International Finance Discussion Papers

Number 328

July 1988

THE TIMING OF CONSUMER ARRIVALS IN EDGEWORTH'S DUOPOLY MODEL

Marc Dudey

ABSTRACT

In his classic Papers relating to Political Economy (1897), Francis Edgeworth demonstrated that when duopolists have limited productive capacity, there may be no Nash equilibrium in prices. One feature of Edgeworth's model is that consumers are assumed to meet with the duopolists at the same time.

This paper analyzes a version of the Edgeworth model in which consumers arrive sequentially instead of simultaneously. This departure from Edgeworth's framework should seem reasonable since there are few markets besides auctions in which buyers all meet with sellers at the same time.

The point of the analysis is to show that when sellers engage in quantity constrained price competition, the timing of consumer arrivals may greatly affect the nature of equilibrium. It turns out that the existence of Nash equilibrium in prices may be restored. It also turns out that the duopolists may be able to maximize joint profits!

The Timing of Consumer Arrivals in Edgeworth's Duopoly Model

Marc Dudey*

I. Introduction

Edgeworth (1897) demonstrated that when duopolists have limited productive capacity, there may be no Nash equilibrium in prices. A number of economists subsequently studied variants of Edgeworth's model, mainly in an effort to obtain existence results. For example, Levitan and Shubik (1972), DasGupta and Maskin (1982) and others obtained equilibria in mixed strategies and Grossman (1981) noted the existence of equilibrium in supply functions. Kreps and Scheinkman (1983) proved that Cournot outcomes are generated when the duopolists choose capacity before setting prices and Hotelling (1929) challenged Edgeworth's conclusion by introducing a product differentiation assumption. Peters (1984) has recently made some progress on the existence question by assuming that sellers are spatially isolated and that each potential customer visits only one location.

This paper analyzes a version of the Edgeworth model in which consumers arrive sequentially instead of simultaneously. Thus, Edgeworth's assumptions of homogeneous products and costless search are retained, but it is assumed that buyers do not arrive at the same time. This departure from Edgeworth's framework should seem like a reasonable

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*The author is a staff economist in the International Finance Division. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff. I would like to thank Dilip Abreu, Karl Dudey, Richard Kihlstrom, Vijay Krishna and Hugo Sonnenschein for helpful discussions.

one since there are few markets besides auctions in which buyers all meet with sellers at the same time.

Here is an outline of what I have in mind. A finite number of consumers come to market one at a time. Upon the arrival of a consumer, duopolists selling inventories of an indivisible good engage in price competition. The consumers purchase at most one unit of the good each and have a common reservation value. In the spirit of Edgeworth's model, the number of units in the inventory of at least one seller is less than the number of buyers but the sum of the inventory levels exceeds the number of buyers. This last assumption guarantees that there would be no Nash equilibrium in prices if the buyers synchronized their arrival times.[1]

This framework is not intended as a realistic description of a duopoly, although it does dispense with the awkward requirement of simultaneous consumer arrival. The point of the analysis is to show that when sellers engage in quantity constrained price competition, the timing of consumer arrivals may greatly affect the nature of equilibrium. One implication of the assumptions listed in the previous paragraph is that the existence of (subgame perfect) Nash equilibrium in prices is restored. Another significant implication is that the duopolists can almost always maximize joint profits.

The paper unfolds in the following way. In section II, I present a more explicit description of the model outlined above. In section III, I informally discuss the special case in which only one seller is quantity constrained. It is in this special case that the results stated in the previous paragraph are most intuitive. In section IV, these results are proved for the cases in which one or both sellers is quantity constrained. Section V considers a variation on the basic model in which duopolists simultaneously choose their capacity (quantity) before meeting consumers. The main conclusion is that the sellers maximize joint profits and split the market as evenly as possible.

Entry deterrence in a market with large fixed costs is the subject of section VI. An incumbent is assumed to precede a potential entrant in choosing capacity. If the potential entrant does enter, the incumbent and entrant compete according to the model described in section II. Otherwise, the problem simply reduces to capacity constrained monopoly. It turns out that when the fixed cost of building capacity is sufficiently large relative to the number of consumers, the incumbent may restrict his productive capacity in order to deter entry. The consequence is that consumers will be rationed at the monopoly price. This result is disturbing since it follows from a fairly natural specification of institutional detail and conflicts with the standard result that "excess capacity deters entry" (see, for example, Spence 1977 and Dixit 1980). The model is also interesting as an example of a market with symmetrically informed participants and no uncertainty which does not clear in equilibrium.

Section VII contains a short summary and an agenda for further research.

II. The Model Without Capacity Choice

Consider a market in which duopolists named A and B sell units of an indivisible good to a collection of n consumers. Each of the n consumers purchases at most one unit and all of them have the same positive reservation value. Without loss of generality, I take this reservation value to be one unit of a numeraire good and assume that all consumers are endowed with at least one unit of the numeraire good. The duopolists A and B are endowed with a and b units respectively, with a $\leq$ b.

Consumers arrive in sequence and, upon the arrival of each consumers, the duopolists make price offers. For convenience, I assume that if a seller has run out of units, he must set a price greater than one. After receiving a pair of price offers, a consumer chooses between sellers in the following way. He buys from the low price seller if the sellers are charging different prices and the low price is less than or equal to one. In case both sellers set the same price and this price is less than or equal to one, each seller is assumed to have an equal chance of attracting the consumer. If both sellers set a price greater than one, the consumer rejects both offers and does not return. Thus, consumers may be identified with n time periods. When setting prices in any period, the duopolists know a, b, and all prices set in previous periods. They also know how inventories have evolved over time.[2] Each seller's objective is to maximize the sum of his earnings in the n periods of trade.

The model just described amounts to an extensive form game played between A and B. A strategy for a seller in this game gives the seller's price in any given period as a function of the prices set and inventories held by both sellers in previous periods. When the duopolists play a pair of strategies, the consumer decision rule specified above may be used to compute the expected total profit for each seller. The payoff function for each seller maps pairs of strategies into the seller's expected total profit.

Subgame perfect equilibrium (Selten, 1975) is the solution concept which I will use to analyze seller behavior in this game. In certain special cases, subgame perfection yields obvious restrictions on seller behavior. If $a \geq n$, the game reduces to repeated Bertrand competition. Subgame perfection therefore requires that prices and profits be driven to zero. On the other hand, there is no competitive tension between the sellers if $a + b \leq n$ or if $a = 0$. In these cases, A earns a and B earns $\min(b, n)$. Sections III and IV deal with the more interesting cases in which $a + b > n$ and $a < n$.

III. Price Competition with One Quantity Constrained Seller

A. Informal Discussion

Suppose that exactly one seller is quantity constrained (that is, the number of units in his initial inventory is less than the number of consumers) and that the quantity constrained seller adopts the following strategy. He sets a price of one whenever his current inventory is less than the number of remaining consumers and he sets a price of zero whenever his inventory is not less than the number of remaining consumers.

This strategy implicitly threatens the unconstrained seller with possibly repeated Bertrand competition unless the constrained seller is allowed to sell all of his units at the monopoly price. The threat is credible (!) since equilibrium requires that both sellers engage in Bertrand competition once the number of consumers remaining is equal to the number of units the constrained seller has left. It follows that when the constrained seller uses the aforementioned strategy, the unconstrained seller may as well let the constrained seller sell all of his units at the monopoly price. The unconstrained seller can then serve remaining consumers at the monopoly price. This reasoning suggests the existence of an equilibrium in which sellers are able to maximize joint profits.

B. The Advantage of Being Quantity Constrained

Notice that if the quantity constrained seller can dispose of his entire inventory at the monopoly price, the unconstrained seller may earn less than the constrained seller. For example, suppose two art dealers own the entire supply of a particular lithograph. One dealer owns 11 of the prints while the other dealer owns 9 of them. The dealers, who have no interest in keeping the lithographs, know that each of 10 art collectors would like to add one of the prints to his collection. The collectors have a common reservation value of $1000. If the art dealers compete according to the model described in section II, then the reasoning of the previous subsection suggests that the constrained seller will earn $9000 while his unconstrained rival will only earn $1000.

This example shows that there may be an advantage to being quantity constrained. It also leads to the question of whether the unconstrained dealer could increase his earnings by destroying some of his prints. Section IV will develop the argument which is needed to answer this question. For now, I will simply observe that there are contexts in which the issue of inventory destruction does not arise. An entrepreneur or worker may find it too costly or impossible to precommit to an artificially low volume of output and sales. For example, it may not be possible for a producing seller to partially disable his means of production. Alternatively, a seller might not be able to prove that the total number of units he is willing or able to supply is less than his initial productive capacity or endowment.

IV. Basic Results

This section studies the game described in section II under the assumptions that at least one seller is quantity constrained and that the sum of the sellers' quantity constraints exceeds the number of consumers. A central finding is that sellers maximize joint profits in equilibrium whenever their quantity constraints are not the same. If A and B are subject to identical quantity constraints, then the sellers set nonpositive prices when they meet the first consumer; all other consumers are served at the monopoly price.

As noted in the previous section, the intuition behind the existence of a joint profit maximizing equilibrium is not hard to understand when exactly one seller is quantity constrained. On the other hand, obtaining uniqueness results and generalizing to the case of two quantity constrained sellers requires a more explicit backward induction argument. To perform this backward induction, it will be helpful to have the following notation concerning subgame payoffs in equilibrium. Consider any subgame with t consumers remaining in which A and B are holding inventories of x and y respectively. Suppose that the payoff to each seller in any such subgame is uniquely determined across subgame perfect equilibria; that is, suppose any pair of equilibrium strategies gives the same pair of subgame payoffs to the sellers. If they exist, call these payoffs $V_t^A(x, y)$ and $V_t^B(x, y)$.

I start with an example which confirms the informal reasoning of section III for the case of two consumers. To begin the backward induction, consider the situation with one consumer remaining in which A and B have inventories x and y. Obviously, if both sellers have at least a unit left, competition for the remaining consumer will force prices and profits to zero in equilibrium. Hence, $V_1^A(x, y) = V_2^B(x, y) = 0$ if x and y are greater than or equal to one. Of course, if exactly one seller has no units left in inventory, then the other seller is free to set the monopoly price. This means $V_1^A(x, 0) = V_1^B(0, y) = 1$ and $V_1^A(0, y) = V_1^B(x, 0) = 0$ if x and y are greater than or equal to one.

Now consider the situation with two consumers remaining in which A and B have initial endowments a and b, where $a = 1$ and $b \geq 2$. If B makes the sale when there are two consumers remaining, then both sellers will have one unit in the last period. As a result, A will earn $V_1^A(1, b - 1) = 0$ and B will earn the sale price plus $V_1^B(1, b - 1) = 0$. If A makes the sale when there are two consumers remaining, A will earn the sale price and B will earn $V_1^B(0, b) = 1$. Thus, making the sale is worth the sale price to A and the sale price minus one to B. It follows that A will set a price equal to one and B will set a price greater than one in equilibrium. Consequently, $V_2^A(1, b) = V_2^B(1, b) = 1$.

This two consumer example gives an indication of what happens in equilibrium when there are n consumers. It turns out that if $a < \min(b, n)$ and $a + b > n$, then the seller with less units in inventory can sell all of his units at the monopoly price. The seller with more units in inventory supplies remaining consumers at the monopoly price. It follows that if $a = b < n$ and $a + b > n$, each seller has an incentive to undercut any positive price set by his rival and become the seller with less units in inventory when there are $n - 1$ consumers remaining. To be more specific, if $a = b < n$ and $a + b > n$, whichever seller makes the sale earns the sale price plus $a - 1$. His rival earns $n - a$. Since the extra profits associated with making the sale must be bid away in equilibrium, each seller will charge a nonpositive price of $n - 2a + 1$ when there are n consumers remaining. Thus, both sellers earn $n - a$. The main results for the n consumer case are summarized in the following proposition.

Proposition 1. There is a pure strategy, subgame perfect equilibrium in the model of section II. If $a < \min(b, n)$ and $a + b > n$, then A earns $V_n^A(a, b) = a$ and B earns $V_n^B(a, b) = n - a$. If $a = b < n$ and $a + b > n$, both sellers earn $V_n^A(a, a) = V_n^B(a, a) = n - a$.

Proof. See the Appendix.

V. The Model with Capacity Choice

Returning to the art dealer game presented at the end of section III, proposition 1 implies that the unconstrained dealer could increase his payoff to \$8000 from \$1000 by publicly destroying three of his lithographs. This demonstrates the need to study a framework in which initial inventories are determined endogenously. Below, I consider a model in which sellers may choose their own productive capacities before meeting consumers. This model can be extended to cover situations in which sellers may credibly destroy portions of existing inventories before meeting consumers.

Consider a two stage game in which sellers simultaneously select their own capacities in the first stage and then play the game described in section II in the second stage. First observe that if the construction of capacity is costly, there may be no subgame perfect equilibrium in pure strategies for this two stage game. For example, suppose the only expense associated with building capacity is a fixed cost c which is greater than n/2 but less than n - 1. This means that the market is a natural monopoly. It follows that, if an equilibrium existed, it would involve one seller choosing a capacity of zero. His rival would therefore choose a capacity of at least n units in this hypothetical equilibrium. However, the strategy pair [0, m] where $m \geq n$ cannot be an equilibrium for the capacity choice game since the seller who is not in the market could earn n - 1 - c instead of zero by choosing a capacity level of n - 1 (use proposition 1).

In this section, I avoid the nonexistence problem by making mild assumptions about the cost of building capacity. Let C(k) denote the cost of building k units of capacity and assume $C((n - 1)/2) < (n - 1)/2$. (the inequality means that the market cannot be a natural monopoly, regardless of whether n is even or odd). As is usual, suppose that the cost of building zero units of capacity is zero and that C is nondecreasing and convex on the positive integers. Under these assumptions, subgame perfect equilibria in pure strategies exist and payoffs in any such equilibrium are approximately symmetric. A more precise statement of this result is contained in the following proposition.

Proposition 2. Given the above assumptions on C, there is at least one pure strategy, subgame perfect equilibrium in the capacity choice game. If $g(k) = k - C(k)$ is maximized at some $k*$ less than or equal to $n/2$, both sellers will earn $g(k*)$ in any equilibrium. If n is odd and all the maximizers of $g(k)$ are greater than $n/2$, then either one seller will earn $g((n + 1)/2)$ while his rival earns $g((n - 1)/2)$ or both sellers will earn $g((n - 1)/2)$ in any equilibrium. If n is even and all the maximizers of $g(k)$ are greater than $n/2$, then each seller will earn $g(n/2)$ in any equilibrium.

Proof. See the Appendix.

The analysis is not much different if, instead of choosing capacity, the sellers may costlessly destroy portions of existing inventories before consumers arrive. If one of the sellers is endowed with less than $n/2$ units, proposition 1 implies that neither seller has an incentive to destroy inventory. On the other hand, if each seller is initially endowed with at least $n/2$ units, an argument like the proof of proposition 2 can be used to show that equilibrium exists and (i) one seller will earn $(n + 1)/2$ while his rival earns $(n - 1)/2$ or both sellers will earn $(n - 1)/2$ if n is odd, and (ii) both sellers will earn $n/2$ if n is even. Thus, in the art dealer game, both dealers will earn $5000 if they can publicly destroy their own lithographs.

VI. Rationing in a Natural Monopoly

In the previous section, sellers were assumed to simultaneously determine capacity before meeting consumers. Restrictions were placed on the cost of building capacity to obtain existence of equilibrium in pure strategies. In this section, the nonexistence problem is circumvented by having the sellers choose capacity sequentially. The framework considered here has a natural interpretation as an entry deterrence model.

I assume that an incumbent seller named A chooses capacity in the first stage of a three stage game. A potential entrant named B chooses capacity in the second stage. In the third stage, A and B compete for n sequentially arriving consumers as in the model of section II. The cost assumption used here is that if either seller builds a positive level of capacity, he incurs a large fixed cost c which is greater than n/2 but less than n - 1. Note that this natural monopoly assumption led to the nonexistence of subgame perfect equilibrium in pure strategies in the section V model.

It is easy to find a subgame perfect equilibrium in pure strategies for the entry deterrence game. There is only enough room in the market for one seller because of the large fixed cost assumption and, according to proposition 1, the seller with smaller capacity can sell all his units at the monopoly price. This means that the incumbent can deter entry and make nonnegative profits by choosing a "small" capacity. The main point is stated in the following proposition.

<u>Proposition 3</u>. There is a subgame perfect equilibrium in the entry deterrence game in which A deters entry by setting a capacity equal to the smallest integer greater than c. In this equilibrium, at least one consumer is rationed at the monopoly price.

<u>Proof</u>. See the Appendix.

The rationing result is of interest because it relies on a reasonable specification of institutional detail and conflicts with a standard result from the entry deterrence literature. Economists (Spence 1977, Dixit 1980, and others) have generally made the point that the presence of a potential entrant may cause an incumbent to set a capacity which is larger than the static monopoly capacity. An exception is the paper by Benoit and Krishna (1987b). These authors use supergame techniques to show that an incumbent may deter entry by choosing a capacity which is smaller than the static monopoly capacity. However, consumers are not rationed in this equilibrium. Footnote 2 mentions another distinguishing feature of the Benoit-Krishna approach.

VII. Concluding Remarks and Directions for Further Work

This paper introduces sequential consumer arrival into a model of quantity constrained price competition. In the context of this model, it is shown that Edgeworth's nonexistence of equilibrium result would no longer hold and that sellers would almost always maximize joint profits. Adding a quantity choice assumption was shown to yield approximately symmetric, joint profit maximizing, equilibrium payoffs. Finally, it is shown that sequential consumer arrival would create the possibility of rationing in a natural monopoly.

Among the most restrictive assumptions made here is the hypothesis that consumers have a common reservation value. The most natural way to introduce downward sloping demand would be to assume that consumers have different reservation values. However, it can be shown that equilibrium in prices need no longer exist when consumers have different reservation values and choose sellers according to the symmetric tie breaking rule used in this paper. Although existence in prices may be restored when consumers use certain asymmetric tie breaking rules, equilibrium payoffs will no longer be unique with at least some of these rules. Such difficulties also arise when consumers have the same reservation value and the discount rate is positive or the number of consumers is random. Furthermore, if consumers have different reservation values, it would seem unreasonable to assume that firms know the order of arrival or even that firms know what these reservation values are. These issues are left for another paper.

Appendix

Proof of Proposition 1:

The inductive step is all that remains. Assume the theorem statement holds for n consumers and any combination of initial endowments (x, y) satisfying $x + y > n$, $x \le y$ and $x < n$. First suppose the number of consumers equals $n + 1$ and that the initial endowments (a, b) satisfy $a + b > n + 1$, $0 < a \le b$ and $a < \min(b, n + 1)$. If A makes the sale, he earns the sale price plus $V_n^A(a - 1, b)$, which equals $a - 1$ by the inductive hypothesis. In this case, B earns $V_n^B(a - 1, b) = n - a + 1$.

If B makes the sale, there are two possibilities. If $b > a + 1$, then B earns the sale price plus $n - a$ and A earns a. This means that making the sale is worth the sale price minus one to both sellers. It follows that either A or B will make the sale at a price of one in equilibrium. Consequently, $V_{n+1}^A(a, b) = a$ and $V_{n+1}^B(a, b) = n - a + 1$ if $b > a + 1$. If $b = a + 1$ and B makes the sale, then B earns the sale price plus $n - a$ and A earns $n - a$. This means that making the sale is worth the sale price minus one to B and the sale price plus $(a - 1) - (n - a)$ to A. Since $a + b = a + (a + 1) > n + 1$ by assumption, the quantity $(a - 1) - (n - a) = 2a - (n + 1)$ is nonnegative. It follows that A must be setting a price equal to one and B must be setting a price greater than one in equilibrium. Thus, $V_{n+1}^A(a, a + 1) = a$ and $V_{n+1}^B(a, a + 1) = n - a + 1$ if $b = a + 1$.

Now suppose $a + b > n + 1$ and $a = b < n + 1$. By the argument in the last paragraph before the proposition statement, both sellers will set a nonpositive price of $n + 1 - 2a + 1$. The equilibrium payoffs are therefore $V_{n+1}^A(a, b) = V_{n+1}^B(a, b) = n + 1 - a$. To finish the proof, notice that if $a = 0$ and $b > n + 1$, A must be setting a price greater than one (by assumption) and B must be setting a price equal to one in equilibrium. Hence, $V^A_{n+1}(a, b) = a$ and $V^B_{n+1}(a, b) = n + 1 - a$. Q.E.D.

Proof of Proposition 2:

A pure strategy, subgame perfect equilibrium in the capacity choice game exists if and only if there is a pair of capacity levels a and b such that $a \le b$, $V^A_n(x, y) - C(x) \ge V^A_n(k, y) - C(k)$ and $V^B_n(x, y) - C(y) \ge V^B_n(x, k) - C(k)$ for all k in $\{1, 2, \ldots, n\}$. In the remainder of this proof, I will abuse terminology by referring to such a pair of capacity levels as an equilibrium. I will also make routine use of proposition 1 and the following observations from section II: $V^A_n(a, b) = a$ and $V^A_n(a, b) = b$ if $a + b \le n$, and $V^A_n(a, b) = V^B_n(a, b) = 0$ if $a \ge n$.

If (a, b) is an equilibrium and g is maximized at some k^* less than or equal to n/2, then $V^A_n(a, b) - C(a) \ge V^A_n(k^*, b) - C(k^*) = g(k^*)$. On the other hand, $V^A_n(a, b) - C(a) \le a - C(a) \le g(k^*)$. It follows that $V^A_n(a, b) - C(a) = g(k^*)$. Similarly, $V^B_n(a, b) - C(b) = g(k^*)$. It is easy to see that (k^*, k^*) is an equilibrium.

Now assume n is odd and all the maximizers of g are greater than n/2. If (a, b) is an equilibrium where $a \le b$, I claim $a \ge (n - 1)/2$. If not, $V^A_n(a, b) - C(a) = g(a) < g((n - 1)/2) = V^A_n((n - 1)/2, b) - C((n - 1)/2)$, contradicting the definition of equilibrium (the inequality follows from the concavity of g and the fact that the smallest maximizer of g is greater than n/2). Next, I assert that $b \ge (n + 1)/2$. If not, then $(n - 1)/2 \ge b \ge a \ge (n - 1)/2$. Hence, $a = b = (n - 1)/2$. However, $V^B_n((n - 1)/2, (n - 1)/2) - C((n - 1)/2) = g((n - 1)/2) < g((n + 1)/2) = V^B_n((n - 1)/2, (n + 1)/2) - C((n + 1)/2)$, contradicting the definition of equilibrium (again, the inequality follows from the concavity of g and the fact that the smallest maximizer of g is greater than n/2). Observe that if $b > (n + 1)/2$, then $a \leq (n + 1)/2$. If not, then $b \geq a > (n + 1)/2$. Thus, $V_n^B(a, b) = \max (0, n - a) < (n - 1)/2$ and $V_n^B(a, (n - 1)/2) - C((n - 1)/2) = (n - 1)/2 - C((n - 1)/2) > V_n^B(a, b) - C(b)$, contradicting the definition of equilibrium.

The above reasoning shows that $(n - 1)/2 \leq a \leq (n + 1)/2$ and $b \geq (n + 1)/2$. It follows that if the sellers do not choose the same capacity level, one seller will earn $g((n + 1)/2)$ and his rival will earn $g((n - 1)/2)$. If both sellers set capacity equal to $(n + 1)/2$, then both sellers will earn $(n - 1)/2 - C((n + 1)/2)$. However, since either seller could earn $g((n - 1)/2)$ by choosing a capacity of $n - 1$, it must be that $C(n - 1)/2) = C((n + 1)/2)$. Hence, both sellers earn $g((n - 1)/2)$. It is easy to check that $[(n + 1)/2, (n - 1)/2]$ is an equilibrium which yields the asymmetric payoffs and $[(n + 1)/2, (n + 1)/2]$ would be an equilibrium which yields the symmetric payoffs if, for example, $C = 0$. The argument is similar if n is even. Q.E.D.

Proof of Proposition 3:

Let $I(c)$ denote the smallest integer greater than c. Suppose the incumbent chooses a capacity level of x and that the potential entrant chooses a capacity of $\min(x - 1, n - 1)$ if $x > I(c)$, 0 if $n - c \leq x \leq I(c)$, and $n - x$ if $x < n - c$. It follows from the results on V_n^A and V_n^B that this (second stage) response function is consistent with subgame perfection. Also, if the entrant uses this response function, $x = I(c)$ is the level of capacity which maximizes profit for the incumbent. At least one seller is rationed in this equilibrium since $I(c) < n$. Q.E.D.

Footnotes

1. Brock and Scheinkman (1985) study an infinitely repeated Edgeworth game in which firms face the same capacity constraint in every period. Benoit and Krishna (1987a, b) also study a related Edgeworth supergame in which capacities are determined endogenously. In all of these papers, the authors are concerned with intraperiod capacity constraints instead of the interperiod constraints imposed here.

2. If a seller knows initial inventories, the prices set by both sellers in previous periods, and the way his own inventory has evolved over time, he can infer the way his rival's inventory has evolved over time. Thus, the theory does not depend on a seller being able to observe his rival's inventory.

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