

**Appendix: Derivation of Equation (24)**  
**Growth-Led Exports: Is Variety the Spice of Trade?**

Take the logarithm of equation (23) and totally differentiate.  
 Make the following notational simplifications:  $P^X/R=PX$ ,  $P^{E^*}=PE^*$ ,  
 $P^{D^*}=PD^*$ .

$$\begin{aligned}
 \text{dlog}(NX) = & \text{(1)} \quad -\sigma \text{ dlog}(PX/PE^*) + \text{(2)} \quad \text{dlog}(E^*/PE^*) + \text{(3)} \quad \text{dlog}[Y/(Y+Y^*)] \\
 & \text{(4)} \quad + (\sigma-1) \text{ dlog}(A^*) + \text{(5)} \quad \text{dlog}[(Y+Y^*)/Y^*] + \text{(6)} \quad \text{dlog}(Z/Z^*) \\
 & \text{(7)} \quad + (1-\sigma) \text{ dlog}(PE^*/PD^*) - \text{(8)} \quad \text{dlog}\{1 + Z Y [PX/(PD^* A^*)]^{1-\sigma}/(Z^* Y^*)\}
 \end{aligned}$$

Terms (1)-(3) above are the same as in equation (24) except that "d" is replaced by "Δ". Making use of  $\text{dlog}(X)=dX/X$ , term (8) can be written:

Term 8

$$\begin{aligned}
 -\{Z Y (1-\sigma) PX [PD^* A^* dpX - PX (PD^* dA^* + A^* dPD^*)]/[Z^* Y^* \\
 (PD^* A^*)^3] + [Z^* Y^* (Y dZ + Z dY) - Z Y (Y^* dZ^* + Z^* dY^*)] \\
 /[(Z^* Y^*)^2]/\{1 + Z Y [PX/(PD^* A^*)]^{1-\sigma}/(Z^* Y^*)\}
 \end{aligned}$$

Use of initial conditions --  $A^*=1$ ,  $PX=PD^*$ ,  $Z=Z^*$  -- allows simplification to

$$\begin{aligned}
 -\{Y (1-\sigma) (dPX - PD^* dA^* - dPD^*)/(Y^* PD^*) + [Y^* Y (dZ - dZ^*) \\
 + Z(Y^* dY - Y dY^*)]/(Z Y^{*2})\}/(1 + Y/Y^*)
 \end{aligned}$$

dividing both numerator and denominator by  $(1+Y/Y^*)$

$$\begin{aligned}
 [Y/(Y^*+Y)](1-\sigma)dA^* \\
 - [Y/(Y^*+Y)](1-\sigma)(dPX - dPD^*)/PD^* \\
 - [Y/(Y^*+Y)](dZ - dZ^*)/Z - (Y^* dY - Y dY^*)/[(Y^*+Y)Y^*]
 \end{aligned}$$

Term 4

$$(\sigma-1)dA^*/A^* \quad (A^*=1)$$

Combine with first term of simplified term 8 to yield

$$(\sigma-1)[Y^*/(Y+Y^*)]dA^*/A^* \text{ which is the fourth term in equation (24).}$$

Term 5

$$\begin{aligned} & [Y^*/(Y+Y^*)][Y^*(dY+dY^*)-(Y+Y^*)dY^*]/Y^{*2} \\ &= [Y^*/(Y+Y^*)][Y^* dY - Y dY^*]/Y^{*2} \\ &= [Y^* dY - Y dY^*]/[(Y+Y^*)Y^*] \end{aligned}$$

which cancels out the fourth term of simplified term 8.

Term 6

$$\begin{aligned} & (Z^*/Z)(Z^* dZ - Z dZ^*)/Z^{*2} \quad (Z=Z^*) \\ &= (dZ - dZ^*)/Z \end{aligned}$$

Combine with third term of simplified term 8 to yield

$$[Y^*/(Y+Y^*)](dZ - dZ^*)/Z \text{ which is the fifth term in equation (24).}$$

Term 7

Use the definition of PE\* in equation (22), defining

$$w = PD^* D^*/E^* \text{ and } (1-w) = PX X/E^*.$$

$$\begin{aligned} & (1-\sigma) d\log\{[w PD^* + (1-w) PX]/PD^*\} \\ &= (1-\sigma)[PD^*(w dPD^* + PD^* dW + dPX - w dPX - PX dW) - w PD^* dPD^* \\ & \quad - PX dPD^* + w PX dPD^*]/PD^{*2} \end{aligned}$$

Substituting the initial condition:  $PX = PD^*$ .

$$(1-\sigma)(dPX - w dPX - dPD^* + w dPD^*)/PD^*$$

$$= (1-\sigma)(1-w)(dPX - dPD^*)/PD^*$$

Under the initial condition of no home bias ( $A^*=1$ ) the share of imports in expenditures  $(1-w)$  equals exporter's share of world output  $[Y/(Y+Y^*)]$ .

$$(1-\sigma)[Y/(Y+Y^*)](dPX - dPD^*)/PD^*$$

which cancels out the second term of simplified term 8.

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