THE DYNAMICS OF INTEREST RATE AND TAX RULES IN A STOCHASTIC MODEL

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ABSTRACT

A simple stochastic equilibrium structure is used to study the implications of monetary and fiscal policy interactions for government intertemporal budget balance. Existence and uniqueness of monetary equilibria are shown to depend on parameters of policy rules. The paper derives closed form solutions for equilibrium inflation and real debt as functions of policy parameters and policy shocks and obtains conditions under which the usual tests that deficits Granger-cause money creation will successfully uncover evidence of monetized deficits. In addition, equilibria are studied in which private agents today know tomorrow's taxes exactly. Coupling this informational assumption with a monetary policy that pegs the nominal interest rate reverses the usual Granger-causal ordering between deficits and monetization, so that money growth (or inflation) may predict higher deficits. This implies that empirical work designed to detect that deficits have been monetized by testing whether deficits Granger-cause money creation, may fail to uncover the monetization.

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1.   Introduction

In recent years the specification of policy behavior as following simple time-invariant rules has moved beyond the realm of abstract analytical structures and is being incorporated into moderately-sized econometric models.[1] Although these econometric models typically include both monetary and fiscal policy, most of the existing theoretical work considers only monetary policy rules, implicitly leaving fiscal policy to respond passively to satisfy the government's budget constraint.

Papers by Aiyagari and Gertler (1985) and Sims (1988) adopt the more symmetric view that fiscal policy need not be a passive instrument of policy. These structures support a continuum of policy interactions ranging from the conventional treatment of fiscal policy as passive to the equally extreme case where the fiscal authority sets the net-of-interest deficit exogenously and monetary policy adjusts to ensure budget balance. Failure to allow for this range of policy interactions may lead to empirical conclusions that mistakenly attribute fiscal effects to monetary policy.

This paper extends the work of Aiyagari and Gertler (1985) and Sims (1988). I embed Sims's specification of policy in a general equilibrium model

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[1]See, for example, Frenkel et. al. (1989), Gagnon (1989), Masson et. al. (1988), McKibbin and Sachs (1989), and Taylor (1989).

and obtain a more complete characterization of the dependence of equilibrium inflation and nominal interest rates on the parameters of monetary and tax rules. I derive conditions on policy behavior under which there exist equilibria with unique solutions for inflation, real balances, and real government debt. The policy rules studied — an interest rate rule that depends on current inflation and a rule for lump-sum taxes that allows for a dependence of taxes on lagged real debt — are closely related to those now being embedded in empirical specifications of the U.S. economy.

The present work differs from much of the recent research on monetary policy rules in its explicit modeling of uncertainty.[2] Thus, policy interactions take the form of one policy authority adjusting its instrument in response to shocks generated by the other authority, where the shocks are modelled as additive components in the policy rules. To highlight the influence of policy shocks (and to contribute analytical simplicity) the model contains only two sources of uncertainty — mutually uncorrelated shocks to the monetary and tax policy rules — but a richer stochastic specification is possible without altering the key results about policy effects.[3]

Policy authorities have access to three types of taxes: direct lump-sum taxes, distorting expected inflation taxes, and lump-sum unanticipated inflation taxes. The degree of reliance on each of these financing schemes is determined by the parameters of the policy rules. By focussing the analysis on how deficit *shocks* are financed, the model supports more realistic equi-

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[2]Calvo and Vegh (1989), Fuhrer and Moore (1989), and Reinhart (1989) are recent examples of deterministic models.

[3]Leeper (1989) simulates a similar structure that includes these policy shocks as well as technology, preference, and government spending shocks.

libria in which direct taxation and money- and debt-financing coexist. For example, even in the extreme case where fiscal *shocks* are fully monetized, the equilibria exhibit positive levels of direct taxation and government debt.

Equilibrium in this stochastic setting can be characterized as a vector autoregression including inflation, real debt, and the random aspects of policy choice. The assumptions on policy behavior place zero restrictions on this VAR, which provides a basis for testing whether actual time series on these variables conform with the model's predictions. These restrictions can frequently be examined with Granger-causality tests.

I entertain two informational assumptions. The model is solved using both the conventional assumption that policy shocks are observed at the time they are realized and the assumption that consumers observe tax, but not monetary, shocks one period before the shocks affect tax revenues. The advanced knowledge of tax policy is intended to reflect observed behavior in many countries where tax rates are known several quarters before the taxes are paid.

King and Plosser (1985) seek evidence that deficits have been monetized by testing whether deficits predict money creation. The model in this paper supports such an approach under certain assumptions on both policy behavior and the information given to private agents. However, if agents know future taxes and monetary policy pegs the nominal interest rate, the usual Granger-causal ordering between deficits and monetization is reversed, so that money growth (or inflation) may predict higher deficits. This implies that empirical work designed to detect that deficits have been monetized by testing whether deficits Granger-cause money creation, may fail to uncover the monetization.

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Section 2 lays out the economic environment and the rules policy authorities are assumed to follow. Section 3 discusses existence and uniqueness of equilibria and characterizes these equilibria under various assumptions on policy. Conditions are derived under which fully monetized deficits will fail to Granger-cause inflation. The implications of providing consumers with foreknowledge of taxes are derived in Section 4. The paper ends with brief remarks.

2.   The Model

At each date t private agents are endowed with a constant y units of the consumption good and the government extracts g < y units of the good for government purchases that yield no utility to consumers.[4] These assumptions imply that the equilibrium real interest rate is constant.

Nominal money balances earn no interest and are valued because they are treated as a durable consumption good whose expected rate of depreciation is the nominal interest rate. That is, current period real balances provide consumers with utility in a way that is separable from the current level of consumption. This implies a simple demand for real balances of the form $M_t/p_t = c_t f(R_t)$, where M_t is nominal money balances at the end of period t, p_t is the price level at time t, R_t is the risk-free gross nominal interest rate on government debt sold at t and redeemed at t+1, and c_t is consumption.

In addition to nominal money balances, private agents may save in the form of one-period nominal government debt obligations, B_t, which have real

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[4]The assumption that g does not yield utility is made solely for notational ease. The constancy of government spending ensures that even if marginal utility depended on g, this dependence would simply take the form of a scale factor.

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value $b_t = B_t/p_t$ and earn interest at rate R_t. In equilibria in which both of these nominal government liabilities are held by consumers, the government has access to anticipated and unanticipated inflation taxes. Anticipated inflation extracts resources by distorting private agents' money demand decisions and works through the nominal interest rate. Unanticipated inflation, which plays a crucial role in this stochastic environment, supplies the government with seignorage revenues by devaluing the existing stocks of government liabilities. These unexpected inflation taxes are lump-sum.

Utility is assumed to be separable in consumption and real balances:[5]

$$u(c_t, m_t) = \log(c_t) + \log(m_t).$$

At each date t consumers discount utility at rate $\beta \in (0,1)$ and choose the decision vector $\{c_t, m_t, b_t\}$ to solve the problem:

$$\max \quad E_1 \sum_{t=1}^{\infty} \beta^t u(c_t, m_t)$$

subject to

$$c_t + m_t + b_t + \tau_t = y + \pi_t^{-1} m_{t-1} + R_{t-1}\pi_t^{-1} b_{t-1} \quad \text{for } t \geq 1, \tag{2.1}$$

given the initial value of assets, $(M_0 + R_0 B_0)/p_0$, and taking as parametric $\{y, \tau_t, R_t, p_t\}$. Consumers pay τ_t units of the consumption good in direct lump-sum taxes at each date. π_t is the gross rate of inflation in the price level from t-1 to t.

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[5]The logarithmic preferences are assumed for simplicity. More general specifications of the utility function that allow for varying degrees of risk aversion would complicate the model without altering the key results.

The government finances the constant level of purchases each period through a combination of direct lump-sum taxes $\{\tau_t\}$ and money creation $\{M_t - M_{t-1}\}$ in order to imply a process for debt $\{b_t\}$ that satisfies the government budget constraint:

$$b_t + m_t + \tau_t = g + \pi_t^{-1} m_{t-1} + R_{t-1}\pi_t^{-1} b_{t-1} \quad \text{for } t \geq 1, \tag{2.2}$$

given initial levels of government liabilities, $(M_0 + R_0 B_0)/p_0$.

The government's financing decisions follow simple rules for the monetary and tax authorities. Monetary policy is posed as obeying a nominal interest rate rule that depends on current inflation, while fiscal policy sets a level of direct taxes that depends on the quantity of real government debt held by the public:

$$R_t = \alpha_1 + \alpha_2 \pi_t + \theta_t, \tag{2.3}$$

$$\text{where} \quad \theta_t = \rho_1 \theta_{t-1} + \epsilon_{1t}, \quad |\rho_1| < 1, \quad \epsilon_{1t} \sim N(0, \sigma_1^2), \quad \text{and}$$

$$\tau_t = \gamma_1 + \gamma_2 b_{t-1} + \psi_t, \tag{2.4}$$

$$\text{where} \quad \psi_t = \rho_2 \psi_{t-1} + \epsilon_{2t}, \quad |\rho_2| < 1, \quad \epsilon_{2t} \sim N(0, \sigma_2^2).$$

The random components to the policy rules represent the assumption that policy behavior depends on aspects of the environment that are not perfectly predictable to private agents. For example, if the technology of policy selection is imperfectly understood by private agents (and the econometrician), individuals will act as if there is a random part of policymaking that is unrelated to the economic variables they observe contemporaneously. From the viewpoint of private agents (and the econometrician), these additive policy shocks appear as exogenous stochastic processes.

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The monetary authority manipulates the nominal money stock to set R according to (2.3). R's dependence on current inflation represents the monetary authority's attempts to "lean against inflationary winds." The specification of policy draws an empirical distinction between two sorts of monetary actions: Those induced by efforts to combat inflationary pressures generated by fiscal shocks, which are R's contemporary reaction to π, and those designed to pursue alternative objectives, which are disturbances to the random term θ.

The coefficient on lagged debt, γ_2 in equation (2.4), reflects the extent to which the fiscal authority ratifies increases in either deficits or debt service with higher future taxes. This specification of the process for lump-sum taxes ensures a positive level of expected direct taxation. Setting $\gamma_2 = 0$ reflects fiscal financing that ignores interest payments on the debt and makes the net deficit, $g - \tau_t$, unresponsive to economic conditions.

Using the assumption on $u(\cdot)$, the equilibrium conditions can be reduced to:

$$c_t + g = y \tag{2.5}$$

$$-(1/R_t) + \beta E_t \ [1/\pi_{t+1}] = 0 \tag{2.6}$$

$$m_t = c[R_t/(R_t - 1)] \tag{2.7}$$

plus (2.1) - (2.4). Equations (2.6) and (2.7) incorporate the constancy of consumption implied by (2.5). Each of these equations is required to hold for $t \geq 1$. E_t is defined as: $E_t x_{t+1} = E[x_{t+1} | \Omega_t]$, where $\Omega_t = \{\theta_{t-s}, \psi_{t-s}, \ s \geq 0\}$ and corresponds with conventional assumptions about when information is revealed to private agents. It is convenient to define an alternative expectations

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operator, E_t^*, which is relevant when private agents today are assumed to observe tomorrow's direct taxes exactly, as: $E_t^* x_{t+1} = E[x_{t+1}|\Omega_t^*]$, where $\Omega_t^* = \Omega_t \cup \{\psi_{t+1}\}$.[6] The solution under this informational assumption is obtained by replacing E_t with E_t^* in the consumer's problem and in the equilibrium conditions above.

In addition to satisfying the equilibrium conditions (2.1) - (2.7), an optimal solution must meet the government's intertemporal budget constraint.[7] The transversality condition for government debt ensures that the real value of debt held by the private sector equals the present value of future surpluses plus seignorage less interest payments on the debt. This intertemporal constraint is:

$$\frac{B_t}{P_t} = \sum_{s=0}^{\infty} \left(\prod_{j=0}^{s} \pi_{t+j+1} R_{t+j}^{-1} \right) [\tau_{t+s+1} - g + m_{t+s+1} - (1/\pi_{t+s+1})m_{t+s}]. \qquad (2.8)$$

I now turn to a discussion of the implications of policy behavior for the satisfaction of (2.8).

3.   Solution Under Conventional Informational Assumptions

The model laid out in section 2 is nearly linear: the feasibility condition and the policy rules are exactly linear, while the Euler equation

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[6]Readers uncomfortable with the idea that this expanded information set attributes to consumers the ability to observe a future-dated variable, may wish to think of the tax rule instead as $\tau_t = \gamma_1 + \gamma_2 b_{t-1} + \psi_{t-1}$. Observing current tax revenues is no longer equivalent to observing only current and past policy shocks since τ_t depends on ψ_{t-1} but not on ψ_t. Under this tax rule the relevant information set at time t is Ω_t.

[7]McCallum (1984) demonstrates the necessity of the transversality condition for debt in a deterministic growth economy.

for debt and the relationship for real balances are linear in logarithms. Only the government budget constraint contains nonlinearities that enter through the valuation of existing nominal government liabilities. I solve the model by linearizing around the deterministic steady state.[8]

Equations (2.1) - (2.7) can be reduced to a first-order recursive system in inflation and real debt. Substitute the interest rate rule into the Euler equation for debt and linearize to obtain a difference equation in expected future inflation and the current realizations of inflation and the monetary policy shock:

$$E_t \widetilde{\pi}_{t+1} = \alpha_2 \beta \widetilde{\pi}_t + \beta \theta_t \quad \text{for } t \geq 1, \tag{3.1}$$

where the "~" denotes deviation from the deterministic steady state.

Next substitute both policy rules and the real balance relation in (2.7) into the government budget constraint to derive the debt equation:

$$\varphi_1 \widetilde{\pi}_t + \widetilde{b}_t + \varphi_2 \widetilde{\pi}_{t-1} - (\beta^{-1} - \gamma_2) \widetilde{b}_{t-1} + \varphi_3 \theta_t + \psi_t + \varphi_4 \theta_{t-1} = 0 \quad \text{for } t \geq 1, \tag{3.2}$$

where $\varphi_1 = c(R-1)^{-1}[(1/\beta\pi) - \alpha_2/(R-1)] + b/\beta\pi$,

$\varphi_2 = \alpha_2 \pi^{-1}[(c/(R-1)^2 - b]$,

$\varphi_3 = -c/(R-1)^2$,

$\varphi_4 = \varphi_2/\alpha_2$,

and c, R, π, and b are deterministic steady state values of consumption, the nominal interest rate, inflation, and real debt. The analysis considers only

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[8]This introduces two sources of error due to the linear approximation to the true nonlinear equations and the fact that the model's stationary distribution is treated as degenerate. It is unlikely, however, that these errors are important for the qualitative implications that are derived.

equilibria in which real government debt outstanding is positive in a steady state.

Note that since private agents do not begin making optimal choices until time 1, equation (3.1) need not hold at the initial date 0. This creates an indeterminacy in the model in the form of a loose condition on the initial inflation rate $\pi_1 = p_1/p_0$. As I now discuss, certain assumptions on policy behavior eliminate this indeterminacy and lead to unique equilibria.

The recursiveness of system (3.1) and (3.2) implies the roots can be read off immediately as $\alpha_2\beta$ and $\beta^{-1}-\gamma_2$ and 0.[9] It is clear from the dependence of these roots on the parameters of the interest rate and tax rules that the model's stability characteristics hinge on policy behavior. When monetary policy reacts sufficiently strongly to inflation ($|\alpha_2\beta| > 1$) the first difference equation is unstable, while when the fiscal authority responds weakly to debt ($|\beta^{-1}-\gamma_2| < 1$) the second equation is unstable.

The (α_2,γ_2) policy parameter space can be divided into four disjoint regions associated with the number of unstable roots in (3.1) and (3.2). Label the regions as:

Region I: $|\alpha_2\beta| > 1$ and $|\beta^{-1}-\gamma_2| < 1$ Active monetary, passive fiscal policy

Region II: $|\alpha_2\beta| < 1$ and $|\beta^{-1}-\gamma_2| > 1$ Passive monetary, active fiscal policy

Region III: $|\alpha_2\beta| < 1$ and $|\beta^{-1}-\gamma_2| < 1$ Passive monetary and fiscal policy

Region IV: $|\alpha_2\beta| > 1$ and $|\beta^{-1}-\gamma_2| > 1$ Active monetary and fiscal policy

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[9]The roots can be obtained more formally. Following Blanchard and Kahn (1980), write the system in terms of $(E_t\pi_{t+1},\pi_t,b_t)$, $(\pi_t,\pi_{t-1},b_{t-1})$, and $(\theta_t,\psi_t, \psi_{t-1})$ by using the assumptions on the processes for the policy shocks, θ_{t+1} and ψ_{t+1}, to evaluate the conditional expectations of these shocks. Then the transition matrix is lower triangular with these three roots along the diagonal. I thank Joe Gagnon for providing this interpretation.

In Region I monetary policy actively pursues price stability by standing ready to contract at the first sign of inflation, while fiscal policy follows the passive course of adjusting direct taxes and debt to balance the budget. This produces equilibria with unique inflation and real debt processes. The second region reverses the stances of the policy authorities. By responding weakly to inflation the monetary authority allows the money stock to adjust to satisfy the budget, while the fiscal authority's refusal to alter taxes prevents deficit shocks from being bond-financed. This again uniquely determines the equilibrium real debt and inflation processes.

In Region III both policy instruments are used for budget balance and there are no unstable roots, resulting in a multiplicity of equilibria. From (3.1) it is clear that for given realizations of the monetary policy shock, the path of inflation depends on the initial inflation rate, π_1. This implies, via the budget constraint, that the path of equilibrium debt also depends on π_1.[10] Finally, when both authorities try to control prices the two unstable roots in Region IV uniquely determine both inflation and real debt but, as demonstrated in Appendix B, an equilibrium exists only if the monetary and fiscal policy shocks are related in a specific way.[11]

I now solve the model under Region I and Region II policy parameters.

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[10]McCallum (1986) introduces terminology to distinguish between price level *indeterminacy* — where the model does not determine any nominal values but real values, in particular real balances, are unique — and price level *nonuniqueness* — where multiple price level solution paths satisfy the model for each given money stock, so there exist many real balance paths. Region III produces many inflation and therefore nominal interest rate solutions, leading to nonuniqueness of equilibrium real balances.

[11]It is also possible to consider cases of borderline stability when one or both roots exactly equal unity. These seem empirically unlikely, however, and are not studied here.

Region I: Active Monetary and Passive Fiscal Policies

Under the assumptions that monetary policy reacts strongly to inflation ($|\alpha_2\beta| > 1$) and fiscal policy raises taxes sharply when debt increases ($|\beta^{-1} - \gamma_2| < 1$), the system has one unstable and one stable root. The solution for inflation from solving (3.1) is:

$$\tilde{\pi}_t = -1/\alpha_2 \sum_{i=0}^{\infty} -(\alpha_2\beta)^{-i} E_t \theta_{t+i}$$

$$= [\beta/(\rho_1 - \alpha_2\beta)]\theta_t,$$

by the assumption that $\{\theta_t\}$ is AR(1), for $t \geq 1$. More generally for positive k,

$$\tilde{\pi}_{t+k} = [\beta\rho_1^k/(\rho_1 - \alpha_2\beta)]\theta_t. \tag{3.3}$$

By the interest rate rule this implies that for $t \geq 1$:

$$\tilde{R}_t = [\rho_1/(\rho_1 - \alpha_2\beta)]\theta_t,$$

which implies for $k > 0$,

$$\tilde{R}_{t+k} = [\rho_1^{k+1}/(\rho_1 - \alpha_2\beta)]\theta_t. \tag{3.4}$$

Thus, in equilibrium, both the inflation and nominal interest rates depend entirely on the parameters of the monetary policy rule, the discount factor, and the monetary policy shock.[12] Monetary policy stabilizes prices by preventing deficit shocks from affecting inflation. Following a monetary

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[12]Although the parameters and variables associated with fiscal policy are absent from expressions (3.3) and (3.4), fiscal policy is far from "irrelevant," because the solutions depend critically on taxes rising sufficiently in the face of real debt expansions to satisfy the government budget constraint.

shock, π and R gradually return to their steady state values along the stable

paths given above. For a particular set of realizations of the policy shocks,

equations (3.3) and (3.4) give the unique solution time paths of $\{\tilde{\pi}_t, \tilde{R}_t\}$ and

consequently of $\{\tilde{m}_t\}$ for $t \geq 1$.

The solution highlights some interesting features of equilibria in

Region I. When monetary policy shocks are serially uncorrelated ($\rho_1 = 0$),

(3.4) implies that the nominal interest rate is constant in equilibrium, so

inflation and the policy shock are perfectly negatively correlated with $\tilde{\pi}_t =$

$-(1/\alpha_2)\theta_t$. Thus the variance of inflation's deviations from steady state

depends inversely on the strength with which the monetary authority reacts to

inflation.[13] With a more general pattern of serial correlation in the

monetary shock, however, the equilibrium nominal interest rate will vary over

time, but only to the extent that the current shock forecasts higher (or

lower) future shocks and therefore inflation (or deflation).

The solution also reveals that given data on equilibrium $\{\tilde{R}_t, \tilde{\pi}_t\}$ it is

not possible to use least squares regressions to recover the monetary policy

rule and back out the time series of implied $\{\theta_t\}$ policy shocks. If an

econometrician with such a data set were to estimate the regression $\tilde{R}_t = \delta\tilde{\pi}_t +$

u_t, he would obtain the estimate $\hat{\delta} = \rho_1/\beta$ with $R^2 = 1$. Since we know that

$|\alpha_2\beta| > 1$ and the policy shock is stationary, this estimate of the true degree

of monetary reaction to inflation cannot be correct. Essentially, least

squares imposes that u_t is orthogonal to inflation, while in the data the two

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[13]The fixity of $\tilde{R}_t$ is special to this model with a constant real interest
rate. In Leeper (1989) the real interest rate varies and when $\rho_1 = 0$ the
solution for the nominal rate depends on taste, technology, and government
spending shocks that affect the real rate. The equilibrium nominal rate's
independence of the monetary shock continues to hold, however.

are correlated. δ will be a consistent estimator for α_2 only if the correlation between θ_t and $\tilde{\pi}_t$ is known *a priori*. To word this differently, we observe only equilibrium sequences of $\{\tilde{R}_t, \tilde{\pi}_t\}$, but the monetary authority reacts only to inflation rates that are off their equilibrium path.

Government debt evolves according to the stable difference equation given by the linearized version of the budget constraint in (3.2). Tax cuts unambiguously raise the level of real debt. With future taxes rising strongly in the face of higher debt, consumers rationally expect these higher taxes and willingly increase their holdings of nominal government debt at the current nominal interest rate. With monetary policy fixing the money stock, the price level is unchanged, and the higher nominal debt is transformed one-for-one into higher real debt. The speed with which real debt returns to steady state depends on the degree of responsiveness of taxes to debt in the tax policy rule.

Region II: Passive Monetary and Active Fiscal Policies

The solution when monetary policy is unresponsive to inflation ($|\alpha_2\beta| < 1$) and taxes do not rise strongly with higher debt ($|\beta^{-1}-\gamma_2| < 1$) is more complicated. The government budget constraint in (3.2) is an unstable difference equation in real debt that can be solved in the usual way to yield:

$$\tilde{b}_{t-1} = (\beta^{-1}-\gamma_2)^{-1} \times$$

$$\sum_{i=0}^{\infty} (\beta^{-1}-\gamma_2)^{-i} E_{t-1}[\varphi_1\tilde{\pi}_{t+i} + \varphi_2\tilde{\pi}_{t+i-1} + \varphi_3\theta_{t+i} + \varphi_4\theta_{t+i-1} + \psi_{t+i}]. \quad (3.5)$$

The first two terms in brackets are expectations of future values of endogenous inflation, which can be evaluated using the stable Euler equation (3.1). For example, the first term is:

$$[\varphi_1/(\beta^{-1}-\gamma_2)]\ \sum_{i=0}^{\infty}\ (\beta^{-1}-\gamma_2)^{-i}E_{t-1}\tilde{\pi}_{t+i}\ =\ \varphi_1\{[\alpha_2\beta/(\beta^{-1}-\gamma_2-\alpha_2\beta)]\tilde{\pi}_{t-1}$$

$$+\ [(1-\beta\gamma_2)/((\beta^{-1}-\gamma_2-\alpha_2\beta)(\beta^{-1}-\gamma_2-\rho_2))]\theta_{t-1}\}.$$

The remaining three expectations in (3.5) involve a straightforward evaluation of exogenous processes.

Performing these evaluations and dating the result at time t gives an expression for real debt in terms of current inflation and the policy shocks:

$$\tilde{b}_t\ =\ [(\varphi_1\alpha_2\beta\ +\ \varphi_2)/(\beta^{-1}-\gamma_2-\alpha_2\beta)]\tilde{\pi}_t\ +$$

$$\{[\beta(\varphi_1(\beta^{-1}-\gamma_2)+\varphi_2)/((\beta^{-1}-\gamma_2-\alpha_2\beta)(\beta^{-1}-\gamma_2-\rho_1))]\ +\ [(\varphi_3\rho_1+\varphi_4)/(\beta^{-1}-\gamma_2-\rho_1)]\}\theta_t$$

$$+\ [\rho_2/(\beta^{-1}-\gamma_2-\rho_2)]\psi_t\ \ \ \ \text{for } t\ \geq\ 1. \tag{3.6}$$

The final step involves simultaneously solving (3.6) and the budget constraint (3.2) to obtain equilibrium real debt and inflation functions in terms of current and lagged exogenous variables and lagged endogenous variables. The general solution is complicated and has been relegated to Appendix A.

Under certain assumptions on policy behavior the solutions for inflation and real debt collapse nicely. Suppose that monetary policy pegs the interest rate by setting $\alpha_2 = 0$ and fiscal policy similarly pegs direct lump-sum taxes by setting $\gamma_2 = 0$. After some simplification the expressions for inflation and real debt become:

$$\tilde{\pi}_t\ =\ -[1/(\varphi_1(1-\beta\rho_2))]\psi_t\ +\ (1/\beta\varphi_1)\tilde{b}_{t-1}\ -\ (\varphi_4/\varphi_1)\theta_{t-1}\ \ \ \ \text{for } t\ \geq\ 1, \tag{3.7}$$

and

$$\tilde{b}_t\ =\ [\beta c(1\ -\ \beta\rho_1)/(\beta\ -\ \pi)^2]\theta_t\ +\ [\rho_2/(\beta^{-1}-\rho_2)]\psi_t\ \ \ \ \text{for } t\ \geq\ 1. \tag{3.8}$$

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Since when $\alpha_2 = 0$, $\varphi_1 > 0$, an unanticipated tax reduction financed by a nominal debt expansion raises the current price level. The monetary authority pegs the nominal interest rate (and therefore expected inflation and real balances) by allowing the money stock to expand in proportion to the price level. When the tax cut is serially uncorrelated ($\rho_2 = 0$), the fixity of future direct taxes implies via the intertemporal budget constraint in (2.8) that real debt remains unchanged at its steady state level.[14] The unanticipated cut in taxes is financed entirely by a lump-sum inflation tax that leaves all real variables unchanged.[15]

Monetary policy shocks that unexpectedly raise the pegged nominal rate have no effect on current inflation but unambiguously raise inflation the next period.[16] The intertemporal constraint translates the expected inflation tax into a higher value of real debt today ($-\varphi_3 > 0$). This monetary shock represents a pure asset exchange, with the decrease in the nominal money stock equalling the increase in nominal debt outstanding. With higher $\tilde{R}_t$ lowering real balances, the monetary shock is a distorting inflation tax that induces consumers to substitute out of money and into debt. The current price level remains unchanged, however, because, as can be seen from the government's flow

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[14]Although tax cuts always increase prices, the price increase is larger the more positively serially correlated is the tax shock, with the smallest price rise occurring when $\rho_2 = -1$. This serial correlation parameter also determines whether a tax cut lowers real debt ($0 < \rho_2 < 1$), raises real debt ($-1 < \rho_2 < 0$) or leaves it unchanged ($\rho_2 = 0$).

[15]It is easy to check that this solution satisfies the stable Euler equation (3.1). Substitute (3.8) into (3.7) dated at t+1 and t and take expectations conditional on time t information to yield $E_t\tilde{\pi}_{t+1} = \beta\theta_t = 0$, for this fiscal experiment.

[16]To see this, again substitute (3.8) into (3.7) dated at t+1 and t to get realized inflation as $\tilde{\pi}_{t+1} = -(1/\varphi_1)[(\varphi_3/\beta)+\varphi_4]\theta_t > 0$ for positive θ_t.

budget constraint, aggregate nominal government liabilities, $B_t + M_t$, are constant. This highlights an important feature of Region II equilibria that is also present in Aiyagari and Gertler (1985). When direct taxes are relatively unresponsive to the level of government debt outstanding, the price level depends on aggregate government liabilities, not merely on the level of the nominal money stock. Moreover, the nominal interest rate depends on the composition of these liabilities, rising as the B_t/M_t ratio increases. If the monetary policy shock is serially uncorrelated, real balances and real debt return to their steady state values at time t+1.

Granger-Causality Tests and Region II Equilibria

I turn now to the implications of Region II equilibria for tests of whether deficits Granger-cause inflation. Without loss of generality, assume taxes are pegged exogenously ($\gamma_2 = 0$) and policy shocks are serially uncorrelated ($\rho_1 = \rho_2 = 0$). Borrowing notation from equation (A.1) in Appendix A, write the moving average representation of the equilibrium debt and inflation processes as:[17]

$$z_t = F(L)B\xi_t + F(L)C\xi_{t-1}, \tag{3.9}$$

where $z_t = (\tilde{b}_t, \tilde{\pi}_t)'$, $\xi_t = (\theta_t, \psi_t)'$, and $F(L) = [I - AL]^{-1}$; L is a lag operator defined as $L^j x_t = x_{t-j}$. The A, B, and C matrices are defined in the appendix. The typical element in F(L) is:

$$f_{ij}(L) = \sum_{k=0}^{\infty} (f_{ij}L)^k.$$

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[17]Stability considerations ensure the vector autoregression in (A.1) is invertible. The eigenvalues of A are $\alpha_2\beta$ and 0. The second root, naturally, comes from zeroing out the unstable $\beta^{-1}-\gamma_2$ root.

Tax shocks will fail to Granger-cause inflation if and only if the (2,1) element of $F(L)B = 0$, since the corresponding element in C is always zero. This requires that

$$b_{22} + [b_{12}a_{21} - b_{22}a_{11}]L = 0. \qquad (3.10)$$

Using the expressions for these elements of A and B that are given in the appendix, a sufficient condition for tax shocks to fail to Granger-cause inflation is that $\alpha_2 = 0$. By pegging the nominal interest rate the monetary authority forces all the monetization of deficits to occur immediately, generating an unanticipated inflation tax. Whenever α_2 differs from zero, the monetary authority allows the effects of a deficit shock to show up in the nominal interest rate, creating a change in expected future inflation. The anticipated depreciation of real money balances induces consumers to sub-stitute out of non-interest-bearing currency and into lower-priced nominal debt. By spreading the monetization over time, monetary policy finances the deficit with a distorting inflation tax, producing the Granger-causal ordering from deficits to inflation that has been tested for by Joines (1985) and King and Plosser (1985), among others.

4.   Implications of Foreknowledge of Taxes

The previous section derives results assuming that current tax shocks provide information about future taxes only to the extent the shocks are serially correlated. I now pursue the alternative assumption that consumers today observe next period's tax rates exactly. It is then possible to demonstrate that when monetary policy pegs the nominal interest rate, in equilibrium, inflation will Granger-cause deficits.

18

Resolve the model applying the E_t^* operator in place of E_t, where E_t^* is the mathematical expectation conditional on the expanded information set including all current and past policy shocks and the next period's tax shock. The general solution is presented in the second part of Appendix A.

Since my present purpose is merely to explore the implications of foreknowledge of taxes for Granger-causality tests, assume both the monetary and the fiscal authorities peg their respective instruments by setting $\alpha_2 = \gamma_2 = 0$. Then the equilibrium inflation and real debt solutions are:

$$\tilde{\pi}_t = -[1/(\varphi_1(\beta^{-1} - \rho_2))]\psi_{t+1} - (1/\varphi_1)\psi_t + [1/(\beta\varphi_1)]\tilde{b}_{t-1} - (\varphi_4/\varphi_1)\theta_{t-1} \quad \text{for } t \geq 1, \quad (4.1)$$

and

$$\tilde{b}_t = [(\beta c(1 - \beta\rho_1)/(\beta - \pi)^2]\theta_t + [1/(\beta^{-1} - \rho_2)]\psi_{t+1} \quad \text{for } t \geq 1. \quad (4.2)$$

Both current and future tax shocks unambiguously raise the current inflation rate and future tax shocks increase current real debt, regardless of the serial correlation pattern of the shock. As under the conventional information assumptions, unanticipated increases in the pegged interest rate raise real debt outstanding but leave the price level unchanged. Such monetary shocks continue to bear the interpretation as pure asset exchanges.

Substitute $\tilde{b}_{t-1}$ from (4.2) into (4.1) to obtain the reduced form for inflation:

$$\tilde{\pi}_t = -[1/(\varphi_1(\beta^{-1} - \rho_2))]\psi_{t+1} + [\beta\rho_2/(\varphi_1(1 - \beta\rho_2))]\psi_t + \beta\theta_{t-1} \quad \text{for } t \geq 1. \quad (4.3)$$

From this expression for inflation it is immediately clear that deficit shocks

will lead inflation temporally, generating a pattern of Granger-causality

running from inflation to national income accounts measures of deficits.[18]

5.   Concluding Remarks

Finally I touch on three remaining issues: (i) the relevance of the

results from this model for interpreting observed time series; (ii) the

welfare implications in this model of various deficit financing schemes; and

(iii) the robustness of the results to modifications in the theoretical

specification.

The most striking outcomes involve the use of Granger-causality tests in

seeking evidence that deficits have been monetized. Such tests will fail to

detect monetization that occurs via a nominal interest rate peg. Under the

conventional assumption that consumers observe only current and past policy

shocks, the fixed nominal rate forces all current and expected future deficits

to be monetized immediately. When consumers are given foreknowledge of tax

realizations the causality test must be performed in reverse because inflation

realizations reflect expectations of, and therefore help to predict, future

deficits. If over some sub-sample monetary policy is well approximated by a

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[18]To follow-up on footnote 6, I prefer to model foreknowledge of taxes by
dating the variables in the tax rule as in (2.4) and applying the E_t^* operator
because this approach retains the notion that at time t the econometrician has
data only on the observable aspects of taxes at t, $\{\tau_s, \ s \leq t\}$, and not on the
unobservable tax shocks. In (4.3) it is clear that the surprise in ψ_{t+1}
affects measurable deficits at time t+1 but not at time t. Under the alterna-
tive tax rule of footnote 6, $\tau_t = \gamma_1 + \gamma_2 b_{t-1} + \psi_{t-1}$, equation (4.3) contains
terms in ψ_t and ψ_{t-1}, but not ψ_{t+1}. In this case it is important to recall
that the realization of ψ_t implies the level of (observable) taxes next
period, τ_{t+1}. Thus, the two approaches have equivalent implications for
actual empirical work.

(stochastic) interest rate peg, combining data from this sub-period with data in which deficits actually do Granger-cause inflation may lead to the mistaken statistical conclusion that deficits were not monetized over the full sample.

In this model both direct taxation and unanticipated inflation are lump-sum taxes that have no effect on consumers' welfare. Only if deficits are monetized over time by the monetary authority's willingness to raise interest rates in the face of inflationary pressures are real balances and, therefore, welfare affected by deficit financing. Of course, these extreme implications stem from the model's simplistic modeling of money demand and should probably not be taken too seriously.

In spite of the model's simplicity, the major implications regarding the role of monetary and fiscal interactions in satisfying the government's intertemporal budget constraint are robust to variations in the structure. The crucial aspect of money demand is that it depend negatively on the nominal interest rate. Modifying this specification to include lagged real balances, for example, (as is popular in empirical demand studies) will not change the qualitative results. In addition, allowing the real interest rate to vary also does not change the policy implications.

Appendix A

General Solution under Conventional Informational Assumptions

Express (3.6) and (3.2) as the system:

$$z_t = A z_{t-1} + B \xi_t + C \xi_{t-1} \quad \text{for } t \geq 1, \tag{A.1}$$

where $z_t = (\tilde{b}_t, \tilde{\pi}_t)'$ and $\xi_t = (\theta_t, \psi_t)'$. Then the general solution for $\tilde{b}_t$ and $\tilde{\pi}_t$ is given by the elements of the A, B, and C matrices in terms of the composite φ_i parameters, defined in Section 3, as:

$$a_{11} = \frac{(1-\beta\gamma_2)(\alpha_2\beta\varphi_1 + \varphi_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2}, \qquad a_{12} = \frac{-\beta\varphi_2(\alpha_2\beta\varphi_1 + \varphi_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2}$$

$$a_{21} = \frac{(\beta^{-1}-\gamma_2)(1-\alpha_2\beta^2-\beta\gamma_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2}, \qquad a_{22} = \frac{-\varphi_2(1-\alpha_2\beta^2-\beta\gamma_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2}$$

$$b_{11} = \frac{-\beta\varphi_3(\alpha_2\beta\varphi_1 + \varphi_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2} +$$

$$\frac{\varphi_1[-\beta^2\varphi_1 + \beta^3\gamma_2\varphi_1 - \beta^3\varphi_2 - \beta\varphi_4 + \alpha_2\beta^3\varphi_4 + \beta^2\gamma_2\varphi_4 - \beta\varphi_3\rho_1 + \alpha_2\beta^3\varphi_3\rho_1 + \beta^2\gamma_2\varphi_3\rho_1]}{(\beta\gamma_2 - 1 + \beta\rho_1)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)},$$

$$b_{12} = \frac{\beta[-\alpha_2\beta\varphi_1 + \alpha_2\beta^2\gamma_2\varphi_1 - \varphi_2 + \beta\gamma_2\varphi_2 + \varphi_1\rho_2 - \beta\gamma_2\varphi_1\rho_2 + \beta\varphi_2\rho_2]}{(1 - \beta\gamma_2 - \beta\rho_2)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)}$$

$$b_{21} = \frac{-\beta^2\varphi_1 + \beta^3\gamma_2\varphi_1 - \beta^3\varphi_2 - \varphi_3 + \alpha_2\beta^2\varphi_3 + 2\beta\gamma_2\varphi_3 - \alpha_2\beta^3\gamma_2\varphi_3 - \beta^2\gamma_2^2\varphi_3 - \beta\varphi_4}{(1 - \beta\gamma_2 - \beta\rho_1)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)} +$$

$$\frac{\alpha_2\beta^3\varphi_4 + \beta^2\gamma_2\varphi_4}{(1 - \beta\gamma_2 - \beta\rho_1)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)} \quad ,$$

$$b_{22} = \frac{-(1-\beta\gamma_2)(1 - \alpha_2\beta^2 - \beta\gamma_2)}{(1 - \beta\gamma_2 - \beta\rho_2)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)}$$

$$c_{11} = \frac{-\beta\varphi_4(\alpha_2\beta\varphi_1 + \varphi_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2} \quad , \qquad c_{12} = 0$$

$$c_{21} = \frac{-\varphi_4(1 - \alpha_2\beta^2 + \beta\gamma_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2} \quad , \qquad c_{22} = 0$$

General Solution under Foreknowledge of Tax Shocks

After solving the system applying the expectations operator E_t^*, relative to an information set that includes current and past policy shocks and next period's tax shock, the system can be written as:

$$z_t = A^*z_{t-1} + B^*\xi_t + C^*\xi_{t-1} + D^*\xi_{t+1} \quad \text{for } t \geq 1. \tag{A.2}$$

As expected, all the differences between the matrices in (A.2) and (A.1) come from the influence of tax shocks, so at most the second columns of the matrices in (A.2) differ from those in (A.1). In particular, $A^* = A$, $b_{11}^* = b_{11}$, $b_{21}^* = b_{21}$, and $C^* = C$. The elements that are different between the two solutions are:

$$b_{12}^* = \frac{-\beta(\alpha_2\beta\varphi_1 + \varphi_2)}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2} \quad , \qquad b_{22}^* = \frac{\alpha_2\beta^2 + \beta\gamma_2 - 1}{\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2}$$

$$d_{11}^* = 0, \qquad d_{12}^* = \frac{\varphi_1(1 - \alpha_2\beta^2 - \beta\gamma_2)}{(\beta^{-1} - \gamma_2 - \rho_2)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)}$$

$$d_{21}^* = 0, \qquad d_{22}^* = \frac{\beta(\alpha_2\beta^2 + \beta\gamma_2 - 1)}{(\beta^{-1} - \gamma_2 - \rho_2)(\varphi_1 - \beta\gamma_2\varphi_1 + \beta\varphi_2)}$$

Appendix B

<u>Region IV: Active Monetary and Fiscal Policies</u>

This appendix demonstrates the conditions under which there exists an equilibrium when policy parameters are drawn from Region IV of the parameter space. This is the case of two unstable roots, which arises when the monetary authority reacts strongly to inflation ($|\alpha_2\beta| > 1$) and the fiscal authority adjusts direct taxes weakly to lagged real debt ($|\beta^{-1}-\gamma_2| < 1$).

In this case the equilibrium inflation rate is given by (3.3), as it is in Region I. The equilibrium debt process comes from solving (3.5), where I now use (3.3) to evaluate expected future inflation rates. Doing this gives:

$$\tilde{b}_t = [(\rho_1\varphi_1\beta + \varphi_2\beta + \rho_1\varphi_3(\rho_1-\alpha_2\beta) + \varphi_4(\rho_1-\alpha_2\beta)) / ((\rho_1-\alpha_2\beta)(\beta^{-1}-\gamma_2-\rho_1))]\theta_t$$
$$+ [\rho_2/(\beta^{-1}-\gamma_2-\rho_2)]\psi_t \quad \text{for } t \geq 1.$$

To see if these are consistent, consider the case of serially uncorrelated shocks. Setting $\rho_1 = \rho_2 = 0$ these expressions become:

$$\tilde{\pi}_t = -(1/\alpha_2)\theta_t \quad \text{for } t \geq 1, \text{ and} \tag{B.1}$$

$$\tilde{b}_t = [(\alpha_2\varphi_4 - \varphi_2)/(\alpha_2(\beta^{-1}-\gamma_2))]\theta_t \equiv 0 \quad \text{for } t \geq 1, \tag{B.2}$$

by the definitions of φ_2 and φ_4. In contrast to Regions I and II equilibria, this doubly forward solution eliminates any connection of inflation and real debt to the past, implying for given realizations of the policy shocks, unique sequences of $\{\tilde{\pi}_t, \tilde{b}_t\}$.

Among other conditions, equations (B.1) and (B.2) must satisfy the government's period budget constraint if they constitute an equilibrium.

Substituting them into (3.2) implies the following restriction on the exogenous policy shock processes:

$$\psi_t = [(\varphi_1/\alpha_2) - \varphi_3]\theta_t + [(\varphi_2/\alpha_2) - \varphi_4]\theta_{t-1}, \qquad (B.3)$$

which obviously violates the assumption that monetary and fiscal shocks are mutually uncorrelated. This proves there cannot exist an equilibrium when policy parameters lie in Region IV.

Intuitively, under this combination of assumptions on policy behavior an equilibrium requires an exact dependence between the policy shocks because neither monetary nor fiscal policy is responding to economic conditions in order to ensure intertemporal budget balance. Consequently, to avoid an explosive path of real debt the policy *shocks* must offset each other in just the right way. The unique inflation and debt processes emerge because both monetary and fiscal policy attempt to stabilize prices, leaving the role of budget balancing to the exogenous processes.

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