Tunnels and Reserves in Monetary Policy Implementation

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Abstract

In recent years, some central banks have implemented monetary policy without reserve requirements by using a ceiling and floor for overnight interest rates established by central bank lending and deposit facilities. This paper develops a theoretical model to explore such a "tunnel" system and the benefits of adding reserve requirements to it. However, reserve requirements may involve social costs owing to the reserve avoidance activities of banks. The paper also depicts a modified model with no reserve avoidance, where banks optimally choose to hold voluntary reserve requirements. The paper highlights the importance for central banks to consider such models in light of idiosyncratic features of their own institutional environment, which may importantly condition the advisability of any particular approach.

Keywords: monetary policy implementation, reserve requirements, overnight interest rates

JEL classifications: E4, E5

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Introduction

Most major central banks conduct monetary policy using a short-term market interest rate as an instrument to achieve their ultimate objectives for the performance of the economy. Traditionally, central banks have tried to hit a target level of the short-term market rate by adjusting the aggregate supply of liquidity through open market operations or other means. In such circumstances, the central bank needs a predictable demand function for the liquidity it supplies. Reserve requirements have been one means of arranging for this, and they are still used for this purpose by some major central banks, including the Federal Reserve, the European Central Bank, and the Bank of Japan.

Reserve requirements create a known or predictable demand for balances held at a central bank on average over a maintenance period. Moreover, the period-averaging induces depository institutions to engage in intertemporal arbitrage of the reserve market interest rate, helping to keep it on target. However, central banks in Switzerland, Sweden, Canada, Australia, and New Zealand have found that they can conduct monetary policy satisfactorily without the aid of reserve requirements. The Bank of England retains a minimal level of reserve requirements, though merely to raise revenue for the central bank independently of the government budget and not for the purpose of implementing monetary policy. Clinton (1997), Guthrie and Wright (2000), and Woodford (2001) have described how a central bank can keep a policy rate on target, without reserve requirements, by relying on standing facilities: a lending facility at a penalty interest rate above the central bank's target rate and a deposit facility (or interest on excess balances held at the central bank) at an interest rate below the target rate. With the lending facility providing a ceiling for overnight rates and the deposit facility a floor, such a system has been called a channel, corridor, or tunnel. Canada, Australia, and New Zealand currently employ a system of this nature with a 50 basis point band between lending and deposit rates.

This paper analyzes further the use of tunnel procedures and any role for period-average requirements in such a system. The model of bank reserve management developed here draws from and contributes to a sizable literature, including work by Orr and Mellon (1961), Poole (1968), Campbell (1987), Kopecky and Tucker (1993), Clouse and Dow (1999), Furfine (2000), Guthrie and Wright (2000), Bindseil (2000), Woodford (2001), Bartolini/Bertola/Prati (2001 and 2002), and Heller/Lengwiler (2003).[1] While some previous work has employed multi-day decision procedures for banks over a reserve-averaging period, no theoretical piece has—to the author's knowledge—incorporated in such models daily account overdrafts at penalty interest rates (using general or Gaussian distribution functions for end-of-day account uncertainties), as developed here.[2]

In reviewing the implications of the model, the paper points out the sensitivity of key findings to detailed features of the relationship of private sector banks to the monetary authority. Seemingly minor modifications in institutional arrangements, such as whether overdrafts count toward reserve requirements, may change the structure of incentives profoundly and alter the behavior of the system. The paper discusses some of these institutional considerations and their relevance to a central bank's use of a tunnel system with or without reserve requirements.

The last section of the paper addresses a well-known trade-off associated with reserve requirements: While they may help reduce the variance of overnight interest rates, they may also generate incentives for reserve avoidance activities that impair financial sector efficiency and that cannot be justified on optimal taxation grounds. The paper concludes with a stylistic model that depicts circumstances in which the inefficiencies could be avoided through a system of voluntary reserve requirements.

A One-Day Tunnel Model

In this section, we present a one-day tunnel model without reserve requirements and then provide motivation for examining a multi-day tunnel model with reserve requirements. In the one-day model, the central bank relies entirely on the tunnel to control the overnight market interest rate, with a ceiling provided by its lending rate and a

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[1] Models of the micro-mechanics of trading in federal funds have also contributed to this literature, including those of Ho and Saunders (1985) and Spindt and Hoffmeister (1988).

[2] Davies (1998) allowed for daily central bank overdrafts, but in a different type of model (with no interbank trading, for example, and an optimal control approach to the first day problem rather than taking expectations of a future day cost function, as here).

floor established by its interest rate on account balances.[3] To motivate the setup, consider a private sector bank with no previous involvement in overnight markets that decides to engage in direct clearing of transactions through the payment system operated by or in association with the central bank. The private bank opens an account at the central bank and begins using the account to clear transactions.[4] The bank tries to track its account position with the central bank during the day, but is nevertheless subject to late payments or delayed accounting information, and therefore can determine its end-of-day position only within a margin of error given by a stochastic term, ε, where $E(\varepsilon) = 0$. During the day, the bank can trade central bank balances with other banks at the market rate, i, and does so to achieve a target account balance of T. If the bank's actual end-of-day account balance of $T + \varepsilon$ is positive, it earns interest at the central bank's deposit interest rate. Any end-of-day overdraft is booked as a loan from the central bank. The central bank has a target for the overnight rate of i^*, which it tries to hit with the help of a lending rate at $i^* + s$ and a deposit rate of $i^* - s$. The symmetry of these spreads around i^* will be seen to be an important feature of the model. A graphic depiction of the representative bank's decision problem is provided in Exhibit 1. More formally, the key assumptions are:

<A1> A representative, competitive bank is risk-neutral at the margin.

<A2> Loans are freely available from the central bank, as perfect substitutes for borrowings from the private market, at an interest rate of $i^* + s$. *End of day account overdrafts are booked as loans at that interest rate.*

<A3> Balances left overnight in an account at the central bank are perfect substitutes for lending in private markets and earn interest at the rate of $i^* - s$.

When choosing its target account balance, T, the private bank's information set is:

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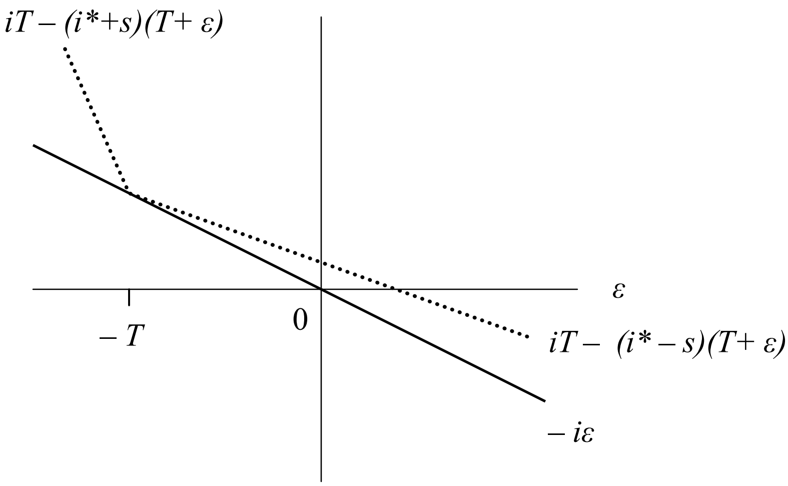
[3] A similar simple tunnel model was discussed briefly by Woodford (2001). The analysis here is more thorough and the subsequent models, which include reserve requirements, are new contributions. Though it was not a tunnel framework, the model of Bartolini/Bertola/Prati (2002) is also similar in some respects to the one developed here.

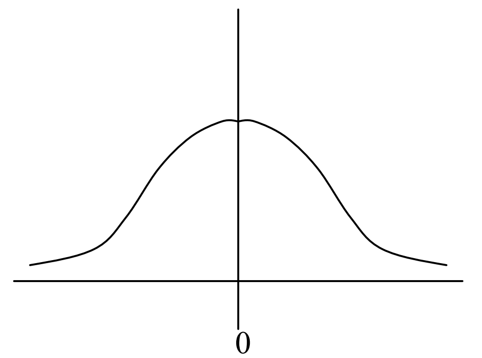
[4] At the Federal Reserve, such transactions could include checks, automated clearinghouse payments, wire transfers, and transfers for cash of book-entry Treasury and agency securities.

Exhibit 1: THE REPRESENTATIVE BANK'S PROBLEM

Cost



Distribution of Account Shock



Note: The bank chooses the amount T to borrow (or withhold) from the market at interest rate i. After an account shock, its end-of-day balance at the central bank is $T + \varepsilon$. It earns $i^* - s$ on any positive balances or pays $i^* + s$ on any account overdraft. With full information, as long as $i^* - s < i < i^* + s,$ the bank chooses $T = -\varepsilon$ and its cost is $-i\varepsilon.$ But knowing only the distribution of ε, it chooses T to minimize the expected opportunity costs given by the probability-weighted vertical distances between the dotted and solid lines on the left.

I_0: i^*, s, the market rate i, the distribution of the account balance shock, $F(\varepsilon)$, and $E(\varepsilon) = 0$.

If the bank had full information, it would set $T = -\varepsilon$, given $i^*-s < i < i^*+s$. Then its end-of-day balance at the central bank would be zero and its net funding cost would be minimized at $-i\varepsilon$. Without knowing ε, the bank chooses T to minimize two types of expected costs: the opportunity cost of holding a positive balance in its account at the central bank, relative to lending funds in the market, given by $i - (i^* - s)$, and the loss, in the case of overdrafts, on borrowing from the central bank rather than from the market, given by $i^* + s - i$. Formally, the bank's problem is:

$$\min_T {}_{I_0} \int_{-T}^{\infty} \left(i - i^* + s\right)(T+\varepsilon)\,dF(\varepsilon) \; - \int_{-\infty}^{-T} (i^* + s \; - \; i)(T+\varepsilon)\,dF(\varepsilon) \qquad [1]$$

The first order condition may be written in the form:

$$F(-T^*) \;=\; \frac{1}{2} \;+\; \frac{i - i^*}{2s}. \qquad [2]$$

where T^* is the optimal choice. Consider interpretations of [2] under the assumption of symmetric or Gaussian distributions:

<A4> The distribution of the account shock, F( ), is symmetric with a zero mean.

<A4'> The account shock has a normal distribution with zero mean and variance σ^2.

If the market rate equals the central bank's desired interest rate, then $F(-T^*) = \frac{1}{2}$, and for a symmetric distribution, this can occur only if banks target a zero balance ($T^* = 0$). Under <A4'>, the interest sensitivity of the representative bank's target balances, relative to either the overnight rate or the deviation of the overnight rate from the central bank's desired rate, is:

$$\frac{\partial T^*}{\partial i} = \frac{\partial T^*}{\partial (i - i^*)} = \frac{-1}{2 s\, n\left(\frac{T^*}{\sigma}\right)} < 0, \qquad [3]$$

where $n()$ denotes the standard normal density. As the overnight rate approaches the central bank's deposit rate $(i \rightarrow i^* - s)$, equation [2] indicates that $T^* \rightarrow \infty$ and [3] reveals that the demand curve flattens out. The elasticity also becomes infinite as the overnight rate approaches the central bank's lending rate of $i^* + s$ and $T^* \rightarrow -\infty$.

The responsiveness of target balances to the spread, s, on the central bank's lending and deposit interest rates can be written as:

$$\frac{\partial T^*}{\partial s} = \frac{1 - 2N\left(\frac{T^*}{\sigma}\right)}{2 s\, n\left(\frac{T^*}{\sigma}\right)}, \qquad [4]$$

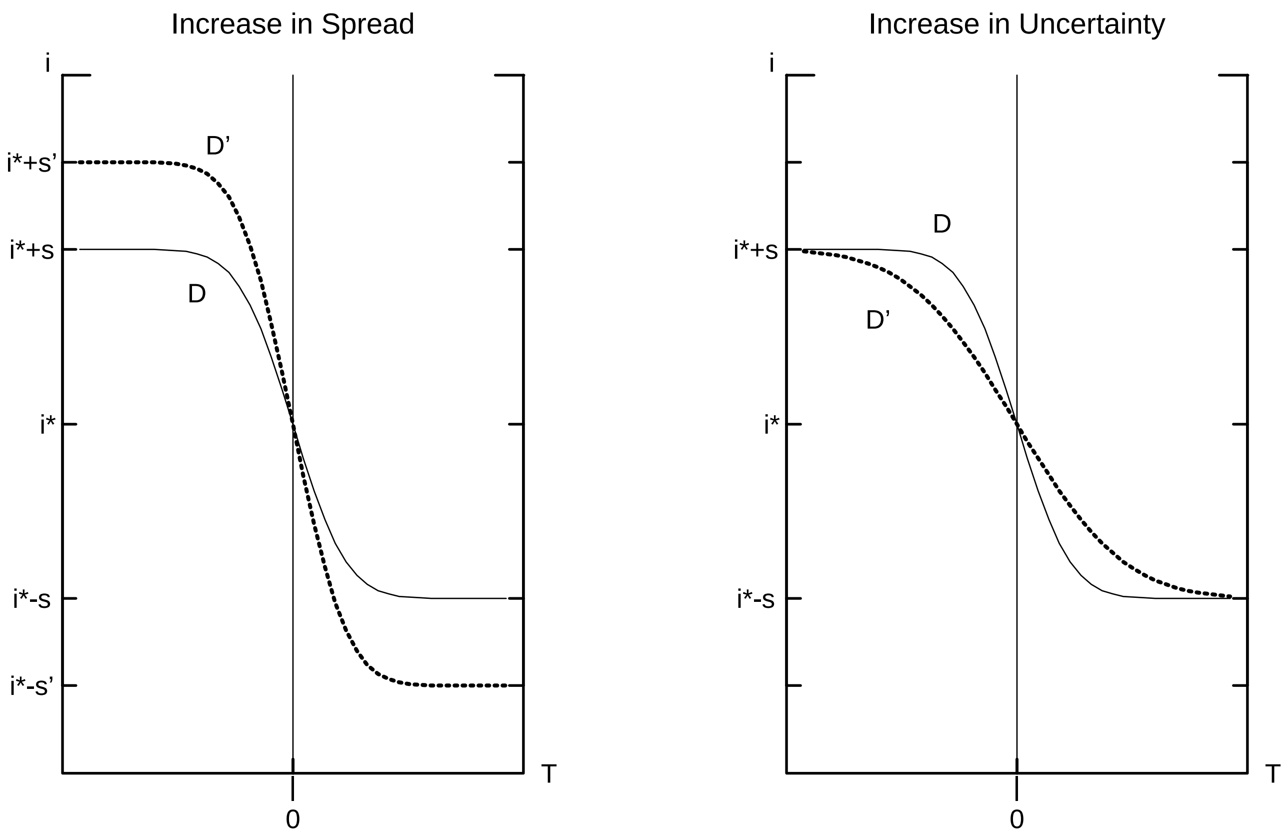
where $N()$ is the cumulative standard normal distribution function. A wider spread of lending and deposit rates around the central bank's desired interest rate implies a generally steeper demand curve, as [4] is positive if $T^* < 0$ and negative if $T^* > 0$. Exhibit 2 depicts a graph of this demand curve and its response to a change in the spread, assuming a normal distribution. Under <A4'>, it can also be shown that the elasticity of T^* with respect to σ is unity. A simulation of the response of the demand curve to an increase in uncertainty is depicted on the right of Exhibit 2.[5]

Woodford (2001) previously pointed out that the demand for balances in this type of model is zero when the market rate equals the central bank's policy target, i^*. Changes in i^* or in the spread, s, do not affect that result, as long as the policy target remains at the midpoint between the central bank's lending and deposit rates. It is especially important that the degree of account uncertainty, σ, also does not alter the intercept of the demand function, for then the aggregate demand for reserves would have the same intercept even if individual banks differ in the account uncertainties they

[5] Poole (1968) first pointed out the flattening of a reserve demand curve with increased dispersion in the distribution of account shocks. His model, which was not a symmetric tunnel framework, employed a bounded support for the distribution.

Exhibit 2: SIMPLE TUNNEL MODEL

Demand for Central Bank Balances



Note: i is the overnight market interest rate, i* is the central bank's desired overnight rate, i* + s is the central bank's lending rate, i* - s is the central bank's interest rate on account balances, D is the demand function for balances by a representative bank, indicated by the quantity targeted, T, on the horizontal axis, while uncertainty is measured by the perceived standard deviation of the end-of-day account balance shock (it doubles in the dotted line in the right panel).

perceive. For these reasons, policy implementation might seem almost automatic once standing facilities are in place to provide a ceiling and floor with symmetric costs around the central bank's desired interest rate.

However, a key quantity is implicitly involved here. The central bank must try to arrange for the aggregate supply of central bank balances to equal zero. Autonomous movements of items on the central bank's balance sheet, such as withdrawals of currency by commercial banks, the movement of funds by the Treasury from the banking system to its account at the central bank, and the implicit provision of central bank liquidity through float arising from check clearings, would still need to be offset by open market operations.

To examine the implications of movements in items on the central bank's balance sheet, we consider the aggregate demand curve. First, T^* is found by inverting [2]:

$$T^* \;=\; -F^{-1}\left(\frac{1}{2} \;+\; \frac{i-i^*}{2s}\right), \qquad [5]$$

where $F^{-1}()$ is the inverse of the distribution function. Then we sum across individual banks, indexed by j:

$$D \;=\; \sum_j T_j^* \;=\; -\sum_j F_j^{-1}\left(\frac{1}{2}+\frac{i-i^*}{2s}\right) \;=\; -N^{-1}\left(\frac{1}{2}+\frac{i-i^*}{2s}\right)\sum_j \sigma_j\;, \qquad [6]$$

where the last equality follows for normal distributions that differ across banks only in the perceived uncertainty regarding end-of-day account balances. Account shocks arising from the failure of expected payments to clear between private banks would aggregate to zero. However, account shocks that arise from the relationship of private banks to the central bank would affect the aggregate quantity of reserves in the market. This paper does not focus on modeling open market interventions. However, for this section, we assume a sequence of daily events that begins, as in Bartolini/Bertola/Prati (2002), with the central bank's open market operation, followed by revelation of an aggregate reserve supply shock, then interbank trading at the market interest rate, followed by the end-of-day account balance shocks. We assume the mid-day reserve supply shock follows a normal distribution with a zero mean and standard deviation of σ_{cb}; it affects the private bank's minimization problem only through the resulting market interest rate for the day. The ratio of the volatility in the central bank's supply of reserves to the sum of the end-of-day account position uncertainties perceived by private banks becomes a key factor in the distribution of the market interest rate:

$$\text{Central/Private Bank Uncertainty} \equiv \frac{\sigma_{cb}}{\sum_j \sigma_j}, \qquad [7]$$

Private banks are uncertain about their positions with both the central bank and other private banks. However, actual central bank uncertainty, σ_{cb}, need not be a subset of the sum of perceived private bank uncertainties, because the two are measured at different times of the day. If the central bank undertakes its open market operation in the morning, when overnight funding markets are more liquid, as in the United States, the uncertainty about its balance sheet may be sizable relative to the uncertainties of private banks regarding their account positions at the end of the day after the completion of market trading.[6] Exhibit 3 depicts histograms of the distribution of market interest rates under different assumptions for the ratio in [7]. The larger the ratio, the fatter the tails of the resulting distribution of overnight interest rates.

The model reveals another key issue for policy implementation as regards the elasticity of demand. In general, if reserve demand is highly elastic, the central bank's job is easy, as a precise estimate of the position of the demand curve is not needed. Errors in assessing reserve demand or in estimating autonomous factors in its own balance sheet would then have little impact on overnight interest rates. By contrast, if the demand for reserves is inelastic, errors in the aggregate reserves supplied by the central bank have substantial effects on the overnight market interest rate.

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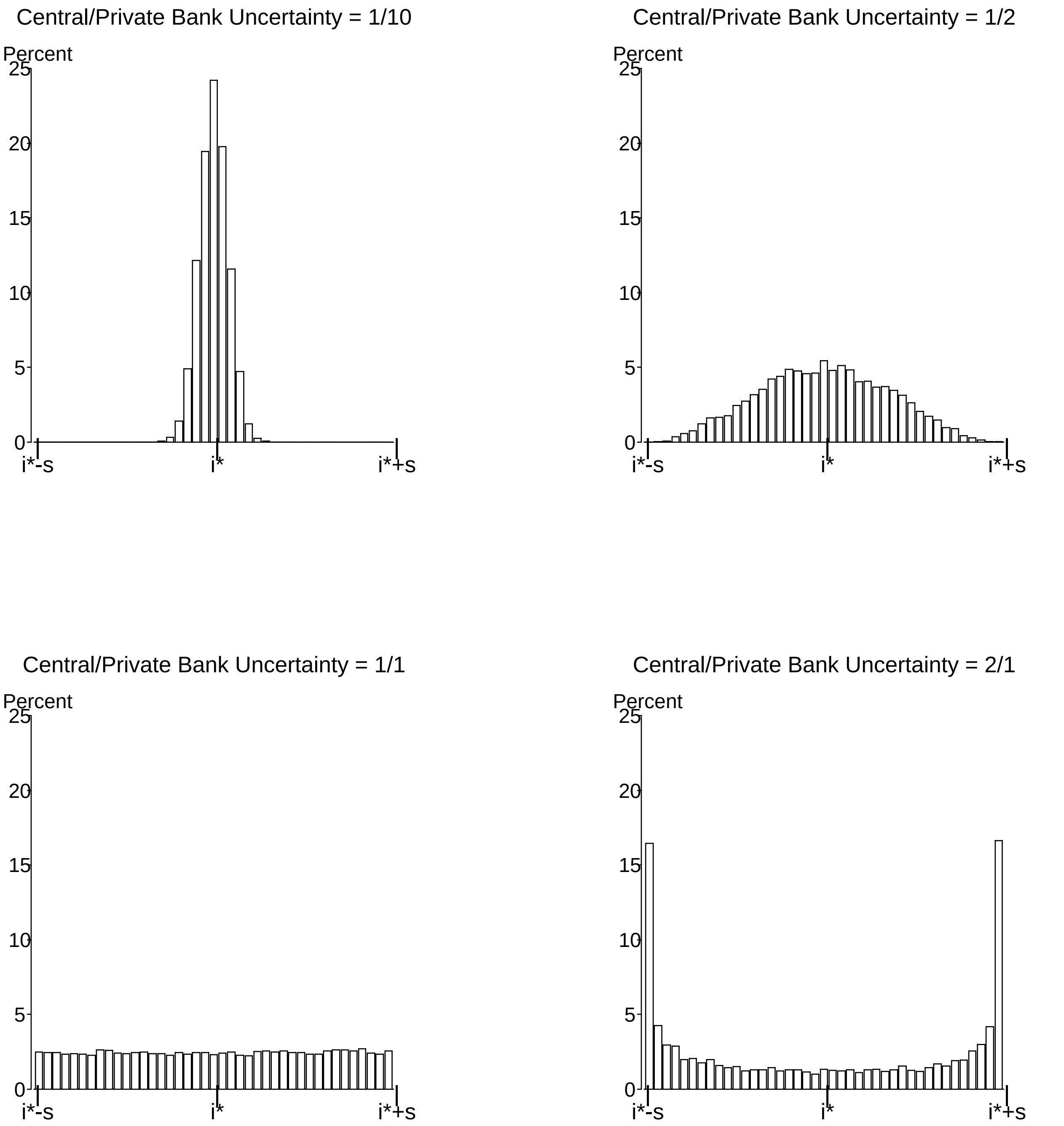
[6] This is an oversimplified account of actual information flows and market frictions late in the day. Uncertainty is likely reduced to fairly low levels for most institutions by the end of trading. However, the shocks modeled here could reflect unexpected late-day payments that occur even before the market closes, if the bank is unable to borrow or lend in sufficient quantities in the late-day market to offset such shocks owing to line limit constraints.

Exhibit 3: TUNNEL MODEL

Simulated Histograms of Market Rates



Note: Each graph is based on 10,000 draws of shocks to the central bank's aggregate supply of reserves.

In the model, demand is highly elastic near the ceiling and floor interest rates. Goodfriend (2002) proposed that a central bank could take advantage of the elastic region for interest rates near the central bank's deposit rate by setting the deposit rate equal to its desired market rate and then supplying a sufficient quantity of reserves to drive the market rate to that floor. However, advocates of tunnel systems generally consider it undesirable for private rates to move all the way to the boundaries of the tunnel. In the above model, at the central bank's desired interest rate of i^*, the demand curve is actually at its steepest, its slope peaking (with a symmetric distribution) at $\frac{\partial i}{\partial T^*} = -s$.

The elasticity issue becomes especially important in light of issues related to the substitutability of market transactions for central bank loans and deposits. For instance, interbank loans are generally unsecured, but collateral must be posted to borrow from a central bank. Thus, the cost of borrowing from the central bank would actually be $i^* + s$ plus the cost of providing collateral. Any stigma effects in borrowing from the central bank would raise the costs further. Moreover, the market rate equivalent to a deposit at the central bank would actually be $i^* - s$ plus a private sector credit risk premium. Such considerations would imply an asymmetry in the opportunity costs around the central bank's desired interest rate. The resulting demand curve would lie above and to the right of the one in the model, but its exact position might vary over time with credit risk perceptions and the opportunity costs of collateral.[7] Because of potential difficulties of estimating the location of the demand curve, particularly in more complex money markets with heterogeneous participants, a central bank might wish to assess the role of time-averaged balance requirements in a tunnel system, to which we now turn.

Time-Averaged Balance Requirements

This section develops a tunnel model in which a representative bank is required to hold an average balance at the central bank over a multi-day reserve maintenance period.

[7] Unexpected implicit asymmetries became evident after implementation of a tunnel system by the Bank of Canada. Despite using standing facilities to create an apparently symmetric spread of 25 basis points above and below the target rate, the central bank found it had to provide a positive level of aggregate reserves to achieve its desired market rate. See Woodford (2001), page 38.

For intuition, consider first the previous one-day tunnel model with no uncertainty. If the market interest rate were between the central bank deposit and lending rates, the private bank would hold a zero balance. If the market rate began moving above the central bank's lending rate, a bank could arbitrage by borrowing from the central bank and selling into the market. If the market rate began moving below the central bank's deposit rate, a bank could arbitrage by borrowing from the market to make deposits at the central bank. Thus, the demand curve would be a single step function as in the left panel of Exhibit 4.

Now suppose that the representative bank, again with no account position uncertainties, is subject to an average reserve requirement of R over a two-day maintenance period, which the bank is compelled to meet. The second day is the reserve "settlement day," and the interest rate on that day, denoted by i_S, is the only uncertainty as of day one. A risk neutral bank would then fund the entire reserve requirement of $2R$ on pre-settlement day, if the interest rate on that day, i_P, were less than the expected market rate on settlement day, $E(i_S)$. If instead $i_P > E(i_S)$, the bank would fund the entire requirement the second day. Thus, the demand curve on day one would be a two-step function as shown on the right in Exhibit 4. Interior solutions on pre-settlement day (with $0 < T_P < 2R$) would all occur at a market rate equal to the expected settlement day interest rate. This is the martingale property employed within a maintenance period by Bartolini/Bertola/Prati (2002) and others. Thus, the period-average balance requirement has increased the elasticity of demand at an interior value for the interest rate. If market participants were confident that the central bank could hit its desired interest rate on settlement day, the demand curve on pre-settlement day would be very elastic at that same rate.

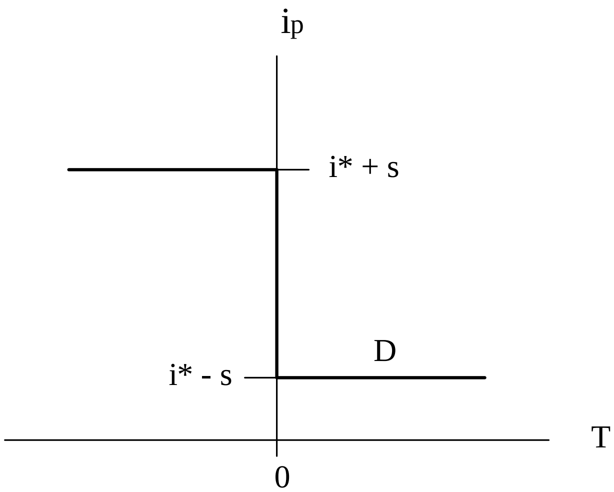
Now we develop a model with uncertainty about daily account positions and two-day balance requirements. Additional assumptions are:

<A5> Reserve requirements are assessed against deposits in a previous period and are met by holding minimum average balances at the central bank over a two-day maintenance period. Shortfalls from requirements are made up through borrowing from the central bank at the standard lending rate of $i^* + s$. *Account shocks on settlement*

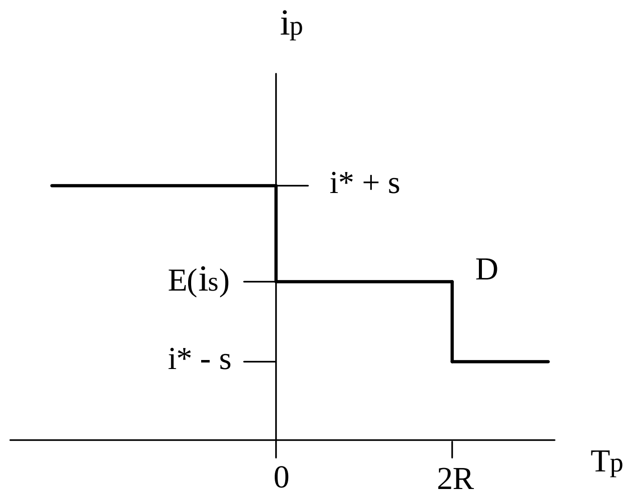
Exhibit 4: TIME-AVERAGED BALANCE REQUIREMENTS
(Demand Curves for Central Bank Balances)

NO ACCOUNT BALANCE UNCERTAINTY

One-Day Model
(no required balance)



First Day of Two-Day Model



Note: R is the two-day average reserve requirement, i^* is the central bank's desired overnight rate, $i^* + s$ is its lending rate, $i^* - s$ is its interest rate on deposit balances, and the market interest rate is represented by i_p on pre-settlement day and i_s on settlement day.

day, denoted by ξ*, may have a different distribution,* $G(\,)$*, than account shocks on pre-settlement day.*

With lagged reserve requirements, account shocks that change the private bank's reservable deposits do not affect its reserve requirement within the same period. The level of uncertainty on settlement day is allowed to differ from that on a pre-settlement day, reflecting the empirical results of Hamilton (1996) and others. Note also that in this model, funds borrowed from the central bank, including overdrafts, can be used to meet reserve requirements. Therefore, a zero period-average requirement is equivalent to the absence of reserve requirements.[8]

Consider initially the problem on settlement day, assuming that the private bank has a remaining balance requirement of b. Banks may purchase balances from the market at the settlement day interest rate, i_S. They earn $i^* - s$ on required or excess balances held at the central bank. Banks choose a target balance T_S with the information set:

I_s: i^*, s, $G(\xi)$, $E(\xi) = 0$, *the settlement day market interest rate,* i_S, *and the remaining required balance as of settlement day,* b.

The bank's settlement day cost minimization problem is then:

$$\min_{T_S} {}_{I_S} \; (i_S - i^* + s)b + \int_{b-T_S}^{\infty} \left(i_S - i^* + s\right)(T_S + \xi - b)dG(\xi) + \int_{-\infty}^{b-T_S} (i^* + s - i_S)(b - T_S - \xi)dG(\xi) \qquad [8]$$

The first expression is the cost of borrowing from the market to meet the balance requirement, the first integral is the opportunity cost of holding excess balances, and the last integral is the net cost of borrowings from the central bank, rather than the market, to

[8] If borrowings undertaken to bring an overdrafted account up to a zero balance did not count towards reserve requirements (as is the case at the Federal Reserve), the symmetry in opportunity costs between overdrafts and excess reserves would be broken. A separate reserve deficiency charge would also need to be created, which could introduce additional asymmetry. As a result, the interest rate at which the demand curve became highly elastic would be more difficult to identify and it could differ across banks.

meet the requirement. If $b = 0$, [8] becomes equivalent to [1]. Similar to [2], the Euler condition associated with [8] is:

$$G(b - T_S^*) \quad = \quad \frac{1}{2} \quad + \quad \frac{i_S - i^*}{2s}. \qquad [9]$$

If $i_S = i^*$ and G is symmetric around the mean of zero, the bank targets its remaining requirement ($T_S^* = b$). We also have:

$$\frac{\partial T_S^*}{\partial b} = 1, \qquad [10]$$

or in other words, $b - T_S^*$ is independent of b. In preparation for setting up the pre-settlement day problem, we write the optimal value of $b - T_S^*$ as a function of i_S:

$$b - T_S^* \quad \equiv \quad k(i_S, i^*, s) \quad = \quad G^{-1}\left(\frac{i_S - i^*}{2s} + \frac{1}{2}\right). \qquad [11]$$

Substituting T_S^* into the objective function gives the expected cost function with information set I_s:

$$\begin{aligned} V(T_S^* \mid I_S) \quad &= \quad (i_S - i^* + s)b \quad + \quad s \int_{k(i_S, i^*, s)}^{\infty} \xi dG(\xi) \quad - \quad s \int_{-\infty}^{k(i_S, i^*, s)} \xi dG(\xi) \\ &= \quad (i_S - i^* + s)b \quad + \quad K(i_S, i^*, s) \end{aligned} \qquad [12]$$

where the upper case $K(i_S, i^*, s)$ is defined as the last two terms on the first line of [12].[9]

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[9] If ξ is normally distributed with mean zero and variance φ^2, then

$$K(i_S, i^*, s) \; = \; 2 s \varphi n\left(\frac{N^{-1}\left(\frac{1}{2} + \frac{i_S - i^*}{2s}\right)}{\varphi}\right).$$

It represents the expected opportunity cost of being above or below the remaining required balance with information set I_s.

In this model, intertemporal arbitrage is limited by the risk of daily account overdrafts. Therefore, the pre-settlement day interest rate, i_P, may differ from the expected settlement day rate. The pre-settlement day information set, I_p, does not include knowledge of either the remaining balance requirement on settlement day, b, or the settlement day interest rate, i_S:

I_p : *i^*, s, $F(\varepsilon)$, $G(\xi)$, $E(\varepsilon) = E(\xi) = 0$, and, i_P, the pre-settlement day market rate.*

For simplicity, we assume that private banks expect the central bank to achieve its desired interest rate on average on settlement day:

<A6> $E_{I_p}(i_S) \;=\; i^*.$

Then we have:

$$E_{I_p}(V(T_S^* \mid I_S)) \;=\; sb \;+\; E_{I_p}(K(i_S, i^*, s)), \qquad [13]$$

where the final term is an expectation of a complicated nonlinear function, but—crucially—it does not depend on the remaining required balance, b. The value of b is determined by both the targeted balance and account uncertainty on pre-settlement day. With a daily average requirement of R and denoting the end-of-day account balance on pre-settlement day by $T_P + \varepsilon$, the value of b is determined as follows:

$$b \;=\; \begin{cases} 2R, & \text{if } T_P + \varepsilon \le 0 \\ 2R - T_P - \varepsilon, & \text{if } 0 \le T_P + \varepsilon \le 2R, \text{ and} \\ 0, & \text{if } T_P + \varepsilon \ge 2R. \end{cases} \qquad [14]$$

Ignoring discounting between the first and second day of the maintenance period, and using [13] and [14], the pre-settlement day target balance is found from:

$$\min_{\substack{I_P \\ T_P}} \int_{-T_P}^{\infty} \left(i_p - i^* + s\right)(T_P + \varepsilon)dF(\varepsilon) \; - \int_{-\infty}^{-T_P} (i^* + s - i_p)(T_p + \varepsilon)dF(\varepsilon)$$

$$+ \quad 2RsF(-T_P) \; + \; s \int_{-T_P}^{2R-T_P} (2R - T_P - \varepsilon)dF(\varepsilon) \; + \; E_{I_p}(K(i_S, i^*, s)). \qquad [15]$$

The first line of [15] is equivalent to [1]. The first term on the second line is the expected cost, with information set I_p, of meeting the period's entire requirement ($2R$) on settlement day, times the probability of having a negative or zero account balance on day one. The integral on the second line is the settlement-day cost of completing the period-average requirement if that requirement is partially met before settlement day. The final term reflects uncertainty under I_p about the settlement day interest rate, but it is unaffected by the pre-settlement day decision.

The Euler condition thus does not depend on settlement day uncertainties:

$$i_p \; - \; i^* \; + \; s[1 - \; F(-T_P^*) \; - \; F(2R - T_P^*)] \; = \; 0. \qquad [16]$$

One implication of [16] is that, if $i_p = i^*$ and if F is symmetric around the mean of zero, the targeted pre-settlement day balance equals the daily average reserve requirement ($T_P^* = R$). With a normal distribution, the key comparative statics are:

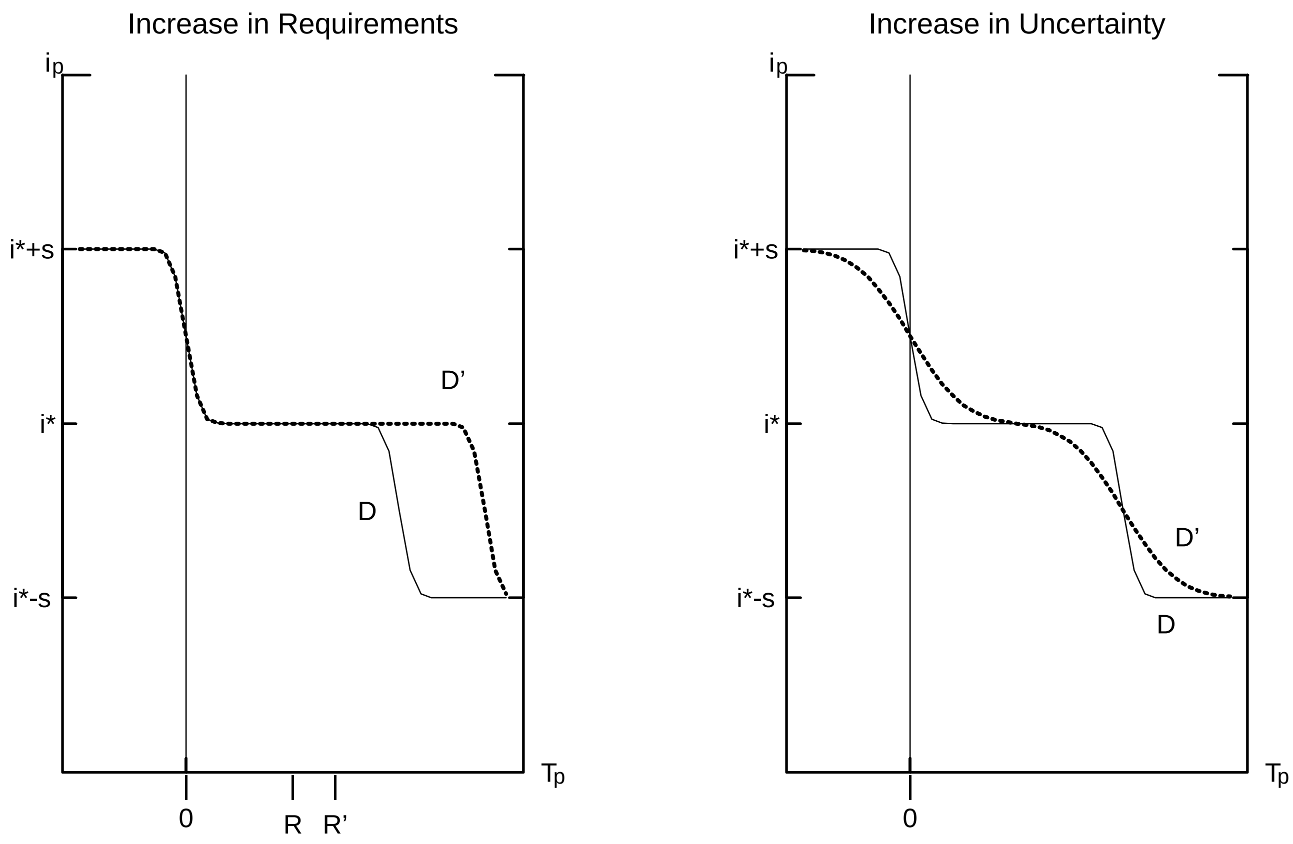
$$\frac{\partial T_p^*}{\partial R} \; = \; \frac{2}{1 \; + \; \dfrac{n(T_p^*/\sigma)}{n\left(\dfrac{2R - T_p^*}{\sigma}\right)}} \; > \; 0 \; . \qquad [17]$$

$$\frac{\partial T_p^*}{\partial i_p} \; = \; \frac{-1}{s\left[n\left(\dfrac{T_p^*}{\sigma}\right) \; + \; n\left(\dfrac{2R - T_p^*}{\sigma}\right)\right]} \; < \; 0 \; . \qquad [18]$$

Under normality, reserve demand curves for two different reserve requirements are depicted in the left panel of Exhibit 5. A key feature is the high elasticity of demand

Exhibit 5: Time-Averaged Balance Requirements

Demand on Pre-Settlement Day



Note: For the baseline (solid lines), the standard deviation of the account shock is 10% of the daily average requirement. The dotted line on the left shows the demand curve with a 40% larger requirement. The dotted line on the right shows demand with a shock standard deviation that is 40% of the requirement.

around the target interest rate for a range of values of T_P^* nearby the daily average requirement. This zone of high elasticity would be extremely helpful for the implementation of policy, as errors in the supply of reserves within the region would have only small effects on the overnight rate. A larger reserve requirement would expand the flat region of the demand curve but would not affect reserve supply uncertainties. The elastic region occurs where the bracketed expression in [16] is close to zero. With a normal distribution, that expression is:

$$N\left(\frac{T_p^*}{\sigma}\right) - N\left(\frac{2R - T_p^*}{\sigma}\right). \qquad [19]$$

The first term is the probability of avoiding an overdraft on pre-settlement day and the second term is the probability of having a positive remaining requirement on settlement day. The first term rises close to unity as T_P^* exceeds 2σ. The second term falls from near unity when T_P^* rises above $2R - 2\sigma$. If the maintenance period were *m* days instead of two days, the latter expression would be $mR - 2\sigma$. This line of reasoning could lead to suggestions for a longer maintenance period in order to engender elastic reserve demand on pre-settlement days in wider regions indicated by:

$$2\sigma < T_P^* < mR - 2\sigma. \qquad [20]$$

However, closed-form solutions of the model quickly become intractable with additional pre-settlement days.[10] With a longer maintenance period, however, some banks that go short, expecting lower interest rates by settlement day, could find themselves running into line limits, which are not modeled here, and as a result, the volatility of the market rate on settlement day could be more elevated. Moreover, with longer maintenance periods,

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10 For instance, the first-order condition for the first day of a three day model, using subscripts for day of period, is:
$i_1 - i^* + s\left\{1 - 2F(-T_1) - \int_{-T_1}^{3R-T_1} F(3R - T_1 - \varepsilon_1 - T_2^*)\,dF(\varepsilon_1)\right\} = 0$, where the optimal second day target, T_2^*, also depends on $3R - T_1 - \varepsilon_1$.

arbitrage of expected changes in the central bank's target interest rate would be more frequent, which the central bank would not find desirable.

The right panel of Exhibit 5 indicates that the elastic region would be smaller with greater account uncertainty in the two-day model but, importantly for aggregation purposes, its location at i^* would not change. The solid line shows the demand curve when σ is 10% of R, while the dotted line gives demand when σ is 40% of R. As defined by [20], the width of the elastic range would shrink from $1.6R$ to $0.4R$ across those two alternatives.

Voluntary Reserve Balance Requirements

In the above model, period-average required balances aid the implementation of monetary policy, but they impose an implicit tax on private banks without conveying any direct benefits to them. The reserve tax could be largely eliminated through the payment of interest on required reserve balances at the market rate of interest. However, despite such interest payments, some banks might prefer to avoid reserve requirements in order to hold assets other than reserve balances. To eliminate completely the inefficiencies associated with reserve avoidance activities, a system of voluntary reserve requirements could be considered.

It is not difficult to imagine an institutional structure in which private banks might obtain some benefit from period-average required balances. A central bank might, for instance, seek to discourage overnight overdrafts by imposing a higher penalty on them than on reserve deficiencies.[11] This penalty structure might reflect true social costs in that providing automatic and potentially unlimited overdrafts could well involve greater risk than offering discount window loans, which may each require the signature of a central bank official to be approved. If the opportunity costs of holding reserves were not

[11] At the Federal Reserve, for example, the charge for overnight overdrafts is 4 percentage points above the effective funds rate on that day, but the penalty for a deficiency in meeting reserve requirements is only 1 percentage point above the primary credit rate, equivalent at present to 2 percentage points above the target funds rate. At the Bank of Japan, overnight overdrafts are charged 6 percentage points above the discount rate, while reserve shortfalls are charged only 3.75 percentage points above that rate. At the ECB, however, overnight overdrafts pay the marginal lending rate, while reserve shortfalls pay 2.5 percentage points higher. See Blenck *et al* (2001).

too high, banks might voluntarily choose to establish a reserve requirement as a precaution against running an overdraft. Below, we explore such a voluntary reserve requirement regime in an analytically convenient, one-period tunnel model. The extension to a two-period framework is a nonlinear simulation exercise.

In this model, the structure of central bank interest rates is:

<A7> Account overdrafts are charged $i^* + s$, *required reserve shortfalls are charged* i^*, *required balances earn* $i^* - \delta$, *and excess reserves earn* $i^* - s$, *where* $0 < \delta < s$.

We write the bank's expected costs as a function of the pre-set reserve requirement, R, with i again the known market rate when the target balance T is chosen:

$$C(R) = \min_T \int_{R-T}^{\infty} \left(i - i^* + s\right)(T + \varepsilon - R)dF(\varepsilon) + \int_{-\infty}^{R-T} (i^* - i)(R - T - \varepsilon)dF(\varepsilon)$$

$$+ \quad (i - i^* + \delta)R \quad - \int_{-\infty}^{-T} s(T + \varepsilon)\,dF(\varepsilon)\,. \qquad [21]$$

The first integral is the opportunity cost on excess reserves; the second integral is the cost of borrowing from the central bank, rather than the market, to fulfill the reserve requirement. On the second line, the first expression is the opportunity cost of holding required reserves and the second is the penalty for account overdrafts. The first order condition is:

$$i - i^* + s[1 - F(R - T^*) - F(-T^*)] = 0. \qquad [22]$$

The response of reserve demand to a higher requirement is:

$$\frac{\partial T^*}{\partial R} = \left[1 + \frac{f(-T^*)}{f(R - T^*)}\right]^{-1} > 0 \quad and \quad < 1. \qquad [23]$$

To determine the bank's optimal choice of a reserve requirement, in principle, the optimal T^* from [22] would be substituted into [21] to obtain a cost function, and the unconditional expectation of that cost function (across the distribution of market interest rates) would be taken. Finally, the optimal value of R would be found by differentiating that unconditional cost function. For convenience, we take the derivative of [21] with respect to R first; the Euler condition for R is:

$$E_{I_u}\left(\frac{\partial C}{\partial R}\right) = E_{I_u}\left(\delta + s\left[F(R^* - T^*) - 1\right]\right) = 0, \qquad [24]$$

where T^* is also a function of R*, given by [22], and I_u is the unconditional information set. Equation [24] is a complex nonlinear function whose solution would depend on the bank's perceived distribution of the market interest rate. That distribution could have a variety of shapes, as suggested by Exhibit 3, reflecting in part the behavior of the central bank. However, some conclusions may be drawn without specializing the model much further. First, with T^* in [22] independent of δ, given R, we can write the comparative static result:

$$\frac{\partial R^*}{\partial \delta} = -\left[E_{I_u}\left(s\,f(R^* - T^*)\left[1 - \frac{\partial T^*}{\partial R^*}\right]\right)\right]^{-1} < 0, \qquad [25]$$

using [23]. Now we assume merely that banks expect the market interest rate to equal the central bank's desired rate on average:

$$I_u\!:\; F(\varepsilon),\ E(i) = i^*.$$

Then, note that, if the opportunity cost of holding required reserves were equal to that for excess reserves ($\delta = s$), [24] would require $F(R^* - T^*)$ to equal zero, which would imply an optimal reserve requirement of $-\infty$.

Next, observe that condition [24] can be written in an alternative form by substituting for $F(R^* - T^*)$ using [22]:

$$E_{I_u}\left(\frac{\partial C}{\partial R}\right) = E_{I_u}\left(\delta + i - i^* - sF(-T^*)\right) = 0. \qquad [24a]$$

If there were no average opportunity cost to holding reserves ($\delta = 0$), [24a] could be satisfied only if $F(-T^*)$ equaled zero, which would imply $T^* \to \infty$ requiring $R^* \to \infty$.

Finally, we can demonstrate the existence of interior solutions:

Proposition: For small positive values of δ, the optimal choice of a reserve requirement in problem [21] is positive and finite.

Proof: First note that the second order condition for R^* to be a minimum holds:

$$E_{I_u}\left(f(R^* - T^*)\left[1 - \frac{\partial T^*}{\partial R^*}\right]\right) > 0, \qquad [26]$$

using [23]. Next, note that, when $R = 0$, the left hand side (LHS) of [24], after substituting for $F(0 - T^*)$ from [22] and taking expectations, is negative when:

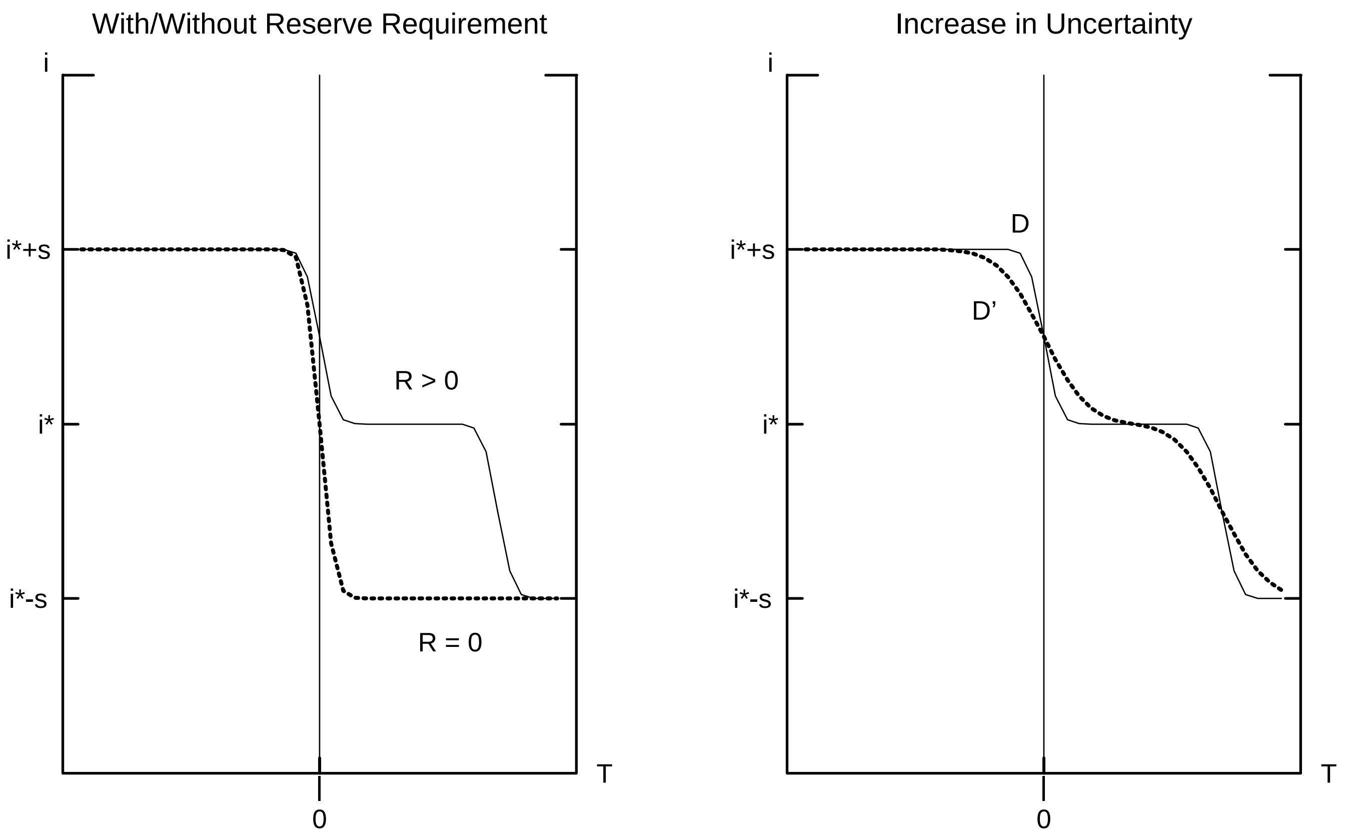
$$\delta < \frac{s}{2}. \qquad [27]$$

Thus, for positive values of δ below this threshold, R^* must be greater than zero. Finally, because $R - T^*$ is increasing in R (from [23]), so is $F(R - T^*)$. As $R \to \infty$, the LHS of [24] goes to δ. Thus, the LHS of [24] must hit zero at a finite positive value for the voluntary reserve requirement. ■

Demand for central bank balances in this model, assuming a normal distribution, is graphed in Exhibit 6. The demand curve without a reserve requirement, given by the dotted line in the left panel, is similar to that in Exhibit 2. Demand for balances equals zero at the central bank's desired interest rate, as the opportunity costs of holding positive balances and of incurring overdraft charges are symmetric around that rate. With a positive reserve requirement, the solid line, the demand curve has an elastic region at the desired interest rate. This elasticity is induced by the lower cost of a reserve deficiency than an account overdraft in this simple one-day model, in contrast to the multi-day model of the previous section, where the elasticity induced by reserve requirements was attributable to intertemporal arbitrage over the reserve-averaging period. As suggested by the right panel, the size of the chosen reserve requirement relative to the account balance uncertainty determines the width of the elastic region, but not its location. The central bank could control the optimal choice of reserve requirement by adjusting the

Exhibit 6: VOLUNTARY RESERVE REQUIREMENTS

Demand for Central Bank Balances



Note: On the left, the dotted line is without a reserve requirement, while the solid line in both charts assumes a voluntary reserve requirement 15 times the standard deviation of the account shock. The dotted line on the right is a requirement that is 5 times the account shock standard deviation. The requirement choice depends on other parameters of the model that are not depicted here.

spread δ. Presumably, the central bank would set the spread to ensure that it had sufficient required balances for the smooth implementation of monetary policy without absorbing too many funds from the banking system.

Conclusion

This paper has discussed a tunnel system for the implementation of monetary policy through standing facilities for lending and interest-bearing deposits at the central bank. It has reviewed the advantages in such a system of symmetric spreads on lending and deposits at the central bank around its desired interest rate. The features of such a model might be useful to consider even for central banks that do not pay interest on deposits but that approach the zero bound on the policy interest rate.

The paper has also analyzed the continuing advantages of period-average account balance requirements, even in the presence of a tunnel system for policy implementation. The evaluation has been conducted through the development of a multi-day model in which private banks subject to account position uncertainties may need to resort to borrowing from the central bank at a penalty interest rate to avoid daily overnight overdrafts and period-average reserve deficiencies.

As shown here, reserve requirements may be beneficial for the implementation of monetary policy. However, they have also been associated with financial market inefficiencies owing to the reserve avoidance activities of banks that may not be justified on the grounds of optimal taxation. One way of addressing the trade-off between the policy implementation benefits and the efficiency costs of reserve requirements would be for a central bank to pay a market rate of interest on required reserve balances. Another possible solution would be a system of voluntary reserve requirements. Elimination of compulsory reserve requirements likely would remove the incentives for reserve avoidance more completely than would the payment of interest on required balances. As shown here, banks may optimally choose to establish a reserve requirement in a tunnel regime if the costs of overdrafts exceed the cost of other borrowings from the central bank. This structure may reflect optimal pricing if overdrafts are riskier than other loans.

The paper has pointed out a few of the many complexities of the relationship of private commercial banks to the central bank. Changes in the parameters of such relationships, such as collateralization policies, non-pecuniary costs, the symmetry of explicit charges, the ability to use overdrafts to satisfy reserve requirements, and the imperfect substitutability of market loans for central bank deposits owing to perceptions of credit risk, may have significant effects on the structure and performance of any policy implementation regime. Moreover, the number and heterogeneity of institutions having accounts at the central bank, the availability of real-time information about account positions, and the market frictions arising from line limits or transaction costs may also importantly condition the functioning of the regime. The level of theoretical modeling in this paper is too stylized to be the basis for recommending a reserve implementation framework appropriate across all the idiosyncratic environments faced by individual central banks. However, with other similar papers, it could be used as a stimulus for further analysis and review of the implications of particular institutional arrangements for central bank policy implementation.

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