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## Sticky Prices, No Menu Costs

David Bowman\*

**Abstract:** A model that contains no costs to changing prices but in which prices do not respond to nominal shocks is presented. In models that do not feature superneutrality of money flexible price equilibria will allow certain types of monetary shocks to affect the real economy. Sticky price behavior may in fact be better at protecting the real economy from the effects of monetary shocks in such environments. This point is demonstrated in a standard monetary model with liquidity effects. An equilibrium in which sticky prices are supported without menu costs is then constructed. In equilibrium firms choose to keep prices fixed in response to nominal shocks because doing so provides a service to their customers, increasing profits by expanding the customer base.

**Keywords:** menu costs, price adjustment, optimality

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In one sense financial economics and macroeconomics face opposing problems. Where financial economists are often pressed to explain specific movements in asset prices when no immediately obvious change in “fundamentals” can be detected, macroeconomists often find themselves trying to explain why nominal goods or labor prices should show no or little apparent response to what seem like obvious monetary and nonmonetary shocks.

Perhaps as a result, macroeconomic models that feature either price or wage stickiness have recently enjoyed a resurgence.<sup>1</sup> The new literature differs from older literature by incorporating sticky prices into otherwise fully optimized infinite-horizon dynamic general equilibrium models, but like the older literature it models price stickiness as arising either from some form of exogenous menu cost or more often simply as an exogenous assumption concerning the frequency at which firms can change prices. Exogenizing the timing of price changes or imposing costs to changing prices are, as modelling techniques, convenient ways of producing a slow response of prices to shocks and have yielded important insights into how an economy with sticky prices responds to various changes in environment, but questions such as welfare analysis are undoubtedly sensitive to the precise reasons that prices move slowly. Whether in the end price or wage stickiness will be agreed upon as an essential feature of macroeconomic models, it seems clear that the lack of a deeper understanding as to why sticky prices might arise is unsatisfying.

The purpose of this paper is to argue that rather than focusing on the costs that may exist to changing prices, economists might begin to consider what possible benefits might arise from price stickiness. Perhaps sticky prices are an optimal response regardless of menu costs. There are several reasons to believe that this may be an intellectually productive approach.

First, many monetary models exhibit economic distortions that cause the second welfare theorem to fail to hold. A standard exercise is to take the flexible price perfectly competitive equilibrium as given and ask what the optimal monetary policy would be — for instance, proving that some form of the Friedman rule would be optimal. On the other hand, in reality governments have not followed the Friedman rule, and it could make as much sense to turn the entire question around by taking monetary policy as given and asking what type of price response would be optimal. As will be seen later, in an economy with monetary distortions there need be no automatic presumption that a flexible price equilibrium will prove better than a sticky price equilibrium at

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<sup>1</sup>See for instance Yun (1996); Rotemberg and Woodford (1997); Dotsey, King and Wolman (1999); Chari, Kehoe and McGratten (2000); or Erceg, Henderson, and Levin(2000). Gali (2000) provides a useful overview.

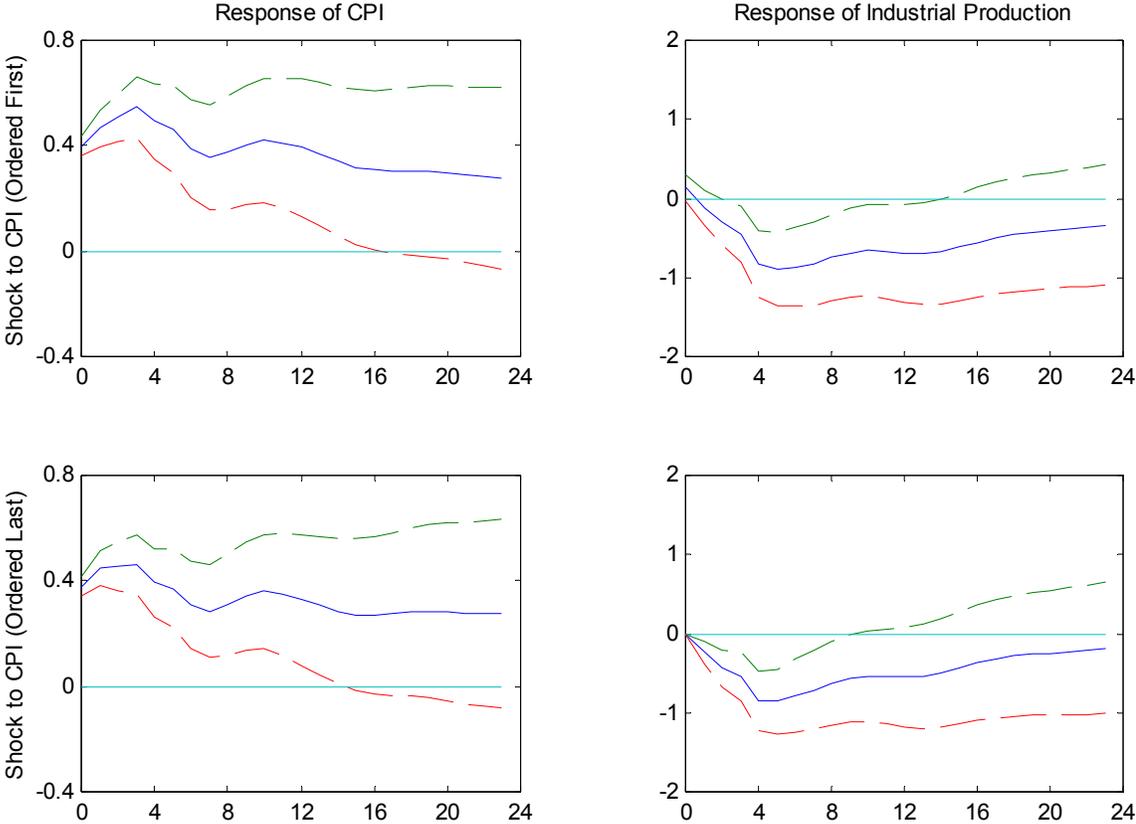
protecting economic agents from monetary shocks.

Second, while it is apparent that many industries do face costs associated with changing prices, what is less apparent is that these costs are large enough to explain the amount of price stickiness we seem to observe or that simple models of menu costs are sufficient to explain price behavior. There are reasons to doubt it. Ball and Romer (1990) argue that menu costs alone are not enough, and Fuhrer and Moore (1995) document that inflation is much more persistent than standard menu cost models tend to predict. Further, even many retail consumer goods that might typically be thought of as sticky in response to monetary shocks can at the same time experience frequent mark-downs or sales (Warner and Barsky(1995)).

Third, menu cost models tend to predict that price responses to *any* shock are sluggish. This is empirically untrue. Over the last decade there have been myriad studies attempting to identify the effects of monetary shocks on output and prices. Leeper, Sims and Zha (1996) provide a comprehensive discussion and large set of original results. While there is obviously disagreement, the standard finding is that a monetary shock has a statistically insignificant impact on output, and only a gradual impact of the price level with little or no immediate impact. In other words, prices are sticky in response to the monetary shocks these models identify. The nonmonetary structural shocks are left unidentified in these exercises, however it is *always* the case that there is at least one nonstructural shock (a combination of the unidentified structural nonmonetary shocks) that has an immediate and statistically significant impact effect on the price level.

Figure 1 demonstrates this point. A five variable vector autoregression was estimated including the log level of services component of the consumer price index, industrial production, M1, a commodity price index, and the federal funds rate with monthly data over the period 1980:1-2000:1. The variables are standard in the literature; the choice of the services subcomponent of the overall CPI is unusual but does not affect the results and was chosen to eliminate any uncertainty that shocks to it reflected movements in commodity prices. The figure graphs the estimated impulse responses to a shock to the error term of the CPI equation when the CPI variable is ordered both first and last within the VAR and shocks are identified using the Cholesky decomposition. As can be seen, the ordering is immaterial: in either case there is a significant immediate impact on the price level very near the peak effect and an eventual negative effect on production. This has the broad outlines of what standard theory predicts in response to a supply shock. Prices are not sticky in response to this shock, and it is economically important. Even when ordered last, the CPI shock accounts for 92% of price variance on impact, 48% of price variance

Figure 1: Empirical Impulse Responses



Impulse Response (in %) to a two standard deviation CPI shock with 95% confidence intervals based on five variable VAR including M1, the federal funds rate, and commodity prices. Monthly data, 1980:1-2000:1.

after one year, and 34% after two years (when ordered first the CPI shock accounts for 69% of price variance after one year and 56% after two years).

The rest of the paper is broken into three sections. The first demonstrates that in even one of the most standard prototype monetary models a sticky nominal price response to monetary shocks can Pareto dominate a flexible price response. The second section introduces search into the flexible price model as a preliminary step to constructing sticky price equilibria. The third section demonstrates that this economic environment is one in which firms are able to internalize enough of the benefits that sticky prices offer to make a sticky price response part of equilibrium behavior.

## 1 Optimal Price Response in a Liquidity Model

The purpose of this section is to establish the point that there need be no automatic presumption that flexible prices are optimal or preferable to sticky prices in standard monetary models. This point can be usefully made by employing a variant of the Lucas (1990) and Fuerst (1992) “liquidity” models, currently one of the more widely employed class of models for understanding the economic effects of monetary policy.<sup>2</sup>

### 1.1 The Flexible-Price Equilibrium

There is a continuum of representative households with preferences over consumption and leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(\bar{l} - l_t)] \quad (1)$$

where the discount factor  $\beta$  is positive but less than unity. Each household is comprised of four members: an asset market participant who at the beginning of each period takes a fraction  $n_t$  of initial cash holdings to purchase risk free debt in the bond market; a shopper who takes the household’s remaining cash and uses it to purchase the consumption good,  $c_t$ ; a worker who goes into the labor market and sells his labor,  $l_t$ ; and an entrepreneur who each period hires

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<sup>2</sup>See also Christiano and Eichenbaum (1992) and Christiano, Eichenbaum and Evans (1997). The model presented is a simple variant of the model without capital presented by Fuerst. It should be noted that there are versions of this model with capital in which prices may not contemporaneously react to monetary shocks, even in the flexible price equilibrium. However this result assumes that investment is a cash good and that the relative price of investment and consumption goods is fixed (ie, that investment and consumption goods are identical.)

labor  $h_t$  and in order to produce the nonstorable consumption good according to the production function

$$y_t = \theta_t f(h_t) \quad (2)$$

where  $f$  is increasing and concave and  $\theta$  is a strictly positive random variable. The markets for the consumption, labor and assets are perfectly competitive.

In addition to the household there is a government whose only activity is to conduct open market operations in the bond market, selling amounts  $x_t^j$  of nominal debt of maturities  $j = 1 \dots J$  at price  $q_t^j$ , each bond promising a sure payment of one unit of currency at the end of period in which it matures.<sup>3</sup>

The timing of activity is important. We assume that the consumption good can only be purchased with money brought into the period and that  $n_t$  is chosen before either  $\theta_t$  or  $x_t = (x_t^1, \dots, x_t^J)$  are made known. Defining  $m_t$  as the household's beginning of period cash holdings scaled by the total amount of cash in the economy,  $M_t$ , and defining  $p_t$  as the price of the consumption good, also scaled by  $M_t$ , the cash-in-advance constraint faced by the shopper is:

$$p_t c_t \leq m_t - n_t \quad (3)$$

We also assume that the firm must pay a fraction  $\lambda$  of its labor costs in advance of the receipt of its sales revenues, so that it must issue an amount of one-period debt in the bond market which it pays off at the end of the period from the proceeds of its sales.<sup>4</sup> It is assumed that the firm chooses its labor input after  $\theta_t$  and  $x_t$  are observed, so that the period's sales revenue is fully known at the time the firm must borrow. Defining  $w_t$  to be the wage rate and  $d_t$  to be the firm's level of debt, both scaled by  $M_t$ , the firm's financing constraint is:

$$\lambda w_t h_t \leq q_t^1 d_t \quad (4)$$

The firm is assumed to act so as to maximize its end of period profit,

$$\pi_t = (p_t y_t - (1 - \lambda) w_t h_t - d_t) \quad (5)$$

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<sup>3</sup>Following Lucas (1990) it would be possible to allow the government to buy and sell a more general set of securities, however the set of zero coupon bonds is flexible enough to accommodate the types of monetary policies I wish to consider.

<sup>4</sup>The assumption that the firm issues only one period debt is without loss of generality and simplifies notation.

Defining  $b_t = (b_t^1, \dots, b_t^J)$  to be the asset market participant's bond purchases (scaled by  $M_t$ ) and  $g_t$  to be the growth rate of money between periods  $t$  and  $t + 1$ , the household's cash accumulation is governed by the equation:

$$m_{t+1} = \frac{(m_t - n_t - p_t c_t) + (n_t + \sum_j (b_{t-j+1}^j - q_t^j b_t^j)) + w_t l_t + \pi_t}{1 + g_t} \quad (6)$$

It will be assumed that the random variables  $(\theta_t, x_t)$  follow a joint Markov process of finite order  $k$  with compact support. Defining  $s_{\theta t} \equiv (\theta_t, \dots, \theta_{t-k})$ ,  $s_{xt} = (x_t, \dots, x_{t-k})$ , and  $s_t = (s_{\theta t}, s_{xt})$ , we restrict analysis to cases in which the monetary growth rate is positive and monetary shocks do not help predict future technology, in the sense that

$$\text{prob}(\theta_t \leq \bar{\theta} \mid s_{t-1}) = \text{prob}(\theta_t \leq \bar{\theta} \mid s_{\theta t-1})$$

The conditional cumulative distribution function for  $s_t$  induced by the process for  $(\theta_t, x_t)$  will be denoted  $G(s_t \mid s_{t-1})$ .

A stationary rational expectations equilibrium is a set of decision functions  $n_t = n(s_{t-1})$ ,  $c_t = c(s_t)$ ,  $l_t = l(s_t)$ ,  $h_t = h(s_t)$ ,  $m_{t+1} = m(s_t)$ ,  $d_t = d(s_t)$ ,  $b_t = b(s_t)$  and positive price functions  $q(s_t) = (q_t^1, \dots, q_t^J)$ ,  $p_t(s_t)$ ,  $w(s_t)$  such that  $0 \leq n_t \leq 1$ ,  $q_t \leq 1$ , the decision functions maximize (1) subject to (2)–(6) when prices are taken as given, and the market clearing conditions  $c(s_t) = \theta_t f(h(s_t))$ ,  $l(s_t) = h(s_t)$ ,  $m(s_t) = 1$ ,  $b(s_t) = (x_t^1 + d_t, x_t^2, \dots, x_t^J)$  are met for all realizations of  $s_t$ .

In addition to (3) and (4), the relevant first order conditions associated with maximizing (1) are:

$$\begin{aligned} v'(\bar{l} - l(s_t)) &= \frac{w(s_t)\mu(s_t)}{1 + g(s_t)} \\ \theta_t f'(h(s_t)) &= \frac{w(s_t)}{p(s_t)} \left( 1 - \lambda + \lambda \frac{1}{q^1(s_t)} \right) \\ \mu(s_{t-1}) &= \beta \int \frac{\mu(s_t)}{q^1(s_t)(1 + g(s_t))} dG(s_t \mid s_{t-1}) \\ \frac{q^j(s_t)\mu(s_t)}{(1 + g(s_t))} &= \int \frac{q^1(s_t)\mu(s_{t+j-1})}{(1 + g(s_{t+j-1}))} dG(s_{t+j-1} \mid s_t) \\ q(s_t) \cdot b(s_t) &\leq n(s_{t-1}) \end{aligned}$$

where  $\mu(s_t) = \beta \int \frac{u'(c(s_{t+1}))}{p(s_{t+1})} dG(s_{t+1} \mid s_t)$ . As is standard, the set of equilibria to

be considered are restricted to those in which the consumer's cash-in-advance constraint is binding. This will be the case if  $\frac{u'(c(s_t))}{p(s_t)} > \frac{\mu(s_t)}{1+g(s_t)}$ , implying that (3) holds with equality.

The following result is standard.

**Claim 1** *Any monetary equilibrium is Pareto suboptimal.*

Proof: From the above conditions one can write

$$\mu(s_{t-1}) = \beta \int \left[ \frac{1-\lambda}{q^1(s_t)} + \frac{\lambda}{(q^1(s_t))^2} \right] \frac{v'(\bar{l} - l(s_t))}{\theta_t f'(l(s_t))} \frac{c(s_t)}{1 - n(s_{t-1})} dG(s_t | s_{t-1})$$

A Pareto Optimal allocation will satisfy the condition  $v'(\bar{l} - l_t) = \theta_t f'(l_t) u'(c_t)$ . Combining this and the above equation yields

$$\int \left[ 1 - \frac{1-\lambda}{q^1(s_t)} + \frac{\lambda}{(q^1(s_t))^2} \right] u'(c(s_t)) c(s_t) dG(s_t | s_{t-1}) = 0$$

which is impossible under the assumption of a positive monetary growth rate  $\square$

The claim does not imply that in this flexible price equilibrium stochastic monetary shocks will immediately affect the price of the consumption good, but this will generally be the case. To see why, note that if consumption is unaffected by money shocks,  $c(s_t) = c(s_{\theta t}, s_{x t-1})$ , then labor supply and the price level must be also unaffected,  $l(s_t) = l(s_{\theta t}, s_{x t-1})$  and  $p(s_t) = p(s_{\theta t}, s_{x t-1})$ . If prices are unaffected, then the firm's labor demand curve,

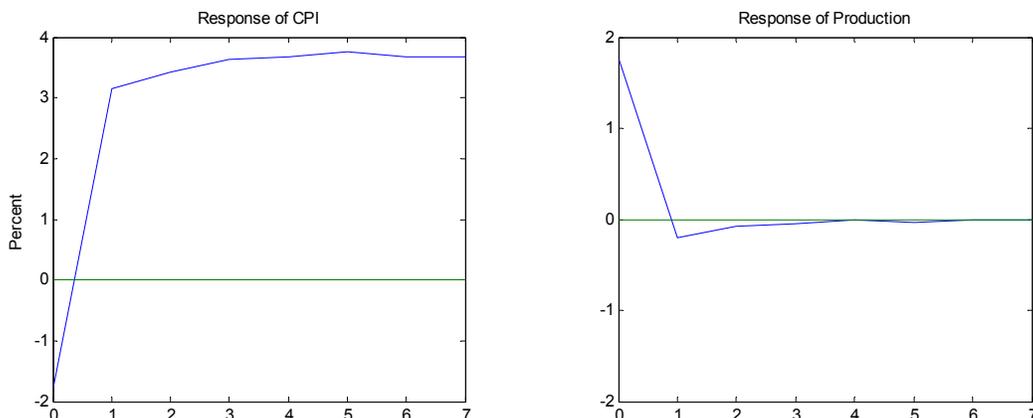
$$h(s_t) = f'^{-1} \left( \frac{w(s_t)}{\theta_t p(s_t)} \left( 1 - \lambda + \lambda \frac{1}{q^1(s_t)} \right) \right)$$

will either shift inward (for  $\lambda > 0$ ) or remain unchanged (for  $\lambda = 0$ ) in response to a standard positive monetary growth shock. The household's labor supply function,

$$l(s_t) = \bar{l} - v'^{-1} \left( \frac{w(s_t) \mu(s_t)}{1 + g(s_t)} \right)$$

will likewise shift inward in response to a positive monetary growth shock, *unless* the expected future marginal utility of money,  $\mu(s_t)$ , rises by enough to offset the effect of the rise in the growth rate of money. If labor demand

Figure 2: Price and Output Impulse Responses to a Money Growth Shock



shifts inward, then labor supply must shift by a precise amount *outward* to keep the equilibrium amount of labor unchanged. Unanticipated money shocks will only leave consumption and output unaffected if it gives rise to expectations of future money growth that raise  $\mu(s_t)$  by exactly the right amount. If, for instance, shocks are i.i.d. then this can never occur. Calibrated versions of the model confirm that this does not occur for plausible positively correlated monetary shocks either. To provide a simple but representative example, Figure 2 graphs the impulse responses to a one standard deviation money growth shock when we set  $u(c) = \ln(c)$ ,  $v(\bar{l} - l) = (\bar{l} - l)$ ,  $\lambda = 1$  (these choices follow Fuerst),  $\beta = .99$  (to calibrate the model at a quarterly frequency),  $f(h) = h^{0.35}$ , and consider an economy subject to the stochastic monetary process  $\Delta \ln(M_t) = \alpha_m + 0.481 \Delta \ln(M_{t-1}) + \varepsilon_{mt}$ , where  $\varepsilon_{mt}$  is a white-noise normally distributed random variable with standard deviation 0.0086 (this choice following Cooley and Hansen (1989)).<sup>5</sup> The model was solved by discretizing the state-space following Tauchen and Hussey (1991).

## 1.2 Consumer Prices at a Ramsey Solution

Throughout this paper the government's monetary rule is taken as given. In essence it is treated as another part of the exogenous structure of the economy,

<sup>5</sup>The mean  $\alpha_m$  is unimportant to the results and was arbitrarily chosen so that the mean inflation rate of the economy is 5%.

like technology. In this subsection we solve a Ramsey tax problem for the economy taking the evolution of the state vector  $s_t$  as exogenous. The purpose is simply to explore what consumer prices would behave like at an optimal allocation without altering the basic features of the model: perfectly competitive behavior and a positive shadow value of money in the consumption market.

Let  $p_t = (1 + \tau_t^c)p_t^*$  now denote the after-tax price of consumption where  $p_t^*$  is the before-tax price received by the firm and let  $w_t = (1 + \tau_t^l)w_t^*$  denote the after-tax wage rate, where  $w_t^*$  is the wage paid by the firm, and let  $\tau_t$  be a lump sum transfer to the household paid at the end of the period, all variables being scaled by beginning of period money holdings. We consider balanced taxes and transfers, so that  $\tau_t = \tau_t^c c_t + \tau_t^l l_t$ .

Only four equations are changed by these additions. (4)-(6) become

$$\lambda w_t^* h_t \leq q_t^1 d_t \quad (4')$$

$$\pi_t = (p_t^* y_t - (1 - \lambda)w_t^* h_t - d_t) \quad (5')$$

$$m_{t+1} = \frac{(m_t - n_t - p_t c_t) + (n_t + \sum_j (b_{t-j+1}^j - q_t^j b_t^j)) + w_t l_t + \pi_t + \tau_t}{1 + g_t} \quad (7)$$

and the first-order condition for the entrepreneur's labor demand decision becomes

$$\theta_t f'(h(s_t)) = \frac{w^*(s_t)}{p^*(s_t)} \left( 1 + \lambda \frac{1 - q^1(s_t)}{q^1(s_t)} \right)$$

The Ramsey solution chooses tax functions  $\tau^c(s_t)$ ,  $\tau^l(s_t)$  to support the optimal allocation, which is determined by the condition

$$v'(\bar{l} - l(s_t)) = \theta_t f'(l(s_t)) u'(c(s_t))$$

Define  $p^r(s_t)$  as the price faced by consumers at the Ramsey solution supporting this allocation. The following is immediate:

**Claim 2** *The price function  $p^r(s_t)$  does not depend on innovations in  $x_t$ ,  $p^r(s_t) = p^r(s_{\theta t}, s_{x t-1})$ .*

*Proof:* The unconstrained Pareto optimal allocation is defined by the condition

$$v'(\bar{l} - l_t) = \theta_t f'(l_t) u'(\theta_t f(l_t))$$

Let  $l^*(\theta)$  be the labor supply that solves this equation. The resulting level of consumption,  $c^*(\theta) = \theta f(l^*(\theta))$ , is obviously unaffected by the monetary state  $s_{xt}$ . Since  $n_t$  is chosen before  $s_{xt}$  is realized, the cash-in-advance implies that  $p_t^r = \frac{1-n(s_{t-1})}{c(\theta_t)}$  cannot respond to contemporaneous innovations in  $x_t$  if it is to support  $c^*(\theta_t)$  as an equilibrium allocation  $\square$

### 1.2.1 Discussion

We have established that in this economy consumer prices will respond to contemporaneous monetary shocks in equilibrium, but they will not respond under a Ramsey solution supporting the optimal allocation. Note that the claim does not rule imply that prices are unresponsive to contemporaneous technology shocks under a Ramsey solution – in fact prices will respond to technology shocks both in equilibrium and under the Ramsey solution supporting the optimal allocation. To obtain some sense what this can imply for model dynamics, we continue the calibration exercise discussed in conjunction with Figure 2 by solving for prices and output under the Ramsey solution supporting the optimal allocation for that specific choice of parameterization. A second version of the model is also considered in which the economy is subject to technology shocks following the process  $\ln(\theta_t) = 0.95 \ln(\theta_{t-1}) + \varepsilon_{\theta t}$ , where  $\varepsilon_{\theta t}$  is a white-noise normally distributed random variable with a standard deviation of 0.00721 (this again follows Cooley and Hansen (1989)). Figure 3 graphs the impulse responses to both a monetary shock and a technology shock in equilibrium and at the Ramsey solution – note that at the Ramsey solution output does not react to the monetary shock.

While the impulse responses to a technology shock are very similar, the Ramsey solution price response to a monetary shock is much more sluggish than the equilibrium response, and looks much closer to the empirical impulse responses reported in Leeper, Sims, and Zha (1996). The impulse responses to the technology shock are in turn quite comparable to the empirical impulse responses reported in our introduction.

Simulating the model indicates that inflation can be well approximated by an AR(1) process. Table 1 compares the AR(1) approximations to inflation in the calibration with money growth shocks. Clearly prices at the Ramsey solution display far greater persistence.

Figure 3: Model Impulse Responses

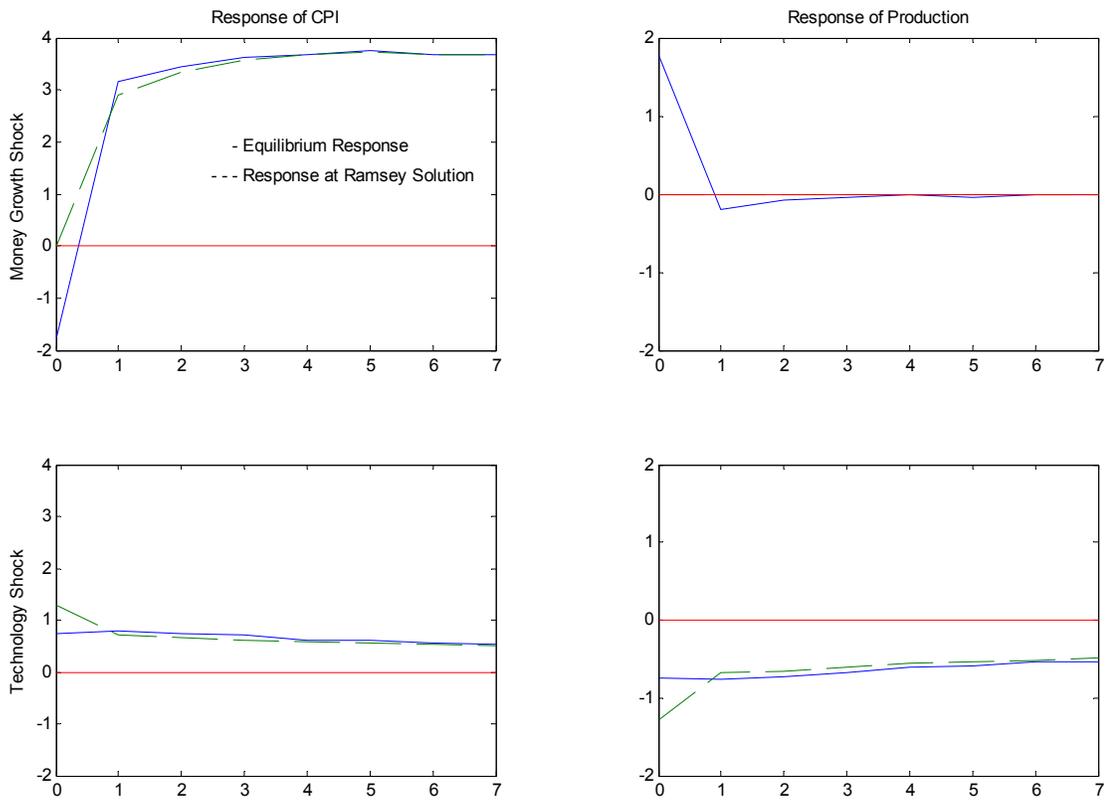


Table 1. Inflation Dynamics in Equilibrium and at Ramsey Solution

Equilibrium	
$\Delta \ln(p_t) = 0.052 - 0.043 \Delta \ln(p_{t-1}) + \varepsilon_{ft}$	$\sigma_{e_f} = 0.00034$
Ramsey Solution	
$\Delta \ln(p_t) = 0.026 + 0.475 \Delta \ln(p_{t-1}) + \varepsilon_{ot}$	$\sigma_{e_o} = 0.00012$

The implication is that sticky prices in response to monetary shocks are optimal in this economy, while flexible prices are suboptimal. This counterintuitive result occurs for a counterintuitive reason: a sticky price response to a monetary innovation is better than a flexible price response at shielding the real economy from the effects of monetary shocks. To understand why, one must distinguish between two types of monetary shocks. Classical results on monetary neutrality concern one time shocks to the current money supply that were previously unexpected – if the money supply unexpectedly doubles and all nominal prices double in response then the real allocation will be unaffected. However, monetary shocks need not be of this type. The  $R^2$  for the growth rate of M1 from the VAR used in the introduction is 56.07%, implying that the majority of movements in the money supply are predictable beforehand.<sup>6</sup> Innovations to expectations of *future* money growth require a quite different price response. An innovation to expectations of future monetary growth will raise expected inflation and in a flexible price equilibrium this will tend to cause current nominal prices to increase. However a rise in the current nominal price level will cause the *current* real value of money balances to decline, leading in most types of models to a decline in real activity. Sticky prices will tend to counteract this effect. Thus, whether flexible or sticky prices are better able to protect the economy from the effects of monetary shocks is to some extent an empirical question.

## 2 Incorporating Search into the Flexible Price Model

The Ramsey solution of the last section required a potentially complicated set of taxes and transfers. While it was a useful construct in demonstrating that a sticky price response to monetary shocks can be Pareto superior to a flexible price response, it still remains to be demonstrated that there may be market equilibria in which firms are able to internalize enough of the benefits offered by a sticky price response for that response to be part of the equilibrium.

<sup>6</sup>The  $R^2$  for the federal funds rate in the same VAR is 97.55%. If this is chosen as the indicator of monetary policy, then nearly all movements in policy are predictable beforehand.

In the Ramsey solution it was the taxes and transfers that allowed firms to internalize these benefits; absent these taxes and transfers the market structure will have to be altered for this internalization to occur. To see why, imagine for a moment that we are in a perfectly competitive equilibrium in which some firms offer a price set one period in advance while other firms offer flexible prices. Although the sticky prices offered by firms may lead to a better equilibrium allocation, those firms will not be able to benefit from it in a perfectly competitive setting because when the flexible price firms offer low prices consumers will abandon the sticky price firms. It is true that when the flexible price firms offer high prices consumers will move demand towards the sticky price firms, however profitability will be lowest at these times, so that sticky price firms will receive a large customer base at precisely the times they would like to supply less at the price they have fixed. In a menu cost model this problem is avoided since flexible prices are rendered unprofitable if the menu cost is high enough. Without menu costs this problem is central in constructing a sticky price equilibrium.

The benefits that sticky prices offer in the economy of the last section are all ex ante: sticky prices help to protect consumption from fluctuations in monetary policy and can allow better cash management decisions — once cash is allocated and the shock is realized neither of these has benefit; consumers will appreciate a more stable level of consumption if the monetary shock would have reduced consumption, but will not appreciate it if the shock would have increased consumption. If consumers are able to freely switch between firms after observing the monetary shock (ex post) then firms will in general be unable to appropriate the ex ante benefits that a sticky price policy might offer. It therefore seems natural to alter the model of the last section by imbedding it in a search model. If consumers must search for price offers then they will be less able to switch to low price firms when monetary shocks are favorable, potentially allowing firms to internalize some of the benefits that sticky prices offer.

In order to render the search model tractable, we limit the analysis to open market operations in one-period bonds only,  $x_t = x_t^1$ , with i.i.d. shocks, and set  $\lambda = 0$  so that there is no cash-in-advance constraint for the firm. We also impose parametric forms for utility and production,  $u(c) = \ln(c)$ ,  $v(\bar{l} - l) = (\bar{l} - l)$ ,  $f(h) = h$ . As before, we constrain the set of equilibria examined to those in which the cash-in-advance constraint is always binding, so that in equilibrium each shopper will choose to make a purchase.

The decision problems of the worker and asset market participant will be unchanged from above, implying the same first-order conditions with respect to the choice of  $n$  and  $l$ . With the simplifying assumptions of this section

$n(s_{t-1})$  is a constant satisfying the equation

$$n = \beta \int \max\left(n, \frac{x_t}{1 + x_t - n}\right) dG(s_t)$$

and the first-order condition for labor supply becomes

$$1 = \frac{w_t \mu}{1 + g_t}$$

where  $\mu = \frac{\beta}{1-n}$  and  $g_t = \max(0, x_t - n)$ .

As before, each representative family is comprised of an asset market participant, a worker, shoppers, and entrepreneurs, however the model will be altered so that there are a continuum of shoppers and entrepreneurs within each family, both with measure 1. Shoppers will now face a search problem: following the nonsequential search model of Burdett and Judd (1983) each has the ability to visit one store at no cost or to visit  $j > 1$  stores at cost  $(j - 1)s$  in terms of utility. The timing of the family's decisions is as follows: before  $s_t$  is observed it chooses  $n$  and  $f(j)$ , where  $f(j)$  is the proportion of shoppers who will visit  $j$  stores,  $j = 1..∞$ . With i.i.d. shocks the same proportion will be chosen each period so that no time subscript is needed. The cash available for consumption,  $(1 - n_t)M_t$ , is distributed amongst the shoppers who are then randomly matched with  $j$  stores according to the probability distribution  $f(j)$ . Given this timing convention and the random allocation of shoppers across firms, cash will be distributed evenly across shoppers.<sup>7</sup> Burdett and Judd show that in the equilibrium of their model no consumer will choose to visit more than two stores even if additional stores can be visited at cost  $s$ . A similar argument holds here and the notation can be simplified so that  $f$  represents the fraction of shoppers who visit one store in equilibrium and  $(1 - f)$  visit two stores.

Each shopper is infinitesimal relative to the overall family, therefore an individual shopper's demand if faced with a lowest price  $p_{it}$  amongst the stores visited this period will be given by

$$p_{it} c_{it} = \begin{cases} (1 - n_{it}) & \text{if } \frac{u'(c(s_t))}{p_{it}} \geq \frac{\mu}{1+g_t} \\ 0 & \text{if } \frac{u'(c(s_t))}{p_{it}} < \frac{\mu}{1+g_t} \end{cases}$$

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<sup>7</sup>The random allocation of shoppers across firms effectively eliminates issues of search with recall. Examining equilibria where the same family member was able to visit the same store each period would be of interest, but would considerably complicate the search decision. Tommasi (1994) analyzes a model of this type, but in a simplified (nonmonetary) setting where the price distribution is essentially exogenous.

where  $u'(c(s_t)) = u'(\int_0^1 c_{it})$  and  $\mu$  are taken as given. This implies that shoppers are willing to purchase at any price below  $\bar{p}_t = \frac{(1-n)(1+g_t)}{\beta c_t}$ .

Search confers a measure of market power to each firm. We assume that prices are set to maximize expected utility, and that the firm meets any demand that occurs at its chosen price. In equilibrium there will be a distribution of prices across firms  $F_t(p)$  such that each firm is indifferent between its own chosen price and any other. Taking  $\bar{p}_t$  as given, the utility-weighted profit of a firm charging price  $p \leq \bar{p}_t$  is

$$\pi(p) = (1-n) [f + 2(1-f)(1-F_t(p))] \frac{\mu_t}{1+g_t} \left(1 - \frac{r_t}{p}\right)$$

where  $r_t = \frac{w_t}{\theta_t}$  is the firm's marginal cost of production. The condition that the firm be indifferent between  $p$  and any other price implies that  $\pi(p) = \pi(\bar{p}_t)$ . This yields the equation

$$1 - F_t(p) = \frac{f \frac{r_t}{\bar{p}_t} \bar{p}_t - p}{2(1-f) \bar{p}_t - r_t}$$

implying that the lowest price charged by any firm is

$$\underline{p}_t = r_t + \frac{f \frac{r_t}{\bar{p}_t} (\bar{p}_t - r_t)}{2(1-f)}$$

Using these equations the inverse of the aggregate price level has a particularly simple representation,

$$\int_{\underline{p}_t}^{\bar{p}_t} \frac{1}{p} [f_t + 2(1-f_t)(1-F_t(p))] dF_t(p) = \frac{f}{\bar{p}_t} + \frac{1-f}{r_t}$$

which is a simple average of the inverses of marginal cost and the equilibrium price ceiling  $\bar{p}$ . Since household consumption is given by

$$c_t = (1-n) \int_{\underline{p}_t}^{\bar{p}_t} \frac{1}{p} [f + 2(1-f)(1-F_t(p))] dF_t(p)$$

we can solve simultaneously for the price ceiling and consumption

$$c_t = \theta_t \frac{1-f}{\beta^{-1}(1+g_t) - f} \tag{8}$$

The household will optimally determine the fraction  $f$  so that it is indifferent between allocating more shoppers to the group visiting one firm or allocating them to the group visiting two firms. This implies that the expected gain in utility from visiting two firms must equal the search cost  $s$ . The expected gain in utility from more intensive search is

$$V = \int u'(c_t) \left[ \int_{\underline{p}}^{\bar{p}} \frac{1-n}{p} 2(1-F(p)) dF(p) - \int_{\underline{p}}^{\bar{p}} \frac{1-n}{p} dF(p) \right] dG(s_t)$$

since  $F(p)$  and  $c_t$  are functions of  $f$ , this defines the function  $V(f)$ . The next claim characterizes  $V(\cdot)$

**Claim 3**  $V(\cdot)$  is continuous, increasing, bounded and  $\lim_{f \rightarrow 0} V(f) = 0$ .

Proof: Solving the integrals involved in  $V$  implies that

$$V = \left( 1 - \int \frac{\beta dG(x_t)}{(1 + \max(0, x_t - n))} \right) \frac{f}{(1-f)^2} \left[ \frac{1}{2(1-f)} \ln \left( \frac{2-f}{f} \right) - 1 \right]$$

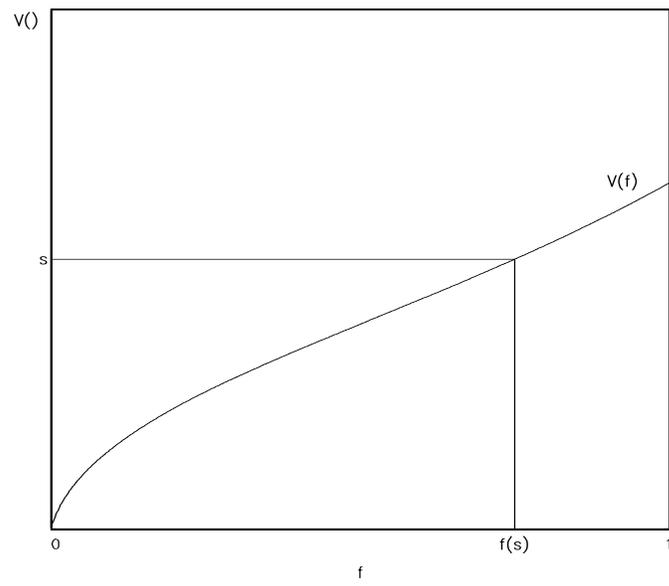
This substantiates the claim:  $V$  is increasing and monotonic,  $\lim_{f \rightarrow 0} V(f) = 0$  and  $\lim_{f \rightarrow 1} V(f) = \frac{1}{3} \left( 1 - \int \frac{\beta dG(x_t)}{(1 + \max(0, x_t - n))} \right) < \frac{1}{3} \square$

Figure 3 graphs the function  $V(f)$ . As can be seen, for higher values of the search cost,  $s$ , a higher equilibrium value of  $f$  will be chosen by the representative family. This differs from the model in Burdett and Judd (1983), where a higher search cost cannot be guaranteed to result in a higher number of consumers visiting one store. The difference results from the fact that  $\bar{p}$  is determined endogenously, where the upper limit on prices is set exogenously in Burdett and Judd. As in Burdett and Judd, there is a  $\bar{s} < \infty$  such that for  $0 < s < \bar{s}$  the household will choose  $0 < f < 1$ , leading to a nondegenerate search equilibrium. As  $s \rightarrow 0$  it is evident that  $f \rightarrow 0$  and it is straightforward to verify from the equations above that the equilibrium of the search model approaches the equilibrium of the perfectly competitive model described in Section 1 for the same preferences, technology and money-injection process.

There is a pre-existing literature on the theoretical links between inflation and search intensity (see for example Benabou (1988, 1992), Benabou and Gertner (1993), Diamond (1993) and Tommasi (1994))<sup>8</sup>. While not the focus

<sup>8</sup>Benabou (1988,1992) examines markets with state-contingent (s,S) trigger strategies by firms. In those models a higher rate of inflation will widen the (s,S) bounds that firms

Figure 4: Determining the fraction of shoppers who visit one store



of this paper, the model just outlined can be usefully compared to this previous work. Unlike the previous work, the current model is a general equilibrium model in which inflation and real interest rates are endogenously determined. With the exception of Diamond (1993), the previous literature does not examine fiat money as is done in this paper, and in the model just described prices are fully flexible which differs from the assumptions in Benabou and Diamond. Taking a second order Taylor approximation to the solution for  $V$ , it is immediate that a rise in expected money growth will increase search intensity at an interior equilibrium, while an increase in the variability of money growth will decrease it. Both results are in line with most of the previous literature. Also in line with the previous literature, an increase in the money growth rate can either increase or decrease welfare: holding search intensity constant it will decrease welfare, however the increase in intensity will lower the market power of firms and increase welfare. At low search costs the welfare reducing aspects of inflation will dominate, but at higher search costs a small increase in inflation may increase welfare.<sup>9</sup>

We conclude this section by re-examining the optimality of flexible prices in the model when it is altered to include search. The incorporation of search costs confers market power to firms, which is itself sub-optimal. The following claim takes the market power of firms in this model as given by constraining firm's expected profits and search costs to remain constant when considering alternative allocations. Even with this constraint allowing prices to respond immediately to money shocks results in a Pareto inferior allocation.

**Claim 4** *The aggregate price level and consumption are functions of the monetary shock  $x_t$ , which is constrained Pareto suboptimal*

Proof: Equation (8). The corresponding aggregate price index is

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choose and increase the incentive for search. Diamond (1993) examines a search model in which firms can choose the price of newly produced goods but cannot change the prices of goods produced earlier and held in inventory. A higher nominal interest rate lowers the return to holding money and thus increases the willingness of consumers to match with firms. Tommasi (1994) models inflation as increasing the relative real variability of firms individual costs, which decreases the gain to search since the value of being a repeat customer is lower. Benabou and Gertner (1993) model the effects of inflation variability in a setting where consumers must solve a signal extraction problem in deciding whether the movement in an individual firm's price is due to the aggregate inflation shock or a firm specific shock. In their model an increase in inflation variance may either increase or decrease search intensity.

<sup>9</sup>Note that an increase in the level of  $x$  will increase the cutoff value  $\bar{s}$ , implying that for high search costs an increase in the expected growth rate of money can move the equilibrium from  $f = 1$  to  $f < 1$ , which unambiguously improves welfare.

$$\begin{aligned}
p_t &= \left[ \int_{\underline{p}_t}^{\bar{p}_t} \frac{1}{p} [f_t + 2(1-f_t)(1-F_t(p))] dF_t(p) \right]^{-1} \\
&= \frac{1-n}{\theta_t} \frac{\beta^{-1}(1+g_t) - f}{1-f}
\end{aligned}$$

Clearly both fluctuate with shocks to  $x_t$ .

The expected utility of a household is

$$E_{t-1}U = E_{t-1} \left( \ln c_t - \frac{c_t}{\theta_t} \right) - (1-f)s$$

and the expected utility-weighted profit of a representative firm is

$$E_{t-1}\pi = E_{t-1} \left( \frac{(1-n)\mu}{1+g_t} - \frac{c_t}{\theta_t} \right)$$

The allocation  $\bar{c}_t = \theta_t E_{t-1} \frac{1-f}{\beta^{-1}(1+g_t)-f}$  will leave firm's expected profits unchanged but will raise the household's expected utility since  $\ln E_{t-1} \frac{1-f}{\beta^{-1}(1+g_t)-f} > E_{t-1} \ln \frac{1-f}{\beta^{-1}(1+g_t)-f}$   $\square$

### 3 Sticky Prices, No Menu Costs

#### 3.1 Characterizing A Sticky Price Allocation

The strategy of this subsection is to characterize what the allocation of a sticky price equilibrium would look like if it existed. As throughout the paper, we study equilibria in which the cash-in-advance constraint is strictly binding and each shopper elects to purchase the good with his available cash. A sticky price equilibrium will be defined to be an equilibrium in which all firms choose to set prices on the basis of some information set  $\Omega_t$ , where  $\Omega_t$  contains the history of shocks occurring in periods  $t-1$  and earlier, but contains only a strict subset of the period  $t$  information set  $s_t = (x_t, \theta_t)$ . Three types of possible sticky price equilibria can be considered:  $m$ -sticky, in which  $\Omega_t$  contains  $\theta_t$  but not  $x_t$ ;  $\theta$ -sticky, in which  $\Omega_t$  contains  $x_t$  but not  $\theta_t$ ; and  $m\theta$ -sticky, in which  $\Omega_t$  contains neither  $x_t$  or  $\theta_t$ . We denote  $E_{\Omega_t}z$  as the expectation of the random variable  $z$  with respect to information set  $\Omega_t$ .

Taking the maximum price  $\bar{p}_t$  as given for the moment and assuming that shoppers are willing to purchase at this price, the expected utility-weighted profit of a firm charging price  $p \leq \bar{p}_t$  is

$$E_{\Omega_t} \pi(p) = (1 - n) [f + 2(1 - f)(1 - F_t(p))] \left[ \left(1 - \frac{r_{\Omega_t}}{p}\right) E_{\Omega_t} \frac{\mu}{1 + g_t} \right]$$

where  $r_{\Omega_t} = \frac{E_{\Omega_t} \frac{1}{\theta_t}}{E_{\Omega_t} \frac{\mu}{1 + g_t}}$  is the expected marginal cost of the firm (measured in terms of the marginal utility of money). The condition that the firm be indifferent between  $p$  and any other price implies a price distribution similar to the last section:

$$1 - F_t(p) = \frac{f \frac{r_{\Omega_t}}{\bar{p}_t}}{2(1 - f)} \frac{\bar{p}_t - p}{p - r}$$

and a similar equation for consumption,

$$c_t = (1 - n) \left( \frac{f}{\bar{p}_t} + \frac{1 - f}{r_{\Omega_t}} \right)$$

In a  $m$ -sticky or  $m\theta$ -sticky price equilibrium the maximum price that a firm could charge while guaranteeing that shopper's would purchase is

$$\bar{p}_t = \frac{(1 + \underline{x} - n)(1 - n)}{\beta c_t}$$

where  $\underline{x}$  is the infimum of the support of  $x$ . No firm will be willing to charge a higher price than this if

$$\frac{1}{c_t} \int_{\underline{x}}^{x'} \frac{1}{1 + x - n} dG(x) > \frac{1}{1 + \underline{x} - n} - \frac{1 - G(x')}{1 + x' - n}$$

for all  $x' > \underline{x}$ . In essence this places an restriction on the variability of  $x$ , however this constraint turns out not to be binding in the conditions for existence established in the next section. In a  $\theta$ -sticky price equilibrium the maximal price has the same form as the last section,  $\bar{p}_t = \frac{(1-n)(1+g_t)}{\beta c_t}$ , and no further constraint need be applied.

Defining  $g_{\Omega_t}$  as  $\underline{x} - n$  in the case of  $m$ - or  $m\theta$ -sticky price equilibria, and as  $x_t - n$  in the case of a  $\theta$ -sticky price equilibrium, the general solution for consumption is

$$c_t = \frac{1-f}{\beta^{-1}(1+g_{\Omega_t})-f} \frac{E_{\Omega_t} \frac{1+g_{\Omega_t}}{1+g_t}}{E_{\Omega_t} \frac{1}{\theta_t}}$$

From which it can be seen that consumption will not depend on money shocks in  $m$ - or  $m\theta$ -sticky price equilibria and will not depend on technology shocks in  $\theta$ - or  $m\theta$ -sticky price equilibria.

The function  $V$  will have the form

$$V = \left(1 - \int \frac{\beta dG(x_t)}{1+g_{\Omega_t}}\right) \frac{f}{(1-f)^2} \left[ \frac{1}{2(1-f)} \ln \left( \frac{2-f}{f} \right) - 1 \right]$$

with the same general properties as it had in the previous section (the function is exactly the same in the  $\theta$ -sticky case).

### 3.2 Existence of an Equilibrium

An equilibrium deviation is defined as any measure-zero set of firms and households such that (1) Given the pricing strategies of deviating firms, households are willing to accept some probability  $\gamma > 0$  that any of its shoppers will be matched with a randomly selected deviating firm when searching. (2) Given the demand curve induced by optimally behaving deviating households, each firm's pricing strategy is such that its expected profits are maximized. (3) The expected utility of deviating households and the expected profits of deviating firms are each at least as large as that obtained from not deviating, and at least one of the two groups is strictly better off. A sticky or flexible price allocation is an equilibrium allocation if there is no equilibrium deviation.

It has been previously proved that at a Pareto optimal allocation prices should respond to technology shocks. The following claim is a consequence of this fact.

**Claim 5** *There is no  $\theta$ - or  $m\theta$ -sticky price equilibrium*

*Proof:* Consider a  $\theta$ -sticky price allocation. From the above equations consumption in this allocation is given by

$$c_t = \frac{1-f}{\beta^{-1}(1+g_t)-f} \frac{1}{E_{t-1} \frac{1}{\theta_t}}$$

The expected profits of a representative firm is

$$E_{t-1}\pi = E_{t-1}\frac{(1-n)\mu}{1+g_t} - E_{t-1}\frac{1-f}{\beta^{-1}(1+g_t)-f}$$

and the expected utility of a representative household is

$$E_{t-1}U = E_{t-1}\ln\frac{1-f}{\beta^{-1}(1+g_t)-f} - \ln E_{t-1}\frac{1}{\theta_t} - E_{t-1}\frac{1-f}{\beta^{-1}(1+g_t)-f} - (1-f)s$$

Now consider a deviating household that sends all its shoppers to a set of deviating firms charging flexible prices. The deviating firms will set the same price distribution described in Section 2 and deviating households will receive the amount (8). The expected profit of a deviating firm will be the same as a nondeviating firm, and the expected utility of a household will be

$$E_{t-1}U = E_{t-1}\ln\frac{1-f}{\beta^{-1}(1+g_t)-f} - E_{t-1}\ln\frac{1}{\theta_t} - E_{t-1}\frac{1-f}{\beta^{-1}(1+g_t)-f} - (1-f)s$$

which is strictly higher than the expected utility of a non-deviating household.

Now consider a  $m\theta$ -sticky price allocation. In the same way it can be proved that a set of firms and households deviating to a  $m$ -sticky price allocation will dominate. Consumption in the  $m\theta$ -sticky price allocation is

$$c_t = \frac{1-f}{\beta^{-1}(1+\underline{x}-n)-f} E_{t-1}\frac{1+\underline{x}-n}{1+x_t-n} \frac{1}{E_{t-1}\frac{1}{\theta_t}}$$

If a set of firms deviates to setting  $m$ -sticky prices and a set of households deviates by sending all of its shoppers to these firms, then consumption will be

$$c_t = \theta_t \frac{1-f}{\beta^{-1}(1+\underline{x}-n)-f} E_{t-1}\frac{1+\underline{x}-n}{1+x_t-n}$$

yielding the same expected profit for deviating firms as received by non-deviating firms, and a higher expected utility for deviating households  $\square$

We now turn to the primary goal of this paper, proving that  $m$ -sticky price equilibria can exist in this environment. Because this is technically difficult, the money growth shock  $x_t$  will be constrained to have a binomial distribution with equal probabilities that  $x_t = x_l$  and  $x_t = x_h$ ,  $x_h > x_l$  from this point forward. Before stating the claim it is helpful to outline a sketch of the proof. It

has already been proved that if given a choice between sending all its shoppers to  $m$ -sticky price firms versus sending all its shoppers to flexible price firms, consumers can receive greater utility going to the sticky price firms, while the sticky price firms can earn the same level of profits as their flexible price counterparts. So what can flexible price deviating firms offer to consumers? The answer is that if shoppers visit both fixed price and flexible price firms, then the flexible price firms can offer consumers the ability to obtain lower prices in states where costs are low, while allowing consumers to purchase from the fixed price firms when costs are high. This provides an option value to shoppers. As is standard, the value of this option will increase as the variance of the underlying shocks increases.

Defining  $g_l = x_l - n$  and  $g_h = x_h - n$ , the coefficient of variation of the gross rate of monetary growth is

$$\varphi = \frac{(1 + g_l) - (1 + g_h)}{(1 + g_l) + (1 + g_h)}$$

The following claim establishes an upper bound on the coefficient of variation for any equilibrium value of  $f$  (any value of the search cost  $0 \leq s \leq \bar{s}$ ) for a sticky price equilibrium to exist, corroborating the intuition that sticky price equilibria cannot exist if the variance of money shocks is high unless search costs are also high.

**Claim 6** *There is an continuous and increasing function  $\varphi_b(f)$ ,  $\varphi_b(0) = 0$  and  $\lim_{f \rightarrow 1} \varphi_b(f) = \infty$ , such that a  $m$ -sticky price equilibrium exists for any  $\varphi \leq \varphi_b$ .*

Proof: Consider any household deviation in which either (1) only shoppers visiting one store visit deviating firms (2) shoppers who visit two stores visit two deviating firms (3) all of the household's deviating shoppers visit deviating firms. Writing a representative firm's expected profits and the household's expected utility as

$$\begin{aligned} E_{t-1}\pi &= E_{t-1} \frac{(1-n)\mu}{1+g_t} - \frac{E_{t-1}c_t}{\theta_t} \\ E_{t-1}U &= E_{t-1} \ln c_t - \frac{E_{t-1}c_t}{\theta_t} \end{aligned}$$

Implies that the deviating firm cannot make the same expected profit without household utility being lower in any of these cases.

This proves that the only possible equilibrium deviation is one in which a measure zero set of shoppers who visit two stores deviate by visiting one firm in the deviating sector and one firm in the non-deviating sector. Each deviating firm will face the same demand curve with a unique unconstrained profit maximizing price depending on the realization of  $x_t$ : if  $x_t = x_l$  then  $p_l^* = \underline{p}$  and if  $x_t = x_h$  then

$$p_h^* = \min \left( \bar{p}, \frac{1 + \sqrt{\varphi \frac{z-1}{z-f}}}{1 - \varphi \frac{z-1}{z-f}} r \right)$$

where  $z = \beta^{-1}(1 + g_l)$ . Inserting these unconstrained prices into the firm's expected profit function, it is possible to solve for the level of  $\varphi$  for which the expected profits of deviating are exactly equal to the expected profits of non-deviating:

$$\varphi_{uc} = \begin{cases} \left( \frac{\frac{1-f}{z-f} + \frac{2}{2-f} \frac{z-1}{z-f} + \sqrt{2}}{\frac{2z-f}{z-1} \frac{1-f}{f} + \sqrt{2}} \right)^2 \frac{z-f}{z-1} & \text{if } p_h^* \leq \bar{p} \\ \frac{f}{(1-f)(2-f)} \frac{z-1}{z-f} & \text{if } p_h^* = \bar{p} \end{cases}$$

Deviating profits are increasing in  $\varphi$ .

In maximizing profits deviating firm's face the constraint that deviating shoppers must be at least as well off as those who do not deviate, otherwise they will have no customers. Setting this to an equality constraint and solving for the prices that maximize expected profits subject to this constraint yields  $p_l^{c*} = \underline{p}$  and

$$p_h^{c*} = \frac{r}{1 - \alpha \frac{z-1}{z-f}}$$

where

$$\alpha = ProductLog \left( \frac{1+f}{1-f} \ln \left( \frac{2-f}{f} \right) - \frac{4-f}{2-f} \right)$$

(the function  $\alpha = ProductLog(x)$  is the solution to  $\alpha - \ln \alpha = x$ ). Inserting these constrained prices into the firms profit function one can solve for the value of  $\varphi$  that sets the expected profit of a deviating firm equal to the expected profit of a non-deviating firm

$$\varphi_c = \frac{\frac{f}{2-f} + \alpha z - 1}{\frac{2-f}{f} - \frac{1}{\alpha} z - f}$$

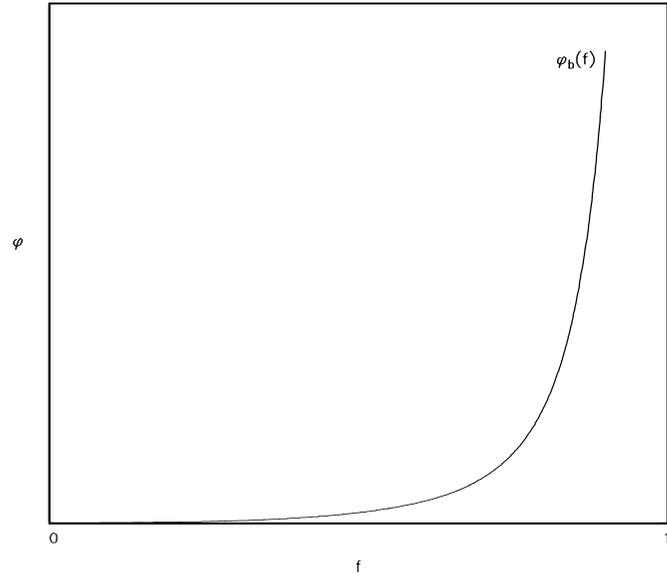
the function  $\varphi_b = \min(\varphi_{uc}, \varphi_c)$  has the properties asserted by the claim, and for any  $\varphi > \varphi_b$  there is an equilibrium deviation in which the expected profits of the deviating firm are higher than the expected profits of a non-deviating firm and the expected utility of a deviating household is at least as high as that of a non-deviating household  $\square$

Figure 4 graphs the function  $\varphi_b$ . Calibrating  $\beta = 0.99$ ,  $z = 1.02$ , and  $f = 0.64$  (corresponding to a 5% markup over costs at the sticky price equilibrium), we have  $\varphi_b = 0.105$  implying that there is a sticky price equilibrium for any standard deviation of 10.5% in the *gross* rate of money growth relative to its mean. If the mean *net* rate of inflation is 5%, then the *gross* mean is 1.05 and a coefficient of variation of 0.105 implies a standard deviation of  $0.105 \cdot 1.05 \approx 0.11$ , implying that a one-standard deviation upper bound on net inflation can contain inflation rates as high as 16% while maintaining the existence of a sticky price equilibrium. Based on this type of back of the envelope calculation, the level of inflation variability experienced in most developed economies appears to be consistent with the existence of a sticky price equilibrium in the model presented here.

## 4 Conclusion

This paper has argued that equilibria in which prices are sticky in response to monetary shocks can actually lead to higher welfare than flexible price equilibria in a fairly standard monetary model. Taking this insight a model has been constructed in which sticky price equilibria can exist even when there are no menu costs. Endogenizing price stickiness in this way yields some insights that are not present when price stickiness is exogenously imposed: the same argument that implies prices should be sticky in response to monetary shocks implies that they should be flexible in response to technology shocks; there is an upper bound on the level of money variability under which a sticky price equilibrium can be supported and this bound depends on the level of search frictions – in economies or markets where search frictions are low sticky prices can only be supported if money variability is low, and in an economy or market where search or other market frictions are absent prices must be flexible.

Figure 5: Boundary coefficient of variation for an  $m$ -sticky price equilibrium



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