Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 848r

April 2006

# Fighting Against Currency Depreciation, Macroeconomic Instability and Sudden Stops

Luis-Felipe Zanna

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors. Recent IFDPs are available on the Web at www.federalreserve.gov/pubs/ifdp/.

# Fighting Against Currency Depreciation, Macroeconomic Instability and Sudden Stops*

Luis-Felipe Zanna†

This Draft: April 2006

**Abstract**

In this paper we show that, in the aftermath of a currency crisis, a government that adjusts the nominal interest rate in response to domestic currency depreciation can induce aggregate instability in the economy by generating self-fulfilling endogenous cycles. We find that, if a government raises the interest rate proportionally more than an increase in currency depreciation, then it induces self-fulfilling cycles that, driven by people's expectations about depreciation, replicate several of the salient stylized facts of the "Sudden Stop" phenomenon. These facts include a decline in domestic production and aggregate demand, a collapse in asset prices, a sharp correction in the price of traded goods relative to non-traded goods, an improvement in the current account deficit, a moderately higher CPI-inflation, more rapid currency depreciation, and higher nominal interest rates. In this sense, an interest rate policy that responds to depreciation may have contributed to generating the dynamic cycles experienced by some economies in the aftermath of a currency crisis.

**Keywords:** Small Open Economy, Interest Rate Policies, Currency Depreciation, Self-fulfilling Cycles, Sudden Stops, Collateral Constraints

**JEL Classifications:** E32, E52, E58, F41

*This paper is based on chapter 3 of my dissertation at the University of Pennsylvania. I am grateful to Martín Uribe, Stephanie Schmitt-Grohé and Frank Schorfheide for their guidance and teaching. I also received helpful suggestions and benefitted from conversations with Roc Armenter, Martin Bodenstein, David Bowman, Sanjay Chugh, Bill Dupor, Bora Durdu, Fabio Ghironi, Chris Gust, Dale Henderson, Sylvain Leduc, Gustavo Suárez and seminar participants at the International Economics Brown Bag Lunch at the University of Pennsylvania, the International Finance Workshop at the Federal Reserve Board and the fall 2005 SCIEA Meetings. All errors remain mine. Previous versions of this paper circulated under the title: "Interest Rate Rules and Multiple Equilibria in the Aftermath of BOP Crises". The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the view of the Board of Governors of the Federal Reserve System or of any other person associated to the Federal Reserve System.

†Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue, NW, Washington, D.C., 20551. Tel.: (202)452-2337. Fax: (202)736-5638. Email: Luis-Felipe.Zanna@frb.gov.

# 1 Introduction

One of the policy debates that emerged out of the Asian Crisis of 1997 was that of the appropriate interest rate policy to fight domestic currency depreciation. On one hand, the IMF advocated for higher interest rates to prevent excessive depreciation. It claimed that such a policy would reduce capital outflows by raising the cost of currency speculation and induce capital inflows by making domestic assets more attractive.[1] On the other hand, some critics of the IMF policy prescription recommended lowering interest rates in the face of depreciation.[2] These critics argued that lower rates would alleviate the subsequent recessions in the Asian economies, and improve the banks' and corporations' balance sheets, which had weakened significantly as a result of the negative effects of rapid currency depreciation on their dollarized liabilities.

These two views had something in common. They both advocated for changes in interest rates in response to some macroeconomic indicators, in this case the currency depreciation rate. They both implied a feedback mechanism, positive or negative, from the current depreciation rate to the nominal interest rate.[3] In this paper, we study some of the possible macroeconomic consequences of these interest rate feedback policies in countries that have been hit by a currency crisis. Our main result is that in the aftermath of a crisis, an interest rate policy that calls for changing the nominal interest rate, positively or negatively, in response to currency depreciation can induce aggregate instability in the economy by generating self-fulfilling endogenous cycles. That is, this policy can cause cycles in the economy that are driven exclusively by people's self-fulfilling beliefs rather than fundamentals, and regardless of whether the response to depreciation is positive or negative.

We find that if a government raises the interest rate proportionally more than an increase in currency depreciation, then it induces self-fulfilling cycles that replicate several of the salient empirical regularities of emerging market crises. In particular, we build a model in which these cycles, which are driven exclusively by people's expectations about currency depreciation, replicate many of the stylized facts following a currency crisis. These facts, labeled by Calvo (1998) as the "Sudden Stop" phenomenon, include a decline in domestic production and aggregate demand, a collapse in asset prices, a sharp correction in the price of traded goods relative to non-traded goods, an improvement in the current account deficit, a moderately higher CPI-inflation, more rapid currency depreciation and higher nominal interest rates. In this sense, interest rate policies that respond to depreciation may have contributed to generating the dynamic cycles experienced by some economies in the aftermath of a currency crisis.

[1] See Fischer (1998) among others.

[2] See for instance Furman and Stiglitz (1998) and Radelet and Sachs (1998) among others.

[3] In fact some works motivated by the debate describe, implicitly or explicitly, the interest rate policy as a feedback rule responding to some measure of nominal depreciation. See for instance Cho and West (2001), Goldfajn and Baig (1998), and Lahiri and Vegh (2003), among others.

As a building block to our full model, we first construct a simple environment of a small open economy in which we are able to derive *analytical* results. We show that interest rate policies that respond *only* to currency depreciation can induce real indeterminacy or multiple equilibria, which in turn opens the possibility of self-fulfilling equilibria.[4] The intuition for this result is the following. Suppose that in response to a "sunspot," agents in the economy expect a higher inflation rate. Because the interest rate policy responds *only* to currency depreciation - and not to inflation - the real interest rate can fall, boosting aggregate demand. As firms see aggregate demand rising, they raise prices, thus validating the original expectations of a higher inflation.[5]

Our full model is a sticky-price small open economy with traded and non-traded goods with two important features: an interest rate policy that responds to currency depreciation, and a collateral constraint which captures the fact that international loans must be guaranteed by physical assets such as capital. Loosely speaking, the crisis is modelled as a time when this constraint is unexpectedly binding.[6] The model also includes other features that have become very useful to match quantitatively some of the stylized facts of the aftermath of a crisis. In particular, we consider non-traded distribution services, and domestic and international loan requirements to hire labor and to purchase an imported intermediate input.[7]

In this full model, an interest rate policy that responds only to depreciation also induces multiple equilibria. However, the interaction of this policy with the collateral constraint induces self-fulfilling *cycles.* This result is related to, but differs from, the seminal work by Kiyotaki and Moore (1997). They show that a binding collateral constraint induces credit cycles that may amplify business cycles driven by fundamentals. In our work this constraint creates cycles, but the interest rate policy makes them self-fulfilling to the extent that they are driven by people's expectations about the economy.

We think our results have at least three important implications. First, they do not support any of the views of the previously mentioned debate. In fact we unveil a peril that may be present in both policy recommendations. What seems crucial is not whether the government increases or decreases the interest rate, but instead the feedback response of the nominal interest rate to currency depreciation that most of the previous studies have ignored.

[4] From now on we will use the terms "multiple equilibria" and "real indeterminacy" (a "unique equilibrium" and "real determinacy") interchangeably. By real indeterminacy we mean that the behavior of one or more (real) variables of the economy is not pinned down by the model. This implies that there are multiple equilibria, which in turn opens the possibility of having fluctuations in the economy generated by endogenous beliefs that are of the "sunspot" type; i.e., they are based on stochastic variables that are extrinsic in Cass and Shell's (1983) terminology.

[5] An interest rate policy that responds to inflation does not necessarily preclude the possibility of self-fulfilling equilibria. But as we discuss below, the response to currency depreciation makes the interest rate policy more prone to induce self-fulfilling equilibria.

[6] This idea of modelling the crisis as an unexpected binding collateral constraint captures the essence of a "Sudden Stop." Other works that introduce such collateral constraints include Caballero and Krishnamurthy (2001), Christiano, Gust and Roldos (2004), Krugman (1999), Mendoza and Smith (2002), and Paasche (2001) among others.

[7] See Burnstein, Eichenbaum and Rebelo (2005a,b) and Christiano, Gust and Roldos (2004), among others.

Second, our results provide a possible explanation of why the empirical literature has not been able to obtain conclusive evidence about whether higher interest rates can cause nominal currency depreciation or, instead, appreciation in the aftermath of a crisis.[8] This literature has tried to control for the variables that influence the nominal exchange rate. But our results suggest that there can be potential influences that may depend on "sunspots," which in turn can induce self-fulfilling cycles in the nominal exchange rate (or the nominal depreciation rate) as well as in other variables. Clearly these influences do not depend on fundamentals, and their effect should be taken into account by this literature.

Third, since interest rate policies that respond to currency depreciation can induce expectations-driven fluctuations, then they can destabilize the economy. Therefore they can be costly in terms of macroeconomic instability and welfare.[9] This has not been studied in the previous literature and deserves further research. For instance, Lahiri and Vegh (2002, 2003) and Flood and Jeanne (2000) focus on the fiscal and output costs of higher interest rates before and after a crisis. In addition Lahiri and Vegh (2003) claim that there is a non-monotonic relationship between welfare and the increase in interest rates. Christiano, Gust and Roldos (2004) explore conditions under which a cut (rise) in the interest rate in the midst of a crisis will stimulate output and improve welfare. Aghion, Bacchetta and Banerjee (2000) find that it might not be optimal to raise interest rates either when the proportion of foreign currency debt is not too large, or when credit provision, domestic investment and production are highly sensitive to changes in nominal interest rates; whereas Braggion, Christiano and Roldos (2005) build a model where, in response to a financial crisis, it is optimal to raise the interest rate immediately, and then reduce it sharply.

The remainder of this paper is organized as follows. In Section 2, we consider a simple economy in "good" times and characterize analytically the equilibrium under interest rate policies that respond only to currency depreciation. In Section 3, we build our full model, which includes, among other things, a collateral constraint. Here we study the determinacy of equilibrium analysis through numerical simulations. In addition, we show that even if the interest rate policy responds to past CPI-inflation, it can still induce multiple equilibria as long as it responds to currency depreciation.[10] In Section 4, we use our full model to construct a self-fulfilling equilibrium that captures the stylized facts of a "Sudden Stop." Section 5 concludes.

[8] See the review of this literature by Montiel (2003).

[9] These policies can lead to "sunspot" equilibria that are characterized by a large degree of volatility of some macroeconomic aggregates such as consumption. Provided that agents are risk averse, then these policies can induce equilibria where agents are worse-off.

[10] The new interest rate rules literature argues that aggressive rules with respect to past inflation are more likely to guarantee a unique equilibrium. See for instance Benhabib, Schmitt-Grohé, and Uribe (2001), Taylor (1999), and Woodford (2003) among others. See also Zanna (2003) for an analysis of interest rate rules in small open economies.

# 2 The Simple Model

In this section we develop a simple infinite-horizon small open economy model. The economy is populated by a continuum of identical household-firm units and a government who are blessed with perfect foresight. Before we describe in detail the behavior of these agents we state a few general assumptions and definitions.

There are two consumption goods: a traded good and a composite non-traded good whose prices are denoted by $P_t^T$ and $P_t^N$ respectively. We assume that the law of one price holds for the traded good. Then $P_t^T = \mathcal{E}_t P_t^{T*}$ where $\mathcal{E}_t$ is the nominal exchange rate, and $P_t^{T*}$ is the foreign price of the traded good. Below we will relax this assumption. We also set $P_t^{T*} = 1$ implying that $P_t^T = \mathcal{E}_t$.

The real exchange rate $(e_t)$ is defined as the ratio between the price of traded goods and the aggregate price of non-traded goods, $e_t = \mathcal{E}_t / P_t^N$. From this definition we deduce that

$$e_t = e_{t-1} \left( \frac{\epsilon_t}{\pi_t^N} \right), \tag{1}$$

where $\epsilon_t = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$ is the gross nominal depreciation, and $\pi_t^N = \frac{P_t^N}{P_{t-1}^N}$ is the gross non-traded goods inflation.

## 2.1 The Government

The government issues two nominal liabilities: money, $M_t^g$, and a domestic bond, $B_t^g$, that pays a gross nominal interest rate $R_t$. It does not have access to foreign debt and makes lump-sum transfers to the household-firm units, $\tau_t$, pays interest on its domestic debt, $(R_t - 1)B_t^g$, and receives revenues from seigniorage. Its budget constraint is described by $m_t^g + b_t^g = \frac{m_{t-1}^g}{\epsilon_t} + \tau_t + \frac{R_{t-1} b_{t-1}^g}{\epsilon_t}$ where $m_t^g = \frac{M_t^g}{\mathcal{E}_t}$ and $b_t^g = \frac{B_t^g}{\mathcal{E}_t}$. It follows a Ricardian fiscal policy by setting the lump-sum transfers in order to satisfy the intertemporal version of its budget constraint and its transversality condition $\lim_{t\to\infty} \frac{b_t^g}{\prod_{s=0}^{t-1} \left( \frac{R_s}{\epsilon_{s+1}} \right)} = 0$.

The description of monetary policy is motivated by the debate that emerged after the Asian crisis about the appropriate interest rate policy to fight against currency depreciation. We assume the government implements an interest rate policy that responds to the deviation of the current depreciation rate from a depreciation target.[11] That is,

$$R_t = \bar{R} \rho \left( \frac{\epsilon_t}{\bar{\epsilon}} \right), \tag{2}$$

where $\rho(.)$ is a continuous, differentiable, and strictly positive function in its argument with $\rho(1) = 1$ and $\rho_\epsilon \equiv \rho'(1) \neq 0$; and $\bar{R}$ and $\bar{\epsilon}$ are the targets of the nominal interest rate and the nominal

[11] Some of the works inspired by this debate, such as Cho and West (2001), Goldfajn and Baig (1998), and Lahiri and Vegh (2003) among others, also describe the interest rate policy, implicitly or explicitly, as a rule that reacts to some measure of nominal depreciation.

depreciation rate that the government wants to achieve.[12]

This policy can respond positively to the deviation of the nominal depreciation rate from its target, $\rho_\epsilon > 0$, capturing the policy recommendation of the IMF; or it can react negatively, $\rho_\epsilon < 0$, describing, to some extent, the policy recommendation of the opposite view. Nevertheless we exclude the cases $\rho_\epsilon = -1, 1$.[13] In other words, the interest rate policy corresponds to (2) with $\rho_\epsilon \equiv \rho'(1) \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$.

## 2.2 The Household-Firm Unit

There is a large number of identical household-firm units. They have perfect foresight, live infinitely and derive utility from consuming, not working, and liquidity services of money. The intertemporal utility function of the representative unit is described by

$$\sum_{t=0}^{\infty} \beta^t \left[ U(c_t^T) + V(c_t^N) + H(h_t^T) + L(h_t^N) + J(m_t) \right], \tag{3}$$

where $\beta \in (0,1)$ corresponds to the subjective discount rate, $c_t^T$ and $c_t^N$ denote the consumption of traded and non-traded goods respectively, $h_t^T$ and $h_t^N$ are the labor allocated to the production of the traded good and the non-traded good, and $m_t$ refers to real money holdings measured with respect to foreign currency. The specification in (3) assumes separability in the single period utility function among consumption, labor, and real money balances. Hence there are no distortionary effects of transactions money demand.[14] Moreover the utility function is separable in $c_t^T$, $c_t^N$, $h_t^T$, and $h_t^N$. This allows us to derive *analytical* results in the determinacy of equilibrium analysis. To complete the characterization we also make the following assumption.

**Assumption 1.** *a) $U(.)$, $V(.)$, $H(.)$, $L(.)$ and $J(.)$ are twice continuously differentiable; and b) $U(.)$, $V(.)$, and $J(.)$ are strictly increasing ($U_T \equiv \frac{dU}{dc_t^T} > 0$, $V_N > 0$, $J_m > 0$) and strictly concave ($U_{TT} < 0$, $V_{NN} < 0$, $J_{mm} < 0$), whereas $H(.)$ and $L(.)$ are strictly decreasing ($H_T \equiv \frac{dH}{dh_t^T} < 0$, $L_N < 0$) and strictly concave ($H_{TT} < 0$, $L_{NN} < 0$).*

The household-firm unit produces a flexible-price traded good and a sticky-price non-traded intermediate differentiated good by employing labor from perfectly competitive markets. For simplicity we assume that there is zero labor mobility.[15] The technologies are described by

$$y_t^T = F\left(\breve{h}_t^T\right) \qquad \text{and} \qquad \tilde{y}_t^N = G\left(\tilde{h}_t^N\right),$$

[12] For simplicity we also assume that these targets correspond to the steady-state levels of these variables.

[13] As we will see below, the cases of $\rho_\epsilon = 1$ or $\rho_\epsilon = -1$ introduce a unit root in the log-linearized system of the economy, precluding the possibility of using the "Theorem of Hartman and Grobman" to derive meaningful conclusions about the dynamics of the non-linear system. See Guckenheimer and Holmes (1985).

[14] Because of this, we can write the real money balances that enter the utility function in terms of foreign currency, $m_t \equiv \frac{M_t}{\mathcal{E}_t}$, without consequences for our results.

[15] As we discuss below, our results do not depend on this assumption.

where $\breve{h}_t^T$ and $\tilde{h}_t^N$ denote the labor hired by the household-firm unit for the production of the traded good and the non-traded good, respectively. The technologies satisfy the following assumption.

**Assumption 2.** *$F(.)$ and $G(.)$ are twice continuously differentiable, strictly increasing ($F_T \equiv \frac{dF}{dh_t^T} > 0$, $G_N > 0$), and strictly concave ($F_{TT} < 0$, $G_{NN} < 0$ ).*

Consumption of the non-traded good, $c_t^N$, is a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit-Stiglitz type. Each household-firm unit is the monopolistic producer of one variety of non-traded intermediate goods. The demand for the intermediate good is of the form $C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ satisfying $d(1) = 1$ and $d'(1) = -\mu$, with $\mu > 1$, where $C_t^N$ denotes the level of aggregate demand for the non-traded good, $\tilde{P}_t^N$ is the nominal price of the intermediate non-traded good produced by the household-firm unit, and $P_t^N$ is the price of the composite non-traded good. The unit sets the price of the good it supplies, $\tilde{P}_t^N$, taking the level of aggregate demand for the good as given. Specifically, the monopolist is constrained to satisfy demand at that price. That is,

$$G\left(\tilde{h}_t^N\right) \geq C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right). \tag{4}$$

Following Rotemberg (1982), we introduce nominal price rigidities for the intermediate non-traded good. The household-firm unit faces a resource cost of the type $\frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}^N\right)^2$, where $\bar{\pi}^N$ is the steady-state level of the gross non-traded goods inflation.

There are incomplete markets. The representative household-firm unit has access to the following two risk free bonds: a government domestic bond, $B_t$, that pays a gross nominal interest rate $R_t$, and a foreign bond, $b_t^*$, that pays a gross foreign interest rate $R_t^*$. In addition, the unit receives income from working, $W_t^T h_t^T + W_t^N h_t^N$, transfers from the government, $\tau_t$, and dividends. Then the flow budget constraint in units of the traded good can be written as

$$m_t + b_t \leq \frac{m_{t-1}}{\epsilon_t} + \frac{R_{t-1}b_{t-1}}{\epsilon_t} + w_t^T h_t^T + w_t^N h_t^N + \tau_t + \Omega_t - c_t^T - \frac{c_t^N}{e_t}, \tag{5}$$

where $b_t = \frac{B_t}{\mathcal{E}_t}$, $w_t^i = \frac{W_t^i}{\mathcal{E}_t}$ with $i = T, N$, and

$$\Omega_t = \left[F\left(\breve{h}_t^T\right) - w_t^T \breve{h}_t^T\right] - \frac{1}{e_t}\left[\frac{\tilde{P}_t^N}{P_t^N} C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) - e_t w_t^N \tilde{h}_t^N - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}\right)^2\right] - R_{t-1}^* b_{t-1}^* + b_t^*. \tag{6}$$

Equation (5) says that the end-of-period real financial domestic assets (money plus domestic bond) can be worth no more than the real value of financial domestic wealth, brought into the period, plus the sum of wage income, transfers, and dividends ($\Omega_t$) net of consumption. The dividends described in (6) correspond to the difference between sale revenues and costs, taking into account that through the firm-side the representative unit can hold foreign debt, $b_t^*$. For holdings of foreign debt the unit pays interests $(R_{t-1}^* - 1)b_{t-1}^*$. Moreover the unit is subject to a Non-Ponzi game condition

$$\lim_{t\to\infty} \frac{n_t}{\prod_{s=0}^{t-1} R_s^*} \geq 0, \tag{7}$$

where $n_t = b_t + m_t - b_t^*$.

The problem of the household-firm unit consists of choosing the set of sequences $\{c_t^T,\, c_t^N,\, h_t^T,\, h_t^N,\, \check{h}_t^T,\, \tilde{h}_t^N,\, \tilde{P}_t^N,\, b_t^*,\, b_t,\, m_t\}_{t=0}^{\infty}$ in order to maximize (3) subject to (4), (5), (6) and (7), given the initial condition $n_{-1}$ and the set of sequences $\{R_t^*,\, R_t,\, \epsilon_t,\, e_t,\, P_t^N,\, w_t^T,\, w_t^N,,\, \tau_t,\, C_t^N\}$. Note that since the utility function specified in (3) implies that the preferences of the agent display non-sasiation, then both constraints (5) and (7) hold with equality. The Appendix contains a detailed derivation of the necessary conditions for optimization. Imposing these conditions along with the market clearing conditions in the labor markets, the equilibrium symmetry ($\tilde{P}_t^N = P_t^N$ and $\tilde{h}_t^N = h_t^N$), the market clearing condition for the non-traded good

$$G\left(h_t^N\right) = c_t^N + \frac{\gamma}{2}\left(\pi_t^N - \bar{\pi}^N\right)^2, \tag{8}$$

and the definitions $\pi_t^N = P_t^N / P_{t-1}^N$, $d(1) = 1$ and $d'(1) = -\mu$, we obtain

$$R_t^* = \frac{R_t}{\epsilon_{t+1}}, \tag{9}$$

$$-\frac{H_T(h_t^T)}{U_T(c_t^T)} = F_T\left(h_t^T\right), \tag{10}$$

$$U_T(c_t^T) = \beta R_t^* U_T(c_{t+1}^T), \tag{11}$$

$$V_N(c_t^N) = \frac{\beta R_t}{\pi_{t+1}^N} V_N(c_{t+1}^N), \tag{12}$$

$$\frac{V_N(c_{t+1}^N)\left(\pi_{t+1}^N - \bar{\pi}^N\right)\pi_{t+1}^N}{V_N(c_t^N)} = \frac{\left(\pi_t^N - \bar{\pi}^N\right)\pi_t^N}{\beta} + \frac{\mu c_t^N}{\beta\gamma}\left(\frac{\mu-1}{\mu} - mc_t\right), \tag{13}$$

where $mc_t = -\frac{L_N(h_t^N)}{V_N(c_t^N)G_N\left(h_t^N\right)}$ corresponds to the marginal cost of producing the non-traded good. In addition equilibrium in the traded good market implies that

$$b_t^* - b_{t-1}^* = \left(R_{t-1}^* - 1\right) b_{t-1}^* + c_t^T - F\left(h_t^T\right). \tag{14}$$

The interpretation of these equations is straightforward. Condition (9) corresponds to an Uncovered Interest Parity condition (UIP) that equalizes the returns of the foreign and domestic bonds. Equation (10) makes the marginal rate of substitution between labor, assigned to the production of the traded good, and consumption of the traded good equal to the marginal product of labor in the production of the traded good. Equations (11) and (12) are the standard Euler equations for consumption of the traded good and consumption of the non-traded good. Equation (13) corresponds to the augmented Phillips curve for the sticky-price non-traded goods inflation. And (14) describes the dynamics of the current account deficit.

## 2.3 Capital Markets

We introduce imperfect international capital markets using the following ad-hoc supply curve of funds

$$R_t^* = R^* f\left(\frac{b_t^*}{\bar{b}^*}\right), \quad \text{with} \quad f'\left(\frac{b_t^*}{\bar{b}^*}\right) > 0, \quad f(1) = 1, \quad f'(1) = \psi > 0, \tag{15}$$

and where $f\left(\frac{b_t^*}{\bar{b}^*}\right)$ corresponds to the country-specific risk premium, and $R^*$ is the risk free international interest rate. This specification captures the idea that the small borrowing economy faces a world interest rate, $R_t^*$, that increases when the stock of foreign debt, $b_t^*$, is above its long run level, $\bar{b}^*$. Then as the external debt grows, so does the risk of default, and in order to compensate the lenders for this risk, the economy has to pay them a premium over the risk free international interest rate. We also assume that the long-run level of foreign stock of debt is positive.

**Assumption 3.** *The long-run level of the foreign stock of debt is positive:* $\bar{b}^* > 0$.

By introducing (15), we "close the small open economy" and avoid the unit root problem, as discussed in Schmitt-Grohé and Uribe (2003). This will allow us to pursue a meaningful analysis of the dynamics of the non-linear equations that describe the economy, using a log-linear approximation.[16]

## 2.4 A Perfect Foresight Equilibrium

We are ready to provide a definition of a perfect foresight equilibrium in this economy.

**Definition 1** *Given the initial condition $b_{-1}^*$, the steady-state level of foreign debt $\bar{b}^*$, and the depreciation target $\bar{\epsilon}$, a symmetric perfect foresight equilibrium is defined as a set of sequences $\{c_t^T,\ c_t^N,\ h_t^T,\ h_t^N,\ b_t^*,\ \epsilon_t,\ \pi_t^N,\ R_t,\ R_t^*\}_{t=0}^{\infty}$ satisfying: a) the market clearing conditions for the non-traded and traded goods, (8) and (14), b) the UIP condition (9), c) the intratemporal efficient condition (10), d) the Euler equations for consumption of traded and non-traded goods, (11) and (12), e) the augmented Phillips curve, (13), f) the interest rate policy (2) and g) the ad-hoc upward-sloping supply curve of foreign funds (15).*

Since fiscal policy is Ricardian, this definition ignores the budget constraint of the government and its transversality condition. The definition also ignores real money balances. This is because monetary

[16] The "unit-root problem" that is commonly present in small open economy models arises because of assuming that $R_t^* = \frac{1}{\beta}$. To see why, use this assumption together with condition (49) to deduce that $\lambda_t = \lambda_{t+1}$. This is an equation that has a unit root and that introduces a unit root in the entire dynamical system of the simple set-up. See Schmitt-Grohé and Uribe (2003).

policy is described as an interest rate policy in (2), and real balances enter in the utility function in a separable way. In addition, note that once we solve for $\{c_t^T,\, c_t^N,\, h_t^T,\, h_t^N,\, b_t^*,\, \epsilon_t,\, \pi_t^N,\; R_t,\, R_t^*\}_{t=0}^{\infty}$, it is possible to retrieve the set of sequences $\{\lambda_t,\, e_t,\, m_t,\, b_t,\, w_t^T,\, w_t^N,\, mc_t\}_{t=0}^{\infty}$ using (1), (5), and equations (41)-(44), (46), and (48) that are presented in the Appendix.

### 2.5 The Determinacy of Equilibrium Analysis

In order to pursue the determinacy of equilibrium analysis, we log-linearize the system of equations that describe the dynamics of this economy around a steady state $\{\bar{c}^T,\, \bar{c}^N,\, \bar{h}^T,\, \bar{h}^N,\, \bar{b}^*,\, \bar{\epsilon},\, \bar{\pi}^N,\; \bar{R},\, \bar{R}^*\}$. In the Appendix we characterize this steady state.

Log-linearizing the equations of Definition 1 around the steady-state yields

$$\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t \qquad \text{with } \; \rho_\epsilon \neq 0 \text{ and either } \; |\rho_\epsilon| > 1 \text{ or } \; |\rho_\epsilon| < 1, \tag{16}$$

$$\hat{R}_t = \hat{R}_t^* + \hat{\epsilon}_{t+1}, \quad \text{with} \quad \hat{R}_t^* = \psi \hat{b}_t^*, \tag{17}$$

$$\hat{c}_t^N = \hat{c}_{t+1}^N - \xi^N \left( \hat{R}_t - \hat{\pi}_{t+1}^N \right), \tag{18}$$

$$\hat{\pi}_t^N = \beta \hat{\pi}_{t+1}^N + \beta \varphi \hat{c}_t^N, \tag{19}$$

$$\hat{b}_t^* = \left( \frac{1+\psi}{\beta} \right) \hat{b}_{t-1}^* + \kappa \hat{c}_t^T, \tag{20}$$

$$\hat{c}_t^T = \hat{c}_{t+1}^T - \xi^T \left( \hat{R}_t - \xi^T \hat{\epsilon}_{t+1} \right), \tag{21}$$

where $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right)$ and

$$\xi^T = -\frac{U_T}{U_{TT}\bar{c}^T} > 0, \qquad \xi^N = -\frac{V_N}{V_{NN}\bar{c}^N} > 0, \qquad \sigma^T = \frac{H_T}{H_{TT}\bar{h}^T} > 0, \qquad \sigma^N = \frac{L_N}{L_{NN}\bar{h}^N} > 0,$$

$$\omega^T = -\frac{F_T}{F_{TT}\bar{h}^T} > 0, \qquad \omega^N = -\frac{G_N}{G_{NN}\bar{h}^N} > 0, \tag{22}$$

$$\kappa = \frac{1}{\bar{b}^*}\left[ \bar{c}^T + \frac{F_T \bar{h}^T \sigma^T \omega^T}{(\sigma^T + \omega^T)\, \xi^T} \right] > 0, \quad \text{and} \quad \varphi = \left[ \frac{(\mu-1)\bar{c}^N}{\beta\gamma\bar{\epsilon}^2} \right] \left[ \frac{\bar{c}^N(\sigma^N + \omega^N)}{G_N \bar{h}^N \sigma^N \omega^N} + \frac{1}{\xi^N} \right] > 0,$$

whose signs are derived using Assumptions 1, 2 and 3. Equations (16)-(21) correspond to the reduced log-linear representations of the interest rate policy, the UIP condition, the Euler equation for consumption of the non-traded good, the augmented Phillips curve, the current account equation, and the Euler equation for consumption of the traded good, respectively.

Our main goal is to show that the policy in (16) is prone to induce multiple equilibria in the economy described by equations (17)-(21). Proving this implies that this policy can cause fluctuations in the economy that are driven by people's self-fulfilling beliefs rather than fundamentals. In fact, before we provide a formal proof, it is worth developing a simple intuition. To do so, it is sufficient to concentrate on equations (16)-(19). Note that given the international interest rate, $\hat{R}_t^*$, then the policy (16) and the UIP condition (17) determine the dynamics of the depreciation rate, $\hat{\epsilon}_t$, and the nominal interest rate, $\hat{R}_t$. Moreover, the nominal interest rate, $\hat{R}_t$, is not affected by either the non-traded good inflation, $\hat{\pi}_t^N$, or the consumption of the non-traded good, $c_t^N$. Taking this into account, we can construct the following self-fulfilling equilibrium.

Assume that agents, in response to a "sunspot," *expect* a higher non-traded good inflation in the next period. Since the interest rate policy does not react to these expectations, then the real interest rate measured with respect to the expected non-traded good inflation, $\hat{R}_t - \hat{\pi}_{t+1}^N$, declines.[17] In response, households increase consumption of non-traded goods - see (18) - which leads firms to raise their prices, inducing a higher non-traded inflation - see (19). But by doing this, firms end validating the original non-traded inflation expectations.

This simple intuition is appealing but incomplete, unless we show that all the equilibrium conditions (16)-(21) are satisfied on the entire equilibrium path. In other words, we need to characterize formally the equilibrium of this economy. To accomplish this, we start by writing equations (16)-(21) as

$$\begin{pmatrix} \hat{\epsilon}_{t+1} \\ \hat{b}_t^* \\ \hat{c}_{t+1}^T \\ \hat{\pi}_{t+1}^N \\ \hat{c}_{t+1}^N \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_\epsilon & -\frac{\psi(1+\psi)}{\beta} & -\psi\kappa & 0 & 0 \\ 0 & \frac{1+\psi}{\beta} & \kappa & 0 & 0 \\ 0 & \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta} & -\varphi \\ \rho_\epsilon \xi^N & 0 & 0 & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right) \end{pmatrix}}_{J^c} \begin{pmatrix} \hat{\epsilon}_t \\ \hat{b}_{t-1}^* \\ \hat{c}_t^T \\ \hat{\pi}_t^N \\ \hat{c}_t^N \end{pmatrix}. \tag{23}$$

Then, we use this system to find and to compare the dimension of its unstable subspace with the number of non-predetermined variables.[18] If the dimension of this subspace is smaller than the number of non-predetermined variables, then, from the results by Blanchard and Kahn (1980), we can infer that there exist multiple perfect foresight equilibria. This forms the basis for the existence of self-fulfilling

[17] Because of perfect foresight $E_t\hat{\pi}_{t+1}^N = \hat{\pi}_{t+1}^N$.

[18] The dimension of the unstable subspace is given by the number of roots of the system that are outside the unit circle. See Blanchard and Kahn (1980).

fluctuations.

The following Proposition states the main result of the determinacy of equilibrium analysis: an interest rate policy that raises or lowers the nominal interest rate in response to current currency depreciation leads to multiple perfect foresight equilibria or, equivalently, to real indeterminacy.

**Proposition 1** *If the government follows an interest rate policy that responds to currency depreciation such as $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ with $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$, then there exists a continuum of perfect foresight equilibria (indeterminacy) in which the sequences $\{\hat{\epsilon}_t,\ \hat{b}^*_t,\ \hat{c}^T_t,\ \hat{\pi}^N_t,\ \hat{c}^N_t\}_{t=0}^{\infty}$ converge asymptotically to the steady state. In addition,*

- ***a)*** *if $|\rho_\epsilon| > 1$ then the degree of indeterminacy is of order 1.*[19]

- ***b)*** *if $|\rho_\epsilon| < 1$ then the degree of indeterminacy is of order 2.*

**Proof.** The eigenvalues of the matrix $J^c$ in (23) correspond to the roots of the characteristic equation $\mathcal{P}^c(v) = |J^c - vI| = 0$. Using the definition of $J^c$ in (23), this equation can be written as

$$\mathcal{P}^c(v) = (v - \rho_\epsilon)\,\mathcal{P}^f(v) = 0, \tag{24}$$

where

$$\mathcal{P}^f(v) = \left[v^2 - \left(1 + \frac{1+\psi}{\beta} + \psi\kappa\xi^T\right)v + \frac{1+\psi}{\beta}\right]\left[v^2 - \left(1 + \frac{1}{\beta} + \varphi\xi^N\right)v + \frac{1}{\beta}\right].$$

By Lemma 4 in the Appendix, $\mathcal{P}^f(v) = 0$ has real roots satisfying $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$, and $|v_4| > 1$. The fifth root of $\mathcal{P}^c(v) = 0$ is $v_5 = \rho_\epsilon$. Clearly, if $|\rho_\epsilon| > 1$ then $|v_5| > 1$, whereas if $|\rho_\epsilon| < 1$ then $|v_5| < 1$. Therefore, using this, the characterization of the roots of $\mathcal{P}^f(v) = 0$, and (24), we can conclude the following. If $|\rho_\epsilon| > 1$ then $\mathcal{P}^c(v) = 0$ has three explosive roots, namely $|v_2| > 1$, $|v_4| > 1$ and $|v_5| > 1$. While if $|\rho_\epsilon| < 1$ then $\mathcal{P}^c(v) = 0$ has two explosive roots, namely $|v_2| > 1$ and $|v_4| > 1$. Hence, regardless of whether $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$, there are at most three explosive roots. Given that there are four non-predetermined variables, $\hat{\epsilon}_t$, $\hat{c}^T_t$, $\hat{\pi}^N_t$ and $\hat{c}^N_t$, then the number of non-predetermined variables is greater than the number of explosive roots. Applying the results of Blanchard and Kahn (1980), it follows that there exists an infinite number of perfect foresight equilibria converging to the steady state. Finally, parts a) and b) follow from the difference between the number of non-predetermined variables and the number of explosive roots, when $|\rho_\epsilon| > 1$ and $|\rho_\epsilon| < 1$, respectively. ■

Proposition 1 has two important implications. First, provided that the fiscal policy is Ricardian, then the interest rate policy induces not only real indeterminacy but also nominal indeterminacy. That

[19] The degree of indeterminacy is defined as the difference between the number of non-predetermined variables and the dimension of the unstable subspace of the log-linearized system.

is, the nominal exchange rate is not pinned-down.[20] This follows from the definition of the nominal depreciation rate, and the fact that the nominal exchange rate is also a non-predetermined variable. Second, and in contrast to the intuition provided above, Proposition 1 suggests that it is possible to construct self-fulfilling equilibria that are based on expectations of a different variable from the non-traded inflation. For instance, we can construct a self-fulfilling equilibrium driven by people's beliefs about currency depreciation. We will pursue this exercise in Section 4.

Table 1: Determinacy of Equilibrium Analysis

| | *The Simple Model* | | *The Full Model* | |
|---|---|---|---|---|
| | *Degree of Responsiveness* | | *Degree of Responsiveness* | |
| *Interest Rate Policy* | $\|\rho_\epsilon\| < 1$ | $\|\rho_\epsilon\| > 1$ | $\|\rho_\epsilon\| < 1$ | $\|\rho_\epsilon\| > 1$ |
| *Forward-Looking* | | | | |
| $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ *with* $\rho_\epsilon \neq 0$ | *M* | *M* | *M* | *M* |
| *Contemporaneous* | | | | |
| $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ *with* $\rho_\epsilon \neq 0$ | *M* | *M* | *M* | *M* |
| *Backward-Looking* | | | | |
| $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ *with* $\rho_\epsilon \neq 0$ | *M* | *U* | *M* | *M or U* |

Note: *M refers to multiple equilibria and U refers to a unique equilibrium*

The results of Proposition 1 also pose the question of whether policies that respond exclusively to either the future depreciation rate, a forward-looking policy $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$, or to the past depreciation rate, a backward-looking policy $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$, can induce multiple equilibria. The answer is yes and the analysis is provided in the Appendix. In particular, we find that forward-looking policies can still lead to multiple equilibria when the interest rate response coefficient to future depreciation satisfies either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. On the contrary, backward-looking policies that are very aggressive with respect to past depreciation, and satisfy $|\rho_\epsilon| > 1$, will guarantee a unique equilibrium; whereas timid policies that satisfy $|\rho_\epsilon| < 1$ can lead to real indeterminacy. These results, as well as the results from Proposition 1, are presented in Table 1 in the columns labeled as "The Simple Model."[21]

[20] A Non-Ricardian fiscal policy combined with the monetary policy under study will determine the level of the nominal exchange rate if $|\rho_\epsilon| > 1$ but not if $|\rho_\epsilon| < 1$.

[21] Our general results still hold if we describe monetary policy as $\Delta R_t = R_t - R_{t-1} = \varphi(\frac{\epsilon_{t+s}}{\epsilon})$ with $s = -1, 0, 1$ and

Note that the assumption of zero labor mobility is not crucial for the results of Proposition 1. In fact, it is possible to obtain similar analytical results under perfect labor mobility and some extra assumptions about the disutility of working.[22] Then, what are the features of the model that drive these results? The crucial features are the following: the description of monetary policy as an interest rate feedback rule, non-traded goods price-stickiness, and the exclusive dependence of the policy on currency depreciation. By Sargent and Wallace (1975), we know that the first feature by itself leads to nominal indeterminacy of the exchange rate level (price level), in a flexible price model under a Ricardian fiscal policy. The second characteristic together with the policy elucidate why nominal indeterminacy turns into real indeterminacy. And finally, the first two features, in tandem with the exclusive response of the policy to depreciation, are what explains why multiple equilibria arise, to some extent, regardless of the degree of responsiveness of the policy.[23]

This simple model suffers from at least two drawbacks. On one hand, there is no feature that captures the fact that the economy is in a crisis. On the other hand, the dynamics of non-traded consumption and inflation, that are supported as a self-fulfilling equilibrium, are completely at odds with the stylized facts of a "Sudden Stop." In particular, the self-fulfilling equilibrium that we constructed above implies that non-traded consumption and inflation are positively correlated. In contrast, the stylized facts suggest that there was a strong decline in consumption accompanied by an increase in inflation. To improve the model, in the next section, we will add some extra features.

# 3 The Full Model

In this section we enrich the simple model in several dimensions. First, in accord with Burnstein, Eichenbaum, and Rebelo (2005a,b), we introduce non-traded distribution services. This together with price stickiness are crucial to explain the large movements in real exchange rates after large devaluations. Second, following Lahiri and Vegh (2002) and Christiano, Gust, and Roldos (2004) we assume that the household-firm unit requires domestic and international loans to hire labor and to purchase an imported intermediate input, respectively. This is important to obtain a decline in output and demand in the midst of the crisis, when interest rates rise. Third, as in the new literature about currency crises, we introduce a collateral constraint. That is, international loans must be guaranteed

$\varphi(1) = 0$. This resembles the implicit descriptions in some of the empirical works mentioned in Montiel (2003).

[22] When labor is mobile across sectors then $w_t^T = w_t^N$, and the equilibrium in the labor market becomes $h_t^T + h_t^N = \check{h}_t^T + \tilde{h}_t^N$. In addition, if we assume that $H_{TT} = 0$ and $L_{NN} = 0$, and keep the rest of Assumptions 1, 2, and 3, then the log-linearized system of equations that describes the economy is *still* (16)-(21). But in this case, $\kappa = \frac{1}{\bar{b}^*}\left[\bar{c}^T - \frac{F_T^2}{F_{TT}\xi^T}\right] > 0$ and $\varphi = \left[\frac{(\mu-1)\bar{c}^N}{\beta\gamma\bar{\epsilon}^2}\right]\left[\frac{1}{\xi^N} - \frac{G_{NN}\bar{c}^N}{G_N^2}\right] > 0$.

[23] In Zanna (2003), we show that in order to guarantee a unique equilibrium, a rule must respond aggressively to the non-traded inflation but timidly to depreciation. A rule that responds aggressively to currency depreciation still opens the possibility of multiple equilibria, regardless of its response to non-traded inflation.

by physical assets such as capital. This will help to provide a definition of a crisis. Fourth, we consider a utility function that is not separable in the two types of consumption. We proceed to explain how we introduce these features and their influence in the previous equations.

### 3.1 The Additional Features

As in Burnstein et al. (2003), we assume that the traded good needs to be combined with some non-traded distribution services before it is consumed. In order to consume one unit of the traded good, it is required $\eta$ units of the basket of differentiated non-traded goods. Let $\breve{P}_t^T$, $P_t^T$ and $P_t^N$ be the producer's price of the traded good, the consumer's price of the traded good, and the general price level of the basket of differentiated non-traded goods, respectively. All of them are expressed in domestic currency. Hence the consumer's price is simply $P_t^T = \breve{P}_t^T + \eta P_t^N$. And since PPP holds at the production level of the traded good ($\breve{P}_t^T = \mathcal{E}_t \breve{P}_t^{T*}$), and the foreign price of the traded good is normalized to one ($\breve{P}_t^{T*} = 1$), we have that $P_t^T = \mathcal{E}_t + \eta P_t^N$.

The production of the differentiated non-traded good is still demand determined by

$$G\left(\tilde{h}_t^N, K^N\right) \geq C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) + \eta C_t^T d\left(\frac{\tilde{P}_t^N}{P_t^N}\right), \tag{25}$$

where $d(1) = 1$, $d'(1) = -\mu$, $C_t^N$ denotes the level of aggregate demand for the non-traded good, $\tilde{P}_t^N$ is the price of the intermediate non-traded good set by the household-firm unit, and $C_t^T$ corresponds to the level of aggregate consumption of the traded good. But now the demand requirements come from two sources.[24] They come from consumption of non-traded goods $C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ that provide utility, and from non-traded distribution services $\eta C_t^T d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ that are necessary to bring one unit of the traded good to the household-firm unit. Note that there is no difference between non-traded consumption goods and non-traded distribution services. As a consequence, in equilibrium the basket of non-traded goods required to distribute traded goods will have the same composition as the non-traded basket consumed by the household-firm unit.

The introduction of the loan requirements and the collateral constraint in the model follows Christiano et al. (2004). The household-firm unit requires domestic loans to hire labor ($\check{h}_t^T$ and $\tilde{h}_t^N$), and international loans to buy an imported input ($I_t$), that will be used in the production of the traded good. These loans are obtained at the beginning of the period and repaid at the end of the period. In this sense, they represent short-term debt and differ from long-term foreign debt $b_{t-1}^*$. We do not model, however, the financial institutions that provide these loans. Instead, we assume that the domestic loans are provided by the government, whereas foreign loans are supplied by foreign creditors.[25]

[24] See Corsetti, Dedola, and Leduc. (2005).

[25] To formalize this point, we could introduce financial institutions in the model that behave in a perfectly competitive way and supply the aforementioned loans. This would not change our main results.

For these loans the unit pays interests $(R_t-1)W_t(\breve{h}_t^T+\tilde{h}_t^N)$ and $(R_t^*-1)\tilde{P}_t^T I_t$, that are accrued between periods, where $R_t$ is the domestic nominal interest rate, and $R^*$ is the international interest rate. The latter is assumed to be constant and equal to $\frac{1}{\beta}$.

The production technology of the traded good uses labor $(\breve{h}_t^T)$, an imported input $(I_t)$, and capital $(K^T)$; whereas the technology for the non-traded intermediate differentiated goods only requires labor $(\tilde{h}_t^N)$ and capital $(K^N)$. That is,

$$y_t^T = F\left(\breve{h}_t^T, I_t, K^T\right) \quad \text{and} \quad \tilde{y}_t^N = G\left(\tilde{h}_t^N, K^N\right).$$

Furthermore, as in Christiano et al. (2004) and Mendoza and Smith (2002), among others, capital is assumed to be time-invariant, does not depreciate, and there is no technology to making it bigger. Under these new features, the dividends that the household-firm unit receives can be written as

$$\begin{aligned} \Omega_t \quad = \quad & F\left(\breve{h}_t^T, I_t, K^T\right) + \frac{1}{e_t}\left[\frac{\tilde{P}_t^N}{P_t^N} G\left(\tilde{h}_t^N, K^N\right) - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}\right)^2\right] \\ & -w_t^T R_t \breve{h}_t^T - w_t^N R_t \tilde{h}_t^N - R^* I_t - R^* b_{t-1}^* + b_t^*, \end{aligned} \tag{26}$$

where $w_t^i = \frac{W_t^i}{\mathcal{E}_t}$ with $i = T, N$, and $e_t = \frac{\mathcal{E}_t}{P_t^N}$.

To model the crisis, we follow Christiano et al. (2004) by imposing the following collateral constraint:

$$R^* b_{t-1}^* + R^* I_t + R_t\left(w_t^T \breve{h}_t^T + w_t^N \tilde{h}_t^N\right) \leq \phi\left(q_t^N K^N + q_t^T K^T\right), \tag{27}$$

where $q_t^N$ and $q_t^T$ represent the real value, in units of foreign currency, of one unit of capital for the production of the non-traded and traded goods, respectively, and $\phi$ is the fraction of these stocks that foreign creditors accept as collateral. The constraint (27) says that the total value of foreign and domestic debt, that the representative household-firm unit has to pay to completely eliminate the debt of the firm by the end of period $t$, cannot exceed the value of the collateral. The crisis makes this constraint unexpectedly binding in every period henceforth without the possibility of being removed.

Finally, we relax the assumption in (3) of a separable utility function between the two types of consumption, $U(c_t^T)+V(c_t^N)$. But we will still assume separability among consumption, labor, and real money balances. We proceed to study how all these features influence the previous optimal conditions.

## 3.2 The New Equilibrium Conditions

The problem that the household firm unit has to solve is similar to the one presented in the simple model. The agent chooses the set of sequences $\{c_t^T, c_t^N, h_t^T, h_t^N, \breve{h}_t^T, \tilde{h}_t^N, I_t, \tilde{P}_t^N, b_t^*, b_t, m_t\}_{t=0}^{\infty}$ in order to maximize

$$\sum_{t=0}^{\infty}\beta^t\left[\tilde{U}(c_t^T, c_t^N) + H(h_t^T) + L(h_t^N) + J(m_t)\right]$$

subject to the budget constraint

$$m_t + b_t \leq \frac{m_{t-1}}{\epsilon_t} + \frac{R_{t-1}b_{t-1}}{\epsilon_t} + w_t^T h_t^T + w_t^N h_t^N + \tau_t + \Omega_t - \left(1 + \frac{\eta}{e_t}\right)c_t^T - \frac{c_t^N}{e_t}$$

and the constraints (7), (25), (26), and (27), given the initial conditions $b_{-1}^*$, $b_{-1}$, and $m_{-1}$ and the set of sequences $\{R_t^*, R_t, \epsilon_t, e_t, P_t^N, w_t^T, w_t^N, \tau_t, C_t^N, C_t^T\}$. The optimization conditions together with symmetry and market clearing conditions can be used to find the laws of motion of the economy. They correspond to (1), (2), (27) with equality,

$$G\left(h_t^N, K^N\right) = c_t^N + \frac{\gamma}{2}\left(\pi_t^N - \bar{\pi}\right)^2 + \eta c_t^T, \tag{28}$$

$$R_t = R^*(1+\zeta_{t+1})\epsilon_{t+1}, \tag{29}$$

$$-\frac{H_T(h_t^T)}{\tilde{U}_T(c_t^T, c_t^N)} = \frac{w_t^T}{1+\frac{\eta}{e_t}}, \tag{30}$$

$$\tilde{U}_T(c_t^T, c_t^N) = \frac{\beta R_t}{\pi_{t+1}^T}\tilde{U}_T(c_{t+1}^T, c_{t+1}^N), \quad \text{where} \quad \pi_{t+1}^T = \epsilon_{t+1}\frac{\left(1+\frac{\eta}{e_{t+1}}\right)}{\left(1+\frac{\eta}{e_t}\right)}, \tag{31}$$

$$\tilde{U}_N(c_t^T, c_t^N) = \frac{\beta R_t}{\pi_{t+1}^N}\tilde{U}_N(c_{t+1}^T, c_{t+1}^N), \tag{32}$$

$$\frac{\tilde{U}_N(c_{t+1}^T, c_{t+1}^N)(\pi_{t+1}^N - \bar{\pi}^N)\pi_{t+1}^N}{\tilde{U}_N(c_t^T, c_t^N)} = \frac{(\pi_t^N - \bar{\pi}^N)\pi_t^N}{\beta} + \frac{\mu\left(c_t^N + \eta c_t^T\right)}{\beta\gamma}\left(\frac{\mu-1}{\mu} - mc_t\right), \tag{33}$$

$$b_t^* - b_{t-1}^* = (R^*-1)b_{t-1}^* + R^* I_t + c_t^T - F\left(h_t^T, I_t, K^T\right), \tag{34}$$

$$F_T\left(h_t^T, K^T, I_t\right) = w_t^T(1+\zeta_t)R_t, \tag{35}$$

$$mc_t = \frac{w_t^N e_t(1+\zeta_t)R_t}{G_N\left(h_t^N, K^N\right)}, \tag{36}$$

$$F_I\left(h_t^T, K^T, I_t\right) = (1+\zeta_t)R^*, \tag{37}$$

$$-\frac{L_N(h_t^N)}{\tilde{U}_N(c_t^T, c_t^N)} = w_t^N e_t, \tag{38}$$

where $\lambda_t \zeta_t$ and $\lambda_t$ are the Lagrange multipliers of the collateral constraint and the budget constraint. The latter multiplier evolves according to the asset pricing equation

$$\lambda_t = \beta R^* (1 + \zeta_{t+1}) \lambda_{t+1}. \tag{39}$$

Equations (28)-(34) are equivalent to equations (8)-(14) in the simple model.[26] Therefore they have a similar interpretation. Equations (35)-(37) correspond to the optimal conditions that determine the demands for labor (for the production of the traded and non-traded good) and for the imported input; and (38) is the intratemporal condition that makes the marginal rate of substitution between labor, assigned to the production of the non-traded good, and consumption of the non-traded good equal to the real salary measured in units of the non-traded good.

The new laws of motion reveal that the additional features have some important consequences. On one hand, distribution services affect the relative price of the traded good at the consumer level with respect to the nominal exchange rate. In the simple model this relative price was equal to one. In the full model this price is $1 + \frac{\eta}{e_t}$, which depends on the distribution costs parameter $\eta$. From (30) and (31), it is clear that through this price, distribution costs affect, in equilibrium, the optimal intratemporal decisions between labor and consumption of the traded good, as well as the optimal intertemporal choices for consumption of the traded good. On the other hand, distribution services generate an extra demand of non-traded goods, as is captured by the last term, $\eta c_t^T$, of the right hand side of (28). This extra demand also influences the dynamics of non-traded goods inflation, as can be seen in (33).

The binding collateral constraint generates an endogenous risk premium, as reflected by the modified UIP condition in (29). In fact, because of this constraint, the effective international interest rate that domestic agents pay becomes $(1 + \zeta_{t+1})R^*$. Thus, raising external debt $b_t^*$ not only requires the payment of interests $(R^* b_t^*)$, but also tightens the binding constraint $(\zeta_{t+1} > 0)$. This generates an additional interest cost.[27]

The requirement of loans to hire labor and the binding collateral constraint affect the labor demand decisions, as can be inferred from (35) and (36). Keeping the rest constant, these two features and the fact that in the short run $\zeta_t > 0$ imply the following. Increases in the effective interest rate, $(1+\zeta_t)R_t$, push the cost of hiring labor up, which in turn discourages the demands for labor for the production of the traded and non-traded goods.

The optimal condition for demand of the imported input, equation (37), equalizes the marginal product of this input to the effective cost of foreign working capital, $(1 + \zeta_t)R^*$, that is necessary to

[26] In the simple model $F_T\left(h_t^T\right) = w_t^T$

[27] Note also that in contrast to the simple model, we have assumed that $R^* = \frac{1}{\beta}$. Nevertheless, in this context, this typical assumption of the small open economy literature does not cause the unit-root problem. Under this assumption and with the binding constraint, condition (39) becomes $\lambda_t = (1 + \zeta_{t+1})\lambda_{t+1}$ which does not introduce a unit root in the system of equations that describes the economy.

import it. As the constraint tightens and $\zeta_t$ goes up, the effective cost raises and the demand for the imported input decreases. Furthermore, the purchases of this input influence the current account equation (34). A decrease in the imported input can cause an improvement in the current account deficit. This improvement is almost immediate, given the assumption that external short term loans to finance the intermediate input have to be repaid at the end of the period, and not at the beginning of next period.

Finally, it is possible to derive the equilibrium value of the prices of capital. The equilibrium value of these asset prices are described by

$$q_t^T = \frac{F_K + \left(\frac{R_t}{\epsilon_{t+1}}\right)^{-1} q_{t+1}^T}{(1-\phi\zeta_t)} \qquad \text{and} \qquad q_t^N = \frac{(\frac{mc_t}{e_t})G_K + \left(\frac{R_t}{\epsilon_{t+1}}\right)^{-1} q_{t+1}^N}{(1-\phi\zeta_t)}, \tag{40}$$

where $F_K$ and $G_K$ are the marginal products of capital in the production of the traded and non-traded goods, respectively.

### 3.3 The Determinacy of Equilibrium Analysis

Using this set-up, the definition of equilibrium under a currency crisis is the following.

**Definition 2** *Given $b_{-1}^*$, $R^*$, $K^N$, $K^T$ and the depreciation target $\bar{\epsilon}$, a perfect foresight equilibrium is defined as a set of sequences $\{c_t^T, c_t^N, \zeta_t, h_t^T, h_t^N, I_t, b_t^*, mc_t, e_t, q_t^T, q_t^N, w_t^T, w_t^N, \epsilon_t, \pi_t^N, R_t\}_{t=0}^{\infty}$ satisfying equations (1), (2), the collateral constraint (27) as an equality, (28)-(37) and (40).*

Note that, as in Christiano et al. (2004) among others, we model the crisis as a collateral constraint that binds in every period. Nevertheless, at the steady state, the shadow price of the collateral constraint is equal to zero. To see this use (39) in tandem with $\beta = \frac{1}{R^*}$ to obtain $\bar{\zeta} = 0$. This implies that the collateral constraint is *marginally* not binding at the steady state. Hence the credit restrictions disappear marginally. In contrast, in the short run the shadow price of the collateral constraint may vary as $\zeta_t$ changes. When $\zeta_t > 0$ is high then the collateral constraint tightens.

As before, to pursue the determinacy of equilibrium analysis, we log-linearize the system of equations around the perfect-foresight steady state. Then we characterize the dimension of the unstable subspace of the system and compare it to the number of non-predetermined variables. By log-linearizing the system, we are following the same approach that Kiyotaki and Moore (1997) adopt to solve for an equilibrium of a model with a binding collateral constraint. This precludes the possibility of exploring non-linear equilibrium dynamics.[28] Moreover, since it is not possible to derive analytical results, we have to rely on numerical simulations. We use the following functional forms. For

[28] To pursue a determinacy of equilibrium analysis in the augmented *non-linear* model is a very challenging task. Most of the works, that include a collateral constraint and that simulate equilibrium dynamics for the non-linear system, do not characterize the equilibrium. They assume that the equilibrium that is found computationally is the relevant one, whose properties must be studied. See Mendoza and Smith (2002) and Christiano et al. (2004), among others.

consumption and labor preferences we use

$$\tilde{U}(c_t^T, c_t^N) = \frac{\left[(\alpha)^{\frac{1}{\mathrm{a}}}\left(c_t^T\right)^{\frac{\mathrm{a}-1}{\mathrm{a}}} + (1-\alpha)^{\frac{1}{\mathrm{a}}}\left(c_t^N\right)^{\frac{\mathrm{a}-1}{\mathrm{a}}}\right]^{\left(\frac{\mathrm{a}}{\mathrm{a}-1}\right)(1-\sigma)} - 1}{1-\sigma}$$

and

$$H(h_t^T) + L(h_t^N) = -\frac{\varsigma}{1+\delta}\left[\left(h_t^T\right)^{1+\delta} + \left(h_t^N\right)^{1+\delta}\right],$$

where $\alpha \in (0,1)$, $\varsigma, \sigma, \mathrm{a} > 0$ and $\delta \geq 0$; whereas for technologies we utilize

$$F\left(h_t^T, K^T, I_t\right) = \left\{\varpi\left[\vartheta_1 (h_t^T)^{\theta^T}(K^T)^{1-\theta^T}\right]^{\frac{\chi-1}{\chi}} + (1-\varpi)\left[\vartheta_2 I_t\right]^{\frac{\chi-1}{\chi}}\right\}^{\frac{\chi}{\chi-1}}$$

and

$$G\left(h_t^N, K^N\right) = (h_t^N)^{\theta^N}(K^N)^{1-\theta^N},$$

where $\theta^N, \theta^T, \varpi \in (0,1)$ and $\vartheta_1, \vartheta_2, \chi > 0$.

**Table 2**

| $R^*$ | $\beta$ | $\bar{R}$ | $\gamma$ | $\mu$ | $\eta$ | $\phi$ | $\alpha$ | a | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1.06 | 0.943 | 1.16 | 17.5 | 6 | 0.85 | 0.185 | 0.7 | 0.4 | 2 |

| $\varsigma$ | $\delta$ | $\varpi$ | $\vartheta_1$ | $\vartheta_2$ | $\theta^T$ | $\chi$ | $\theta^N$ | $K^T$ | $K^N$ |
|---|---|---|---|---|---|---|---|---|---|
| 4.59 | 5 | 0.6 | 1.4 | 3.5 | 0.5 | 0.7 | 0.64 | 1 | 2 |

The values of the parameters are mainly borrowed from the calibration by Christiano et al. (2004), except for the intratemporal elasticity of substitution (a), the parameter related to distribution services ($\eta$), the parameter that governs the degree of price stickiness ($\gamma$), and the parameter associated with the degree of imperfect competition ($\mu$).[29] We do not pick any particular value for the interest rate response coefficient to currency depreciation ($\rho_\epsilon$), since we will study how this parameter affects the determinacy of equilibrium. We choose values for "a" and $\eta$ that are in line with similar values used in the distribution services literature.[30] Since there are no robust estimates of a New-Keynesian Phillips curve for emerging economies, we choose values for $\gamma$ and $\mu$ that are consistent with the values used in the closed economy literature about nominal price rigidities.[31] Table 2 summarizes the parametrization.

Using this parametrization, we study how the determinacy of equilibrium varies with respect to the response coefficient to depreciation ($\rho_\epsilon$) of the policy (2) and other structural parameters. As an

[29] Note that target nominal depreciation rate, $\bar{\epsilon}$, can be found by evaluating (29) at the steady state. That is, $\bar{\epsilon} = \frac{\bar{R}}{R^*}$. The value of $\bar{R}$ that we take is close to the one in Christiano et al. (2004).

[30] See Burnstein et al. (2005a,b).

[31] See Schmitt-Grohé and Uribe (2004), among others.

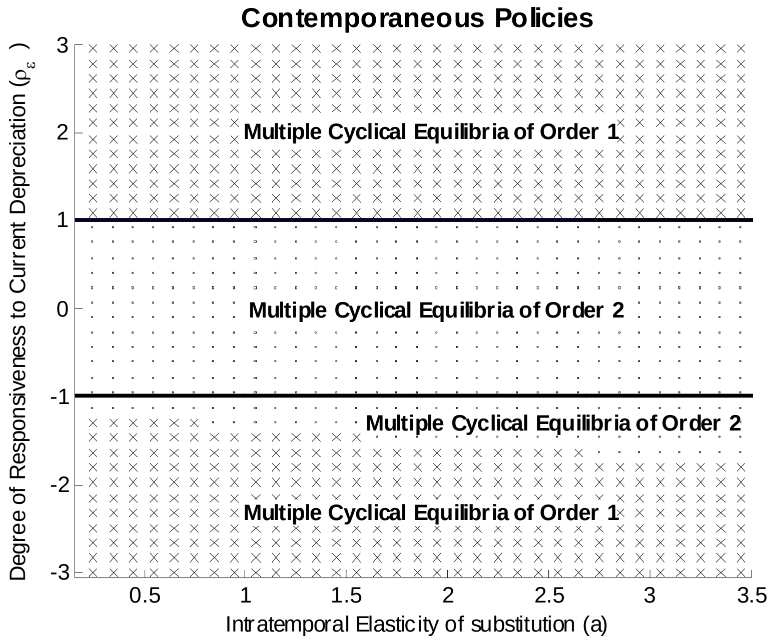


Figure 1: Characterization of the equilibrium for interest rate policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ varying the degree of responsiveness to currency depreciation ($\rho_\epsilon$) and the intratemporal elasticity of substitution (a). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order one. A dot "." represents parameter combinations under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order two.

illustrative case, we focus on the experiment of varying the intratemporal elasticity of substitution (a), while keeping the other structural parameters constant. The results are presented in Figure 1, where a cross "x" denotes combinations of $\rho_\epsilon$ and "a" under which the policy induces multiple cyclical equilibria, whose degree of indeterminacy is of order one; and a dot "." represents parameter combinations under which the policy induces multiple cyclical equilibria, whose degree of indeterminacy is of order two.[32] As can be seen in this figure, a policy that responds to current currency depreciation, by raising ($\rho_\epsilon > 0$ with $\rho_\epsilon \neq 1$) or lowering ($\rho_\epsilon < 0$ with $\rho_\epsilon \neq -1$) the nominal interest rate, can induce multiple equilibria regardless of the intratemporal elasticity of substitution "a."

Experiments with respect to other structural parameters lead to similar results.[33] This suggests that the results in the full set-up are similar to the ones in the simple set-up. But there is an important distinction. Now because of the binding collateral constraint, there exist self-fulfilling *cycles* or, equivalently, multiple *cyclical* equilibria. This is just a consequence of two mechanisms working together. On one hand, from the results in the simple model we have that this policy can induce self-fulfilling "non-cyclical" fluctuations. On the other hand, from Kiyotaki and Moore (1997) we know that

[32] Cyclical equilibria are associated with the existence of non-explosive complex roots of the log-linearized system.

[33] The results are available from the author upon request.

the introduction of a binding collateral constraint can cause credit *cycles.* Hence the combination of the two mechanisms can lead to self-fulfilling *cyclical* equilibria. The following Proposition summarizes these results.

**Proposition 2** *Under a currency crisis, if the government follows an interest rate policy that responds to currency depreciation, such as $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ with $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$, then there exists a continuum of perfect foresight cyclical equilibria (indeterminacy), in which the sequences $\{\hat{c}_t^T$, $\hat{c}_t^N$, $\hat{\zeta}_t$, $\hat{h}_t^T$, $\hat{h}_t^N$, $\hat{I}_t$, $\hat{b}_t^*$, $\hat{m}c_t$, $\hat{e}_t$, $\hat{q}_t^T$, $\hat{q}_t^N$, $\hat{w}_t$, $\hat{\epsilon}_t$, $\hat{\pi}_t^N$, $\hat{R}_t\}_{t=0}^{\infty}$ converge to the steady state.*

These results are not specific to the particular policy that we consider. In the Appendix we study forward-looking policies, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$, and backward-looking policies, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$. We find that forward-looking policies always induce multiple cyclical equilibria, as long as the response coefficient to future depreciation satisfies either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. Except for the presence of cycles these results coincide, to some extent, with the ones from the simple model. On the other hand, for backward-looking policies the coefficient of response to past depreciation plays an important role in the characterization of the equilibrium. That is, timid rules satisfying $|\rho_\epsilon| < 1$ always induce multiple equilibria, while aggressive rules with $|\rho_\epsilon| > 1$ can guarantee a unique equilibrium. Nevertheless, being aggressive with respect to past depreciation ($|\rho_\epsilon| > 1$) is a necessary but not a sufficient condition to guarantee a unique equilibrium. Therefore, backward-looking policies can *still* destabilize the economy by inducing self-fulfilling cyclical fluctuations.

These results, as well as the results for a contemporaneous policy $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$, are summarized in Table 1 in the columns labeled as “The Full Model.” Note that our results do not support any of the views of the debate about the right interest rate policy in the aftermath of the Asian crisis. In fact, we unveil a peril that may be present in both policy recommendations. What is crucial in our analysis is not whether the government increases or decreases the interest rate; instead, it is the feedback response of the nominal interest rate to current, future, or past currency depreciation, that most of the previous studies have ignored.

It is possible to argue that, in the aftermath of a crisis, governments may also adjust the nominal interest rate in response to inflation. This raises the question of whether a policy that reacts to both the CPI-inflation and the depreciation rate can still induce aggregate instability in the economy. The answer to this question is affirmative making our previous results stronger. To some extent, the response to currency depreciation is crucial to explain the possibility of multiple equilibria. To see this, we can study the following interest rate policy

$$\hat{R}_t = \rho_\pi \hat{\pi}_{t-1}^{cpi} + \rho_\epsilon \hat{\epsilon}_t \qquad \text{with} \qquad \rho_\pi > 1,\ \rho_\epsilon \neq 0 \text{ and either } |\rho_\epsilon| > 1 \text{ or } |\rho_\epsilon| < 1$$

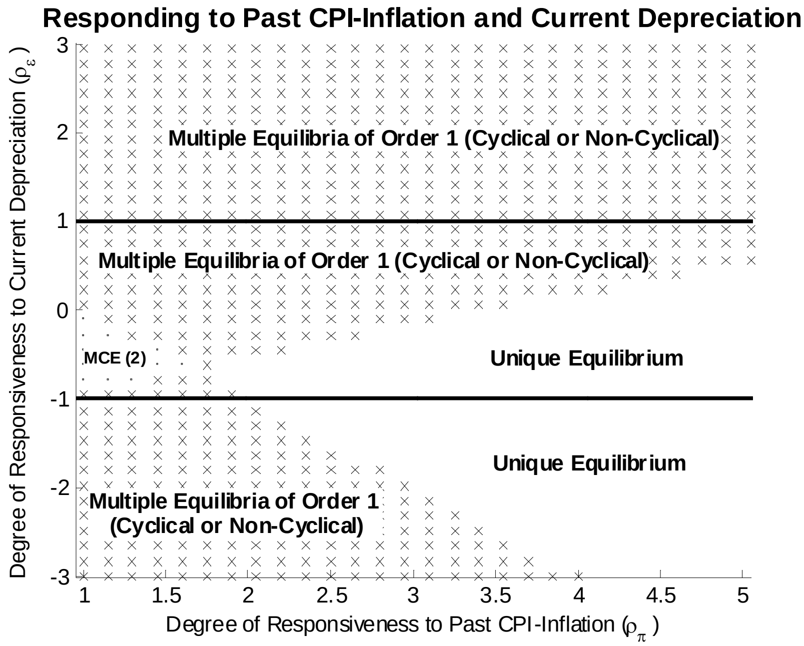


Figure 2: Characterization of the equilibrium for the rule $\hat{R}_t = \rho_\pi \hat{\pi}^{cpi}_{t-1} + \rho_\epsilon \hat{\epsilon}_t$ varying the degrees of responsiveness to past CPI-inflation ($\rho_\pi$) and to current currency depreciation ($\rho_\epsilon$). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations associated with multiple equilibria whose degree of indeterminacy is of order one. These equilibria can be cyclical or non-cyclical. A dot "." represents parameter combinations associated with multiple equilibria whose degree of indeterminacy is of order two. These equilibria are cyclical and we name these combinations as "MCE(2)". The white regions represent parameter combinations under which there exists a unique equilibrium.

that reacts, positively or negatively, to current currency depreciation and, aggressively and positively, to past CPI-inflation.[34]

The results of the analysis are shown in Figure 2. We study how variations of the degrees of responsiveness to past CPI-inflation and current depreciation, $\rho_\pi$ and $\rho_\epsilon$, affect the characterization of the equilibrium. We observe that policies that respond to both past CPI-inflation and currency depreciation can *still* induce multiple equilibria. In fact, using the parametrization in Table 2 and under the celebrated "Taylor coefficient", $\rho_\pi = 1.5$, any policy that responds to currency depreciation will lead to real indeterminacy. Note also that if the policy is positively aggressive with respect to current depreciation, $\rho_\epsilon > 1$, then excessively aggressive policies with respect to inflation, say any $\rho_\pi \in [3, 5]$, will continue to induce self-fulfilling equilibria. Nevertheless, this analysis also highlights the importance of reacting aggressively to past CPI-inflation in order to avoid destabilizing the economy.

There is still a relevant question that has not been answered. If the governments of the Asian economies followed these interest rate policies, then would it be possible to support the stylized facts

[34] The literature of interest rate rules claims that an aggressive backward-looking rule with respect to inflation is more prone to guarantee a unique equilibrium than forward-looking and contemporaneous policies. See Woodford (2003).

of the aftermath of the crisis, the "Sudden Stop" phenomenon, as one of these self-fulfilling equilibria? An affirmative answer to this question would make our previous results more credible. This defines the goal of the next section.

# 4 Constructing a Self-fulfilling Cyclical Equilibrium

In this section, we use the full model in tandem with an interest rate policy that responds to currency depreciation, in order to construct a self-fulfilling cyclical equilibrium that replicates some of the stylized facts of the "Sudden Stop" phenomenon. In this equilibrium, the only source of cyclical fluctuations is the self-validation of people's beliefs about currency depreciation.

We assume the government follows an interest rate policy, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$, that responds aggressively, $|\rho_\epsilon| > 1$, to current depreciation, $\hat{\epsilon}_t$.[35] This policy captures the immediate reaction to currency depreciation. Unfortunately, in the empirical literature that emerged after the Asian crisis, there are no *robust* estimates for the parameter $\rho_\epsilon$. For illustrative purposes we set $\rho_\epsilon = 2$, implying that the government increases the interest rate proportionally more than the increase in depreciation.

From Proposition 2 we know that varying $\rho_\epsilon$ will not preclude the possibility of multiple equilibria. This, however, changes the degree of indeterminacy. For $\rho_\epsilon > 1$ the degree of indeterminacy is one.[36] Then we can construct a self-fulfilling cyclical equilibrium in which a "sunspot" affects people's expectations of *only one* variable of the economy, such as currency depreciation.[37] Moreover, as long as $\rho_\epsilon > 1$, increasing or reducing $\rho_\epsilon$ will not change the qualitative results that we will present, and that capture some of the stylized facts of the "Sudden Stops."

We assume the crisis occurs exogenously. That is, the binding collateral constraint is exogenously imposed at time $t = 0$, as in Christiano et al. (2004). In this sense, we are only interested in studying what happens in the economy at and in the aftermath of the crisis. In what follows, therefore, we concentrate exclusively in the equilibrium dynamics of the economy at and after $t = 0$.

Imagine that when the crisis hits the economy at time $t = 0$, people develop expectations, in response to a "sunspot", of a 10% higher domestic currency nominal depreciation. By the determinacy of equilibrium analysis, we know that these expectations will be self-validated. In addition, they induce cycles as described by the impulse response functions presented in Figures 3 and 4. In these figures, all the variables but the multiplier of the collateral constraint are measured as percentage deviations from the steady state. A quick inspection reveals that at time $t = 0$, and for some subsequent periods,

[35] It is also possible to construct self-fulfilling equilibria with the forward-looking and backward-looking policies that, in principle, replicate most of the stylized facts.

[36] That is, in this case the dynamic log-linearized system that describes the economy has complex and non-explosive eigenvalues, and the number of non-predetermined variables exceeds the number of explosive eigenvalues by one.

[37] If the degree of indeterminacy were 2, then we would have an extra degree of freedom. We could assume that a "sunspot" affects the expectations of an additional variable different from the depreciation rate. In this sense, we are being conservative.

the self-fulfilling equilibrium captures the following stylized facts of the "Sudden Stops": a decline in the aggregate demand (consumptions of traded and non-traded goods and aggregate consumption), a collapse in the domestic production (traded output and non-traded output), a collapse in asset prices (prices of traded and non-traded capital), a sharp correction in the price of traded goods relative to non-traded goods, an improvement in the current account deficit, a moderately higher CPI-inflation, a more rapid currency depreciation, and higher nominal interest rates. After some periods, domestic output and aggregate demand increase, the current account deficit deteriorates, and CPI-inflation, currency depreciation and interest rates decrease. Then, the cycles are quickly dampened, and the economy converges to the steady state.

We proceed to explain these results. The self-validated increase in the nominal depreciation rate implies an increase in the nominal exchange rate and, by the policy, leads to an increase in the nominal interest rate at $t = 0$. Since the rule is aggressive, the nominal interest rate rises by more than both the expected non-traded good inflation rate and the expected traded good inflation rate at time $t = 1$.[38] Hence, the real interest rate at $t = 0$, measured in terms of either the expected traded inflation or the non-traded inflation at $t = 1$, goes up. Provided that this induces an intertemporal substitution effect in consumption that more than offsets any intratemporal substitution effect, then consumption of the traded good and consumption of the non-traded good decline at $t = 0$. This can be inferred from (31) and (32). As a result of this, aggregate consumption also decreases implying that the model is able to capture the initial decline in aggregate demand present in the "Sudden Stops."

Since the real value of capital as a collateral is expressed in terms of foreign currency, then the previously mentioned increase of the nominal exchange rate at $t = 0$ reduces this value. Thus, there is an incentive to reduce the demand for the imported input at $t = 0$, as can be deduced from (27) as an equality, and taking into account that $b^*_{t-1}$ is a predetermined variable. At the same time, a higher interest rate in response to depreciation also pushes up the costs of loans to hire labor utilized in the production of traded goods. This creates an incentive for the household-firm unit to cut back labor in the production of the traded good.[39] The simulations show that in fact these incentives are materialized leading to a decrease in traded output at $t = 0$.

On the other hand, the supply of the non-traded good is demand determined. This supply satisfies both consumption of non-traded goods and distribution services for traded goods. Consequently, the previously mentioned decrease in demand of both goods causes a decrease in non-traded output (labor) at $t = 0$. Thus the model is able to capture the decline in output of both non-traded and traded goods. Provided that the decrease in traded output is smaller than the contractions in consumption of the

[38] This is probably the case because there is sluggish price adjustment for the price of the non-traded good that, in turn, affects the price of the traded good through the existence of non-traded distribution services.

[39] This can be deduced from (35) and the production technology of the traded good. Labor costs increase not only because the nominal interest rate increases but also because the collateral constraint tightens. That is, there is an increase in the "effective" nominal interest rate $(1 + \zeta_t)R_t$.

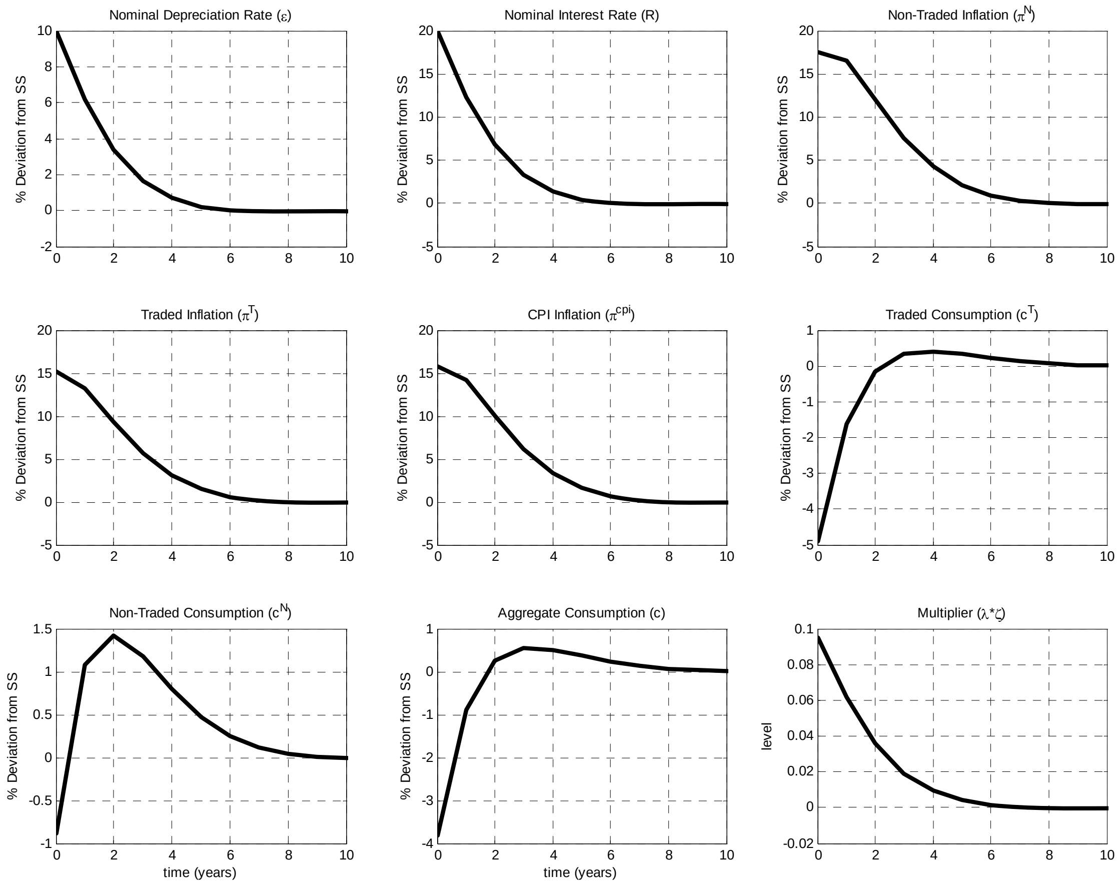


Figure 3: Impulse responses of a self-fulfilling equilibrium, when at time $t = 0$ people expect a higher nominal depreciation rate ($\hat{\epsilon}_0 = 10\%$). This equilibrium replicates some of the "Sudden Stops" stylized facts: a decline of consumption of traded and non-traded goods, a relatively moderate increase in CPI-inflation, a higher depreciation rate, and a higher nominal interest rate. All the variables are measured as percentage deviations from the steady state except for the multiplier.

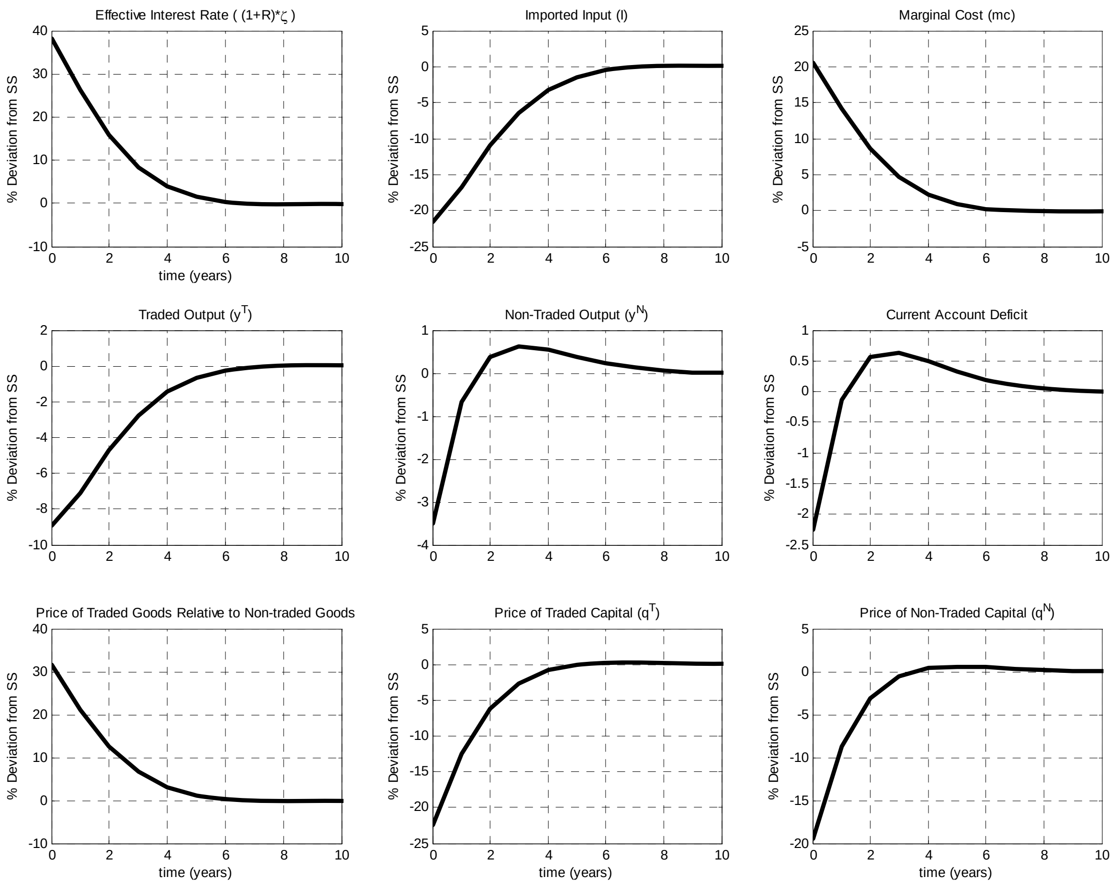


Figure 4: Impulse responses of a self-fulfilling equilibrium, when at time $t = 0$ people expect a higher nominal depreciation rate ($\hat{\epsilon}_0 = 10\%$). This equilibrium replicates some of the "Sudden Stops" stylized facts: a collapse in the domestic production (of the traded and non-traded good), a collapse in asset prices (prices of capital), a sharp correction in the price of traded goods relative to non-traded goods, and an improvement in the current account deficit. All the variables are measured as percentage deviations from the steady state.

traded good and the imports of the intermediate input, then the current account improves, as can be seen in (34).

As the collateral constraint tightens and the nominal interest rate rises, the "effective" nominal interest rate $(1+\zeta_t)R_t$ increases. As a result of this, marginal costs of producing the non-traded good go up, forcing the household-firm unit to raise the price of this good. This, in turn, leads to a higher non-traded goods inflation rate, as can be deduced from (33) and (36). Higher currency depreciation rates and non-traded inflation rates induce higher CPI-inflation rates.

In addition, the presence of price-stickiness and distribution services implies that, as a consequence of large depreciations, there is a sharp correction in the price of the traded good relative to the price of non-traded good at the consumer level $\left(\frac{P_t^T}{P_t^N}\right)$. This captures another empirical regularity of the aftermath of a crisis.

The real value of a unit of capital for traded output (respectively, non-traded output), in terms of foreign currency, is determined by the net present value of the flows of the marginal product of capital in the production of the traded good (respectively, non-traded good). This can be seen by iterating forward equations (40). Provided that capital is constant in the analysis, the decreases in labor and in the intermediate input cause a decline in the marginal product of capital, reducing the real value of capital. A similar mechanism reduces the real value of capital for non-traded output. Therefore, asset prices fall capturing another stylized fact of the "Sudden Stops."

# 5 Concluding Remarks

In this paper we study interest rate policies that, in the aftermath of a currency crisis, call for adjusting the nominal interest rate in response to domestic currency depreciation. We show that these policies can induce macroeconomic instability by generating self-fulfilling cycles. We find that, if a government raises the interest rate proportionally more than an increase in currency depreciation, then it induces self-fulfilling cycles that, driven by people's expectations about depreciation, replicate several of the salient stylized facts of the "Sudden Stop" phenomenon. In this sense, interest rate policies that respond to depreciation may have contributed to generating the dynamic cycles experienced by some economies in the aftermath of a currency crisis.

Our results have the following implications. Previous works have emphasized that these interest rate policies can cause fiscal and output costs. We suggest that these policies can be also costly to the extent that they can induce macroeconomic instability in the economy by opening the possibility of "sunspot" equilibria. These equilibria, that are not driven by fundamentals, can be associated with a large degree of volatility for some macroeconomic aggregates such as consumption. Provided that agents are risk averse, then volatile consumption can be costly in terms of welfare.

Our results also provide a possible explanation of why the empirical literature has not been able to disentangle the relationship between interest rates and the nominal exchange (depreciation) rate in the aftermath of a crisis. This literature has tried to control for the variables that influence the nominal exchange rate. But our results suggest that there can be potential influences that may depend on "sunspots", which in turn can induce self-fulfilling cycles in the nominal exchange rate (or the nominal depreciation rate) as well as in other variables. Clearly, these influences do not depend on fundamentals, and their effect is something that the empirical literature should take into account.

An issue that we have not addressed is whether the "sunspot" equilibria are learnable by the agents of the economy. It was implicitly assumed that agents could learn and coordinate their actions on any particular equilibrium. In future research, we plan to relax this assumption and use the Expectational Stability concept developed in Evans and Honkapojha (2001) to pursue a learnability analysis.

# A Appendix

This Appendix has two parts. The first part includes material that supports the analysis for the simple model of Section 2. The second part includes the simulations of the determinacy of equilibrium analysis for forward-looking and backward-looking rules for the full model of Section 3.

## A.1 The Simple Model

### A.1.1 The First Order Conditions of the Household-Firm Unit Problem in the Simple Model

The representative household-firm unit chooses the set of sequences $\{c_t^T,\, c_t^N,\, h_t^T,\, h_t^N,\, \check{h}_t^T,\, \tilde{h}_t^N,\, \tilde{P}_t^N,$ $b_t^*,\, b_t,\, m_t\}_{t=0}^{\infty}$ in order to maximize (3) subject to (4), (5), (6) and (7), given the initial condition $n_{-1}$ and the set of sequences $\{R_t^*,\, R_t,\, \epsilon_t,\, e_t,\, P_t^N,\, w_t^T,\, w_t^N,\, \tau_t,\, C_t^N\}$. The first order conditions correspond to (5) and (7) with equality and

$$U_T(c_t^T) = \lambda_t, \tag{41}$$

$$\frac{U_T(c_t^T)}{V_N(c_t^N)} = e_t, \tag{42}$$

$$-\frac{H_T(h_t^T)}{U_T(c_t^T)} = w_t^T, \tag{43}$$

$$-\frac{L_N(h_t^N)}{V_N(c_t^N)} = w_t^N e_t, \tag{44}$$

$$1 = \frac{w_t^T}{F_T\left(\check{h}_t^T\right)}, \tag{45}$$

$$mc_t = \frac{w_t^N e_t}{G_N\left(\tilde{h}_t^N\right)}, \tag{46}$$

$$\begin{aligned} 0 &= \frac{\lambda_t C_t^N}{e_t} d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) - \gamma \frac{\lambda_t}{e_t}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}^N\right)\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} + \left(\frac{\tilde{P}_t^N}{P_t^N} - mc_t\right)\frac{\lambda_t C_t^N}{e_t} d'\left(\frac{\tilde{P}_t^N}{P_t^N}\right) \\ &\quad + \beta\gamma\frac{\lambda_{t+1}}{e_{t+1}}\left(\frac{\tilde{P}_{t+1}^N}{\tilde{P}_t^N} - \bar{\pi}^N\right)\frac{\tilde{P}_{t+1}^N}{\tilde{P}_t^N}, \end{aligned} \tag{47}$$

$$J_m(m_t) = U_T(c_t^T)\left(\frac{R_t - 1}{R_t}\right), \tag{48}$$

$$\lambda_t = \beta R_t^* \lambda_{t+1}, \tag{49}$$

$$\lambda_t = \frac{\beta R_t}{\epsilon_{t+1}}\lambda_{t+1}, \tag{50}$$

where $\frac{mc_t \lambda_t}{e_t}$ and $\lambda_t$ correspond to the Lagrange multipliers of (4) and (5) respectively.

We will focus on a symmetric equilibrium in which all the monopolistic producers of sticky-price non-traded goods pick the same price. Hence $\tilde{P}_t^N = P_t^N$. Since all the monopolists face the same wage rate, $W_t^N$, and the same production function, $G\left(h_t^N\right)$, then they will demand the same amount of labor $\tilde{h}_t^N = h_t^N$. In equilibrium the money market, the domestic bond market, the labor markets, the non-traded goods market, and the traded good market clear. In particular, for the non-traded good market and traded good market we have

$$G\left(h_t^N\right) = c_t^N + \frac{\gamma}{2}\left(\pi_t^N - \bar{\pi}^N\right)^2 \tag{51}$$

and

$$b_t^* = R_{t-1}^* b_{t-1}^* + c_t^T - F\left(h_t^T\right). \tag{52}$$

Combining (49) and (50) yields the uncovered interest parity condition (9). From (43) and (45) we can derive equation (10). Using conditions (41) and (49) we obtain the Euler equation for consumption of the traded good that corresponds to (11). Utilizing (1), (41), (42), and (50) we derive the Euler equation (12) for consumption of the non-traded good. And finally using the notion of a symmetric equilibrium, conditions (4), (42), (44), (46), (47), the labor market equilibrium conditions, and the definitions $\pi_t^N = P_t^N / P_{t-1}^N$, $d(1) = 1$ and $d'(1) = -\mu$, we can derive the augmented Phillips curve described by equation (13).

### A.1.2 Characterization of the Steady State in the Simple Model

We use $\beta R^* = 1$, (1), (8)-(15), and the condition that at the steady state $x_t = \bar{x}$ for all the variables to derive

$$\bar{\pi}^N = \bar{\epsilon}, \qquad \beta\bar{R} = \bar{\epsilon}, \qquad 1 = \beta\bar{R}^*, \qquad \bar{R}^* = R^*,$$

$$\left(\frac{\mu-1}{\mu}\right)V_N(G(\bar{h}^N))=-\frac{L_N(\bar{h}^N)}{G_N\left(\bar{h}^N\right)},$$

$$U_T\left(\left(1-1/\beta\right)\bar{b}^*+F\left(\bar{h}^T\right)\right)=-\frac{H_T(\bar{h}^T)}{F_T\left(\bar{h}^T\right)},$$

$$\bar{c}^N=G\left(\bar{h}^N\right)\qquad \bar{c}^T=\left(1-1/\beta\right)\bar{b}^*+F\left(\bar{h}^T\right).$$

Then it is simple to prove that under some assumptions that include Assumptions 1, 2, and 3, and given $\mu>1$, $\beta\in(0,1)$ and $\bar{\epsilon}>1$, there exists a steady state $\{\bar{c}^N,\ \bar{c}^T,\ \bar{h}^T,\ \bar{h}^N,\ \bar{\pi}^N,\ \ \bar{R},\ \bar{R}^*\}$ for the economy that provided a particular $\bar{b}^*$ satisfies these equations with $\bar{c}^T,\bar{c}^N,\bar{h}^T,\bar{h}^N>0$ and $\bar{\pi}^N,\bar{R},\bar{R}^*>1$. In particular to guarantee that there exist $\bar{c}^T,\bar{c}^N,\bar{h}^T,\bar{h}^N>0$ we need some Inada-type assumptions such as $U_T(0)=V_N(0)=\infty$, $U_T(\infty)=V_N(\infty)=0$, $F_T(0)=G_N(0)=\infty$, and $F_T(\infty)=G_N(\infty)=0$.[40]

### A.1.3 Forward-Looking and Backward-Looking Policies in the Simple Model

In order to pursue the determinacy of equilibrium analysis for a forward-looking policy, $\hat{R}_t=\rho_\epsilon\hat{\epsilon}_{t+1}$, we use this and equations (17)-(21) to obtain

$$\begin{pmatrix}\hat{b}_t^*\\ \hat{c}_{t+1}^T\\ \hat{\pi}_{t+1}^N\\ \hat{c}_{t+1}^N\end{pmatrix}=\underbrace{\begin{pmatrix}\frac{1+\psi}{\beta} & \kappa & 0 & 0\\ \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0\\ 0 & 0 & \frac{1}{\beta} & -\varphi\\ \frac{\rho_\epsilon\psi(1+\psi)\xi^N}{\beta(\rho_\epsilon-1)} & \frac{\rho_\epsilon\psi\kappa\xi^N}{(\rho_\epsilon-1)} & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right)\end{pmatrix}}_{J^f}\begin{pmatrix}\hat{b}_{t-1}^*\\ \hat{c}_t^T\\ \hat{\pi}_t^N\\ \hat{c}_t^N\end{pmatrix}. \tag{53}$$

Then the determinacy of equilibrium analysis delivers the results stated in the following proposition.

**Proposition 3** *If the government follows a forward-looking interest rate policy such as $\hat{R}_t=\rho_\epsilon\hat{\epsilon}_{t+1}$ with $\rho_\epsilon\neq 0$ and either $|\rho_\epsilon|>1$ or $|\rho_\epsilon|<1$, then there exists a continuum of perfect foresight equilibria (indeterminacy) in which the sequences $\{\hat{b}_t^*,\ \hat{c}_t^T,\ \hat{\pi}_t^N,\ \hat{c}_t^N\}_{t=0}^\infty$ converge asymptotically to the steady state. The degree of indeterminacy is of order 1.*[41]

**Proof.** The eigenvalues of the matrix $J^f$ in (53) correspond to the roots of the characteristic equation $\mathcal{P}^f(v)=\left|J^f-vI\right|=0$. By Lemma 4 we know that $\mathcal{P}^f(v)=0$ has real roots satisfying $|v_1|<1$, $|v_2|>1$, $|v_3|<1$, and $|v_4|>1$. Therefore $\mathcal{P}^f(v)=0$ has only two explosive roots, which means that the matrix $J^f$ in (53) has two explosive eigenvalues. Given that there are three non-predetermined variables namely $\hat{c}_t^T$, $\hat{\pi}_t^N$ and $\hat{c}_t^N$, then the number of non-predetermined variables is greater than the number of explosive roots. Applying the results of Blanchard and Kahn (1980), it follows that there exists an infinite number of perfect foresight equilibria converging to the steady

[40] Details are available from the author upon request.

[41] The degree of indeterminacy is defined as the difference between the number of non-predetermined variables and the dimension of the unstable subspace of the log-linearized system.

state. In addition, the difference between the number of non-predetermined variables and explosive roots implies that the degree of indeterminacy is of order 1. ■

To derive the results for a backward-looking policy we use the log-linearized version of this policy, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ together with equations (20)-(17) to obtain the log-linearized system

$$\begin{pmatrix} \hat{R}_{t+1} \\ \hat{\epsilon}_{t+1} \\ \hat{b}^*_t \\ \hat{c}^T_{t+1} \\ \hat{\pi}^N_{t+1} \\ \hat{c}^N_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & \rho_\epsilon & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{\psi(1+\psi)}{\beta} & -\psi\kappa & 0 & 0 \\ 0 & 0 & \frac{1+\psi}{\beta} & \kappa & 0 & 0 \\ 0 & 0 & \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\beta} & -\varphi \\ \xi^N & 0 & 0 & 0 & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right) \end{pmatrix}}_{J^b} \begin{pmatrix} \hat{R}_t \\ \hat{\epsilon}_t \\ \hat{b}^*_{t-1} \\ \hat{c}^T_t \\ \hat{\pi}^N_t \\ \hat{c}^N_t \end{pmatrix}. \tag{54}$$

Then the determinacy of equilibrium analysis delivers the results stated in the following proposition.

**Proposition 4** *Assume the government follows a backward-looking policy such as $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ with $\rho_\epsilon \neq 0$.*

> ***a)*** *if $|\rho_\epsilon| > 1$ then there exists a unique perfect foresight equilibria in which the sequences $\{\hat{R}_t, \hat{\epsilon}_t, \hat{b}^*_t, \hat{c}^T_t, \hat{\pi}^N_t, \hat{c}^N_t\}_{t=0}^\infty$ converge to the steady state.*
>
> ***b)*** *if $|\rho_\epsilon| < 1$ then there exists a continuum of perfect foresight equilibria (indeterminacy) in which the sequences $\{\hat{R}_t, \hat{\epsilon}_t, \hat{b}^*_t, \hat{c}^T_t, \hat{\pi}^N_t, \hat{c}^N_t\}_{t=0}^\infty$ converge asymptotically to the steady state. In addition, the degree of indeterminacy is of order 2.*

**Proof.** The eigenvalues of the matrix $J^b$ in (54) correspond to the roots of the characteristic equation $\mathcal{P}^b(v) = \left|J^b - vI\right| = 0$. Using the definition of $J^b$ in (54), this equation can be written as

$$\mathcal{P}^b(v) = \left(v^2 - \rho_\epsilon\right)\mathcal{P}^f(v) = 0, \tag{55}$$

where $\mathcal{P}^f(v)$ is defined in Lemma 4. Using this Lemma we know that $\mathcal{P}^f(v) = 0$ has real roots satisfying $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$, and $|v_4| > 1$. The fifth and the sixth roots of $\mathcal{P}^b(v) = 0$ are $v_5 = \sqrt{\rho_\epsilon}$ and $v_6 = -\sqrt{\rho_\epsilon}$. Clearly, if $|\rho_\epsilon| > 1$ then $|v_5| > 1$ and $|v_6| > 1$, whereas if $|\rho_\epsilon| < 1$ then $|v_5| < 1$ and $|v_6| < 1$. Using this, the characterization of the roots of $\mathcal{P}^f(v) = 0$ and (55) we can conclude the following. If $|\rho_\epsilon| > 1$ then $\mathcal{P}^b(v) = 0$ has four explosive roots, namely $|v_2| > 1$, $|v_4| > 1$, $|v_5| > 1$, and $|v_6| > 1$. While if $|\rho_\epsilon| < 1$ then $\mathcal{P}^b(v) = 0$ has two explosive roots, namely $|v_2| > 1$ and $|v_4| > 1$. Therefore if $|\rho_\epsilon| > 1$ the number of explosive roots is equal to the number of non-predetermined variables ($\hat{\epsilon}_t$, $\hat{c}^T_t$, $\hat{\pi}^N_t$ and $\hat{c}^N_t$). Hence, applying the results of Blanchard and Kahn (1980), it follows that there exists a unique equilibrium. This completes the proof for a).

On the contrary, by the previous analysis, if $|\rho_\epsilon| < 1$ then the number of explosive roots, 2, is less than the number of non-predetermined variables ($\hat{\epsilon}_t$, $\hat{c}^T_t$, $\hat{\pi}^N_t$ and $\hat{c}^N_t$), 4. Applying the results of Blanchard and Kahn (1980), it follows that there exists an infinite number of perfect foresight equilibria converging to the steady state. The degree of indeterminacy is the difference between the number of non-predetermined variables and the number of explosive roots. This completes the proof for b). ■

### A.1.4 Lemmata and Proofs for the Results in the Simple Model

**Lemma 1** *Define $\mathcal{P}\ (v)= v^2 + Tv + D$ and consider the characteristic equation $\mathcal{P}\ (v)= 0$. If either a) $P(1) < 0$ or b) $P(-1) < 0$ then the roots $v$ are real.*

**Proof.** First recall from Azariadis (1993) that a sufficient condition to have real roots is that $T^2 - 4D \geq 0$. To prove a) note that $\mathcal{P}(1) < 0$ means that $\mathcal{P}(1) = 1 - T + D < 0$. But this implies that $4T - 4 > 4D$, which in turn leads to $T^2 - 4D > T^2 - 4T + 4 = (T-2)^2 \geq 0$. Hence the roots are real. Next we prove b). $\mathcal{P}(-1) < 0$ means that $\mathcal{P}(1) = 1 + T + D < 0$. But this implies that $-4T - 4 > 4D$, that in turn leads to $T^2 - 4D > T^2 + 4T + 4 = (T+2)^2 \geq 0$. Hence the roots are real. ■

**Lemma 2** *Define $\mathcal{P}_{bc^T}(v) = v^2 - \left(1 + \frac{1+\psi}{\beta} + \psi\kappa\xi^T\right) v + \frac{1+\psi}{\beta}$. The roots of the characteristic equation $\mathcal{P}_{bc^T}(v) = 0$ are real and satisfy $|v_1| < 1$ and $|v_2| > 1$.*

**Proof.** First using the definition of $\mathcal{P}_{bc^T}(v)$, Assumptions 1, 2 and 3 and definitions in (22) we obtain that $\mathcal{P}_{bc^T}(1) = -\psi\kappa\xi^T < 0$ and $\mathcal{P}_{bc^T}(-1) = 2\left(1 + \frac{1+\psi}{\beta}\right) + \psi\kappa\xi^T > 0$. Since $\mathcal{P}_{bc^T}(1) < 0$, then by Lemma 1 we know that the two roots are real. In addition, from Azariadis (1993), having $\mathcal{P}_{bc^T}(1) < 0$ and $\mathcal{P}_{bc^T}(-1) > 0$ imply that one root lies inside of the unit circle and the other one lies outside the unit circle. Without loss of generality we can conclude that $|v_1| < 1$ and $|v_2| > 1$. ■

**Lemma 3** *Define $\mathcal{P}_{\pi^N c^N}(v) = v^2 - \left(1 + \frac{1}{\beta} + \varphi\xi^N\right) v + \frac{1}{\beta}$. The roots of the characteristic equation $\mathcal{P}_{\pi^N c^N}(v) = 0$ are real and satisfy $|v_3| < 1$ and $|v_4| > 1$.*

**Proof.** First using the definition of $\mathcal{P}_{\pi^N c^N}(v)$, Assumptions 1, 2 and 3 and definitions in (22) we obtain that: $\mathcal{P}_{\pi^N c^N}(-1) = -\varphi\xi^N < 0$ and $\mathcal{P}_{\pi^N c^N}(1) = 2\left(1 + \frac{1}{\beta}\right) + \varphi\xi^N > 0$. Since $\mathcal{P}_{\pi^N c^N}(-1) < 0$ then by Lemma 1 we know that the two roots of $\mathcal{P}_{\pi^N c^N}(v) = 0$ are real. In addition, from Azariadis (1993), having $\mathcal{P}_{\pi^N c^N}(-1) < 0$ and $\mathcal{P}_{\pi^N c^N}(1) > 0$ imply that one root lies inside the unit circle and the other one lies outside the unit circle. Without loss of generality we can conclude that $|v_3| < 1$ and $|v_4| > 1$. ■

**Lemma 4** *The roots of the characteristic equation*

$$\mathcal{P}^f(v) = \mathcal{P}_{bc^T}(v)\mathcal{P}_{\pi^N c^N}(v) = 0, \tag{56}$$

*where*

$$\mathcal{P}_{bc^T}(v) = v^2 - \left(1 + \frac{1+\psi}{\beta} + \psi\kappa\xi^T\right) v + \frac{1+\psi}{\beta}$$

$$\mathcal{P}_{\pi^N c^N}(v) = v^2 - \left(1 + \frac{1}{\beta} + \varphi\xi^N\right) v + \frac{1}{\beta}$$

*are real and satisfy* $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$ *and* $|v_4| > 1$.

**Proof.** By Lemma 2 we know that $\mathcal{P}_{bc^T}(v) = 0$ has two real roots satisfying $|v_1| < 1$ and $|v_2| > 1$. On the other hand, by Lemma 3, we know that $\mathcal{P}_{\pi^N c^N}(v) = 0$ has two real roots satisfying $|v_3| < 1$ and $|v_4| > 1$. Using these and (56), the result of the Lemma follows. ■

## A.2 The Full Model

### A.2.1 Forward-Looking and Backward-Looking Policies in the Full Model

We characterize the equilibrium for forward-looking policies ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$) and backward looking policies ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$), using the parametrization of Table 2. As an illustrative case, we focus on the experiment of varying the degree of responsiveness to future (past) currency depreciation ($\rho_\epsilon$) and the intratemporal elasticity of substitution (a), keeping the rest constant. The results are presented in Figure 5. The top panel shows the results for a forward-looking policy. The bottom panel presents the results for a backward-looking policy.

From the top-panel, we can infer that forward-looking policies always induce multiple cyclical equilibria, as long as $\rho_\epsilon \neq 0$, and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. On the other hand, for backward-looking rules, the coefficient of response to past depreciation, $\rho_\epsilon$, plays an important role. In particular, timid rules with respect to past depreciation ($|\rho_\epsilon| < 1$) always induce multiple equilibria, regardless of the intratemporal elasticity of substitution (a). In contrast, aggressive rules ($|\rho_\epsilon| > 1$) can guarantee a unique equilibrium. Nevertheless, being aggressive with respect to past depreciation ($|\rho_\epsilon| > 1$) is not a sufficient condition to guarantee a unique equilibrium. It is only a necessary condition.

Varying other structural parameters different from the intratemporal elasticity of substitution (a), in tandem with $\rho_\epsilon$, lead to similar results.[42] The following proposition summarizes these results.

**Proposition 5** *Under a currency crisis,*

1. *If the government follows a forward-looking policy such as* $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ *with* $\rho_\epsilon \neq 0$ *and either* $|\rho_\epsilon| > 1$ *or* $|\rho_\epsilon| < 1$, *then there exists a continuum of perfect foresight "cyclical" equilibria (indeterminacy), in which the sequences* $\{\hat{c}_t^T,\ \hat{c}_t^N,\ \hat{\zeta}_t,\ \hat{h}_t^T,\ \hat{h}_t^N,\ \hat{I}_t,\ \hat{b}_t^*,\ \hat{mc}_t,\ \hat{e}_t,\ \hat{q}_t^T,\ \hat{q}_t^N,\ \hat{w}_t,\ \hat{\epsilon}_t,\ \hat{\pi}_t^N,\ \hat{R}_t\}_{t=0}^{\infty}$ *converge to the steady state.*

2. *If the government follows a backward-looking policy such as* $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ *with* $\rho_\epsilon \lesseqgtr 0$, *then*

   ***a)*** $|\rho_\epsilon| < 1$ *is a sufficient condition for the existence of a continuum of perfect foresight equilibria, possibly "cyclical", in which the sequences* $\{\hat{c}_t^T,\ \hat{c}_t^N,\ \hat{\zeta}_t,\ \hat{h}_t^T,\ \hat{h}_t^N,\ \hat{I}_t,\ \hat{b}_t^*,\ \hat{mc}_t,\ \hat{e}_t,\ \hat{q}_t^T,\ \hat{q}_t^N,\ \hat{w}_t,\ \hat{\epsilon}_t,\ \hat{\pi}_t^N,\ \hat{R}_t\}_{t=0}^{\infty}$ *converge to the steady state.*

[42] The results are available from the author upon request.

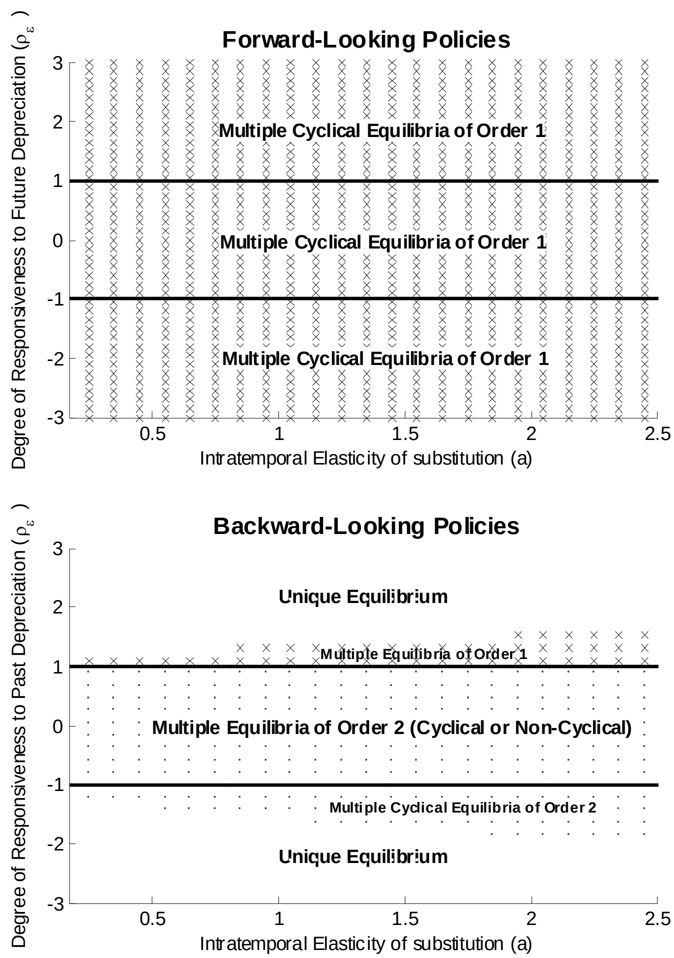


Figure 5: Characterization of the equilibrium for forward-looking (top-panel) and backward-looking (bottom-panel) policies varying the degree of responsiveness to currency depreciation ($\rho_\epsilon$) and the intratemporal elasticity of substitution (a). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations under which the policy induces multiple equilibria whose degree of indeterminacy is of order one. A dot "." represents parameter combinations under which the policy induces multiple equilibria whose degree of indeterminacy is of order two. The white regions represent parameter combinations under which there exists a unique equilibrium.

***b)*** $|\rho_\epsilon| > 1$ *is a necessary but not a sufficient condition for the existence of a unique perfect foresight equilibrium, where the sequences* $\{\hat{c}_t^T,\ \hat{c}_t^N,\ \hat{\zeta}_t,\ \hat{h}_t^T,\ \hat{h}_t^N,\ \hat{I}_t,\ \hat{b}_t^*,\ \hat{m}c_t,\ \hat{e}_t,\ \hat{q}_t^T,\ \hat{q}_t^N,\ \hat{w}_t,\ \hat{\epsilon}_t,\ \hat{\pi}_t^N,\ \hat{R}_t\}_{t=0}^{\infty}$ *converge to the steady state.*

# References


[1] Aghion, P., P Bacchetta and A. Banerjee (2000), "A Simple Model of Monetary Policy and Currency Crises," *European Economic Review,* 44:728-738.

[2] Azariadis, C. (1993), *Intertemporal Macroeconomics* (Blackwell, Cambridge, Massachussetts).

[3] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2001), "Monetary Policy Rules and Multiple Equilibria", *American Economic Review* 91(1):167-184.

[4] Blanchard, O. and C. Kahn (1980), "The Solution of Linear Difference Models Under Rational Expectations", *Econometrica* 48(5):1305-1312.

[5] Braggion, F., Christiano L. and J. Roldos (2005), "The Optimal Monetary Response to a Financial Crisis," Northwestern Univesrity, Manuscript.

[6] Burnstein, A., Neves J. and S. Rebelo (2003), "Distribution Costs and Real Exchange Rate Dynamics," *Journal of Monetary Economics*, 50:1189–1214.

[7] Burnstein, A., Eichenbaum M. and S. Rebelo (2005a), "Large Devaluations and the Real Exchange Rate," forthcoming *Journal of Political Economy.*

[8] Burnstein, A., Eichenbaum M. and S. Rebelo (2005b), "Modeling Exchange Rate Passthrough After Large Devaluations," forthcoming *Journal of Monetary Economics.*

[9] Caballero, R. and A. Krishnamurthy (2001), "International and Domestic Collateral Constraints in a Model of Emerging Market Crises," *Journal of Monetary Economics, 48:513-548.*

[10] Calvo, G. (1998), "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops," *Journal of Applied Economics,* 1:35-54.

[11] Cass, D. and K. Shell (1980), "Do Sunspots Matter?," *Journal of Political Economy*, 91:103–227.

[12] Cho, D. and K. West (2003), "Interest Rates and Exchange Rates in the Korean, Phillipine and Thai Exchange Rate Crises," in: M. Dooley abd J. Frankel eds., *Managing Currency Crises in Emerging Markets,* University of Chicago Press, 11-30.

[13] Christiano L., C. Gust and J. Roldos (2004), "Monetary Policy in a Financial Crisis," *Journal of Economic Theory,* 119:64-103.

[14] Corsetti, G., L. Dedola and S. Leduc (2005), "DSGE Models of High Exchange-Rate Volatility and Low Pass-Through," Manuscript, Federal Reserve Board.

[15] Evans, G, and S. Honkapohja (2001), *Learning and Expectations*, Princeton University Press, New Jersey.

[16] Fischer, S. (1998), "In Defense of the IMF: Specialized Tools for a Specialized Task", *Foreign Affairs* 77(2).

[17] Flood, R. and O. Jeanne (2000), "An Interest Rate Defense of a Fixed Exchange Rate", Working Paper No. 00/159, International Monetary Fund.

[18] Furman, J. and J. Stiglitz (1998), "Economic Crises: Evidence and Insight from East Asia", *Brookings Papers on Economic Activity* 2:1 -114.

[19] Goldfajn, I. and T. Baig (1998), "Monetary Policy in the Aftermath of Currency Crises: The Case of Asia", Working Paper No. 98/170, International Monetary Fund.

[20] Gould, D. and S. Kamin (2000), "The Impact of Monetary Policy on Exchange Rates During Financial Crises", IFDP No.699, Board of Governors of the Federal Reserve System.

[21] Guckenheimer J. and P. Holmes (1983), *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Field,* Springer-Verlag, New York.

[22] Kiyotaki, N. and J. Moore (1997), "Credit Cycles," *The Journal of Political Economy,* 105(2):211-348.

[23] Krugman, P. (1999) "Balance Sheets, the Transfer Problem and Financial Crises", in: P. Isard, A. Razin and A. Rose, eds., *International Finance and Financial Crises* (Kluwer Academic Publishers).

[24] Lahiri, A. and C. Vegh (2002), "Output Costs, BOP Crises and Optimal Interest Rate Policy", Manuscript, UCLA.

[25] Lahiri, A. and C. Vegh (2003), "Delaying the Inevitable: Interest Rate Defense and Balance of Payment Crises", *Journal of Political Economy,* 111(2), 404-424.

[26] Mendoza, E. and K. Smith (2002), "Margin Calls, Trading Costs, and Asset Prices in Emerging Markets: The Financial Mechanics of The 'Sudden Stop' Phenomenon," NBER Working Paper No. 9286.

[27] Montiel, P. (2003), "Tight Money in a Post-Crisis Defense of the Exchange Rate: What Have We Learned?," *The World Bank Research Observer,* 18(1), 1-23.

[28] Paasche, B. (2001), "Credit Constraints and International Financial Crises," *Journal of Monetary Economics,* 28:623-650.

[29] Radelet, S. and J. Sachs (1998), “The East Asian Financial Crisis: Diagnosis, Remedies, Prospects”, *Brookings Papers on Economic Activity* 1:1-90.

[30] Rotemberg, J. (1982), “Sticky Prices in the United States”, *Journal of Political Economy* 90(6):1187-1211.

[31] Sargent, T. and N. Wallace, (1975), “Rational Expectations, the Optimal Monetary Instrument, amd the Optimal Money Supply Rule,” *Journal of Political Economy* 83(2):241-254.

[32] Schmitt-Grohé, S. and M. Uribe (2003), “Closing Small Open Economy Models,” *Journal of International Economics*, 61, 163-185.

[33] Schmitt-Grohé, S. and M. Uribe (2004), “Optimal Fiscal and Monetary Policy under Sticky Prices,” *Journal of Economic Theory*, 114, 198-230.

[34] Taylor, J. B.(1999), *Monetary Policy Rules*, Chicago, National Bureau of Economic Research.

[35] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy,* Princeton, Princeton University Press.

[36] Zanna, L. F. (2003), “Interest Rate Rules and Multiple Equilibria in the Small Open Economy,” IFPD No. 785, Board of Governors of the Federal Reserve System.