A Cointegration Analysis of Monetary Policy Rules
in the United States 1988-2002*

Anders Møller Christensen  
_Economics Department_  
Danmarks Nationalbank  
amc@nationalbanken.dk

Heino Bohn Nielsen  
_Institute of Economics_  
University of Copenhagen  
heino.bohn.nielsen@econ.ku.dk

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Abstract

Using cointegration techniques it is rejected that US monetary policy 1988-2002 can be described by a traditional Taylor-type rule. Instead we find a stable long-term relationship between the policy interest rate, the Federal funds rate, the unemployment rate and the long-term interest rate, with deviations from the long-term relation being corrected primarily via changes in Federal funds rate. This is taken as an indication that the FOMC sets interest rates with a view to activity, expressed via the unemployment rate, and to expected inflation and other conditions available in financial markets, expressed via the bond rate.

**Keywords:** Taylor rule; Bond rate; Cointegration.

**JEL Classification:** C32; E52.

1 Introduction

In recent years a lot of research has been carried out with respect to monetary policy rules. One line of research has actual behavior of central banks in a given period in focus. Another line deals with issues regarding the optimal monetary policy given the central banks objectives and a model for the economy. The present paper is mainly of the empirical kind and considers the monetary policy in the United States since 1988.

The paper differs from most other research on so-called Taylor rules in at least two respects.

First, the econometric technique of cointegration analysis is applied, allowing for a simultaneous investigation of long-term relationships between the variables entering the analysis and the short-term dynamics, including the important issue whether deviations from the long-term relationships are corrected via changes in what is considered the monetary policy instrument.

The second difference concerns the choice of variables entering the analysis. Inspired by the role of the yield-curve in recent monetary analysis and in the literature on leading indicators,

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the long-term bond rate is included in the analysis in parallel with the inflation rate and the unemployment rate.

The extended information set and the multivariate nature of the analysis makes it possible to scrutinize the monetary policy reaction function and the role of financial market information in monetary policy. The analysis concludes that the monetary policy of the Federal Open Market Committee (FOMC) has not followed a traditional Taylor rule. The short-term rate is set as if the information on the economy available in the capital market, here represented by the bond rate, has played an important role in addition to developments in unemployment. We do not consider this a real-time policy rule but a better representation than the Taylor rule of the kind of factors that have entered the decision making process.

The rest of the paper is organized as follows. Sections 2 and 3 briefly discuss the basic concepts regarding Taylor rules and measurement. Section 4 presents the econometric tools involved in cointegration analysis and some important hypotheses implied by the Taylor rule. Section 5 presents the empirical evidence on simple monetary policy rules for the US since 1988, while Section 6 concludes and suggests issues for further research.

2 Taylor Rules

In an influential article Taylor (1993) suggests that the FOMC has managed the Federal funds rate according to the simple linear formula

$$ f_t = \pi_t + \lambda_1 \cdot \tilde{u}_t + \lambda_2 \cdot \tilde{\pi}_t + \kappa_0, $$

where $f_t$ denotes the Federal funds rate, $\pi_t$ and $\tilde{\pi}_t$ denote the inflation and the deviation of inflation from a specified target respectively, $\tilde{u}_t$ denotes deviation of economic activity from a natural level, and the constant $\kappa_0$ is interpretable as the target real interest rate in equilibrium. If the inflation gap is measured as the deviation from a constant target, $\tilde{\pi}_t = \pi_t - \pi^*$, as it is usually the case, $\pi^*$ is not empirically identifiable and the relation collapses to

$$ f_t = \lambda_1 \cdot \tilde{u}_t + (1 + \lambda_2) \cdot \pi_t + \kappa_1, $$

(1)

where $\kappa_1 = \kappa_0 - \lambda_2 \pi^*$. The original rule in Taylor (1993) was based on the current-quarter output gap and the change in the GDP deflator, and the coefficients $\lambda_1 = \lambda_2 = 0.5$ and $\kappa_1 = 1$ produces an influential description of the monetary policy 1987–1992.

Inter alia Orphanides (2001) emphasizes the importance of using only real-time and not final, revised data, and Evans (1998) and Ball and Tchaidze (2002) consider policy rules based on the deviation of unemployment from an estimated natural rate and consumer price inflation, possibly after excluding some volatile components. That allows for an analysis based on monthly rather than quarterly data, and is also the approach taken in this paper.

In the basic formulation, the relation (1) is contemporaneous and the Federal funds rate could at any time be approximated by the right hand side. Some empirical applications, therefore, use (1) directly as a regression equation, see e.g. Evans (1998) and Ball and Tchaidze (2002). Alternatively the right hand side of (1) can be considered a notional target, $f^*_t$, and a model for
partial adjustment of $f_t$ to $f_t^*$ can be considered, e.g.

$$f_t = \rho \cdot f_{t-1} + (1 - \rho) \cdot (\lambda_1 \cdot \tilde{u}_t + (1 + \lambda_2) \cdot \pi_t + \kappa_1),$$  (2)

see Judd and Rudebusch (1998) or Orphanides (2001). For $\rho = 0$ (2) collapses to the usual Taylor rule while some degree of interest rate smoothing prevails if $\rho > 0$.

In empirical analyses, inflation is often found to be best approximated by a unit root process. In that case, simple inference on the parameters in (1) and (2) is only valid if the variables cointegrate, a hypothesis which is rarely tested in this literature. In the present paper we take a different route and consider the relation (1) as a candidate for an equilibrium relation and estimate the parameters within a multivariate dynamic framework. This approach has several advantages. First, it is possible to test if the Federal fund rate is related to the explanatory variables in a relation like (1) such that the deviations, $f_t - f_t^*$, are stationary. Second, the multivariate approach allows us to test for the endogeneity of the included variables with respect to the parameters in (1). For the relation to be interpreted as a policy rule, deviations $f_t - f_t^*$ should be corrected by $f_t$ - with the interpretation that the FOMC seeks to eliminate misalignment from the target rate. If there are no dynamic forces in the model making $f_t$ correct to $f_t^*$, then there is no natural interpretation of $f_t^*$ as a target value for $f_t$. Furthermore, we can test if the variables of interest, unemployment and inflation, react to the stance of monetary policy and test if the variables are actually controllable by the monetary policy instrument as defined in Johansen and Juselius (2001), see further in Section 4.

Bernanke and Blinder (1992) suggest to use the spread between the Federal fund rate, $f_t$, and a long term bond rate, $r_t$, as an indicator of the stance of monetary policy. The bond rate naturally incorporates information on inflation expectations, and at the same time it is insensitive to short-run variations in monetary conditions. The bond rate could also contain other relevant information. A sudden increase in the bond rate could reflect a declining credibility of monetary policy and the FOMC could react by a preemptive increase in the Federal funds rate, see also Carey (2001). Mehra (2001) and Carey (2001) include the bond rate as an additional variable in a Taylor rule like (1). Since $r_t$ will react with an impact of one-to-one from inflation expectations, and inflation is already present, we insert only the 'new' information as measured by the real bond rate, $r_t - \pi_t$, and correct for the average 'tilt' of the yield curve, $\tau$, to obtain

$$f_t = \lambda_1 \cdot \tilde{u}_t + (1 + \lambda_2 - \lambda_3) \cdot \pi_t + \lambda_3 \cdot r_t + \kappa_2,$$

where $\kappa_2 = \kappa_0 - \lambda_2 \pi^* - \lambda_3 \tau$. If there is a one-to-one impact from the bond rate to the Federal funds rate, $\lambda_3 = 1$, we get a simple Taylor rule for the interest rate spread:

$$f_t = r_t + \lambda_1 \cdot \tilde{u}_t + \lambda_2 \cdot \pi_t + \kappa_2.$$  (3)

The interest rate spread is often considered to be a predictor of future inflation or activity, cf. Mishkin (1990), Estrella and Hardouvelis (1991), and the survey by Estrella and Mishkin (1996). Taking the information on expectations of future activity and inflation in the long-term rates into consideration when setting short-term rates is therefore straightforward in a certain sense. However, in theory the relationship is normally turned the other way round, making

Several applications have emphasized the forward looking nature of monetary policy, see Clarida, Gali, and Gertler (1998), Clarida, Gali, and Gertler (2000), Orphanides (2001) and Svensson (2003). In the present study this feature is mainly implicit, in the sense that this type of vector autoregression is consistent with the concept of forward looking expectations using data based projection functions. However, when data on long-term interest rates are included in the dataset, information on expected future inflation and expected alternative real yields are included more directly without restrictions on the way expectations are formed.

3 Data Measurements
To analyze monetary policy reaction functions like (1) and (3) we consider a monthly data set, 

\( Y_t = (f_t : r_t : u_t : \pi_t)' \),

comprising the effective Federal funds rate, \( f_t \), a constant maturity 10 year Treasury bill rate, \( r_t \), the unemployment rate corrected for a linear trend, \( u_t \), and core inflation measured as 100 times the year-on-year change in the log transformed consumer price index excluding food and energy, \( \pi_t \)\(^1\). The considered sample covers the period since Alan Greenspan began as the chairman of the Federal Reserve Board. The effective sample is 1988 : 1 – 2002 : 12, and we condition the analysis on the last months of 1987.

Graph (A) in Figure 1 depicts the Federal funds rate and the Treasury bill rate and graph (B) depicts the spread, \( f_t - r_t \). The interest rates have some similarities, but have been far from parallel. On average \( f_t \) has been lower than \( r_t \) but on three occasions, the Federal funds rate has exceeded the bond yield. Graph (C) depicts the unemployment rate. Several authors have suggested a fall in the natural rate of unemployment in the period under consideration, see \textit{inter alia} Ball and Tchaidze (2002). To allow for a decline in the natural rate in a transparent way and to avoid a linear trend in the empirical analysis of the monetary policy rules we take a very simple approach and correct \textit{a priori} for a linear trend in unemployment using least squares. We make no assumptions on the level of the natural rate and include in the empirical analysis the variable

\[ u_t = u_t^* - 0.00996 \cdot (t - \bar{t}), \]

where \( u_t^* \) is the observed unemployment rate and \( (t - \bar{t}) \) is the demeaned linear trend such that \( u_t \) and \( u_t^* \) have same means over the period. The estimated linear trend is also depicted in graph (C) and assumes that the natural rate has fallen approximately 2% in the sample period. We are of course aware that the linear correction creates problems if extrapolations are made. However, in this way we avoid making more subjective manipulations of data. We are confident that this specific choice is not material for the results reported below. The sample period covers a slack

\(^1\)All data series are taken from the EcoWin data base. The unemployment rate is calculated from the total number of unemployed and the total civilian labor force, both seasonally adjusted.
in the early 1990s and a subsequent long upturn ending sharply in 2000. A comparison of (C) and (A) indicates a negative correlation between \( u_t \) and \( f_t \), and there is also a clear correlation between \( u_t \) and the interest rate spread, \( f_t - r_t \), in (B). Finally graph (D) depicts core inflation. Inflation has been steadily decreasing over the period with bouts of rising inflation. One in early 1990s and one in early 2000s. Comparing developments in core inflation with the Federal funds rate and the interest rate spread indicates a weaker simple correlation.

### 4 Econometric Tools

To analyze the interaction between the interest rates, the unemployment, and inflation, we assume that the variables are integrated of at most first order, and use a \( p \)-dimensional vector autoregression (VAR):

\[
H(r) \Delta Y_t = \alpha (\beta' Y_{t-1} + \mu) + \sum_{i=1}^{k-1} \Gamma_i \Delta Y_{t-i} + \epsilon_t, \quad t = 1, 2, ..., T. \tag{4}
\]

The innovations \( \epsilon_t \) are assumed to be independently and identically Gaussian distributed, \( N(0, \Omega) \), and the initial values, \( Y_{-k+1}, ..., Y_0 \), are considered fixed. If the levels \( Y_t \) are cointegrated with \( r \) long-run relations then \( \alpha \) and \( \beta \) are of dimension \( p \times r \) such that the rank of \( \Pi = \alpha \beta' \) is \( r \leq p \). The remaining autoregressive parameters, \( \Gamma_1, ..., \Gamma_{k-1} \), are of dimension \( p \times p \), see Johansen (1996). Based on theoretical considerations we do not allow for linear trends in the variables and include only a constant restricted to the cointegrating relations.
Maximum Likelihood (ML) estimation of $H(r)$ is given by reduced rank regression, which reduces to solving a certain eigenvalue problem, see Johansen (1996, chapter 6). To determine the number of long-run relations, different nested models, $H(0) \subset \ldots \subset H(r) \subset \ldots \subset H(p)$, can be compared using likelihood ratio (LR) tests, the so-called Trace test statistics, which have non-standard asymptotic distributions due to the unit roots in the processes under the null.

Conditional on the number of long-run relations, $r$, it is possible to test restrictions on the long-run coefficients, $\beta^* = (\beta' : \mu)'$, and on the short run adjustment coefficients, $\alpha$. In this paper we consider hypotheses of the form

$$H : \alpha = (A_1 \phi_1 : \ldots : A_r \phi_r) \text{ and } \beta^* = (H_1 \varphi_1 : \ldots : H_r \varphi_r),$$

where $\phi_i$ and $\varphi_i$ contain the free parameters in columns $i$ of $\alpha$ and $\beta^*$ respectively. The formulation imposes individual restrictions on the columns in $\alpha$ and $\beta^*$ and can be used to impose structure on the long-run interactions between the variables and the directions of correction of possible disequilibrium. Under $H$, the model can be estimated using e.g. the switching algorithms of Johansen (1996) or Boswijk (1995) or a general optimization procedure; and LR test statistics for $H$ are distributed $\chi^2$ under the null.

The solution of (4) for the levels, $Y_t$, in terms of the innovations and initial values is given by the so-called Granger representation

$$Y_t = C \sum_{i=1}^{r} \epsilon_i + C(L) \epsilon_t + \tau_0,$$

where $C = \beta_L (\alpha'_L (I - \Gamma_1 - \ldots - \Gamma_{k-1}) \beta_L)^{-1} \alpha'_L$ is the $p \times p$ dimensional long-run impact matrix of rank $p - r$, $C(L)$ is a convergent polynomial in the lag operator $L$, and $\tau_0$ are coefficients depending on $\mu$ and the initial values, see Johansen (1996, Theorem 4.2). The interpretation of a coefficient $C_{ij}$ in $C$ is the long-run effect on variable $i$ from an innovation to $\epsilon_j$.

Johansen and Juselius (2001) analyze how to implement monetary policy control rules in a cointegrated vector autoregression. They consider a target variable $b'Y_t$ and a given instrument $a'Y_t$, where $a$ and $b$ are $p$-dimensional vectors (often unit vectors). The definition of controllability of $b'Y_t$ with $a'Y_t$ in this context is that $b'Y_t$ can be made stationary around a target value $b^*$ by intervening in $a'Y_t$ at all point in time. The necessary control rule and the properties of the controlled process are derived in Johansen and Juselius (2001, Theorem 7). To analyze if such a control rule has been in action, a necessary condition is that $b'Y_t$ is stationary. The condition for controllability is that $b'Ca \neq 0$, such that interventions to the instruments give a non-zero long-run impact on the target, see Johansen and Juselius (2001, Theorem 7).

**Implied Hypotheses for Monetary Policy Rules.** For the present data $Y_t$ and $r = 1$, which is the main case considered in the empirical analysis, the first part of (4) can be written as

$$\begin{pmatrix}
\Delta f_t \\
\Delta r_t \\
\Delta u_t \\
\Delta \pi_t
\end{pmatrix} = \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4
\end{pmatrix} \begin{pmatrix}
f_{t-1} + \beta_2 r_{t-1} + \beta_3 u_{t-1} + \beta_4 \pi_{t-1} + \mu + \ldots
\end{pmatrix}.$$
where the long-run relation is normalized on the Federal funds rate, $f_{t-1}$. For the empirical model to be interpretable as a characterization of monetary policy we propose two requirements:

1. That the coefficients $(1: \beta_2: \beta_3: \beta_4: \mu)^\prime$ are interpretable as a policy rule. From theory we expect $\beta_2 \geq 0$, $\beta_3 \geq 0$, and $\beta_4 \leq 0$. If $\beta_2 = 0$ the relation collapse to a conventional Taylor rule. If $\beta_2 = -1$ the relation is a simple rule for the interest rate spread.

2. That $\alpha_1 < 0$ such that deviations of the Federal funds rate from the target is reduced by monetary policy actions.

Besides tests of these requirements, it is also possible to test the effects of the misalignments of the policy rate from the equilibrium rate on unemployment and inflation. This corresponds to inference on $\alpha_3$ and $\alpha_4$ respectively. In particular, we expect high interest rates to put downward pressure on inflation, $\alpha_4 \leq 0$, and upward pressure on unemployment, $\alpha_3 \geq 0$. This involves a Phillips-curve trade-off between the two goals in the optimal policy setting.

Controllability of the inflation rate, $\pi_t$, with the Federal funds rate, $f_t$, can be tested as the hypothesis that $C_{41} \neq 0$, where $C_{41}$ denotes element $(4,1)$ in the long-run impact matrix $C$. A priori we expect $C_{41} < 0$.

5 Empirical Analysis of US Monetary Policy

In this section we look at the empirical evidence on monetary policy in the US based on the monthly data set, $Y_t = (f_t: r_t: u_t: \pi_t)^\prime$, for the effective sample $t = 1988:1,...,2000:12$.

First step in the analysis is to determine the lag length $k$ of the VAR. Information criteria and successive testing for removal of lags from a maximum 12 point towards $k = 3$ or $k = 4$. Since there are some residual autocorrelation in the model with three lags, we base the analysis of the long-run structure on a VAR with $k = 4$ lags. By and large similar results as the ones presented below are obtained for $k = 3$.

Table 1 reports a battery of misspecification tests. The only deviation from the different nulls of a well specified model is a marginal autoregressive conditional heteroscedasticity (ARCH) in the equation for core inflation. This is not unusual for monthly data and is not easily remedied within the VAR framework. Work of Rahbek, Hansen, and Dennis (2002) indicates that moderate ARCH effects do not disturb the analysis of the cointegrated VAR, and we choose to ignore this potential problem in the following. It is interesting to note that there are no extreme outliers in the data and the null of normality of the residuals is accepted. This is also the case for the Federal funds rate which is managed by the FOMC.

Long-Run Structure. Next step in the analysis is to determine the cointegration rank. It is known from simulation studies that it is no easy task to select the cointegration rank in empirical applications. The small sample distribution of the Trace test for a cointegration rank of $\text{Rank}(\Pi) \leq r$ against the stationary alternative, $H(p)$, is typically displaced to the right.

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2The empirical analysis was carried out using a set of procedures programmed in Ox 3.0, see Doornik (2001), and PcGiv, see Doornik and Hendry (1997).
Table 1: Tests for misspecification of the unrestricted VAR(4)

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th>AR(1-7)</th>
<th>ARCH(7)</th>
<th>Normality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f_t$</td>
<td>.63 [.43]</td>
<td>1.04 [.41]</td>
<td>1.96 [.06]</td>
<td>4.30 [.11]</td>
</tr>
<tr>
<td>$\Delta r_t$</td>
<td>.91 [.34]</td>
<td>1.27 [.27]</td>
<td>.78 [.60]</td>
<td>4.31 [.12]</td>
</tr>
<tr>
<td>$\Delta u_t$</td>
<td>.05 [.82]</td>
<td>1.05 [.40]</td>
<td>.51 [.82]</td>
<td>.91 [.64]</td>
</tr>
<tr>
<td>$\Delta \pi_t$</td>
<td>.02 [.88]</td>
<td>.75 [.10]</td>
<td>2.43 [.02]</td>
<td>.45 [.80]</td>
</tr>
<tr>
<td>Multivariate tests:</td>
<td>.82 [.66]</td>
<td>1.09 [.26]</td>
<td>...</td>
<td>9.96 [.27]</td>
</tr>
</tbody>
</table>

Note: Figures in square brackets are $p$-values. AR(1) are the $F$-tests for first order autocorrelation and are distributed as F(1,162) and F(36.477) for the single equation and vector tests respectively. AR(1-7) are tests for up to seventh order autocorrelation and are distributed as F(7,156) and F(112,526) respectively. ARCH (7) tests for ARCH effects up to the seventh order and is distributed as F(7,149). The last column reports results of the Doornik and Hansen (1994) test for normality, distributed as $\chi^2(2)$ and $\chi^2(8)$ respectively.

Table 2: Trace tests for the cointegration rank

<table>
<thead>
<tr>
<th>$H(r)$</th>
<th>$r = 0$</th>
<th>$r \leq 1$</th>
<th>$r \leq 2$</th>
<th>$r \leq 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalues</td>
<td>.144</td>
<td>.109</td>
<td>.069</td>
<td>.017</td>
</tr>
<tr>
<td>LR statistic</td>
<td>64.59</td>
<td>36.62</td>
<td>15.89</td>
<td>3.10</td>
</tr>
<tr>
<td>Asymptotic $p$-value</td>
<td>[.00]</td>
<td>[.03]</td>
<td>[.18]</td>
<td>[.59]</td>
</tr>
<tr>
<td>Bartlett factor</td>
<td>1.11</td>
<td>1.11</td>
<td>1.69</td>
<td>1.35</td>
</tr>
<tr>
<td>Corrected $p$-value</td>
<td>[.02]</td>
<td>[.08]</td>
<td>[.70]</td>
<td>[.73]</td>
</tr>
</tbody>
</table>

Note: Likelihood Ratio tests for $H(r)$ against $H(p)$. Case with a restricted constant. Figures in square brackets are asymptotic $p$-values based on the approximate critical values derived from $\Gamma$-distributions by Doornik (1998).

relative to the asymptotic distribution. To take account of the resulting size distortion in finite samples, Johansen (2002) suggests a Bartlett correction factor for the Trace test. This is applied to the current data in Table 2. The model $H(0)$ with no cointegration is rejected, with a $p$-value based on the Bartlett corrected test of 2%. The model $H(1)$ with $r = 1$ long-run relation is a borderline case, with a $p$-value of 8% according to the Bartlett corrected test. However, as this is in line with the theoretical setup we choose this for the main analysis.

Based on the model with $r = 1$ we want to analyze the information in the data of the structure of the long-run relation. The unrestricted estimates of $\alpha$ and $\beta^*$ are reported in Table 3 under $H_0$, with $t$-values based on the asymptotic standard deviations in parentheses. The relation is normalized on the Federal funds rate and the $t$-values indicate a significant coefficient to the Treasury bill rate. A magnitude in the proximity of one is also found in Mehra (2001) for a longer sample period. The coefficient to unemployment is 1.7, indicating that a high unemployment is associated with a low Federal funds rate. The coefficient is clearly significant, with a $t$-value of 9.6. The coefficient for core inflation is also positive, which is the opposite of the expected for a monetary policy rule, but it is not significantly different from zero. The adjustment coefficients in $\alpha$ clearly suggest an interpretation of the relation as
a monetary reaction function. In particular, the adjustment coefficient in the Federal funds equation is negative and clearly significant with a \( t \)-value of -3.9. There is also a significantly negative impact in the inflation equation, indicating that a high interest rate relative to the target lower inflation. The adjustment in the equation for Treasury bill rate is close to zero and the adjustment in the unemployment relation is negative but not significantly different from zero.

Based on the unrestricted coefficients, the long-run relation thus looks like a monetary policy rule. A conventional Taylor rule would imply a zero coefficient for the Treasury bill rate, \( \beta_2 = 0 \). Imposing this restriction gives the results reported under \( H_1 \). First it could be noted that the restriction is formally rejected at a 5% level, indicating that the simple Taylor rule does not seem to be adequate for the present sample. In the restricted relation, both the coefficient to \( u_t \) and \( \pi_t \) are significant with the expected signs and the magnitude of the coefficient to inflation is close to the 1.5 suggested by the original Taylor rule. A static least squares regression of \( f_t \) on \( u_t, \pi_t \) and a constant term, often seen in the empirical literature on Taylor rules, yields

\[
f_t = -1.773 \cdot u_t + 1.206 \cdot \pi_t + 11.500, \tag{6}
\]

where \( t \)-values are in parentheses and \( R^2 \) is 0.90. The estimated equation (6) is close to the quarterly results in Ball and Tchaidze (2002) and is not too far from the long-run relation in \( H_1 \), although the coefficient to unemployment is somewhat smaller. At a first glance the results look like a monetary policy reaction function, but the adjustment coefficients to the long-run relation in \( H_1 \) do not give much support for this interpretation, since there is no feedback to the Federal funds rate. Deviations from the relation are corrected by \( u_t \) and \( \pi_t \) but not by the Federal funds rate, \( f_t \). This highlights the dangers of estimating structural Taylor rules from static regressions. Firstly, with likely unit roots in the variables inference on (4) is difficult, and secondly the dynamic adjustment to a possible equilibrium is not modelled, making the interpretation of the nature of the relation hazardous.

The above results suggest an important role for the bond rate. The theoretical relation (3) gives a simple interpretation as a Taylor-type rule for the interest rate spread if the coefficient to \( r_t \) is restricted to minus one. Under \( H_2 \) we have reported the results after imposing \( \beta_2 = -1 \).
The restriction produces a LR test statistic of 0.49 corresponding to a $p$-value of 0.48 in a $\chi^2(1)$ distribution. In this relation the coefficient to inflation has the expected sign, but it is clearly insignificantly with a $t$-value of 0.67. Imposing the additional restriction, $\beta_4 = 0$, does only marginally change the likelihood and produces the results reported under $H_3$.

The coefficients under $H_3$ still suggest that the feedbacks to $r_t$ and $u_t$ are very weak, and imposing the two additional restrictions that $r_t$ and $u_t$ are weakly exogenous for the long-run coefficients, $\alpha_2 = \alpha_3 = 0$, produces the preferred results reported under $H_4$.

**A Characterization of US Monetary Policy 1988-2002.** In the preferred model, the long-run relation can be written as

$$f_t - r_t = -1.637 \cdot u_t + 7.797,$$

which is a Taylor-type rule for the interest rate spread with a significant impact from unemployment and a zero impact from inflation beyond that contained in the expected inflation via $r_t$. The constant contains the average value of unemployment and the average shape of the yield curve. Deviations from this relation is corrected primarily by the Federal funds rate, eliminating 8% of a misalignment each month. There is also a negative coefficient in the equation for $\Delta\pi_t$, indicating that a high funds rate suppress inflation. The effect is not terribly strong, with a $t$-value of $-2.4$. The preferred structure is accepted as a reduction of the unrestricted specification with a LR statistic of 2.86 corresponding to a $p$-value of 0.58 in a $\chi^2(4)$.

The result that actual inflation is not directly present in the empirical rule could reflect that the period under consideration has been characterized by little inflationary pressure and therefore limited information on the impact of actual inflation in the policy rule. As mentioned before, expected inflation is present with a coefficient of 1 via the long-term interest rate.

For the Federal funds rate to be a valid instrument for controlling inflation, it is required that $\hat{C}_{11}$ is significantly negative, where $\hat{C}$ is the estimated counterpart to $C$. The estimated long-run impact matrix is given by

$$\hat{C} = \begin{pmatrix}
0.167 & 1.820 & -2.748 & -0.336 \\
(0.19) & (3.70) & (-3.05) & (-0.67) \\
-0.025 & 1.143 & -0.141 & 0.051 \\
(-0.06) & (5.10) & (-0.34) & (0.22) \\
-0.118 & -0.414 & 1.593 & 0.236 \\
(-0.32) & (-2.00) & (4.20) & (1.13) \\
-0.376 & 0.299 & -0.397 & 0.755 \\
(-1.86) & (2.63) & (-1.90) & (0.54)
\end{pmatrix},$$

with standard normal distributed asymptotic $t$-values in parentheses. The coefficient is $-0.376$, indicating that a one percentage point innovation to the Federal funds rate lowers the long-run core inflation rate with slightly less than 0.4 percentage points on average. The parameter is not particularly well-determined, however, with a $t$-value of $-1.86$. Again this could reflect that the variation in inflation in the sample period and the amount of information on the monetary transmission channel are limited. For a longer sample, 1985 : 8 – 1999 : 2, and a data set comprising real money, (interpolated) real GDP, monthly inflation as well as 5 interest rates, Johansen and Juselius (2001) also find little support for the controllability of inflation with the Federal funds rate. In that analysis the impact $\hat{C}_{11}$ is actually positive.
Graph (A) in Figure 2 depicts deviations from the estimated policy rule together with the deterministic component comprising the effects of the initial values 1987 : 9 – 1987 : 12. The deviations show that interest rates in the initial period and during 1988 was lower than suggested by the relationship. A common interpretation relates this to concern for the financial stability after the crash in the stock market in late 1987. We have tried to recalculate the analysis starting the effective sample in 1989 : 1, but the estimation results are only marginally affected. The same type of concern might explain the relatively low interest rates in 1999 after the financial crisis in Russia and the problems related to LTCM. Such effects are clearly outside the information set of the current simple model. In 1995/1996 rates were higher than suggested. Along with other observers a likely interpretation is that this was due to a real-time belief that natural rate of unemployment had not fallen significantly from the level of the early 1990’s. It is interesting to note that by the end of the sample, 2002 : 12, where the Federal finds rate is at a historic low, the policy rate is still above its equilibrium value according to our estimates.

To evaluate the stability of the relation we depict in graph (B) and (C) the recursively estimated parameters in the long-run relation, the coefficient to unemployment and the constant respectively. We use three and a half years to initialize the estimates and fix the short-run parameters at the full sample values, see Hansen and Johansen (1999). By and large similar results are obtained if the short-run parameters are reestimated in each sub-sample, although a larger initial sample is necessary to initialize the procedure. The estimates look remarkably stable, and the narrowing of the 95% confidence bands indicate an increasing information on the parameters. Finally Graph (D) depicts the recursively calculated test statistic for the overidentifying restrictions. The identifying structure is clearly acceptable in all sub-samples.

**Short-Run Structure.** The VAR used to characterize the long-run properties is heavily over-parametrized, with many insignificant parameters. To illustrate the short-run adjustment we use a general-to-specific modelling strategy, see Hendry and Mizon (1993), to find a more parsimonious representation. Using a conventional 5% critical level and retaining the adjustment coefficient $\alpha_4$ to inflation yield the following representation

$$
\begin{pmatrix}
\Delta f_t \\
\Delta r_t \\
\Delta u_t \\
\Delta \pi_t
\end{pmatrix} = 
\begin{pmatrix}
-0.085 \\
0 \\
0 \\
-0.024
\end{pmatrix}
\begin{pmatrix}
(0.149) \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\Delta f_{t-1} - f^*_{t-1} \\
\Delta r_{t-1} \\
\Delta u_{t-1} \\
\Delta \pi_{t-1}
\end{pmatrix} + 
\begin{pmatrix}
0.188 \\
0 \\
0.167 \\
-0.257
\end{pmatrix}
\begin{pmatrix}
(2.88) \\
(4.57) \\
(3.47) \\
(-3.21)
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\Delta f_{t-1} \\
\Delta r_{t-1} \\
\Delta u_{t-1} \\
\Delta \pi_{t-1}
\end{pmatrix} + 
\begin{pmatrix}
0.177 \\
0 \\
0.145
\end{pmatrix}
\begin{pmatrix}
(3.08) \\
(2.02)
\end{pmatrix}
\begin{pmatrix}
0 \\
0
\end{pmatrix}
\begin{pmatrix}
\Delta f_{t-3} \\
\Delta r_{t-3} \\
\Delta u_{t-3} \\
\Delta \pi_{t-3}
\end{pmatrix},
$$

which produces a LR test statistic of 39.89 compared to the unrestricted vector error correction model (VECM), corresponding to $p$-value of 0.48 in a $\chi^2(40)$. 

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Figure 2: Cointegrating relation and recursive results. The recursive estimation is done for sub-sample \( t = 1988:1, \ldots, T_0 \), where the endpoints take the values \( T_0 = 1991:7, \ldots, 2002:12 \). In each sub-sample the short-run parameters are fixed at their full-sample estimates. (D) depicts the test statistics for the 4 over-identifying restrictions in \( H_4 \) and the 5% critical value for individual tests, see Kongsted (1998).

Considering the equation for the Federal Funds rate, which is the main focus here, indicates a simple behavior. Besides the autoregressive terms in \( \Delta f_{t-1}, \Delta f_{t-2}, \text{and} \Delta f_{t-3} \), which describe the interest rate smoothing, there are additional short-run terms only for one period lagged bond rate and unemployment. The coefficient to \( \Delta r_{t-1} \) is 0.17, well below the long-run impact of one. The coefficients to the lagged change in the unemployment rate, \( \Delta u_{t-1} \), is –0.26. In the parsimonious system the adjustment coefficient \( \alpha_4 \) in the inflation equation is smaller and less significant than in the unrestricted VECM, giving less support for the short-run controllability of inflation.

6 Concluding Remarks

We have found that the interest-rate setting of the FOMC in the period 1988-2002 has been somewhat different to that implied by other research. A long-term relationship between the Federal funds rate, the unemployment rate and the long-term interest rate is found. Deviations from this relationship are mainly corrected via changes in the Federal funds rate. This implies that in this small system the Federal funds rate can be considered the endogenous variable, indicating that the relationship has the character of a monetary policy rule. Rates are set as if
FOMC reacts to unemployment and long-term interest rates. From a decision-making point of view a likely interpretation is a reaction to activity expressed by the unemployment rate and to information derived from financial markets expressed by the long-term interest rate.

We are fully aware that the decision-making process in real time has been far more complicated that a literal reading of our results suggest. Forecasts of inflation and activity using different models have been important as has a careful digestion of recent statistics. As a simple way to summarize factors entering the interest rate setting using \textit{ex post} data the results nevertheless provide a better description than the traditional Taylor rule.

The analysis is carried out using a cointegrated vector autoregression, allowing for a simultaneous modelling of the long-term relationships and short-run dynamics. When testing against a more traditional Taylor-rule specification, our model suggests that inflation enters the relationship via its impact on expected inflation through the long-term interest rate. We are unable to find a significant role for inflation beyond that. A specification without information from the financial market is clearly statistically rejected, and the dynamic adjustment, where the Federal funds rate is weakly exogenous for the long-run parameters, indicates that although the relation looks like a simple Taylor (1993) rule, it is far more difficult to interpret as a policy reaction function.

There is nothing trivial about these results. In the dominant theory of the yield-curve, the expectation theory of the term structure, long-term rates are considered the product of expected future short-term rates, while we find that the long-term rate exerts influence on the Federal funds rate. At the same time visual inspection for other countries does not suggest that monetary policy has corresponded to such a rule in other central banks, cf. Christensen (2002), even if the interest rate spread often seems to be informative when the monetary policy stance is subject to discussions.

A more thorough investigation using other central banks is an obvious subject for future research. From a theoretical point of view, however, the suggested role for the long-term interest rate points in the direction, that research on the relationship between the return on real assets, the market for stocks and the market for bonds might be rewarding and lead to a better understanding of the obtained empirical results in this paper.
References


