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ABSTRACT

This paper investigates term premia behavior in U.S., German, and Japanese markets. Onshore returns are evaluated in order to focus on the co-movement of the term premia across a set of potentially heterogeneous markets. The paper extends the work of Campbell and Clarida [1987], who find that the term premia within the Euromarket appear to move together. In keeping with their approach, Hansen and Hodrick's [1983] latent variable model is used. The model constrains expected returns, conditional on an information set, to be proportional to one another. These restrictions are not rejected for the markets examined here, implying that the term premia behave as if in a single market.

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## 1. Introduction

This paper investigates term premia behavior in U.S., German, and Japanese markets.[2] Onshore returns are evaluated in order to focus on the co-movement of the term premia across a set of potentially heterogeneous markets. The paper extends the work of Campbell and Clarida [1987], who find that the term premia within the Euromarket appear to move together. In keeping with their approach, the latent variable model of Hansen and Hodrick [1983] is used.[3] The model constrains expected excess returns, conditional on an information set, to be proportional to one another. These restrictions are not rejected for the markets examined here, implying that the term premia behave as if in a single market.

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1. The author is a staff economist in the International Finance Division. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. This research is based on a dissertation chapter, done at the University of California at Berkeley, and I am grateful to my thesis advisors, Roger Craine, Jeffrey Frankel, and Greg Connor for their comments. I would also like to thank Hali Edison, Neil Ericcson and the participants of the Federal Reserve System's October 1989 Conference on International Trade and Finance for valuable discussion.
2. Turnovsky [1989] provides a detailed discussion of the relationship between macroeconomic policies and the term structure.
3. In contrast to Campbell and Clarida's [1987] results, Campbell [1987] rejects this model for a broad range of domestic excess returns (including the term premium) in the United States. His rejection may reflect the heterogeneity of the assets chosen. A more homogeneous set of assets is chosen in this paper in order to focus on the issue of international capital market integration by eliminating one competing source of rejection of the model.

The next section of the paper discusses models of the term structure of interest rates and currency risk, and introduces the latent variable model. The empirical implications of the model are discussed in Section 3, and it is tested in Section 4. A summary is presented in Section 5. Some econometric issues and alternative test statistic formulations are discussed in the appendix.

## 2. Models of the Term Structure

Both the pure expectations hypothesis of the term structure and the hypothesis of uncovered interest rate parity narrowly constrain an expected risky return to equal a nominally riskfree one: the expectations hypothesis equates the expected return to holding a series of short-term assets with a known long-term return, and uncovered interest parity equates a default-free domestic return with the expected home-currency return to an otherwise identical asset denominated in a foreign currency.[4] Repeated empirical rejections of these hypotheses and their slightly less restrictive variants that allow for a non-zero constant risk premium imply that both the term premium and the exchange rate risk premium are non-zero and frequently time-varying.[5] The latent variable model nests both hypotheses by

4. Shiller and McCulloch [1986] present simple relationships between numerous empirical formulations of the expectations hypothesis, including the roll-over term premium described here. Campbell, Shiller and Shoenholtz [1983] derive a discrete-time linearized version with which several theoretical formulations are consistent.

5. Tests of the expectations theory of the term structure are surveyed in Shiller and McCulloch [1986]. Hodrick [1987] surveys tests of the hypothesis that forward exchange rates are unbiased predictors of future spot rates. Both theories are rejected in most of the tests surveyed.

treating them as restrictions on specific asset returns in a general model of asset prices.

A number of representative-agent capital asset pricing models give rise to Euler equations restricting the relationships between asset prices and the marginal rate of substitution of nominal consumption between periods.[6] These relationships have been used to characterize returns to assets of differing maturities to give a generalized version of the expectations theory of the term structure.[7] Relating real and nominal variables, this version and the corresponding theory for exchange risk premia have been difficult to evaluate empirically. The latent variable model provides an alternative specification using the construct of an unobservable return to derive testable restrictions on the relationships among only the observable asset returns, themselves.

The solution to the representative consumer's intertemporal optimization problem requires the expected marginal utility from the purchase of an asset to equal the marginal utility of the consumption foregone for its purchase. The Euler equation giving the relationship between the marginal rate of intertemporal substitution and an asset return embodies this requirement in the following expected product:

$$E\,(\,Q_{t,t+n} \cdot {}_iR_{t,t+n}\,|\Omega_t) = 1.$$

6. In these models, the representative agent maximizes some form of:

$$E\{\,\sum_{i=0}^{\infty} \delta^t u(c_{t+i})\ \mid \Omega_t\},$$

subject to an intertemporal budget constraint, where $\delta$ is a discount factor, $0 < \delta < 1$, and u is a concave utility function of period t consumption, $c_t$. Lucas [1978] provides a general equilibrium version of the representative-agent asset pricing model, and Hansen and Hodrick [1983] explain its relation to the latent variable model in detail.

7. The version discussed here is a discrete-time version of Cox, Ingersol, and Ross [1985].

Here, $Q_{t,t+n}$ is the intertemporal marginal rate of substitution of money; ${}_iR_{t,t+n}$ is the "$i^{th}$" asset's gross nominal return from period t to period t+n; and, $\Omega_t$ is the agent's information at period t.[8]

Hansen and Hodrick [1983] point out that for a pair of expected returns, the difference between the products equals zero. In terms of a nominally risk-free return, $R_{t,t+n}$, and any other asset i, subtracting one Euler equation from the other gives:

$$E[Q_{t,t+n} \cdot ( {}_iR_{t,t+n} - R_{t,t+n}) | \Omega_t] = 0.$$

Denoting the excess return by ${}_ir_{t,t+n} \equiv {}_iR_{t,t+n} - R_{t,t+n}$, the above expression may be rewritten to show that the excess return is proportional to the return's covariance with the intertemporal marginal rate of substitution of money:[9]

$$E({}_ir_{t,t+n} | \Omega_t) = -\text{cov}_t(Q_{t,t+n}, {}_ir_{t,t+n}) \cdot R_{t,t+n}, \tag{1}$$

where the factor of proportionality is the risk-free rate, $R_{t,t+n}$.

Equation 1 characterizes the term premium and the exchange risk premium since both are examples of an expected difference between a risky and a nominally riskless return. The term premium, denoted $\Phi_{t,n,1}$, is the expected difference between the return on the risky strategy of rolling over n consecutive 1-period assets and the

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8. $Q_{t,t+n}$ weights the marginal rate of substitution of consumption by the relative purchasing power of money. More specifically,

$$Q_{t,t+n} \equiv \delta^n \cdot [u'(c_{t+n})/u'(c_t)] \cdot 1/(p_{t+n}/p_t)$$

where: $u'(c)$ is the marginal utility of consumption, and $p_t$ is the $t^{th}$ period price of a unit of consumption .

9. Since the risk-free rate, $R_{t,t+n}$, is known, the original Euler equation defines it implicitly, $R_{t,t+n} \equiv 1/E(Q_{t,t+n} | \Omega_t)$. This definition and the definition of a covariance give Expression 1.

nominally riskless, n-period domestic rate, $R_{t,t+n}$:[10]

$$\Phi_{t,n,1} = E\left( \prod_{i=1}^{n} R_{t+i-1,t+i} - R_{t,t+n} \mid \Omega_t \right).$$

Likewise, the exchange risk premium, denoted ${}_j\Gamma_{t,t+n}$, is the expected difference between the return to a domestic asset and the net return to exchanging domestic currency for foreign currency at the spot rate, purchasing a foreign asset with return ${}_jR_{t,t+n}$, then re-exchanging the gross amount into domestic currency at the period t+n spot rate, $S_{t+n}$:

$$\Gamma_{t,t+n} = E\left[ ({}_jR_{t,t+n} \cdot S_t/S_{t+n}) - R_{t,t+n} \mid \Omega_t \right].$$

Substituting these definitions into Equation 1 gives general expressions for the term and the exchange risk premia:

$$\Phi_{t,n,1} = -\mathrm{cov}_t(Q_{t,t+n}\ ,\ \prod_{i=1}^{n} r_{t+i-1,t+i}) \cdot R_{t,t+n}. \tag{2}$$

$${}_j\Gamma_{t,t+n} = -\mathrm{cov}_t(Q_{t,t+n}\ ,\ {}_jr_{t,t+n} S_t/S_{t+n}) \cdot R_{t,t+n}. \tag{3}$$

The pure expectations hypothesis and uncovered interest parity restrict the covariances in Equation 2 and 3 to equal zero. More common empirical variants of the hypotheses restrict the covariances to be only time-invariant, rather than zero. Most empirical tests of both versions of the term premium hypothesis reject it, finding that the term *spread*, $R_{t,t+n} - R_{t,t+1}$, known at period t, has predictive value for the term *premium*. Similarly, uncovered interest parity is often rejected empirically.

10. The use of a nominally riskless rate is more restrictive than is necessary. More generally, $R_{t,t+n}$ may have a zero-$\beta$ return, where $\beta$ is the ratio of the return's covariance with $Q_{t,t+n}$ to its variance.

Without these rejected restrictions, direct empirical tests of Equations 1 through 3 require evaluation of the nominal marginal rate of substitution of consumption, $Q_{t,t+n}$.[11] The latent variable model reformulates the Euler equations in terms of asset returns only, so that measures of consumption are not required. The model uses the construct of a benchmark asset (or portfolio) with an unobservable return, ${}_qR_{t,t+n}$, perfectly correlated with $Q_{t,t+n}$.[12] Perfect correlation implicitly defines this asset's return.[13] This allows the expected excess return of Equation 1 to be expressed as in terms of asset returns.[14] Specifically, the excess return is proportional to the expected excess return perfectly correlated with $Q_{t,t+n}$. Letting $L_{t,t+n} \equiv {}_qR_{t,t+n} - R_{t,t+n}$, gives[15]

$$E({}_ir_{t,t+n}|\Omega_t) = \beta_{t,i,q} \cdot E(L_{t,t+n}|\Omega_t) \tag{4}$$

where:

$$\beta_{t,i,q} \equiv \mathrm{cov}_t({}_iR_{t,t+n}, {}_qR_{t,t+n})/\mathrm{var}_t({}_qR_{t,t+n}), \text{ and}$$

Similarly, the term premium and the exchange risk premium are proportional to the latent expected excess return, $E(L_{t,t+n}|\Omega_t)$:

11. Hansen and Singleton [1983] and others have used consumption data to determine $Q_{t,t+n}$. However, like most measures of real economic activity, observations of consumption are typically available infrequently and measured imprecisely relative to observations of asset prices.
12. Campbell [1987] uses the more general framework of k hedge portfolios rather than a single benchmark asset.
13. Equation 1 and the requirement that $\mathrm{Corr}(Q_{t,t+n}, {}_qR_{t,t+n}) = 1$ together define ${}_qR_{t,t+n} = Q_{t,t+n}/E(Q^2_{t,t+n}|\Omega_t)$.

14. With the riskfree rate, $R_{t,t+n} = 1/E(Q_{t,t+n}|\Omega_t)$, excess returns may be expressed: $E({}_ir_{t,t+n}|\Omega_t) = -\mathrm{cov}_t({}_ir_{t,t+n}, {}_qR_{t,t+n}) / E({}_qR_{t,t+n}|\Omega_t)$.

15. Note ${}_qR_{t,t+n} = {}_iR_{t,t+n} \Rightarrow \mathrm{var}_t({}_qR_{t,t+n}) = -({}_qR_{t,t+n} - R_{t,t+n})E({}_qR_{t,t+n}|\Omega_t)$.

$$\Phi_{t,n,1} = \beta_{\Phi(t,n,1),q} E(L_{t,t+n}|\Omega_t), \tag{5}$$

$$_j\Gamma_{t,t+n} = \beta_{\Gamma(j),q} E(L_{t,t+n}|\Omega_t) \tag{6}$$

where:

$$\beta_{\Phi(t,n,1),q} = \mathrm{cov}_t( \prod_{i=1}^{n} R_{t+i-1,t+i},\ {}_qR_{t,t+n} ) \ /\ \mathrm{var}_t({}_qR_{t,t+n}), \text{ and}$$

$$\beta_{\Gamma(j),q} = \mathrm{cov}_t( {}_jr_{t,t+n}S_t/S_{t+n},\ {}_qR_{t,t+n} ) \ /\ \mathrm{var}_t({}_qR_{t,t+n})$$

Equations 4, 5, and 6 parallel the structure of Equations 1,2, and 3, where the expected excess return and the term premium are proportional to covariances with $Q_{t,t+n}$. However, in the first equations, the factors of proportionality, $R_{t,t+n}$, are equal; while, the factors of proportionality in the subsequent equations depend on the covariances of the risky returns with the unobservable return, $_qR$.

3. Observable Implications of the Latent Variable Model

As discussed above, the latent variable model implies that excess returns move in proportion to an unobservable return. Hansen and Hodrick [1983] give empirical content to this by characterizing the restrictions in terms of observable variables, $x_t$, available to the investor at period t, $x_t \in \Omega_t$. If expectations are formed rationally, Equation 4 becomes:

$$_ir_{t,t+n} = \beta_{t,i,q} E(L_{t,t+n}|\ \Omega_t) + \upsilon_t$$

where $E(\upsilon_t|\Omega_t) = 0$.

The latent variable may itself be linearly projected onto the information variables, $x_t$; so, $L_{t,t+n} = \alpha' x_t + \epsilon_t$. Substituting this projection into the above expression gives the $i^{th}$ excess return as a function of observable variables:[16]

$$_i r_{t,t+n} = \beta_{t,i,q} \alpha' x_t + e_t$$

where, $e_t = \beta_{i,q} \epsilon_t + \upsilon_t$.

This may be directly estimated in the case where $\beta_{t,i,q}$ is a constant, by defining the reduced form parameters:

$$\theta_i \equiv \beta_{i,q} \alpha.$$

Then:

$$_i r_{t,t+n} = \theta_i' x_t + e_t. \qquad (7)$$

The definition of $\theta_i$ and Equation 7 together describe the $i^{th}$ excess return as a linear combination of the information variables, where the weights, $\theta$, have particular restrictions. The coefficients, $\theta_i$, for each excess return must be proportional to the coefficients for any other excess return. That is, for any two assets, i and j, the coefficients are a scalar multiple of each other: $\theta_i = k\theta_j$, where $k = \beta_i/\beta_j$. In all, for j asset returns and h information variables, this provides (j-1)(h-1) over-identifying restrictions on the estimation of Equation 7 for j assets.

While Equation 7 is written in terms of an expected excess return, the restrictions of the model also apply to differences between excess returns. For the foreign term premium, the first risky return is the n-period foreign return evaluated in the domestic currency, and the second risky return is the return from rolling over consecutive

16. Collect terms from: $_i r_{t,t+n} = \beta_{t,i,q}(\alpha' x_t + \epsilon_t) + \upsilon_t$.

1-period foreign assets, also evaluated in domestic currency at the final period:

$$({}_jS_t / {}_jS_{t+n})({}_jR_{t,t+n} - \prod_{i=1}^{n} {}_jR_{t+i-1,t+i}) = (\beta_{i,q} - \beta_{j,q})\alpha' x_t + {}_je_t.$$

Or, more simply,

$${}_j\Phi_{t,1,n} = {}_j\beta\, \alpha' x_t + {}_{i,j}e_t, \qquad \text{where: } {}_j\beta = {}_j\beta_{1,q} - {}_j\beta_{n,q}.$$

In its reduced form,

$${}_j\Phi_{t,1,n} = {}_j\theta' x_t + {}_je_t \tag{8}$$

Unrestricted, this equation simply predicts the term premia in all the countries on the basis of a common set of information variables, $x_t$. The power of the test to reject the restrictions imposed by the latent variable model (or by any of the versions of the expectations hypothesis) depends on the predictive value of these information variables in explaining the term premia. The choice of information variables used here is discussed in the following section.

## 4. Testing the Latent Variable Model

This section discusses the estimation of the term premia as well as tests of the latent variable model and the expectations hypothesis. The estimated coefficients and test statistics are given in Tables 1 through 4.

The roll-over premium is estimated for each country using 1-month and 3-month assets. With each time period being one month long, and n=3, the domestic term premium is defined as follows:

$$\Phi_{t,3,1} \equiv E( R_{t,t+3} - \prod_{i=1}^{3} R_{t+i-1,t+i} | \Omega_t).$$

While, the foreign term premium is[17]:

$$ {}_j\Phi_{t,3,1} \equiv E[({}_jR_{t,t+3} - \prod_{i=1}^{3} {}_jR_{t+i-1,t+i})\, {}_jS_t / {}_jS_{t+3} \mid \Omega_t]. $$

This definition is used to estimate Equation 8. Weekly observations of annual yields on 30-day and 90-day certificates of deposit are used to construct the series of returns for the United States and Japan. For Germany, interbank rates are used. The sample period extends from October, 1986 to July, 1988.

The information variables, $x_t$ in Equation 8, are all available to investors at period t. They include: the annualized U.S. term spread, $R_{t,t+3} - R_{t,t+1}$; the annualized spread between the yield on U.S. 90-day C.D.s and the yield on the German 3-month interbank rate, ${}_{us}R_{t,t+3} - {}_{wg}R_{t,t+3}$; and, the spread between the annualized yields on the U.S. C.D. and the Japanese 3-month C.D., ${}_{us}R_{t,t+3} - {}_{ja}R_{t,t+3}$.

These variables are the domestic analogues of those used by Campbell and Clarida [1987], where they show that the differences between the 3-month nominal returns across currencies, along with the term spread, help to predict eurocurrency term premia. The use of the onshore term spread is also consistent with the many domestic studies rejecting a constant term premium by showing that the term spread predicts the term premium.

The fact that the observations are drawn weekly results in the overlapping observations structure modeled by Hansen and Hodrick [1980] and in the corresponding moving average error process.

17. Recall ${}_jS_{t+j}$ is the spot rate at period t+j (not the forward rate).

Weekly observations of the domestic and foreign realized returns give rise to error processes that are MA(8) and MA(13), respectively.[18] Hansen and Hodrick's [1980] procedure for consistent estimation of the variance-covariance matrix addresses this problem, and a modification allows for conditional heteroskedasticity as well.[19] The tests of both the expectations hypothesis and the latent variable model use this heteroskedastic consistent version, modified as in Newey and West [1985] to ensure positive definiteness.[20]

Table 1 gives the unrestricted estimates of the parameters of Equation 6 and the heteroscedastic-serial correlation consistent estimates of their standard errors. Unrestricted parameter estimates are given, rather than the constrained estimates, because of some undesirable small sample properties inherent in the usual constrained estimation procedures.[21] As shown on the table, the coefficient estimates are not large, but their corrected standard errors (all much larger than the corresponding uncorrected standard errors) are small, and a number of the coefficients are individually statistically significant. In each of the three countries, the coefficients are

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18. In the case of the domestic asset, agents' expectational errors remain unresolved for 2 months, at which time $R_{t+2,t+3}$ becomes known. In the case of the foreign assets, the uncertainty remains until the asset matures in 3 months and the exchange rate uncertainty is resolved.
19. Cumby and Obstfeld [1984], Hodrick and Srivastava [1984], and Giovannini and Jorion [1987] all find strong evidence of conditional heteroskedasticity.
20. The heteroskedastic version of Hansen and Hodrick's [1980] variance-covariance estimator used here is $(X'X)^{-1}X'WX(X'X)^{-1}$, where X is a (Txk) matrix of information variables. The (i,j)th element of W equals $\hat{e}_i\hat{e}_j g_{i,j}$, when (|i-j|+1) is less than the order of the moving average lag, k, and equals zero otherwise; $\hat{e}_i$ is a consistent estimate of $e_i$. The weight $g_{i,j}$ is chosen to equal [ 1 - |i-j|/(k+1) ] by Newey and West [1985].
21. The GMM estimators are not unique in this case, and using Maximum Likelihood, convergence is difficult.

jointly statistically significant (rendering the expectations hypothesis rejected).

Table 2 gives the test statistics for evaluating the latent variable model and for the various versions of the expectations hypothesis. As above, the $\chi^2$ statistic presented is constructed using a Wald test, rather than using Generalized Method of Moments (GMM) to constrain the estimates and calculate the test statistic. Because the correct formulation of the Wald statistic is ambiguous, a number of different formulations are presented in the appendix, as is the GMM test statistic. As the upper panel of the table shows, the hypothesis that the coefficients in the term premium equations are proportional to one another cannot be rejected. In the context of the latent variable model, this provides evidence of a high degree of co-movement of the term premia across the onshore markets of the United States, Germany, and Japan.

This result extends that of Campbell and Clarida [1987], where they find that the latent variable model cannot be rejected for returns to such assets within the Euromarket, while it contrasts with Campbell's [1987] rejection of the latent variable model for the term premium in the United States. Campbell's [1987] rejection may, in part, be due to the wide range of assets included in his study. An attempt has been made here to choose assets that differ in only three ways: maturity, currency of denomination, and political jurisdiction of issue. By examining the behavior of the term premia across countries, one is asking whether a particular relative price, that of capital for one period in terms of capital for another period, behaves similarly across these markets. In the sense of the constant-$\beta$ latent variable model, the relative prices examined here do appear to move together.

The joint hypotheses of the expectations theory and uncovered interest parity are tested using the same parameter estimates given in Table 1, and the resulting test statistics are given on Table 2. As nested within the model, these hypotheses together imply that for each country, $\theta_i = 0$ for all i. These constraints are strongly rejected for the three countries together and are rejected for each country, including the United States, where the test of the pure expectations hypothesis cannot be rejected in a simpler test, given in Table 3, that does not include these information variables. The somewhat less restrictive hypothesis of constant exchange risk and term premia implies that, for each country, $\theta_i = 0$ for i not equal to 1. This hypothesis is rejected jointly and is also strongly rejected for assets in the United States. But, it cannot be rejected individually for yields on assets in Japan at standard confidence levels or in Germany, at the 95 percent confidence level; though, it can be rejected in Germany at the 90 percent level.

Table 3 presents the results of the estimation of the term premia restricting the coefficients on the information variables, except the constant, to equal zero: $\theta_i = 0$, $i \neq 1$. The top panel gives the term premia estimates for the three countries, their corrected standard errors, and the probability values associated with the tests that each term premia estimate is not zero. The point estimates of the term premia are all positive, though small: between 10 and 25 basis points, at an annual rate. Their corrected standard errors are also fairly small, though they are much larger than the uncorrected ones. As a result, among individual countries, the pure expectations/rational expectations hypothesis is rejected at the 95 percent confidence level for both German and Japanese assets. At the same time, the expectations hypothesis cannot be rejected for U.S. assets without the information variables at the 95 percent confidence level, but it can be rejected at the 90 percent level.

The test statistic for the hypothesis that rational expectations and the pure expectations hypothesis hold in all three countries is shown in the middle panel of the table. In this case, the combined hypothesis for all three countries can also only be rejected at the 90 percent level.[22] The final panel of the table gives the hypothesis that the three term premia are identical. It, too, cannot be rejected without information variables.

## 7. Summary

This paper uses the framework of the latent variable model to investigate the extent of term-structure co-movement across onshore markets in the United States, Germany, and Japan. The term premia of these markets are evaluated in terms of a single domestic currency so that the their co-movement may be viewed as reflecting the extent to which the premia, from the domestic investor's point of view, behave as if assets are trading within a single market. The restrictions of a constant-$\beta$ latent variable model are not rejected for the onshore

22. One alternative to invoking rational expectations is to use survey data to separate the forward rate into an expected future rate and a term premium. Froot [1987] does this and finds, for short-term maturities in the United States (such as those examined here), that expectations of future rates appear to be formed rationally and that the term premium is time-varying. Froot and Frankel [1987] find the opposite for exchange rates.

markets in these countries, and the term premia appear to move together.

Nested within this framework is a test of uncovered interest parity and the pure expectations hypothesis of the term structure of interest rates. More restrictive than the latent variable model, this joint hypothesis is rejected for the three countries as a whole, and individually for the United States and Japan at the 95 percent significance level. For German assets, it is rejected at the 90 percent confidence level.

## Appendix: Formulation of the Wald Statistic

This Appendix discusses the construction of the Wald statistic. Gregory and Veall [1985] show that the statistic is not unique in any finite sample when non-linear transformations of the null hypothesis are allowed, despite their asymptotic equivalence.[23] For tests of the latent variable model, the conclusions of Section 4 are sensitive to the particular chosen formulation of the hypothesis. Some guidance in choosing a formulation is taken from the scalar results of both simulations by Gregory and Veall [1985] and the analytical work of Phillips and Park [1988]. In addition, Monte Carlo simulations are used. Finally, the model is tested using GMM.

Gregory and Veall's [1985] classic example is the hypothesis that one parameter, a, is the inverse of another, b. That is $H_0$: a = 1/b. This may be written either as $H_0$: a - 1/b = 0, or as $H_0$: ab - 1 = 0. The two formulations yield distinct test statistics. Gregory and Veall [1985] provide Monte Carlo evidence showing that the differences can be numerically substantial and depend on the parameter values. This point is amplified by the analytical work of Phillips and Park [1988] and by Lafontaine and White [1986].

The latent variable model similarly restricts parameters to be inversely related. From Equation 6, the hypothesis may be written as either:

$$ {}_j\Phi_{t,1,3}/{}_j\beta \quad - \quad \alpha' x_t = 0, \tag{A2} $$

or,

$$ {}_j\Phi_{t,1,3} \quad - \quad {}_j\beta\, \alpha' x_t = 0. \tag{A1} $$

23. The problem stems from the fact that the asymptotic properties of the Wald statistic rely on a Taylor approximation of the null hypothesis. The approximation can be a bad one when the hypothesis or its reparameterization is extremely non-linear.

While two formulations are algebraically equivalent, they are nonlinear reparameterizations of each other and yield numerically different Wald statistics. Table 4 gives the $\chi^2$ statistics calculated from A1. Three $\chi^2$ statistics are given, since the choice of the country for normalization is also arbitrary. In contrast to the results presented in Section 4, the single-beta latent variable model is rejected at any standard significance level in all three cases. The Wald statistics constructed using the A2, the multiplicative formulation, are given in Table 5. The latent variable model cannot be rejected in any of the three multiplicative formulations.

The intuition of Gregory and Veall [1985] provides some insight into why these results might differ so sharply and guidance regarding the choice of parameterization. They point out that approximations of inverse functions that rely on derivatives might be very bad when the value of the function is close to zero, suggesting the multiplicative alternative in that case.[24] Many of the coefficient estimates in Equation 6 are small. Phillips and Park [1988] analytically compare alternative parameterizations similar to those of Gregory and Veall [1985] by evaluating the higher order terms of the Taylor approximations. While their work has not been extended to multivariate cases, it reinforces the conclusions of Gregory and Veall.

A Monte Carlo experiment provides additional support for choosing the multiplicative formulation of the hypothesis. Using the unrestricted parameter estimates, given in Table 2, for the coefficients in the U.S. term premium equation and for the constants

24. Their Monte Carlo work, as well as the work of Phillips and Park, substantiates this.

in the term premium equations for Germany and Japan, the Wald statistics were calculated with the hypothesis as true using both the multiplicative and the proportionate formulations.

Table 6 gives the results of this experiment. At the 95 percent confidence level, the Wald statistic constructed from A2 rejects the latent variable model (true by construction) 29 times in 1000 draws using normal random variables. In contrast, A1 resulted in 845 rejections of the true model. At the 50 percent confidence level, there were 627 rejections using the A2, and 957 rejections using A1. These results are entirely consistent with the interpretation that the highly non-linear reparameterization changes the results dramatically, and that approximations will be particularly poor when values close to zero appear in the denominator the hypothesis formulation.

Finally, Table 7 provides the GMM estimation.[25] The results of this estimation are given in Table 1. The test statistics given at the bottom of the table show that, in both cases, the latent variable model of the term premia cannot be rejected at any standard levels of significance.[26]

25. The J·k sample moment conditions are:

$$\sum_{t=1}^{T} (\Phi_t - \alpha\beta' x_t) \otimes x_t = (0\ 0\ \ldots 0).$$

Starting values are taken from the unrestricted estimates of the term structure equation given Table 1.

26. This confirms the results of Section 4.

Table 1

Latent Variable Model of the Term Structure
Unrestricted Coefficient Estimates

$$ {}_j\Phi_{t+1,t+3} = \theta_1 + \theta_2 ts_t + \theta_3 dms_t + \theta_4 jas_t + e_t $$

| | j = | | |
|---|---|---|---|
| | United States | Germany | Japan |
| $\hat{\theta}_1$ | -.5697* | .1462 | .1818* |
| s.e.($\hat{\theta}_1$) | (.1673) | (.2365) | (.0911) |
| pval($\theta_1 \neq 0$) | .9993 | .4637 | .9540 |
| $\hat{\theta}_2$ | .3076 | .6392* | .3045 |
| s.e.($\hat{\theta}_2$) | (.4163) | (.4158) | (.2239) |
| pval($\theta_2 \neq 0$) | .5400 | .8757 | .8262 |
| $\hat{\theta}_3$ | -.0317 | -.2990 | .0022 |
| s.e.($\hat{\theta}_3$) | (.1334) | (.1335) | (.0416) |
| pval($\theta_3 \neq 0$) | .1879 | .9749 | .0423 |
| $\hat{\theta}_4$ | .2941* | .2989 | -.0552 |
| s.e.($\hat{\theta}_4$) | (.1290) | (.1871) | (.0612) |
| pval($\theta_4 \neq 0$) | .9774 | .8898 | .6329 |

Notes:

1. $ {}_j\Phi_{t+1,t+3} = (S_t/S_{t+3})( {}_jR_{t,t+3} - \prod_{i=0}^{2} {}_jR_{t+i,t+i+1} )$

$$ts_t = R_{t,t+3} - R_{t,t+1}$$

$$dms_t = {}_{us}R_{t,t+3} - {}_{wg}R_{t,t+3}$$

$$jas_t = {}_{us}R_{t,t+3} - {}_{ja}R_{t,t+3}.$$

2. This table presents estimates based on weekly observations from October 22, 1986 to July 27, 1988.
3. Heteroskedastic and moving-average consistent standard error estimates and $\chi^2$ statistics are given here.
4. An asterisk indicates a coefficient statistically different from zero.

Table 2

Tests of the Latent Variable Model
and
the Expectations Hypothesis of the Term Structure

$${}_j\Phi_{t+1,t+3} = \theta_1 + \theta_2 ts_t + \theta_3 dms_t + \theta_4 jas_t + e_t$$

The Latent Variable Model

$H_0$: ${}_j\theta_i = ({}_{us}\theta_1/{}_j\theta_1)\,{}_j\theta_i$, i = 2 to 4, j = Germany, Japan

| | |
|---|---|
| $\chi^2(6)$ | 9.40 |
| prob | .848 |

The Expectations Hypothesis

$H_0$: ${}_j\theta_i = 0$, $\forall i$

| | United States | Germany | Japan |
|---|---|---|---|
| $\chi^2(4)$ | 18.35 | 9.63 | 59.04 |
| prob | .9989 | .9529 | .9999 |

| | Combined |
|---|---|
| $\chi^2(12)$ | 87.01 |
| prob | .9999 |

$H_0$: ${}_j\theta_i = 0$, $\forall i \neq 1$

| | United States | Germany | Japan |
|---|---|---|---|
| $\chi^2(3)$ | 16.83 | 6.64 | 2.60 |
| prob | .9992 | .9155 | .5431 |

| | Combined |
|---|---|
| $\chi^2(12)$ | 26.07 |
| prob | . |

Notes: See Table 1.

Table 3

Tests of the Expectations Hypothesis when $\theta_i = 0,\ \forall i \neq 1$

$$ {}_jS_t / {}_jS_{t+3} \ ( {}_jR_{t,t+3} - \prod_{i=0}^{2} {}_jR_{t+i,t+i+1}^{1/3} ) = {}_j\Phi + u_t $$

Coefficient Estimates

j =

| | United States | Germany | Japan |
|---|---|---|---|
| ${}_j\hat{\Phi}$ | 0.1380 | 0.2147 | 0.1006 |
| s.e.$({}_j\hat{\Phi})$ | (0.1024) | (0.1036) | (0.0343) |
| pval$({}_j\hat{\Phi} \neq 0)$ | .9111 | .9809 | .9983 |

Pure Expectations Hypothesis

| | $H_0$ : ${}_{US}\Phi = {}_{WG}\Phi = {}_{JA}\Phi = 0$ |
|---|---|
| $\chi^2(3)$ | 14.721 |
| prob | .936 |

Single Term Premium Hypothesis

| | $H_0$ : ${}_{US}\Phi = {}_{WG}\Phi = {}_{JA}\Phi$ |
|---|---|
| $\chi^2(2)$ | 1.1533 |
| prob | .562 |

Notes:

1. This table presents estimates based on weekly observations from October 10, 1987 to May 18, 1988.
2. Heteroskedastic and moving-average consistent standard error estimates and $\chi^2$ statistics are given in the table. The uncorrected standard errors of ${}_{US}\Phi$, ${}_{WG}\Phi$, and ${}_{JA}\Phi$ are:

   0.0479, 0.04462 and 0.0242, respectively.

Table 4

Reparameterizations of Tests
of the
Latent Variable Model

$$ {}_{j}\Phi_{t+1,t+3} = \theta_1 + \theta_2 ts_t + \theta_3 dms_t + \theta_4 jas_t + e_t $$

Wald Statistics
using the
Proportionate Hypothesis Formulation

$$ H_0: {}_{k}\theta_i - {}_{k}\theta_1 \cdot ( {}_{j}\theta_i / {}_{j}\theta_1 ) = 0 $$

$i = 2$ to 4

j = U.S.; k = Germany, Japan:

| | |
|---|---|
| $\chi^2(6)$ | 30.77 |
| prob | .9999 |

j = Germany; k = U.S., Japan:

| | |
|---|---|
| $\chi^2(6)$ | 22.94 |
| prob | .9992 |

j = Germany; k = U.S., Japan:

| | |
|---|---|
| $\chi^2(6)$ | 39.39 |
| prob | .9999 |

Notes: See Table 3.

Table 5

Reparameterizations of Tests
of the
Latent Variable Model

$$ {}_{j}\Phi_{t+1,t+3} = \theta_1 + \theta_2 ts_t + \theta_3 dms_t + \theta_4 jas_t + e_t $$

Wald Statistics
using the
Multiplicative Hypothesis Formulation

$$ H_0: ({}_{j}\theta_i \cdot {}_{k}\theta_1) - ({}_{j}\theta_1 \cdot {}_{k}\theta_i) = 0 $$

$i = 2$ to $4$

j = U.S.; k = Germany, Japan:

| | |
|---|---|
| $\chi^2(6)$ | 9.40 |
| prob | .848 |

j = Germany; k = U.S., Japan:

| | |
|---|---|
| $\chi^2(6)$ | 5.65 |
| prob | .536 |

j = Germany; k = U.S., Japan:

| | |
|---|---|
| $\chi^2(6)$ | 5.66 |
| prob | .538 |

Notes: See Table 3.

Table 6

Reparameterizations of Wald Tests
of the Latent Variable Model
of the Term Structure of Interest Rates

Monte Carlo Simulations

$$ {}_j\Phi_{t+1,t+3} = \theta_1 + \theta_2 ts_t + \theta_3 dms_t + \theta_4 jas_t + e_t $$

Multiplicative Formulation of Hypothesis:

$$ H_0: ({}_j\theta_i \cdot {}_k\theta_1) - ({}_j\theta_1 \cdot {}_k\theta_i) = 0, \quad i = 2 \text{ to } 4, \quad j = \text{U.S., Germany, Japan} $$

Proportionate Formulation of Hypothesis:

$$ H_0: {}_j\theta_i - {}_j\theta_1 \cdot ({}_k\theta_i / {}_k\theta_1) = 0, \quad i = 2 \text{ to } 4, \quad j = \text{U.S., Germany, Japan} $$

| Confidence Level | Number of Rejections from 1000 Trials: Multiplicative Formulation | Number of Rejections from 1000 Trials: Proportionate Formulation |
|---|---|---|
| 95 percent | 29 | 845 |
| 90 percent | 72 | 881 |
| 75 percent | 263 | 926 |
| 50 percent | 627 | 957 |
| 25 percent | 907 | 982 |

Notes:

1. 93 draws in each sample.
2. Simulations are made using the unconstrained parameter estimates for the U.S. term premium equation and the estimated constants for the other two equations.

Table 7

GMM Estimation of the Latent Variable Model
Alternate Specifications

$$E_t(\Phi_t - \alpha\beta' x_t) \otimes x_t = (0\ 0\ \ldots\ 0).$$

| | $H_{0,1}$: $\alpha=(1, \alpha_2, \alpha_3)$ $\beta=(\beta_1, \beta_2, \beta_3, \beta_4)$ | $H_{0,2}$: $\alpha=(\alpha_1, \alpha_2, \alpha_3)$ $\beta=(1, \beta_2, \beta_3, \beta_4)$ |
|---|---|---|
| **Parameter Estimates** | | |
| **U.S. Coefficients** | | |
| Constant | -0.059 | -0.015 |
| Term Spread | 0.358 | 0.086 |
| DM Spread | -0.058 | -0.016 |
| Yen Spread | 0.104 | 0.125 |
| **Proportionality Factors** | | |
| Germany | 1.0027 | 1.066 |
| Japan | 1.169 | 1.017 |
| **Test Statistics** | | |
| $\chi^2(6)$ | 6.162 | 5.830 |
| (probability) | .595 | .557 |

Notes:

1. This estimation used modified versions of a very useful GAUSS GMM proceedure written by Lars P. Hansen, John C. Heaton, and Masao Ogaki, under NSF Grant No. SES-8512371.

2. See Table 1.

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