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Abstract: Galí and Gertler (1999) are the first to find that the baseline sticky price model fits the U.S. data well. I examine the robustness of their estimates along two dimensions. First, I show that their IV estimates are not robust to an alternative normalization of the moment condition being estimated. However, when using a Monte-Carlo study to investigate small-sample properties, I show that the normalization chosen by Galí and Gertler (1999) yields a superior estimator. Second, I check whether or not the proportion of backward-looking firms augmenting the baseline model to fit the data is dependent on the type of contracting assumed. I find that using Taylor-style contracts, rather than Calvo-style contracts, this proportion jumps to 50 percent.

Keywords: Phillips Curve, Staggered Contracts, Monte Carlo

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1 Introduction

The study of the Phillips curve has been recognized as an important activity since Phillips (1958) identified a negative correlation between inflation and unemployment. King and Watson (1994) give a comprehensive discussion of the evolution of the traditional empirical literature.

The typical setup of recent research is an environment of monopolistically competitive intermediate producers, as in Dixit and Stiglitz (1977), coupled with sticky prices. To simplify the aggregation of prices, a contracting framework developed by Calvo (1983), an put into an optimizing, general-equilibrium environment, by Yun (1996), is commonly employed. In this framework, firms fix their prices until they receive a random signal. This simplification, however, leads to a new Phillips curve being solely forward-looking. As a result, inflation persistence is absent from the new specification. As a remedy, researchers have appended lags of inflation, or postulated a departure from optimizing behavior.

As Galí and Gertler (1999) note, the motivation for appending lags of inflation is largely empirical. Fuhrer and Moore (1995) appeal to a relative wage hypothesis that, however, does not evolve from a general equilibrium set-up[1]. Roberts (1997) introduces adaptive expectations for a subset of agents. Galí and Gertler (1999) similarly assume that a fraction of the economic actors in their model do not optimize, as Campbell and Mankiw (1989) do in their test of the permanent income hypothesis of consumption.

Galí and Gertler (1999) are the first to report a good fit of the baseline sticky-price model to the U.S. data. Rudd and Whelan (2001) argues that the methodology of Galí and Gertler (1999) is particularly sensitive to errors in model specification. In this paper, I check its robustness along two dimensions. First, I show that the instrumental-variable (IV) estimates reported by Galí and Gertler are not robust to an alternative normalization of the moment condition. This is a standard issue encountered when using IV estimators. In small samples, normalizing the moment condition by the coefficient of one of its variables can affect the estimation results. In the case of the new Phillips curve specification, it would be natural to normalize the moment condition by the coefficient of current inflation. However, when I setup a Monte Carlo study to check the small sample properties

[1] In fact I show that the endogenous persistence of this specification is in the same order of magnitude as that produced by assuming the more standard Taylor (1980) contracts.

of the normalized estimator, I find that it is inferior to its non-normalized counterpart.

Second, I check for robustness to the choice of contracting assumption. Galí and Gertler (1999), for algebraic simplicity, choose a Calvo-style contracting structure. In that setup, firms reset prices when hit by a price-renewal signal that follows a Poisson distribution. It is possible that a small number of firms, not receiving a price-renewal signal, could keep their prices so low as to capture a wide share of the market. By forcing firms to reset prices every N periods, Taylor-style contracts avoid this problem.

Surprisingly, despite the fact that lags of inflation are already present in the Phillips curve implied by Taylor contracting, when using a specification test as in Campbell and Mankiw (1989), the proportion of backward-looking firms needed to fit the US data is estimated to be much higher than the level reported by Galí and Gertler (1999) (whose theoretical model does not imply lags of inflation in the Phillips curve).

As in Galí and Gertler (1999), I run the specification test after linearizing the model. One side-effect of the linearization is that, in the baseline theoretical model, lags of inflation enter the Phillips curve with a negative coefficient. This could be an explanation for why a much higher proportion of backward-looking firms is needed to fit the U.S. data with Taylor contracts than with Calvo contracts. Sbordone (2001), using a test that does not require linearizing the first order conditions, does not find that the contracting specification matters. The test proposed here, however, is still relevant for calibration purposes, as routine solution methods for dynamic general equilibrium models require linear conditions.

The plan of the paper is as follows: section 2 gives an overview of the standard setup in the new Phillips curve literature; section 3 investigates the small sample properties of two IV estimators that only differ by a normalization; section 4 compares estimates for Calvo-style and Taylor-style prices; section 5 concludes.

2 The new Phillips curve

Galí and Gertler (1999) give a good review of the recent state of the literature. I will only attempt to summarize the salient points.

The structure behind the new Phillips curve is an environment of monopolistically competitive firms that are faced with a constraint on price adjustment. Following Calvo (1983), the literature has postulated that every period a fraction θ of the firms, randomly chosen following a Poisson process, readjusts its price. Thus, on average, prices remain fixed for $1/\theta$ periods.

Profit maximization, for a firm chosen to adjust its price at time t, implies a first order condition for price that, log-linearized and expressed in terms of inflation, leads to:

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{V}_t + \beta E_t \pi_{t+1} \tag{1}$$

where π is inflation, $\hat{V}$ is the percent deviation of the the firm's real marginal cost from its steady state, and β is the discount factor.

The traditional work on the Phillips curve chose unemployment or the output gap (following Okun's law) as the indicator of economic activity. More recently, alternative measures of real activity have been explored. In the standard sticky price framework, as in Rotemberg and Woodford (1998), there is an approximate log-linear relationship between marginal cost and the output gap so that

$$\hat{V}_t = \kappa x_t \tag{2}$$

where x_t is the difference between the log of output and the log of the natural rate of output, i.e. the level of output would take if prices were perfectly flexible. κ is the output elasticity of marginal cost.

Combining equation (1) and (2), one obtains an equation for inflation in the same spirit as the original Phillips curve.

$$\pi_t = \lambda\kappa x_t + \beta E_t \pi_{t+1}. \tag{3}$$

Notice that lags of inflation are conspicuously absent from the equation above. Galí and Gertler (1999) assume that a fraction ω of firms follow a simple rule of thumb that entails setting prices according to:

$$P_t^b = P_{t-1}^f + \pi_{t-1}$$

where P_t^b is the price set by a backward-looking firm when hit by a price-renewal shock, P_{t-1}^f is the price set last period by a forward-looking firm hit by a price renewal shock, while π_{t-1} is last period's inflation. Then

Equation (3) becomes

$$\gamma_0 \pi_t = \lambda \hat{V}_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} \tag{4}$$

where γ_0, λ and γ_b are given by:

$$\gamma_0 = \theta + \omega \left[1 - \theta(1-\beta)\right] \tag{5}$$

$$\lambda = (1-\omega)(1-\theta)(1-\beta\theta) \tag{6}$$

$$\gamma_f = \beta\theta \tag{7}$$

$$\gamma_b = \omega \tag{8}$$

A testable moment condition is easily obtained from the equation above. Assuming rational expectations, $E_t\pi_{t+1}$ can be rewritten as $E_t\pi_{t+1} = \pi_{t+1} - \epsilon_t$, where ϵ_t is a forecast error. Substituting into the equation above:

$$\gamma_0 \pi_t = \lambda \hat{V}_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} - \gamma_b \epsilon_t \tag{9}$$

Notice however, that two normalizations are possible. One can choose whether or not to divide the left-hand side of equation (9) by γ_0. Asympotically, IV-based estimators would yield the same estimates. In small samples, however, the normalization chosen turns out to significantly affect the estimation results.

3 Comparing Estimators

Under the assumption of rational expectations, any variable dated $t-1$ or earlier would be a valid instrument to estimate equation (4). Following Galí and Gertler, I use four lags of inflation (as measured by using the GDP deflator), four lags of the share of income to labor, four lags of the interest rate spread, and four lags of a measure of wage inflation. The dataset ranges from 1959 quarter 1 to 2001 quarter 2.

The estimation results of equation 3 are reported in Table 1. While under one normalization the estimate of ω – the fraction of backward-looking firms – is numerically small, in the order of 10%, for an alternative normalization the same estimate is in the order of 40%. While this disparity was not initially reported by Galí and Gertler (1999), Galí and Gertler (2000) addresses this issue by comparing the predictive power of the two sets of estimates. It is argued that the non-normalized estimator has greater predictive power and is, therefore, preferable. This kind of selection criterion, however, does not say much about how close to the truth the two estimators get. This could be achieved investigating their small sample properties, which is what I do in the next section.

3.1 A Monte Carlo Experiment

In order to compare the small sample properties of the two alternative estimators considered above, one can rely on Monte Carlo analysis. Given that ϵ_t in equation (9) is a rational-expectations forecast error, it will be independent over time. One can then apply the following procedure. After fixing the true values of the parameters β, θ, and ω, given data for π_t and $\hat{V}_t$, one is in a position to generate data for ϵ_t. Given this initial series for ϵ_t, one can then generete new synthetic data for the forecast error, by sampling with replacement from the initial ϵ_t series. This new synthetic series, together with the true model (i.e. the choice of β, θ and ω), allows one to dynamically generate new synthetic data for π_t spanning the original size of the dataset.

I repeat this process one hundred times for different values of θ and ω. I let ω range from 0 to 1 in 0.05 increments. I let θ take the following values: 0.666, 0.750, 0.800, 0.833 , corresponding respectively to an average price contract length of 3, 4, 5, 6 quarters. As can be evinced from Figure 1, the non-normalized estimator produces estimates that have a confidence interval whose width is comparable to that of the normalized estimates. However, the average of the estimates for ω produced by the non-normalized estimator is signifcantly closer to the true value of ω. Therefore, I conclude that, regardless of the "true" value fixed for θ and ω, the normalized estimator is inferior to the non-normalized estimator.

4 Derivation of the Phillips curve with Taylor-style prices

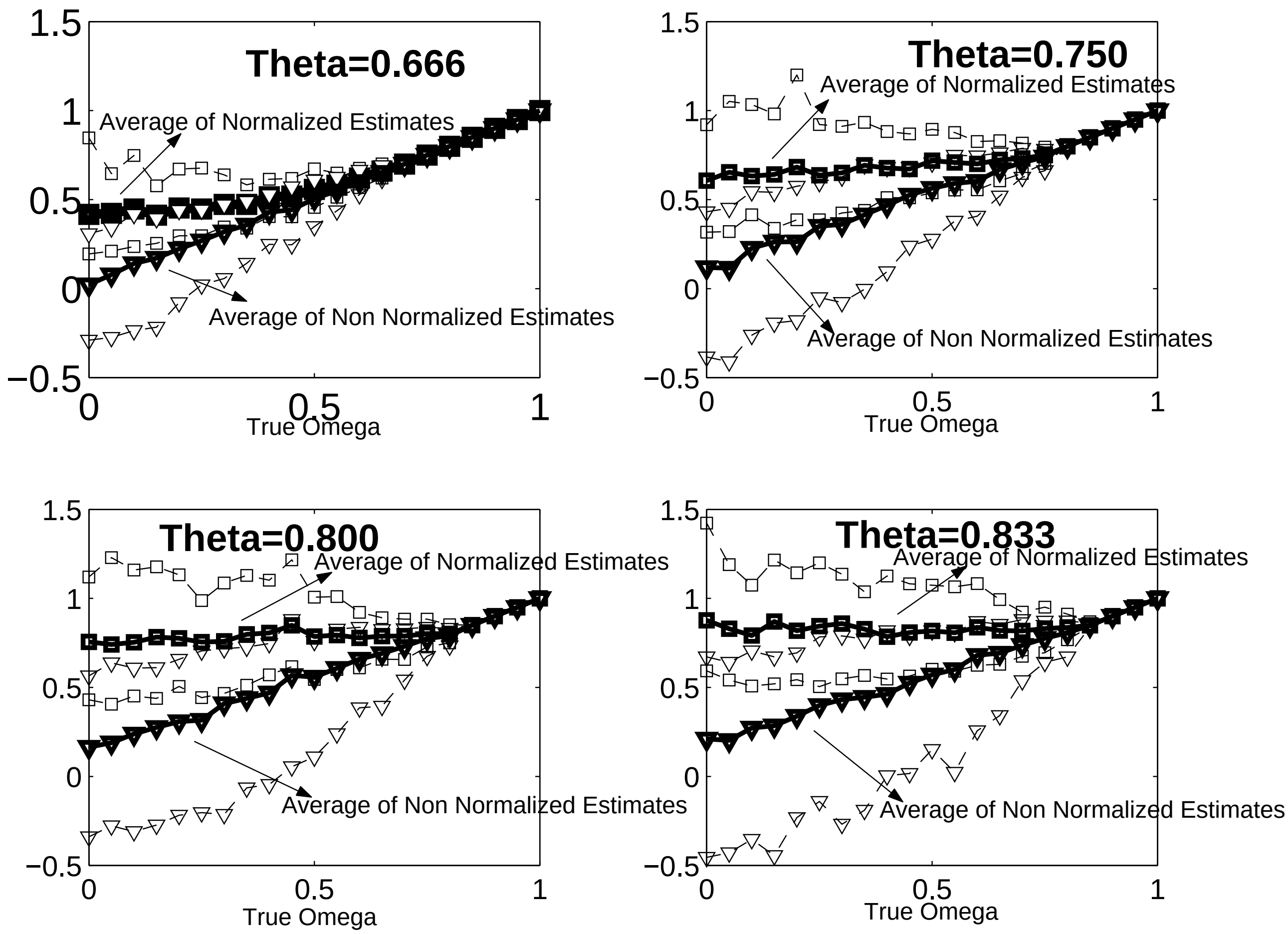
In this section I develop a second test of robustness for the results reported by Galí and Gertler (1999). Rather than focusing on a Calvo-style pricing mechanism, I introduce prices of fixed duration, as in Taylor (1980).

Table 1: **Estimation results using Calvo-style price contracts**

Specification	Galí-Gertler 1	Galí-Gertler 2
Normalize by γ_0	yes	no
Contract length	Random	Random
ω	0.371	0.142
	(0.000)	(0.057)
θ	0.921	0.914
	(0.000)	(0.000)
$\frac{\gamma_f}{\gamma_0}$	0.713	0.865
	(0.000)	(0.000)
$\frac{\gamma_b}{\gamma_0}$	0.287	0.135
	(0.000)	(0.030)
$\frac{\lambda_0}{\gamma_0}$	0.003	0.005
	(0.399)	(0.188)
R^2	0.849	-
Test of Overidentifying Restrictions	(0.917)	(0.898)

Probability values in parentheses.

Figure 1: **Comparison of normalized and non-normalized estimators**



The chart reports the average of 100 estimates and a 90% confidence interval.

The setup follows that of Chari, Kehoe, and McGrattan (2000). In the production sector there is a final market and an intermediate market. The final market is competitive. The intermediate market is imperfectly competitive and firms there set prices for N periods in a staggered fashion. There is a continuum of intermediate firms that can be normalized to 1. Products in the intermediate sector are imperfect substitutes. Their elasticity of substitution is governed by the parameter ϵ. All intermediate products are necessary for the production of final products.

The zero profit condition in the final market implies that the final product price or aggregate price at time t, $\bar{P}_t$, is given by

$$\bar{P}_t = \left[\int_0^1 (P_{j,t})^{\frac{\epsilon}{\epsilon-1}} dj \right]^{\frac{\epsilon-1}{\epsilon}} \tag{10}$$

where $P_{j,t}$ is the price of intermediate product j at time t.

Intermediate firms can be identified by when they set their price. Let the subscript j be 1 for firms that set their price at time t, let j be 2 for firms that set their price at time $t+1$, and so on. Tracking firms in this new way, equation 10 can be rewritten as:

$$\bar{P}_t = \left[\sum_{j=1}^{N} \omega \frac{1}{N} \left(P_{j,t}^{b}\right)^{\frac{\epsilon}{\epsilon-1}} + (1-\omega) \frac{1}{N} \left(P_{j,t}^{f}\right)^{\frac{\epsilon}{\epsilon-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \tag{11}$$

The superscripts f and b distinguish between forward and backward-looking firms. Furthermore, notice that in a symmetric setup, an equal number of firms $\frac{1}{N}$ will change price in each period. If N is chosen to be 1, every firm resets its price every period. If N is chosen to be 2, prices are fixed for two periods and they are reset every other period. For estimation purposes, the equation above can be generalized, so that different firms, fixing their price for a varying length of time, could coexist. Such a setup can be achieved by having a fraction θ_1 of the intermediate firms fixing its price for one period only. A fraction θ_2 fixes its price for 2 periods. A fraction θ_3 fixes its price for 3 periods. And finally a fraction θ_4 fixes its price for 4 periods. Under this modified setup a fraction ω of the intermediate firms would still be backward-looking. Equation (11) would then become:

$$\bar{P}_t = \sum_{i=1}^{4} \theta_i \left[\sum_{j=0}^{i-1} \omega \frac{1}{i} \left(P_{j,t}^{bi}\right)^{\frac{\epsilon}{\epsilon-1}} + (1-\omega) \frac{1}{i} \left(P_{j,t}^{fi}\right)^{\frac{\epsilon}{\epsilon-1}} \right]^{\frac{\epsilon-1}{\epsilon}} \tag{12}$$

The superscript next to the P that indicates a firm's price denotes whether the firm is backward b, or forward-looking f, as well as the contract length, i. Dividing equation 12 by $\bar{P}_{t-1}$, and defining $\pi_t = \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1$ and $q^{fi}_{j,t} = \frac{P^{fi}_{j,t}}{\bar{P}_{t-1}}$, one obtains:

$$(\pi_t + 1)^{\frac{\epsilon}{\epsilon-1}} = \left[\theta_1 \left[\omega + (1-\omega)(q^{f1}_{0,t})^{\frac{\epsilon}{\epsilon-1}}\right] + \theta_2 \left[\frac{1}{2}\omega \left(1 + \left(\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}}\right) + \frac{1}{2}(1-\omega)\left(q^{f2}_{0,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f2}_{1,t}{}^{\frac{\epsilon}{\epsilon-1}}\right)\right] + \right.$$
$$\theta_3 \left[\frac{1}{3}\omega \left(1 + \left(\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}} + \left(\frac{1}{1+\pi_{t-2}}\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}}\right) + \frac{1}{3}(1-\omega)\left(q^{f3}_{0,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f3}_{1,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f3}_{2,t}{}^{\frac{\epsilon}{\epsilon-1}}\right)\right] +$$
$$\theta_4 \left[\frac{1}{4}\omega \left(1 + \left(\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}} + \left(\frac{1}{1+\pi_{t-2}}\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}} + \left(\frac{1}{1+\pi_{t-3}}\frac{1}{1+\pi_{t-2}}\frac{1}{\pi_{t-1}+1}\right)^{\frac{\epsilon}{\epsilon-1}}\right) + \right.$$
$$\left.\left. + \frac{1}{4}(1-\omega)\left(q^{f4}_{0,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f4}_{1,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f4}_{2,t}{}^{\frac{\epsilon}{\epsilon-1}} + q^{f4}_{3,t}{}^{\frac{\epsilon}{\epsilon-1}}\right)\right]\right]$$

Linearizing the above around a zero inflation steady state:

$$\pi_t = \left[\sum_{i=2}^{4} \theta_i \sum_{j=1}^{i-1} -\frac{i-j}{i}\omega\pi_{t-j}\right] + \left[\sum_{i=1}^{4} \theta_i \sum_{j=0}^{i-1} (1-\omega)\frac{1}{i}\hat{q}^{fi}_{j,t}\right] \tag{13}$$

This is the new Phillips curve equation. Below I describe how to obtain $\hat{q}^{fi}_{j,t}$. From the first order conditions for profit maximization for intermediate producers, forward-looking firms that renew their price fix it according to:

$$P^{fn}_{i,t} = \frac{1}{\epsilon} \frac{E_t \sum_{\tau=t}^{\tau=t+n-1} \beta^{\tau-t} \frac{U_c(\tau)}{U_c(t)} \frac{\bar{P}_t}{\bar{P}_\tau} y_\tau \bar{P}_\tau^{\frac{2-\epsilon}{1-\epsilon}} V^{fn}_{i,\tau}}{E_t \sum_{\tau=t}^{\tau=t+1} \beta^{\tau-t} \frac{U_c(\tau)}{U_c(t)} \frac{\bar{P}_t}{\bar{P}_\tau} y_\tau \bar{P}_\tau^{\frac{1}{1-\epsilon}}} \tag{14}$$

Dividing both sides of the equation by $\bar{P}_{t-1}$ and log linearizing around a zero inflation steady state, one obtains:

$$\hat{q}^{fn}_{0,t} = \frac{1}{\sum_{i=0}^{n-1}\beta^i} E_t \left[\sum_{i=0}^{n-1} \hat{V}^{fn}_{i,t+i}\beta^i + \sum_{i=0}^{n-1} \pi_{t+i} \left(\sum_{j=i}^{n-1} \beta^j\right)\right] \tag{15}$$

The final step is to combine equation (15) with equation (13).

4.1 Econometric specification

Rather than estimating the whole model using full information methods, I will restrict the focus of the exercise to estimation of only one equation that can be related, as shown above, to the traditional Phillips Curve.

In order to estimate equation (13), one needs to relate the log deviation from steady state of the unit cost of production, $\hat{V}_t$, to an observable series. Following the simplification introduced by Yun when formalizing the Calvo model, the assumption of perfectly competitive markets for labor and capital (under CRS) ensures a constant marginal cost across intermediate product firms. Thus $V_{i,t} = V_t$ for all i. Furthermore, following

Galí and Gertler, the unit cost can be expressed in terms of the labor share. Real marginal cost, V_t, is given by the ratio of the wage rate to the marginal product of labor. Given the intermediate production technology, $y_{i,t} = K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}$, the marginal product of labor is $1 - \alpha \frac{y_{i,t}}{L_{i,t}}$. Thus, V_t can be written as:

$$V_t = \frac{\frac{w_t}{P_t}}{(1-\alpha) y_{i,t}} L_{i,t} \tag{16}$$

Rearranging (16) one can see that

$$V_t = \frac{1}{1-\alpha} s_t \tag{17}$$

where s_t is the labor income share. Linearizing (17) around steady state, one can see that

$$\hat{V}_t = \hat{s}_t \tag{18}$$

where the firm subscript has been dropped and the "hat" indicates relative deviation from steady state. Finally one can arrive at a testable equation by assuming rational expectations. Then $\pi_{t+1} = E_t \pi_{t+1} + \mu_{t+1}$ and $\pi_t = E_{t-1}\pi_t + \mu_t$. Similarly, $\hat{s}_{t+1} = E_t \hat{s}_{t+1} + \nu_{t+1}$ and $\hat{s}_t = E_{t-1}\hat{s}_t + \nu_t$ and so on.

This leads to the following regression equation:

$$\begin{aligned} \gamma_0 \pi_t &= \lambda_{f(3)} \hat{s}_{t+3} + \lambda_{f(2)} \hat{s}_{t+2} + \lambda_{f(1)} \hat{s}_{t+1} + \lambda_0 \hat{s}_t + \lambda_{b(1)} \hat{s}_{t-1} + \lambda_{b(2)} \hat{s}_{t-2} + \lambda_{b(3)} \hat{s}_{t-3} + \\ &+ \gamma_{f(3)} \pi_{t+3} + \gamma_{f(2)} \pi_{t+2} + \gamma_{f(1)} \pi_{t+1} + \gamma_{b(1)} \pi_{t-1} + \gamma_{b(2)} \pi_{t-2} + \gamma_{b(3)} \pi_{t-3} + \epsilon_t \end{aligned} \tag{19}$$

where the λs and γs are a function of β, ω, θ_1, θ_2, θ_3, θ_4. The exact form of these functions is relegated to an appendix. For clarity of exposition, a much simpler form of the restrictions can be obtained, for instance, by restricting the model to include only those firms fixing their price for two periods. This is achieved by imposing $\theta_1 = 0, \theta_2 = 1, \theta_3 = 0, \theta_4 = 0$. Then equation (19) reduces to the following:

$$\pi_t = \frac{1}{\gamma_0}(\lambda_f \hat{s}_{t+1} + \lambda_0 \hat{s}_t + \lambda_b \hat{s}_{t-1} + \gamma_b \pi_{t-1} + \gamma_f \pi_{t+1} + \epsilon_t) \tag{20}$$

$$\frac{\lambda_f}{\gamma_0} = -\frac{(-1+\omega)\beta}{1+\omega+2\omega\beta} \tag{21}$$

$$\frac{\lambda_0}{\gamma_0} = -\frac{(-1+\omega)(1+\beta)}{1+\omega+2\omega\beta} \tag{22}$$

$$\frac{\lambda_b}{\gamma_0} = -\frac{-1+\omega}{(1+2\beta)\omega+1} \tag{23}$$

$$\frac{\gamma_f}{\gamma_0} = -\frac{(-1+\omega)\beta}{1+\omega+2\omega\beta} \tag{24}$$

$$\frac{\gamma_b}{\gamma_0} = -\frac{\omega(1+\beta)}{1+\omega+2\omega\beta} \tag{25}$$

$$\frac{\epsilon_t}{\gamma_0} = -\frac{\beta}{1+\beta}\frac{1}{2}(1-\omega)\left[\mu_{t+1}+\mu_t+\nu_{t+1}+\nu_t\right] \tag{26}$$

equation (19) can be estimated using the generalized method of moments. Under the assumption of rational expectations, any variable dated $t-1$ or earlier would be a valid instrument [2]. Following Galí and Gertler, I use four lags of inflation (as measured by using the GDP deflator), four lags of the share of income to labor, for lags of the interest rate spread, and four lags of a measure of wage inflation. The dataset ranges from 1959 quarter 1 to 2001 quarter 2.

The appendix shows the structural restrictions on the parameters when different contract lengths are selected and a comparison with the restrictions imposed by having Calvo contracts, rather than contracts as in here à la Taylor.

4.2 Estimation results

The estimation results of the Phillips curve, as expressed in equation (19), are in Table 2. I have imposed that θ_1 be zero, and that θ_2, θ_3, and θ_4 –the proportion of firms setting their prices for 2, 3 and 4 quarters respectively– sum to 1 and lie within 0 and 1. All remaining functional restrictions on the parameters, as dictated by the model, are described in detail in the appendix.

As for Calvo-style pricing, when specifying the moment conditions, two different normalizations are possible. Under the first one, the moment condition takes the form:

$$\begin{aligned}\gamma_0\pi_t &= \gamma_{f3}\pi_{t+3}+\gamma_{f2}\pi_{t+2}+\gamma_{f1}\pi_{t+1}+\gamma_{b1}\pi_{t-1}+\gamma_{b2}\pi_{t-2}+\gamma_{b3}\pi_{t-3}+\gamma_{b4}\pi_{t-4} \\ &\quad +\lambda_{f3}\hat{s}_{t+3}+\lambda_{f2}\hat{s}_{t+2}+\lambda_{f1}\hat{s}_{t+1}+\lambda_0\hat{s}_t+\lambda_{b1}\hat{s}_{t-1}+\lambda_{b2}\hat{s}_{t-2}+\lambda_{b3}\hat{s}_{t-3}+\epsilon_t\end{aligned}$$

To get the alternative normalization, divide the above equation by γ_0. Then:

$$\begin{aligned}\pi_t &= \frac{1}{\gamma_0}\left[\gamma_{f3}\pi_{t+3}+\gamma_{f2}\pi_{t+2}+\gamma_{f1}\pi_{t+1}+\gamma_{b1}\pi_{t-1}+\gamma_{b2}\pi_{t-2}+\gamma_{b3}\pi_{t-3}+\gamma_{b4}\pi_{t-4}\right. \\ &\quad \left.+\lambda_{f3}\hat{s}_{t+3}+\lambda_{f2}\hat{s}_{t+2}+\lambda_{f1}\hat{s}_{t+1}+\lambda_0\hat{s}_t+\lambda_{b1}\hat{s}_{t-1}+\lambda_{b2}\hat{s}_{t-2}+\lambda_{b3}\hat{s}_{t-3}+\epsilon_t\right]\end{aligned}$$

[2] When the contract length is extended to 3 periods, then any variable dated t-2 or earlier would be a valid instrument. Similarly, for contracts lasting 4 periods, instruments need to be dated t-3 or erlier.

The results reported in Table 2 are not affected by the normalization. With the rule of thumb $P_t^b = \bar{P}_{t-1}$, ω is estimated at 0.437 when normalizing by γ_0 and at 0.411 when not applying the normalization. Both estimates are different from 0 at the 0.1 percent significance level. When the rule of thumb is extended to $P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$ then the estimate of ω jumps up to 0.728 using the normalization by γ_0. This estimate is also statistically significant at the 0.1% level. The estimation routine did not achieve convergence when normalizing by γ_0. The validity of the restrictions can only be tested jointly with the validity of the instruments, which I perform using a standard test of overidentifying restrictions. The null hypothesis that the model is well specified and that the instruments are valid fails to be rejected for all specifications. In Table 2, I report the upper tail of the distribution for the test statistic.

The estimates for $\theta_2 - \theta_4$ are not statistically significant. This suggests restricting the model to incorporate only contracts of a single duration. In Table 3, I report the GMM estimates of the parameters in equation (19), imposing the restriction that $\theta_2 = 1$. Tables 3 to 5 report the estimates imposing $\theta_3 = 1$ and $\theta_4 = 1$ respectively.

The proportion of backward-looking firms (ω) estimated using either normalization (dividing or not the left-hand side of equation 19 by γ_0), when the backward-looking firms set prices by considering the previous period's price level only ($P_t^b = \bar{P}_{t-1}$) lies between 35% and 45% according to which contract length is chosen. Not surprisingly, lengthening the contract brings down the proportion of backward-looking firms. When I change the rule of thumb to $P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$, the estimate of ω ranges from 45% (when contracts last four periods) to 75% (when contract last two periods). These results are of a different order of magnitude from the ones reported by Galí and Gertler (2000), and reproduced earlier on in this paper, whose estimate of ω is in the order of 10%.

As a comparison, Table 3 to 5 report the estimates of ω obtained adopting, as in Galí and Gertler, contracts of randomly variable length. The parameter restrictions imposed under that setup are in equations (5)-(8). Under the assumption of rational expectations, the forecast errors entering equation 19, included in the term ϵ_t, are uncorrelated with instruments lagging three periods behind. In the model of Galí and Gertler, this lag is shorter. To control for the instruments when comparing estimates coming from the two models, I have used the same instruments throughout specifications.

An explanation for these significantly higher estimates for ω takes into account the effects of linearizing the model prior to the estimation of the Phillips curve. For a contract length of three quarters, when backward firms use the rule of thumb $P_t^b = \bar{P}_{t-1}$, substituting $\omega = 0$ and $\beta = 1$ into equation (19) yields:

$$\pi_t = \frac{1}{3}\hat{s}_{t+2} + \frac{2}{3}\hat{s}_{t+1} + \hat{s}_t + \frac{2}{3}\hat{s}_{t-1} + \frac{1}{3}\hat{s}_{t-2} + \frac{1}{3}\pi_{t+2} + \pi_{t+1} - \frac{1}{3}\pi_{t-1} + \epsilon_t \quad (27)$$

Remarkably, the sign on the coefficient for the lag of inflation is negative. The intuition for this is the following. At time t, the firms allowed to renew their price determine the inflation rate. If inflation was high in past quarters, then current prices (and thus inflation) should be kept low to maximize profits by capturing a wider share of the competitors's market. The positive sign on the cofficients for future inflation reflect the fact that, if competitors are expected to raise prices in the future, then it will be advantageous to increase prices now, thus pushing inflation up. One, however, would expect the elasticity of substitution to enter this equation and influence the sign of the parameters. It is exactly in the linearization that the elasticity of substitution is dropped.

Given that the linearized baseline model is stacked against inflation persistence, it seems plausible that a higher fraction of backward-looking firms would be needed to generate any persistence at all. This line of reasoning is confirmed by Sbordone (2001), who uses an estimation method that follows Campbell and Shiller (1988). This method allows a comparison of the pricing mechanisms without linearizing. Sbordone's conclusion is that the fit to the US data of the two models is good in both cases. While Sbordone's approach has some obvious advantages, fitting the linearized models to the data still has a purpose. It is useful in calibrating dynamic general equilibrium models whose routine solution methods involve linearizing the necessary conditions for an equilibrium.

Table 6 reports the results of some sensitivity analysis. The table reports various estimates of the fraction of backward-looking firms for different sample sizes. Restricting the sample size even to include only the 1980s and the 1990s does not affect the findings reported.

Table 2: **Estimation results using price contracts lasting 2, 3 and 4 periods**

	Spec. 1	Spec. 2	Spec. 3	Spec. 4
Normalize by γ_0	yes	no	yes	no
Rule of thumb	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$
ω	.437	.411	-	.728
	(.000)	(.007)	-	(.000)
θ_2	.337	.361	-	.976
	(.801)	(.780)	-	(.001)
θ_3	.662	.639	-	.0235
	(.771)	(.798)	-	(.958)
θ_4	.000177	.000639	-	.0000
	(1.00)	(1.00)	-	(1.00)
R^2	.175	-	-	-
Test of Overidentifying Restrictions	(.598)	(.661)	-	(.436)

Probability values in parentheses. The regression equation is the following:

$$\begin{aligned}\gamma_0\pi_t &= [\gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4} \\ &\quad +\lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \epsilon_t]\end{aligned}$$

The parameters in the regression equation above are functions of the fraction of backward-looking firms ω, and the proportion of firms fixing their prices for 2, 3 and 4 quarters, respectively θ_2, θ_3 and θ_4. The exact functional forms are spelled out in the appendix. θ_2, θ_3 and θ_4 are restricted to sum to 1 and to lie within 0 and 1.

Table 3: **Estimation results using price contracts lasting two quarters**

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Galí-Gertler
Normalize by γ_0	yes	no	yes	no	no
Rule of thumb	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$
Contract length	2 quarters	2 quarters	2 quarters	2 quarters	random
ω	.471 (.000)	.457 (.000)	.749 (.000)	.738 (.000)	.285 (.019)
θ	- -	- -	- -	- -	.842 (.000)
$\frac{\gamma_{f(1)}}{\gamma_0}$	.218 (.000)	.229 (.000)	.0774 (.000)	.0875 (.000)	.755 (.000)
$\frac{\gamma_{b(1)}}{\gamma_0}$	-.391 (.000)	-.385 (.000)	- -	- -	.245 (.002)
$\frac{\gamma_{b(2)}}{\gamma_0}$	- -	- -	.461 (.000)	.459 (.000)	- -
$\frac{\lambda_{f(1)}}{\gamma_0}$	.218 (.000)	.229 (.000)	.0774 (.000)	.0817 (.000)	- -
$\frac{\lambda_0}{\gamma_0}$	.437 (.000)	.458 (.000)	.155 (.000)	.163 (.000)	.00957 (.075)
$\frac{\lambda_{b(1)}}{\gamma_0}$	.218 (.000)	.229 (.000)	.0774 (.000)	.0817 (.000)	- -
R^2	.202	-	.509	-	-
Test of Over Identifying Restrictions	(.739)	(.885)	(.524)	(.624)	(.870)

Probability values in parentheses. The regression equation is the following:

$$\begin{aligned}\gamma_0\pi_t &= [\gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4} \\ &\quad + \lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \epsilon_t]\end{aligned}$$

For specification 1 to 4, the parameters in the above equation are functions of ω and β (fixed at 1). For Galí's and Gertler's specifications, they are functions of ω, β (fixed at 1) and θ. The exact functional forms are reported in the appendix.

Table 4: **Estimation results using price contracts lasting three quarters**

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Galí-Gertler
Normalize by γ_0	yes	no	yes	no	no
Rule of thumb	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$
Contract length	3 quarters	3 quarters	3 quarters	3 quarters	random
ω	.408 (.000)	.385 (.000)	.596 (.000)	.577 (.000)	.285 (.019)
θ	- -	- -	- -	- -	.842 (.000)
$\frac{\gamma_{f(2)}}{\gamma_0}$	.109 (.000)	.116 (.000)	0.0614 (.000)	.0654 (.000)	- -
$\frac{\gamma_{f(1)}}{\gamma_0}$	.326 (.000)	.347 (.000)	.184 (.000)	.196 (.000)	.755 (.000)
$\frac{\gamma_{b(1)}}{\gamma_0}$	-.558 (.000)	-.551 (.002)	-.333 (.000)	-.333 (.000)	.245 (.002)
$\frac{\gamma_{b(2)}}{\gamma_0}$	-.225 (.204)	-.0218 (.064)	- -	- -	- -
$\frac{\gamma_{b(3)}}{\gamma_0}$	- -	- -	.272 (.000)	.268 (.000)	- -
$\frac{\lambda_{f(2)}}{\gamma_0}$	.109 (.000)	.116 (.000)	.0614 (.000)	.0654 (.000)	- -
$\frac{\lambda_{f(1)}}{\gamma_0}$	.218 (.000)	.231 (.000)	.123 (.000)	.131 (.000)	- -
$\frac{\lambda_0}{\gamma_0}$	.326 (.000)	.347 (.000)	.184 (.000)	.196 (.000)	.00957 (.075)
$\frac{\lambda_{b(1)}}{\gamma_0}$	.218 (.000)	.231 (.000)	.123 (.000)	.131 (.000)	- -
$\frac{\lambda_{b(2)}}{\gamma_0}$	.109 (.000)	.116 (.000)	.0614 (.000)	.0654 (.000)	- -
R^2	.173	-	.302	-	-
Test of OIR	(.883)	(.896)	(.635)	(.832)	(.870)

Probability values in parentheses. The regression equation is the following:

$$\begin{aligned}\gamma_0\pi_t \quad = \quad & [\gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4} \\ & + \lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \epsilon_t]\end{aligned}$$

For specification 1 to 4, the parameters in the above equation are functions of ω and β (fixed at 1). For Galí's and Gertler's specification, they are functions of ω, β (fixed at 1) and θ. The exact functional forms are reported in the appendix.

Table 5: **Estimation results using price contracts lasting four quarters**

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Galí-Gertler
Normalize by γ_0	yes	no	yes	no	no
Rule of thumb	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{P_{t-1}}{P_{t-2}}$
Contract length	4 quarters	4 quarters	4 quarters	4 quarters	random
ω	.346 (.000)	.327 (.000)	.475 (.000)	.453 (.000)	.285 (.019)
θ	- -	- -	- -	- -	.842 (.000)
$\frac{\gamma_{f(3)}}{\gamma_0}$	.0691 (.000)	.0727 (.000)	.0488 (.000)	.0519 (.000)	- -
$\frac{\gamma_{f(2)}}{\gamma_0}$	.207 (.000)	.218 (.000)	0.146 (.000)	.156 (.000)	- -
$\frac{\gamma_{f(1)}}{\gamma_0}$	.415 (.000)	.436 (.000)	.293 (.000)	.311 (.000)	.755 (.000)
$\frac{\gamma_{b(1)}}{\gamma_0}$	-.646 (.000)	-.641 (.000)	-.500 (.000)	-.500 (.000)	.245 (.002)
$\frac{\gamma_{b(2)}}{\gamma_0}$	-.362 (.000)	-.355 (.000)	-.226 (.000)	-.224 (.000)	- -
$\frac{\gamma_{b(3)}}{\gamma_0}$	-.146 (.177)	-.141 (.077)	- -	- -	- -
$\frac{\gamma_{b(4)}}{\gamma_0}$	- -	- -	.177 (.000)	.172 (.000)	- -
$\frac{\lambda_{f(3)}}{\gamma_0}$	.0691 (.000)	.0727 (.000)	.0488 (.000)	.0519 (.000)	- -
$\frac{\lambda_{f(2)}}{\gamma_0}$	.138 (.000)	.145 (.000)	.0976 (.000)	.104 (.000)	- -
$\frac{\lambda_{f(1)}}{\gamma_0}$	.207 (.000)	.218 (.000)	.146 (.000)	.156 (.000)	- -
$\frac{\lambda_0}{\gamma_0}$	.276 (.000)	.291 (.000)	.195 (.000)	.114 (.000)	.00957 (.075)
$\frac{\lambda_{b(1)}}{\gamma_0}$	.207 (.000)	.218 (.000)	.146 (.000)	.156 (.000)	- -
$\frac{\lambda_{b(2)}}{\gamma_0}$	.138 (.000)	.145 (.000)	.0976 (.000)	.104 (.000)	- -
$\frac{\lambda_{b(3)}}{\gamma_0}$	.0691 (.000)	.0488 (.000)	.0287 (.000)	.0519 (.000)	- -
R^2	.162	-	.227	-	-
Test of OIR	(.885)	(.896)	(.708)	(.860)	(.941)

Probability values in parentheses. The regression equation is the following:

$$\begin{aligned}\gamma_0\pi_t \quad = \quad & [\gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4} \\ & +\lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \epsilon_t]\end{aligned}$$

For specification 1 to 4, the parameters in the above equation are functions of ω and β (fixed at 1). For Galí's and Gertler's specifications, they are functions of ω, β (fixed at 1) and θ. The exact functional forms are reported in the appendix.

Table 6: **Estimates of Fraction of Backward-Looking Firms for Various Sample Sizes**

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Galí-Gertler
Normalize by γ_0	yes	no	yes	no	no
Rule of thumb	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}$	$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$	$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$
Sample: 1960q1-2001q2					
n=2	.471	.457	.748	.737	
	(.000)	(.000)	(.000)	(.000)	
n=3	.408	.385	.596	.577	.285
	(.000)	(.000)	(.000)	(.000)	(.019)
n=4	.346	.327	.475	.453	
	(.000)	(.000)	(.000)	(.000)	
Sample: 1970q1-2001q2					
n=2	.468	.453	.719	.719	
	(.000)	(.000)	(.000)	(.000)	
n=3	.399	.383	.574	.558	.324
	(.000)	(.000)	(.000)	(.000)	(.007)
n=4	.345	.332	.467	.451	
	(.000)	(.000)	(.000)	(.000)	
Sample: 1980q1-2001q2					
n=2	.467	.466	.887	.729	
	(.000)	(.000)	(.000)	(.000)	
n=3	.403	.395	.595	.588	.0439
	(.000)	(.000)	(.000)	(.000)	(.719)
n=4	.346	.340	.479	.474	
	(.000)	(.000)	(.000)	(.000)	

Probability values in parentheses. n indicates the number of periods for which prices are fixed. The regression equation is the following:

$$\gamma_0 \pi_t = [\gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4}$$
$$+\lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \epsilon_t]$$

For specification 1 to 4, the parameters in the above equation are functions of ω and β (fixed at 1). For Galí's and Gertler's specification, they are functions of ω, β (fixed at 1) and θ. The exact functional forms are reported in the appendix.

5 Conclusion

A standard problem with IV estimation is that, in small samples, normalizing the moment condition by the coeffiecient of one of the variables can affect the estimation results. In the case of the new Phillips Curve, for estimates obtained using Calvo-style contracts, using a Monte Carlo experiment I have shown that one particular normalization produces a clearly superior estimator. This is the same estimator adopted by Galí and Gertler (1999). When repeating the exercise with Taylor-style contracts, the estimates are robust to the choice of normalization for the moment condition.

One of the reasons for embarking on this line of work was the intuition that Taylor-style contracts, by producing a Phillips curve incorporating lags of inflation, would have made explaining the inflation persistence in the data easier. The results I obtained falsify this original hypothesis. They show that when the baseline model is augmented to include backward-looking price setting, the fraction of these non-optimizing firms is statistically significant and numerically important. It is estimated to be in the order of 50%. The baseline Phillips curve derived from optimal Taylor-style contracts fails to provide a good description of inflation dynamics, at least in its linear form. I have shown that the linearization, by eliminating the elasticity of substitution from the regression equation, stacks the baseline model against the generation of inflation persistence. Given this intuition, it seems plausible that a greater fraction of backward-looking firms is needed to fit the U.S. data using Taylor-style contracts rather than Calvo-style contracts.

The results presented here add urgency to the refinements of non-linear solution algorithms for large-scale models. These results also stand as a strong warning against the application of the estimates of Galí and Gertler (1999) in the calibration of linear dynamic general-equilibrium models using contracts à la Taylor.

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A Parameter restrictions of the Phillips curve equation

In this appendix, I report in detail the parameter restrictions imposed on the Phillips curve equation by the Taylor staggered contracting specification. Tables 7 and 8 summarize these restrictions, and help in the comparison with the Calvo contracting specification.

The notation follows that of the main body of the paper. π and s denote, respectively, inflation and the unit cost of production. ϵ is an independently and identicaly distributed rational-expectation forecast error. ω is the fraction of backward-looking firms. β is the discount factor. θ_1, θ_2, θ_3, and θ_4 are, respectively, the proportion of firms fixing prices for 1, 2, 3, and 4 quarters.

Log-linearizing around a zero-inflation steady state the first order conditions for the profit maximization problem, one can obtain the following moment condition:

$$\begin{aligned}\gamma_0\pi_t &= \gamma_{f3}\pi_{t+3}+\gamma_{f2}\pi_{t+2}+\gamma_{f1}\pi_{t+1}+\gamma_{b1}\pi_{t-1}+\gamma_{b2}\pi_{t-2}+\gamma_{b3}\pi_{t-3}+\gamma_{b4}\pi_{t-4}\\ &\quad +\lambda_{f3}\hat{s}_{t+3}+\lambda_{f2}\hat{s}_{t+2}+\lambda_{f1}\hat{s}_{t+1}+\lambda_0\hat{s}_t+\lambda_{b1}\hat{s}_{t-1}+\lambda_{b2}\hat{s}_{t-2}+\lambda_{b3}\hat{s}_{t-3}+\epsilon_t\end{aligned}$$

where the parameters are defined as follows:

$$\gamma_{f3}=\frac{\theta_4\ (1/4-1/4\,\omega)\,\beta^3}{1+\beta+\beta^2+\beta^3}$$

$$\gamma_{f2}=\frac{\theta_3\ (1/3-1/3\,\omega)\,\beta^2}{1+\beta+\beta^2}+\theta_4\left(\frac{(1/4-1/4\,\omega)\left(\beta^2+\beta^3\right)}{1+\beta+\beta^2+\beta^3}+\frac{(1/4-1/4\,\omega)\,\beta^3}{1+\beta+\beta^2+\beta^3}\right)$$

$$\begin{aligned}\gamma_{f1} &= \theta_2\frac{(1/2-1/2\,\omega)\,\beta}{1+\beta}+\theta_3\left(\frac{(1/3-1/3\,\omega)\left(\beta+\beta^2\right)}{1+\beta+\beta^2}+\frac{(1/3-1/3\,\omega)\,\beta^2}{1+\beta+\beta^2}\right)\\ &\quad +\theta_4\left(\frac{(1/4-1/4\,\omega)\left(\beta+\beta^2+\beta^3\right)}{1+\beta+\beta^2+\beta^3}+\frac{(1/4-1/4\,\omega)\left(\beta^2+\beta^3\right)}{1+\beta+\beta^2+\beta^3}+\frac{(1/4-1/4\,\omega)\,\beta^3}{1+\beta+\beta^2+\beta^3}\right)\end{aligned}$$

$$\begin{aligned}\gamma_0 &= 1-\theta_1\ (1-\omega)-\theta_2\left(1/2-1/2\,\omega+\frac{(1/2-1/2\,\omega)\,\beta}{1+\beta}\right)\\ &\quad -\theta_3\left(1/3-1/3\,\omega+\frac{(1/3-1/3\,\omega)\left(\beta+\beta^2\right)}{1+\beta+\beta^2}+\frac{(1/3-1/3\,\omega)\,\beta^2}{1+\beta+\beta^2}\right)\\ &\quad -\theta_4\left(1/4-1/4\,\omega+\frac{(1/4-1/4\,\omega)\left(\beta+\beta^2+\beta^3\right)}{1+\beta+\beta^2+\beta^3}+\frac{(1/4-1/4\,\omega)\left(\beta^2+\beta^3\right)}{1+\beta+\beta^2+\beta^3}+\frac{(1/4-1/4\,\omega)\,\beta^3}{1+\beta+\beta^2+\beta^3}\right)\end{aligned}$$

$$\gamma_{b1} = (\theta_1 - 1/3\,\theta_3 - 1/2\,\theta_4)\,\omega + \theta_2\ (1/2 - 1/2\,\omega) + \theta_3\ \left(1/3 - 1/3\,\omega + \frac{(1/3 - 1/3\,\omega)\left(\beta + \beta^2\right)}{1 + \beta + \beta^2}\right)$$
$$+\theta_4\ \left(1/4 - 1/4\,\omega + \frac{(1/4 - 1/4\,\omega)\left(\beta + \beta^2 + \beta^3\right)}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\left(\beta^2 + \beta^3\right)}{1 + \beta + \beta^2 + \beta^3}\right)$$
$$-1/2\,\theta_2\ (1 - \omega) - 2/3\,\theta_3\ (1 - \omega) - 3/4\,\theta_4\ (1 - \omega)$$

$$\gamma_{b2} = 1/4\,\frac{2\,\theta_2\,\omega + 2\,\theta_2\,\beta\,\omega + 2\,\theta_2\,\beta^2\omega + 2\,\theta_2\,\beta^3\omega - \theta_4\,\omega\,\beta - \theta_4\,\beta^2\omega - \theta_4\,\beta^3\omega - \theta_4}{(\beta + 1)\,(1 + \beta^2)}$$

$$\gamma_{b3} = 1/3\,\theta_3\,\omega$$

$$\gamma_{b4} = 1/4\,\omega\,\theta_4$$

$$\lambda_{f3} = \frac{\theta_4\ (1/4 - 1/4\,\omega)\,\beta^3}{1 + \beta + \beta^2 + \beta^3}$$

$$\lambda_{f2} = \frac{\theta_3\ (1/3 - 1/3\,\omega)\,\beta^2}{1 + \beta + \beta^2} + \theta_4\ \left(\frac{(1/4 - 1/4\,\omega)\,\beta^2}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^3}{1 + \beta + \beta^2 + \beta^3}\right)$$

$$\lambda_{f1} = \frac{\theta_2\ (1/2 - 1/2\,\omega)\,\beta}{1 + \beta} + \theta_3\ \left(\frac{(1/3 - 1/3\,\omega)\,\beta}{1 + \beta + \beta^2} + \frac{(1/3 - 1/3\,\omega)\,\beta^2}{1 + \beta + \beta^2}\right)$$
$$+\theta_4\ \left(\frac{(1/4 - 1/4\,\omega)\,\beta}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^2}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^3}{1 + \beta + \beta^2 + \beta^3}\right)$$

$$\lambda_0 = \theta_1\ (1 - \omega) + \theta_2\ \left(\frac{1/2 - 1/2\,\omega}{1 + \beta} + \frac{(1/2 - 1/2\,\omega)\,\beta}{1 + \beta}\right)$$
$$+\theta_3\ \left(\frac{1/3 - 1/3\,\omega}{1 + \beta + \beta^2} + \frac{(1/3 - 1/3\,\omega)\,\beta}{1 + \beta + \beta^2} + \frac{(1/3 - 1/3\,\omega)\,\beta^2}{1 + \beta + \beta^2}\right)$$
$$+\theta_4\ \left(\frac{1/4 - 1/4\,\omega}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^2}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^3}{1 + \beta + \beta^2 + \beta^3}\right)$$

$$\lambda_{b1} = \frac{\theta_2\ (1/2 - 1/2\,\omega)}{1 + \beta} + \theta_3\ \left(\frac{1/3 - 1/3\,\omega}{1 + \beta + \beta^2} + \frac{(1/3 - 1/3\,\omega)\,\beta}{1 + \beta + \beta^2}\right)$$
$$+\theta_4\ \left(\frac{1/4 - 1/4\,\omega}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta^2}{1 + \beta + \beta^2 + \beta^3}\right)$$

$$\lambda_{b2} = \frac{\theta_3\ (1/3 - 1/3\,\omega)}{1 + \beta + \beta^2} + \theta_4\ \left(\frac{1/4 - 1/4\,\omega}{1 + \beta + \beta^2 + \beta^3} + \frac{(1/4 - 1/4\,\omega)\,\beta}{1 + \beta + \beta^2 + \beta^3}\right)$$

$$\lambda_{b3} = \frac{\theta_4\ (1/4 - 1/4\,\omega)}{1 + \beta + \beta^2 + \beta^3}$$

Restrictions on the parameters θ_1 - θ_4

$$\theta_1 = 0$$

$$\theta_2 = \frac{1}{1 + e^{z_3} + e^{z_4}}$$

$$\theta_3 = \frac{e^{z_3}}{1 + e^{z_3} + e^{z_4}}$$

$$\theta_4 = \frac{e^{z_4}}{1 + e^{z_3} + e^{z_4}}$$

Table 7: **Baseline model with homogenous contracts of fixed length**

Parameter restrictions on $\pi_t = \lambda_{f3}\hat{s}_{t+3} + \lambda_{f2}\hat{s}_{t+2} + \lambda_{f1}\hat{s}_{t+1} + \lambda_0\hat{s}_t + \lambda_{b1}\hat{s}_{t-1} + \lambda_{b2}\hat{s}_{t-2} + \lambda_{b3}\hat{s}_{t-3} + \gamma_{f3}\pi_{t+3} + \gamma_{f2}\pi_{t+2} + \gamma_{f1}\pi_{t+1} + \gamma_{b1}\pi_{t-1} + \gamma_{b2}\pi_{t-2} + \gamma_{b3}\pi_{t-3} + \gamma_{b4}\pi_{t-4} + \epsilon_t$ for contracts lasting n quarters. Rule of thumb:$P_t^b = \bar{P}_{t-1}\frac{\bar{P}_{t-1}}{\bar{P}_{t-2}}$

n	λ_{f3}	λ_{f2}	λ_{f1}	λ_0	λ_{b1}	λ_{b2}	λ_{b3}	
2			$\frac{1-\omega}{1+3\omega}$	$2\frac{1-\omega}{1+3\omega}$	$\frac{1-\omega}{1+3\omega}$			
3		$1/3\,\frac{1-\omega}{1+2\omega}$	$2/3\,\frac{1-\omega}{1+2\omega}$	$\frac{1-\omega}{1+2\omega}$	$2/3\,\frac{1-\omega}{1+2\omega}$	$1/3\,\frac{1-\omega}{1+2\omega}$		
4	$1/2\,\frac{1-\omega}{3+5\omega}$	$\frac{1-\omega}{3+5\omega}$	$3/2\,\frac{1-\omega}{3+5\omega}$	$2\frac{1-\omega}{3+5\omega}$	$3/2\,\frac{1-\omega}{3+5\omega}$	$\frac{1-\omega}{3+5\omega}$	$1/2\,\frac{1-\omega}{3+5\omega}$	
n	γ_{f3}	γ_{f2}	γ_{f1}		γ_{b1}	γ_{b2}	γ_{b3}	γ_{b4}
2			$\frac{1-\omega}{1+3\omega}$		0	$\frac{2\omega}{1+3\omega}$		
3		$1/3\,\frac{1-\omega}{1+2\omega}$	$\frac{1-\omega}{1+2\omega}$		$-\frac{1}{3}$	0	$\frac{\omega}{1+2\omega}$	
4	$1/2\,\frac{1-\omega}{3+5\omega}$	$3/2\,\frac{1-\omega}{3+5\omega}$	$3\frac{1-\omega}{3+5\omega}$		$-\frac{1}{2}$	$\frac{-3\omega-1}{10\omega+6}$	0	$2\,\frac{\omega}{3+5\omega}$

Table 8: **Model with contracts of random length as in Galí and Gertler**

Parameter restriction on $\pi_t = \frac{1}{\gamma_0}\left[\lambda \hat{s}_t + \gamma_f \pi_{t+1} + \gamma_b \pi_{t-1} + \epsilon_t\right]$			
γ_0	λ	γ_f	γ_b
$\theta + \omega(1-\theta(1-\beta))$	$(1-\omega)(1-\theta)(1-\beta\theta)$	$\beta\theta$	ω