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## **Inflation and the Great Ratios: Long Term Evidence from the U.S.**

Shaghil Ahmed\* and John H. Rogers

### **ABSTRACT**

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Using over 100 years of U.S. data, we find that the long-run effects of inflation on consumption, investment, and output are positive. Thus, models generating long-term negative effects of inflation on output and consumption (including endogenous growth and RBC models with money) seem to be at odds with data from the moderate inflation rate environment we consider. Also, great ratios like the consumption and investment rates are not independent of inflation, which we interpret in terms of the Fisher effect. However, in the full sample, the variability of the stochastic inflation trend is small relative to the variability of the productivity and fiscal trends, so inflation accounts for little of the movements in real variables. By comparison, we find in the post-WWII sub-period that although significant "permanent" shocks to inflation are a more regular feature of the data, the long-run real effects of a *given size* inflation shock are much smaller.

**Key Words:** Inflation, Investment, Great Ratios, Tobin Effect, Fisher effect.

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## 1. Introduction

Consider a situation in which, with the economy in a low-inflation steady state, the rate of inflation falls *permanently*, say by 2 percentage points. What would be the *long-run* effects on real economic variables such as output, consumption, the real interest rate, investment, and the capital stock? Would the long-run path of the so-called "great ratios", such as the investment rate and the consumption-output ratio get altered? Economic theory provides no clear-cut prediction. On the one hand there is the famous superneutrality result due to Sidrauski (1967). Yet, Sidrauski's result emerges from a very specific theoretical set-up, requiring in particular the strong assumption that consumption and leisure are separable in utility. In several theoretical models, the superneutrality result breaks down as inflation can have either positive or negative effects on real variables such as output and investment, depending on the exact assumptions concerning preferences and how money is introduced into the economy. Additionally, in these models the real interest rate, and, therefore, implicitly the great ratios (investment/output and consumption/output), may or may not be independent of inflation in the long run. (see Orphanides and Solow [1990] for a survey).

Therefore, whether the long-run effects of inflation on real economic aggregates are positive or negative, and whether the real interest rate is independent of inflation in the long run are empirical issues. Recently, there has been considerable interest in the existence and nature of a long-run trade-off between inflation and unemployment (or output gap) [e.g. King and Watson (1994, 1997) and Akerlof, Dickens, and Perry (1996)] as well as in the effects of inflation on economic growth. Understanding these effects is, among other things, crucial for evaluating monetary policy, especially in light of the debate about moving from the current low inflation rate to price stability.

Existing empirical results are mixed. Cross-country growth regressions suggest that the effects of inflation on output (growth) or investment are negative. However, these results may be driven largely by the presence of high inflation countries and, for reasons discussed by Levine and Renelt (1992), lack robustness. On the other hand, King and Watson (1994, 1997) find that results on neutrality and superneutrality are sensitive to the short-run identification assumptions made, although

for the assumptions they consider plausible, departures from neutrality tend to be insignificant. It should be noted that the King-Watson findings are based on bivariate systems and do not use a multivariate structural model.

With these considerations in mind, we re-examine the empirical evidence on the long-term interactions between inflation and the real economy. Our goal is to sort out which of several theoretical channels characterizing these interactions are empirically more relevant. Using long-term U.S. data, we ask whether a once-and-for-all permanent increase in inflation leads to an upward or a downward jump in the balanced-growth paths of output and investment. We use the label "Tobin-effect" to denote an upward jump and "reverse-Tobin effect" to indicate a downward jump. Although we do not test for the precise transmission channel that Tobin (1965) originally had in mind, we use these labels because they are convenient and used often. Additionally, we present indirect evidence on the "Fisher effect" (Fisher[1930]): the hypothesis that inflation rate has a one-to-one positive effect on the nominal interest rate and, consequently, does not affect the real interest rate.<sup>1</sup> Since we use U.S. data, our results should not be used to infer what would happen in high inflation environments.

Our empirical findings are organized in three parts. First, the univariate properties of the data are described and cointegration (CI) vectors are estimated. Hypothesis testing on these CI vectors reveals whether the data are consistent with a long-run Fisher effect. Our test relies on the direct long-run correspondence between the investment-output ratio and the real rate of interest; this implies that, if the real interest rate is independent of inflation, the latter should have equal effects on investment and output over a long enough horizon. This is testable using cointegration analysis. Our approach to examining the Fisher effect is thus different from previous studies in that *we avoid explicitly modelling inflationary expectations*. However, our method does not shed light on the short-to-medium run validity of the Fisher effect.

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<sup>1</sup>As will become clear, the absence of a Tobin or reverse-Tobin effect and the Fisher effect holding are not one and the same thing when leisure is endogenous.

Second, under certain restrictions, we are able to identify and estimate additional structural parameters, which allow us to compute the effects of *exogenous* changes in the *long-run* component of inflation on the *levels* of consumption, investment, and output (as opposed to the effects on ratios to GDP obtained from the cointegration analysis). We do this by estimating a fully identified structural vector error correction model (VECM), similar in spirit to King, Plosser, Stock and Watson (1991), but with the major difference that we allow and test for long-run nonneutralities.

Finally, we examine the robustness of our findings over different sub-periods of the data. This allows for the possibility that there may have been structural breaks in the interactions between inflation and real variables over the long period covered in the full sample.

Our estimates indicate the presence of a Tobin-type effect and also indicate that the Fisher effect does not hold in annual U.S. data from 1889-1995. However, the variance decompositions also show that the stochastic trend in inflation is not particularly important for explaining real economic fluctuations. We use these results to assess the potential usefulness of different theoretical models for understanding the long-run real effects of inflation, and compare our results to others in the literature.

The remainder of our paper is organized as follows: In section 2, we set up a general framework that nests the different types of effects of inflation on the real economy found in the theoretical literature. In this framework, the long-run paths of the variables are driven by three stochastic trends: a productivity (or output) trend, a fiscal trend, and an inflation trend. Section 3 links the theoretical framework to our empirical estimation. In this section, we also discuss the identification assumptions and present and interpret our empirical results. Section 4 concludes.

## **2. Theoretical Predictions on the Long-Term Real Effects of Inflation**

Consider a simple, deterministic, optimization problem of an infinitely-lived, integrated household-firm unit.<sup>2</sup> While this is a standard framework that does not yield major new theoretical

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<sup>2</sup>Sticky prices and/or imperfect information (e.g., Ball, Mankiw, and Romer [1988]) models can also generate nonneutralities and highly *persistent* real effects of inflation. However, in such models,

insights, it does help to serve two purposes. First, the problem is set up in a general enough way that most of the theoretical results on the real effects of inflation emerge as special cases. Second, it gives us an opportunity to introduce a fiscal trend (which the typical theoretical literature on inflation and growth does not have) in addition to the usual inflation and productivity trends. Incorporating our fiscal variable is necessary in order to adequately characterize the data in our empirical work.

The representative agent's optimization problem is:

$$\max_{\{C_t, K_{t+1}, M_t^d\}} \sum_{t=0}^{\infty} \beta^t u \left( C_t, 1 - N_t, \frac{M_t^d}{P_t} \right) \equiv \max_{\{C_t, K_{t+1}, M_t^d\}} \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \phi_L \ln(1 - N_t) + \phi_M \ln \left( \frac{M_t^d}{P_t} \right) \right\} \quad (1)$$

subject to the sequence of constraints (2) and (3) below:

$$Z_t^\theta F(N_t, K_t) + \frac{M_t^d + Q_t}{P_t} - [K_{t+1} - (1 - \delta)K_t] - C_t - \frac{M_{t+1}^d}{P_t} = 0 \quad (2)$$

$$\frac{M_t^d + Q_t}{P_t} - a_C C_t - a_K [K_{t+1} - (1 - \delta)K_t] \geq 0 \quad (3)$$

where  $\beta$  is a subjective discount rate,  $C$  = consumption,  $N$  = the fraction of time spent working,  $M^d$  ( $M$ ) = desired (initial) holdings of money,  $Q$  = lump-sum transfer of money from the government,  $P$  is the price level,  $\phi_L, \phi_M > 0$  are preference parameters,  $K$  is the capital stock,  $\delta$  is the depreciation rate ( $0 < \delta < 1$ ),  $\theta, a_C, a_K$  are parameters satisfying  $0 < \theta < 1, 0 \leq a_C, a_K \leq 1$ ,  $F$  represents technology, changes in  $Z$  represent shifts to the production function, and where investment =  $K_{t+1} - (1 - \delta)K_t$ .

In equation (1), utility is log-linear. Equation (2) is the representative agent's budget constraint implying an equality of the sources of funds--which are private sector output (the first term), initial real money holdings, and the real value of lump-sum transfers from the government--to the uses of

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these effects do *not last forever* and, strictly speaking, the long-run neutrality propositions apply.

funds--which are consumption, investment, and holdings of real balances.<sup>3</sup> It is assumed that agents internalize the government's budget constraint and treat the transfers of money from the government as lump-sum, although monetary policy sets these transfers proportional to existing money holdings.<sup>4</sup> Equation (3) is a cash-in-advance (CIA) constraint, with a fraction  $a_c$  of consumption and a fraction  $a_k$  of investment required to be financed by cash holdings. It seems rather awkward to have money in the utility function *as well as* a CIA constraint. However, this is for convenience only: in the special cases we consider below *either* money provides utility ( $\phi_M > 0$ ) *or* the CIA constraint is relevant ( $a_c$  and/or  $a_k > 0$ ) but never both.

The output available to private agents,  $Y-G$ , is given by:

$$(1-g_t)Y_t = (1-g_t)\exp\{\phi g_t\}A_t^\theta F(N_t, K_t) \equiv Z_t^\theta N_t^\theta K_t^{1-\theta} \quad (4)$$

where  $g = G/Y$  represents the size of the government with  $G$  being aggregate government purchases of goods and services,  $A$  is the technology shift variable,  $Z = [\exp\{\phi g\}(1-g)]^{1/\theta}A$ , and  $F(N,K) = N^\theta K^{1-\theta}$ . The above specification of production is standard Cobb-Douglas, except that we focus on *private* output and also allow government size to directly affect private output, with no particular stance taken on the sign of this effect ( $\phi \leq$  or  $\geq 0$ ).<sup>5</sup>

The equations describing steady-state paths for the above model are of the standard form and are relegated to Appendix A for the sake of brevity. We highlight here the main properties of three

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<sup>3</sup>Writing (2) as an equality, rather than an inequality, recognizes that we have imposed standard conditions on preferences and technology that lead to the option of free disposal never being exercised.

<sup>4</sup>While we have abstracted from government debt, with Ricardian equivalence holding our long-run properties would be robust to the introduction of such debt.

<sup>5</sup>Barro and Sala-i-Martin (1995, p.158) argue that it is appropriate to have private production depend on government size (rather than the level of government purchases) if public goods use is subject to congestion effects.

well-known models of the long-term effects of inflation on real variables that emerge as special cases of the above framework:

*Model 1: Sidrauski Model:* This arises if money enters the utility function ( $\phi_M > 0$ ), but there is no CIA constraint ( $a_C = 0 = a_K$ ). In this case constraint (3) is not binding, and we get Sidrauski's well-known superneutrality result.<sup>6</sup>

*Model 2: CIA-for-Consumption Model:* In this case, money provides no direct utility ( $\phi_M = 0$ ), but cash is needed in advance to finance consumption expenditures ( $a_C=1, a_K=0$ ). The model resembles that of Cooley and Hansen (1989): inflation acts as a tax on market activities and induces households to switch from market to non-market activity (leisure). As a result, consumption and work effort fall in response to a permanent rise in inflation. However, in the long run the real interest rate is still independent of inflation. With constant returns to scale, the real rate depends only on the productivity-adjusted-capital-labor ratio ( $K/ZN$ ), which is still constant in the steady state if the CIA constraint applies only to consumption. The model also generates a reverse-Tobin effect: with  $N$  falling and  $K/ZN$  constant, the productivity-adjusted capital stock ( $K/Z$ ), and hence productivity-adjusted investment ( $I/Z$ ), must fall in the long run. The intuition is that the fall in work effort decreases the marginal productivity of capital. Put in another way, the long-run negative effects of inflation on investment and output are equal, leaving the ratio, and hence the real rate unchanged.

*Model 3: CIA-for-Consumption-and-Investment Model:* As in model 2, money is not allowed to enter the utility function ( $\phi_M=0$ ), but the CIA constraint now applies to both consumption and investment ( $a_C=I=a_K$ ). In this case, the negative effects on consumption, investment, and work effort noted above still apply. But, since inflation now represents an additional cost to investment,  $K/Z$  falls by more than  $I/Z$  in response to a rise in inflation, and consequently the real interest rate rises and the

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<sup>6</sup>Alternatively, superneutrality arises within a cash-in-advance framework if the CIA constraint applies only to consumption and labor supply is *exogenous*. However, in fairness to Sidrauski (1967), which is the classic reference in this area, we have chosen to present the result in this way.

investment rate falls in the steady-state. Thus, the Fisher effect does not apply, even in the long run. Some of these results are discussed in Abel (1985), who (abstracting from the labor/leisure choice) compares the dynamic accumulation of capital in models in which the CIA constraint applies only to consumption with those in which it applies to both consumption and investment.

Before turning to models in which inflation has positive effects on capital accumulation, two other points are worth emphasizing. First, as the above results illustrate, when labor supply is endogenous, having the Fisher effect holding and inflation not affecting investment and output are not one and the same thing. Second, there is also the more modern genre of *endogenous* growth models with money, which we have not discussed. These models also generate a reverse-Tobin effect of the type discussed above. But they display the important additional feature that a once-and-for all rise in inflation has a negative effect on the steady-state *growth rate* of the economy as well. (See, for example, Gomme [1993]).

*Model IV: Tobin Model:* The final model we discuss generates a positive effect of inflation on the steady-state capital stock and investment, and is exemplified by the portfolio adjustment model of Tobin (1965). The intuition behind the original Tobin-effect is that, with a fixed savings rate, higher inflation increases the opportunity cost of holding money, inducing savers to shift from holding real balances to holding physical capital. This permanently lowers the marginal product of capital and thus the real interest rate falls; the Fisher effect does not hold, but the prediction goes in the opposite direction from the CIA-for-consumption-and-investment model.

The Tobin effect, as originally formulated, was criticized on the grounds that it assumed an exogenous savings rate.<sup>7</sup> This criticism led to a literature that has shown that a "Tobin-type effect"

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<sup>7</sup>Moreover, the mechanism in Tobin's original formulation cannot possibly lead to a very large effect of inflation on capital (in terms of percent change), given plausible values of the interest elasticity of money demand and the ratio of non-interest-bearing money to capital. We thank Joe Gagnon for pointing this out to us.

can arise even in optimizing models, although not in the type of framework used above. For example, it can arise in two-period OLG models, in infinite horizon models with individual heterogeneity and family disconnectedness due to uncertain lifetimes, and in models with consumption and money entering utility in a nonseparable way with particular assumptions about how the marginal utility of consumption is affected by money.<sup>8</sup> A positive relationship between inflation and investment can also arise if there are distortions in the tax system. Specifically, Bayoumi and Gagnon (1996) argue that if it is nominal, rather than real, capital income, that is taxed, as in Feldstein (1976), higher inflation countries should have greater investment. They find this to be consistent with the data.

The widely different predictions on the real effects of inflation generated by the four models discussed above are summarized in table 1. This table encapsulates just how varied is the theoretical literature on the link between inflation and real variables (including the so-called "great ratios" such as the investment rate, consumption-output ratio, and the capital-labor ratio).<sup>9</sup> The empirical work that follows should be helpful in distinguishing between the different types of models.

### **3. The Empirical Framework and Results**

#### **3.1 The Data**

We use annual U.S. data from 1889 to 1995. Output, consumption, investment, and government spending on goods and services are expressed in per capita billions of 1987 dollars. Inflation is the rate of change of the GDP deflator. Total resident population is used to obtain per capita values. The notation used in reporting the results is as follows:  $y$ ,  $c$ , and  $i$  are the log-levels of per capita real values of output, consumption, and investment, respectively;  $G/Y$  is the ratio of real government spending to output; and  $\pi$  is inflation. Appendix B gives sources of the data.

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<sup>8</sup>See Orphanides and Solow (1990) for a survey. See also Wang and Yip (1992) for the role of nonseparability in utility.

<sup>9</sup>There are also models in which inflation has an *ambiguous* effect on the steady-state capital stock (e.g. Brock [1974] and Fischer [1983]).

Table 2 reports summary statistics. Over the full sample period 1889-1995, inflation averages about 3 percent, while the shares in total GDP of government spending, consumption, and investment are 20 percent, 64 percent, and 16 percent, respectively. As is well-known (and can be seen from the data plots in figure 1), there have been dramatic shifts in inflation and the real variables during sub-periods of our sample. For example, while the Great Depression era was a time of deflation and very low investment, the pre-WWI period was characterized by low inflation (an annual average rate of under 1 percent) and high investment (23 percent of GDP). The post-WWII period was just the opposite from the pre-WWI period, with a relatively high inflation rate (greater than 4 percent) and a relatively low investment share (16 percent). Of course, these summary statistics are simple correlations and should not be given any deep structural interpretation. Note also from table 2 that the share of consumption in total output has varied much less than that of investment over the different sub-periods, although the WWII years 1942-45 were marked by a large negative spike in the consumption-output ratio (see figure 1).

### **3.2 Univariate Properties**

The use of per capita data for output, consumption, and investment amounts to deflating the aggregate quantities of these variables by the deterministic component of the trend in work effort driven by population growth. The representative agent theoretical framework laid out earlier presumes that the variables  $y$ ,  $c$ ,  $i$ ,  $G/Y$  and  $\pi$  have stochastic trends embedded in them (i.e. have unit roots). We conduct three types of univariate analysis to evaluate this: looking at plots of the data, examining the autocorrelations, and conducting two formal tests for unit roots.

Examining the plots in figure 1, the deterministically detrended logs of per capita values of output, consumption, and investment appear to be nonstationary. The figure also plots government size, inflation, and the great ratios  $(c-y)$  and  $(i-y)$ . The question of the nonstationarity of government size and inflation is not so clear-cut from the plots of the data. Plots of the first differences (not reported) give a strong indication that the differences are stationary.

The autocorrelations of the variables are plotted in figure 2. The autocorrelations of the detrended per capita levels of consumption, investment, output, and government size do not die away quickly, indicating nonstationarity. The inflation autocorrelations decay at a rate quicker than that of the autocorrelations of the detrended levels of output, consumption, and investment, but slower than that of the autocorrelations of the first differences (not shown). This reinforces that the issue of the nonstationarity of the inflation rate is not clear-cut.

Table 3 reports the results of two tests for unit roots: the augmented Dickey Fuller (ADF) test, which has the unit root as the null hypothesis (Dickey and Fuller [1979]), and the KPSS test, for which the null is trend-stationarity or stationarity (Kwiatowski, Phillips, Schmidt and Shin [1992]). (The trend is not included in the case of the government size variable, since this ratio is bounded between 0 and 1.) The results are presented in the first four columns. There is substantial evidence for unit roots in per capita values of output, consumption, investment, and government size, but the results on inflation are very borderline and sensitive to the exact sample period used. Because some of the series display unusual dynamics during the wars, we also conducted ADF tests that allowed war-time dynamics to be different (by including variables that interact the lagged first differences with a war-time dummy in the Dickey-Fuller regressions). For this specification, the results (reported in the final column) indicate the presence of unit roots in all the variables.

We thus proceed with the maintained hypothesis of unit roots in  $y$ ,  $c$ ,  $i$ , and  $\pi$ . In the case of  $G/Y$ , we alternatively report results both under the assumption of a unit root and stationarity. Two factors influenced our decision in this respect. First, many RBC models that incorporate fiscal policy assume that  $G/Y$  is mean-reverting, although deviations from the mean are modelled as being very persistent (e.g. Baxter and King [1993]). Second, although our univariate tests do support the unit root in  $G/Y$ , these standard tests are based on an assumption of a *linear* process, whereas this variable--being a ratio bounded between 0 and 1--cannot be a restriction-free linear unit root process.

We also realize from our own results and those of others that the question of a unit root in inflation is controversial. However, there is a vast theoretical literature analyzing the real effects of once-and-for-all unanticipated changes in inflation and whether a long-run inflation-unemployment tradeoff exists or not. This literature seems to put a strong prior on the presence of a unit root in inflation. Moreover, there are empirical results that are sympathetic to unit roots in inflation (e.g. Ball and Cecchetti [1990] and Mishkin [1992]). Barsky (1987) has also argued that since 1914, shocks to inflation have become more persistent, particularly in the post-war period. However, more recently, Culver and Papell (1996) find that, while in individual country time-series data inflation appears to have a unit root, in a panel setting inflation appears to be stationary. We proceed with the assumption of a unit root in inflation, but recognize that the empirical evidence is mixed.

### 3.3 The General Model in a Stochastic Environment

The univariate analysis suggests the presence of stochastic trends. Our general model allows for three stochastic trends: a productivity (output) trend (the long-run component of the technology shift variable  $A$  in the production function), a fiscal trend (the long-run component of government size) and an inflation trend. It is convenient to separate out the trend and cyclical components:

$$\ln A_t = \ln \bar{A}_t + \tilde{a}_t, \quad g_t = \bar{g}_t + \tilde{g}_t, \quad \pi_t = \bar{\pi}_t + \tilde{\pi}_t, \quad (5)$$

where an overbar represents the trend component and tilde denotes the stationary component. The theoretical results summarized in table 1 suggest that steady-state paths (denoted by asterisks) depend on the stochastic trend in inflation. In particular,

$$\begin{aligned} c_t^* &\equiv \left( \frac{C_t}{Z_t N_t} \right)^* = c(\bar{\pi}_t); & i_t^* &\equiv \left( \frac{I_t}{Z_t N_t} \right)^* = i(\bar{\pi}_t); \\ y_t^* &\equiv \left[ \frac{Y_t(1-g_t)}{Z_t N_t} \right]^* = y(\bar{\pi}_t); & N_t^* &= N(\bar{\pi}_t); \end{aligned} \quad (6)$$

where, note that  $y^*$  is long-run effective per capita *private* (not total) output. In (6), invoking certainty equivalence, we have replaced the previously constant steady-state inflation rate by the expected value

of its permanent component, which from the random walk property of stochastic trends is just the current long-run component. In general, the theory discussed earlier implies that the relationships in (6) will be nonlinear. However, in our empirical work we postulate linear relationships between the logs of the variables on the left hand side of (6) and the permanent component of inflation. This can be viewed as a linear approximation to the underlying nonlinear processes for the purposes of estimation. Given this, (5), (6), and the definition of  $Z$  [which implies  $\ln Z = (\phi/\theta)g + (1/\theta) \ln (I-g) + \ln A \approx \ln A - [(1-\phi)/\theta] g$ ], yield:

$$\ln C_t = \ln \bar{A}_t - \alpha_C \bar{g}_t + (\beta_N + \beta_C) \bar{\pi}_t + \{\tilde{a}_t + \tilde{c}_t + \tilde{n}_t - \alpha_C \bar{g}_t\} \quad (7)$$

$$\ln I_t = \ln \bar{A}_t - \alpha_I \bar{g}_t + (\beta_N + \beta_I) \bar{\pi}_t + \{\tilde{a}_t + \tilde{i}_t + \tilde{n}_t - \alpha_I \bar{g}_t\} \quad (8)$$

$$\ln Y_t = \ln \bar{A}_t + (1 - \alpha_Y) \bar{g}_t + (\beta_N + \beta_Y) \bar{\pi}_t + \{\tilde{a}_t + \tilde{y}_t + \tilde{n}_t - (1 - \alpha_Y) \bar{g}_t\}, \quad (9)$$

where  $\beta_N, \beta_C, \beta_I, \beta_Y$  represent the long-run effects of the inflation on the logarithms of  $N^*, c^*, i^*, y^*$ , respectively, and  $\alpha_C, \alpha_I, \alpha_Y$  represent the long-run effects of government size on the logs of  $c^*, i^*, y^*$ , respectively.<sup>10</sup> Our particular theoretical set-up of section 2 implies that  $\alpha_C = \alpha_I = \alpha_Y = (1-\phi)/\theta$ .

However, in a more general model (e.g. when government spending enters utility in a nonseparable way), these restrictions will not necessarily hold, and hence we test them. The terms in curly brackets in (7)-(9) represent stationary components that are constant along steady-state paths.

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<sup>10</sup>We treat  $N, K$  as unobserved variables due to lack of long-term data on them and thus do not report the corresponding equations for these variables.

Eqs. (7)-(9) imply the two independent structural cointegrating (CI) relationships given below:

$$\alpha'X_t \equiv \begin{bmatrix} \alpha_C + (1 - \alpha_Y) & -(\beta_C - \beta_Y) & -1 & 1 & 0 \\ \alpha_I + (1 - \alpha_Y) & -(\beta_I - \beta_Y) & -1 & 0 & 1 \end{bmatrix} \begin{pmatrix} g_t \\ \pi_t \\ \ln Y_t \\ \ln C_t \\ \ln I_t \end{pmatrix} \sim I(0) \quad (10)$$

These long-term relationships can be estimated and we can do hypothesis testing on them. We estimate the following CI relationships using the maximum-likelihood Johansen method (see Johansen and Juselius [1992]):

$$\beta_{11}g_t + \beta_{12}\pi_t + \beta_{13} \ln Y_t + \ln C_t = e_{1t} ; \quad \beta_{21}g_t + \beta_{22}\pi_t + \beta_{23} \ln Y_t + \ln I_t = e_{2t} \quad (11)$$

where  $e_i$ ,  $i=1,2$ , represent stationary deviations from long-run paths. These estimates are conditional on a cointegrating rank of 2, which we test for. Comparing (10) with (11), our structural model implies  $\beta_{13} = -I = \beta_{23}$ . In addition, if  $\alpha_C = \alpha_Y = \alpha_I$ , we have  $\beta_{11} = I = \beta_{21}$ . When the Fisher effect holds, we also have  $\beta_{22} \equiv \beta_I - \beta_Y = 0 = \beta_{12} \equiv \beta_C - \beta_Y$  (i.e. inflation has no effect on the shares of investment and consumption in GDP). These are the restrictions we test.

### 3.4 Cointegration Test Results: Evidence on the Relationship Between Inflation and Great Ratios

We first examine the cointegrating rank for both the 3-trend and 2-trend specifications of the model. In the former, all five variables are treated as having unit roots. The 2-trend specification also contains all five variables, but treats G/Y as a stationary, deterministic variable so there are four variables with unit roots. In each case we should have two CI vectors, since the CI rank will be the number of variables with unit roots less the number of stochastic trends.

The results are in table 4. The lag length in the VAR was selected starting with a lag length of five and sequentially eliminating lags with F-tests used to check the validity of each reduction. The null hypothesis of  $p$  cointegrating vectors (CI rank =  $p$ ) is tested against the alternative of  $p+1$  cointegrating vectors using the maximum eigenvalue test statistic, and the more general alternative of

at least  $p+1$  cointegrating vectors using the trace statistic. Test statistics are reported both with and without a small sample correction due to Reimers (1992). Consistent with the theoretical set-up, results fairly strongly support the null of two CI vectors for both specifications. The exception is a borderline rejection when the trace statistic is used without the degrees-of-freedom correction.

Table 5 displays our estimates of the cointegrating vectors for the 3-trend and 2-trend specifications. Recall that, according to the theoretical model of section 2, in both vectors the coefficients on output and government size should be -1 and 1, respectively. We begin by testing these overidentifying restrictions. For the 3-trend model, the restrictions on the government size variable are rejected at conventional significance levels. (In the 2-trend model  $G/Y$  does not enter the CI vector.)

The last row of the table shows that the restrictions on output are not rejected in the investment vector, but rejected in the consumption vector for both specifications. The joint hypothesis that the output coefficient is -1 in both vectors is also rejected (see the second last row of the table). Although the estimate of the output coefficient in the consumption vector for both specifications (about -1.08) is not far from the predicted value of -1, the standard errors are very small, making the deviation from the predicted value statistically significant at conventional levels. Based on economic significance, however, it could reasonably be claimed that the output restrictions are satisfied. Moreover, economic theory considerations put a very strong prior on the -1 coefficient on output: it arises from permanent productivity innovations leading to balanced growth. By contrast, it is possible for a more general specification of our theoretical model in which the coefficient on  $G/Y$  is not unity (e.g. if government spending enters utility in a nonseparable manner). In light of this, we proceed to estimate the cointegrating vectors with the output variable restricted and  $G/Y$  unrestricted.

Next turn to the inflation coefficients in the estimated CI vectors. For the 3-trend model, keeping fixed the effects of government size, a permanent increase in inflation is associated with a drop in the consumption-output ratio and a rise in the investment-output ratio. The coefficient

estimates are statistically significant at customary levels. They indicate that a permanent one percentage point increase in inflation is associated with a long-run drop in the consumption-output ratio of about 4 percent and rise in the investment-output ratio of about 5 1/2 percent. This translates into a drop in the share of consumption in total GDP of about 2 1/2 percentage points and rise in the investment share of about 1 percentage point, using as initial shares the full-sample means reported in table 2. These estimates are large, as they imply that a permanent, one standard deviation change in the rate of inflation (5.5 percent according to table 2) is associated with approximately a one standard deviation change in  $I/Y$  and considerably more than a one standard deviation change in  $C/Y$ .

The estimates from the 2-trend specification also show that the relationship between inflation and the investment-output ratio is positive and that between inflation and the consumption-output ratio is negative. The point estimates are again statistically significant and the coefficient on consumption is noticeably larger than in the 3-trend system.

The results in table 5 also imply that the *Fisher effect does not hold* in the long run: as inflation rises permanently, the long-run investment-output ratio goes up and, therefore, the capital-output ratio goes up and the real interest falls.<sup>11</sup> Since, typically, models in which the real rate falls

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<sup>11</sup>Suppose we parameterize  $\theta$ ,  $\delta$ ,  $\mu$ , which represent the labor share of output, the depreciation rate, and the long-term growth rate of the economy. Then, we can use our estimated effect of  $\pi$  on  $I/Y$ , the steady-state relationship  $I/Y = (\mu + \delta)K/Y$ , and the equality of the real interest rate to the net marginal product of capital to determine the effect of inflation on the steady-state real rate of interest. For plausible parameter values, the computed effect on the real interest rate is implausibly large. However, introducing quadratic adjustment costs to changing the capital stock can yield more reasonably sized effects. We do not report these numbers because there is quite a wide range, depending on the size of the adjustment costs, which are difficult to pin down. Also, our estimates for the post-WWII sub-period (discussed later) yield effects on the real rate of more plausible magnitudes.

with higher inflation are ones in which a Tobin-type effect holds, indirectly this provides some evidence in favor of this effect.

### 3.5 Modelling the Stochastic Trends: Evidence on the Effects of Inflation on Y, I, and C

The analysis so far, based on estimated CI vectors, does not tell us about the direction of causation between inflation and the great ratios, nor about the relationship between inflation and the *levels* of consumption and investment. Therefore, in order to interpret the results more cleanly in light of economic theory, we need to identify exogenous shocks to the inflation trend, which (as always) comes at the expense of further restrictions. To this end, assume that the stochastic trends behave according to:

$$\bar{g}_t = \bar{g}_{t-1} + \boldsymbol{\varepsilon}_{gt} \quad (12)$$

$$\ln \bar{A}_t = \mu + \ln \bar{A}_{t-1} + \boldsymbol{\varepsilon}_{At} \quad (13)$$

$$\bar{\pi}_t = \bar{\pi}_{1t} + \beta_G \bar{g}_t ; \quad \bar{\pi}_{1t} = \mu_\pi + \bar{\pi}_{1,t-1} + \boldsymbol{\varepsilon}_{\pi t} \quad (14)$$

where  $\mu$  is the long-term growth rate of the economy,  $\beta_G$ ,  $\mu_\pi$  are fixed parameters, and the  $\boldsymbol{\varepsilon}$ 's are zero-mean, serially uncorrelated, independent disturbance terms. Eqs. (13) and (14) imply that the stochastic parts of the inflation and productivity trends are independent. Also, we have assumed that the fiscal trend is exogenous and allowed to affect steady-state inflation ( $\beta_G \neq 0$ ). The sign of  $\beta_G$  tells us whether the inflation tax is complementary to ( $\beta_G > 0$ ) or substitutable with ( $\beta_G < 0$ ) general income taxation. King *et al* (1991) also estimate a 3-trend model, including an inflation trend, but they *impose* long-run neutrality as an identification assumption. Later, we provide a comparison of our results to theirs and also examine robustness to changing some of the causality assumptions on the stochastic trends.

A key issue before we proceed is whether the identification restrictions embedded in (12)-(14) are consistent with economic theory or not. In particular, is it plausible that the stochastic trends in inflation and productivity are independent? First, economic theory clearly suggests that it is

reasonable to assume that the stochastic part of the productivity trend does not affect inflation in the long run. Once-and-for-all permanent shocks to the supply of output affect the long-run price *level* but not its long-run *rate of change*.

Second, and more controversially, is it reasonable to assume that a permanent change in inflation can have a long-run effect on the investment rate--and hence the real interest rate--but no long-run effect on productivity growth--and hence the growth rate of the economy? At first pass, this would seem inconsistent with the standard intertemporal efficiency condition for consumption: with log-utility, this is given by  $\beta(1+r) = u'(C_{t+1})/u'(C_t) = (1+\mu)$ . From this equation, anything that affects the real interest rate in the long run, including inflation, should also affect the long-term growth rate of the economy. The puzzle can be solved by realizing that the above form of intertemporal efficiency condition holds only when the CIA constraint does *not* apply to investment, which are precisely the cases in which the real interest rate is independent of inflation. In general, if the CIA constraint applies to investment also ( $a_K = 1$ ), the appropriate intertemporal efficiency condition includes the Lagrange multipliers associated with constraints (2) and (3).<sup>12</sup> Also, as an empirical matter, if output growth is stationary (as the data strongly seem to suggest), shocks to the random walk component of inflation cannot have any significant *permanent* effect on growth.

Considering the fundamental shocks, one natural interpretation of the permanent shock to inflation ( $\epsilon_\pi$ ) is that it represents changes in the monetary authority's target inflation rate. However,

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<sup>12</sup>The general version of the intertemporal efficiency condition for consumption in the CIA model can be shown to be:  $\beta(1+r) = (1+\mu) + [1+\mu-\beta(1-\delta)](\gamma/\lambda)a_K$ , where  $\lambda, \gamma$  are the Lagrange multiplier associated with constraints (2) and (3), respectively. This reduces to the familiar  $\beta(1+r) = (1+\mu)$  only when  $a_K = 0$ . There are also other models where the investment rate can have a unit root, yet with suitable parameterization the growth rate effects of this unit root are small (e.g. Mendoza, Milesi-Feretti, and Asea [1995]).

following most of the theoretical literature on inflation and growth, we will, for the most part, refer to it as an inflation shock, rather than explicitly as a money growth shock.

*The Estimated VECM.* The vector of our five observed variables,  $X = (g, \pi, \ln Y, \ln C, \ln I)'$  is determined by the three permanent innovations to the stochastic trends,  $\boldsymbol{\varepsilon}_{gp}, \boldsymbol{\varepsilon}_{\pi p}, \boldsymbol{\varepsilon}_{At}$  and two transitory disturbances, which we label  $\boldsymbol{\varepsilon}_{1t}^T, \boldsymbol{\varepsilon}_{2t}^T$ . In moving average form, the *structural* model is:

$$\Delta X_t = \theta(L)\boldsymbol{\varepsilon}_t \quad (15)$$

where  $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_g, \boldsymbol{\varepsilon}_\pi, \boldsymbol{\varepsilon}_A, \boldsymbol{\varepsilon}_1^T, \boldsymbol{\varepsilon}_2^T)'$ . The CI relationships given in (10) imply that the matrix of long-run multipliers,  $\theta(1)$ --obtained by setting  $L=1$  in (15)--is:

$$\theta(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \beta_G & 1 & 0 & 0 & 0 \\ \beta_G(\beta_N + \beta_Y) + (1 - \alpha_Y) & \beta_N + \beta_Y & 1 & 0 & 0 \\ \beta_G(\beta_N + \beta_C) - \alpha_C & \beta_N + \beta_C & 1 & 0 & 0 \\ \beta_G(\beta_N + \beta_I) - \alpha_I & \beta_N + \beta_I & 1 & 0 & 0 \end{pmatrix} \equiv [\boldsymbol{\theta} : \mathbf{0}] \equiv [\tilde{\boldsymbol{\theta}}\boldsymbol{\Gamma} : \mathbf{0}] \quad (16)$$

where  $\mathbf{0}$  is the 5x2 null matrix and

$$\tilde{\boldsymbol{\theta}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ -(\alpha_C - \alpha_Y) & (\beta_C - \beta_Y) & 1 \\ -(\alpha_I - \alpha_Y) & (\beta_I - \beta_Y) & 1 \end{pmatrix}; \quad \boldsymbol{\Gamma} = \begin{pmatrix} 1 & 0 & 0 \\ \beta_G & 1 & 0 \\ \beta_G(\beta_N + \beta_Y) - \alpha_Y & \beta_N + \beta_Y & 1 \end{pmatrix} \quad (17)$$

Eqs. (16)-(17) imply that the matrix  $\theta$  is the product of a matrix consisting of known coefficients (since  $\beta_C - \beta_Y, \beta_I - \beta_Y, \alpha_Y - \alpha_C$  and  $\alpha_I - \alpha_Y$  can be obtained from the estimates of the CI vectors) and a lower triangular matrix. Given lower triangularity and the independence of the permanent innovations, it can be shown that the parameters  $\beta_G, (\beta_N + \beta_Y), (\beta_N + \beta_C), (\beta_N + \beta_I), \alpha_Y, \alpha_C, \alpha_I$  are identified and can be retrieved from the reduced-form VECM. Also, under the assumption that the

transitory disturbances are independent of the permanent disturbances, variance decompositions and impulse responses with respect to the permanent shocks can be computed.

The formal proof of identification is very similar to that in King *et al.* (1991) and is given in Appendix C. The intuition for why these parameters are identified lies in the recursive nature of the long-run model. Specifically, since the fiscal trend is causally prior to the inflation trend and independent of the productivity trend, the long-term behavior of  $g$  will identify this trend. Accounting for the effect of this trend on inflation, the long-term behavior of inflation then identifies the inflation trend. Similarly, accounting for the long-run effects of inflation and government size on consumption, investment, and output, the long-run behavior of *any* one of these three variables identifies the productivity trend. (Note that the long-run responses of these three variables to the productivity trend have been constrained to be equal from the CI vectors imposed, which just reflects the property that the productivity trend by itself leads to balanced growth .)

*Coefficient Estimates.* The point estimates and standard errors of the  $\beta$  parameters are reported in table 6. Estimates from both the 3-trend and 2-trend models indicate that a Tobin-type effect is present: a permanently higher inflation rate increases output, investment, and consumption ( $\beta_Y + \beta_N$ ,  $\beta_C + \beta_N$ ,  $\beta_I + \beta_N > 0$ ). With the exception of  $\beta_C + \beta_N$ , which is borderline, these are all statistically significant, using two standard-deviation confidence bands. These results do *not* represent effects on lifetime utility and, therefore, should *not be given any welfare connotations*. The issue of the optimal rate of inflation is beyond the scope of our paper.

The results in table 6 also indicate that  $\beta_I > \beta_Y > \beta_C$ , so that a rise in inflation leads to an increase in the investment-output ratio and a drop in the consumption-output ratio. The effects on these ratios are of exactly the same magnitude as those reported in table 5, since the CI vectors estimated there have been imposed in estimating the VECM. For the 3-trend model, we also find that  $\beta_G > 0$ , indicating that the revenue creation function of inflation is used in a complementary fashion to

other taxes. ( $\beta_G$  is not identified from the long-run restrictions in the 2-trend model, since  $G/Y$  is treated as stationary in that model.)

*Variance Decompositions.* Table 7 displays the fraction of the forecast error variance of each variable that is attributable to the three permanent shocks in the 3-trend model. The inflation shock (panel B) accounts for under 20 percent of the forecast error variance of either output or investment, and no more than 6 percent of the error variance of consumption, at any horizon. Most of these point estimates are less than twice the standard error. Fiscal shocks (panel A) account for most of the variance of  $G/Y$ , and a sizable amount of the variance of inflation and output. The error variance of consumption is almost entirely accounted for by the permanent output (productivity) shock (panel C), while investment is explained by a combination of the permanent and transitory shocks. The lack of importance of transitory shocks in explaining consumption is consistent with predictions of the life-cycle permanent-income hypotheses of consumption behavior.<sup>13</sup>

Table 8 reports the variance decompositions from 2-trend specification. Generally the results are the same as above. In particular, inflation shocks explain a very small percentage of the forecast error variance of consumption or investment. However, the contribution of inflation shocks to output variability (above 30 percent) is higher than in the 3-trend specification, but standard errors are large.

The point estimates of the structural parameters and the variance decompositions are suggestive of the following interpretation: Over the entire 1889-1995 period, permanent shocks to inflation, (when they do occur), appear to have fairly large positive long-run effects on output, investment, and consumption. However, significant shocks to inflation appear *not* to occur very

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<sup>13</sup>To examine robustness, we also estimated a system in which inflation is placed after the consumption, investment, and output block in the long-run causal ordering. In the 3-trend specification, the contribution of inflation shocks to the variability of output is slightly less and to the variability of consumption is slightly more under the new ordering than the original ordering. The shock's contribution to investment is approximately the same under either ordering.

frequently during much of the sample. In other words, the unit root in inflation is small compared to the unit roots in the productivity and fiscal trends. This suggests that superneutrality may not be too bad an approximation when analyzing *historical* U.S. economic data on real variables. However, this should not be taken to imply that when we are discussing hypothetical or proposed significant changes in inflation, we can ignore the long-run effects of inflation on the real economy. This is because our results also suggest that when inflation shocks do occur, they have significant real effects and that models that generate the reverse-Tobin effect (including endogenous growth models and RBC models that incorporate money) appear to be at odds with the data.

### **3.6 Comparison with other results**

One popular source of existing evidence on the output effects of inflation is cross-country growth regressions. Examples are Kormendi and Meguire (1985), Barro (1991), and DeGregorio (1993). Typically, research in this area finds a negative impact of inflation on output *growth*, although in some cases (e.g. Barro[1991]), the effects are quite small. When large negative effects of inflation on growth are found, the sample generally includes high inflation countries. For example, DeGregorio (1993) focuses on Latin American countries. Bruno and Easterly (1998) also find that it is during "discrete" high inflation crises that growth falls sharply. Thus, one way to reconcile our results with these cross-country studies is to argue that the relationship between inflation and output is markedly different in low to moderate inflation environments than in high inflation environments. Moreover, Levine and Renelt (1992) have found the results of these cross-country growth regressions to be "fragile", in the sense that estimates are very sensitive to the set of conditioning variables used.

Another type of evidence on the real effects of inflation comes from estimates of dynamic Phillips curves. This work is exemplified by King and Watson (1994, 1997). They use bivariate systems to examine both neutrality and superneutrality propositions, depending on the assumed order of integratedness of prices (or money). The identification method is to make an assumption about the magnitude of some impact elasticity. King and Watson find that the results on the superneutrality

proposition are sensitive to the assumption made about the magnitude of the impact effects, although for the range of short-run effects they consider plausible (based on Keynesian, monetarist, and RBC types of models), the departures from superneutrality are not big.<sup>14</sup> They also test for the Fisher effect in a two-variable system (nominal interest rate and inflation) and find mixed evidence.

King and Watson's findings are based on *bivariate* systems, using (deliberately) only *minimal* structural information. Our results, based on a multivariate model whose structure is derived from equilibrium growth theory, appear to lead to different conclusions about the long-run real effects of inflation; however, if the full range of the King/Watson estimates is considered, there is some overlap.

King *et al* (1991) (after which we have closely patterned the empirical methodology used in this paper) *do* use a structural multivariate model and include a stochastic inflation trend. In contrast to our paper, King *et al* impose long-run neutrality as part of an identification scheme, rather than test for it. Nevertheless, in the short run, shocks to the inflation trend contribute almost nothing to output fluctuations, which is consistent with our full-sample variance decomposition results. Also, their impulse responses indicate virtually no short-to-medium-run response of consumption, output, and investment to the inflation shock. Although their impulse responses do not represent long-run properties, they appear to be inconsistent with our full-sample results showing significant real effects of inflation. However, as discussed in the next section, the King *et al* results are more consistent with our results from the post-war sample.

King *et al* do find that their "real interest rate" trend is very important in the variance decompositions and impulse responses. However, it is difficult to give a fundamental interpretation to an *exogenous* real interest rate trend (in particular, their real interest rate trend is independent of their inflation trend). Our identification scheme allows for inflation to affect the investment rate (and thus

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<sup>14</sup>There are also papers that have used the assumption of *zero* impact elasticities to test long-run neutrality (e.g. Geweke [1986], Stock and Watson [1988], and Fisher and Seater [1993]).

implicitly, the real interest rate) allows for long-run departures from the Fisher effect, in line with at least some of the growth theory literature.

### 3.7 Analysis of sub-periods

Are our results driven by special sub-periods such as wars or the Great Depression, which was a period of deflation and low investment? As noted above, table 1 and figure 1 indicate that the processes for inflation and real variables have been quite different across the pre-WWI, interwar, and post-1949 periods. Therefore, to examine robustness, we also estimate the models over two subsamples: the post-war period (1950-1995) and the inter-war period (1918-1941).<sup>15</sup>

These results are presented in table 9. The first row of results (reporting the estimated CI vectors) shows that both sub-periods are characterized by a positive relationship between inflation and the investment-output ratio and a negative one between inflation and the consumption-output ratio. The long-run relationship between inflation and the investment rate is much stronger for the inter-war period than the full sample: a permanent 1 percentage point drop in inflation leads to a long-run increase of nearly 30 percent in the investment rate (which translates into a 4 percentage point increase taking as initial shares the sample mean). This 30 percent change should be contrasted with a 5 1/2 percent change obtained from the full sample. This large effect probably is the consequence of the dramatic movements in inflation and investment during the Great Depression.<sup>16</sup> In the post-WWII period, the long-run relationship between inflation and investment, although still positive and

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<sup>15</sup>The results from the sub-periods should be interpreted with some caution, however, since CI techniques are more appropriate for longer spans of data.

<sup>16</sup>Eichengreen (1992) argues persuasively that the direction of causation in this interwar relationship likely goes from monetary contraction (and hence a fall in inflation) to investment and output. This is consistent with our variance decompositions for the inter-war period (reported later), which indicate that inflation shocks are important over this period in explaining the forecast error variance of real variables.

significant, is considerably weaker than in the full sample: the estimate implies that a permanent 1 percentage point drop in inflation is associated with an increase in the investment rate of only 1 percent (which translates into only a two-tenths of a percentage point increase). This is more consistent with the findings of King *et. al.* (1991) discussed above.

The variance decomposition results are reported in the final row of table 9. In contrast to the full-sample results, the inflation shock accounts for a large percentage of the forecast error variances of output, consumption, and investment in the inter-war and especially the post-WWII periods. This suggests that the inflation trend became increasingly important relative to the other permanent trends in the latter part of the sample, a result that probably reflects the increased persistence of inflation itself, as noted by Barsky (1987) and others.

We conclude from the above that the post-WWII period is noticeably different than other sub-periods. Although significant "permanent" shocks to inflation are a more regular feature of the data, the long-run effects of a *given size* shock are much smaller. For instance, the decade-average inflation rate was about 5 percentage points lower in the 1980s than in the 1970s. Despite this large permanent drop in the inflation rate, neither investment nor the investment rate fell by as much as would be implied by the full sample estimates.

#### **4. Concluding Remarks**

Understanding the long-run real effects of changes in inflation is essential to debates concerning the channels of monetary policy transmission and the goal of price stability. Long-term U.S. data indicates that a permanent unanticipated rise in inflation is associated with a rise in the share of investment in GDP in the long run and also has positive long-run effects on the levels of output, consumption, and investment. These results are consistent with the real interest rate falling in response to a permanent rise in inflation and the existence of a "Tobin-type effect". However, the results should not be given a welfare connotation, as we have not examined the implications for utility.

Our empirical approach does not tell us the exact mechanism that generates a Tobin-effect and we leave this as an open question. Possible factors are finite lifetimes, individual heterogeneity, and uncertain lifetimes highlighted in the literature cited in the Orphanides and Solow (1990) survey paper, tax distortions of the type highlighted by Feldstein (1976) and more recently by Bayoumi and Gagnon (1996), or downward rigidity of nominal wages with individual firms experiencing stochastic shocks to the demand for their output, as emphasized by Akerlof *et al* (1996).

Our results are inconsistent with a variety of theoretical models that generate a reverse-Tobin effect, models that at the present time seem to be slightly favored in the inflation and growth literature. However, our full-sample variance decompositions indicate that, although significant non-neutralities are found, the role of inflation in explaining fluctuations in real variables is very limited, compared to the role played by productivity and fiscal trends. This suggests that real-business-cycle models and endogenous growth models without money might be useful approximations when analyzing *historical* U.S. data on real variables, *but only because the permanent component of inflation is relatively small and not because long-run superneutrality applies.*

The results from the post-war (1950-1995) and inter-war (1918-1941) subsamples confirm the existence of a "Tobin-type effect", but differ from the full-sample estimates in two ways. First, the estimated long-run effects on output, investment, and consumption are much larger in the inter-war period and much smaller in the post-war period than in the full sample. Second, as measured by the variance decompositions, the inflation trend is quite important in the inter-war and post-war periods, unlike the full-sample results. The results suggests that in those periods when permanent changes in inflation are estimated to have large long-run real effects such as pre-WWII, such shocks did not occur often; by contrast, in the post-WWII period when permanent shocks to inflation are a more regular feature of the data, such shocks are estimated to have smaller long-run effects.

Three avenues of further research in this area seem to us to be particularly worthy of pursuit: First, it would be useful to try to identify the exact mechanism by which an exogenous increase in

inflation leads to a rise in consumption, investment, and output. Second, what accounts for the finding that the observed response of real variables to a *given size* inflation shock is smaller in the post-WWII period? Finally, it would be interesting to estimate these types of structural VECMs using panel data and look for differences in the real effects of inflation across high inflation and low inflation countries.

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## Appendix A: Steady-State Paths

The optimization problem is to choose the sequence  $\{C_t, K_{t+1}, M_{t+1}^d\}$  to maximize (1) subject to the sequence of constraints (2) and (3) in the text. If, in the long run,  $g$  is constant and technology,  $A$  grows at rate  $\mu$ , then the first order conditions and market equilibrium imply that the steady-state paths satisfy the following conditions:

$$\frac{1}{C_t} = \lambda_t + a_c \gamma_t \quad (\text{A1})$$

$$\frac{\phi_L}{1-N_t} = \lambda_t Z_t^\theta N_t^{\theta-1} K_t^{1-\theta} \quad (\text{A2})$$

$$\beta \phi_M \left( \frac{P_{t+1}}{M_{t+1}^d} \right) + \beta \lambda_{t+1} + \beta \gamma_{t+1} = (1+\pi) \lambda_t \quad (\text{A3})$$

$$\beta \left[ (1-\theta) Z_{t+1}^\theta N_{t+1}^\theta K_{t+1}^{1-\theta} + (1-\delta) \right] \lambda_{t+1} + \beta \gamma_{t+1} a_k (1-\delta) = \lambda_t + a_k \gamma_t \quad (\text{A4})$$

$$Z_t^\theta F(N_t, K_t) - (\mu + \delta) K_t - C_t = 0 \quad (\text{A5})$$

$$\gamma_t \left\{ \frac{M_t (1 + \mu_M)}{P_t} - a_c C_t - a_k (\mu_A + \delta) K_t \right\} = 0 \quad (\text{A6})$$

where  $\lambda_t, \gamma_t$  represent the Lagrange multipliers associated with (2) and (3) respectively,  $\mu_M$  is the long-run money growth rate, and  $\pi$  represents the constant steady-state inflation rate, which can be shown to be  $\mu_M - \mu$ .

## Appendix B: Data Sources

(1) Y = real gross domestic product in billions of 1987 dollars. The sources are Kendrick (1961) table A-IIa from 1889-1928, and the National Income and Product Accounts (NIPA) from 1929-1995 (U.S. Department of Commerce (1993) and various issues of the Survey of Current Business).

(2) C = real consumption expenditures in billions of 1987 dollars. Sources are the same as for Y.

(3) I = real gross private domestic investment in billions of 1987 dollars. Sources are the same as for Y.

(4) G = real federal government expenditures on goods and services in billions of 1987 dollars. Sources are the same as for Y.

(5) P = GDP deflator, taken as the ratio of nominal GDP to real GDP (1987 = 1.00). Nominal GDP data are taken from Kendrick (1961) table A-IIb from 1889-1928, and NIPA from 1929-1995.

(6) POP = total resident population of the United States, taken from U.S. Bureau of the Census (1976 and 1992) and updates.

## REFERENCES

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United States Department of Commerce, Bureau of Economic Analysis (1993) National Income and Product Accounts, Volume I, 1929-58 and Volume II, 1959-88 (Washington, DC).

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----- (1992) Statistical Abstract of the United States (Washington, DC).

## Appendix C: Identification and Estimation Strategy

The structural model in *MA* form is (15) in the text and reproduced below for convenience:

$$\Delta X_t = \theta(L)\boldsymbol{\varepsilon}_t; \quad \text{var}(\boldsymbol{\varepsilon}_t) = S \equiv \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix}, \quad \theta(1) \equiv [\theta \quad 0] \equiv \begin{bmatrix} \theta_1 & 0 \\ \theta_2 & 0 \end{bmatrix}, \quad (\text{C1})$$

where recall  $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_g \quad \boldsymbol{\varepsilon}_\pi \quad \boldsymbol{\varepsilon}_A \quad \boldsymbol{\varepsilon}_1^T \quad \boldsymbol{\varepsilon}_2^T)$  consists of the three permanent and two transitory shocks and  $X = (g, \pi, \ln Y, \ln C, \ln I)$ .  $S_{11}$  (3x3),  $S_{22}$  (2x2) are the *diagonal* covariance matrices of the *structural* permanent and transitory disturbances respectively with  $S_{12} = S_{21} = 0$ , implying the *independence* of the permanent and transitory disturbances. The matrix  $\theta$  (5x3) is the product of the two matrices in (17) in the text; we have partitioned  $\theta$  further into  $\theta_1$  (3x3) and  $\theta_2$  (2x3), with  $\theta_1$  *lower triangular*.

The reduced-from *VECM* can be used to obtain the following reduced-form *MA* representation:

$$\Delta X_t = C(L)e_t; \quad \text{var}(e_t) = V, \quad C(1) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (\text{C2})$$

where  $C(1)$  has been partitioned into its first three rows,  $C_1$  (3x5), and its last two rows,  $C_2$  (2x5).

Next, express the structural disturbances as linear combinations of reduced-form disturbances:

$$\boldsymbol{\varepsilon}_t = P^{-1}e_t; \quad P^{-1} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (\text{C3})$$

where  $P^{-1}$  has been partitioned for convenience into its first three rows,  $P_1$  (3x5), and its last two rows,  $P_2$  (2x5). To show that our model identifies the permanent structural disturbances, we have to demonstrate that, under the assumptions given above (that  $S_{11}$ ,  $S_{22}$  are diagonal matrices,  $S_{12}$ ,  $S_{21}$  are null matrices, and  $\theta_1$  is lower triangular),  $P_1$  is determined uniquely.

*Obtaining the First Three Rows of  $P^{-1}$  ( $P_1$ )*

From (C1)-(C3),

$$\begin{aligned} \theta(1)\boldsymbol{\varepsilon}_t &= C(1)e_t \\ \Rightarrow \theta(1)S\theta(1)' &= C(1)VC(1)' \equiv W \\ \Rightarrow \theta_1 S_{11} \theta_1' &= W_{11}, \end{aligned} \quad (\text{C4})$$

where  $W_{11}$  is the upper left-hand (3x3) submatrix of  $W$ . It follows directly from the last line of (C4) that the Choleski factor of  $W_{11}$  will give the unique  $\theta_1$  such that  $S_{11}$  is the identity matrix. Then, from the first line of (C4), (C3), and the partitions for  $\theta(1)$  and  $C(1)$  given in (C1), (C2) respectively, it in turns follows that  $P_1 = \theta_1^{-1}C_1$ .

*Obtaining the Last Two Rows of  $P^{-1}$  ( $P_2$ )*

We need to do this to complete our identification. This is to ensure that we impose the assumption that the permanent disturbances are independent of the transitory disturbances, so that the

impulse responses and variance decompositions with respect to the permanent disturbances are valid and not mixed up with the effects of the transitory disturbances.

From (C3), it follows that:

$$\begin{aligned} S &= P^{-1}VP^{-1'} \\ \Rightarrow S_{12} &= P_1VP_2' \\ S_{22} &= P_2VP_2' \end{aligned} \tag{C5}$$

We must choose  $P_2$  such that  $S_{12} = 0$  and  $S_{22}$  is invertible. This can be done by picking any two *linearly independent* solutions to  $P_1Vx = 0$ , where  $x$  is a (5x1) vector of unknowns being solved for. One way to do this is to pick the two independent eigenvectors associated with the non-zero eigenvalues of the matrix  $M$ , where  $M = I - A'(AA')^{-1}A$ , with  $A \equiv P_1V$ . Since  $Mx = \lambda_i x$ , where the  $\lambda_i$ s are the eigenvalues of  $M$ ,  $AM = 0$  (which is true by construction) implies  $Ax = 0$ . Note that this is *just one way* to obtain  $P_2$  and that is why the *transitory disturbances are not individually identified*.

Right now,  $S_{11}$  is the identity matrix and  $\theta$  is not in the exact form given by (16); it does not have the unit normalizations shown there. To put  $\theta$  in the required form, we can renormalize the size of the shocks, such that the long-run response of a variable to its own shock is normalized to be unity. This will put  $\theta$  in the form of (16) and make  $S_{11}$  diagonal only, rather than the identity matrix.

**Table 1: Summary of Theoretical Results on the Long-Term Real Effects of Inflation**

| Model     | Effect of a once-and-for-all change in $\pi$ on the steady-state value of |          |          |          |                |     | Super-neutral? | Tobin Effect? | Reverse Tobin? | Fisher Effect? |
|-----------|---|----------|----------|----------|----------------|-----|----------------|---------------|----------------|----------------|
|           | c = C/ZN  | k = K/ZN | i = I/ZN | y = Y/ZN | N              | I/Z |                |               |                |                |
| Sidrauski | 0   | 0        | 0        | 0        | 0              |     | Yes            | No            | No             | Yes            |
| CIA (C)   | 0   | 0        | 0        | 0        | -              | -   | No             | No            | Yes            | Yes            |
| CIA(C,I)  | -   | -        | -        | -        | -              | -   | No             | No            | Yes            | No             |
| Tobin     | ?   | +        | +        | +        | 0 <sup>1</sup> | +   | No             | Yes           | No             | No             |

Notes: + indicates positive effect, - indicates negative effect, 0 indicates no effect, and ? indicates ambiguous effect. <sup>1</sup>Models generating a Tobin effect typically focus on capital accumulation. In general, a negative effect of inflation on work effort could still be present in these models.

**Table 2: Summary Statistics - Means and Standard Deviations**

|            | Full Sample<br>(1889-1995) | Pre-WWI<br>(1889-1914) | Inter-War<br>(1918-41) | Post-War<br>(1950-95) | Post-OPEC<br>(1973-95) |
|------------|----------------------------|------------------------|------------------------|-----------------------|------------------------|
| G/Y        | 19.4<br>(8.56)             | 11.6<br>(0.92)         | 18.3<br>(5.66)         | 21.7<br>(3.19)        | 19.0<br>(0.69)         |
| $\pi$      | 3.05<br>(5.54)             | 0.81<br>(2.72)         | 0.36<br>(7.03)         | 4.30<br>(2.42)        | 5.56<br>(2.48)         |
| C/Y        | 63.8<br>(5.40)             | 64.1<br>(2.10)         | 67.8<br>(4.52)         | 63.3<br>(3.27)        | 66.1<br>(1.52)         |
| I/Y        | 16.6<br>(5.41)             | 22.9<br>(3.13)         | 13.7<br>(4.98)         | 15.8<br>(1.25)        | 16.1<br>(1.32)         |
| $\Delta y$ | 3.13<br>(5.92)             | 3.49<br>(5.82)         | 2.92<br>(7.29)         | 3.08<br>(2.40)        | 2.39<br>(2.15)         |

Notes: G/Y, C/Y, and I/Y denote, respectively, the ratios of real government purchases, real consumption, and real investment to real GDP;  $\pi$  denotes the annual percentage change in the GDP deflator, and  $\Delta y$  denotes real GDP growth. Reported above is the mean and standard deviation (in parenthesis) of each series, in percent.

**Table 3: Unit Roots Tests**

| Variable | ADF<br>(level) | ADF<br>(1st diff.) | KPSS<br>(level) | KPSS<br>(1901-95) | ADF- level<br>[war dynamics] |
|----------|----------------|--------------------|-----------------|-------------------|------------------------------|
| c        | -2.14          | -4.24*             | 0.24**          | 2.12**            | -2.05                        |
| i        | -2.24          | -4.80**            | 0.28**          | 0.47**            | -2.24                        |
| y        | -2.95          | -4.57**            | 0.08            | 2.02**            | -2.82                        |
| $\pi$    | -3.51*         | -6.00**            | 0.05            | 0.25**            | -2.62                        |
| G/Y      | -2.28          | -5.98**            | 0.65*           | 0.65**            | -0.49                        |

Notes: c, i, and y denote, respectively, the logs of real per capita consumption, investment, and GDP;  $\pi$  denotes the annual percentage change in the CPI, and G/Y is the ratio of government purchases to GDP. ADF denotes the Augmented Dickey-Fuller test statistic for the unit root null hypothesis. KPSS denotes the Kwiatkowski, Phillips, Schmidt, and Shin test of the null of stationarity. ADF [war dynamics] refers to the ADF tests that allow the short-run dynamics for the world war years to be different. The sample period is 1889-1995, unless otherwise noted. A #, \*, and \*\* indicates rejection of the null at 10%, 5%, and 1%, respectively. A time trend is included in all tests for all variables except (G/Y). A lag length of 5 is used in all tests.

**Table 4: Tests of Cointegrating rank****Results for 3-trend model**

| H <sub>0</sub> : CI rank=p | Max eigenvalue statistic | Max eigenvalue statistic (df) | 95% critical value | Trace statistic | Trace statistic (df) | 95% critical value |
|----------------------------|--------------------------|-------------------------------|--------------------|-----------------|----------------------|--------------------|
| p = 0                      | 44.84**                  | 38.31*                        | 33.5               | 106.1**         | 90.69**              | 68.5               |
| p ≤ 1                      | 28.08**                  | 24.0*                         | 27.1               | 61.3**          | 52.38*               | 47.2               |
| p ≤ 2                      | 19.85                    | 16.96                         | 21.0               | 33.23*          | 28.39                | 29.7               |
| p ≤ 3                      | 13.26                    | 11.33                         | 14.1               | 13.38           | 11.43                | 15.4               |
| p ≤ 4                      | 0.12                     | 0.10                          | 3.8                | 0.12            | 0.11                 | 3.8                |

System: G/Y,  $\pi$ , y, c, i; Sample = 1893-1995; Lag length = 3.  
Constant included in the deterministic component.

**Results for 2-trend model**

| H <sub>0</sub> : CI rank=p | Max eigenvalue statistic | Max eigenvalue statistic (df) | 95% critical value | Trace statistic | Trace statistic (df) | 95% critical value |
|----------------------------|--------------------------|-------------------------------|--------------------|-----------------|----------------------|--------------------|
| p = 0                      | 42.3**                   | 37.4**                        | 27.1               | 82.6**          | 73.0**               | 47.2               |
| p ≤ 1                      | 27.4**                   | 24.2**                        | 21.0               | 40.3**          | 35.6**               | 29.7               |
| p ≤ 2                      | 12.7                     | 11.2                          | 14.1               | 12.8            | 11.4                 | 15.4               |
| p ≤ 3                      | 0.22                     | 0.20                          | 3.8                | 0.22            | 0.20                 | 3.8                |

System:  $\pi$ , y, c, i; Sample = 1893-1995; Lag length = 3.  
Constant, G/Y and 3 lags of G/Y are included in the deterministic component.

NOTES: (1) \* (\*\*) indicates significance at the 5% (1%). (2) The maximum eigenvalue statistic (df) and trace statistic (df) apply a simple small-sample correction to Johansen's statistics (replacing T by T-nm, where T = number observations, n=number of variables, m = number of lags), as recommended by Reimers (1992).

**Table 5: Estimates of Structural Cointegrating Vectors**

| Variable  | Coefficient              |                         |                            |                         |
|---|--------------------------|-------------------------|----------------------------|-------------------------|
|   | 3-trend model            |                         | 2-trend model <sup>1</sup> |                         |
|   | Vector 1:<br>consumption | Vector 2:<br>investment | Vector 1:<br>consumption   | Vector 2:<br>investment |
| c   | 1.00 <sup>r</sup>        | 0.00 <sup>r</sup>       | 1.00 <sup>r</sup>          | 0.00 <sup>r</sup>       |
| i   | 0.00 <sup>r</sup>        | 1.00 <sup>r</sup>       | 0.00 <sup>r</sup>          | 1.00 <sup>r</sup>       |
| y   | -1.00 <sup>r</sup>       | -1.00 <sup>r</sup>      | -1.00 <sup>r</sup>         | -1.00 <sup>r</sup>      |
| $\pi$   | 4.02<br>(0.72)           | -5.58<br>(1.17)         | 11.0<br>(1.95)             | -6.52<br>(1.26)         |
| G/Y   | -0.63<br>(0.39)          | 3.84<br>(0.63)          | ---                        | ---                     |
| Lag length                                      | 3                        |                         | 3                          |                         |
| $\chi^2(2)$ [p-value] <sup>2</sup>              | 16.5 [.00]               |                         | 17.7 [.00]                 |                         |
| Coefficient on y<br>(unrestricted) <sup>3</sup> | -1.075<br>(0.024)        | -0.99<br>(0.09)         | -1.08<br>(0.03)            | -0.97<br>(0.10)         |

Notes: Standard errors are in parentheses. A " r " indicates that the coefficient was constrained to the value shown. <sup>1</sup>In the four-variable system, G/Y is treated as stationary and exogenous; its contemporaneous value and three lagged values are included as deterministic components in the system. <sup>2</sup>This is the Chi-squared statistic associated with the likelihood ratio test of the null hypothesis that the restrictions imposed on the output variable in the two vectors are jointly satisfied. "P-value" refers to the marginal significance level of the  $\chi^2$  statistic. <sup>3</sup>The coefficient on y from a separate estimate in which the unit coefficient restriction is relaxed.

**Table 6: Estimates of Structural Parameters**

| Model         | Coefficient         |                     |                     |                |
|---------------|---------------------|---------------------|---------------------|----------------|
|               | $\beta_N + \beta_Y$ | $\beta_N + \beta_C$ | $\beta_N + \beta_I$ | $\beta_G$      |
| 3-trend model | 7.47<br>(2.14)      | 3.45<br>(1.77)      | 13.1<br>(2.44)      | 0.48<br>(0.18) |
| 2-trend model | 20.7<br>(5.84)      | 9.74<br>(5.09)      | 27.3<br>(5.98)      | ---            |

Notes: Standard errors, shown in parenthesis, were computed by Monte Carlo simulation using 1,000 replications.

**Table 7: Variance Decompositions: 3-Trend Model**

| A. Fraction of the forecast error variance attributed to the fiscal shock    |                |                |                |                |                |  |
|--|----------------|----------------|----------------|----------------|----------------|--|
| Horizon:   | G/Y            | $\pi$          | y              | c              | i              |  |
| 1  | 77.8<br>(17.6) | 42.9<br>(14.9) | 49.7<br>(16.3) | 0.17<br>(7.35) | 0.18<br>(8.43) |  |
| 2  | 71.1<br>(17.4) | 37.9<br>(13.5) | 53.2<br>(14.6) | 1.86<br>(6.96) | 1.45<br>(8.05) |  |
| 5  | 62.3<br>(15.4) | 32.8<br>(11.4) | 51.1<br>(13.3) | 1.82<br>(6.52) | 1.94<br>(7.23) |  |
| 20   | 60.8<br>(14.9) | 32.4<br>(11.1) | 51.0<br>(13.2) | 1.92<br>(6.47) | 2.87<br>(6.97) |  |
| B. Fraction of the forecast error variance attributed to the inflation shock |                |                |                |                |                |  |
| Horizon:   | G/Y            | $\pi$          | y              | c              | i              |  |
| 1  | 1.57<br>(4.33) | 2.68<br>(6.64) | 17.5<br>(12.0) | 5.13<br>(8.31) | 1.94<br>(7.01) |  |
| 2  | 9.37<br>(6.30) | 8.89<br>(6.56) | 18.5<br>(10.6) | 5.35<br>(7.58) | 3.31<br>(6.48) |  |
| 5  | 18.3<br>(6.95) | 14.8<br>(7.01) | 17.8<br>(9.54) | 5.63<br>(6.64) | 14.1<br>(6.27) |  |
| 20   | 19.0<br>(7.03) | 15.3<br>(7.04) | 17.9<br>(9.43) | 5.77<br>(6.66) | 15.7<br>(6.68) |  |
| C. Fraction of the forecast error variance attributed to the output shock    |                |                |                |                |                |  |
| Horizon:   | G/Y            | $\pi$          | y              | c              | i              |  |
| 1  | 1.93<br>(6.54) | 4.06<br>(6.83) | 28.8<br>(15.8) | 90.9<br>(14.1) | 25.1<br>(13.0) |  |
| 2  | 3.71<br>(7.08) | 7.27<br>(7.79) | 24.2<br>(12.4) | 83.9<br>(12.7) | 28.6<br>(11.8) |  |
| 5  | 3.61<br>(6.26) | 6.81<br>(6.87) | 23.9<br>(11.2) | 81.4<br>(11.7) | 24.7<br>(9.95) |  |
| 20   | 3.90<br>(6.16) | 6.74<br>(6.75) | 23.9<br>(11.1) | 81.0<br>(11.6) | 24.3<br>(9.46) |  |

Notes: Standard errors, shown in parenthesis, were computed by Monte Carlo simulation using 1,000 replications.

**Table 8: Variance Decompositions: 2-Trend Model**


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A. Fraction of the forecast error variance attributed to the inflation shock

| Horizon: | $\pi$          | y              | c              | i              |
|----------|----------------|----------------|----------------|----------------|
| 1        | 2.80<br>(7.70) | 34.1<br>(18.6) | 5.23<br>(8.98) | 1.89<br>(7.63) |
| 2        | 8.04<br>(7.22) | 34.7<br>(18.1) | 7.80<br>(8.44) | 1.86<br>(7.12) |
| 5        | 10.8<br>(7.91) | 38.8<br>(14.5) | 7.92<br>(7.44) | 4.78<br>(6.74) |
| 20       | 11.0<br>(8.01) | 39.6<br>(14.2) | 8.16<br>(7.44) | 6.31<br>(6.65) |

B. Fraction of the forecast error variance attributed to the output shock

| Horizon: | $\pi$          | y              | c              | i              |
|----------|----------------|----------------|----------------|----------------|
| 1        | 0.11<br>(3.89) | 56.6<br>(19.1) | 94.4<br>(12.0) | 37.1<br>(15.8) |
| 2        | 2.89<br>(5.24) | 55.5<br>(18.0) | 86.2<br>(11.0) | 39.0<br>(14.7) |
| 5        | 3.24<br>(5.13) | 50.0<br>(14.6) | 83.5<br>(10.2) | 36.6<br>(13.3) |
| 20       | 3.21<br>(5.13) | 49.0<br>(14.4) | 83.2<br>(10.3) | 36.5<br>(13.0) |

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Notes: Standard errors, shown in parenthesis, were computed by Monte Carlo simulation using 1,000 replications.

**Table 9: Analysis of Sub-periods**

| Estimates   | Model                            |                                  |                                 |                                 |
|---|----------------------------------|----------------------------------|---------------------------------|---------------------------------|
|   | 3-trend<br>Post-War (1950-95)    | 3-Trend<br>Inter-War (1918-41)   | 2-Trend<br>Post-War (1950-95)   | 2-Trend<br>Inter-War (1918-41)  |
| CI vectors:   |                                  |                                  |                                 |                                 |
| (1) [(c-y), $\pi$ , G/Y]  | [1.0 <sup>r</sup> , 0.71, 1.87]  | [1.0 <sup>r</sup> , 6.38, -3.27] | [1.0 <sup>r</sup> , 1.01, ---]  | [1.0 <sup>r</sup> , 2.24, ---]  |
| (2) [(i-y), $\pi$ , G/Y]  | [1.0 <sup>r</sup> , -1.00, 0.67] | [1.0 <sup>r</sup> , -29.2, 22.6] | [1.0 <sup>r</sup> , -0.95, ---] | [1.0 <sup>r</sup> , -11.3, ---] |
| Structural parameters:<br>( $\beta_N + \beta_Y$ , $\beta_N + \beta_C$ , $\beta_N + \beta_I$ , $\beta_G$ ) | (2.69, 1.98, 3.69, -1.49)        | (11.7, 5.35, 40.9, 0.78)         | (1.51, 0.50, 2.46, ---)         | (4.90, 2.66, 16.2, ---)         |
| Variance decompositions <sup>1</sup> :<br>(2-Year horizon)  |                                  |                                  |                                 |                                 |
| (1) Pct. due to fiscal shock<br>[G/Y, $\pi$ , y, c, i]  | [65.6, 20.6, 6.55, 22.8, 16.7]   | [64.4, 38.1, 17.5, 5.94, 35.3]   | [ --- ]                         | [ --- ]                         |
| (2) Pct. due to inflation shock<br>[G/Y, $\pi$ , y, c, i]   | [15.6, 3.62, 62.4, 43.8, 31.2]   | [15.9, 6.49, 50.2, 50.7, 51.8]   | [--- , 10.7, 64.1, 24.8, 48.4]  | [--- , 45.9, 89.9, 58.0, 84.9]  |
| (3) Pct. due to output shock<br>[G/Y, $\pi$ , y, c, i]  | [13.6, 70.9, 28.6, 32.2, 35.8]   | [7.80, 9.12, 27.9, 41.7, 7.76]   | [--- , 24.4, 25.0, 65.2, 23.7]  | [--- , 22.9, 6.28, 39.0, 6.18]  |

Notes: A " r " indicates that the coefficient was constrained to the value shown. <sup>1</sup>The fraction of the forecast error variance of each variable attributed to the fiscal, inflation, and output shocks.

# Figure 1. Data in Levels

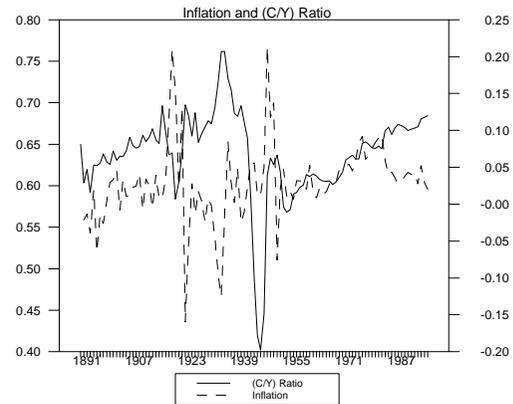
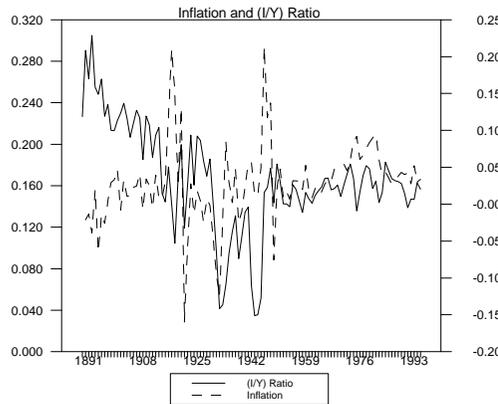
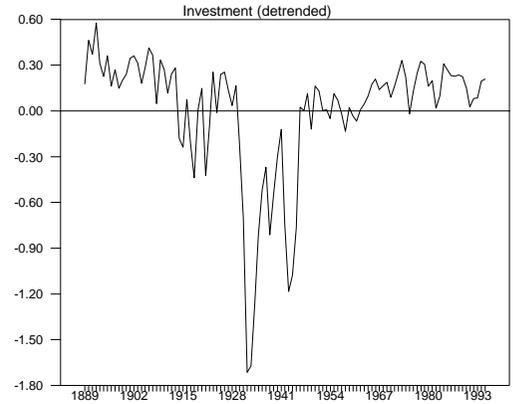
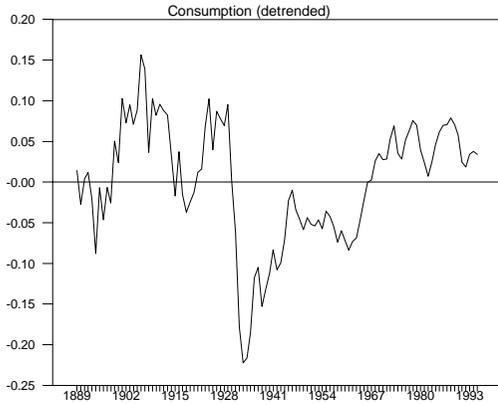
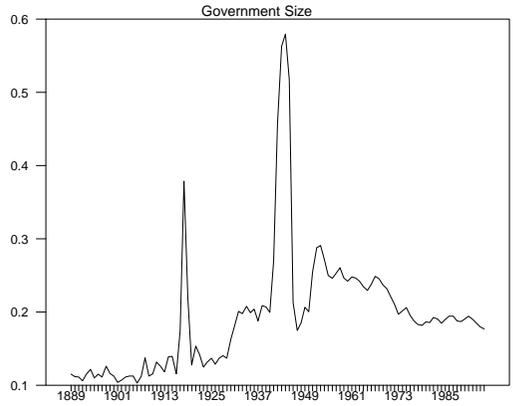
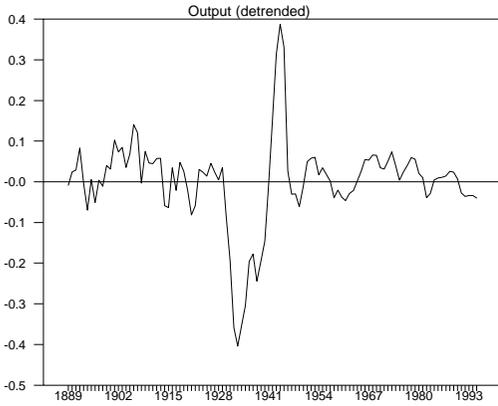


Figure 2. Autocorrelations - Levels

