

Appendix: Derivation of Equation (24)
 Growth-Led Exports: Is Variety the Spice of Trade?

Take the logarithm of equation (23) and totally differentiate. Make the following notational simplifications: $P^X/R=PX$, $P^{E^*}=PE^*$, $P^{D^*}=PD^*$.

$$\begin{aligned}
 \text{dlog}(NX) = & \text{(1)} \quad -\sigma \text{ dlog}(PX/PE^*) + \text{(2)} \quad \text{dlog}(E^*/PE^*) + \text{(3)} \quad \text{dlog}[Y/(Y+Y^*)] \\
 & \text{(4)} \quad + (\sigma-1) \text{ dlog}(A^*) + \text{(5)} \quad \text{dlog}[(Y+Y^*)/Y^*] + \text{(6)} \quad \text{dlog}(Z/Z^*) \\
 & \text{(7)} \quad + (1-\sigma) \text{ dlog}(PE^*/PD^*) - \text{(8)} \quad \text{dlog}\{1 + Z Y [PX/(PD^* A^*)]^{1-\sigma}/(Z^* Y^*)\}
 \end{aligned}$$

Terms (1)-(3) above are the same as in equation (24) except that "d" is replaced by "Δ". Making use of $\text{dlog}(X)=dX/X$, term (8) can be written:

Term 8

$$\begin{aligned}
 -\{Z Y (1-\sigma) PX [PD^* A^* dpX - PX (PD^* dA^* + A^* dPD^*)]/[Z^* Y^* \\
 (PD^* A^*)^3] + [Z^* Y^* (Y dZ + Z dY) - Z Y (Y^* dZ^* + Z^* dY^*)] \\
 /[(Z^* Y^*)^2]/\{1 + Z Y [PX/(PD^* A^*)]^{1-\sigma}/(Z^* Y^*)\}
 \end{aligned}$$

Use of initial conditions -- $A^*=1$, $PX=PD^*$, $Z=Z^*$ -- allows simplification to

$$\begin{aligned}
 -\{Y (1-\sigma) (dPX - PD^* dA^* - dPD^*)/(Y^* PD^*) + [Y^* Y (dZ - dZ^*) \\
 + Z(Y^* dY - Y dY^*)]/(Z Y^{*2})\}/(1 + Y/Y^*)
 \end{aligned}$$

dividing both numerator and denominator by $(1+Y/Y^*)$

$$\begin{aligned}
 [Y/(Y^*+Y)](1-\sigma)dA^* \\
 - [Y/(Y^*+Y)](1-\sigma)(dPX - dPD^*)/PD^* \\
 - [Y/(Y^*+Y)](dZ - dZ^*)/Z - (Y^* dY - Y dY^*)/[(Y^*+Y)Y^*]
 \end{aligned}$$

Term 4

$$(\sigma-1)dA^*/A^* \quad (A^*=1)$$

Combine with first term of simplified term 8 to yield

$$(\sigma-1)[Y^*/(Y+Y^*)]dA^*/A^* \text{ which is the fourth term in equation (24).}$$

Term 5

$$\begin{aligned} & [Y^*/(Y+Y^*)][Y^*(dY+dY^*)-(Y+Y^*)dY^*]/Y^{*2} \\ &= [Y^*/(Y+Y^*)][Y^* dY - Y dY^*]/Y^{*2} \\ &= [Y^* dY - Y dY^*]/[(Y+Y^*)Y^*] \end{aligned}$$

which cancels out the fourth term of simplified term 8.

Term 6

$$\begin{aligned} & (Z^*/Z)(Z^* dZ - Z dZ^*)/Z^{*2} \quad (Z=Z^*) \\ &= (dZ - dZ^*)/Z \end{aligned}$$

Combine with third term of simplified term 8 to yield

$$[Y^*/(Y+Y^*)](dZ - dZ^*)/Z \text{ which is the fifth term in equation (24).}$$

Term 7

Use the definition of PE* in equation (22), defining

$$w = PD^* D^*/E^* \text{ and } (1-w) = PX X/E^*.$$

$$\begin{aligned} & (1-\sigma) d\log\{[w PD^* + (1-w) PX]/PD^*\} \\ &= (1-\sigma)[PD^*(w dPD^* + PD^* dW + dPX - w dPX - PX dW) - w PD^* dPD^* \\ & \quad - PX dPD^* + w PX dPD^*]/PD^{*2} \end{aligned}$$

Substituting the initial condition: PX = PD*.

$$(1-\sigma)(dPX - w dPX - dPD^* + w dPD^*)/PD^*$$

$$= (1-\sigma)(1-w)(dPX - dPD^*)/PD^*$$

Under the initial condition of no home bias ($A^*=1$) the share of imports in expenditures $(1-w)$ equals exporter's share of world output $[Y/(Y+Y^*)]$.

$$(1-\sigma)[Y/(Y+Y^*)](dPX - dPD^*)/PD^*$$

which cancels out the second term of simplified term 8.

Note: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to IFDPs (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author. Recent IFDPs are available on the Web at www.federalreserve.gov/pubs/ifdp/. The views expressed here are my own and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.