Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 673

July 2000

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Abstract: The general inability of sticky-price monetary business cycle models to generate liquidity effects has been noted in the recent literature by authors such as Christiano (1991), Christiano and Eichenbaum (1992a, 1995), King and Watson (1996), and Bernanke and Mihov (1998b). This paper develops a sticky-price monetary business cycle model that is capable of generating an empirically plausible liquidity effect. Time-to-build and time-to-plan in investment together with habit-persistence in consumption are the features of the model that allow it to produce this result.

Keywords: sticky-price monetary business cycle models, time-to-build, time-to-plan, multiple capital stocks, habit-persistence, liquidity effect.

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This paper develops a sticky-price monetary business cycle model that is capable of generating a liquidity effect (a decline in nominal interest rates in response to an expansionary monetary shock).

The difficulties encountered by standard monetary business cycle models in generating liquidity effects have been noted by several authors (for example, Christiano (1991), Christiano and Eichenbaum (1992a, 1995), King and Watson (1996), and Bernanke and Mihov (1998b)). The general inability of these models to produce a liquidity effect significantly detracts from the attractiveness of the monetary business cycle literature: The liquidity effect is so fundamental to the received view of how monetary policy affects the economy that generating such an effect should be a basic hurdle for any monetary business cycle model to clear.

A number of monetary business cycle models have, under specific assumptions, been able to generate the required response in nominal interest rates (see Grossman and Weiss (1983), Rotemberg (1984), Lucas (1990), Christiano (1991), Fuerst (1992), and Christiano and Eichenbaum (1992, 1995) for examples). These models (all of which are modified cash-in-advance models) fall into the class of monetary business cycle models known as limited participation models; they generate real responses to monetary policy by assuming that unanticipated monetary injections impact differently on different agents of the model. As a result, after a monetary expansion, certain agents in the model end up holding disproportionately large money stocks relative to their steady-state share of aggregate money holdings; this then affects the real economy through various channels.

In these limited participation models, a major source of difficulty in generating a liquidity effect arises from the fact that money growth is assumed to follow an AR(1) process. A positive monetary shock therefore leads to higher future money growth, which raises anticipated inflation and puts upward pressure on nominal interest rates. As an illustration, in models of the type used by Christiano (1991) and Christiano and Eichenbaum (1992a, 1995) demand for money balances arises from households' holding money balances to finance nominal consumption and firms' holding money balances to pay nominal wages. If an x percent increase in the money supply affects both classes of agents identically, there would be no change in the real interest rate or real economy as households would increase their money demand by x percent (since prices increase by x percent) and firms would increase their money demand by x percent (since nominal wages increase by x percent). If instead the monetary injection initially only affects firms (say because households cannot adjust their desired money balances), firms must be persuaded to hold the entire increment to money balances, which puts downward pressure on interest rates. Of course, at the same time there is the increase in future inflation that results from the monetary shock; this puts *upward* pressure on nominal interest rates. Whether a liquidity effect results depends on which of the two effects dominates. Naturally, mechanisms that limit agents' ability to absorb additional money balances tend to make a decline in interest rates more likely; the challenge, then, for these models' proponents is to identify and incorporate these sorts of mechanisms.[1]

Generating a liquidity effect in the class of monetary business cycle models that achieves real responses to money policy through the assumption of sticky prices and/or wages has received somewhat less attention in the literature. Like their liquidity model counterparts, these models also encounter difficulties in generating a liquidity effect; once again, an important source of difficulty involves the response of expected inflation to a monetary shock. Moreover, these models typically face a second hurdle in trying to produce a liquidity effect: The predicted response of fixed investment is often very strong—so much so that *real* interest rates can actually rise following a monetary shock. As King (1991) and Kimball (1995) have noted, this feature renders these models incapable of generating a liquidity effect in their basic form. Put another way, since a positive monetary shock necessarily leads to an increase in anticipated inflation, the only way to obtain a liquidity effect in a sticky-price monetary business cycle model is to generate a decline in real interest rates in response to a monetary shock that is larger than the increase in anticipated inflation.

This paper demonstrates that a liquidity effect obtains in an otherwise standard sticky-price monetary business cycle model when assumptions are made about the economy's investment and consumption technologies. The assumptions for investment—that capital takes time to build and to plan, and that investment plans are costly to change once they are underway—act to reduce the response of investment following a monetary shock. Similarly, the assumption of habit-persistence in consumption weakens the response of consumption to a monetary shock while strengthening the response of saving. When combined, the weaker response of investment and stronger response of saving force a decline in real interest rates that exceeds the increase in anticipated inflation.

The model is developed in the first three sections of the paper. Section one constructs the basic version of the model (with habit-persistence in consumption but without time-to-build or time-to-plan in investment) that is unable to generate a liquidity effect for reasonable parameter values. Section two describes how time-to-build, time-to-plan, and ex-post investment inflexibilities are incorporated into the model, and constructs a simple model that includes these features. Section three outlines a more detailed version of this model that when calibrated is capable of generating a liquidity effect.

The paper does not report any results from the model with time-to-build and time-to-plan in investment but without habit-persistence in consumption. King (1991) notes that attempting to generate a liquidity effect by dampening the response of investment

[1] This overview does not really do justice to the complexity of these models; in particular, their predicted paths for interest rates are complicated by the effect of monetary shocks on labor supply and investment. The basic tension in these models, however, involves the interplay between the effect of anticipated inflation on interest rates and the reduction in rates that obtains as agents reequilibrate their money holdings.

typically results in a very strong initial response of consumption; this runs contrary to what is observed empirically. The results that obtain from the model with only time-to-build and time-to-plan in investment provide support for King's observation. I note also that the model with just time-to-build and time-to-plan is capable of generating only a very modest and short-lived liquidity effect. The inclusion of habit-persistence in consumption is thus important inasmuch as it yields a more empirically consistent response of consumption to a monetary shock as well as a stronger and more persistent liquidity effect.[2]

In order to simplify the exposition, I assume in developing the model that in the steady state each type of capital accounts for the same share of total capital, and that spending for multi-stage investment projects is split evenly across all the periods over which the spending takes place. This assumption is unrealistic and is relaxed in section four, where I calibrate the model's steady state capital shares and investment spending profiles to match the data as closely as possible. Section five compares the responses of the model's key variables to their empirical counterparts, section six discusses the robustness of the model's results to the calibrated parameter values, and section seven concludes.

1 The Benchmark Sticky-Price Monetary Business Cycle Model with Habit-Persistence in Consumption

This section outlines the benchmark sticky-price monetary business cycle model with habit-persistence in consumption around which the paper's analysis is built. The model is developed around a discrete-time version of Kimball's (1995) model; given his results, it is unsurprising that this model also turns out to be incapable of generating a decline in either the real or nominal interest rate in response to an expansionary monetary shock.

The economy is characterized by the following set of agents: a continuum of monopolistically competitive intermediate-goods producers, each of which hires labor and capital to produce a differentiated product for which they set the price (so meeting demand at the posted price);[3] a competitive representative final-good producing firm that uses all of the differentiated intermediate goods to produce a final composite market good that is used for consumption and investment in the economy; a representative household, who consumes some of the final market good, supplies labor and rents capital to firms, undertakes investment in the economy's capital stock, and holds money balances to finance its consumption and investment transactions; and a monetary authority who sets the growth rate of the nominal money supply.

[2] Fuhrer (1998) also reports that adding habit-persistence to an otherwise standard specification of consumer behavior noticeably improves the empirical fit of consumption.

[3] Slightly differentiated products are necessary in this context since they provide firms with some degree of market power, in turn allowing them to set prices that differ from the competitive market-clearing level.

1.1 Setup of the Model

1.1.1 Consumers

The household sector of the economy is characterized by an infinitely lived representative household with preferences defined over effective consumption $\mathbf{C}_t$ (which I describe in the next section), "habit" $\mathbf{G}_t$, and total hours worked H_t:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln \left(\mathbf{C}_t - \mathbf{G}_t \right) - \left(\frac{1}{1+s} \right) \left(H_t \right)^{1+s} \right] \right\} \tag{1}$$

where β and s denote the consumer's discount factor and the inverse of its elasticity of intertemporal labor substitution.[4] The variable $\mathbf{G}_t$ is the household's "habit," which evolves according to:

$$\mathbf{G}_t = h\mathbf{G}_{t-1} + b\mathbf{C}_{t-1}. \tag{2}$$

where h and b are parameters in the habit-formation process.

The household's flow budget constraint is given by:

$$B_t \leq R_{t-1}B_{t-1} + \omega_t H_t + \rho_t^A K_t^A + profits_t - T_t - \mathbf{P}_t\mathbf{C}_t - \mathbf{P}_t\mathbf{I}_t^A \tag{3}$$

where B_t denotes the household's end-of-period t holdings of nominal bonds; R_t is the gross nominal interest rate in period t; K_t^A denotes the household's holdings of type-A capital at the start of period t; ω_t and ρ_t^A are the nominal wage and rental rate for type-A capital; T_t is the net nominal government transfer to the household; $\mathbf{I}_t^A$ is effective investment in type-A capital; and $\mathbf{P}_t$ is the effective price of output (again, I postpone discussion of "effective" quantities and prices until the next section).[5]

The household owns the economy's capital stock, which in the benchmark model is defined to be a single type of factor that takes only one period to be put in place:

$$K_{t+1}^A = (1-\delta)K_t^A + K_t^A J\left(\frac{\mathbf{I}_t^A}{K_t^A}\right). \tag{4}$$

Here δ is the rate of depreciation and $K_t^A J\left(\frac{\mathbf{I}_t^A}{K_t^A}\right)$ describes the adjustment costs for changing the capital stock. Following Hayashi (1982), the function $J(.)$ is assumed to have the properties $J(\delta) = \delta$, $J'(\delta) = 1$, and $J''(\delta) \leq 0$. Note that the capital stock for any period is known with certainty in the preceding period.

The household chooses $\left\{\mathbf{C}_t, K_{t+1}^A, H_t\right\}_{t=0}^{\infty}$ to maximize its utility (equation (1)) subject to its habit-formation process (equation (2)), its budget constraint (equation (3)), and the

[4] Here and throughout the paper, boldface variable names denote effective quantities.

[5] The economy's capital stock is called type-A capital for reasons that will become obvious when the more detailed versions of the model are presented in subsequent sections.

evolution of the capital stock (equation (4)). This yields the following first-order conditions for the household:

$$
\begin{aligned}
\frac{1}{(\mathbf{C}_t-\mathbf{G}_t)} &= \beta E_t\left[\left(\frac{1}{\mathbf{C}_{t+1}-\mathbf{G}_{t+1}}\right)\left((h+b)+\left(\frac{\frac{R_t}{\mathbf{\Pi}_{t+1}}-h}{\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}-h}\right)\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}\right)\right)\right] \\
&\quad -\beta^2 E_t\left[\left(\frac{1}{\mathbf{C}_{t+2}-\mathbf{G}_{t+2}}\right)\left((h+b)\left(\frac{\frac{R_t}{\mathbf{\Pi}_{t+1}}-h}{\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}-h}\right)\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}\right)\right)\right]
\end{aligned}
\tag{5}
$$

$$
\frac{(H_t)^s}{\widetilde{\boldsymbol{\omega}}_t}=\left(\frac{1}{\mathbf{C}_t-\mathbf{G}_t}\right)-\beta(b+h)E_t\left[\frac{1}{\mathbf{C}_{t+1}-\mathbf{G}_{t+1}}\right]+\beta h E_t\left[\frac{(H_{t+1})^s}{\widetilde{\boldsymbol{\omega}}_{t+1}}\right] \tag{6}
$$

$$
\begin{aligned}
& E_t\left[-\frac{R_t}{\mathbf{\Pi}_{t+1}}J^{-1'}\left(\left(\frac{K^A_{t+1}}{K^A_t}\right)-(1-\delta)\right)+\widetilde{\boldsymbol{\rho}}_{t+1}\right] \\
= \; & E_t\left[J^{-1}\left(\left(\frac{K^A_{t+2}}{K^A_{t+1}}\right)-(1-\delta)\right)-J^{-1'}\left(\left(\frac{K^A_{t+2}}{K^A_{t+1}}\right)-(1-\delta)\right)\left(\frac{K^A_{t+2}}{K^A_{t+1}}\right)\right]
\end{aligned}
\tag{7}
$$

where $\widetilde{\boldsymbol{\omega}}_t=\frac{\omega_t}{\mathbf{P}_t}$, $\widetilde{\boldsymbol{\rho}}_t=\frac{\rho_t}{\mathbf{P}_t}$, and $\mathbf{\Pi}_{t+1}=\frac{\mathbf{P}_{t+1}}{\mathbf{P}_t}$ (note that I use tildes to denote real values). The first-order conditions have the standard interpretations.[6]

1.1.2 Effective Consumption and Investment

Effective consumption is defined as a combination of actual consumption C_t (which comprises the goods bought for consumption by the household) together with the real money balances that are used for buying these consumption goods $\frac{M_t^C}{P_t}$. Note that because nominal money balances $\left(M_t^C\right)$ are used towards the purchase of these goods, the relevant price deflator is the price of the final good (P_t). Effective investment in type-A capital is defined similarly.

I assume a CES aggregator function:

$$
\mathbf{X}_t=\left[a\,(X_t)^\eta+(1-a)\left(\frac{M_t^X}{P_t}\right)^\eta\right]^{\frac{1}{\eta}} \tag{8}
$$

where $X=C$ or I^A. The parameters η $(\in(-\infty,1])$ and a $(\in[0,1])$ determine the importance of real money balances in generating effective consumption (type-A investment)

[6] Equation (5) is the intertemporal IS equation, equation (6) gives the household's labor supply, and equation (7) defines the household's supply of capital in the following period.

flows from actual consumption (type-A investment). Equation (8) has the interpretation that larger money balances allow the household to shop with greater precision so that it effectively consumes (invests) more goods from those that it actually purchases.

The household's optimization problem separates into a dynamic component, which determines its optimal choice of $\left\{\mathbf{C}_t, K_{t+1}^A, H_t\right\}_{t=0}^{\infty}$ and, by equation (4), $\left\{\mathbf{I}_t^A\right\}_{t=0}^{\infty}$, and a within-period static problem that determines $\left\{C_t, I_t^A, \frac{M_t^C}{P_t}, \frac{M_t^{I^A}}{P_t}\right\}_{t=0}^{\infty}$. The static problem takes the following form:

$$\min_{\left\{X_t, M_t^Z\right\}} P_t X_t + \left(1 - \frac{1}{R_t}\right) M_t^X \text{ s.t. } \left[a\left(X_t\right)^{\eta} + (1-a)\left(\frac{M_t^X}{P_t}\right)^{\eta}\right]^{\frac{1}{\eta}} \leq \mathbf{X}_t \tag{9}$$

for $X = C, I^A$. This implies that the price of effective consumption and investment is given by:

$$\mathbf{P}_t = P_t \left[a^{\frac{1}{1-\eta}} + (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{\eta}{1-\eta}}\right]^{-\frac{1-\eta}{\eta}} \tag{10}$$

and yields the following solutions for actual consumption and investment and the money balances that are used for these purchases:

$$X_t = \mathbf{X}_t a^{\frac{1}{1-\eta}} \left[a^{\frac{1}{1-\eta}} + (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{\eta}{1-\eta}}\right]^{-\frac{1}{\eta}} = \mathbf{X}_t \left(a \frac{\mathbf{P}_t}{P_t}\right)^{\frac{1}{1-\eta}}, \text{ and} \tag{11}$$

$$\begin{aligned} \frac{M_t^X}{P_t} &= \mathbf{X}_t (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{1}{1-\eta}} \left[a^{\frac{1}{1-\eta}} + (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{\eta}{1-\eta}}\right]^{-\frac{1}{\eta}} \\ &= \mathbf{X}_t (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{1}{1-\eta}} \left(\frac{\mathbf{P}_t}{P_t}\right)^{\frac{1}{1-\eta}}, \end{aligned}$$

where $X = C, I^A$.

It is also useful to note that:

$$\frac{M_t^X}{P_t} = X_t \left(\frac{1-a}{a}\right)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R_t}\right)^{-\frac{1}{1-\eta}} \tag{12}$$

for $X = C, I^A$, which can be thought of as a consumption- and investment-transactions money demand equation. The two equations for consumption and investment can be aggregated to yield a money demand equation which is expressed in terms of aggregate output.

1.1.3 Final-Good Producers

The representative final-good producing firm uses all of the differentiated intermediate goods to produce a composite final good that is used for consumption and investment. Following Kimball (1995), the production function Y_t is defined implicitly as:

$$1 = \int_0^1 \Psi\left(\frac{Y(z)_t}{Y_t}\right) dz$$

where $Y(z)_t$ is the quantity of the differentiated good z that the final-good producer demands. The function $\Psi(\cdot)$ satisfies $\Psi(1) = 1$ and is increasing and concave.[7]

The final-good producing firm's problem is to choose $\{Y(z)_t \ \forall\ z \in [0,1]\}_{t=0}^{\infty}$ so as to minimize costs given the set of prices $\{P(z)_t \ \forall\ z \in [0,1]\}_{t=0}^{\infty}$ posted by the intermediate-good producers. The final-good producing firm's cost-minimization problem yields a demand function for each of the intermediate goods, defined as:

$$Y(z)_t = Y_t \Psi^{-1\prime}\left(\frac{P(z)_t}{P_t}\Psi'(1)\right). \tag{13}$$

The demand functions for the intermediate goods imply that the competitive price P_t for the final (actual) good is defined implicitly as:

$$1 = \int_0^1 \Psi\left[\Psi^{-1\prime}\left(\frac{P(z)_t}{P_t}\Psi'(1)\right)\right] dz. \tag{14}$$

1.1.4 Intermediate-Good Producers

The intermediate-goods sector is composed of a continuum of monopolistically competitive firms who hire labor and rent capital competitively from the household but set the price of their differentiated good subject to the demand curve that they face for their product.

The intermediate-good producing firms each choose $\left\{H(z)_t, K^A(z)_t\right\}_{t=0}^{\infty}$ to minimize their production costs given the following Cobb-Douglas production function:

$$Y(z)_t = \left(K^A(z)_t\right)^{\alpha} \left(H(z)_t\right)^{(1-\alpha)} - FC$$

where α is the elasticity of output with respect to capital and FC represents a fixed cost faced by the firm.

The firms' cost-minimization problem yields the following set of factor demands:

$$H(z)_t = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(Y(z)_t + FC\right) \left(\frac{\widetilde{\rho}_t^A}{\widetilde{\omega}_t}\right)^{\alpha}$$

[7] Bergin and Feenstra (1999, 2000) study persistence in the real effects of monetary shocks using a translog aggregator function; this is a more restrictive variant of Kimball's (1995) specification.

$$K^A(z)_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} (Y(z)_t + FC) \left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^A}\right)^{1-\alpha}$$

and the real marginal cost function:

$$\widetilde{MC}(z)_t = \left(\frac{\widetilde{\omega}_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{\widetilde{\rho}_t^A}{\alpha}\right)^{\alpha} \tag{15}$$

where $\widetilde{\omega}_t = \frac{\omega_t}{P_t}$, $\widetilde{\rho}_t^A = \frac{\rho_t^A}{P_t}$, and $\widetilde{MC}\,(z)_t = \frac{MC(z)_t}{P_t}$. Firms choose the price of their differentiated good, $P(z)_t$, to maximize profits given their cost function and demand curve.

The model developed in this paper uses a specification of price stickiness due to Calvo (1983) in order to generate real and persistent effects of monetary policy on real output. Calvo pricing assumes that in each period a proportion $(1-\gamma)$ of firms are able to change their prices while the remaining γ are constrained to charge the previous period's price regardless of the time elapsed since the last time they changed their price. I further assume that when prices are reset they are done so one period in advance, which implies that the economy-wide price level (and hence the inflation rate) is known one period in advance.

The firms' profit-maximization problem involves choosing $\{P(z)_t\}_{t=1}^{\infty}$ to maximize the present discounted value of profits, defined as:

$$\sum_{k=0}^{\infty} \gamma^k \beta^{k+1} E_t \left[\frac{\lambda_{t+k+1}}{\lambda_t} \left(\left(\frac{P(z)_{t+1}}{P_{t+k+1}} - \widetilde{MC}(z)_{t+k+1}\right) Y(z)_{t+k+1} - FC\right)\right]$$

subject to equation (13), where λ_t denotes the marginal utility of consumption in period t. For firms who do change their prices in period t, the profit-maximization problem implicitly defines a price $P(z)_{t+1}$ of:

$$1 = \frac{\sum_{k=0}^{\infty} \gamma^k \beta^{k+1} E_t \left[\lambda_{t+k+1} \widetilde{MC}(z)_{t+k+1} \left(\frac{P(z)_{t+1}}{P_{t+k+1}}\right)^{1-\theta_{t+k+1}} \theta_{t+k+1} Y(z)_{t+k+1}\right]}{\sum_{k=0}^{\infty} \gamma^k \beta^{k+1} E_t \left[\lambda_{t+k+1} \left(\frac{P(z)_{t+1}}{P_{t+k+1}}\right)^{2-\theta_{t+k+1}} (\theta_{t+k} - 1)\, Y(z)_{t+k+1}\right]} \tag{16}$$

where θ_{t+k} is the time-varying elasticity of substitution, defined as:

$$\theta_{t+k} = -\frac{\Psi'\left(\frac{Y(z)_{t+k}}{Y_{t+k}}\right)}{\Psi''\left(\frac{Y(z)_{t+k}}{Y_{t+k}}\right) \frac{Y(z)_{t+k}}{Y_{t+k}}}.$$

Economy-wide factor demand schedules are derived by integrating the individual firms' factor demands over the unit interval:

$$H_t = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(Y_t \int_0^1 \Psi^{-1\prime} \left(\frac{P(z)_t}{P_t} \Psi'(1)\right) dz + FC\right) \left(\frac{\widetilde{\rho}_t^A}{\widetilde{\omega}_t}\right)^{\alpha} \tag{17}$$

$$K_t^A = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(Y_t \int_0^1 \Psi^{-1\prime} \left(\frac{P(z)_t}{P_t} \Psi'(1)\right) dz + FC\right) \left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^A}\right)^{1-\alpha}. \tag{18}$$

With all firms facing the same wage and rental rates, economy-wide marginal cost is equal to the marginal cost of an individual firm, which is given by equation (15). The economy-wide price level is determined in the final-goods market by equation (14).

1.1.5 Monetary Authority

The money supply is assumed, for the time being, to evolve according to a general ARMA(r,s) process:

$$\mu_t = (\mu_{t-1})^{\rho_1^\mu} (\mu_{t-2})^{\rho_2^\mu} \ldots (\mu_{t-r})^{\rho_r^\mu} (\mu^*)^{1-\rho_1^\mu-\rho_2^\mu-\ldots-\rho_r^\mu} e^{\varepsilon_t+\phi_1\varepsilon_{t-1}+\ldots+\phi_s\varepsilon_{t-s}}, \tag{19}$$

where $\mu_t = \frac{M_t}{M_{t-1}}$ is the gross growth in the nominal money supply in period t, μ^* is the steady-state growth in the nominal money supply, $\rho_1^\mu,\rho_2^\mu,\ldots,\rho_r^\mu,\phi_1,\ldots,\phi_s$ are parameters of the ARMA(r,s) process, and ε_t represents a stochastic component of the process.[8]

1.1.6 Market Clearing

A final condition for the model is that of market clearing, namely, that the volume of actual consumption and actual investment undertaken be equal to the volume of output produced:

$$C_t + I_t^A = Y_t. \tag{20}$$

1.2 Equilibrium

Equilibrium is an allocation $\{\mathbf{C}_t,\mathbf{I}_t^A,C_t,I_t^A,Y_t,\frac{M_t^C}{P_t},\frac{M_t^I}{P_t},H_t,K_{t+1}^A\}_{t=0}^\infty$ and a sequence of values $\left\{\mathbf{\Pi}_{t+1},\widetilde{\boldsymbol{\omega}}_t,\widetilde{\boldsymbol{\rho}}_t^A,\Pi_{t+1},\widetilde{\omega}_t,\widetilde{\rho}_t^A,R_t,\widetilde{MC}_t\right\}_{t=0}^\infty$ that satisfy equations (2), (4) to (7), (10) to (12), and (14) to (20), given K_0^A, Π_0, $\mathbf{G}_0$, M_{-1}, and μ_{-1}, and the sequence of money growth shocks $\{\varepsilon_t\}_{t=0}^\infty$. The variables that constitute the equilibrium have been defined so as to ensure that their steady-state values remain constant over time.

1.3 Log-linearizing the Model

In order to examine how the economy responds to monetary policy shocks, I follow the method of King, Plosser, and Rebelo (1988) and use log-linear approximations to characterize fluctuations about a stationary steady-state equilibrium. The log-linearized first-order conditions are given in Appendix A.1.

[8] Here and throughout the paper, I use an asterisk to denote steady-state values.

The set of log-linearized equations can be reduced to a system of eight linear difference equations of the form:

$$AE_t y_{t+1} = By_t + Cx_t, \tag{21}$$

where y_t is a column vector containing the eight endogenous variables $\widehat{K}^A_{t+2}$, $\widehat{K}^A_{t+1}$, $\widehat{\mathbf{\Pi}}_{t+1}$, $\widehat{K}^A_t$, $\widehat{\mathbf{\Pi}}_t$, $\widehat{\mathbf{G}}_t$, $\widehat{\mu}_{t-1}$, and $\widehat{M}_{t-1}$, x_t is a scalar that equals the monetary policy shock ε_t, A and B are 8 x 8 matrices containing the coefficients on the endogenous variables in the model, and C is an 8 x 1 matrix containing the reduced-form coefficients on the monetary policy shock. As shown by Blanchard and Kahn (1980), a unique stationary equilibrium exists when the number of stable eigenvalues possessed by the matrix $A^{-1}B$ is equal to the number of predetermined variables, which in this case is five.

1.4 Calibrating the Model

Before calibrating the model I note three equilibrium relationships: The first-order conditions (5) and (7) imply that $\beta = \frac{\mathbf{\Pi}^*}{R^*}$ and $\widetilde{\boldsymbol{\rho}}^{A*} = \frac{1}{\beta} - (1-\delta)$, respectively, while zero profits in the long run implies that $\frac{y^*}{y^*+FC} = \frac{\theta-1}{\theta}$.

I follow Kimball (1995) and assume a steady-state gross real interest rate of 1.0049 per quarter (1.02 percent per year), which for a zero steady-state actual and effective inflation rate (Π^* and $\mathbf{\Pi}^*$) implies the same value for the steady-state gross nominal interest rate (R^*). I set the depreciation rate of type-A capital (δ) to Kimball's (1995) rate of 2.1 percent per quarter (8 percent per year).

In calibrating the household's preferences I adopt Bernanke, Gertler, and Gilchrist's (1999) assumption of an intertemporal elasticity of labor substitution (s^{-1}) of 3. Habit-persistence requires the calibration of only two new parameters: $\frac{C^*}{G^*}$ and h (since $b = \left(\frac{G^*}{C^*}\right)(1-h)$). I assume $\frac{C^*}{G^*} = 1.2048$ and $h = 0.3$, which are the values preferred by Boldrin, Christiano, and Fisher (1995).

The effective consumption and investment aggregator functions (equation (8) for $X = C$ and I^A) imply a unit income elasticity of money demand and a nominal interest elasticity of money demand equal to $\left(\frac{-1}{R^*-1}\right)\left(\frac{1}{1-\eta}\right)$. There are few estimates of the short-run interest elasticity of money demand.[9] I calibrate a short-run interest elasticity of money demand by examining the relationship between the responses of the Federal funds rate and the velocity of money following a monetary shock (which I identify using Bernanke and Mihov's (1998a) method). Figure 1 plots these responses (note that the Federal funds rate is expressed at a quarterly rate in the diagram).[10] The implied short-run interest elasticity of money demand

[9] King and Watson (1996) assume a value of -1 based on the observation that the interest elasticity of money demand over business-cycle frequencies should be substantially smaller than the long-run elasticity, which Lucas (1988, 1999) and Stock and Watson (1993) estimate to be approximately -10.

[10] The responses shown in figure 1 are from a VAR that is specified exactly as in Bernanke and Mihov (1998b) and estimated over their sample period of January, 1966 to December, 1996. The results in figure

is consistent with the value of -1 that King and Watson (1996) assume. I therefore adopt their unit short-run interest elasticity of money demand, which results in a value of η equal to -200.5. The high value of η implies that the value assumed for a in equation (8) is virtually immaterial.

To calibrate the goods-market parameters I follow Kimball (1995) and assume the capital elasticity of output (α) to be 0.3; the steady-state elasticity of demand (θ) to be 11; the elasticity of θ with respect to the firm's market share (ϵ) to be -42.8 and the curvature of the adjustment cost technology in equilibrium $\left(J^{-1''}(\delta)\right)$ to be -9.7.[11] For the fraction of firms in any period able to reset their prices, I use Bernanke, Gertler, and Gilchrist's (1999) value of one-quarter (that is, $\gamma = 0.75$).

The shares of output accounted for by consumption and investment are calibrated by noting that $i^A = \frac{\delta\alpha}{\widetilde{\rho}^{A*}} = \frac{\delta\alpha}{\widetilde{\rho}^{A*}} a^{\frac{1}{1-\eta}} \left[a^{\frac{1}{1-\eta}} + (1-a)^{\frac{1}{1-\eta}} \left(1 - \frac{1}{R^*}\right)^{\frac{-\eta}{1-\eta}} \right]^{-\frac{1}{\eta}}$. Since consumption accounts for the remaining component of output, $c = 1 - i^A$.

To calibrate the money growth rule I note that in the steady state nominal money supply growth (μ^*) must equal the steady-state inflation rates ($\mathbf{\Pi}^*$ and Π^*), which I have assumed to be zero. For the parameters $\rho_1^\mu, \rho_2^\mu, ..., \rho_r^\mu, \phi_1, ..., \phi_s$ I find it necessary to consider two different money growth processes. Recent literature addressing the liquidity effect has almost universally favored an AR(1) specification for the money growth process—see for example, King (1991), Christiano (1991), and Christiano and Eichenbaum (1992a, 1995)—with ρ_1^μ, the only non-zero parameter, generally calibrated to equal 0.32 or 0.50.[12] This assumption is made in spite of the fact that the empirically observed responses of money growth to an exogenous monetary policy shock display pronounced higher-order dynamics. In figure 2, I plot the estimated response of money growth to a monetary policy shock (identified using Bernanke and Mihov's (1998a) method), along with the money growth profile implied by AR(1) processes with ρ_1^μ equal to either 0.32 or 0.5. In contrast, figure 3 plots the estimated response of money growth together with fitted AR(2), AR(3), and ARMA(1,1) representations of the response. It is evident from these figures that allowing for higher-order dynamics yields a calibrated process for money growth that is much closer

1 remain essentially the same under alternative methods of identification, such as those used by Bernanke and Blinder (1992), Christiano and Eichenbaum (1992b), and Strongin (1995).

[11] I obtain this value from Kimball's (1995) assumption that $\frac{-\delta J''(\delta)}{J'(\delta)} = 0.2$ by noting that the inverse function theorem implies $J^{-1'}(\delta) = \frac{1}{J'(\delta)} = 1$ and $J^{-1''}(\delta) = -J''(\delta) = 9.7$.

[12] Christiano (1991) and Christiano and Eichenbaum (1992a,1995) use a persistence parameter of 0.32 following Christiano (1991), who finds that the persistence of base money growth in a first-order autoregressive model is 0.32 over the period 1970:1 to 1984:1. King (1991) uses a persistence parameter of 0.5, having found that he cannot reject the hypothesis that nominal M1 growth followed a first-order autoregressive process with a persistence parameter of 0.5 over the period 1953:1 to 1987:4. Similarly, Christiano, Eichenbaum and Evans (1998) are unable to reject the hypothesis that $\rho_1^\mu = 0.5$ for the response of nominal M2 growth to an identified monetary shock and report substantially more evidence against the hypothesis that $\rho_1^\mu = 0.3$.

to the empirically observed response.[13]

Calibrating equation (19) so as to match the response of money growth that results from an identified monetary policy shock is sensible if I wish to make use of Christiano, Eichenbaum, and Evans' (1998) method to compare the responses of key variables in my model to their empirical counterparts.[14] That said, however, the widespread use of the AR(1) money growth process in this literature—which as Christiano and Eichenbaum (1995) have pointed out is partially responsible for the difficulties involved in generating a liquidity effect in standard models—suggests that it is informative for me to at least consider my model's performance when such a process is employed.

In calibrating equation (19) as an AR(1) process, I set ρ_1^μ equal to 0.32. I calibrate the higher-order version of equation (19) as a ARMA(1,1) process with ρ_1^μ and ϕ_1 equal to 0.53 and 0.93; as figure 3 demonstrates, this provides the best fit to the empirically observed process. The robustness of the model's results to other money growth processes is discussed in section six.

In developing the model, I present only the results from the model that assumes an AR(1) process for money growth. The higher-order specification of the money growth process will not be used until section five when the calibrated model's responses are compared to those found in the empirical literature.

1.5 Response to a Money Shock

The responses of the model's key variables to a one percent shock to money supply growth are presented in figure 4. These responses demonstrate the problem which this paper addresses, namely, that the benchmark sticky-price monetary business cycle model with habit-persistence—which for most variables produces impulse responses to a money growth shock that are broadly consistent with empirical estimates—is unable to generate a decline in real and nominal interest rates.

It is apparent from Panels (f) and (h) that the strong positive response of investment to the money shock forces the household to reduce its demands on the economy's output and postpone some of its consumption. Inducing the household to save more and invest less requires an increase in real interest rates (seen in Panel (b)). Since an increase in inflation follows a positive shock to money growth (shown in Panels (c) and (e)), an increase in real interest rates necessarily implies an increase in nominal interest rates (seen in Panel (a)).

The following sections incorporate a time-to-build and time-to-plan technology in the evolution of the economy's capital stock. This serves to suppress the initial response of investment, which abates the onus on the household to increase its saving and reduces the upward pressure on real interest rates. The introduction of time-to-build and time-to-

[13] The results of figures 2 and 3 are largely robust to alternative methods of monetary policy identification.

[14] I discuss the Christiano, Eichenbaum and Evans method in detail in section five.

plan is preferable to simply assuming greater investment adjustment costs: While increased adjustment costs reduce initial investment (thus yielding a more desirable response for real and nominal interest rates), it also results in a very slow adjustment path for the economy back to the steady state. Time-to-build and time-to-plan, on the other hand, reduces the response of investment only for the length of time it takes to complete all the projects that were begun or planned before the shock. It does not, therefore, slow the economy's adjustment back to the steady state.

2 The Sticky-Price Monetary Business Cycle Model with up to Two-Period Time-to-Build

In this section I develop the model with time-to-build and time-to-plan. An economy's capital stock consists of many different varieties of capital which take different lengths of time to come on-line. I therefore develop my model assuming multiple capital types, each of which has different project planning and completion times. In this section I present the simplest possible version of the model with two types of capital: one taking one period to build (called type-A capital) and the other taking two periods to build (called type-B capital). I present the model in detail in order to give an idea of the modeling strategy involved and the basic nature of the problem. In section three a more detailed version of this model is outlined which assumes a distribution of capital types that ranges from those taking one period to build (type-A projects) to those taking six periods to build (type-F projects).

I introduce two types of capital by assuming that there are two steps in producing the intermediate good, both of which take place within the same intermediate-good producing firm. In the first stage the intermediate-good producing firm combines type-A capital with some of the homogeneous labor input to produce the type-A sub-intermediate good, while also combining type-B capital with some of the homogeneous labor to produce a type-B sub-intermediate good. In the second stage of production these two sub-intermediate goods are combined to produce the intermediate differentiated good over which the firm has market power.

The agents in the model presented in this section are the same as those in section one, although the introduction of multiple capital stocks with varying project completion times changes their objective functions and constraints somewhat. The problem solved by the final-good producing firm is, however, completely unchanged and so is not repeated in this section. The assumed process for money growth also remains as given by equation (19).

2.1 Setup of the Model

2.1.1 Consumers

The principal modification to the household's problem is in the evolution equation for the economy's two capital stocks. Type-A capital, which takes only one period to build, still evolves according to equation (4). Type-B capital, however, which takes two periods to build, requires the household to make its decision as to the optimal type-B capital stock two periods in advance. Multi-stage investment projects are also assumed to involve ex-post investment inflexibilities: that is, the household is required to commit to the amount of effective investment that will take place in each stage of the two-period investment project. This investment profile, and therefore the realized capital stock, can be altered; however, any deviation from the initial profile entails some cost.

I adapt Christiano and Eichenbaum's (1995) approach to modeling ex-post output inflexibilities in order to capture ex-post inflexibilities in investment. I assume that there is a non-linear relationship (specifically, a CES aggregator function) that relates the type-B effective investment that takes place over the two periods to the amount of type-B capital that finally obtains; this in turn is embedded in the standard cost-of-adjustment framework. The realized market capital stock is therefore given by:

$$K_{t+2}^{B} = (1-\delta)\,K_{t+1}^{B} + K_{t+1}^{B} J\left(\frac{\phi^{B}}{K_{t+1}^{B}}\left(\sum_{i=1}^{2}\Psi_{i}^{B}\left(\mathbf{I}_{i,t+i-1}^{B,t+i-1}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right) \tag{22}$$

where

$$\Psi_{i}^{B} = \frac{\beta^{i-1}\left(\mathbf{I}_{i}^{B*}\right)^{1-\sigma}}{\sum_{j=1}^{2}\beta^{j-1}\left(\mathbf{I}_{j}^{B*}\right)^{1-\sigma}}, \text{ and } \phi^{B} = \left(\frac{\sum_{i=1}^{2}\beta^{i-1}\left(\mathbf{I}_{i}^{B*}\right)^{1-\sigma}}{\sum_{i=1}^{2}\beta^{i-1}\left(\mathbf{I}_{i}^{B*}\right)}\right)^{\frac{1}{\sigma}}\sum_{i=1}^{2}\mathbf{I}_{i}^{B*}$$

and where $-\infty < \sigma \leq 1$ and $\mathbf{I}_{1}^{B*}$ and $\mathbf{I}_{2}^{B*}$ are the steady-state levels of the first- and second-period type-B effective investment spending. The superscript t on effective investment indicates the period in which the effective investment spending is planned, while the subscript $i\,(=1,2)$ indicates whether the effective investment is for the first or second stage of the project. As in section one, equation (22) implies that the type-B capital stock in any period is known as of the preceding period.

As discussed, the household begins to make its decisions about future type-B capital and the type-B investment plan two periods in advance. In deciding on how to spread its effective investment profile over the two periods, the household minimizes the expected discounted cost of effective investment conditional on a desired value for the $t+2$ capital stock. The household's problem in period t is thus:

$$\min_{\left\{\mathbf{I}_{1,t}^{B},\mathbf{I}_{2,t+1}^{B}\right\}} \mathbf{P}_{t}\mathbf{I}_{1,t}^{B} + R_{t}^{-1}\mathbf{P}_{t+1}\mathbf{I}_{2,t+1}^{B}$$

$$-R_t^{-1}E_t\left[R_{t+1}^{-1}\mathbf{P}_{t+2}\right]K_{t+1}^{B}J\left(\frac{\phi^B}{K_{t+1}^B}\left(\sum_{i=1}^{2}\Psi_i^B\left(\mathbf{I}_{i,t+i-1}^{B,t+i-1}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right)$$

subject to:

$$J\left(\frac{\phi^B}{K_{t+1}^B}\left(\sum_{i=1}^{2}\Psi_i^B\left(\mathbf{I}_{i,t+i-1}^{B,t+i-1}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right)\geq\frac{E_tK_{t+2}^B}{K_{t+1}^B}-(1-\delta) \tag{23}$$

where $E_tK_{t+2}^B$ is yet to be determined. The cost-minimization problem yields the following optimal effective investment plan:

$$\mathbf{I}_{1,t}^{B,t}=\left(\frac{\mathbf{I}_1^{B*}}{\mathbf{I}_1^{B*}+\mathbf{I}_2^{B*}}\right)\left(\frac{\mathbf{I}_1^{B*}+\mathbf{I}_2^{B*}\beta}{\mathbf{I}_1^{B*}+\mathbf{I}_2^{B*}(\beta)^{\frac{1}{1-\sigma}}\left(\frac{\mathbf{\Pi}_{t+1}}{R_t}\right)^{\frac{-\sigma}{1-\sigma}}}\right)^{\frac{1}{\sigma}}K_{t+1}^BE_t\left[J^{-1}\left(\frac{K_{t+2}^B}{K_{t+1}^B}-(1-\delta)\right)\right] \tag{24}$$

$$\mathbf{I}_{2,t+1}^{B,t}=\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)(\beta)^{\frac{1}{1-\sigma}}\left(\frac{\mathbf{\Pi}_{t+1}}{R_t}\right)_{1,t}^{\frac{-1}{1-\sigma}}\mathbf{I}_{1,t}^{B,t}. \tag{25}$$

Given the distribution of type-B effective investment over the two periods, the household then chooses its desired period $t+2$ type-B capital stock.

The model is solved in the same way as in the benchmark case: Household utility (equation (1), where now $H_t = H_t^A + H_t^B$) is maximized subject the household's budget constraint, now defined as:

$$B_t\leq R_{t-1}B_{t-1}+\omega_tH_t+\rho_t^AK_t^A+\rho_t^BK_t^B+profits_t-T_t-\mathbf{P}_t\mathbf{C}_t-\mathbf{P}_t\mathbf{I}_t^{A,t}-\mathbf{P}_t\mathbf{I}_{1,t}^{B,t}-\mathbf{P}_t\mathbf{I}_{2,t}^{B,t} \tag{26}$$

and the evolution of the two types of capital, defined in equations (4) and (23).

The household's first-order conditions with respect to its choice of $\mathbf{C}_t$, C_t, and $\frac{M_t^C}{P_t}$ are identical to those of section one (that is, equation (5), and equations (11) and (12) with $X = C$) as are the first-order conditions for the choice of K_{t+1}^A, $\mathbf{I}_t^A$, I_t^A, and $\frac{M_t^{I^A}}{P_t}$ (equations (7) and (4), and (11) and (12) with $X = I^A$) and H_t (equation (6)).

The household's first-order condition with respect to its choice of $E_tK_{t+2}^B$ is:

$$\begin{aligned}
&-\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\left(\frac{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{\mathbf{\Pi}_{t+1}}{R_t}\right)^{\frac{-\sigma}{1-\sigma}}}\right)\left(\frac{R_t}{\mathbf{\Pi}_{t+1}}+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{R_t}{\mathbf{\Pi}_{t+1}}\right)^{\frac{1}{1-\sigma}}\right)\\
&\times E_t\left[\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}J^{-1'}\left(\frac{K_{t+2}^B}{K_{t+1}^B}-(1-\delta)\right)\right]+E_t\left[\widetilde{\boldsymbol{\rho}}_{t+2}^B\right]\\
=\;&\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta\right)^{\frac{1}{\sigma}}
\end{aligned}$$

$$
\begin{aligned}
&\times E_t\left[\frac{\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}\right)^{\frac{1}{1-\sigma}}\right)}{\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{\mathbf{\Pi}_{t+2}}{R_{t+1}}\right)^{\frac{-\sigma}{1-\sigma}}\right)^{\frac{1}{\sigma}}}J^{-1}\left(\frac{K_{t+3}^B}{K_{t+2}^B}-(1-\delta)\right)\right] \\
&-\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta\right)^{\frac{1}{\sigma}} \qquad (27) \\
&\times E_t\left[\frac{\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{R_{t+1}}{\mathbf{\Pi}_{t+2}}\right)^{\frac{1}{1-\sigma}}\right)}{\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{\mathbf{\Pi}_{t+2}}{R_{t+1}}\right)^{\frac{-\sigma}{1-\sigma}}\right)^{\frac{1}{\sigma}}}J^{-1\prime}\left(\frac{K_{t+3}^B}{K_{t+2}^B}-(1-\delta)\right)\left(\frac{K_{t+3}^B}{K_{t+2}^B}\right)\right].
\end{aligned}
$$

This optimal choice of $E_t K_{t+2}^B$, along with equation (24), and equations (11) and (12) (for $X = I_1^B$), yields current first-stage effective investment ($\mathbf{I}_{1,t}^{B,t}$), current first-stage actual investment $\left(I_{1,t}^{B,t}\right)$, and required money balances $\left(\frac{M_t^{I_1^B}}{P_t}\right)$.

In period t the household also has the opportunity to revise its period $t-1$ investment plan for type-B capital that is expected to come on-line in period $t+1$. While a revised plan may yield a preferred level for the final type-B capital stock, the revision requires the household to incur a cost (this cost is captured by the assumed form of the aggregator function). Second-stage effective investment planned and undertaken in period t is defined as:

$$
\begin{aligned}
\mathbf{I}_{2,t}^{B,t} \quad = \quad &\left(\left(\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta\right)\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}K_t^B J^{-1}\left(\frac{K_{t+1}^B}{K_t^B}-(1-\delta)\right)\right)\right)^{\sigma}-\left(\mathbf{I}_{1,t-1}^{B,t-1}\right)^{\sigma}\right)^{\frac{1}{\sigma}} \\
&\times\left(\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)^{1-\sigma}\beta\right)^{\frac{-1}{\sigma}}. \qquad (28)
\end{aligned}
$$

Given $\mathbf{I}_{2,t}^{B,t}$, the household chooses its desired period $t+1$ capital stock. The household's first-order condition with respect to its choice of K_{t+1}^B is:

$$
\begin{aligned}
&-\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)^{\sigma}\left(\frac{\mathbf{I}_{2,t}^{B,t}}{K_t^B J^{-1}\left(\frac{K_{t+1}^B}{K_t^B}-(1-\delta)\right)}\right)^{1-\sigma}\left(\frac{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta}{\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)^{1-\sigma}\beta}\right) \\
&\times\left(\frac{R_t}{E_t\mathbf{\Pi}_{t+1}}\right)J^{-1\prime}\left(\frac{K_{t+1}^B}{K_t^B}-(1-\delta)\right)+E_t\left[\widetilde{\boldsymbol{\rho}}_{t+1}^B\right]
\end{aligned}
$$

$$
\begin{aligned}
= & \left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta\right)^{\frac{1}{\sigma}} \frac{\left(\left(\frac{R_t}{E_t\mathbf{\Pi}_{t+1}}\right)+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{R_t}{E_t\mathbf{\Pi}_{t+1}}\right)^{\frac{1}{1-\sigma}}\right)}{\left(1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\beta^{\frac{1}{1-\sigma}}\left(\frac{E_t\mathbf{\Pi}_{t+1}}{R_t}\right)^{\frac{-\sigma}{1-\sigma}}\right)^{\frac{1}{\sigma}}} \\
& \times E_t\left[J^{-1}\left(\frac{K_{t+2}^B}{K_{t+1}^B}-(1-\delta)\right)-J^{-1'}\left(\frac{K_{t+2}^B}{K_{t+1}^B}-(1-\delta)\right)\left(\frac{K_{t+2}^B}{K_{t+1}^B}\right)\right].
\end{aligned} \tag{29}
$$

This optimal choice of $E_tK_{t+1}^B$, along with equation (28) and equations (11) and (12) (for $X = I_2^B$), yields current second-stage effective investment ($\mathbf{I}_{2,t}^{B,t}$), current second-stage actual investment $\left(I_{2,t}^{B,t}\right)$, and required money balances $\left(\frac{M_t^{I_2^B}}{P_t}\right)$.

2.1.2 Intermediate-Good Producers

The technology for producing each of the sub-intermediate goods is given by:

$$
Y^X(z)_t = \left(K^X(z)_t\right)^\alpha \left(H^X(z)_t\right)^{(1-\alpha)} \tag{30}
$$

where X denotes the type of sub-intermediate good (A or B). The technology for producing the intermediate good from the two sub-intermediate goods is given by:

$$
Y(z)_t = \kappa\left(\sum_{i\in S}\Psi_i^Y\left(Y^X(z)_t\right)^\phi\right)^{\frac{1}{\phi}} - FC \tag{31}
$$

where $S = \{A, B\}$, $-\infty < \phi \leq 1$, $\kappa > 0$, and where the weights in the CES aggregator are equal to:

$$
\Psi_i^Y = \frac{\left(\widetilde{\rho}^{i*}\right)^{\alpha+(1-\alpha)(1-\phi)}\left(K^i(z)^*\right)^{1-\phi}}{\sum_{j\in S}\left(\widetilde{\rho}^{j*}\right)^{\alpha+(1-\alpha)(1-\phi)}\left(K^j(z)^*\right)^{1-\phi}}. \tag{32}
$$

The intermediate-good producers' problem involves solving the following cost-minimization problem:

$$
\min_{\{K^j(z),H^j(z)\}_{j\in S}} \omega_t\left(\sum_{i\in S}H^i(z)_t\right)+\sum_{i\in S}\rho_t^iK^i(z)_t
$$

$$
\text{s.t. } \kappa\left(\sum_{i\in S}\Psi_i^Y\left(\left(K^i(z)_t\right)^\alpha\left(H^i(z)_t\right)^{1-\alpha}\right)^\phi\right)^{\frac{1}{\phi}} - FC \geqslant Y(z)_t\,.
$$

This yields demand curves for labor and capital of the form:

$$H^X(z)_t = \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{Y(z)_t + FC}{\kappa}\right) \left(\frac{\left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^X}\right)^{-\alpha} \left(\frac{\Psi_X}{\widetilde{\rho}_t^X}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j=S} \left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^j\right)^{\phi}}\right)^{\frac{1}{1-\phi}}\right)^{\frac{1}{\phi}}}\right), \; X = A, B \tag{33}$$

$$K^X(z)_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{Y(z)_t + FC}{\kappa}\right) \left(\frac{\left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^X}\right)^{1-\alpha} \left(\frac{\Psi_X}{\widetilde{\rho}_t^X}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j=S} \left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^j\right)^{\phi}}\right)^{\frac{1}{1-\phi}}\right)^{\frac{1}{\phi}}}\right), \; X = A, B \tag{34}$$

and a marginal cost function:

$$\widetilde{MC}(z)_t = \left(\frac{1}{\kappa}\right) \left(\frac{\widetilde{\omega}_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{\sum_{i=S} \left(\frac{\widetilde{\rho}_t^i}{\alpha}\right)^{\alpha} \left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^i\right)}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j=S} \left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^j\right)^{\phi}}\right)^{\frac{1}{1-\phi}}\right)^{\frac{1}{\phi}}}\right). \tag{35}$$

The firms' profit-maximizing choice of $\{P(z)_t\}_{t=1}^{\infty}$ is the same as that given in section one, so the first-order condition for $P(z)_{t+1}$ for firms who are able to change their prices in period t remains as in equation (16). Finally, the economy-wide factor-demand schedules are:

$$H_t^X = \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{Y_t \int_0^1 \Psi^{-1\prime} \left(\frac{P_t(z)}{P_t} \Psi'(1)\right) dz + FC}{\kappa}\right) \left(\frac{\left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^X}\right)^{-\alpha} \left(\frac{\Psi_X}{\widetilde{\rho}_t^X}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j=S} \left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^j\right)^{\phi}}\right)^{\frac{1}{1-\phi}}\right)^{\frac{1}{\phi}}}\right) \tag{36}$$

$$K_t^X = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{Y_t \int_0^1 \Psi^{-1\prime}\left(\frac{P_t(z)}{P_t}\Psi'(1)\right) dz + FC}{\kappa}\right) \left(\frac{\left(\frac{\widetilde{\omega}_t}{\widetilde{\rho}_t^X}\right)^{1-\alpha} \left(\frac{\Psi_X}{\widetilde{\rho}_t^X}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j=S}\left(\frac{\Psi_i}{\left(\widetilde{\rho}_t^j\right)^{\phi}}\right)^{\frac{1}{1-\phi}}\right)^{\frac{1}{\phi}}}\right) \tag{37}$$

where $X = A, B$. Economy-wide marginal cost (which is identical to the marginal cost of an individual firm) is given by equation (35). As before, the economy-wide price level is determined in the final-goods market by equation (14).

2.1.3 Market Clearing

With two types of capital (one of which takes two periods to build), the economy's market-clearing condition becomes:

$$C_t + I_t^A + I_{1,t}^{B,t} + I_{2,t}^{B,t} = Y_t. \tag{38}$$

2.2 Equilibrium

Equilibrium is an allocation $\{\mathbf{C}_t, \mathbf{I}_t^A, \mathbf{I}_{1,t}^{B,t}, \mathbf{I}_{2,t}^{B,t}, C_t, I_t^A, I_{1,t}^{B,t}, I_{2,t}^{B,t}, Y_t, \frac{M_t^C}{P_t}, \frac{M_t^{I^A}}{P_t}, \frac{M_t^{I_1^B}}{P_t}, \frac{M_t^{I_2^B}}{P_t}, H_t^A, H_t^B, K_{t+1}^A, K_{t+1}^B, K_{t+2}^B\}_{t=0}^{\infty}$ and a sequence of values $\{\mathbf{\Pi}_{t+1}, \widetilde{\boldsymbol{\omega}}_t, \widetilde{\boldsymbol{\rho}}_t^A, \widetilde{\boldsymbol{\rho}}_t^B, \Pi_{t+1}, \widetilde{\omega}_t, \widetilde{\rho}_t^A, \widetilde{\rho}_t^B, R_t, \widetilde{MC}_t\}_{t=0}^{\infty}$ that satisfy equations (4) to (7), (10) to (12), (14), (24), (27) to (29), and (35) to (38), given K_0^A, K_0^B, Π_0, $\mathbf{G}_0$, $I_{1,-1}^{B,-1}$, M_{-1}, and μ_{-1}, and the sequence of monetary shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

2.3 Log-linearizing the Model

The economy's log-linearized first-order conditions are given in Appendix A.2. The set of log-linearized equations can be reduced to a system of twelve linear difference equations and written in a form similar to equation (21). In this case y_t is a column vector containing the twelve endogenous variables $E_t\widehat{K}_{t+3}^B$, $E_t\widehat{K}_{t+2}^B$, $E_t\widehat{\Pi}_{t+2}$, $\widehat{K}_{t+1}^B$, $\widehat{\Pi}_{t+1}$, $\widehat{K}_t^B$, $\widehat{K}_t^A$, $\widehat{\Pi}_t$, $\widehat{\mathbf{G}}_t$, $\widehat{I}_{1,t-1}^{B,t-1}$, $\widehat{\mu}_{t-1}$, and $\widehat{M}_{t-1}$, x_t is a scalar that equals the monetary policy shock ε_t, A and B are 12 x 12 matrices containing the coefficients on the endogenous variables in the model, and C is a 12 x 1 matrix containing the reduced-form coefficients on the monetary policy shock. In this case a unique stationary equilibrium exists when $A^{-1}B$ possesses seven stable eigenvalues.

2.4 Calibrating the Model

The household sector of the model has only two new parameters that need to be calibrated: σ, the CES aggregator parameter, and $\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)$, the ratio of second- to first-period market investment spending in the steady-state equilibrium. I assume a fairly low degree of substitution between type-B investment spending over the two periods and thus calibrate σ to equal -20. I also assume for the time being that in the steady state, investment spending in type-B capital is evenly split over the two periods, that is, $\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right) = 1$. The assumption that $\mathbf{I}_1^{B*} = \mathbf{I}_2^{B*}$ simplifies many of the equations of the two-period case and could have been assumed at the beginning of the section to allow a much simpler exposition of the model. The advantage, however, of deriving the model with $\mathbf{I}_1^{B*} \neq \mathbf{I}_2^{B*}$ is that it provides a convenient way to describe time-to-plan. Specifically, the two-period time-to-plan model derived in this section can be easily converted into a model involving one-period time-to-plan and one period time-to-build by simply letting the parameter $\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)$ approach ∞. This method of capturing time-to-plan, used also by Christiano and Todd (1996), is employed in deriving the fully specified model of section four.

For the intermediate-good producing firms it is necessary to calibrate the parameter ϕ, which reflects the degree of substitution between the sub-intermediate goods used to produce the intermediate good. Again I assume a fairly low degree of substitution and calibrate ϕ to equal -40.

To derive the weights in the intermediate-good production function (equation (31)), it is necessary to make an assumption as to the proportions of the economy's capital stock accounted for by each type of capital. In developing the model, I assume that all types of capital account for an equal share of the economy's total capital stock (I relax this assumption later). The proportions of the economy's capital stock accounted for by each type of capital are required to determine the shares $\left(\frac{H^{A*}}{H^{A*}+H^{B*}}\right)$ and $\left(\frac{H^{B*}}{H^{A*}+H^{B*}}\right)$ in appendix equation (54) since $\frac{H^{B*}}{H^{A*}} = \frac{\rho^{B*}}{\rho^{A*}}\frac{K^{B*}}{K^{A*}} = \frac{\boldsymbol{\rho}^{B*}}{\boldsymbol{\rho}^{A*}}\frac{K^{B*}}{K^{A*}}$ (where $\widetilde{\boldsymbol{\rho}}^{B*} = \left(\frac{\sum_{i=1}^{2}\beta^{i-1}\mathbf{I}_i^{B*}}{\beta\sum_{i=1}^{2}\mathbf{I}_i^{B*}}\right)\left(\frac{1}{\beta} - (1-\delta)\right)$ by equations (27) and (29)). The share of each type of capital in the economy's capital stock also determines the shares of output accounted for by each type of investment. Hence:

$$
\begin{aligned}
i^A &= \frac{\delta\alpha}{\widetilde{\rho}^{A*}}\left(\frac{1}{1+\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}\right) \\
&= \frac{\delta\alpha}{\widetilde{\boldsymbol{\rho}}^{A*}}a^{\frac{1}{1-\eta}}\left[a^{\frac{1}{1-\eta}} + (1-a)^{\frac{1}{1-\eta}}\left(1-\frac{1}{R^*}\right)^{\frac{-\eta}{1-\eta}}\right]^{-\frac{1}{\eta}}\left(\frac{1}{1+\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
i^B &= \frac{\delta\alpha}{\widetilde{\rho}^{B*}}\left(\frac{\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}{1+\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}\right) \\
&= \frac{\delta\alpha}{\widetilde{\boldsymbol{\rho}}^{B*}}a^{\frac{1}{1-\eta}}\left[a^{\frac{1}{1-\eta}}+(1-a)^{\frac{1}{1-\eta}}\left(1-\frac{1}{R^*}\right)^{\frac{-\eta}{1-\eta}}\right]^{-\frac{1}{\eta}}\left(\frac{\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}{1+\left(\frac{\widetilde{\rho}^{B*}}{\widetilde{\rho}^{A*}}\right)^{1-\alpha}\frac{K^{B*}}{K^{A*}}}\right).
\end{aligned}
$$

2.5 Response to a Monetary Shock

The responses of the model's key variables to a one percentage point shock in the growth rate of the money supply are presented in figure 5.[15] To demonstrate the changes brought about by the introduction of time-to-build with ex-post investment inflexibilities the responses from the benchmark model are also presented.

Figure 5 shows that the introduction of time-to-build with ex-post investment inflexibilities reduces the positive response of nominal interest rates and generates a decline in real interest rates. The inclusion of time-to-build shows promise, and as will be subsequently shown, a simple extension of the model that incorporates a richer variety of project completion times can indeed generate a liquidity effect.

3 The Sticky-Price Monetary Business Cycle Model with Six-Period Time-to-Build

In this section I outline a version of the model that incorporates six distinct types of capital. The completion times for the six varieties of capital range from one period (type A) to six periods (type F).

The agents in the model presented in this section are the same as those in sections one and two. The objective functions and constraints faced by the agents in the model change in much the same way as they did for the model in section two, and as before the money growth process (equation (19)) and the problem faced by the final-good producing firm remain unchanged from section one.

Again, the greatest modification to the household's problem involves the evolution of the economy's six capital stocks. Type-A and type-B capital, which still take one and two periods to build, evolve according to equations (4) and (22). Type-C, type-D, type-E, and type-F capital, however, which take three, four, five, and six periods to build evolve according to:

[15] The response of the nominal money supply from this model (and from all future models) is identical to the response displayed in figure 4.

$$K_{t+l}^{X} = (1-\delta)\,K_{t+l-1}^{X} + K_{t+l-1}^{X} J\left(\frac{\phi^{X}}{K_{t+l-1}^{X}}\left(\sum_{i=1}^{l}\Psi_i^X\left(\mathbf{I}_{i,t+i-1}^{X,t+i-1}\right)^{\sigma}\right)^{\frac{1}{\sigma}}\right)$$

where

$$\Psi_i^X = \frac{\beta^{i-1}\left(\mathbf{I}_i^{X*}\right)^{1-\sigma}}{\sum_{j=1}^{l}\beta^{j-1}\left(\mathbf{I}_j^{X*}\right)^{1-\sigma}}, \text{ and } \phi^X = \left(\frac{\sum_{i=1}^{l}\beta^{i-1}\left(I_i^{X*}\right)^{1-\sigma}}{\sum_{i=1}^{l}\beta^{i-1}\left(I_i^{X*}\right)}\right)^{\frac{1}{\sigma}}\sum_{i=1}^{l} I_i^{X*}$$

and where for $X=C$, $l=3$, for $X=D$, $l=4$, for $X=E$, $l=5$, and for $X=F$, $l=6$.

In any period t the household now also chooses the utility-maximizing levels of $\left\{E_t K_{t+j}^F\right\}_{j=1}^{6}$, $\left\{E_t K_{t+j}^E\right\}_{j=1}^{5}$, $\left\{E_t K_{t+j}^D\right\}_{j=1}^{4}$, and $\left\{E_t K_{t+j}^C\right\}_{j=1}^{3}$ given the cost-minimizing investment spending profiles associated with the desired levels of the of the capital stock, namely $\left\{\left\{\mathbf{I}_{i,t+i-1}^{F,t}\right\}_{i=j}^{6}\right\}_{j=1}^{6}$, $\left\{\left\{\mathbf{I}_{i,t+i-1}^{E,t}\right\}_{i=j}^{5}\right\}_{j=1}^{5}$, $\left\{\left\{\mathbf{I}_{i,t+i-1}^{D,t}\right\}_{i=j}^{4}\right\}_{j=1}^{4}$, and $\left\{\left\{\mathbf{I}_{i,t+i-1}^{C,t}\right\}_{i=j}^{3}\right\}_{j=1}^{3}$. This yields eighteen first-order conditions for the household as well as eighteen equations relating current period-t effective investment spending to desired capital. There are a further thirty-six equations of the form (11) and (12) that relate effective investment to actual investment and required money balances. (These equations, are in addition to those discussed in section two.)

To accommodate six types of capital in the model it is necessary to assume six sub-intermediate goods whose production functions are given by equation (30). The technology for producing the intermediate good from the six sub-intermediate goods is still given by equation (31) (where $S = \{A,B,C,D,E,F\}$), while the weights Ψ_i^Y are still given by equation (32). The firm's cost-minimization problem is also as defined in section two. The factor demands for each firm thus remain unchanged from equations (33) and (34) (where $X = \{A,B,C,D,E,F\}$). As in section 2, each firm's factor demands are aggregated across all firms to give the economy-wide factor demands for each input $\left\{H_t^i, K_t^i\right\}_{i=S}$. These are the same as in equations (36) and (37) (where $X = \{A,B,C,D,E,F\}$). Note that the factor demands for $\left\{H_t^i, K_t^i\right\}_{i=A,B}$ replace those of section two, while the factor demands for $\left\{H_t^i, K_t^i\right\}_{i=C,D,E,F}$ are in addition to those of section two.

The definition of real marginal cost (both for the individual firm and for all intermediate firms in total) when there are six types of capital is the same as in equation (35) (again with $S = \{A,B,C,D,E,F\}$). This new expression for real marginal cost replaces that of section two. The optimal price set by firms who are able to change their price is still given by equation (16).

Equilibrium is an allocation $\{\mathbf{C}_t,\mathbf{I}_t^A,\left\{\mathbf{I}_{i,t}^{B,t}\right\}_{i=1}^{2}$ to $\left\{\mathbf{I}_{i,t}^{F,t}\right\}_{i=1}^{6},C_t,I_t^A,\left\{I_{i,t}^{B,t}\right\}_{i=1}^{2}$ to $\left\{I_{i,t}^{F,t}\right\}_{i=1}^{6}$, $Y_t,\frac{M_t^C}{P_t},\frac{M_t^{I^A}}{P_t},\left\{\frac{M_t^{I_i^B}}{P_t}\right\}_{i=1}^{2}$ to $\left\{\frac{M_t^{I_i^F}}{P_t}\right\}_{i=1}^{6}$, H_t^A, H_t^B, to H_t^F,K_{t+1}^A,$\{K_{t+i}^B\}_{i=1}^2$ to $\{K_{t+i}^F\}_{i=1}^6\}_{t=0}^{\infty}$ and a sequence of values $\{\mathbf{\Pi}_{t+1},\widetilde{\boldsymbol{\omega}}_t,\widetilde{\boldsymbol{\rho}}_t^A$ to $\widetilde{\boldsymbol{\rho}}_t^F,\Pi_{t+1},\widetilde{\omega}_t,\widetilde{\rho}_t^A$ to $\widetilde{\rho}_t^F,R_t,\ \widetilde{MC}_t\}_{t=0}^{\infty}$ that satisfy equations (4) to (7),[16] (10) to (12), (24), (27) to (29), and the seventy-two additional household first-order conditions just discussed, as well as equations (14), (35) to (37), and the new market-clearing condition of:

$$C_t + I_t^A + \sum_{i=1}^{2} I_{i,t}^{B,t} + \sum_{i=1}^{3} I_{i,t}^{C,t} + \sum_{i=1}^{4} I_{i,t}^{D,t} + \sum_{i=1}^{5} I_{i,t}^{E,t} + \sum_{i=1}^{6} I_{i,t}^{F,t} = Y_t. \tag{39}$$

given K_0^A, K_0^B, K_0^C, K_0^D, K_0^E, K_0^F, Π_0, $\mathbf{G}_0$, $\mathbf{I}_{1,-1}^{B.-1}$, $\left\{\mathbf{I}_{i,-1}^{C,-1},\right\}_{i=1}^{2}$, $\left\{\mathbf{I}_{i,-1}^{D,-1},\right\}_{i=1}^{3}$, $\left\{\mathbf{I}_{i,-1}^{E,-1},\right\}_{i=1}^{4}$, $\left\{\mathbf{I}_{i,-1}^{F,-1},\right\}_{i=1}^{5}$, $\mathbf{I}_{1,-2}^{C,-2}$, $\left\{\mathbf{I}_{i,-2}^{D,-2},\right\}_{i=1}^{2}$, $\left\{\mathbf{I}_{i,-2}^{E,-2},\right\}_{i=1}^{3}$, $\left\{\mathbf{I}_{i,-2}^{F,-2},\right\}_{i=1}^{4}$, $\mathbf{I}_{1,-3}^{D,-3}$, $\left\{\mathbf{I}_{i,-3}^{E,-3},\right\}_{i=1}^{2}$, $\left\{\mathbf{I}_{i,-3}^{F,-3},\right\}_{i=1}^{3}$, $\mathbf{I}_{1,-4}^{E,-4}$, $\left\{\mathbf{I}_{i,-4}^{F,-4},\right\}_{i=1}^{2}$, $\mathbf{I}_{1,-5}^{F,-5}$, M_{-1}, and μ_{-1}, and the sequence of monetary shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

As with the previous models the first-order conditions are log-linearized and reduced to a set of 58 linear equations that can be written in a form similar to equation (21). In this case y_t is a column vector containing $E_t\widehat{K}_{t+7}^F$, $E_t\widehat{K}_{t+6}^F$, $E_t\widehat{\Pi}_{t+6}$, $E_t\widehat{K}_{t+5}^F$, $E_t\widehat{\Pi}_{t+5}$, $E_t\widehat{K}_{t+4}^F$, $E_t\widehat{\Pi}_{t+4}$, $E_t\widehat{K}_{t+3}^F$, $E_t\widehat{\Pi}_{t+3}$, $E_t\widehat{K}_{t+2}^F$, $E_t\widehat{\Pi}_{t+2}$, $\widehat{K}_{t+1}^F$, $\widehat{\Pi}_{t+1}$, $\widehat{K}_t^A$, $\widehat{K}_t^B$, $\widehat{K}_t^C$, $\widehat{K}_t^D$, $\widehat{K}_t^E$, $\widehat{K}_t^F$, $\widehat{\Pi}_t$, $\widehat{\mathbf{G}}_t$, $\widehat{\mathbf{I}}_{1,-1}^{B.-1}$, $\left\{\widehat{\mathbf{I}}_{i,-1}^{C,-1},\right\}_{i=1}^{2}$, $\left\{\widehat{\mathbf{I}}_{i,-1}^{D,-1},\right\}_{i=1}^{3}$, $\left\{\widehat{\mathbf{I}}_{i,-1}^{E,-1},\right\}_{i=1}^{4}$, $\left\{\widehat{\mathbf{I}}_{i,-1}^{F,-1},\right\}_{i=1}^{5}$, $\widehat{\mathbf{I}}_{1,-2}^{C,-2}$, $\left\{\widehat{\mathbf{I}}_{i,-2}^{D,-2},\right\}_{i=1}^{2}$, $\left\{\widehat{\mathbf{I}}_{i,-2}^{E,-2},\right\}_{i=1}^{3}$, $\left\{\widehat{\mathbf{I}}_{i,-2}^{F,-2},\right\}_{i=1}^{4}$, $\widehat{\mathbf{I}}_{1,-3}^{D,-3}$, $\left\{\widehat{\mathbf{I}}_{i,-3}^{E,-3},\right\}_{i=1}^{2}$, $\left\{\widehat{\mathbf{I}}_{i,-3}^{F,-3},\right\}_{i=1}^{3}$, $\widehat{\mathbf{I}}_{1,-4}^{E,-4}$, $\left\{\widehat{\mathbf{I}}_{i,-4}^{F,-4},\right\}_{i=1}^{2}$, $\widehat{\mathbf{I}}_{1,-5}^{F,-5}$, $\widehat{\mu}_{t-1}$, and $\widehat{M}_{t-1}$. A unique stationary equilibrium exists when the matrix $A^{-1}B$ has 44 stable eigenvalues.

In calibrating the model I note that $\widetilde{\boldsymbol{\rho}}^{A*}$ and $\widetilde{\boldsymbol{\rho}}^{B*}$ are as defined in sections one and two while $\widetilde{\boldsymbol{\rho}}^{C*} = \left(\frac{\sum_{i=1}^{3}\beta^{i-1}\mathbf{I}_i^{C*}}{\beta^2\sum_{i=1}^{3}\mathbf{I}_i^{C*}}\right)\left(\frac{1}{\beta}-(1-\delta)\right)$, $\widetilde{\boldsymbol{\rho}}^{D*} = \left(\frac{\sum_{i=1}^{4}\beta^{i-1}\mathbf{I}_i^{D*}}{\beta^3\sum_{i=1}^{4}\mathbf{I}_i^{D*}}\right)\left(\frac{1}{\beta}-(1-\delta)\right)$, $\widetilde{\boldsymbol{\rho}}^{E*} = \left(\frac{\sum_{i=1}^{5}\beta^{i-1}\mathbf{I}_i^{E*}}{\beta^4\sum_{i=1}^{5}\mathbf{I}_i^{E*}}\right)\left(\frac{1}{\beta}-(1-\delta)\right)$, and $\widetilde{\boldsymbol{\rho}}^{F*} = \left(\frac{\sum_{i=1}^{6}\beta^{i-1}\mathbf{I}_i^{F*}}{\beta^5\sum_{i=1}^{6}\mathbf{I}_i^{F*}}\right)\left(\frac{1}{\beta}-(1-\delta)\right)$. I maintain the assumption that all six types of capital account for an equal share of the economy's total capital stock and that for each type of capital, investment spending is evenly split across the periods for which the investment spending takes place.

The responses of the key variables to a one percentage point shock in the growth rate of the money supply are presented in figures 6 and 7. Adding additional capital stocks with longer project completion times suppresses the economy-wide response of investment, thus putting less upward pressure on real and nominal interest rates. It is interesting to observe the gradual effect that adding capital stocks with longer project completion times has on real and nominal interest rates. I therefore also plot the responses emerging from

[16] Note that in equation (6) $H_t = \sum_{i=A}^{F} H_t^i$.

the models presented in sections one and two as well as from models with three to five types of capital.

It is clear from panels (a) and (c) of figure 6 and panel (c) of figure 7 that introducing capital stocks with longer project completion times does indeed reduce the response of investment and the upward pressure on real and nominal interest rates. Furthermore, the version of the model that includes capital stocks that take up to six periods to complete is capable of generating a liquidity effect that persists for almost a year.

The introduction of capital stocks with longer project completion times leads to little change in the responses of variables other than real and nominal interest rates and to a lesser extent investment and output. The response of consumption is slightly larger as a result of the introduction of the additional capital stocks. The assumption of habit-persistence, however, prevents the undesirable result that would otherwise obtain, namely, a very large increase in consumption in the periods immediately following the monetary shock (not shown).

4 Recalibrating the Steady-State Capital Shares and Investment Spending Profiles

In developing and extending the model with time-to-build I assumed that in the steady state each type of capital accounted for the same share of the economy's capital stock, and that investment spending was split evenly across the periods over which the spending was undertaken. This assumption, which was made primarily to facilitate the comparison across models, is clearly unrealistic; in this section, therefore, I present results for the model of the previous section (with six types of capital and habit-persistence) calibrated with more sensible steady-state capital shares and investment spending profiles. To calibrate the capital shares I calculate the shares of total fixed investment accounted for by projects of different lengths. (This is roughly equivalent to calculating the capital shares since in equilibrium investment just replaces depreciation.)

Over the last 30 years equipment investment has on average accounted for 41 percent of total fixed investment while structures and residential investment have on average accounted for 27 percent and 32 percent, respectively. Of these three classes of fixed investment structures investment typically involves investment projects of the longest lengths. Based on a study of project completion patterns in 1991, the U.S. Bureau of the Census (1992) estimated that the average project completion time for private structures building projects is 14 months. Maffertone's (1998) discussion of the stages through which a building project progresses from its initial conception to its start of building suggests planning times of at least six months, if not more. I therefore assume four-period time-to-build and two-period time-to-plan for structures investment and thus use type-F capital to represent structures capital. In the steady-state, the expenditure involved in planning a structures project is evenly split over the two planning periods, as is the expenditure involved in the construction over the four building periods. In terms of recalibrating the type-F investment spending profile this implies $\left(\frac{\mathbf{I}_2^{F*}}{\mathbf{I}_1^{F*}}\right) = 1$ and $\left(\frac{\mathbf{I}_4^{F*}}{\mathbf{I}_3^{F*}}\right) = \left(\frac{\mathbf{I}_5^{F*}}{\mathbf{I}_4^{F*}}\right) = \left(\frac{\mathbf{I}_6^{F*}}{\mathbf{I}_5^{F*}}\right) = 1$. Since the first two periods are planning stages which involve relatively small levels of expenditure, I assume $\left(\frac{\mathbf{I}_3^{F*}}{\mathbf{I}_2^{F*}}\right) = 20,000$.

Residential capital is represented by type-C capital. The March release of the Census Bureau's Survey of Housing Starts contains a supplement that reports project completion times in private residential buildings. Over the last 10 years the time involved in constructing the average residential building has averaged a little over six-and-a-half months; I therefore, calibrate two-period time-to-build for household investment. In a further supplement to the March release of the Survey of Housing Starts the length of time between when a building permit is issued and when construction begins is reported to average one month. However, since time-to-plan involves more that just the time between the permit's issue and the building's start, I assume one-period time-to-plan for household investment. In the steady state, the expenditure involved in building a residential capital investment project is evenly split over the two construction periods, which implies that $\left(\frac{\mathbf{I}_3^{C*}}{\mathbf{I}_2^{C*}}\right) = 1$. Again, since the planning stage involves relatively little expenditure, I assume $\left(\frac{\mathbf{I}_2^{c*}}{\mathbf{I}_1^{c*}}\right) = 20,000$.

Most studies on equipment project completion times cover the manufacturing sector only. Abel and Blanchard (1986) find that manufacturing firms undertaking equipment investment face an average delivery lag of three quarters, during which time they pay installments for the purchase of the investment good. Since the 1973 productivity slowdown, equipment investment in manufacturing has averaged about 25 percent of total equipment investment (that is, 10.5 percent of the economy's total fixed investment). I assume that this industrial equipment investment requires one period to plan and three periods to build and is thus represented by type-D capital. This investment spending profile implies that $\left(\frac{\mathbf{I}_2^{D*}}{\mathbf{I}_1^{D*}}\right) = 20,000$, and $\left(\frac{\mathbf{I}_3^{D*}}{\mathbf{I}_2^{D*}}\right) = \left(\frac{\mathbf{I}_4^{D*}}{\mathbf{I}_3^{D*}}\right) = 1$.

The are few studies on completion times for equipment investment projects undertaken outside the manufacturing sector. This share of equipment investment (which accounts for 30.5 percent of total fixed investment) is comprised primarily of information processing and related equipment, and transportation and related equipment. I assume that this type of investment requires one period to plan and one period to build, and is thus represented by type-B capital. This investment spending profile implies that $\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right) = 20,000$.

The recalibrated capital shares for the six types of capital are given in Table 1.

Table 1 - Calibrated Capital Shares

Type of Capital:	A	B	C	D	E	F
Share of Capital Stock:	0%	30.5%	32%	10.5%	0%	27%

I also assume substantially less substitution across the investment expenditures taking place over the various stages of a multi-period project and set $\sigma = -5,000,000$. A lower degree of substitution across the sub-intermediate goods used to produce the intermediate good is also assumed by setting $\phi = -100$.

The responses of the key variables to a one percentage point shock in money growth for the model with six types of capital and habit-persistence, recalibrated with more realistic steady-state capital shares and investment spending profiles, are presented in figure 8. The responses demonstrate that the more realistic model is still capable of generating a liquidity effect and continues to provide sensible qualitative responses for the other variables in the model.

5 Comparison of the Model's Responses to Those Observed Empirically

In this section I compare the responses of the calibrated model's key variables to their empirical counterparts. In doing so, I employ the more realistic ARMA(1,1) specification of the money growth process described in section one. I note that I am following a method suggested by Christiano, Eichenbaum, and Evans (1998) whereby the parameters of a univariate money growth process are calibrated so as to match the empirically observed response of money growth to a policy shock. As Christiano, *et al.* demonstrate, this is fully equivalent to modeling monetary policy in the form that it is usually thought of—that is, as an interest rate rule that depends on endogenous variables as well as an exogenous monetary policy shock. The specific process I use, however, differs from Christiano, *et al.*'s AR(1) process, which, as demonstrated in section one, has only limited success in capturing the observed dynamics of money growth.

As noted above the parameters of the ARMA(1,1) process were calibrated so as to match the response of money growth that was generated by the just-identified specification of Bernanke and Mihov's (1998a, 1998b) monetary policy VAR.[17] Figure 9 presents the responses for nominal M2, real output, the price level, and nominal interest rates (expressed at a quarterly rate) from the VAR model along with those from my model.[18] Panels (b) and

[17] The monetary policy shock in the just-identified model of Bernanke and Mihov (1998b) is defined so as to induce a 1 percent increase in nonborrowed reserves; it is thus slightly different to the monetary policy shock in the calibrated model which generates a 1 percent increase in the rate of growth of the nominal money supply. The responses from the calibrated model are rescaled so as to generate a long-run response of the nominal money supply that is comparable in magnitude to that from the VAR model.

[18] My choice of identification methods is informed by the difficulties that the empirical literature has encountered in establishing support for the liquidity effect, along with Bernanke and Mihov's (1998b) finding that these difficulties are largely due to the inaccurate identification of monetary policy shocks. The fact that the response of nominal interest rates is so sensitive to the accurate identification of monetary policy

(c) of figure 9 show that the theoretical model is able to produce responses of real output and the price level that are similar in magnitude to the responses generated from the VAR model. The model's ability to capture the timing of the responses is also reasonable.[19] Panel (d) of figure 9 shows that the model implies an initial response of the nominal interest rate that is about one-third the size of that observed empirically; the response generated in the subsequent period is closer to its empirical counterpart.[20] The model's response of the nominal interest rate is able to match better its empirical counterpart when alternative methods of identification—in particular those that use nonborrowed reserves to measure the Fed's policy stance—are assumed (see figures 2 and 3 of Christiano, Eichenbaum, and Evans (1999)). The response of nominal interest rates from the calibrated model is less persistent than the VAR response presented in panel (d) of figure 9 but compares more favorably to the empirical responses generated using alternative methods of monetary policy identification (see again figures 2 and 3 of Christiano, Eichenbaum, and Evans (1999)).

The exercise performed in this section suggests that the model with time-to-build and time-to-plan in investment and habit-persistence in consumption is not only capable of generating a liquidity effect, but is also able to deliver responses that are empirically plausible.

6 Robustness to Parameter Values

In this section I discuss how the paper's results vary with changes in some of the calibrated parameter values. I limit my discussion only to the parameters which I feel could be somewhat contentious; these are parameters—such as $\rho_1^\mu, \rho_2^\mu, ..., \rho_r^\mu, \phi_1, ..., \phi_s$ in the money growth rule—for which there exist no consensus estimate in the literature, as well as parameters—such as σ, ϕ, and a few of the calibrated investment spending profiles—that are specific to the model presented in this paper and have therefore not been used in other models. The purpose of this section is to establish that it is not merely a convenient choice of these parameters that allows me to obtain a liquidity effect.

shocks—so much so as to alter the sign of the response for some sample periods—leads me to prefer the Bernanke-Mihov method of identification.

I have nonetheless compared the responses from my calibrated model to those implied by alternative methods of identification (not shown). Of the VAR-generated responses presented in this section, only the response of nominal interest rates changes materially when alternative methods of monetary policy identification are assumed.

[19] The response of output peaks five quarters earlier in the calibrated model than in the VAR. The response of prices in the VAR are flat for the first two years while in the calibrated model they begin to rise almost immediately.

[20] When the AR(1) money growth process with $\rho_1^\mu = 0.32$ is assumed, the magnitude of the initial response of nominal interest rates generated by the calibrated model is very similar to its empirical counterpart (see panel (d) of figure 10). This money growth process, however, implies an initial response of the nominal money supply that is more than twice as large as the response generated from the VAR (see panel (a) of figure 10).

My calibrated model is able to generate a liquidity effect for a wide range of sensible money growth processes. When money growth is calibrated following King (1991) as an AR(1) process with $\rho_1^{\mu} = 0.5$, a liquidity effect (not shown) still results. However, since a monetary policy shock now signals even greater future money growth, the resulting liquidity effect is slightly smaller and less persistent. When money growth is calibrated using the estimated AR(2) and AR(3) processes shown in figure 3, a liquidity effect (not shown) still results, although in these cases it is somewhat smaller and less persistent.

Greater ex-post investment inflexibilities (which arise from lower values of the parameter σ) generate a stronger liquidity effect since it is this feature of the investment technology that induces firms to continue with any investment plans formulated before the monetary shock. While the calibrated value of $\sigma = -5,000,000$ might appear to be very low, it is in keeping with existing strategies for modeling time-to-build: in Kydland and Prescott (1982), there is no scope for revising the investment plan and hence an implicit assumption that assumes $\sigma \rightarrow -\infty$.

Higher rates of substitution between the various sub-intermediate outputs (which arises from lower values of the parameter ϕ) will limit the generation of a greater liquidity effect: If the intermediate-good producing firm is able to substitute easily between sub-intermediate goods, then a monetary shock will induce the firm to use more of the sub-intermediate goods that employ the capital stocks requiring shorter building times. Consequently, most investment taking place after the shock will be in the type-A capital stock, which will imply that the introduction of multiple capital stocks with varying building times will do nothing to suppress the response of investment nor to generate a liquidity effect. The fact that multiple capital stocks (which are quite dissimilar) exist suggests that the economy's production technology requires different types of capital with different characteristics. It is therefore reasonable to expect a very low degree of substitutability between the different capital stocks in the model and thus the low degree of substitutability (captured by $\phi = -100$) between the sub-intermediate outputs.

The depth of the empirical foundations upon which investment spending profiles are calibrated in section four varies considerably from one type of capital to the other. For type-C and type-F capital stocks, for example, the calibrated investment profiles are soundly based on empirical estimates of time-to-build and time-to-plan for residential and structures investment. In contrast, for type-B capital (which represents information processing and related equipment and transportation and related equipment) the profile of one-period time-to-build and one-period time-to-plan was calibrated entirely based on judgement. In between these extremes is type-D capital, whose assumed three-period time-to-build profile draws from empirical studies of equipment project completion times in manufacturing while its one-period time-to-plan is again based on judgement. In the absence of any estimates of investment spending profiles for type-B capital and time-to-plan for type-D capital, my only available course of action is examine whether the liquidity effect is preserved when the judgement-based investment profile parameters (and implicitly some capital share parameters) are altered in such a way as to hinder the decline in nominal interest rates. I modify my assumptions for type-B capital to assume that it requires one period to build with no prerequisite planning time. This reduces the resultant liquidity effect quite substantially—since it allows a large immediate increase in investment—but does not eliminate it entirely. I also modify my assumptions for type-D capital to assume that it takes three periods to build without any required planning time. This diminishes the liquidity effect by only a small amount inasmuch as the immediate increase in industrial equipment investment is tempered by ex-post inflexibilities, and because this form of capital accounts for a reasonably small share of total capital.

7 Conclusion

This paper has constructed a sticky-price monetary business cycle model with time-to-build and time-to-plan in investment and habit-persistence in consumption that has the desirable property that real and nominal interest rates decline in response to a positive money shock. The development of a model that displays a liquidity effect does not come at the expense of the model's other variables: if anything, the introduction of capital stocks with longer project completion times leads to improved responses for several variables—most notably output and investment—while generating few changes in the responses of others.

An examination of how the response of nominal interest rates implied by the theoretical model changes when some of the nonstandard calibrated parameters are altered indicates that the liquidity effect generated by the model is robust and is in no way merely a result of a conveniently chosen set of parameter values.

A comparison of the responses of key variables from the theoretical model—such as real output, the price level, and nominal interest rates—to those that emerge from a monetary VAR model finds that the calibrated model's responses are similar in magnitude and timing to those observed empirically. While the initial response of nominal interest rates in the calibrated model is smaller than the response generated by the identified VAR considered here, the fact that the shortcoming of the model lies in its inability to generate the correct magnitude of the response of nominal interest rates—rather than the correct sign—suggests a considerable improvement over the benchmark model.

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A Appendix

A.1 Log-linearizing the Benchmark Model

The first-order conditions from the household's utility maximization problem (equations (5) to (7) and (4)) log-linearize to:[21]

$$\begin{aligned}
& -\left[\left(\frac{C^*}{C^*-G^*}\right)\widehat{\mathbf{C}}_t-\left(\frac{G^*}{C^*-G^*}\right)\widehat{\mathbf{G}}_t\right] \\
= & \left(\frac{1-\beta b-\beta h}{1-\beta h}\right)\left(\widehat{R}_t-E_t\widehat{\mathbf{\Pi}}_{t+1}\right)-\left(\frac{\beta h\,(1-\beta b-\beta h)}{1-\beta h}\right)\left(E_t\widehat{R}_{t+1}-E_t\widehat{\mathbf{\Pi}}_{t+2}\right) \\
& -(1+\beta b+\beta h)\left[\left(\frac{C^*}{C^*-G^*}\right)E_t\widehat{\mathbf{C}}_{t+1}-\left(\frac{G^*}{C^*-G^*}\right)E_t\widehat{\mathbf{G}}_{t+1}\right] \\
& +(\beta b+\beta h)\left[\left(\frac{C^*}{C^*-G^*}\right)E_t\widehat{\mathbf{C}}_{t+2}-\left(\frac{G^*}{C^*-G^*}\right)E_t\widehat{\mathbf{G}}_{t+2}\right], \qquad (40)
\end{aligned}$$

$$\widehat{\widehat{\boldsymbol{\omega}}}_t = s\widehat{H}_t+\left(\frac{h\beta}{1-h\beta}\right)\left(\widehat{R}_t-E_t\widehat{\mathbf{\Pi}}_{t+1}\right)$$

[21] In what follows a caret over a variable signifies the variable's log-deviation from its steady-state value while a star represents the steady-state value of the variable.

$$+\left(\frac{1}{1-h\beta-b\beta}\right)\left[\left(\frac{C^*}{C^*-G^*}\right)\widehat{\mathbf{C}}_t-\left(\frac{G^*}{C^*-G^*}\right)\widehat{\mathbf{G}}_t\right] \tag{41}$$
$$-\left(\frac{h\beta+b\beta}{1-h\beta-b\beta}\right)\left[\left(\frac{C^*}{C^*-G^*}\right)E_t\widehat{\mathbf{C}}_{t+1}-\left(\frac{G^*}{C^*-G^*}\right)E_t\widehat{\mathbf{G}}_{t+1}\right],$$

$$\left\{\frac{1}{1+\delta\beta-\beta}\right\}\widehat{R}_t-\left\{\frac{1}{1+\delta\beta-\beta}\right\}E_t\widehat{\mathbf{\Pi}}_{t+1}$$
$$= E_t\widehat{\boldsymbol{\rho}}^A_{t+1}+\frac{J^{-1''}(\delta)}{1+\delta\beta-\beta}\left(\beta E_t\widehat{K}^A_{t+2}-(1+\beta)\widehat{K}^A_{t+1}+\widehat{K}^A_t\right), \tag{42}$$

$$\widehat{\mathbf{I}}^A_t=\left(\frac{1}{\delta}\right)\widehat{K}^A_{t+1}-\left(\frac{1-\delta}{\delta}\right)\widehat{K}^A_t,\text{ and} \tag{43}$$

$$\widehat{\mathbf{G}}_t=h\widehat{\mathbf{G}}_{t-1}+(1-h)\widehat{\mathbf{C}}_{t-1},$$

where ρ_{cc} is the intertemporal rate of substitution of effective consumption.

The first-order condition from the household's static problem of choosing the cost-minimizing combinations of actual consumption, actual investment, and money balances (equation (11)) log-linearizes to:

$$\widehat{X}_t=\widehat{\mathbf{X}}_t+\left(\frac{1}{1-\eta}\right)\widehat{\left(\frac{\mathbf{P}_t}{P_t}\right)} \tag{44}$$

where $X=C$ for consumption and $X=I^A$ for type-A investment, and where:

$$\widehat{\left(\frac{\mathbf{P}_t}{P_t}\right)}=\left(\frac{(1-a)^{\frac{1}{1-\eta}}\left(1-\frac{1}{R_t}\right)^{\frac{-\eta}{1-\eta}}}{a^{\frac{1}{1-\eta}}+(1-a)^{\frac{1}{1-\eta}}\left(1-\frac{1}{R_t}\right)^{\frac{-\eta}{1-\eta}}}\right)\left(\frac{1}{R^*-1}\right)\widehat{R}_t.$$

Equation (12) linearizes to:

$$\begin{aligned}\widehat{\left(\frac{M_t}{P_t}\right)} &= c\widehat{\left(\frac{M^C_t}{P_t}\right)}+i^A\widehat{\left(\frac{M^{I^A}_t}{P_t}\right)}\\ &= c\widehat{C}_t+i^A\widehat{I}^A_t-\left(\frac{1}{R^*-1}\right)\left(\frac{1}{1-\eta}\right)\widehat{R}_t\\ &= \widehat{Y}_t-\left(\frac{1}{R^*-1}\right)\left(\frac{1}{1-\eta}\right)\widehat{R}_t. \end{aligned}\tag{45}$$

The economy's factor demands (equations (18) and (17)) linearize to:

$$\widehat{K}^A_t=\left\{\frac{Y^*}{Y^*+FC}\right\}\widehat{Y}_t+(1-\alpha)\widehat{\widehat{\omega}}_t-(1-\alpha)\widehat{\widehat{\rho}}^A_t \tag{46}$$

$$\widehat{H}_t = \left\{ \frac{Y^*}{Y^* + FC} \right\} \widehat{Y}_t - \alpha \widehat{\widehat{\omega}}_t + \alpha \widehat{\widehat{\rho}}_t^A \tag{47}$$

while marginal cost (equation (15)) linearizes to:

$$\widehat{\widehat{MC}}(z)_t = (1 - \alpha) \widehat{\widehat{\omega}}_t + \alpha \widehat{\widehat{\rho}}_t^A .$$

The log-linearization of equation (16) combined with equation (14) implies that deviations of the gross inflation rate from its steady-state value of unity evolve according to:

$$\widehat{\Pi}_t = \left\{ \left[\frac{1-\theta}{1 - \theta(1-\epsilon)} \right] \left[\frac{(1-\gamma)(1-\gamma\beta)}{\gamma} \right] \right\} \widehat{\widehat{MC}}_t + \beta \widehat{\Pi}_{t+1} \tag{48}$$

where ϵ is the elasticity of θ with respect to the firm's market share.

The monetary authority's money growth rule (equation (19)) log-linearizes to:

$$\widehat{\mu}_t = \rho_M \widehat{\mu}_{t-1} + \varepsilon_t \tag{49}$$

while the market clearing condition (equation (20)) implies:

$$\widehat{Y}_t = c\widehat{C}_t + i^A \widehat{I}_t^A \tag{50}$$

where c and i^A are the steady-state shares of output devoted to consumption and investment in market capital.

The following relationships between actual and effective prices should also be noted:

$$\widehat{\widehat{\boldsymbol{\omega}}}_t = \widehat{\widehat{\omega}}_t - \widehat{\left(\frac{\mathbf{P}_t}{P_t} \right)} \tag{51}$$

$$\widehat{\widehat{\boldsymbol{\rho}}}_t^A = \widehat{\widehat{\rho}}_t^A - \widehat{\left(\frac{\mathbf{P}_t}{P_t} \right)}, \text{ and} \tag{52}$$

$$E_t \widehat{\boldsymbol{\Pi}}_{t+1} = \widehat{\left(\frac{\mathbf{P}_{t+1}}{P_{t+1}} \right)} - \widehat{\left(\frac{\mathbf{P}_t}{P_t} \right)} - \widehat{\Pi}_{t+1}. \tag{53}$$

A.2 Log-linearizing the Model with Two-Period Time-to-Build

The household's log-linearized first-order conditions for its choice of $\widehat{\mathbf{C}}_t$, $\widehat{K}_{t+1}^A$, and $\widehat{\mathbf{I}}_t^A$ remain as in the previous section (that is equations (40), (42) and (43)). For $\widehat{H}_t^A$ and $\widehat{H}_t^B$, however, they change slightly to:

$$
\begin{aligned}
\widehat{\widetilde{\boldsymbol{\omega}}}_t &= s\left(\frac{H^{A*}}{H^{A*}+H^{B*}}\right)\widehat{H}_t^A + s\left(\frac{H^{B*}}{H^{A*}+H^{B*}}\right)\widehat{H}_t^B + \left(\frac{h\beta}{1-h\beta}\right)\left(\widehat{R}_t - E_t\widehat{\mathbf{\Pi}}_{t+1}\right) \\
&+ \left(\frac{1}{1-h\beta-b\beta}\right)\left[\left(\frac{C^*}{C^*-G^*}\right)\widehat{\mathbf{C}}_t - \left(\frac{G^*}{C^*-G^*}\right)\widehat{\mathbf{G}}_t\right] \\
&- \left(\frac{h\beta+b\beta}{1-h\beta-b\beta}\right)\left[\left(\frac{C^*}{C^*-G^*}\right)E_t\widehat{\mathbf{C}}_{t+1} - \left(\frac{G^*}{C^*-G^*}\right)E_t\widehat{\mathbf{G}}_{t+1}\right]. \qquad (54)
\end{aligned}
$$

The new log-linearized first-order conditions for the choice of $E_t\widehat{K}_{t+2}^B$ and $\widehat{\mathbf{I}}_{1,t}^{B,t}$ are:

$$
\begin{aligned}
E_t\widehat{\widetilde{\boldsymbol{\rho}}}_{t+2}^B &= \left\{\frac{1}{\left(1+\beta\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\right)(1+\beta\delta-\beta)}\right\}(\widehat{R}_t - \widehat{\mathbf{\Pi}}_{t+1}) \\
&+ \left\{\frac{1+\beta\delta+\beta\left(\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)-1\right)}{\left(1+\beta\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\right)(1+\beta\delta-\beta)}\right\}(E_t\widehat{R}_{t+1} - E_t\widehat{\mathbf{\Pi}}_{t+2}) \\
&- \frac{J^{-1'}(\delta)}{(1+\beta\delta-\beta)}\left(\beta E_t\widehat{K}_{t+3}^B - (1+\beta)E_t\widehat{K}_{t+2}^B + \widehat{K}_{t+1}^B\right) \text{ and} \qquad (55)
\end{aligned}
$$

$$
\widehat{\mathbf{I}}_{1,t}^{B,t} = \left(\frac{1}{\sigma-1}\right)\left(\frac{\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}{1+\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}\right)\left(\widehat{R}_t - \widehat{\mathbf{\Pi}}_{t+1}\right) + \left(\frac{1}{\delta}\right)E_t\widehat{K}_{t+2}^B - \left(\frac{1-\delta}{\delta}\right)\widehat{K}_{t+1}^B
$$

and for $E_t\widehat{K}_{t+1}^B$ and $\widehat{\mathbf{I}}_{2,t}^{B,t}$:

$$
\begin{aligned}
E_t\widehat{\widetilde{\boldsymbol{\rho}}}_{t+1}^B &= \left\{\frac{1+\beta\delta+\beta\left(\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)-1\right)}{\left(1+\beta\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\right)(1+\beta\delta-\beta)}\right\}(\widehat{R}_t - \widehat{\mathbf{\Pi}}_{t+1}) \\
&- \frac{J^{-1'}(\delta)}{(1+\beta\delta-\beta)}\left(\beta E_t\widehat{K}_{t+2}^B - (1+\beta)\widehat{K}_{t+1}^B + \widehat{K}_t^B\right) - \frac{(\sigma-1)}{(1+\beta\delta-\beta)}\widehat{\mathbf{I}}_{2,t}^{Y,t} \\
&+ \frac{(\sigma-1)}{(1+\beta\delta-\beta)}\left(\frac{1}{\delta}\right)\widehat{K}_{t+1}^B - \frac{(\sigma-1)}{(1+\beta\delta-\beta)}\left(\frac{1-\delta}{\delta}\right)\widehat{K}_t^B, \text{ and} \qquad (56)
\end{aligned}
$$

$$
\widehat{\mathbf{I}}_{2,t}^{B,t} = \left(\frac{1+\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}{\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}\right)\left(\frac{1}{\delta}\right)\widehat{K}_{t+1}^B - \left(\frac{1+\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}{\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}\right)\left(\frac{1-\delta}{\delta}\right)\widehat{K}_t^B + \left(\frac{1}{\left(\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}\right)\beta}\right)\widehat{\mathbf{I}}_{1,t-1}^{B,t-1}.
$$

C_t, I_t^A,$I_{1,t}^{B,t}$, and $I_{2,t}^{B,t}$ are given by equation (44) while:

$$
\begin{aligned}
\frac{\widehat{M_t}}{P_t} &= c\frac{\widehat{M_t^C}}{P_t} + i^A \frac{\widehat{M_t^{I^A}}}{P_t} + \frac{i^B}{2}\frac{\widehat{M_t^{I_1^B}}}{P_t} + \frac{i^B}{2}\frac{\widehat{M_t^{I_2^B}}}{P_t} \\
&= \widehat{Y}_t - \left(\frac{1}{R^* - 1}\right)\left(\frac{1}{1-\eta}\right)\widehat{R}_t.
\end{aligned}
$$

The new log-linearized factor demands (equations (36) and (37)) are:

$$
\begin{aligned}
\widehat{H}_t^X &= \left\{\frac{Y^*}{Y^* + FC}\right\}\widehat{Y}_t - \alpha\widehat{\widetilde{\omega}}_t + \sum_{i\in\{S/X\}}\left[\frac{\left(\frac{1}{1-\phi}\right)\left(\frac{\Psi_i}{(\widetilde{\rho}_t^i)^\phi}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^j)^\phi}\right)^{\frac{1}{1-\phi}}\right)}\right]\widehat{\widetilde{\rho}}_t^i \\
&+ \left[\alpha - \frac{\left(\frac{1}{1-\phi}\right)\left(\frac{\Psi_X}{(\widetilde{\rho}_t^X)^\phi}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^j)^\phi}\right)^{\frac{1}{1-\phi}}\right)}\right]\alpha\widehat{\widetilde{\rho}}_t^X \text{ and}
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\widehat{K}_t^X &= \left\{\frac{Y^*}{Y^* + FC}\right\}\widehat{Y}_t + (1-\alpha)\widehat{\widetilde{\omega}}_t + \sum_{i\in\{S/X\}}\left[\frac{\left(\frac{1}{1-\phi}\right)\left(\frac{\Psi_i}{(\widetilde{\rho}_t^i)^\phi}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^j)^\phi}\right)^{\frac{1}{1-\phi}}\right)}\right]\widehat{\widetilde{\rho}}_t^i \\
&+ \left[-(1-\alpha) - \frac{\left(\frac{1}{1-\phi}\right)\left(\frac{\Psi_X}{(\widetilde{\rho}_t^X)^\phi}\right)^{\frac{1}{1-\phi}}}{\left(\sum_{j\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^j)^\phi}\right)^{\frac{1}{1-\phi}}\right)}\right]\widehat{\widetilde{\rho}}_t^X
\end{aligned} \tag{58}
$$

where $X = A, B$, while marginal cost is:

$$
\widehat{\widetilde{MC}}(z)_t = (1-\alpha)\,\widehat{\widetilde{\omega}}_t + \alpha\left[\frac{\sum_{i\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^i)^\phi}\right)^{\frac{1}{1-\phi}}\widehat{\widetilde{\rho}}_t^i}{\sum_{j\in S}\left(\frac{\Psi_i}{(\widetilde{\rho}_t^j)^\phi}\right)^{\frac{1}{1-\phi}}}\right].
$$

The log-linearized market clearing condition changes to:

$$
\widehat{Y}_t = c\widehat{C}_t + i^A\widehat{I}_t^A + i^B\left(\frac{1}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\widehat{I}_{1,t}^{B,t} + i^B\left(\frac{\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}{1+\frac{\mathbf{I}_2^{B*}}{\mathbf{I}_1^{B*}}}\right)\widehat{I}_{2,t}^{B,t}. \tag{59}
$$

In addition to the relationships between actual and effective prices listed in equations (51) to (53), the model with two types of capital also has:

$$\widehat{\widetilde{\boldsymbol{\rho}}}_t^B = \widehat{\widetilde{\rho}}_t^B - \widehat{\left(\frac{\mathbf{P}_t}{P_t}\right)}. \tag{60}$$

Figure 2.1 - Responses of the Federal Funds Rate & Velocity of Money to an Identified Monetary Shock

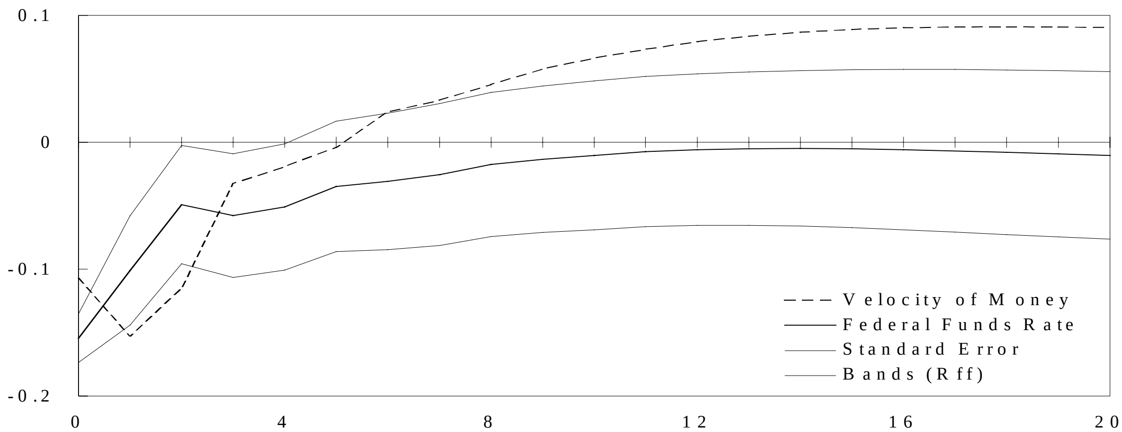


Figure 2.2 - Responses of Money Growth to an Identified Monetary Shock and Money Growth implied by Candidate AR(1) Processes

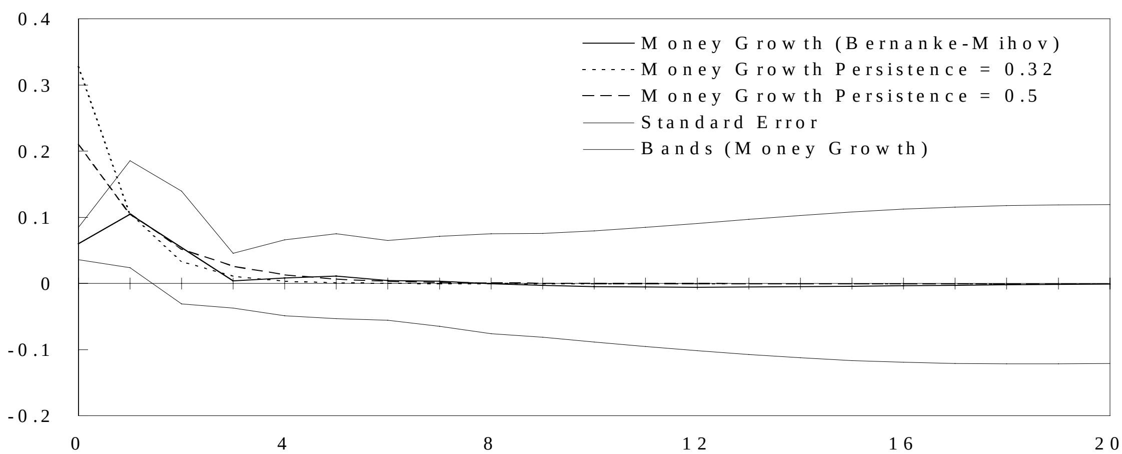


Figure 2.3 - Responses of Money Growth to an Identified Monetary Shock and Money Growth implied by ARMA(1,1), AR(2) and AR(3) Processes

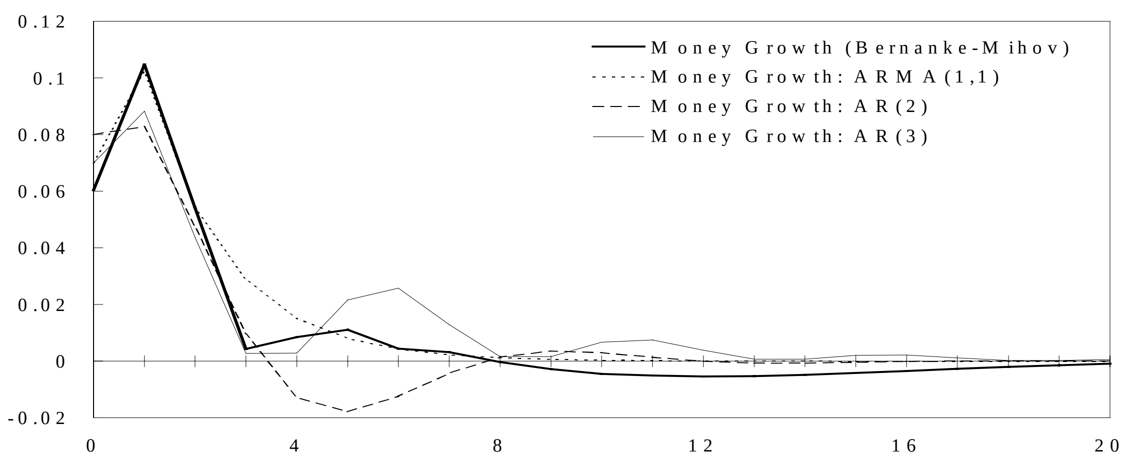


Figure 2.4 - Responses to a Money Growth Shock in the Benchmark Model

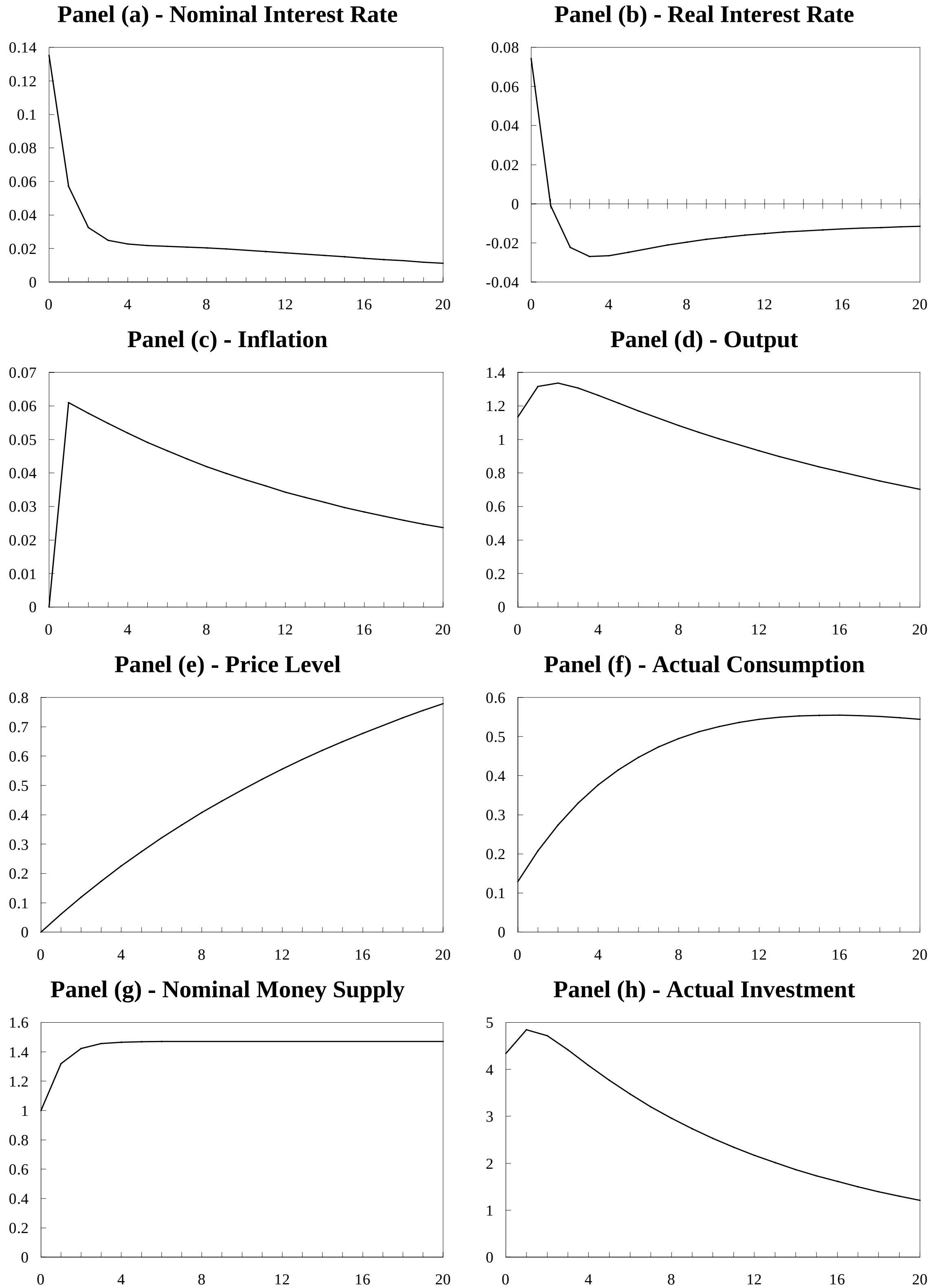


Figure 2.5 - Responses to a Money Growth Shock in the Models with One-Period and with One- and Two-Period Time-to-Build

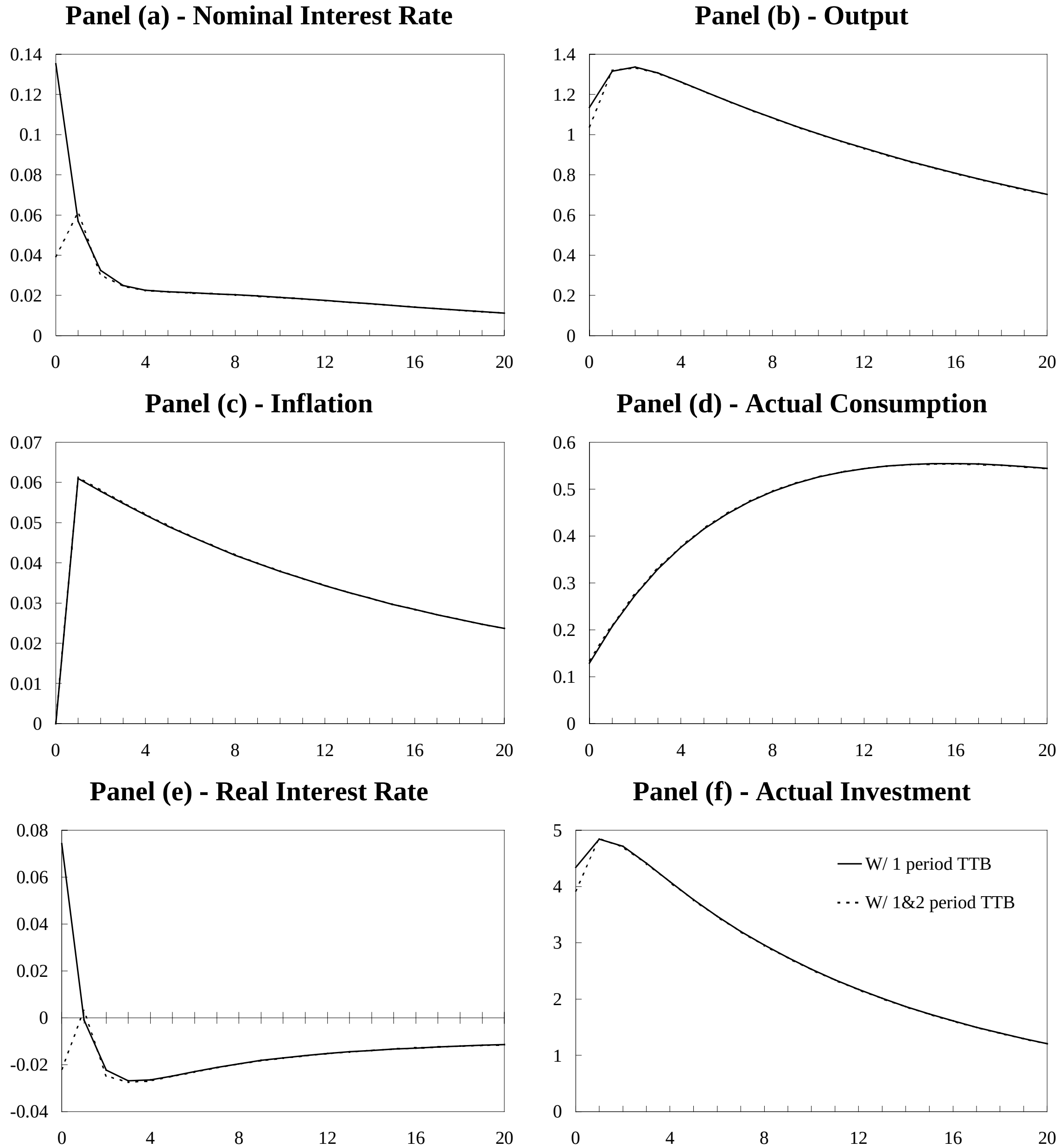


Figure 2.6 - Responses to a Money Growth Shock, Fully Specified Models

Panel (a) - Nominal Interest Rate

Panel (b) - Inflation

Panel (c) - Real Interest Rate

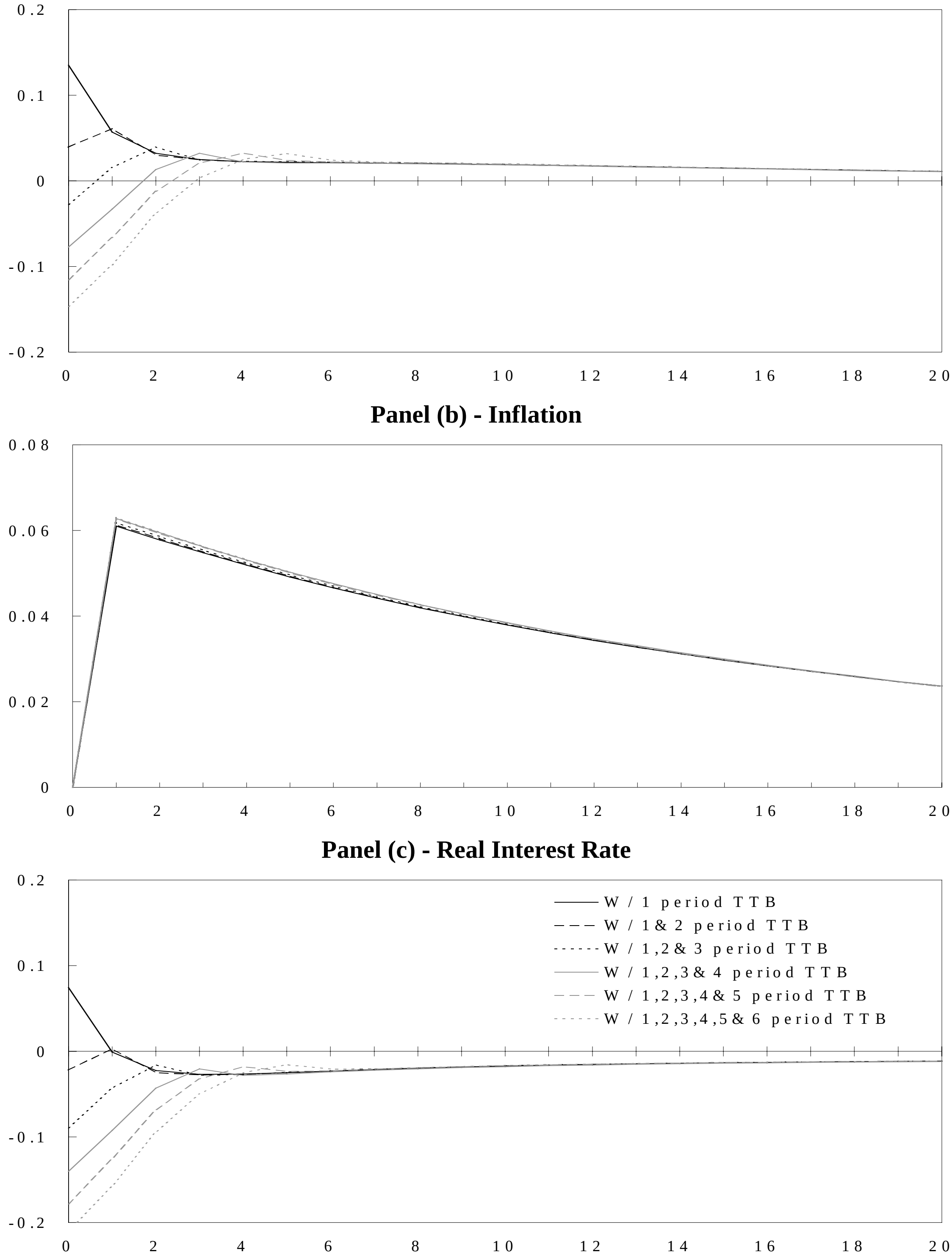


Figure 2.7 - Responses to a Money Growth Shock, Fully Specified Models

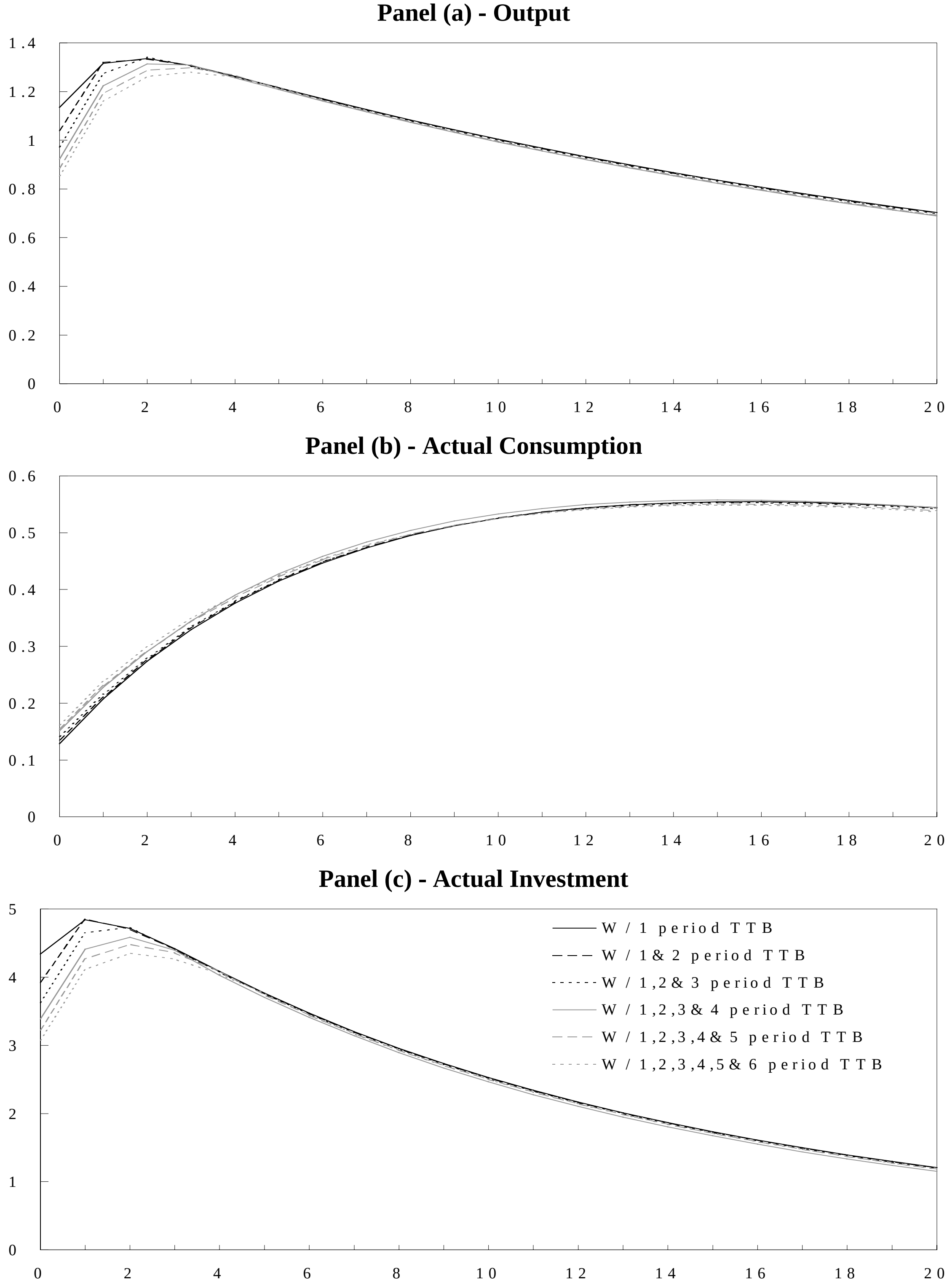


Figure 2.8 - Responses to a Money Growth Shock in the Model with Recalibrated Capital Shares and Investment Profiles

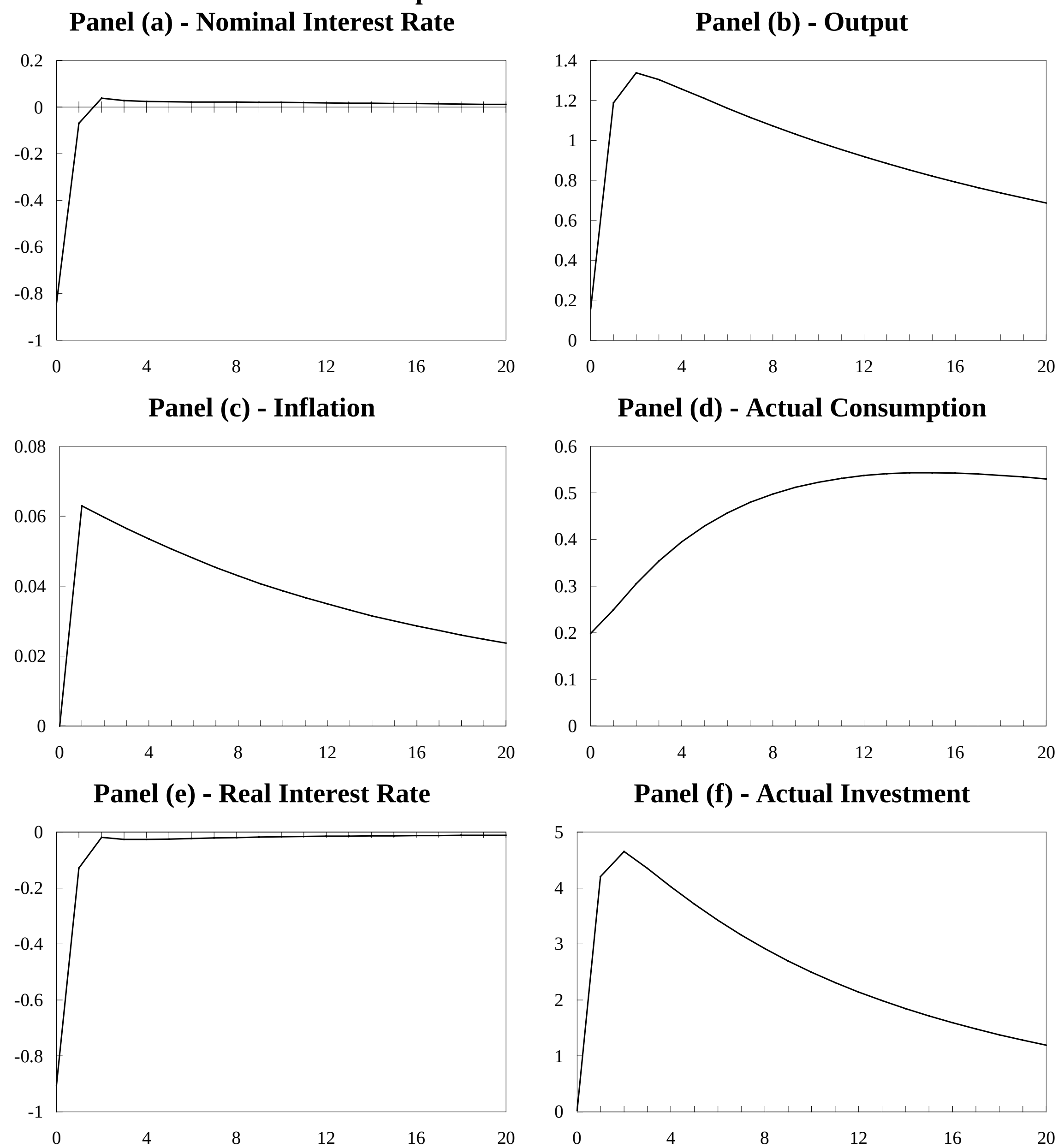
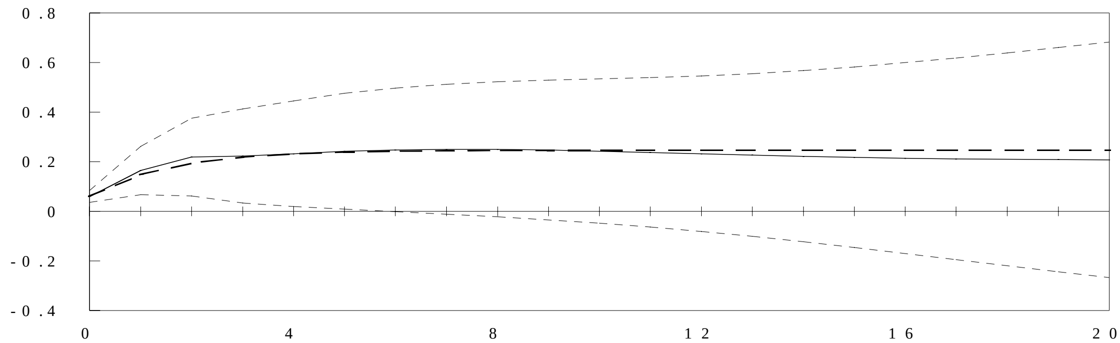
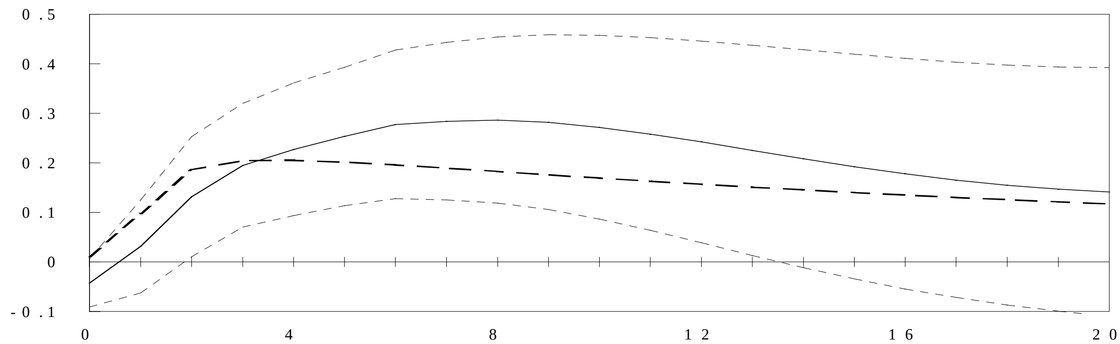


Figure 2.9 - Responses to a Money Growth Shock: VAR and Theoretical Models
(ρ^{μ}_{1} = 0.53 and ϕ_1 = 0.93)

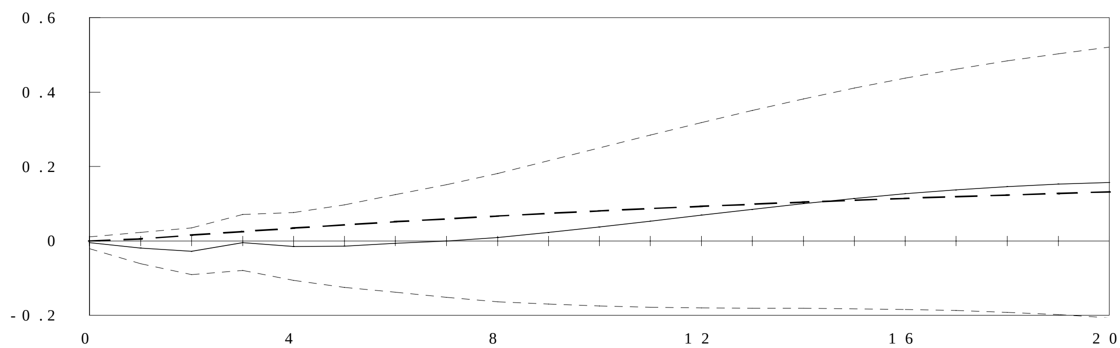
Panel (a) - Nominal Money Supply



Panel (b) - Output



Panel (c) - Price Level



Panel (d) - Nominal Interest Rate

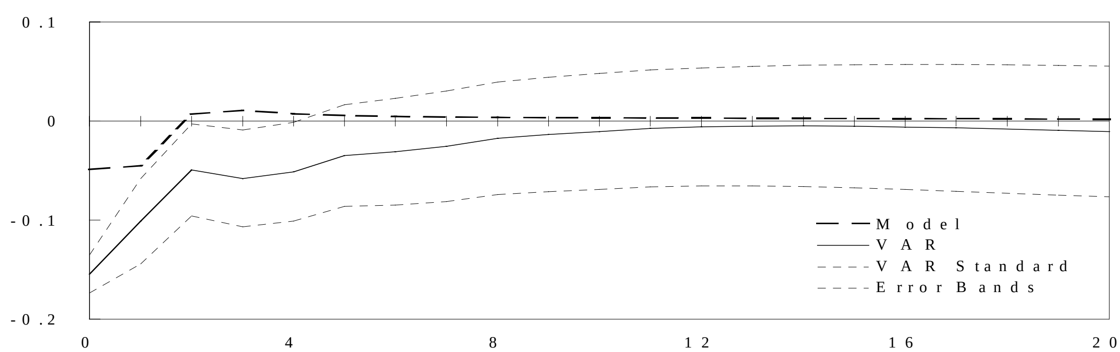
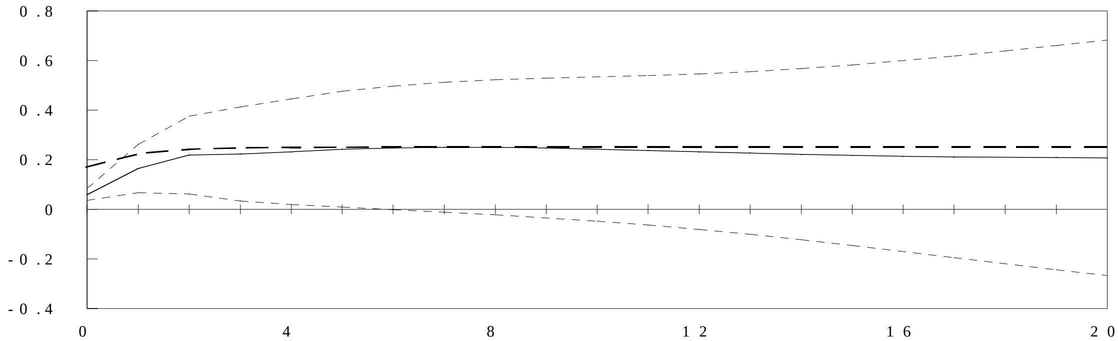
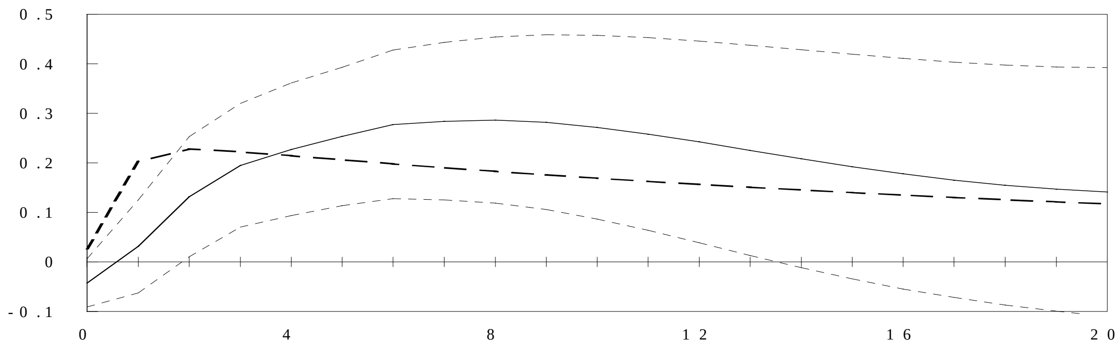


Figure 2.10 - Responses to a Money Growth Shock: VAR and Theoretical Models ($\rho^{\mu}_1 = 0.32$)

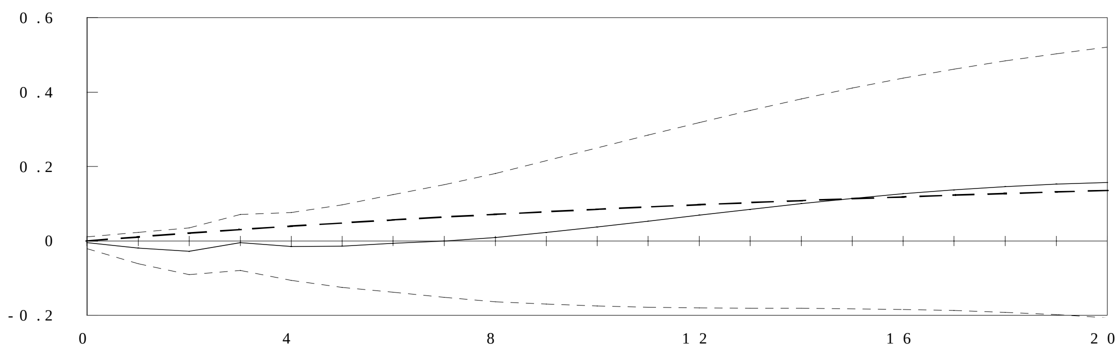
Panel (a) - Nominal Money Supply



Panel (b) - Output



Panel (c) - Price Level



Panel (d) - Nominal Interest Rate