Then clearly (A.1) has a solution on A , denoted by $\Delta_c(\alpha)$, for $c \in]0, \Delta_U]$. Denote the right-hand side of (A.1) by $g(\alpha, \Delta)$. By construction of \tilde{W} and the assumption that W''/W' is uniformly bounded, $\partial g(\alpha, \Delta) / \partial \Delta < M$ for some $M < \infty$, $\forall \alpha \in A$, $\forall \Delta \in [c, \bar{\Delta}[$. Thus, by the mean-value theorem, g satisfies a Lipschitz condition on A . By theorem 10.1 in Ross (1965), the solution $\Delta_c(\alpha)$ is unique.

It remains to show that $\lim_{c \rightarrow 0} \Delta_c(\alpha)$ exists and satisfies (A.1) for $c = 0$. If c decreases to zero, $(\Delta_c(\alpha))$ is non-increasing and bounded from below by zero. Thus $\Delta_0(\alpha) = \lim_{c \rightarrow 0} \Delta_c(\alpha)$ exists. $\Delta_0(\alpha)$ is clearly unique and satisfies (A.1). $\forall \alpha' > 0$, (Δ_c) is equicontinuous on $\alpha \geq \alpha'$. Hence, Δ_0 is continuous on $\alpha > 0$. $\Delta_0(\alpha)$ is strictly increasing in α (for small α) and bounded from below by zero. Thus, $\lim_{\alpha \rightarrow 0} \Delta_0 = 0$ and Δ_0 is continuous on A . The first and second

derivatives of Δ_0 are given by (A.1). They exist and are continuous except at $\alpha = 0$ (if $W'(0) = 0$) and α_M .

We have treated the case where $\alpha_L = 0$, but clearly the case $\alpha_L > 0$ is a trivial extension. The second part of proposition 3 is a straightforward consequence of the fact that Δ^S obeys (18) for all $\alpha_L \leq \alpha < \alpha_M$, and that $W'' > 0$. Note that the proof of Theorem 1 required, in addition to the assumptions on W given below eq.(2) of the text, that W''/W' be uniformly bounded on $[c, \bar{\Delta}[$ for $c > 0$.

Appendix B

Here we provide an example for which eq.(18) has a closed-form solution. The example has $U'' < 0$, which requires that $q(t)$ be asymmetrically distributed and/or that the incumbent's current popularity ν be high. (We can have $q(t)$ being asymmetrically distributed and yet retain eq.(6) if q is viewed as an incumbent-specific shock rather than a party-specific shock.)

Example. $U[E^P(\alpha|\tau)] = \ln[\nu + E^P(\alpha|\tau)]$, and $W(\Delta) = \ln B - \ln(B - |\Delta|)$. Note that $W'' > 0$, so the example satisfies the second-order conditions derived in the proof of proposition 1. For $\Delta \geq 0$, substituting into (25) gives

$$(B1) \quad \Delta' + \frac{x\Delta}{(1-x)(\nu+\alpha)} = \frac{xB}{(1-x)(\nu+\alpha)} - 1,$$

a linear first-order differential equation with solution

$$(B2) \quad \Delta(\alpha) = B - (1-x)(\nu+\alpha) + C(\nu+\alpha)^{-x/(1-x)}.$$

First we observe that if the arbitrary constant $C = 0$, then cheating is linear in α . This may seem like a natural equilibrium, but $\Delta(0) = 0$ requires $C = (1-x)\nu^{1/(1-x)} - B\nu^{x/(1-x)}$. It is easy to confirm that $\Delta = 0 \forall \alpha$ is an equilibrium if $\frac{1-x}{B} > \frac{x}{\nu}$. If $\frac{x}{\nu} > \frac{1-x}{B}$, then $\frac{\partial \Delta}{\partial \alpha} \Big|_{\Delta=0} > 0$, and there is a range of α over which the candidate cheats. It is also straightforward to confirm proposition 4 with this example.

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FOOTNOTES

¹There continues to be a significant amount of empirical work on the topic, especially in the political science literature. See, for example, Kirchgässner (1985), Hibbs (1985), or Jonung (1985).

²McCallum (1978) makes this point. The conventional rationale for political business cycles is also questioned by Stigler (1973). Nordhaus (1975) notes that the cycle will disappear in his model, once voters understand the process.

³Cukierman and Meltzer (1985) have independently developed a related line of research, though their approach is very different from ours. Backus and Driffill (1985) adopt Kreps and Wilson's (1982) model of reputation to provide another rationale for political business cycles. [Barro (1985) and Tabellini (1983) present similar models.] In the Backus-Driffill model, the probability the incumbent will inflate increases towards the end of his final term in office. He inflates precisely because he is not worried about the future, and has no incentive to maintain his reputation as an inflation fighter. By inflating (more than anticipated), the benevolent and rational elected official is able to temporarily reduce the effect of distortions which keep employment below its socially optimal value. This repeated game "reputational" model of political business cycles seem at odds with Tufte's (1978) evidence that the political business cycle is more pronounced when the incumbent is up for re-election.

⁴McCallum (1978), and Golden and Poterba (1980), find little evidence of a political business cycle in employment. The evidence of a political business cycle in variables such as transfers and money supply growth is stronger (see Tufte (1978), or the Hibbs and Fassbender volume (1981).)

⁵See Barro (1979).

⁶Our main results do not depend on the assumption that W is minimized at zero, or that W is independent of τ . There is however a general argument, due to Kimbrough (1985), that the seignorage tax should always be set at zero even if taxes on labor and consumption are distorting. Kimbrough argues that for any reasonable transactions technology, money should be viewed as an intermediate good in the production function, and not as a final consumption good. By a standard theorem in public finance, it is suboptimal to tax intermediate goods (since it causes a distortion on two margins), unless there are increasing marginal tax collection costs.

⁷If ϵ follows a higher-order stochastic process, then the public may take into account the opposition's performance when last in office. The main effect this will have on the analysis is to alter the probability the incumbent will win; the resulting political business cycle is qualitatively similar.

⁸In (2), the representative voter is risk-neutral with respect to ϵ . If the public were risk averse, then the fact that it knows more about the incumbent's competency than the opposition's would make the incumbent's re-election more likely. Allowing for this possibility should not alter the general nature of the results.

⁹When R is the opposition party, then $U^R = 1 - U^D$. The winner receives all the votes only because we have not allowed for different voters to have different values of q (party-specific preferences related to exogenous non-economic factors). Our later analysis requires only that a party be more likely to win the more competent it is perceived to be.

¹⁰The analysis would have to be modified slightly if parties also cared directly about the competency of the government. The main qualitative effect would be that when the incumbent knows it has a high level of competency relative to the mean value of the opposition's competency shock, then it will place a greater weight on being elected. If on the other hand, the incumbent knows its competency to be very low, then there is the (perverse) possibility that it might prefer to see the opposition win.

¹¹The main effect of relaxing the assumption that both parties place the same weight (x) on social welfare would be that expected inflation in $t+2$ is no longer the same for both parties. See proposition 4 in section III below. Because we allow for fairly general U functions, the analysis can be generalized to where parties care about their plurality, and not just their probability of winning.

¹²We discuss the possible existence of pooling equilibria later on.

¹³In the finite horizon case, (12) is the only possible form for rational expectations. Expectations conditioned on past behavior "unravel backwards". Other equilibria are possible in the infinite horizon case, however; see section IV below.

¹⁴If $g - \tau^{\max} = \Delta^*(0)$, then the type zero agent will not (cannot) defect by setting Δ lower. (He is so incompetent that he cannot make ends meet without relying on seignorage.) It is straightforward to show that no type $\alpha > 0$ will defect if $\Delta^* = \Delta^S$ follows (18).

¹⁵It is also straightforward to examine the case where there is some maximum level of cheating, Δ^{\max} . Δ may have a maximum if, for example, there is a limit to how much seignorage can be extracted from money holders. Suppose Δ^{\max} exists, and the solution to (18) reaches Δ^{\max} at some α_z . Then there is an equilibrium where Δ remains at Δ^{\max} for $\alpha > \alpha_z$ until the lowest α such that $xU'(\alpha) \leq (1-x)W'(\Delta^{\max})$. Denote this α as α_y . For $\alpha \geq \alpha_y$, the equilibrium path is again governed by (18), and Δ begins to decline. The proof is analogous to the proof of proposition 3.

¹⁶The analysis of this section bears a resemblance to the information theory models surveyed in Riley (1979). The question arises as to why our model is not plagued by the existence problems familiar from that literature. "Unraveling in the middle" does not occur here because the voters face a single large agent. The issue of having pooling equilibria which bid away subcoalitions of types (because everyone in the coalition gets a better vote/social welfare package than in the separating equilibrium) does not arise.

¹⁷ See, for example, Frey and Schneider (1978). There are several other empirical studies which also attempt to relate presidential popularity to the severity of the political business cycle. Golden and Poterba (1980) find no evidence of a relationship.

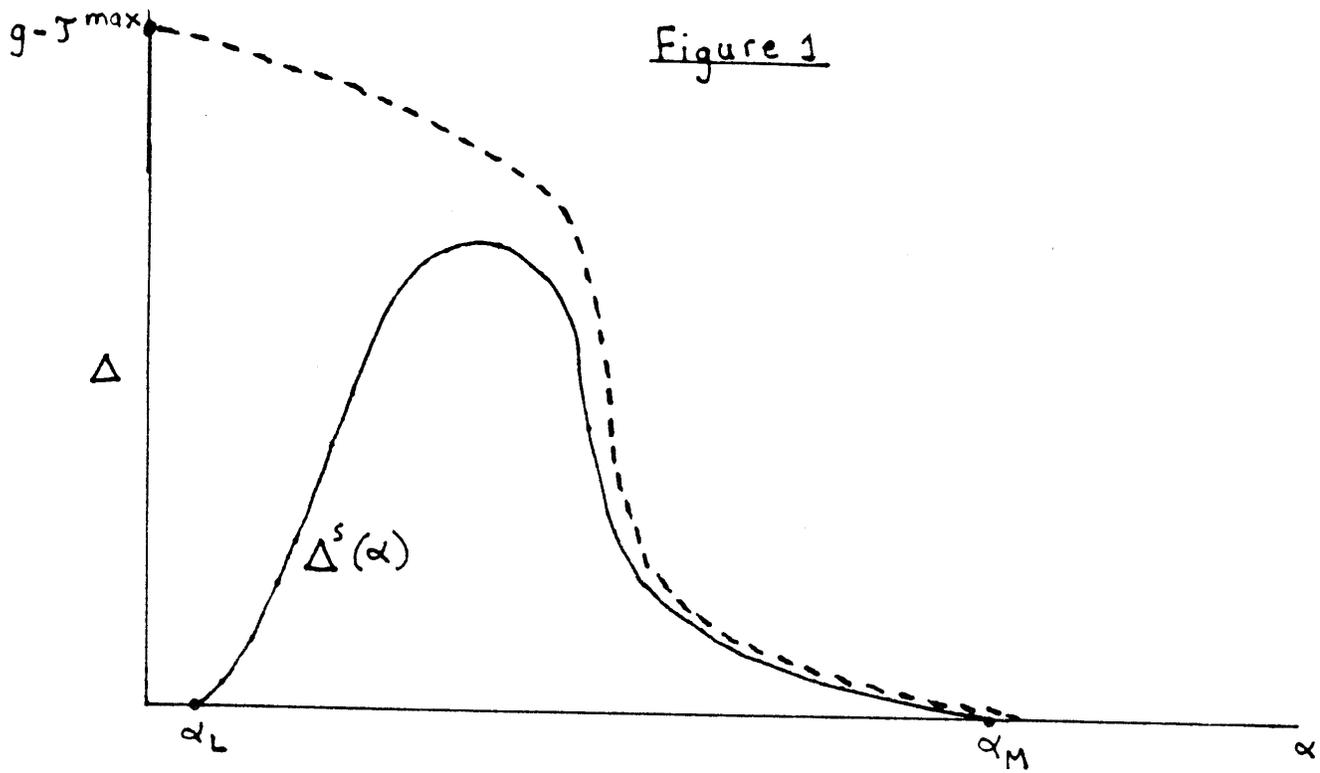
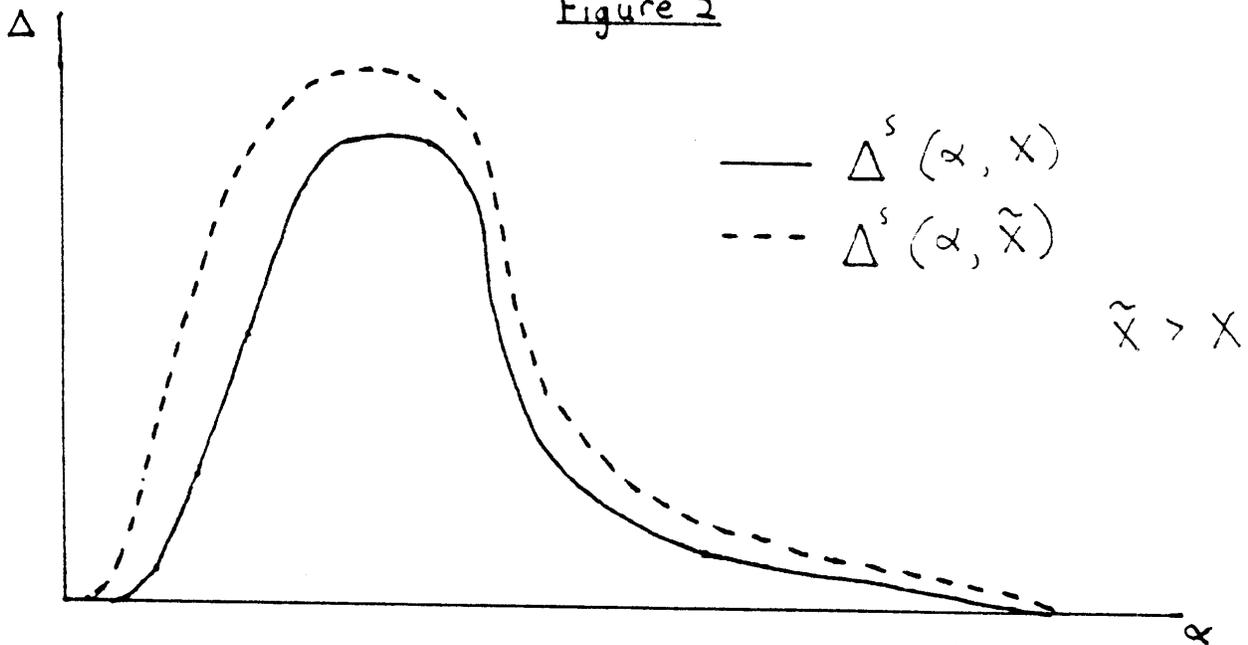
¹⁸ That the infinite horizon case may yield outcomes with higher social welfare is well-known. In the macroeconomics literature, see Barro and Gordon (1983), or Canzoneri (1985), for example. The "election game" may end with some positive probability each period; this effectively raises the incumbent's discount rate.

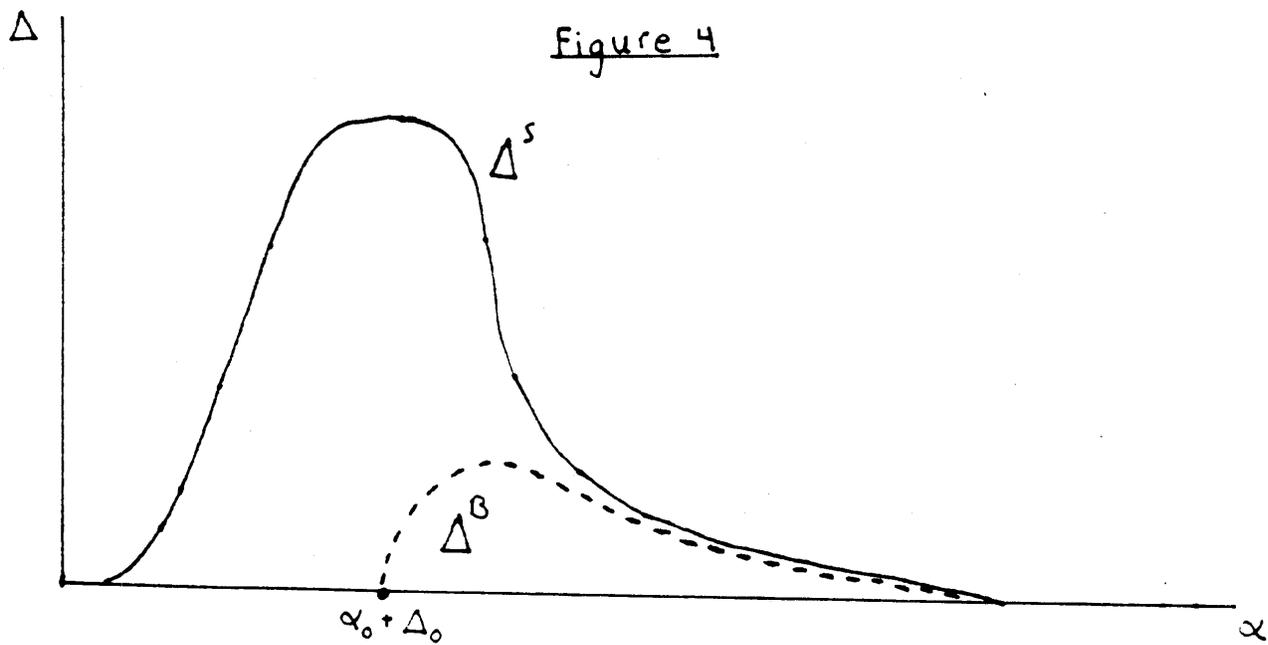
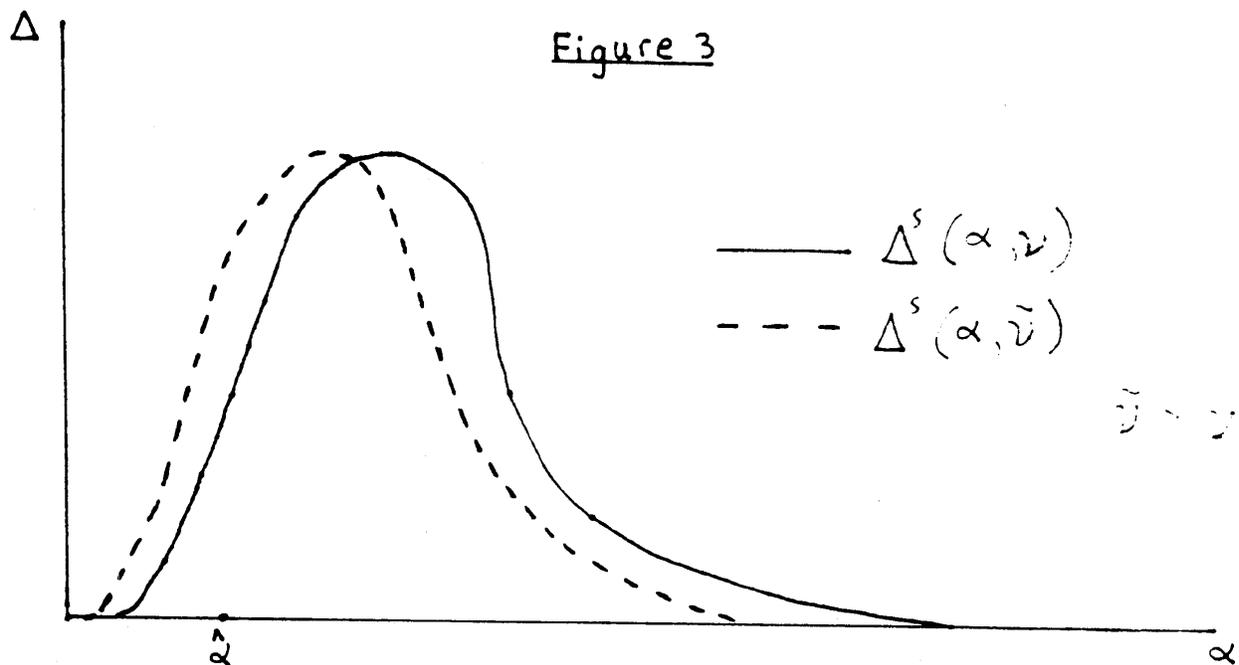
¹⁹ Alesina (1985a,b), in the context of a Barro-Gordon (1983)/Kydland-Prescott (1977) model, has shown how cooperation between two political parties can serve to dampen post-election volatility in prices and employment. Wage setters aren't sure which party is going to win, and hence cannot predict prices. The two parties may implicitly "cooperate" and always choose the same post-election monetary policy despite their different preferences vis-a-vis inflation and unemployment. The winner of the election will follow the cooperative strategy if the long-term gain in reduced volatility outweighs the short-run costs. The "third" channel we refer to in the text is similar to Alesina's, except that our analysis pertains to pre-election volatility.

²⁰ The trigger-strategy equilibria are analogous to those discussed in Friedman (1971).

²¹ This is a fairly general result in infinitely repeated games. The class of punishment strategies considered here involve a reversion to the finite horizon equilibrium. It is possible that there exist more severe punishment strategies in our model, analogous to those described by Abreu (1985).

²² Canzoneri (1985) models private information in the framework of Barro and Gordon (1983). His analysis is an application of the trigger strategy equilibrium proposed by Green and Porter (1984). The model here is different in part because there are a continuum of types (instead of one known type), each of whom must be presented with the appropriate incentives.

Figure 1Figure 2



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