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COMPETITION BY CHOICE

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ABSTRACT

This paper relates firm location choice and consumer search. Firms that cluster together attract consumers by facilitating price comparison, but clustering increases the intensity of local competition. I construct a simple model which shows that firms may choose head-on competition by locating together. In special cases, this can be the unique equilibrium outcome. I also use the model to show that price setting firms may earn more in equilibrium than quantity setting firms.

Competition by Choice

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If consumers find it more convenient to compare the offerings of firms that are located close together, then firm locations can influence consumer search patterns. As emphasized in the marketing literature (see, in particular, Paul Nystrom, 1930, and Richard Nelson, 1958), firms should take this into account when choosing location. This paper analyzes firm location choice under the assumption that consumers are limited in their ability to compare prices across locations.

My analysis applies to markets in which consumers visit firms to learn prices; advertising and telephone search are assumed to play a negligible role in transmitting price information. One can picture a woman who is looking for a pair of shoes to match the color of a particular dress. It could well be easier for her to visit a store than to describe what she wants over the phone, while the cost to a shoe store of supplying sufficiently detailed advertising about its product line could

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be prohibitive.[1] One might therefore expect shoe store locations to affect the woman's search pattern. The woman might, for example, lower her search costs by obtaining price quotes from all the shoe stores in one shopping center before visiting the shoe stores at another shopping center. This view of the search process is different from the random search process envisioned by Stigler (1961).

My analysis focuses on how firms choose location when they know how consumer search patterns will be affected by their location decisions. Consumers may be attracted to locations occupied by a relatively large number of firms because they expect a relatively high degree of competition there. As a result, firms have an incentive to cluster together. On the other hand, to the extent that clustering promotes competition, firms have another, opposing incentive to locate apart. In the words of Paul Nystrom, "[s]tores that sell exactly the same kinds of goods and that are clearly competitive do not necessarily merely divide the business that might have been done if there were but one store. Known competition in itself attracts trade, and people come from farther away." He adds that "[i]t may safely be presumed, however, that there is a limit to the good that can come to the individual store from the clustering of competitive shops. While the group secures a greater total volume than could be secured by widely scattered individual stores, it is quite another question as to whether the individual stores may not suffer distinct losses from competition..."

I develop a location-search game to study the tension between these incentives. The game is played by finitely many firms and consumers (to begin with, some form of limited access to the production or retailing technology is assumed to restrict entry into the industry).

In the simplest version of the game, firms produce at the same constant marginal cost and consumers have identical demand correspondences.

Each firm in the game locates in one of many shopping centers. A "shopping plan" for a consumer in the game specifies a shopping center which the consumer will visit for any distribution of firms across shopping centers. The interpretation is that consumers know where firms are located and have enough time to visit only one shopping center. However, a consumer deciding where to shop cannot directly observe the prices or quantities available at any shopping center. After consumers have decided where to shop, firms either all choose a price or all choose a quantity. The terms of trade at any shopping center that is visited by at least one consumer are determined as follows. If firms choose price, consumers divide any purchases equally between the firms charging the lowest price. If firms choose quantity, consumers buy at a price which equates market demand and supply.[2]

Subgame perfect Nash equilibrium (see Reinhard Selten, 1975) is used to solve the game. This requires that firms play Bertrand's (1883) price setting game or Cournot's (1838) quantity setting game in post-location competition. It also requires that each consumer's shopping plan maximize, for any distribution of firms across shopping centers, the consumer's utility given the manner in which firms choose prices or quantities and given the shopping plans of other consumers. Finally, it requires that each firm's choice of shopping center maximize the firm's profit, given the shopping center choices of other firms, the consumers' shopping plans and the manner in which firms choose prices or quantities.

My analysis shows that an equilibrium in which all firms locate in the same shopping center exists. Thus, the model can be used to explain why firms selling similar or even homogeneous products (for example, gas stations, fast food restaurants, car dealers) often cluster together. This explanation differs from the celebrated Hotelling (1929) story in that it focuses on consumer search, rather than transportation costs, as the force driving the clustering phenomenon.[3,4]

It turns out, however, that there may be other equilibria in which firms locate apart and thereby increase industry profits. Multiple equilibrium outcomes are "more likely" if firms choose prices than if firms choose quantities. As a result, price setting firms may earn more in equilibrium than quantity setting firms. The general point is that the level of industry profits may increase with the intensity of the form of local competition.

The paper is divided into five sections. Section I contains a formal description of the model outlined above. The results mentioned above are established in section II. Section III tests the robustness of the main results with several natural extensions of the basic model: fixed costs as a barrier to entry, search or transportation costs, consumers who are imperfectly informed about firm locations, sequential firm location choice, and heterogeneous consumers. Introducing fixed costs as a barrier to entry causes price setting firms to locate apart in any equilibrium if post-location competition is sufficiently intense. Although search costs have no effect on the set of equilibrium distributions of price setting firms across locations, they have an (ambiguous) effect on the incentive quantity setting firms have to locate together. If consumers are imperfectly informed about firm locations or if firms choose location sequentially, there will exist equilibria in which all firms locate together. However, these conditions can produce equilibria in which industry profits are higher than in any equilibrium of the basic model. Introducing heterogeneous consumers can lead to nonexistence of equilibrium in pure strategies. Moreover, if consumers are heterogeneous and are allowed to choose mixed strategies, firms may locate apart in any equilibrium. Section IV relates the basic model to the product standardization literature. Section V contains some concluding remarks.

I. The Basic Model

The analysis is based on a four-stage game played by a collection of firms and consumers. Limited access to the production or retailing technology fixes the number of firms at n. In the first stage of the game, firms simultaneously select a shopping center. Throughout the paper, I will assume that a shopping center can accomodate at least n firms and that there are at least n shopping centers. In the second stage, m consumers with the same demand correspondence learn where firms are located and decide where to shop. In the third stage, either all firms choose price or all firms choose quantity. In the fourth stage, each consumer learns the terms of trade which are available at the shopping center he decided to visit and makes his purchases. Each consumer's demand correspondence $f: R_+ \twoheadrightarrow R_+$ satisfies the following conditions: (i) There exists some $p^* > 0$ such that $f(p) = 0$ for all $p \geq p^*$, (ii) f is differentiable and monotonically decreasing on $(0, p^*]$, and (iii) $f(0) = [b, \infty)$, where $b = \lim_{p \downarrow 0} f(p)$. If the firms are price setters, local aggregate demand is divided evenly among the firms charging the lowest price. Now, suppose the firms are quantity setters and consider a location with $w > 0$ consumers. If the firms collectively choose a total quantity which does not exceed wb, consumers make their purchases at a price which clears the market. If the firms collectively choose a total quantity that exceeds wb, price equals zero. If quantity setting firms at some shopping center attract no consumers, assume that the price at this shopping center equals zero. Firms produce at the same constant marginal cost c, where $0 < c < p^*$.

Formally, if S denotes the set of shopping centers, each firm's strategy set is $S \times \{g: S^n \times S^n \to R_+\}$. Since consumer behavior in the fourth stage has already been specified, each consumer's strategy set may be reduced to $\{h: S^n \to S\}$ without loss of generality.

Payoffs can be derived from strategies as follows. Any collection of firm and consumer strategies generates a distribution of firms and consumers across shopping centers. Given this distribution, the terms of trade at each shopping center may be calculated in the manner described above. Each firm's payoff is its profit, and each consumer's payoff is the value he received from buying at the price at which trade occurs.

II. Main Results

This next section characterizes and identifies subgame perfect Nash equilibrium outcomes for special cases of the model.

A. Quantity Setting Firms

If firms choose quantity, equilibrium requires that they behave as Cournot competitors at any location attracting at least one consumer.[5] Suppose that a unique, symmetric Cournot equilibrium exists for any combination of firms and consumers.[6] Let $q^C(x, y)$ and $C(x, y)$ denote the Cournot equilibrium quantity and profit per firm at a shopping center with x consumers and y firms. Assume $C(x, y) > 0$ for all $x \geq 1$ and $y \geq 1$, and that $yq^C(x, y)$, the aggregate quantity supplied in equilibrium, is monotonically increasing in y for any $x \geq 1$.[7]

The first main result of this subsection is stated below.

Proposition 1: There is an equilibrium of the quantity setting model in which all firms locate in the same shopping center.

The proof of proposition 1 makes use of the following preliminary.

Lemma 1: In the quantity setting model, the Cournot equilibrium price at any shopping center that attracts at least one consumer does not depend on the number of consumers. In addition, the Cournot equilibrium profit per firm is linearly homogeneous in the number of consumers.

Proof: In a symmetric Cournot equilibrium with x identical consumers and y firms which produce at constant marginal cost c, each firm solves:

$$\max_q \; f^{-1}\left[\frac{q + (y - 1)q^C(x, y)}{x}\right] q - cq$$

where f^{-1} is each consumer's inverse demand correspondence. The first order condition is:

$$f^{-1}\left[\frac{yq^C(x, y)}{x}\right] + f^{-1\prime}\left[\frac{yq^C(x, y)}{x}\right]\frac{q^C(x, y)}{x} = c.$$

Hence, by the uniqueness of the symmetric Cournot equilibrium, $q^C(x, y) = xq^C(1, y)$. Consequently, the price, $f^{-1}[\frac{yq^C(x, y)}{x}] = f^{-1}[yq^C(1, y)]$ is independent of x and $C(x, y) = xC(1, y)$. Q.E.D.

The practical implication of lemma 1 is that an individual consumer does not need to take into account the search patterns of other consumers when deciding where to shop. The consumer only needs to consider where firms are located.

Proof of Proposition 1: The proof is by construction. Let each firm's strategy specify the same shopping center s. Let each consumer's strategy specify s if it is occupied by at least one firm and if there are equal numbers of firms at each firm-occupied shopping center. In all other cases, let consumer strategies specify one of the shopping center occupied by the largest number of firms. Given such consumer strategies, a single firm locating outside s would attract no consumers, given that all rival firms are located in s. Now, the assumptions that demand is monotonically decreasing and that $yq^C(x, y)$ is monotonically increasing in y for any $x \geq 1$ imply that the equilibrium price at a shopping center attracting a given positive number of consumers will be monotonically decreasing in the number of firms that occupy the shopping center. This observation together with lemma 1 implies that a consumer cannot improve his payoff by deviating from the strategy described above. Q.E.D.

The main intuition behind Proposition 1 is that, if all firms are located together, it will not pay any single firm to move to another location because consumers will expect such a firm to charge the monopoly price. This explains why firms selling similar products may locate together even though the result will be an increase in the intensity of local competition.[8, 9]

The question of whether there may be _other_ equilibrium outcomes in which firms locate in different shopping centers still remains. Such other equilibrium outcomes might be viewed as more reasonable, since they lead to higher industry profits. It turns out that the equilibrium outcome identified in Proposition 1 may - but need not be - unique. I will present three examples to demonstrate this point. To organize discussion of these examples, I will begin by isolating conditions on the equilibrium profit function under which the equilibrium identified in Proposition 1 _is_ unique.

Proposition 2: If the number of firms is at least 3, if $3C(1, 2) > C(1, 1)$, and if $2C(1, y + 1) > C(1, y)$ for $y \geq 2$, then equilibrium requires that all firms locate in the same shopping center. For a given number of firms greater than or equal to 3, one may omit the conditions $2C(1, y + 1) > C(1, y)$ for all y which do not divide the number of firms.

Proof: Let n denote the number of firms. Suppose that $n \geq 3$ and $t \geq 2$ shopping centers are occupied by at least one firm in equilibrium. Recall from the proof of proposition 1 that the price at any shopping center attracting at least one consumer is monotonically decreasing in the number of firms at the shopping center. It follows that a shopping center with less than the largest number of firms will attract no consumers. Any firm in a shopping center that is not occupied by the largest number of firms could earn positive (instead of zero) profit by moving to a shopping centers that is occupied by the largest number of firms. Therefore, each firm-occupied shopping center must be occupied by the same number of firms in equilibrium.

Suppose this number is one, which implies $t = n$. Some firm must have no more than $\lfloor m/n \rfloor$ consumers at its location. By the definition of equilibrium, it must be unprofitable for this firm to locate with another group of firms and attract all the consumers. Hence, $C(\lfloor m/n \rfloor, 1) > C(m, 2)$, which implies $C(1, 1) > nC(1, 2) \geq 3C(1, 2)$, a contradiction.

Now, assume there are two firms at each firm-occupied shopping center, which means $t = n/2$. Some firm has no more than $\lfloor 2m/n \rfloor$ consumers at its shopping center. Since this firm has no incentive to move, $C(\lfloor 2m/n \rfloor, 2) > C(m, 3)$. Thus, $C(1, 2) > 2C(1, 3)$, a contradiction.

Applying induction, one discovers that there is no number of firms in an occupied shopping center which is consistent with the hypothesis and equilibrium. Therefore, all firms must be in the same shopping center. The second statement follows from the fact that each shopping center must contain the same number of firms in equilibrium. Q.E.D.

The conditions on $C(x, y)$ require that Cournot duopoly with three consumers be more profitable than quantity setting monopoly with one consumer and, for $y \geq 2$, $y + 1$ firm Cournot oligopoly with two consumers be more profitable than y firm Cournot oligopoly with one consumer. Intuitively, this means that firms always locate together in equilibrium if $y + 1$ firm Cournot oligopoly is not much more intense than y firm Cournot oligopoly.

The following example uses the above mentioned conditions on $C(x, y)$ to show that the equilibrium outcome identified in proposition 1 may be unique.

Example 1: If $f^{-1}(q) = a - bq$ on $(0, a/b)$, then

$$C(1, y) = \frac{(a - c)^2}{b} \left[\frac{1}{(y + 1)^2} \right]$$

Verifying the inequalities in Proposition 2 is straightforward. Thus, in the much studied case of linear demand, firms always choose to compete by locating together.

The next two examples emphasize that the conditions of proposition 2 may not be satisfied; that is, multiple equilibrium outcomes are possible if firms are quantity setters. The first of these examples violates the assumption that the number of firms exceed two. The second violates the profit function inequalities.

Example 2: Suppose there are two consumers, a and b, and two quantity setting firms A and B. Consider a collection of strategies for the firms and consumers which satisfy the following conditions. First, the firms locate apart. Second, both consumers' strategies specify the location occupied by both firms if the firms locate in the same place. Third, consumer a locates with firm A and consumer b locates with firm B if the firms locate apart. These conditions may be satisfied in equilibrium as long as each firm profits as much from being a monopolist facing one consumer as from being a duopolist facing two consumers. That is,

$$f(q^C(1, 1))q^C(1, 1) - cq^C(1, 1) \geq f(q^C(2, 2))q^C(2, 2) - cq^C(2, 2).$$

Since $q^C(2, 2)$ is available to the monopolist, this inequality must hold.

Example 3: Suppose there are six quantity setting firms that produce at zero marginal cost and four consumers with inverse demand $f^{-1}(q) = (1 - q)^8$ on $(0, 1)$. In addition, suppose the consumers and firms play strategies which satisfy the following conditions. Three firms, A, B and C, locate together in one shopping center and three firms, D, E, and F, locate at another shopping center. Consumers visit a shopping center with the largest number of firms. In case more than one shopping center is occupied by the largest number of firms, consumers a, b, d and e visit the shopping center containing firms A, B, D and E, respectively. To check that these conditions may be satisfied in equilibrium, it suffices to check that each firm profits as much from being a triopolist facing two consumers as from being a quadropolist facing four consumers. In fact, the symmetric Cournot equilibrium profit in the case of three firms and two consumers is .01423 and the symmetric Cournot equilibrium profit with four firms and four consumers is .01301.

B. Price setting Firms

If firms choose price instead of quantity, the form of local competition is Bertrand competition in equilibrium. Thus, if a shopping center is occupied by at least two firms, firms earn zero profits and consumers capture all gains from trade. On the other hand, if a shopping center is occupied by one firm and at least one consumer, that firm earns monopoly profits while consumer(s) obtain the value of buying at the monopoly price. The main results of this subsection are contained in the following proposition.

Proposition 3: In the price setting model, there are precisely two equilibrium resource allocations, which are the perfect competition and monopoly allocations. However, equilibrium places no restrictions on the distribution of firms across shopping centers.

Proof: It is easy to see that a consumer cannot improve on a strategy of visiting a shopping center that is occupied by the largest number of firms. Suppose all consumers adopt such a strategy. Now specify an arbitary distribution of firms across shopping centers.

If no shopping center is occupied by more than one firm, then every firm earns monopoly profits on the consumers who arrive at its location. If the largest number of firms in a shopping center is at least two, then all firms will earn zero profits.

If no shopping center is occupied by more than one firm, any attempt by a firm to attract more consumers by competing with another firm will be met with zero profits. Thus, it is possible for every shopping center to be occupied by no more than one firm in equilibrium.

If there are at least two locations with at least two firms or one shopping center with at least three firms, an individual firm cannot change its location and cause the largest number of firms in a shopping center to fall below two. Hence, no individual firm can increase its profits by changing its location.

If there is exactly one shopping center s with exactly two firms and other shopping centers are occupied by no more than one firm, assume consumer strategies specify s if all shopping centers are occupied by the same number of firms and one of the occupied shopping centers is s. Clearly, no firm can raise its profits above zero by changing its location.[10] Q.E.D.

Observe that clustering reduces industry profits in both the price and quantity setting models. However, a quantity setting firm does not take the equilibrium implications of clustering into account when choosing location. This is why quantity setting firms may have an incentive to locate together even though this reduces industry profits. On the other hand, due to the ferociousness of price competition, a price setting firm can fully take into account the consequences of locating with one or more of its rivals. It recognizes that it has nothing to gain by locating with one of its rivals in the same shopping center. Consequently, if firms choose price, a larger set of equilibrium outcomes can be sustained.

III. Extensions

This section is divided into five parts. Parts A and B consider the effects of fixed costs as a barrier to entry and consumer search costs, respectively. Part C introduces consumers who are imperfectly informed about firm locations. Part D develops a version of the basic model in which firms locate sequentially rather than simultaneously. Part E introduces heterogeneous consumers.

The findings do not imply that clustering is an unlikely equilibrium outcome, but they do show why certain modifications in the basic model may cause firms to locate apart in equilibrium. The results of parts A, B, and E suggest that no strong case can be made for existence of an equilibrium outcome in which all firms locate together. The extensions considered in parts C and D reinforce the idea that an equilibrium outcome in which all firms locate together will not, in general, be unique.

A. Fixed Costs as a Barrier to Entry

In sections I and II, access to the production or retailing technology was assumed to be limited to n firms. This subsection allows unrestricted access to the technology, but it assumes that firms that enter the market incur a fixed cost $F > 0$ in production. The resulting complication is that the ability of a given number of firms to cover fixed costs depends on how the firms are distributed across locations.

The basic model is easily modified to incorporate unrestricted access to the technology and fixed costs. Suppose there is a countable infinity of potential entrants and shopping centers. Each potential entrant either decides not to enter and thereby avoids the fixed cost, or chooses a shopping center and automatically incurs the fixed cost. Formally, each potential entrant's strategy set is S x {enter, don't enter} x R_+. Consumer strategy sets are unchanged. If the potential entrant decides not to enter, its payoff is zero. Payoffs to entrants and consumers are computed as in section I except that each entrant's payoff is reduced by the fixed cost F.

The presence of fixed costs and unrestricted access to the technology obviously eliminates perfect competition as an equilibrium outcome if firms are price setters. As long as F is less than the profit that a single price setting firm could earn, the only outcome of equilibrium which allows entrants to cover their fixed costs involves all the entrants locating apart from each other. In this case, monopoly is the only outcome of equilibrium.

On the other hand, the implications of the quantity setting version of the model that were derived in II. A. are robust to the introduction of fixed costs as a barrier to entry. The analog of proposition 1 is stated and proved below.

Proposition 4: If it is not unprofitable for at least one firm to enter ($F \leq mC(1, 1)$), there is an equilibrium in the quantity setting model with unrestricted access to the technology and fixed costs in which all entrants locate in the same shopping center.

<u>Proof</u>: There is a positive integer n such that $mC(1, n) \geq F > mC(1, n + 1)$ because F is positive, the area underneath the market demand curve is bounded, and $F \leq mC(1, 1)$. Suppose n firms enter and locate in the same shopping center and all other firms don't enter. It is unprofitable for any nonentrant to enter because $F > mC(1, n + 1)$. Any entrant has nothing to gain by becoming inactive because $mC(1, n) \geq F$. The rest of the argument is essentially the same as the proof of proposition 1. Q.E.D.

An analog to proposition 2 can be obtained by replacing the word "firm" with the word "entrant" in the proposition statement and proof. In addition, examples 1, 2, and 3 may be extended with appropriate choices of F.

The general point here is that a given level of fixed costs will cause firms to locate apart if the form of post-location competition is sufficiently intense. Price setting firms selling differentiated products might locate together. And if the difference between the Cournot and Bertrand equilibrium price is sufficiently small (for example, because the number of firms is large), proposition 4 will hold simply because there is only one entrant.

B. Search Costs

The basic model can be extended to include search costs by redefining consumer strategy sets and payoffs. A consumer who must pay search costs may prefer to "pass" and not go shopping. Consequently, each consumer's strategy set should be enlarged to the set of functions mapping S^n to S x {shop, pass}. A consumer who decides to pass receives a payoff of zero. A consumer who decides to shop receives a payoff equal to the value received from buying at the price at which trade occurs less search costs (the cost of visiting a particular shopping center may be different for different consumers).

In the context of this revised framework, any distribution of firms across locations is consistent with equilibrium if firms set prices.[11] Hence, search costs do not affect the incentives of price setting firms to locate together. On the other hand, the introduction of positive consumer search costs can magnify or diminish the incentive quantity setting firms have to cluster. Example 4 shows that the clustering incentive can be magnified if all consumers incur the same search cost when visiting any firm. Example 5 shows that if search costs differ across consumers, firms may locate apart in any equilibrium.

<u>Example 4</u>: Assume that consumers incur a positive search cost when visiting any shopping center. If there are two quantity setting firms that locate apart, then consumers should expect the monopoly price at each location. If the search cost is greater than the surplus consumers receive from buying at the monopoly price, both consumers pass and the firms will make no sales. If the two quantity setting firms locate together, then consumers should expect the Cournot duopoly price. If the search cost is less than the surplus consumers receive from buying at the Cournot duopoly price, then consumers will visit the duopolists and the duopolists will earn positive profit. Thus, the unique equilibrium outcome involves both firms locating together.[12]

<u>Example 5</u>: Consider a situation with two quantity setting firms and equal numbers of two types of consumers. Suppose type I consumers have a low cost c_L of visiting a shopping center I and a high cost c_H of visiting shopping center II. Suppose type II consumers have a low cost c_L of visiting shopping center II and a high cost c_H of visiting shopping

center I. The natural interpretation is that type I consumers live close to shopping center I and type II consumers live close to shopping center II. Provided c_L is less than the value consumers receive from buying at the monopoly price, each firm will attract one half of the consumers if the two firms locate in shopping centers I and II respectively. It follows that there is an equilibrium in which the firms locate apart. On the other hand, each firm can increase its profit by locating apart from its rival since, by doing so, it will certainly earn monopoly profits on half the consumers. Hence, there can be no equilibrium in which the firms locate together.

Notice that, in example 5, a lone outside firm is able to attract consumers in spite of the fact that consumers recognize its monopoly power. This is because some consumers find it less costly to visit the outside firm. This argument loses some of its force as the number of firms in the industry increases. Suppose there are n firms instead of 2. The difference between the monopoly price and the Cournot equilibrium price at a shopping center with n - 1 firms is increasing in n. If this difference exceeds $c_H - c_L$ for large n, an outside firm will not attract any consumers and a clustering equilibrium will exist.

C. Imperfect Information About Firm Locations

This subsection assumes that consumers are imperfectly informed about firm locations in the sense that no firm expects consumers to react to changes in its own location. Formally, the only modification of the basic model of section II is that each consumer's strategy set is assumed to be S instead of $\{g: S^n \rightarrow S\}$.

In this version of the basic model, a clustering outcome is possible. Suppose that all firms and consumers locate in shopping center

s. Clearly, no individual firm or consumer has any incentive to move from s since there are no other firms or consumers outside s. It follows that there is an equilibrium in which all firms and consumers locate together.

However, there are other equilibrium outcomes. For example, there is a different equilibrium outcome associated with every common divisor of m and n. To see this, suppose that an equal number of firms and an equal number of consumers locate in each of k shopping centers. No consumer has an incentive to move to another shopping center by lemma 1. In addition, firms have nothing to gain from moving to a shopping center with n/k - 1 other firms to a shopping center with no consumers or to a more competitive shopping center with n/k other firms.

D. Sequential Firm Location Choice

Suppose that firms locate sequentially instead of simultaneously. Let the firms be indexed by their order of movement; that is, firm i is the i^{th} firm to choose location. Then firm 1's strategy set is $S \times \{h: S^n \times S^n \rightarrow R_+\}$ and, for $i \geq 2$, firm i's strategy set is $\{g: S^{i-1} \rightarrow S\} \times \{h: S^n \times S^n \rightarrow R_+\}$. Suppose S is countable, so that it is possible to index shopping centers by j, $j = 1, \ldots, \infty$. The rest of the game is defined as in section I.

Since firms that locate simultaneously ignore the effect of their own strategy on the strategies of other firms, one might expect sequential firm location choice to increase industry profits. Nevertheless, sequential firm location choice does not rule out the existence of a clustering equilibrium.

<u>Proposition 5</u>: If quantity or price setting firms locate sequentially, an equilibrium in which all firms locate together exists.

Proof: (Outline) Since the firms choose quantities or prices after all firms and consumers have located, equilibrium requires that quantity or price setting firms engage in Cournot or Bertrand competition in post-location competition. For $i = 1, \ldots, n$ and $j = 1, \ldots, \infty$, let the variable $u_j(s_1, \ldots, s_i)$ represent the number of the first i firms locating in shopping center j if the first i firms are using the strategies $s_1, \ldots, s_i$, respectively. Suppose that each consumer's strategy is to choose shopping center r, where r solves:

$$\text{Min } k$$
$$\text{subject to } k \in \underset{j}{\text{argmax}}\ u_j(s_1, \ldots, s_n).$$

and that, for $i = 1, \ldots, n$, firm i locates in shopping center t(i), where t(i) solves:

$$\text{Min } k$$
$$\text{subject to: } u_k(s_1, \ldots, s_{i-1}) + (n - i + 1) \geq \max_j u_j(s_1, \ldots, s_{i-1}).$$

The consumers' strategy picks out one of the shopping centers occupied by the largest number of firms - this is sufficient for consistency with equilibrium. To see why the description of firm location choice is consistent with equilibrium, consider the alternatives which are available to firm n. If it locates in shopping center t(n), it will earn $C(m, u_{t(n)}(s_1, \ldots, s_n)) > 0$. If it located in some shopping center t* with a smaller number of firms, $u_{t*}(s_1, \ldots, s_{n-1}) + 1 < \max_j u_j(s_1, \ldots, s_n)$. Hence, no consumers would locate in shopping center t* and firm n would earn 0. On the other hand, firm n would have nothing to gain by locating in a shopping center with at least as many firms as shopping center t(n). The same type of argument can be used to show that, if firms $i + 1, \ldots, n$ locate in the manner described above, then firm i has no reason not to locate in shopping center t(i). Now observe that if firms and consumers locate in the manner described above, they will all locate in shopping center 1. Q.E.D.

Proposition 4 shows that sequential firm location will not alter the basic conclusion that firms may locate together in equilibrium. The weakness of proposition 4 is that its proof depends on the specification of consumer search behavior when there are equal numbers of firms at each firm-occupied location (propositions 1 and 3 are essentially immune to this criticism - see footnotes 9 and 11). This point is made clear in the example below.

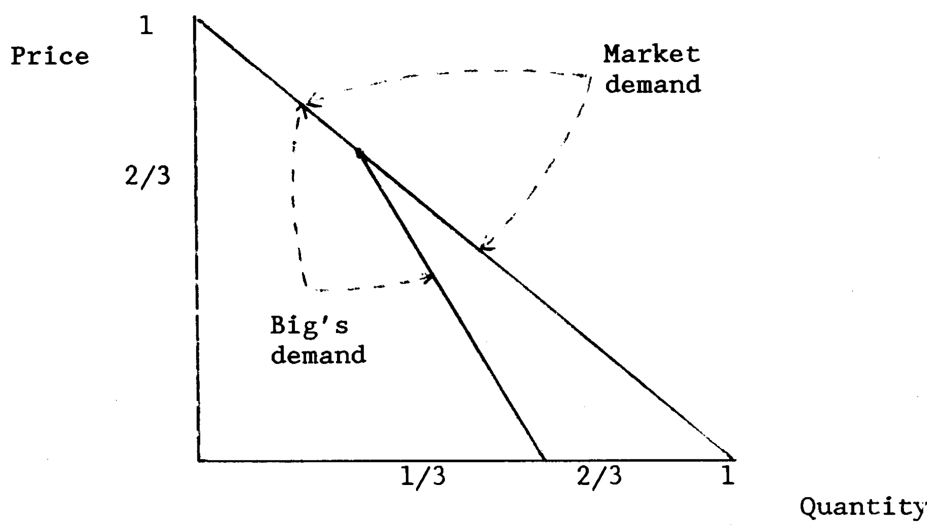
Example 7: Suppose there are three quantity setting firms and six consumers. Suppose firm 1 locates in shopping center 1. Firm 2 locates in shopping center 1 (2) if firm 1 locates in shopping center 2 (1). Firm 3 locates with firm 2. The consumers visit the shopping center occupied by the largest number of firms; in case of ties, the consumers distribute themselves evenly across locations. All of this is consistent with equilibrium as long as $6C(1, 2) > 2C(1, 1)$. By contrast, $6C(1, 2) > 2C(1, 1)$ guarantees that all firms locate together in any equilibrium of the quantity setting model with simultaneous location choice (apply proposition 2).

E. Heterogeneous Consumers

Proposition 1 was proved using the assumption of identical consumers. Although this assumption is stronger than necessary, it does play an important role in the analysis. The next example shows why an equilibrium in pure strategies may not exist if consumers are heterogeneous. It also shows that, if consumers are heterogeneous and mixed strategies are allowed, firms may locate apart in any equilibrium.

Example 8: Suppose there are two firms producing at zero marginal cost and two consumers. The first consumer, Mr. Big, has the inverse demand function $f^{-1}(q) = \begin{cases} 1 - q & \text{if } q \in (0, 1/3) \\ 4/3 - 2q & q \in [1/3, 2/3) \end{cases}$ on (0, 2/3). The second consumer, Mr. Small, has the inverse demand function $f^{-1}(q) = 2/3 - 2q$ on (0, 1/3). If both consumers visit the same shopping center, the market demand at that shopping center is linear, as in figure 1.

[Figure 1]



The following statements can be verified with routine calculations.

(1) If firms and consumers all locate in the same shopping center, each firm earns an equilibrium profit of 1/9 if the firms are quantity setters and 0 if the firms are price setters. In this case, Big's consumer surplus is 7/36 and Small's consumer surplus is 1/36.

(2) If the firms locate apart and the consumers locate together with one of the firms, then (i) the firm attracting neither consumer earns 0 while the firm attracting both consumers earns 1/4, (ii) Big's consumer surplus is 17/144, and (iii) Small's consumer surplus is 1/144.

(3) If the firms locate apart and each firm attracts one consumer, then (i) the firm attracting Small earns 1/18 and the firm attracting Big earns 2/9, (ii) Big's consumer surplus is 1/18, (iii) Small's consumer surplus is 1/36.

By (2) and (3), if firms locate apart in shopping centers I and II, then the consumers are (in effect) forced to play the 2 x 2 matrix game shown in figure 2.

[Figure 2]

Mr. Big

Mr. Small		I	II
	I	1/144, 17/144	1/36, 1/18
	II	1/36, 1/18	1/144, 17/144

Observe that, although this game has no equilibrium in pure strategies, there is a unique mixed strategy equilibrium in which consumers choose each shopping center with probability 1/2. Thus, if firms locate apart, each earns an expected profit of 19/144. Hence, equilibrium requires that risk neutral firms locate apart.

The idea behind the example is straightforward. Small wishes to separate himself from Big to get a lower price. However, Big can get a lower price by joining Small. In equilibrium, Small randomizes between the shopping centers in an effort to hide from Big. Moreover, the firms can earn more by locating apart because this allows them to take advantage of local market power.

It is not hard to check that increasing the number of firms from 2 to 3 in example 8 would cause the argument to fail. This is because Small would prefer to face price or quantity setting duopolists with Big over facing a monopolist by himself. In fact, a consumer would always prefer to face price setting duopolists over facing a monopolist. The general point is that, if consumers are heterogeneous, the existence of a clustering equilibrium will depend on the number of firms and on the intensity of the form of post-location competition.

IV. Product Standardization

My approach to location choice is related to the informal discussions of product standardization in Farrell and Saloner (1985) and Fisher, McGowan and Greenwood (1983). Farrell and Saloner write that "Standardization can commit producers to compete in an 'aftermarket' for spare or replacement parts, complementary inputs or peripheral devices" and observe that "in the auto industry, it is notorious that spare parts have a much higher profit margin than cars do."

The parallel between this view of product standardization and my approach to location choice is that the standardization of products corresponds to the choice of the same location. A slight modification in the price setting model can be used to illustrate this parallel. Suppose that instead of simply choosing location in stage 1 of the price setting model, car producing firms choose location and price. Interpret the location as an auto design and the price as an auto price. Suppose that firms selecting the same design commit themselves to price competition in an aftermarket for spare parts. The idea is that, if two or more firms choose the same design, their spare parts are interchangeable. In stage 2, consumers choose between firms on the basis of available design/price combinations. Consumers do not directly observe spare part prices. In stages 3 and 4, aftermarket competition takes place and consumers make their purchases.

Arguments similar to those used in Proposition 3 can also be used to show that any distribution of firms across auto designs is consistent with subgame perfection. However, if the firms are monopolists in the aftermarket, they will compete away aftermarket profits in stage 1 by lowering auto prices. As a result, Farrell and Saloner's remark concerning high profit margins in the market for spare auto parts is consistent with zero profits in the auto industry.

V. Conclusion

Classical models of perfect and imperfect competition are generally based on the assumption that consumers have complete information about the offerings of different sellers. However, in reality, the transmission of such information is usually not costless and the degree of competition between firms will therefore depend (in part) on what firms do to facilitate price comparison by consumers. This appears to be what Tibor Scitovsky (1950) had in mind when he wrote "I believe that the market's perfection depends on the buyer's expertness...

lines and pricing, but, on the other hand, it may be best to have a third store in your center than floating away to form the nucleus of another, competing center." This statement is quite easily understood in the context of the quantity setting model.

10. The argument of this paragraph depends on what consumers do when they see more than one shopping center with the largest number of firms. Thus, the existence of equilibria in which one shopping center is occupied by two firms and all other shopping centers are occupied by at most one firm depends on consumer tie breaking rules. In particular, existence of a clustering equilibrium depends on consumer tie breaking rules if $n = 2$.

11. The proof is similar to the proof of proposition 3.

12. Example 4 shows that clustering may not be inconsistent with higher industry profits.

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