

Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 400

June 1991

DETERMINING FOREIGN EXCHANGE RISK AND BANK CAPITAL REQUIREMENTS

Michael P. Leahy

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## ABSTRACT

This paper examines three alternative measures of exchange rate risk that could be used to develop a risk-based capital requirement for banks with foreign-exchange exposure. One measure, the standard deviation of the portfolio, is constructed under the assumption that exchange rate changes are distributed normally. While this measure is widely used in a variety of financial applications, it is subject to the criticism that it fails to capture well the behavior of exchange rate changes in the tails of their density function. A second possible measure is developed that combines the standard deviation and a method used by the Bank of England to assess foreign exchange exposure. This measure fails to represent the tail behavior and correlation patterns of exchange rates. The third measure uses nonparametric methods to determine capital requirements. The third measure does not suffer from the deficiencies of the other two: it allows for a rich pattern of exchange rate correlations and for non-normal characteristics in the tails of the density function.

Because of the generality of the nonparametric method, it is used to quantitatively assess the deficiencies of the other two measures. In a sample of simulated portfolios of marks, yen, and sterling, it is shown that the standard deviation measure is likely to yield capital requirements that are too small relative to the nonparametric measure. The second measure behaves on average like the standard deviation measure but the capital requirement is more erratic: it generates too much capital for some portfolios and too little capital for others in larger proportions than the standard deviation measure.

## Determining Foreign Exchange Risk and Bank Capital Requirements

Michael P. Leahy<sup>1</sup>

### 1. Introduction

Bank regulators face the following problem: how much capital should banks be required to hold against their foreign exchange positions? In its simplest form, the problem can be posed as one of balancing the benefits of bank participation in foreign exchange markets, to the extent that that participation requires banks to take positions in foreign exchange, against the risks of bank failures. In practice, however, it can be very difficult to determine with any precision the benefits or the risks that enter into the decision problem, much less the appropriate social utility function to use to evaluate the different choices.

One step towards solving this larger problem is the development of an appropriate measure of the risks associated with taking foreign exchange positions. The Basle Committee on Banking Supervision has been considering various measures of the market or price risk associated with banks' foreign exchange positions. That work is part of a larger effort by the Basle Committee to develop risk-based capital adequacy standards.

This paper examines three possible measures of foreign exchange risk. One measure is the standard deviation of the portfolio, computed

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1. The author is a staff economist in the Division of International Finance. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff. I am grateful to Neil Ericsson, David Howard, Eric Leeper, Jeffrey Marquardt, Christopher McCurdy, Helen Popper, Andrew Rose, Kelly Shaw, Ralph Smith, and Charles Thomas for helpful discussions and suggestions. Maya Larson provided able research assistance.

under the assumption that the distribution of exchange rate changes is multivariate normal. This measure of risk is widely used in a variety of financial applications in which it is important to gauge the volatility of prices or returns.<sup>2</sup> However, for the purposes of determining bank capital requirements, this measure has two potential flaws.

One widely cited feature of the distribution of the exchange rate changes is that its density appears to be leptokurtic, or fat-tailed, relative to that of the normal distribution.<sup>3</sup> Consequently, the normal distribution may understate the true probability of drawing large, unfavorable exchange rate changes that could make a bank insolvent. Therefore, assuming exchange rate changes are distributed normally may lead regulators to set bank capital requirements too low in some cases.

Another potential problem with the assumption of normality in this analysis is that the normal distribution can assign positive probability to impossible events. Because an exchange rate cannot decline by more than 100 percent, assuming percent changes are drawn from a normal distribution has the clearly false implication that declines of more than 100 percent are possible. Many studies of exchange rate variability do not have to deal with this issue of a bounded lower support for the distribution of the percent change because they use the change in the natural logarithm of the exchange rate. However, as shown below in the formal statement of the regulator's problem, the percent change rather than the log change is the more natural measure in this context. This problem is less severe when the variance of the normal

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2. For a recent citation applied to exchange-rate risk, see the 1988 Annual Report of the Foreign Exchange Committee, pp. 19-21.

3. See Westerfield [1977] for some of the early work on exchange rate distributions. See also Boothe and Glassman [1987] and the references therein.

distribution is relatively small, as it would be for the currencies considered here, because the probability of extreme declines in exchange rates would be negligible.<sup>4</sup> However, it is conceivable that the distinction between log and percent changes might matter for currencies that experience large changes, as do some Latin American currencies, or for longer horizons than those considered here.

A second possible measure of foreign exchange risk is based in part on a method that has been used by the Bank of England to assess foreign exchange exposure. This method takes as a measure of exposure the larger of two components: (i) the sum of the long net currency positions in the portfolio and (ii) the absolute value of the sum of the short net currency positions.<sup>5</sup> To make this measure comparable to the other measures studied, I have adapted it so that on average across portfolios this second measure yields the same capital requirement the standard deviation would yield. Because of the scaling, this measure shares some of the same deficiencies of the standard deviation measure. However, because it takes no account of historical patterns of exchange rate changes, it implies an arbitrary and unrealistic pattern of correlations between of exchange rates.<sup>6</sup>

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4. More precisely, if the change in the natural logarithm is distributed normally, the percent change is distributed lognormally. These distributions can have quite different-looking densities when the variances are large. However, as the variance approaches zero, the distribution of the lognormal approaches that of the normal. See Johnson and Kotz [1970], p. 117.

5. See Bank of England [1981].

6. Implicit in this measure are some extreme assumptions about the correlations between exchange rates. Moreover, the measure implies that the correlations should vary as the banks' portfolios vary. For this measure to be correct, pairs of exchange rates corresponding to short currency positions must be perfectly positively correlated, pairs of exchange rates corresponding to long currency positions must be perfectly

A third measure uses nonparametric methods to assess the riskiness of banks' portfolios. This measure is flexible enough to allow for a rich pattern of exchange rate correlations and the possibility that the density function for exchange rate changes has fatter tails than the normal or bounded support.

Because the nonparametric measure is the most general, it can be used to assess the magnitude of the deficiencies of the other measures. It is shown that capital requirements for dollar-based banks tend to be too small for portfolios of marks, yen, and sterling when the standard deviation measure is used. The adapted Bank of England measure behaves on average like the standard deviation measure but the capital requirement is more erratic: capital requirements are too large for some portfolios and too small for others in proportions that exceed those for the standard deviation measure. A similar set of results hold for Swiss franc-based banks.

The next section of this paper contains a more formal presentation of the analysis of foreign exchange risk and bank capital requirements. The third section describes the estimators used. The fourth section presents results, and the fifth contains some concluding remarks.

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positively correlated, and pairs of exchange rates corresponding to pairs of short and long positions must be perfectly negatively correlated. This configuration of exchange rate correlations is unlikely for any given portfolio under consideration, and it cannot be true simultaneously for portfolios with different mixtures of net positions.

## 2. The Risk of Insolvency and Capital Requirements

Consider a dollar-based bank that has assets and liabilities denominated in a number of foreign currencies indexed by  $i$ , where  $i = 1, \dots, n$ . Let  $a_i$  and  $l_i$  be assets and liabilities denominated in a particular foreign currency  $i$ . Define the net foreign currency position in currency  $i$  as  $f_i = a_i - l_i$ . A positive value for  $f_i$  indicates a long position in currency  $i$  and a negative value a short position. Similarly, let  $a$  and  $l$  be assets and liabilities denominated in dollars, and define the dollar position as  $d = a - l$ . Finally, let  $e_{i,0}$  be an exchange rate on day 0, where  $e_{i,0}$  is expressed as dollars per unit of currency  $i$ . Then the day-0 value,  $P_0$ , of the bank's portfolio expressed in dollars is given by:

$$(1) \quad P_0 = \sum_{i=1}^n f_i e_{i,0} + d = \sum_{i=1}^n z_i + d,$$

where  $z_i$  is defined to be the day-0 dollar value of the position in foreign currency  $i$ . Because  $P_0$  is the difference of the dollar value of the bank's assets and the dollar value of its liabilities, it can also be taken as a measure of the bank's capital on day 0.

Abstracting from interest earnings on long positions or interest expenses on short positions and assuming the bank does not alter the quantities of foreign currencies and dollars in its portfolio, we can calculate the dollar value of its portfolio on some later day, day 1:

$$(2) \quad P_1 = \sum_{i=1}^n f_i e_{i,1} + d = \sum_{i=1}^n (f_i e_{i,0}) (e_{i,1}/e_{i,0}) + d$$

$$= \sum_{i=1}^n z_i(1 + x_{i,1}) + d,$$

where  $x_{i,1}$  is the percent change in  $e_i$  between day 0 and day 1. One additional simplification yields  $P_1$  in terms of  $P_0$ :

$$(3) \quad P_1 = P_0 + \sum_{i=1}^n z_i x_{i,1} = P_0 + z'x,$$

where  $z' = (z_1, z_2, z_3, \dots, z_n)$  and  $x' = (x_{1,1}, x_{2,1}, x_{3,1}, \dots, x_{n,1})$ .

The regulator's problem can now be expressed as requiring the bank to take steps at day 0 to ensure that the probability  $P_1$  will be negative is small:

$$(4) \quad \text{Prob}(P_1 < 0) \leq \alpha,$$

where  $\alpha$  is "small." The choice of  $\alpha$  comes from the larger problem of determining how much risk is too much. It will depend on the benefits of allowing banks to take foreign exchange positions and the costs of bank failures. This paper does not address the question of choosing  $\alpha$ , but takes  $\alpha$  as a parameter and focuses instead on the question of determining the appropriate capital requirement for a given value of  $\alpha$ .

Using (3), we can rewrite the probability inequality as:

$$(5) \quad \text{Prob}(z'x < -P_0) \leq \alpha.$$

The regulator can reduce the probability of insolvency on day 1 by requiring a larger value for  $P_0$ , the bank's day-0 capital. As can be

seen from equation (1),  $P_0$  can be adjusted without altering net foreign currency positions by requiring the bank to issue equity and take the increase in capital as an increase in  $d$ , the net dollar position in the portfolio on day 0. The regulator could also reduce the probability of insolvency by requiring adjustments to the components of  $z$  and  $d$ . At day-0 exchange rates, appropriate adjustments can decrease  $z'x$  without changing  $P_0$ .

Cast this way, the problem of determining an appropriate capital requirement is reduced to an estimation problem. The regulator needs to estimate the  $\alpha$ -quantile of the distribution of  $z'x$ , conditional on  $z$ . Given some such distribution for  $z'x$ , the  $\alpha$ -quantile is the value  $q_\alpha(z)$  defined by:

$$(6) \quad \text{Prob}(z'x \leq q_\alpha(z)) = \alpha.$$

If  $q_\alpha(z)$  can be estimated, then setting  $-P_0 \leq q_\alpha(z)$  implies requiring the bank to have enough capital so that  $P_0 \geq -q_\alpha(z)$  or to allocate a large enough quantity of dollars to the portfolio so that  $d \geq -q_\alpha(z) - \sum_{i=1}^n z_i$ . Let  $P^*(z)$  be the minimum value of  $P_0$  that satisfies the inequality  $\text{Prob}(z'x < -P_0) \leq \alpha$ , i.e.,  $P^*(z)$  is the minimum capital requirement, and let  $d^*(z)$  be the associated minimum dollar position.

Looking at the regulator's problem as a problem in estimating a "small" quantile, we can see how assumptions about the distribution of  $z'x$  that restrict the tails of the probability density function to be too lean can lead to capital requirements that are too small. Consider the left panel of figure 1. The regulator's problem is to find the value of  $q_\alpha$  such that the  $\text{Prob}(z'x < q_\alpha)$  is exactly  $\alpha$ . The fatter the tails of the

density function for  $z'x$ , the lower  $q_\alpha$  will be and the higher  $P_0$  should be.

Furthermore, assumptions about the conditional distribution of  $z'x$  that require some positive probability of very large losses when they are impossible can lead to requirements that are too large. For example, when each foreign currency position in the portfolio is long, i.e., when each of the components of  $z$  is positive,  $z'x$  has a finite lower bound of  $L = - \sum_{i=1}^n z_i$ , as shown in the right panel of figure 1. The largest possible loss on those foreign currency positions would occur when the values of all the foreign currencies declined to zero. Assuming some positive probability in the distant reaches of the tail of the density may generate values of  $q_\alpha$  that are too low and consequently values of  $P_0$  that are too high.

### 3. The Two Quantile Estimators

#### Parametric Approach

One approach to determining the capital requirement begins by assuming that  $x$  has a multivariate normal distribution with mean  $\mu$  and variance-covariance matrix  $V$ . Under this assumption, the conditional distribution of the value of the portfolio  $P_1(z)$  is univariate normal with mean  $z'\mu + P_0$  and variance  $z'Vz - (z'\mu)^2$ . Because  $P_1(z)$  is distributed normally, one can use a standard normal table to determine the probability  $\alpha$  that  $P_1(z)$  will be less than  $k_\alpha$  standard deviations below its mean. Thus,  $k_\alpha$  will satisfy the following relation:

$$(7) \quad \text{Prob}(P_1(z) \leq z'\mu + P_0 - k_\alpha \{z'Vz - (z'\mu)^2\}^{1/2}) \leq \alpha.$$

The regulator desires to set  $P_0$  so that

$$(8) \quad z'\mu + P_0 - k_\alpha \{z'Vz - (z'\mu)^2\}^{1/2} \geq 0.$$

Thus, the minimum capital requirement consistent with a probability of insolvency less than or equal to  $\alpha$  under the assumption that exchange rate changes are distributed normally is given by  $P_N^*(z)$ :

$$(9) \quad P_N^*(z) = k_\alpha \{z'Vz - (z'\mu)^2\}^{1/2} - z'\mu = -q_\alpha(z).$$

Figure 2 shows the normal density for  $z'x$  conditional on a given set of foreign currency positions and the  $\alpha$ -quantile that would be used to set the minimum capital requirement for those positions. The corresponding minimum dollar position in the portfolio, again assuming the foreign-currency positions are unchanged, is given by  $d_N^*(z)$ :

$$(10) \quad d_N^*(z) = k_\alpha \{z'Vz - (z'\mu)^2\}^{1/2} - z'\mu - \sum_{i=1}^n z_i.$$

With these formulas, the estimator for capital requirements under the assumption of normality requires only estimates of the mean vector  $\mu$  and the variance-covariance matrix  $V$  to become easily useful. Once the means and the variance-covariance matrix are estimated, capital requirements can be calculated for any portfolio by plugging values of  $z$  into the formula (9).

### Nonparametric Approach

The nonparametric approach **does not** begin with assumptions about the distribution of exchange rate **changes**,  $x$ , and derive the implied distribution of foreign currency **positions**,  $z'x$ . Instead it focuses directly on the distribution of  $z'x$ . The simplest nonparametric quantile estimator is the sample quantile, which is constructed as follows. Given a set of foreign currency positions  $z$  and a random sample of  $T$  sets of corresponding exchange rate **changes**  $x$ , we can order the  $T$  values of  $z'x$ . The sample quantile for  $q_\alpha$  is then the  $j$ th of the ordered values of  $z'x$ , where  $j$  is the integer part of the product of  $\alpha$  and  $T$ . Thus, the sample quantile estimate of  $q_{0.2}$  given a random sample of 100 is the 20th of the ordered values.

More formally, let  $Y_1(z) < Y_2(z) < \dots < Y_T(z)$  denote the order statistics of the sample. Then, the sample quantile  $Q_\alpha(z)$  is given by:

$$(11) \quad Q_\alpha(z) = Y_j(z) \quad \text{for } j/T \leq \alpha < (j+1)/T, \quad j = 1, \dots, T.$$

Because this quantile estimator is piecewise constant as  $\alpha$  moves from 0 to 1, other nonparametric estimators **have been** proposed that smooth the estimated quantile function. However, Sheather and Marron [1990] found little difference between various **nonparametric** quantile estimators in Monte Carlo studies and suggested **that** the sample quantile estimator, because of its simplicity, will often be a reasonable choice as a quantile estimator. Furthermore, Yang [1985] found that the smoothed estimators he considered did not perform as well as the sample quantile in estimating extreme quantiles of **heavy-tailed** distributions. In light

of this evidence, the only nonparametric quantile estimator I consider here is the sample quantile.

Using this method to estimate the quantile yields the following estimate for the minimum capital requirement  $P_S^*(z)$  and the corresponding minimum dollar position  $d_S^*(z)$ :

$$(12) \quad P_S^*(z) = -Y_j(z), \text{ and}$$

$$d_S^*(z) = -Y_j(z) - \sum_{i=1}^n z_i,$$

for  $j/T \leq \alpha < (j+1)/T$ ,  $j = 1, \dots, T$ .

A useful feature of the nonparametric method is that confidence intervals for quantiles are straightforward to derive. Because the probability that any single draw of  $z'x$  falls below  $q_\alpha$  is  $\alpha$ , the probability exactly  $k$  of  $T$  observations will fall short of  $q_\alpha$  is  $\binom{T}{k} \alpha^k (1-\alpha)^{T-k}$ . Extending this analysis yields the following probability statement:<sup>7</sup>

$$(13) \quad \text{Prob}(q_\alpha \leq Y_s) = \sum_{k=0}^{s-1} \binom{T}{k} \alpha^k (1-\alpha)^{T-k}.$$

Expression (13) can be used to establish criteria with which to determine when a particular capital requirement is statistically "too small" or "too large." Consider figure 3. For a given probability  $\alpha$  and sample size  $T$ , one can find the smallest order statistic  $Y_h$  such that the

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7. For further details, see, for example, DeGroot, Morris H., Probability and Statistics, Addison-Wesley, 1975, pp.471-473.

probability  $q_\alpha \leq Y_h$  is at least  $\tau$ . Thus, any quantile estimate  $\tilde{q}$  larger than  $Y_h$  implies we could reject the hypothesis that  $\tilde{q} \leq q_\alpha$  with confidence level  $\tau$  in favor of the hypothesis that  $\tilde{q} > q_\alpha$ . Putting this in terms of capital requirements, we could say that any capital requirement  $-\tilde{q}$  smaller than  $-Y_h$  is statistically less than the actual amount of capital required to keep the probability of capital exhaustion at  $\alpha$  or below. Similarly, one can find the smallest order statistic  $Y_1$  such that the probability  $q_\alpha \leq Y_1$  is at least  $1-\tau$  and use  $Y_1$  as the criterion to determine when any capital requirement is statistically more than the actual amount of necessary capital.

#### 4. Data, Portfolios, and Results

Using daily exchange rate data for the U.S. dollar against the mark, yen, and sterling from March 1, 1973 to the end of 1990, I computed percent changes over the set of horizons from 1 day to 30 days. Missing observations for Saturdays, Sundays, and holidays were incorporated in the horizon calculations so that, for example, the change from a Friday to the following Monday is considered a 3-day change.

Because capital requirements are sensitive to the choice of portfolios (z-vectors, as described in section 2), I considered a number of them. I constructed a set of 182 vectors evenly dispersed on a unit sphere. The coordinates of the vectors were taken as the dollar amounts of the foreign currencies.

For each z-vector of the three currencies, I computed the nonparametric minimum capital requirement,  $P_S^*(z)$ , and the minimum capital requirement under the normality assumption,  $P_N^*(z)$ . Using the method of

high and low order statistics  $Y_h(z)$  and  $Y_l(z)$  described in the previous section, I also calculated 90 percent confidence bounds on the appropriate level of capital for each portfolio. These calculations were made for two typical levels of risk tolerance:  $\alpha = 0.01$  and  $\alpha = 0.025$ .

The top panel of chart 1 shows for  $\alpha = 0.01$  the average capital requirements produced by the nonparametric and the parametric normal methods at each horizon along with 90 percent confidence bounds.<sup>8</sup> The solid line, labeled "normal," is the average of capital requirements generated using the normality assumption. It is given by  $\bar{P}_N^* = (1/182) \sum_{z=1}^{182} P_N^*(z)$ . The dashed line, between the two dotted confidence bounds, is the average of capital requirements produced using the sample quantiles across portfolios. It is given by  $\bar{P}_S^* = (1/182) \sum_{z=1}^{182} P_S^*(z)$ . The confidence bounds are also averages of the upper and lower bounds across portfolios at each horizon. The bottom panel shows the same calculations for  $\alpha = 0.025$ .

Three points are apparent from this chart. The first is that the capital requirement increases significantly with the horizon. The requirement for a 30-day horizon is, depending on the risk tolerance, from 5 to 7 times that for the 1-day horizon. The choice of horizon will thus have a large effect on the amount of capital regulators will want to ask banks to hold. Factors important in determining the appropriate

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8. I should note that because of an insufficient number of observations at horizons from 22 days to 30 days, it was not possible to construct the upper bound in the top panel so that there was only a 5 percent probability the true capital requirement lay above the bound. Therefore, in the top panel only, the confidence bounds decline slowly from 90 percent or more at the horizon of 21 days to 72 percent at the horizon of 30 days. The bounds in the lower panel were not subject to this constraint.

horizon to evaluate the riskiness of any given portfolio include the frequency with which banks and regulators can monitor exposure and the ability of banks to unwind their portfolio positions over time. While the analysis here has little to say about determining the best horizon, it does show how significant an effect horizon can have on capital requirements and it shows that the normal-based and nonparametric methods appear to agree in general on the sensitivity of capital requirements to horizon.

The second point is that at all horizons capital requirements produced using the normality assumption fall short on average of those based on nonparametric estimation. The degree by which the normal-based capital requirements understate the nonparametric capital requirements tends to increase as risk tolerance declines and the quantile to be estimated is further out into the tail of the distribution.

Finally, one should notice that the spread between the dotted confidence bounds tends to widen as the horizon lengthens. This reflects the decline in sample size associated with increase in horizon. At the 1-day horizon, there are 3469 exchange rate changes in the sample; at the 30-day horizon, only 203 nonoverlapping exchange rate changes are available. The decline in sample size reduces the power of the nonparametric procedure to reject the hypothesis that  $P_N^*(z) \geq -q_\alpha(z)$  at any given horizon. However, in looking at the results across horizons, the failure to reject at any given horizon seems less convincing. Because  $\bar{P}_N^*$  lies consistently below  $\bar{P}_S^*$  at all horizons and is not randomly dispersed on either side of  $\bar{P}_S^*$ , it is likely that more data at longer horizons would only tend to shrink the confidence interval around  $\bar{P}_S^*$  without reducing the shortfall in  $\bar{P}_N^*$ .

Chart 2 summarizes some further results across z-vectors. The left half of the table presents results from quantile estimates made under the assumption that the regulator is willing to tolerate a 0.01 probability of insolvency. The right half shows comparable results made under the assumption the regulator is willing to tolerate a probability of bank failure of 0.025.

The top panels present data that provide additional support for the conclusion that  $P_N^*(z)$  tends to understate the amount of capital required relative to the nonparametric estimator  $P_S^*(z)$ . The lines marked "mean" show at each horizon the average of the ratios of the two measures across portfolios (  $\text{mean}(z) = (1/182) \sum_{z=1}^{182} P_S^*(z)/P_N^*(z)$  ). These averages are consistently greater than 1 at all horizons, with no strong tendency to rise or fall as the horizon increases. For a risk tolerance of 1 percent,  $P_S^*(z)$  is between 9 and 20 percent higher on average than  $P_N^*(z)$  and averages 16 percent higher across horizons; for a risk tolerance of 2-1/2 percent, the range is from 2 to 10 percent with an average of 6 percent. The lines marked "high" and "low" show the maximum and minimum values of the ratio  $P_S^*(z)/P_N^*(z)$  over the portfolios at each maturity. For individual portfolios, the nonparametric method yields capital requirements that are as much as 74 percent higher than the normal-based method for 1 percent capital requirements and as high as 49 percent for 2-1/2 percent requirements. As indicated by the lows,  $P_N^*(z)$  does rise above  $P_S^*(z)$  for some portfolios:  $P_S^*(z)$  is much as 25 percent below  $P_N^*(z)$  for 1 percent capital requirements and as much as 21 percent below for 2-1/2 percent capital requirements.

The second row of charts provides another measure of the extent to which  $P_N^*(z)$  understates  $P_S^*(z)$ . It shows for each horizon the

proportion of the total number of portfolios for which the ratio  $P_S^*(z)/P_N^*(z)$  is greater than 1. For 1 percent capital requirements, normal-based methods understate the nonparametric capital requirement in about 70 percent or more of the portfolios. For 2-1/2 percent capital requirements, the proportion declines somewhat, although at most horizons it is still well above 50 percent.

The third row shows at each horizon the proportion of portfolios for which  $P_N^*(z)$  is less than the true capital requirement ( $-q_\alpha$  from section 3) with a confidence level of 0.95 or more (indicated by the line labeled "too little"). It also shows the proportion for which  $P_N^*(z)$  is greater than the true capital requirement with a confidence level of 0.95 or more (indicated by the line labeled "too much"). For 1 percent capital requirements, the proportion of statistically significant shortfalls in  $P_N^*(z)$  is higher than the proportion of statistically significant surpluses at all horizons except 29 days. The number of portfolios for which assuming a normal distribution results in too much capital is relatively small at most horizons for both levels of  $\alpha$ . On the other hand, the number of portfolios for which the normal results in too little capital is quite high at shorter horizons and decreases as the horizon lengthens and  $\alpha$  increases. While this result is consistent with the findings of other researchers that the fat tails in distributions of exchange rate changes tend to diminish at longer horizons,<sup>9</sup> it is more likely to be the result of the decline in sample size and diminished power.<sup>10</sup>

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9. See Diebold [1988] or Baillie and Bollerslev [1989].

10. Koedijk, Schafgans, and de Vries [1990] also argue that the observed tendency towards normality may reflect the loss in efficiency due to reduced sample size.

The bottom row shows the proportions of statistically significant deviations of capital requirements produced using my adaptation of the Bank of England (BoE) method. Because the measure of exposure used by the Bank of England<sup>11</sup> does not vary with the horizon or the level of risk the regulator is willing to tolerate, I scaled the measure at each horizon so that on average it yields the capital requirement given by  $P_N^*(z)$ . The resulting capital requirement is sensitive to the horizon and the level of risk.

Since the adapted BoE method is constructed to equal  $P_N^*(z)$  on average, it is not surprising that the general trends in the statistical deviations over the horizons are similar. The method tends to understate the amount of capital required more frequently than it tends to overstate the amount, and the proportions by which the method understates capital tend to fall as the horizon increases. However, in contrast to the normal-based method, the adapted BoE method fails to show as much improvement as the horizon lengthens. It generates too little capital for some portfolios and too much capital for others in larger proportions at the longer horizons and consistently generates too much capital in higher proportions than the normal-based method.

Charts 3 and 4 show a parallel set of calculations for portfolios of marks, yen, and sterling over the same time period using the Swiss franc as the home currency rather than the dollar. In general, the results are similar. The normal-based and BoE methods tend to consistently understate the amount of capital required, and the shortfall

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11. As stated in the introduction, the Bank of England method is the larger of two components: (i) the sum of the long net currency positions in the portfolio and (ii) the absolute value of the sum of the short net currency positions.

is statistically significant at shorter horizons. What is more striking in the Swiss franc results than in the dollar results is the improvement made in the normal-based estimator at the shorter horizons as the risk tolerance shifts from 0.01 to 0.025. For a risk tolerance of 1 percent,  $P_S^*(z)$  is between 16 and 29 percent higher on average than  $P_N^*(z)$  and averages 22 percent higher; for a risk tolerance of 2-1/2 percent, the range is from 3 to 14 percent with an average of 8 percent.

##### 5. Summary and Concluding Remarks

This study provides a quantitative assessment of two possible measures of exchange rate risk that could be used to develop a risk-based capital requirement for banks with foreign exchange exposure. One measure is constructed under the assumption that exchange rate changes are distributed normally and is subject to two potential criticisms. The first is that it fails to take into account the evidence that exchange rate changes appear to be drawn from distributions whose densities have fatter tails than do those of the normal. This deficiency may lead to bank capital requirements that are too small. The second is that, in contrast to the normal distribution, the distribution of percent changes in exchange rates, which is the appropriate measure to use in the analysis of portfolio risk, is asymmetric and bounded from below. This distortion may lead to bank capital requirements that are too large. A second measure merges the normal-based method with a method used by the Bank of England to assess foreign exchange exposure. This method suffers from the some of the same shortcomings as the normal-based method and

also fails to allow for a realistic pattern of exchange rate correlations.

After presenting a simplified model of the regulator's problem and showing how it can be seen as a problem in quantile estimation, this paper uses nonparametric methods, which can allow for fat tails and bounded support, to assess quantitatively the degree to which the normal-based measure fails to estimate the appropriate capital requirement. While the issue of bounded support does not appear to be quantitatively significant for the exchange rates considered here, the issue of fat tails does. In a sample of simulated portfolios of marks, yen, and sterling, it is shown that normal-based capital requirements tend to understate the appropriate requirements from 9 to 20 percent with a central tendency of roughly 16 percent on average across dollar-based portfolios for a risk tolerance of 1 percent. For Swiss franc-based portfolios, the range is from 16 to 29 percent with an average of 22 percent. At a 2-1/2 percent level of risk tolerance, normal-based methods understate capital requirements somewhat less because the quantile to be estimated is not as far into the tail of the distribution. For dollar-based portfolios, the range is from 2 to 10 percent with an average of 6 percent; for Swiss franc-based portfolios, the range is from 3 to 14 percent with a mean of 8 percent. All these shortfalls appear to be sustained consistently over horizons from 1 day to 30 days.

Some assessment of an adaptation of the Bank of England method was also presented. Because the BoE method was scaled to match the performance of the normal-based measure on average across portfolios, its average performance was similar to that of the normal-based method. However, its performance was otherwise clearly worse than the normal's.

It generates too little capital for some portfolios and too much capital for others in larger proportions at the longer horizons and consistently generates too much capital in higher proportions than the normal-based method.

Under the assumption that the nonparametric methods can and do describe the true distribution of the data more accurately than the normal-based methods, I would recommend using nonparametric methods to determine bank capital requirements, rather than either of the two alternatives. The nonparametric methods are statistically superior. Furthermore, they are analytically straightforward. Once bank regulators have agreed on the level of risk tolerance and the appropriate horizon, a database of exchange rate changes could be constructed and stored on diskette. Then, given a portfolio, a regulator could run a relatively simple PC program that would calculate the changes in portfolio values given by the database of exchange rate changes, sort the changes in portfolio values, and select the appropriate sample quantile.

This study is limited in a number of dimensions. First, the study does not address the issues of determining the appropriate level of risk or the appropriate horizon. In particular, the assumption that the bank does not change its position between day 0 and day 1 becomes more difficult to justify as interval between the days gets longer. A useful but perhaps difficult extension of this analysis would allow the bank the opportunity to unwind positions over time.

Second, a fuller study of bank capital requirements would include other currencies and other instruments in the banks' portfolios. Interest rates and prices of other assets are correlated with exchange rates, and an appropriate measure of the riskiness of any portfolio

should consider all the assets in that portfolio. The basic structure of this analysis allows for the incorporation of all the bank's assets and liabilities into the calculation of its portfolio value in as much detail as is desired. The vector  $z$  can be expanded to include different types of positions denominated in each currency, as long as the vector  $x$  includes the appropriate percent change in the home currency price of that type of instrument. Expanding the list of instruments may require other adjustments as well. For example, if the instrument does not pay on day 1 a fixed nominal price known on day 0 (as one might expect for, say, real estate holdings as opposed to a bank deposit), then  $x$ , which should include the change in the dollar price of the instrument, must take into account any change in local-currency price as well as the change in the exchange rate. Furthermore, by adding net interest flows between day-0 and day-1 to the components of  $z$ , the analysis can be generalized to take into account net interest earnings on long positions and net interest expenses on short positions. Net interest flows may be particularly important to the extent that they compensate for expected exchange rate changes.

Third, both the normal and nonparametric methods are based on the premise that the distribution of exchange rate changes is stable. These methods require that historical changes in exchange rates be drawn from the same distribution as future changes in exchange rates. If the process generating exchange rates were to change significantly, then both methods would be subject to criticism on the grounds of irrelevance: the distributions they estimate may not describe the current exchange rate process.

Finally, it may be possible to construct better measures of exchange rate risk and more accurate capital requirements by considering the conditional distribution of changes in portfolio values. From recent studies finding ARCH effects in the distribution of exchange rate changes, we can infer that setting capital requirements conditional on current and past observations of some measure of the dispersion of the process as well as on the positions in the portfolio may improve the efficiency of the estimates.

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Figure 1  
Probability Density Function for  $z/x$  Conditional on  $z$

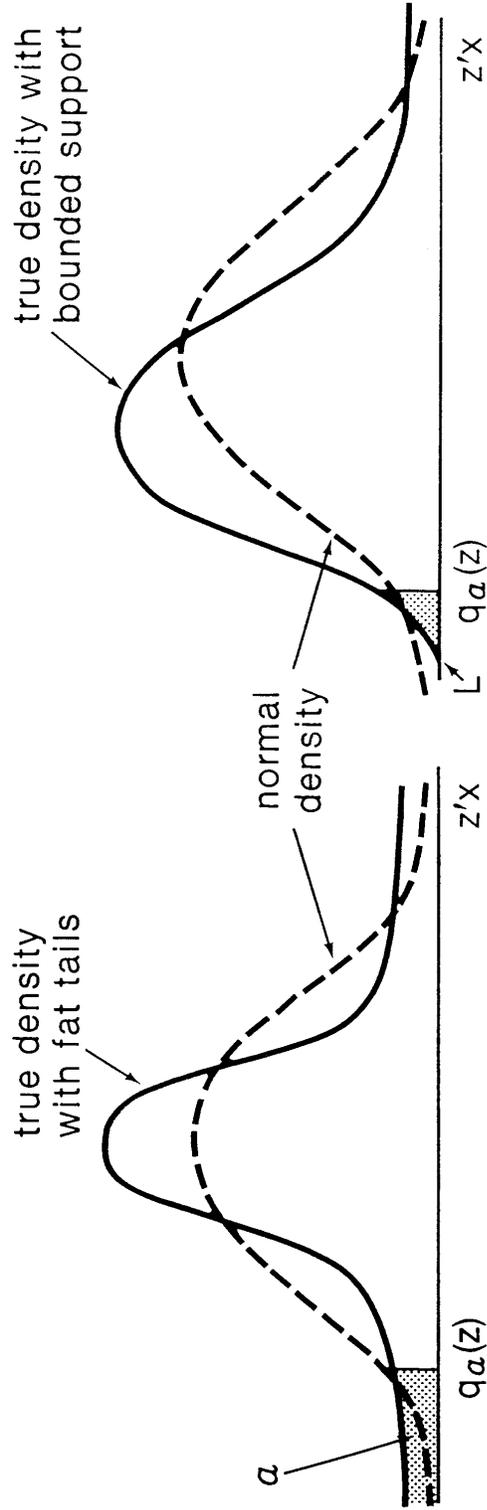


Figure 2  
Normal Density for  $z'x$  Conditional on  $z$

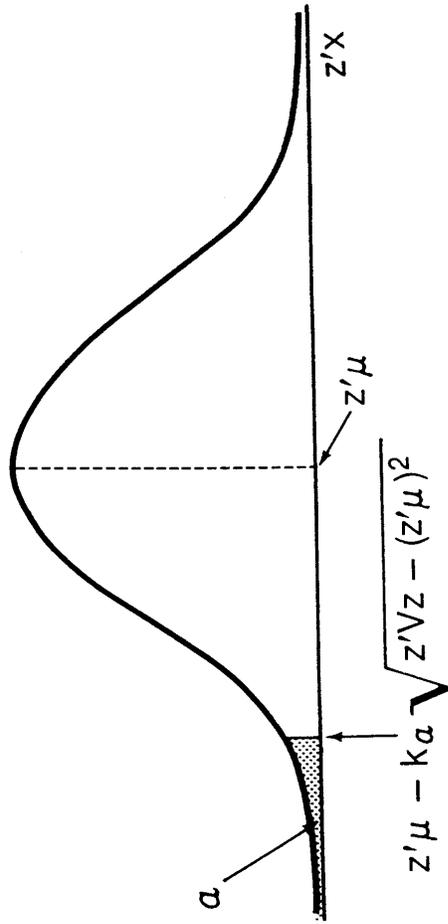


Figure 3  
Probability Densities for Order Statistics  $Y_1$  and  $Y_h$

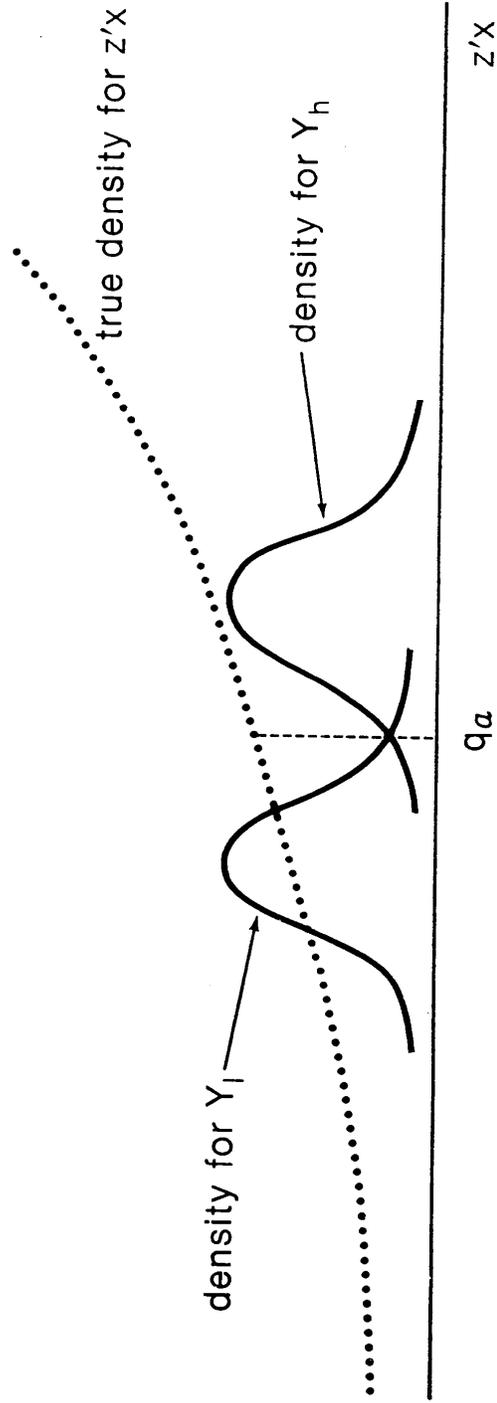
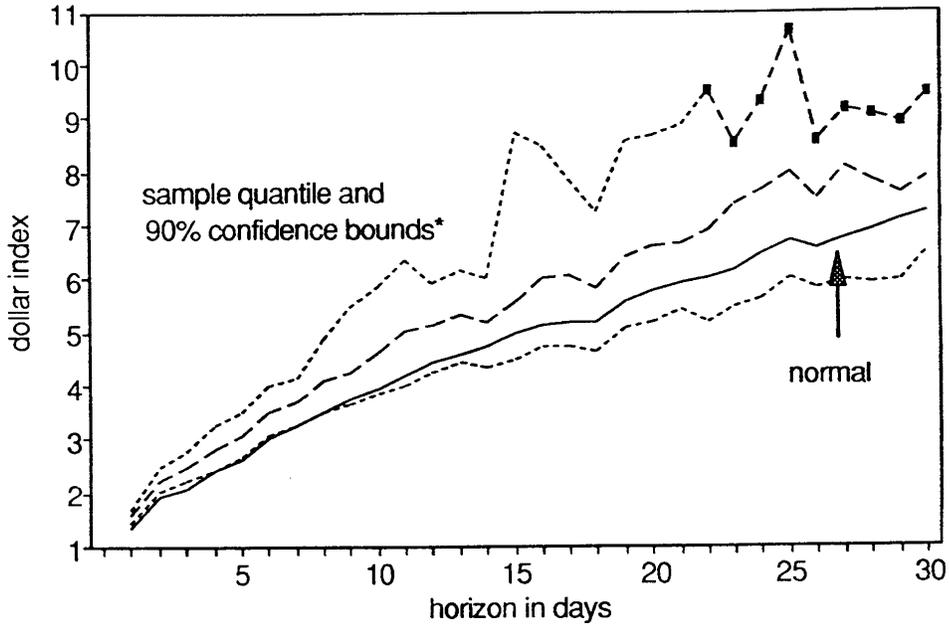


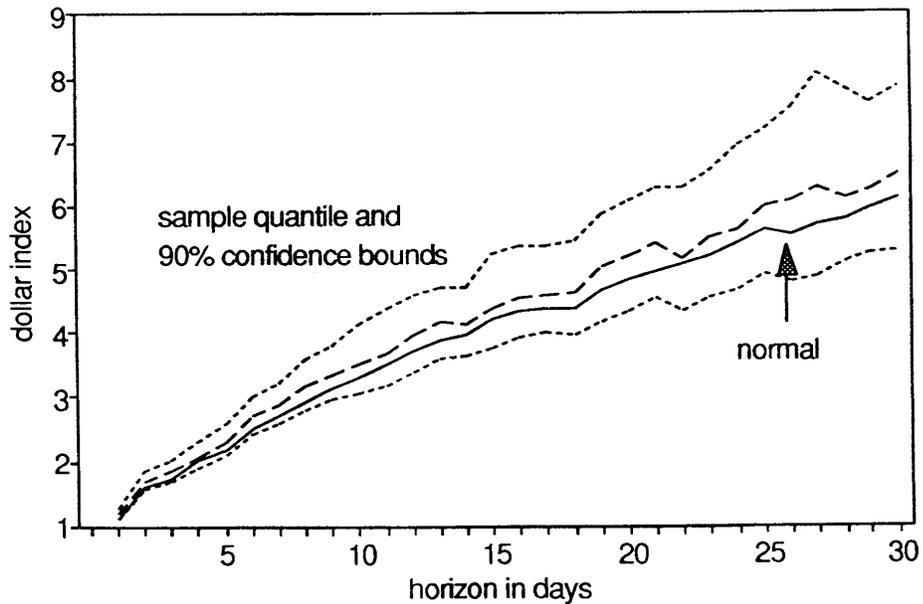
Chart 1

## Average Capital Requirements 0.01 quantile

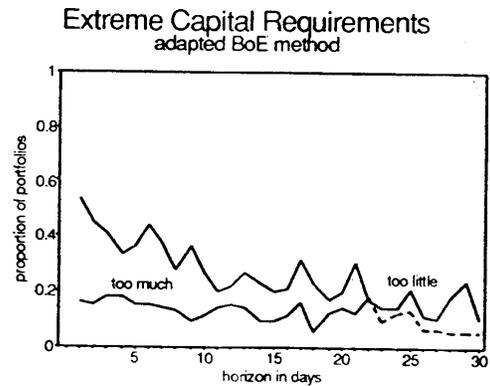
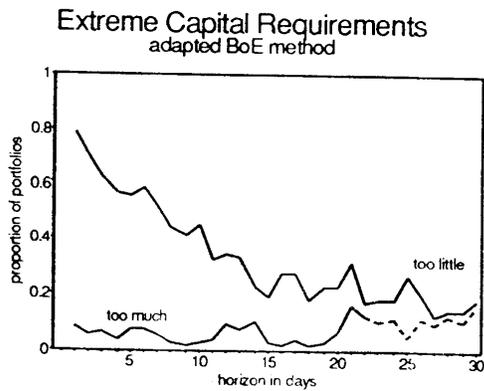
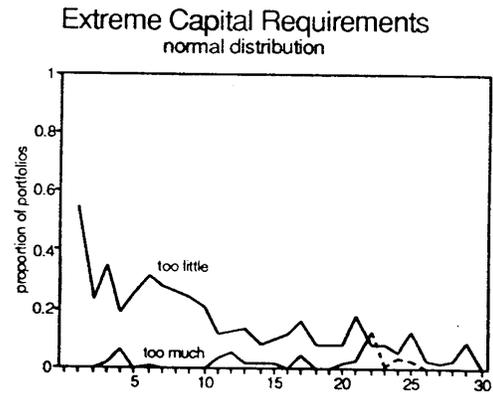
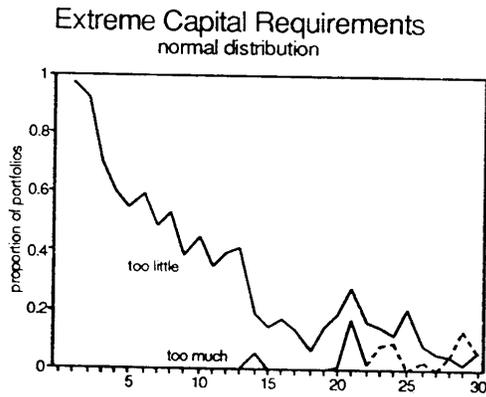
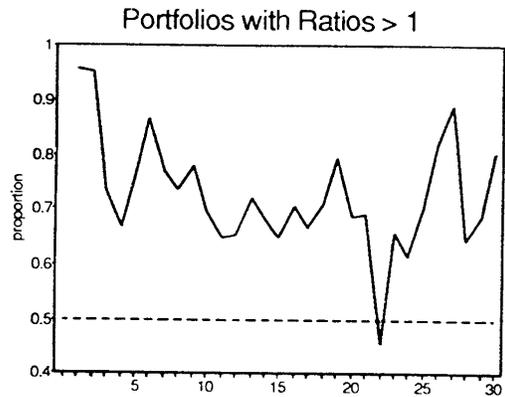
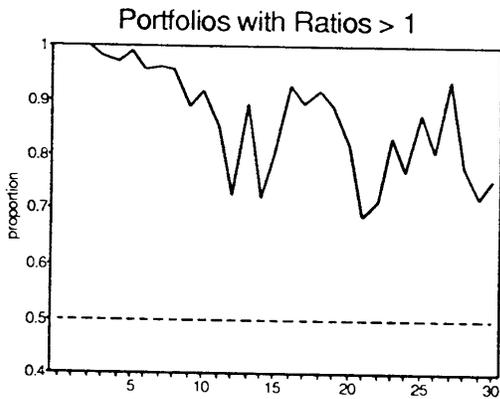
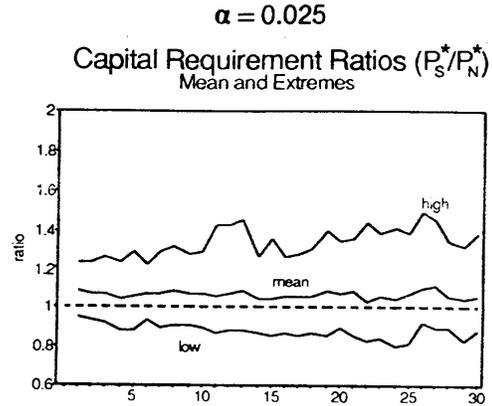
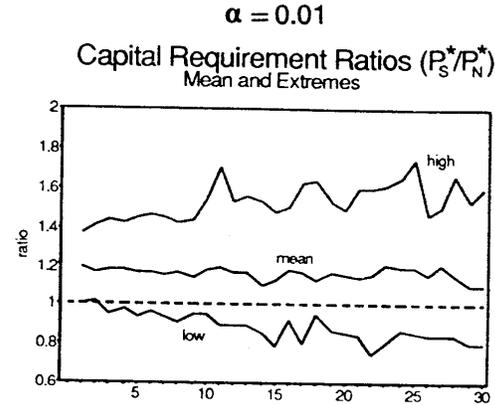


\*The upper bounds past 21 days decline from 95 percent to 77 percent.

## Average Capital Requirements 0.025 quantile



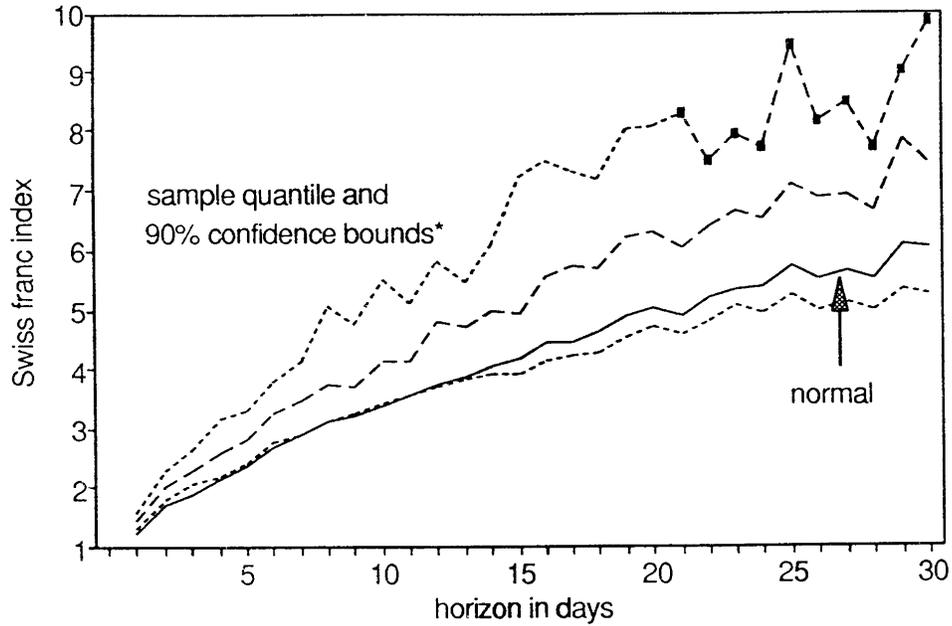
### Comparison of Capital Requirements for Portfolios of Marks, Yen, and Sterling U.S. Dollar is Home Currency



Note: The "too much" lines in the bottom four charts become dashed after 21 days to represent the required change in confidence levels due to insufficient data. The confidence levels decline from 95 percent to 77 percent as the horizon increases.

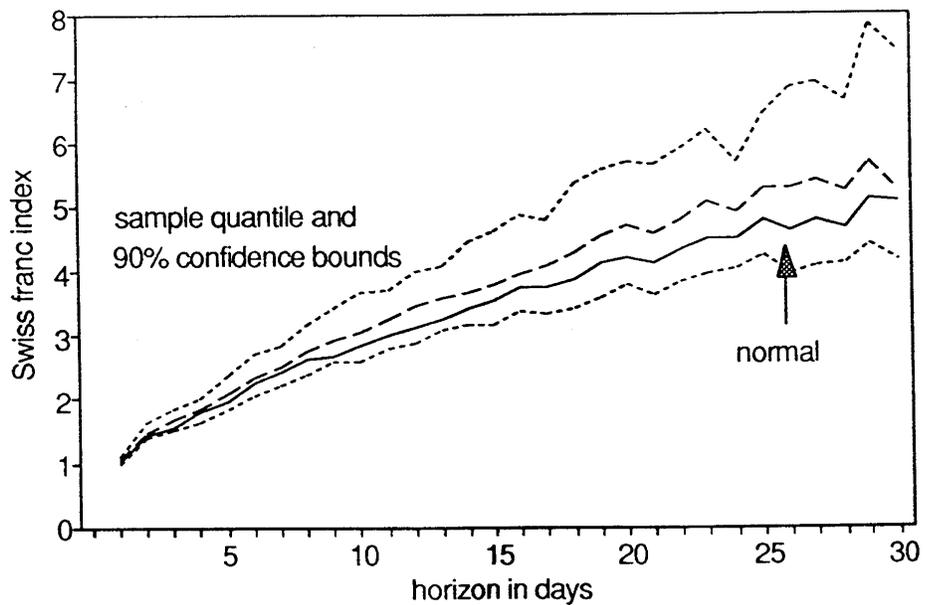
Chart 3

# Average Capital Requirements 0.01 quantile

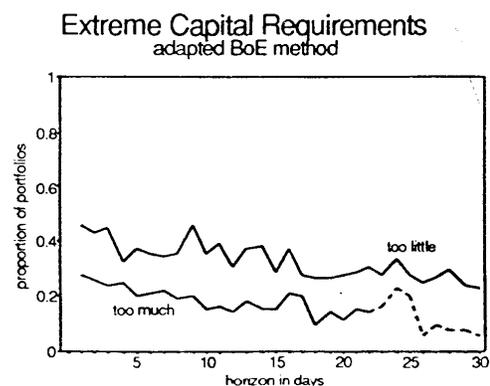
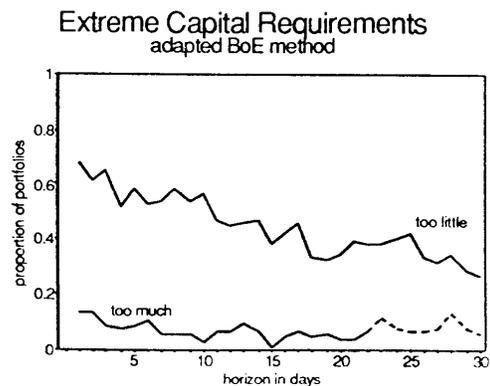
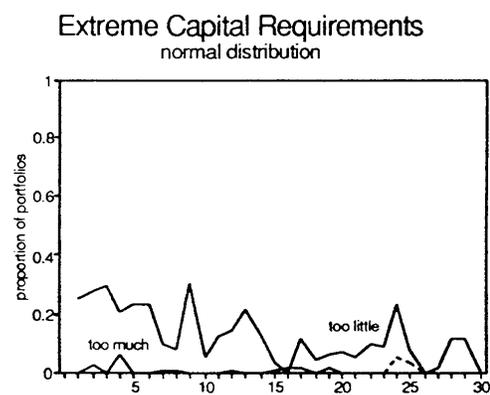
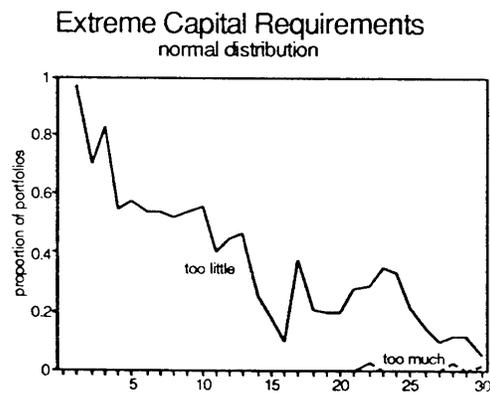
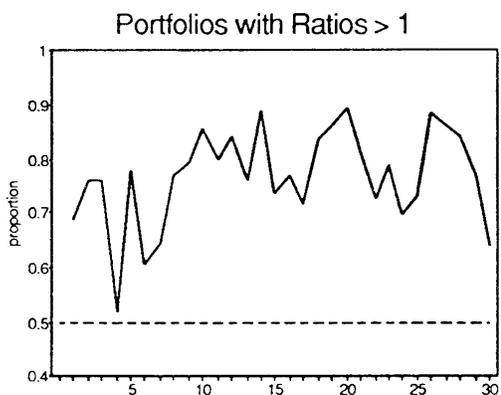
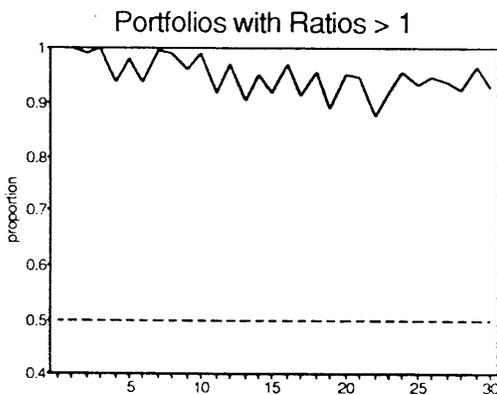
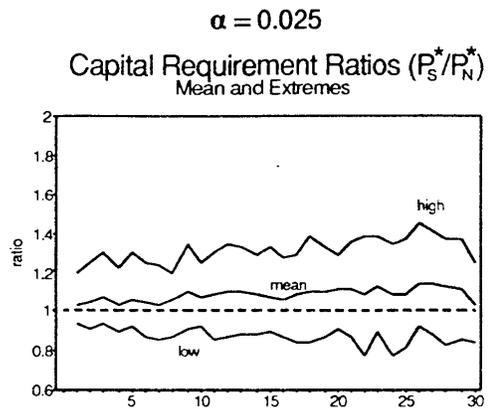
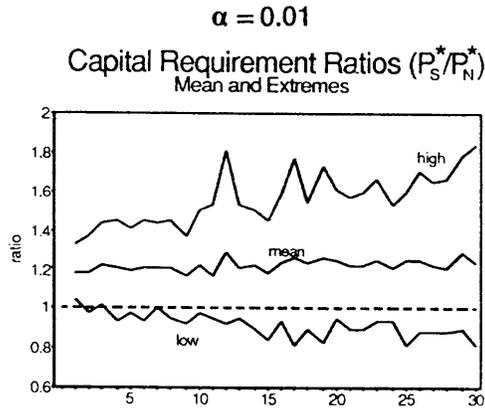


\*The upper bounds past 21 days decline from 95 percent to 77 percent.

# Average Capital Requirements 0.025 quantile



### Comparison of Capital Requirements for Portfolios of Marks, Yen, and Sterling Swiss Franc is Home Currency



Note: The "too much" lines in the bottom four charts become dashed after 21 days to represent the required change in confidence levels due to insufficient data. The confidence levels decline from 95 percent to 77 percent as the horizon increases.