Money and Inflation

Bennett T. McCallum  
Carnegie Mellon University

Edward Nelson  
Federal Reserve Board

Preliminary

October 1, 2009

Email addresses of authors: bmccallum@cmu.edu; edward.nelson@frb.gov.  
This paper is being prepared for the Elsevier/North-Holland Handbook of Monetary Economics, Volume 3, edited by Benjamin M. Friedman and Michael Woodford. We thank Richard Anderson for useful discussions and Kathleen Easterbrook for research assistance. The views expressed in this paper are solely the responsibility of the authors, and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.
1. Introduction

Extensive and well-publicized developments of the past two decades, most of which are amply documented in contributions to the present Handbook, have greatly reduced the role of monetary aggregates in basic monetary theory and especially in monetary policy analysis. Thus, as is well known, today’s mainstream approach to monetary policy analysis presumes that policy rules reflect period-by-period adjustments of a short-term interest rate—not any monetary aggregate. In addition, the model of private sector behavior is typically written in a manner that includes no reference to any monetary aggregate; this is an approximation, in economies that possess a medium of exchange, but one that seems to be satisfactory for policy purposes.1 Consequently, policy models need not refer to monetary aggregates at all, even when the economy in question does utilize a medium of exchange. Since these models are intended to explain behavior of inflation, as well as movements in aggregate demand and the policy interest rate, current analysis typically ignores the relationship between money and inflation.

The task of the present paper is, accordingly, to consider what if any relationship there is between these variables, and whether there is any substantial reason for modifying the current mainstream mode of policy analysis. The paper’s outline is as follows. In Section 2, we begin with some reflections on the body of thought known as the Quantity Theory of Money. Sections 3 to 6 are then concerned with related theoretical topics and with empirical regularities relating to money growth and inflation. In Section 7 we turn to the implications of a declining demand for a medium of exchange, and in Section 8 we consider analyses of price level determination that focus on interest rates. Section 9 concludes.

2. The quantity theory of money

Any exploration of the relationship between money and inflation almost necessarily begins with a discussion of the venerable “quantity theory of money”—hereafter abbreviated as QTM. There is, nevertheless, considerable disagreement over the meaning of this body of analysis. Popular treatments, and some textbooks, often begin by associating the QTM with the equation of exchange, $MV = PY$, where $M$, $Y$, and $P$ respectively denote measures of the nominal quantity of money, real transactions or physical output per period, and the price level, with $V$ then being the corresponding monetary “velocity.” An outline of the equation

---

1 On this topic, see Ireland (2004), Woodford (2003), and McCallum (2001).
of exchange is perhaps acceptable as the beginning of an exposition of the QTM. But it would be unfortunate to take the QTM and the equation of exchange as interchangeable. The equation of exchange is an identity—it might appropriately be thought of as a definition of velocity. Being an identity, the equation of exchange is consistent with any proposition concerning monetary behavior, and in the absence of restrictions on the behavior of any terms in the equation, cannot be used to characterize a specific monetary theory. To take the QTM as equivalent to the equation of exchange would, consequently, be to deprive it of any empirical or theoretical content.

That somewhat different meanings are assigned to the QTM by different writers can be seen by consulting the writings of Hume (1752), Wicksell (1906/1935), Fisher (1913), Keynes (1936), Friedman (1956, 1987), Patinkin (1956, 1972), Samuelson (1967), Niehans (1978), and Lucas (1980). In fact, the later writers have had in mind quantities of fiat (paper) money whereas the earlier ones were discussing quantities of metallic money. David Hume’s treatments (such as Hume, 1752) considered both the case where an increase in (metallic or paper) money leads to a gradual, proportional rise in prices, and the case of an open-economy where the expansion in metallic money results in an export of that money. Nevertheless, for the currently-relevant case of fiduciary money, there seems to be one basic proposition characterizing the QTM, that is, one common thread that unites various definitions and applications. This proposition is that if a change in the quantity of (nominal) money were exogenously engineered by the monetary authority, then the long-run effect would be a change in the price level (and other nominal variables) of the same proportion as the money stock, with no change resulting in the value of any real variable. This proposition pertains to “long-run” effects, i.e., effects that would occur hypothetically after all adjustments are completed. In real time, there will always be changes occurring in tastes or technology before full adjustment can be effected, so no experiment of this kind can literally be carried out in actual economies. Furthermore, in most actual economies the monetary authority does not conduct monetary policy so as to generate exogenous changes in the stock of money, so nothing even approximating the hypothetical experiment is ever attempted in reality.

Does the foregoing imply that no statement with empirical content can be made about the QTM? We suggest not; the essential point is that the basic QTM proposition given above holds in a model economy if, and only if, the model exhibits the property known as long-run “neutrality of money.” Indeed, the latter concept is defined so as to satisfy the stated proposition. Accordingly,

---

2 The statement is *ceteris paribus*—it concerns effects of the single postulated change.
we argue that the QTM amounts to the claim that actual economies possess the properties that imply long-run monetary neutrality.

This position is closer to that of Patinkin (1972) than that of Friedman (1972), in their celebrated exchange, since Friedman (1956, 1972) preferred to regard the quantity theory of money as a proposition about the demand function for money. Other expositions of Friedman’s—such as Friedman (1987)—did, however, treat the QTM as centering on the distinction between the nominal quantity of money (whose path is implied by the choices of the monetary authority) and the real quantity of money (whose path is determined by the choices of the private sector). The model property that separates the determination of the real and nominal quantities of money corresponds to the long-run monetary neutrality property. Friedman’s emphasis on the demand function for money is therefore reconcilable with an identification of the QTM with monetary neutrality, in the sense that price homogeneity of the money demand function is crucial for long-run monetary neutrality.³

Indeed, long-run monetary neutrality is dependent on homogeneity properties holding across the private sector’s main behavioral relations. Basically, private agents’ objective functions and technology constraints should be formulated entirely in terms of real variables—there is no concern by rational private agents for the levels of nominal magnitudes.⁴ Then implied supply and demand equations will also include only real variables—they will be homogenous of degree zero in nominal variables.⁵ Since supply and demand relations can be estimated econometrically, the QTM has empirical content for structural modeling—it requires that all supply and demand equations have the stated homogeneity property. These equations, if properly formulated, are structural relations that do not depend upon the policy rule in effect.⁶ Their validity or invalidity therefore has nothing to do with the operating procedures of the monetary authority. The QTM does not, consequently, have anything to do with “the exogeneity of money” in actual practice. In particular, it does not matter whether the central bank is using an interest rate or a monetary aggregate (or, say, the price of foreign exchange) as its instrument variable.

---

³ In addition, Friedman (1956) argues that an infinite interest elasticity of the demand function for money is inconsistent with the quantity theory. This constitutes a further overlap of Friedman’s conception of the QTM and that used here, as an infinite interest elasticity must be ruled out to produce the monetary neutrality result.

⁴ The government’s tax regime might imply that budget constraints cannot be written entirely in real terms. For simplicity, we abstract from this case.

⁵ Note that in this (standard) case, the monetary authority must follow a rule that depends upon some nominal variable. Otherwise, nominal indeterminacy will prevail—the model will fail to determine the value of any nominal variable. This is substantially different from the type of “indeterminacy” featured in the recent literature, which is the existence of more than one dynamically stable rational-expectations solution.

⁶ Here we have in mind behavioral relations—e.g., Euler equations.
One of the relations in any complete model for a monetary economy is a demand function for real money balances. As noted above, one condition for long-run neutrality to prevail is that this function must relate the demand for real balances only to real variables (usually including a real rate of return differential that is the opportunity cost of holding money\(^7\) and a real transactions quantity). The money demand relation then implies that the steady-state inflation rate will equal the steady-state rate of growth of the money stock minus a term pertaining to the rate of growth of output or real transactions. An exogenous change (if it somehow occurred) in the rate of growth of the money stock would, therefore, induce an equal change of the same in the inflation rate unless it induced a change in the *rate of growth* of real transactions or the real interest differential. Neither of these possibilities seems at all likely, so the QTM essentially implies that steady-state inflation rates move one-for-one with steady-state money growth rates.

The exposition of the QTM above, in terms of private reactions to an exogenous policy action, would appear at first glance to leave out what is widely regarded as an important policy implication of the QTM. Many observers have noted that the QTM rules out autonomous factors such as increases in the prices of specific types of good (such as food or energy) from being sources of sustained movements in prices. The position is that, by holding the money stock constant in the face of an increase in the price of a specific good, the monetary authority can prevent total nominal spending and so the aggregate price level from undergoing a sustained increase. A stress on the critical importance of monetary “accommodation” in price level determination is embedded in Samuelson’s (1967) definition of the QTM and in many textbook treatments (for example, Mishkin, 2007). In fact, this element is encompassed by the QTM definition given above. Though our statement focused on a policy-induced monetary increase, the process described in the wake of that increase involves a price level reaction that is complete once prices have restored their proportional relation to money. A model in which prices are unrestrained by monetary accommodation would imply that an initial price level increase can trigger an indefinite price level spiral. Thus, our QTM definition, although expressed in terms of exogenous policy actions, involves restrictions on model behavior that imply that the monetary policy response to nonpolicy shocks is crucial in determining the implications of those shocks for price level behavior.

A discussion of several approaches to empirical analysis of the QTM implications appears in Section 4.

---

\(^7\) This differential is the difference between the real—and nominal—rates of return on money and interest-bearing assets. For simplicity, we assume that money is, like actual currency, not interest-bearing.
3. Related concepts

Other concepts, related to but distinct from the QTM’s long-run monetary neutrality, deserve brief mention. The first of these is the *superneutrality* of money. The QTM proposition, with its implication that steady-state inflation rates move one-for-one with steady-state money growth rates, does not imply that different maintained money-growth (and inflation) rates have no lasting effect on real variables. In particular, it does not rule out permanent effects on *levels* of output, consumption, real interest rates, etc. A higher inflation rate, for example, typically implies an increased nominal interest rate and therefore an elevated spread between the rates of return on money and securities. Such a change raises the interest income foregone when holding real money balances, so rational agents will reduce the fraction of their assets held in the form of money. In many cases, the implied type of portfolio readjustment will lead to changes in the steady-state capital/labor and capital/output ratios, which are key real variables.

In the case where *no* change in real variables occurs with altered steady-state inflation rates, the economy is said to possess the property of “superneutrality.” From what has been said, however, it should be clear that superneutrality should not be expected to hold in economies in which money provides transaction-facilitating services, as it does normally in most actual economies. It is plausible that the departures from superneutrality in practice will be small, for reasons discussed in McCallum (1990). Thus, for example, a shift in the steady-state inflation rate from 0 percent (per annum) to 5 percent would imply a fall in the steady-state real rate of interest of perhaps only about 0.04 percent. Superneutrality will therefore be a property that holds approximately.

One of the variables that is insensitive to alternative ongoing inflation rates when superneutrality holds is the real rate of interest (for example, the one-period rate). The absence of superneutrality, on the other hand, implies that a change in the steady-state inflation rate may change the steady-state real rate of interest. It should be noted such a change is entirely consistent with the so-called “Fisher equation,” which in its linearized form may be written as \( r_t = R_t - E_t \pi _{t+1} \) (with \( \pi \) being the net rate of inflation). The latter should be thought of as an identity—that is, as a definition of \( r_t \). The literature arguably contains some confusion on this matter, with some writers treating the Fisher equation as a behavioral equation that separates nominal from real variables, going on to claim that the Fisher equation is contradicted if an

\[ (1 + R_t) = (1 + r_t)(1 + E_t \pi _{t+1}). \]

---

8 For this calculation, involving specific assumptions about functional forms and quantitative magnitudes, see McCallum (2000, pp. 876–879).

9 Actually, the exact discrete-time expression is \( (1 + R_t) = (1 + r_t)(1 + E_t \pi _{t+1}) \).
altered inflation rate produces a (steady-state) shift in the real interest rate. In the Sidrauski-Brock model, the steady-state real rate of interest is indeed independent of the steady-state rate of inflation, but the same feature is not true in a typical overlapping-generations model, even though the Fisher equation holds in both models (McCallum, 1990).

There is another widely used concept involving long-run relationships, a distinct property in its own right but sometimes incorrectly regarded as part and parcel of superneutrality. This is the “natural rate hypothesis” (NRH), introduced by Friedman (1966, 1968) and refined by Lucas (1972). Friedman’s version of this hypothesis states that differing steady-state inflation rates will not keep output (or employment) permanently high or low relative to the “natural-rate” levels that would prevail in the absence of nominal price stickiness. Lucas’s version is stronger; it states that there is no monetary policy that can permanently keep output (or employment) away from its natural-rate value, not even an ever-increasing (or ever-decreasing) inflation rate. Note the distinction between these concepts and superneutrality: an economy could be one in which superneutrality does not obtain—in the sense that different permanent inflation rates lead to different steady-state levels of capital and thus natural levels of output—but the economy could satisfy the natural-rate hypothesis.

The validity of the NRH, or Friedman’s weaker version called the “accelerationist” hypothesis, was a subject of considerable debate starting in the late 1960s. Lucas (1972) and Sargent (1971) pointed out that the initial tests (such as those of Solow, 1969) were inconsistent with rational expectations, and later evidence favored the NRH, which by the early 1980s had become integrated even into Keynesian treatments (see, for example, Gordon, 1978, or Baumol and Blinder, 1982). In the last decade and half, however, what is in effect an overturning of this consensus has occurred, thanks to the widespread adoption of the Calvo (1983) specification of nominal price adjustment. The basic discrete-time form of the Calvo specification implies that in any period only a fraction of sellers may make price adjustments, with all others compelled to hold their nominal prices at their prior values. This assumption leads to the following economy-wide relationship, in which $y_t$ is the log of output, and $\bar{y}_t$ the natural (i.e., flexible-price) level of output:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - \bar{y}_t).$$

(2)

Here $\beta$ is a discount factor satisfying $0 < \beta < 1$. If we take this relation to describe the level of inflation, it implies a steady-state relationship between inflation and the (constant) output gap, i.e., each value of $E[\pi_t]$ is associated with its own constant value of $y_t - \bar{y}_t$. The Calvo
adjustment scheme consequently fails to satisfy even the accelerationist hypothesis, still less the stronger NRH. A minimal step toward remedying this situation would be to replace (2) with something like the following:

\[ \pi_t - \pi = \beta(E_t \pi_{t+1} - \pi) + \kappa(y_t - \bar{y}_t), \quad (2') \]

(as in Yun, 1996, or Svensson, 2003, for example). Here \( \pi \) represents the steady-state inflation rate under an existing policy rule, assumed to be one that admits a steady-state inflation rate. A relationship such as equation (2’) would prevail if those sellers who are not given an opportunity (in a given period) to reset their prices optimally, have their prices rise at the trend rate (rather than holding them constant). Equation (2’) would imply that on average \( y_t - \bar{y}_t \) is zero, thereby satisfying the accelerationist hypothesis, Friedman’s weaker version of the NRH. (Even so, specification (2’) does not imply the stronger Lucas version, which pertains to inflation paths more general than steady states.)

4. Historical behavior of monetary aggregates

Some perspective on the behavior of monetary aggregates in the United States is provided by Figure 1, which plots quarterly observations on four-quarter growth rates of M1 and M2 since 1959. The modern M1 and M2 series were introduced by the Federal Reserve Board in 1980 (with some minor redefinitions thereafter). These series replaced narrower official definitions of each series. Despite their broader coverage, the pre–1980 growth rates of the modern definitions of M1 and M2 closely match those of the prior definitions. A partial demonstration of this fact is given in Figure 2, which plots growth in annual averages of the former M1 aggregate against the corresponding growth in the modern M1 series.

On the choice between M1 and M2 definitions, Friedman and Schwartz (1970, pp. 2, 92) stated: “important substantive conclusions seldom hinge on which definition is used… We have tried to check many of our results to see whether they depend critically on the specific definition used. Almost always, the answer is that they do not…” This conclusion has not

---

10 See Hafer (1980) on the differences between old and new monetary aggregate definitions, and Anderson and Kavajecz (1994) on the history of money stock estimates in the United States. Anderson and Kavajecz credit Abbot (1962) with the invention of the “M1” label. The label “M2” for a broad definition that includes time deposits dates at least to Friedman and Meiselman (1963).

11 The source for the data on old M1 used in Figure 2 is Lothian, Cassese, and Nowak (1983); the vintage of the M1 series tabulated there is close to that used by Lucas (1980).

12 Similarly, Meltzer (1969, p. 97) stated, “I don’t know of any period in which there would be a substantial difference… using one rather than the other definition of money as an indicator of monetary policy.”
proved to be durable. For much of the period since 1970, the M1 and M2 series have moved differently. Regulation Q was cited as a factor promoting discrepancies between M1 and M2 growth in the late 1960s and 1970s. But the abolition of Regulation Q did not bring an end to the discrepancies between M1 and M2 growth. On the contrary, the deregulated environment prevailing since the early 1980s seems to have perpetuated the differences in the behavior of the own rates on M1 and M2. The result has been an intensification of the discrepancies between the growth rates of the M1 and M2 aggregates.

A change in the own rate on the deposits included in a monetary aggregate, holding constant the interest rates on securities, tends to change the real demand for that aggregate. Whether this affects the growth rate of the nominal quantity of money depends on the operating procedure of the monetary authority. When the Federal Reserve uses an interest-rate instrument, it must acquiesce to the implications for money growth of its interest-rate choices. Consequently, the discrepancies between M1 growth and M2 growth in practice frequently reflect the different opportunity costs associated with each aggregate.

Discussions of the effect of financial deregulation and innovation on the behavior of monetary aggregates routinely make the claim that the advent of payment of interest on M1 deposits has greatly changed the character of M1. While this argument appears to be important for the analysis of the international experience with deregulation, it has limited validity for the United States. The prohibition on interest on demand deposits has in fact never been lifted in the United States. The M1 series, as redefined in 1980, does include, in addition to currency and demand deposits, the category of other checkable deposits (OCDs), i.e., certain non-demand, checkable deposits that can legally bear interest. The OCD component of M1 rose relative to the demand deposit portion of M1 during most of the 1980s, suggesting that the interest return on OCDs had some attraction to bank customers. But, on the whole, it seems that explicit interest on M1 deposits has not proved to be a major factor affecting portfolio decisions. Convention, surviving regulations, and continuing differences in the transactions services provided by M1 funds compared to non-M1 M2, have all meant that the own rate on M1 has rarely been attractive relative to other deposit rates even in the era of deregulation.

---

13 For example, the discussion in Lucas (2000, p. 170) suggests that U.S. demand deposits formerly could not bear interest, but now can do so. Many similar statements by other authors could be cited.

14 For example, the table of rates on M1 deposits in the United Kingdom provided in Hendry and Ericsson (1991, p. 876) indicates that U.K. transactions deposits went from non-interest-bearing at the start of 1984 to earning 7.5% annual interest rates on average at the end of the year.
Figure 1. Growth in M1 and M2

Percent change on previous year

M1  M2


Figure 2. Pre-1980 and new definition of M1
(annual averages, percent change)

M1 growth (new)  M1 growth (old)

The fall in M1 velocity in the 1980s has occasionally been attributed to the payment of interest on M1. But M1 velocity movements up to the late 1980s appear to be well captured by the declining opportunity cost of holding money as recorded in market interest rates, without recourse to an explanation that involves a changing own rate on M1 (Lucas, 1988; Hoffman and Rasche, 1991; Stock and Watson, 1993).

Generally speaking, therefore, the whole of M1 is interest sensitive, and a rise in securities market interest rates promotes flows out of M1 balances. By contrast, from the late 1970s onward, the proportion of non-M1 M2 deposits bearing market-related interest rates rose considerably, standing at over 60% by early 1982 (Gramley, 1982). The overall interest sensitivity of M2 comes primarily from the fact that the rates on several classes of deposit within M2 adjust to securities market interest rates only with a delay.

A different means through which financial innovation affects M1 behavior has proved to be much more significant in practice. “Sweeps” programs allow routine transfers, at the banks’ initiative, between M1 deposits and non-M1 deposits. An embryonic version of this arrangement developed during the 1970s in the form of the automatic transfer system (ATS) (see Hafer, 1980), but extensive adoption of retail sweep deposit programs on the part of banks did not take effect until January 1994 (Anderson, 2003). The arrangement is attractive to depositors because of the better returns on non-M1 M2 deposits, and appeals to banks as a means of avoiding the more onerous reserve requirement on M1 deposits. The resulting portfolio behavior is believed to have created variations in M1 that have little macroeconomic meaning, with Anderson (2003, p. 1) arguing, “Retail-deposit sweep programs are only accounting changes: they do not affect the amounts of transaction deposits that banks’ customers perceive themselves to own.” (Italics in original.) A series of studies (including Jones, Dutkosky, and Elger, 2005, and Cynamon, Dutkowsky, and Jones, 2006) has attempted to correct the U.S. monetary aggregates for the effect of the sweeps program. Figure 3 plots growth in M1 against growth in an adjusted M1 series. The deposits component of this adjusted series, following Ireland (2009), replaces reported M1 deposits after 1993 with the Cynamon-Dutkowsky-Jones M1 deposit series that corrects for sweeps. In addition, the adjusted series used in Figure 3 excludes the Federal Reserve Board’s estimates (available from 1964 onward) of U.S. currency held abroad, as reported in the flow of funds. We see from Figure 3 that these adjustments, on balance, lead to a more moderate decline in M1 growth in the late 1990s.
Figure 4 plots the velocities of M1 and M2. As is well known, the combination prevailing before the early 1980s was of an upward-trending M1 velocity and a stationary M2 velocity. As is also well known, M1 velocity underwent a major break in trend after 1981. (The apparent resumption of an upward M1 velocity trend in the late 1990s is largely illusory, reflecting the sweeps programs.) The presentation of both series on the same scale in Figure 4 means that M2 velocity appears very stable over the whole sample. But on closer inspection there are several notable shifts in the series—including a fall in M2 velocity with the introduction of money market deposit accounts in 1983 Q1, followed by a major velocity rise in the mid-1990s, and a decline, not fully reversed, that occurred during the monetary policy easing and international turmoil of 2001–02.

One argument that has been advanced to explain the stability of M2 velocity relative to M1 velocity is that M2 constitutes a more homogeneous aggregate. The sweeps program itself tends to produce variations in M1 that cancel within M2, and so provides a more or less mechanical basis for why M2 should be considered the more homogeneous series. But it is also possible that M2 might be preferable even from the perspective of standard theories of money demand. While the M1 definition was intended to capture the concept of transactions balances, much of the non-M1 component of M2 might perform transactions services. In that case, the medium-of-exchange concept of money might better be represented by M2. Dorich (2009) argues that M2 should be used as the empirical measure of transactions money, and Reynard (2004) does so excluding one class of M2 deposit (namely, small time deposits, in recent years about one-seventh of M2). Arguing somewhat against the use of M2-type series as measures of transactions money, at least for studies using long sample periods, are the empirical results coming from the Divisia procedure, which Lucas (2000) argues is the best way to construct monetary aggregates. The Divisia approach in practice produces a series that downweights much of the non-M1 component of M2, and leads to quite different behavior of M2 and Divisia M2 during key episodes in the 1970s and 1980s (see Barnett and Chauvet, 2008).

---

15 The behavior of M2 demand during the 1990s has been the subject of numerous studies, including Duca (1995), Lown, Peristiani, and Robinson (1999), and Carlson, Hoffman, Keen, and Rasche (2000).
Figure 3. Growth in M1 and adjusted M1

Figure 4. Quarterly values of M1 and M2 velocity
5. Flawed evidence on money growth-inflation relations

A number of procedures have been widely advanced as yielding evidence—pro or con—regarding quantity-theory relations between money growth and inflation. Three of the most prominent test procedures, however, are flawed. These are tests based on: (i) determining long-run money demand stability; (ii) regressions of inflation on money growth (or scatter plots of the series) using country-average data); (iii) long moving averages of time series data. We discuss each in turn.

Evidence on money demand stability: Quantity-theory relations between money growth and inflation do not depend on constancy of all parameters in an estimated money demand function, nor on cointegration among the components of the money demand function. To see this, let us write down a standard money demand equation:

$$ \log \left( \frac{M}{P} \right)_t = c_0 + c_1 \log(Y_t) + c_2 R_t + c_3 t + \epsilon_t $$

(3)

where $c_1 > 0$, $c_2 < 0$. This is the typical specification (possibly with aggregate consumption $C_t$ substituting for aggregate output $Y_t$) that would emerge from utility analysis (e.g., McCallum and Goodfriend, 1987; Lucas, 1988, 2000), other than our inclusion of the $c_3 t$ term. This linear trend term is designed to capture smooth progress in payments technology. If the financial system develops in a way that allows agents to economize on their money holdings over time, then $c_3 < 0$. With a unitary income elasticity and a stationary nominal interest rate, the trend term implies a rising trend in velocity, i.e., real balances grow at a slower rate than real income.

Money demand and cointegration studies are often motivated by the claim that money demand stability is a condition for quantity-theory relations between money growth and inflation. Lucas (1980), however, rejects the alleged dependence of a money growth/inflation link on money demand stability. There are several reasons to support Lucas’ position. For example, a unit root in $\epsilon_t$, the money demand shock in equation (3), would be considered a violation of dynamic stability in the money demand function, implying no cointegration and, by some definitions, money demand instability; but it would imply a first-difference relation,

$$ \Delta \log \left( \frac{M}{P} \right)_t = c_1 \Delta \log Y_t + c_2 \Delta R_t + c_3 + \Delta \epsilon_t $$

(4)
and hence a unitary money growth/inflation relationship, conditional on other variables. In particular, with stationary \( R_t \) behavior,

\[
E[\Delta \log M_t] = E[\pi_t] + c_3 + c_1 E[\Delta \log Y_t]
\]  

so that there is on average a one-for-one relation between money growth, adjusted for output growth, and inflation. Hence, as argued by McCallum (1993), lack of cointegration between the levels of money (or money per unit of output) and prices is not a problematic result for the quantity theory.

Likewise, a change in the intercept term in the money demand function would permanently shift the relationship between the levels of money and prices, but would, once the shift to the new intercept was complete, wash out entirely from the first-differenced money demand function which is the underpinning of the money growth/inflation relationship. Furthermore, a one-time shift in the long-run interest semielasticity of money demand, such as has been argued by Ireland (2009) to have occurred in recent years in the case of M1 demand, does not affect the longer-term relation between money growth and inflation, provided \( \Delta R_t \) averages zero. Summing up, while the price level homogeneity of the money demand function is crucial for delivering quantity-theory relations, instability in several other aspects of the long-run money demand relation does not preclude a close relation between money growth and inflation.

It should furthermore be clear that, as Lucas (1980) also argued, money demand stability is consistent with a weak relationship between inflation and monetary growth. The case of M1 in the United States is perhaps the best example. As noted above, long-run M1 demand behavior up to the late 1980s appeared explicable via a standard demand function for money. But the M1 growth/inflation relationship seemed to break down in the early 1980s. The discrepancy between M1 growth rates and inflation is attributable to the sustained change in the opportunity cost of holding money. The \( \Delta R_t \) term in equation (4) above, instead of averaging zero, was negative on average, and this declining opportunity cost of holding money promoted a recovery of real money balances. To be sure, a nonzero \( \Delta R_t \) was not exceptional by postwar standards. The \( \Delta R_t \) term had been on average positive in the 1950s, 1960s, and 1970s. This led Barro (1982) to dispute the way that contributions of velocity growth to inflation were typically characterized in presentations of the quantity theory. These expositions tended to treat velocity growth arising from interest-rate increases as a “one-time” factor, affecting the price level but not the trend of prices. Barro pointed out that,
with $R_t$ in practice trending upward, the contribution that velocity growth made to U.S. inflation, when measuring money with the M1 definition, was in fact substantial. The contribution of $\Delta R_t$ to velocity growth over these decades was, however, steady enough that it did not prevent a close correlation between inflation and prior monetary growth. After 1981, the trend of $R_t$ turned downward. But the actual decline in $R_t$ and associated fall in velocity, came in spurts. For example, the decline in the federal funds rate that took place in the second half of 1982 was almost entirely reversed in the course of the Federal Reserve’s tightening over most of 1983 and 1984; but in 1985 and 1986, interest rates fell to levels not seen since the early 1970s. Thus, instead of the interest-rate decline contributing to a more or less constant difference between M1 growth and inflation, it affected M1 velocity growth markedly in specific periods, notably mid-1982 to mid-1983 and 1985–86, essentially wiping out the correlation between inflation and money growth once these periods were incorporated into calculations.

The downward trend in nominal interest rates has continued in the 1990s and 2000s, with both the real interest rate and the expected-inflation component declining. While financial developments such as sweeps have undoubtedly contributed to distortions to both M1 growth and M1 demand, one should not expect a close money growth/inflation relation even in the absence of such distortions, because of the uneven but substantial shifts in the opportunity cost of holding money.

_Evidence with country-average data:_ One popular way of scrutinizing quantity-theory relations is to construct per-country average observations on money growth and inflation, for use in scatter plots or in panel-data regressions of inflation on money growth. When high double-digit inflation countries are included, scatter plots of annual averages of money growth and inflation tend to bring out an impressive relation (see, for example, Friedman, 1973, p. 18; Lucas, 1980, Figure 1; and McCandless and Weber, 1995, Chart 1). Results for countries which have experienced average inflation in single digits tend to be more mixed. For example, Issing, Gaspar, Angeloni, and Tristani (2001, p. 11) display a scatter of money growth/inflation per-country averages for “low-inflation” countries; they treat the quantity theory of money as implying a unitary slope for the plot, and fail to reject this slope restriction. De Grauwe and Polan (2005), on the other hand, find a poor relation between averages of money growth and inflation for low-inflation countries, although much better results have been reported in an exercise by Frain (2004) using the same sources for data as De Grauwe and Polan.
Favorable or unfavorable, these results using cross-country data are flawed as evidence on the quantity theory. A limiting case brings out the point. Consider two countries, $A$ and $B$, in both of which there is no change in real income or nominal interest rates over time, and no money demand shocks. Then the first-differenced money demand equation implies that the money growth/inflation correlation is perfect in each country, i.e., $\Delta \log M_{ti} = \Delta \log P_{ti} + c_{3i}$, for $i = A, B$. But the non-inflationary rate of money growth will not be identical across countries, except in the special case of identical trends in payment technology, $c_{3A} = c_{3B}$. The flaw in tests of the quantity theory based on cross-country averages is that they impose a constant $c_3$ value across each country—in essence, a common trend to velocity across countries.

Studies of money growth and inflation across countries have rarely recognized this point; an exception is Parkin (1980, p. 172), who correctly noted for six major countries that “there is virtually no association between averages of inflation and money growth,” owing not to the absence of a within-country money growth/inflation link, but to “different trend changes in the demand for M1 balances arising from financial innovations.” The point is of crucial quantitative significance when it comes to studying low-inflation countries. To take an example, Germany had lower inflation in the United States over 1962–79: 3.7% CPI inflation in Germany, 4.9% in the United States. But M1 growth over 1962–79 averaged 8.3% in Germany (with 4.6% growth in M1 per unit of output) and 5.3% in the United States (1.4% growth in per-unit terms). An approach that focused on these cross-country averages would suggest that inflation was not closely related to money growth. But, in each country, inflation was in fact highly correlated with prior M1 growth over the 1962–79 period, with time series evidence supporting an approximately unitary relation. The cross-country approach neglects the different velocity trends across countries and fails to bring out the money growth/inflation relation that is obtainable from time series evidence.16

Admittedly, under very high inflation conditions, the trend in velocity due to exogenous improvements in payments technology is typically swamped by other factors: the inflation rates associated with rapid rates of money growth are large relative to the exogenous velocity trend.17 This accounts for why high inflation results often look impressive in cross-country average evidence despite the flawed nature of this evidence.

16 For studies that use the monetary base or reserve money as the empirical measure of money (such as Haldane, 1997), an additional factor distorting comparisons across countries is the failure to adjust for changes in reserve requirements. McCallum and Hargraves (1995) provide illustrations of the historical importance of this factor.

17 In cases of hyperinflation, trends in velocity may continue to reflect developments in financial processes, but it would no longer be appropriate to treat this development as taking place smoothly and exogenously. Steep
Evidence using moving averages of time series: It was noted above that an implication of the QTM is that steady-state money growth rates and steady-state inflation rates are linked one-for-one, once allowance is made for output growth. Lucas (1980, 1986) argues that, in studying time series of a particular country, this steady-state relation can be brought out by taking long moving averages of monetary growth and inflation. Lucas (1986, p. S405) goes so far as to say, “Without such averaging, the quantity theory… does not provide a serviceable account of comovements in money and inflation.” The argument that taking long moving averages of time series is the way to recover close money growth/inflation relations is also made in empirical studies such as Dewald (2003).

One objection to this procedure, which is not the criticism on which we focus here, is examined in detail by Sargent and Surico (2008). The interpretation of coefficient estimates in a regression of inflation (or its moving average) on a moving average of monetary growth will depend on whether past quarters’ money growth rates (which enter the calculation of the moving average) are actually standing in for expectations of future money growth. If that is so, then the coefficient estimate associated with the average-money-growth term will not tend to 1.0 even in an environment where the quantity theory is valid; it will be a function of the policy rule parameters, for the same reason as that discussed in the natural rate literature.

Sargent and Surico explore the behavior of the coefficient on the money growth term in moving-average regressions from simulations of a variety of models. Some of the models and parameter values contemplated do deliver large departures from a unitary money growth/inflation relation, and hence serve as one argument against the Lucas moving-average approach. But the practical relevance of their results for monetary policy models used in practice is open to question. Even under the conditions contemplated by Sargent and Surico, the coefficient on average money growth does tend to unity if long-term inflation is a unit root process, as it is assumed to be in Smets and Wouters (2007) and Woodford (2008), for example. Moreover, as detailed below, when we simulate a standard New Keynesian model with a standard interest-rate rule, the money growth/inflation relation is approximately unitary even when money growth and inflation are stationary.

trends in velocity can emerge as holders of money balances make more intensive efforts to reduce the fraction of their assets in the form of money. These trends tend to reinforce the money growth/inflation correlation, but also to push the slope describing their relationship away from unity; the induced reaction of velocity growth leads to a more than one-for-one reaction of inflation to monetary growth.

18 The unconditional means of inflation and monetary growth, however, retain a unitary relationship with one another.
Our criticism of the Lucas (1980) procedure is somewhat different. Time-averaging is advertised as a means of allowing for lags—especially by McCandless and Weber (1995)—but in practice it may do so poorly. In particular, long averaging does not appear in practice to deliver any greater improvement in fit of the QTM than be obtained by retaining the non-averaged time series data.

To see this, consider the data Lucas (1980) used in studying the United States. He used second-quarter observations for M1 growth and CPI inflation for 1955–75. Using the modern vintage of CPI data and the Lothian-Cassese-Nowak (1983) data on old M1 (which are close to the data used by Lucas), and taking the four-quarter log differences for each second-quarter observation, we present three regressions in Table 1. The first regresses inflation on money growth for 1955–75. This was the relationship which Lucas characterized as loose and which motivated his use of moving averages. The second regression replaces the annual data with (overlapping) five-year averages of the data (the average for 1956–60 being the first observation, 1957–61 the second, etc., for a total of 16 observations). The third and fourth regressions return to the annual data (with sample periods 1955–75 and 1960–75, respectively), but instead of regressing inflation on the current year’s money growth, they regress inflation on money growth two years earlier.

Table 1. M1 growth/ CPI inflation relationship using different degrees of time aggregation, United States, 1955–1975

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Explanatory variable</th>
<th>Sample period</th>
<th>Coefficient on money growth term</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual inflation</td>
<td>Annual money growth</td>
<td>1955–1975</td>
<td>0.515 (0.236)</td>
<td>0.200</td>
</tr>
<tr>
<td>Five-year moving average of inflation</td>
<td>Five-year moving average of money growth</td>
<td>1960–1975</td>
<td>0.832 (0.134)</td>
<td>0.732</td>
</tr>
<tr>
<td>Annual inflation</td>
<td>Annual money growth  lagged two years</td>
<td>1955–1975</td>
<td>0.809 (0.178)</td>
<td>0.518</td>
</tr>
<tr>
<td>Annual inflation</td>
<td>Annual money growth  lagged two years</td>
<td>1960–1975</td>
<td>0.829 (0.214)</td>
<td>0.517</td>
</tr>
</tbody>
</table>

Note: The annual data underlying the regressions are for four-quarter growth rates of M1 and the CPI for the second quarter of the year.

Moving averages do have the effect of moving the coefficient on money growth from significantly below unity to above 0.80—and insignificantly different from unity. But so does the procedure of retaining the annual data while replacing current money growth with
lagged money growth. It is clear from the regressions that the improvement in the performance of the QTM as one moves from time series to averaged data is no better than that delivered by a time series calculation that allows for an interval between movements in money growth and in inflation.

We suggest that this result is not special to Lucas’ example. On the contrary, the timing relationships between money growth, nominal income growth, and inflation mean that similar results are likely to show up using other sample periods and other countries. Replacing a regression of inflation on money growth with moving averages of the same series changes the right-hand-side variable from current money growth to an average of current, prior, and future money growth terms. But movements in money growth tend on average to lead movements in inflation—a regularity noted even in classic contributions on the quantity theory by Hume (1752) and Wicksell (1906/1935), and stressed in the monetarist literature, especially by Milton Friedman from 1970 onward (for example, Friedman, 1972, 1987). It is a regularity that continues to be found in studies using more recent data (see Batini and Nelson, 2001; Christiano and Fitzgerald, 2003).

Superficially, time-averaging might seem to go in the right direction in allowing for the lag, as the averaging introduces prior money growth into the right-hand-side monetary term. But it is inadequate if inflation regularly follows money growth. A regression of time-averaged inflation on time-averaged money growth still implies a relationship between inflation and money growth that is on average contemporaneous; future money growth rates enter the right-hand-side expression with the same weight. In practice, taking moving averages seems a poor way for controlling for the lag in the relationship between money growth and inflation. It is preferable to continue to use non-averaged time series data. With this background in mind, let us consider the empirical relationship between money growth and inflation in U.S. time series.

6. Money growth and inflation in time series data

A useful starting point for a discussion of money growth/inflation relations in time series data is Sargent and Wallace’s (1981, p. 1) characterization: “Friedman did assert that a monetary authority could exert substantial control over the inflation rate, especially in the long run.” Taken together with Lucas’ identification of the long run with long averages of data, this interpretation could deprive the quantity theory of much of its policy content at the business cycle frequency—one might argue, for example, that quantity-theory considerations should
not be a factor in those monetary policy decisions that are concerned with inflation’s behavior two to five years ahead. Such a line of attack is made by Svensson (1999, p. 215) who states that “this long-run correlation is irrelevant at the horizon relevant for monetary policy.” It is seemingly supported by Assenmacher-Wesche and Gerlach’s (2007, p. 535) observation that “money growth and inflation are closely tied only in the long run.” Svensson’s claim that a very long-run relationship lacks any policy relevance is doubtful, since policymakers are concerned with very long-term inflation expectations. But the more general notion that quantity-theory considerations only “bite” at very long horizons does seem to reduce the QTM’s relevance for monetary policy decisions.

We would argue that to consign the money growth/inflation relation to a far horizon—to accept it as applying to long run relationships, but not to medium-run relationships with which policymakers are also concerned—is not justified either by Milton Friedman’s work or by the empirical evidence. Certainly, a money growth/inflation relationship at very low frequencies is implied by the quantity theory, and some of Friedman’s writings, e.g., Friedman (1960, p. 86), emphasized the influence of monetary policy on the trend of prices rather than on inflation at shorter horizons. But the “always” in Friedman’s (1963) proposition that “inflation is always and everywhere a monetary phenomenon” indicates that he did not confine his vision of the money growth/inflation relation to very long horizons. Indeed, Friedman (1950) argued against treating the quantity theory as affecting data patterns in the long run only; and Friedman (1956, para. 25) argued that the theory explained “substantial changes over short periods in the stock of money and in prices.”

This is not to deny that empirical problems with the measurement of money, such as have occurred due to the sweeps program in the United States and to the existence of nontransactions deposit categories in M2, will distort the relationship between monetary growth and inflation. But this is not, so far as we can see, a low-frequency vs. high-frequency data issue per se; it seems unrealistic to expect that measurement problems matter only for the cyclical relationship and wash out of the long-run relationship. Even for the defective measures of money that practitioners have to work with, we would argue that to recover the relation between money growth and inflation, a filter to the data that extracts low-frequency information is not required.

The argument that a relationship between money growth and inflation exists at the business cycle frequency does not, incidentally, rest on any claim that money appears in the structure of the IS or Phillips curves that describe spending and pricing decisions. Neither New
Keynesian nor monetarist analyses imply the presence of money in the structural IS and Phillips curve equations, even though quantity-theory relations do prevail in models featuring these equations. Indeed, since as Lucas (1986, p. S405) observes, “a change in money does not automatically cause prices to move equiproportionally in any direct sense,” one important function of models of monetary policy analysis is to spell out the indirect process that tends to produce an equiproportionate relation between prices and money.

Different models have been designed for this purpose—ranging from models featuring nominal price stickiness due to nominal contracts, to flexible-price models with imperfect information. Across models, the response to a monetary expansion invariably involves a rise in nominal spending along the way to higher prices. But the more realistic models also need to account for the well-established dynamic relations between nominal spending and its output and price components. Two important aspects of these relations, discussed presently, are: (i) the short-run covariation of real and nominal spending; and (ii) the delay, on average, between changes in nominal spending and in the price level. The robustness of these regularities across sample periods and countries argues against trying to explain them by appealing to a particular policy regime, and instead for an approach to model specification that delivers these regularities for a variety of assumed monetary policy rules. These regularities have implications not only for appropriate model specification but for empirical procedures for ascertaining the medium-run relation between money growth and inflation.

Regularity (i): Nominal and real spending move together in the short run

In their study of U.S. monetary history, Friedman and Schwartz (1963, p. 678) observed that “real income tends to vary over the cycle in the same direction as money income does…” This observation holds true for U.S. data beyond the period covered by Friedman and Schwartz. McCallum (1988, p. 176) reports a correlation above 0.8 for 1954–85 quarterly changes in U.S. nominal and real GNP. Likewise, Brown and Darby (1985, p. 192) conclude from a study of annual data for several major countries that, contemporaneously, “the course of money income is much more closely related to that of real income than of price.”

---

19 This attitude is also voiced by Dotsey and King (2005, p. 227).
20 Likewise, the correlation between quarterly real GDP growth and quarterly nominal GDP growth for the United States for the period 1954 Q3–2009 Q2 is 0.82. This calculation, like those in Tables 1 to 7, uses log-differences to measure percentage changes.
Table 2. Correlations of inflation and nominal income growth
(Inflation in year $t$, nominal income growth in year $t-k$)

<table>
<thead>
<tr>
<th></th>
<th>GDP deflator inflation</th>
<th></th>
<th>CPI inflation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 0$</td>
<td>$k = 1$</td>
<td></td>
<td>$k = 0$</td>
</tr>
<tr>
<td>Germany 1957–1998</td>
<td>0.587</td>
<td>0.753</td>
<td>0.209</td>
<td>0.443</td>
</tr>
<tr>
<td>Germany 1980–1998</td>
<td>0.544</td>
<td>0.767</td>
<td>0.182</td>
<td>0.461</td>
</tr>
<tr>
<td>Japan 1959–2008</td>
<td>0.837</td>
<td>0.829</td>
<td>0.720</td>
<td>0.795</td>
</tr>
<tr>
<td>Japan 1980–2008</td>
<td>0.843</td>
<td>0.851</td>
<td>0.716</td>
<td>0.770</td>
</tr>
<tr>
<td>United States 1959–2008</td>
<td>0.624</td>
<td>0.708</td>
<td>0.541</td>
<td>0.709</td>
</tr>
<tr>
<td>United States 1980–2008</td>
<td>0.574</td>
<td>0.661</td>
<td>0.505</td>
<td>0.662</td>
</tr>
<tr>
<td>United Kingdom 1957–2008</td>
<td>0.923</td>
<td>0.834</td>
<td>0.893</td>
<td>0.845</td>
</tr>
<tr>
<td>United Kingdom 1980–2008</td>
<td>0.862</td>
<td>0.902</td>
<td>0.767</td>
<td>0.893</td>
</tr>
<tr>
<td>United Kingdom 1977–2008</td>
<td>0.785</td>
<td>0.860</td>
<td>0.761</td>
<td>0.808</td>
</tr>
<tr>
<td>United Kingdom 1957–1972</td>
<td>0.911</td>
<td>0.929</td>
<td>0.841</td>
<td>0.917</td>
</tr>
</tbody>
</table>

Regularity (ii): Inflation tends to follow nominal spending growth

The second regularity, consistent with but not implied by the first, is that inflation rates tend to be more closely related to prior nominal income growth than to same-period nominal income growth. This phenomenon was noted for the United States by Nelson (1978, p. 4) who stated, “An important conclusion is that the price level is very slow to respond to changes in nominal income.” It is illustrated for several major countries in Table 2, which presents correlations of inflation with current and prior nominal GDP growth, for two measures of inflation (GDP deflator and CPI), using annual data for selected sample periods. The table documents a pronounced tendency for nominal income growth to have a better correlation with the following year’s inflation than with current inflation. The lagged character of this relation is especially notable in the case of deflator inflation, a series which is biased toward having a close contemporaneous correlation with nominal GDP growth because of their connection via an identity.

The full-sample correlations for the United Kingdom in Table 2 would appear to contradict the claim that nominal income growth leads inflation, but in fact do not do so. For most of the first quarter of 1974, the U.K. government imposed restrictions on days worked as an
energy-conservation measure. As a result, recorded rates of both nominal and real U.K. GDP growth were artificially low in 1974, and nominal GDP growth did not peak until the inflation peak of 1975. Correlations for the United Kingdom that omit the mid-1970s observations reestablish a lead of nominal income growth over inflation, as the table shows.

**Implications for money growth per unit of output**

The preceding considerations have implications for the use of money-per-unit-of-output series in studying the money/inflation relationship. With a unitary income elasticity, the demand for money function provides a connection of money to nominal income. As we have seen, the empirical relation between growth rates in nominal income and in prices seems to be close, but with nominal GDP growth tending to lead inflation. Taking these points together leads to the implication that, when money growth is closely related to inflation, it is usually also closely related to nominal income growth. But different lags are relevant in each case; in annual data, money growth tends to be most closely related to current year’s nominal income growth; but its maximum correlation with inflation is typically with inflation one or more years ahead. Consequently, there are problems with the procedure of adjusting money growth for output growth so as to obtain a measure of inflationary pressure. Over long periods, such an adjustment is appropriate, but over short periods, money growth adjusted for output growth may be an inferior indicator to money growth proper.

If correlations of money growth per unit of output growth and inflation are actually roundabout measures of the association between nominal income growth and monetary growth, they fail to capture the lead of money growth over inflation. That this is not simply a hypothetical issue is brought out by considering data for M1 growth and inflation in the United States in the 1960s and 1970s (Figures 5 and 6). The raw M1 growth data clearly lead movements in inflation; but adjusting for output growth delivers merely a contemporaneous money growth/inflation relationship.

Another problem inherent in comparisons of inflation with output-adjusted money growth is that the short-run nonneutrality of money may disguise the inflationary pressure implied by a given amount of money growth. In the late 1970s, for example, loose U.S. monetary policy led to rapid growth in both money and output. The strength of output disguised the longer-
Figure 5. CPI inflation in the 1970s and M1 growth two years earlier, United States

Figure 6. CPI inflation and M1 growth per unit of output in the 1970s, United States
term weakness in output implied by the productivity slowdown. Subtracting output growth from monetary growth in these years gave false comfort, suggesting that policy settings were not as inflationary as they in fact were.

**Money growth/inflation relations in a New Keynesian model**

Having sketched some major aspects of the dynamic money growth/inflation relation in the data, let us turn to insights regarding this relation that come from a structural model. We use the Bernanke-Woodford (1997) model, appended by a money demand function. Other than featuring lagged expectations in the spending and pricing relations, the Bernanke-Woodford model corresponds to the standard New Keynesian specification. Thus, instead of equation (2), the Phillips curve consists of:

\[
\pi_t = \beta E_{t-1} \pi_{t+1} + \kappa (E_{t-1} [y_t - \bar{y}_t]) + e_{\pi_t}. \tag{6}
\]

The IS equation is:

\[
y_t = E_y y_{t+1} - \sigma (E_{t-1} [R - E_{t-1} \pi_{t+1}]) + e_{y_t}. \tag{7}
\]

Here \(\sigma > 0\), and \(e_{\pi_t}\) and \(e_{y_t}\) are Phillips curve and IS shocks, respectively. We retain the money demand function (4), so portfolio decisions are based on realized output and interest rates. To complete the model, we assume that monetary policy follows, up to a white noise shock, a Taylor (1993) rule with smoothing:

\[
R_t = \rho_R R_{t-1} + (1 - \rho_R) (\phi_y y_t + \phi_{\pi} \pi_t) + e_{Rt}. \tag{8}
\]

We set the parameters as follows: \(\beta = 0.99\), \(\kappa = 0.05\), \(\sigma = 0.6\), \(\rho_R = 0.7\), \(\phi_y = 0.125\), \(\phi_{\pi} = 1.5\), \(c_1 = 1\), \(c_3 = 0\). The money demand interest semielasticity \(c_2\) is kept to 4, corresponding to the value suggested for the business cycle frequency by King and Watson (1996). We assume that the IS and Phillips curve shocks are AR(1) with parameter 0.9 and that the monetary policy shock is white noise.

We solve the model and compute impulse responses. Figure 7 plots the response to a monetary policy shock of money growth, inflation, nominal interest rates, and nominal income growth (\(\Delta x\), defined as \(\pi + \Delta y\)). The monetary policy shock lowers the nominal interest rate and leads to an immediate rise in money growth. Because of the delays implied
Figure 7. Responses to a monetary policy shock, Bernanke-Woodford model

Fig. 7a: Dm response to policy shock

Fig. 7b: pi response to policy shock

Fig. 7c: R response to policy shock

Fig. 7d: Dπ response to policy shock
Figure 8. Responses to an IS shock, Bernanke-Woodford model

Fig. 8a: \( D_m \) response to IS shock

Fig. 8b: \( \pi \) response to IS shock

Fig. 8c: \( R \) response to IS shock

Fig. 8d: \( D_x \) response to IS shock
Figure 9. Responses to a Phillips curve shock, Bernanke-Woodford model

Fig. 9a: Dm response to PC shock

Fig. 9b: pi response to PC shock

Fig. 9c: R response to PC shock

Fig. 9d: Dx response to PC shock
by the lagged-expectation terms, real spending (not shown) and inflation react with a delay to interest-rate movements. Thus money growth leads inflation in the responses, even though the term that drives inflation (i.e., the sum of current and expected future output gaps) is wholly forward-looking.

Figure 8 plots the model response to an IS shock. Again, money growth reacts ahead of inflation.

Figure 9 plots responses to a Phillips curve shock. This is the only type of shock that can raise inflation immediately in this model. A policy tightening serves to reverse the initial rise in inflation. The contemporaneous money growth/inflation relation is negative in this case, and the peak in inflation precedes the peak in money growth. These patterns contrast with the lead of money growth over inflation observed in the previous responses. On the other hand, the nominal income growth/inflation relation in the responses is contemporaneous. This suggests that the Phillips curve shock is relatively unimportant empirically, since the tendency in the data, on average, is for nominal income growth to lead inflation.

Four aspects of these results are worth cataloguing. First, money growth and inflation seem to be closely related—indeed, they seem to enjoy an approximately unitary relationship. This is despite the fact that the responses describe dynamics rather than steady-state relations. This standard New Keynesian model suggests that a great deal of the relationship between money growth and inflation is manifested at the business cycle frequency.

Second, money growth tends to have a contemporaneous or leading relation with inflation in this model. The Lucas (1980) approach to extracting quantity-theory relations can be thought of as implying a dependence of inflation on a two-sided distribution (i.e., both lags and leads) of money growth rates (see McCallum, 1984). The responses above suggest that in practice the future-money terms are less important for the study of the relation between inflation and money growth. This is despite the fact that, in the model, inflation is forward-looking when expressed in terms of the output gap. The decision delays built into the model confer on money a leading relationship. Also note that, in principle, after a shock that raises the level of money, the proportionality between money and prices can be restored by a return of the money stock to its original level; but that is not how the proportionality is principally restored for the shocks we consider. Rather, for IS and policy shocks, prices tend to move after the shift in money in a manner that restores the original level of real balances.
Third, the results with the Bernanke-Woodford model contradict the position that successful monetary policy should so stabilize inflation as to wipe out the money growth/inflation relation. The reasons this argument, which has appeared widely since the 1960s, does not appear relevant are, first, that the delays built into the model prevent complete stabilization of inflation, and, second, the inflation response coefficient of 1.5 implied by the Taylor rule still leaves some muted variation in inflation, which in turn has its counterpart in muted variation in monetary growth.

Fourth, while none of the responses depict the experiment we referred to in our definition of the QTM, i.e., an exogenous change in the money stock, they have several features common with the QTM experiment; the shocks contemplated in Figures 8 to 10 produce permanent changes in the levels of nominal money and prices, but only temporary movements in output and interest rates, and with the levels of money and prices restored to their original proportional relationship with one another.

These results reinforce the suggestion that quantity-theory relations should be recoverable from business-cycle data; that recovering the relation between inflation and money growth mainly involves looking at the relation between inflation and prior, not future, money growth; and that environments where policymakers follow a firm interest-rate rule should still deliver traditional quantity-theory patterns in the reduced-form behavior of money and prices.

Thus fortified by these model results, let us now examine the some examples of the empirical relation between money growth and inflation. The first example we use uses data from Japan. Our data for Japan’s M1 growth and CPI inflation, constructed from annual averages of data from IFS. Regressions of inflation and money growth with this dataset are reported in Table 3.

We consider first the sample period 1959−1989. A static regression of inflation on money growth delivers an insignificant and low coefficient estimate. But this reflects not the absence of a relation in the time series relation, but the failure to allow for lags; adding lags 1 to 3 raises the $R^2$ from 0.08 to 0.58. The coefficient sum on monetary growth is, however, only 0.44. The post-1973 slowdown in Japan’s real growth rate, which lowered the noninflationary rate of monetary growth, appears to be having a major impact on the results. Adding an intercept dummy equal to 1.0 after 1973 greatly improves the fit and interpretability of the regression, with the coefficient sum on money growth now 0.825 and
### Table 3. Regressions for Japan CPI Inflation

**Sample Period: 1959–1989**

| Monetary Variable | Constant 0 1 2 3 Sum D74 R² SEE |
|-------------------|-----------------|---------|---------|--------|--------|--------|
| M1 growth         | 0.031 (0.015)   | 0.178   | ---     | ---    | 0.178  | ---    | 0.082  | 0.040 |
|                   | (0.110)   |         |         |         | (0.110) |         |         |
| M1 growth         | -0.002 (0.012) | -0.279  | 0.214   | 0.311  | 0.196  | 0.441  | ---    | 0.582  | 0.028 |
|                   | (0.119)   | (0.131) | (0.131) | (0.117) | (0.093) |         |         |
| M1 growth         | -0.079 (0.023) | 0.067   | 0.278   | 0.313  | 0.166  | 0.825  | 0.058  | 0.735  | 0.023 |
|                   | (0.133)   | (0.108) | (0.107) | (0.095) | (0.126) | (0.015) |         |
| M1 growth per unit of output | 0.029 (0.010) | 0.413   | ---     | ---    | 0.413  | ---    | 0.274  | 0.035 |
|                   | (0.125)   |         |         |         | (0.125) |         |         |
| M1 growth per unit of output | 0.005 (0.009) | 0.171   | 0.106   | 0.217  | 0.317  | 0.811  | ---    | 0.639  | 0.026 |
|                   | (0.120)   | (0.125) | (0.123) | (0.118) | (0.122) |         |         |

**Sample Period: 1959–2008**

| Monetary variable | Constant 0 1 2 3 Sum D74 R² SEE |
|-------------------|-----------------|---------|---------|--------|--------|--------|
| M1 growth         | 0.016 (0.010)   | 0.177   | ---     | ---    | 0.177  | ---    | 0.091  | 0.038 |
|                   | (0.081)   |         |         |         | (0.081) |         |         |
| M1 growth         | -0.011 (0.010) | -0.058  | 0.165   | 0.120  | 0.188  | 0.415  | ---    | 0.365  | 0.033 |
|                   | (0.100)   | (0.122) | (0.122) | (0.101) | (0.089) |         |         |
| M1 growth         | -0.009 (0.023) | -0.061  | 0.164   | 0.119  | 0.187  | 0.409  | -0.001 | 0.365  | 0.034 |
|                   | (0.116)   | (0.124) | (0.124) | (0.103) | (0.128) | (0.015) |         |
| M1 growth per unit of output | 0.027 (0.008) | 0.131   | ---     | ---    | 0.131  | ---    | 0.037  | 0.040 |
|                   | (0.097)   |         |         |         | (0.097) |         |         |
| M1 growth per unit of output | 0.016 (0.010) | 0.074   | 0.043   | 0.064  | 0.128  | 0.309  | ---    | 0.108  | 0.039 |
|                   | (0.120)   | (0.135) | (0.134) | (0.119) | (0.135) |         |         |
insignificantly different from unity, and the coefficient on the dummy suggesting a rise in the inflation rate for given money growth (and corresponding slowdown in potential growth, assuming a unit income elasticity of money demand) of 5.8%.

We also present results with money growth per unit of output as the explanatory variable. For the coefficient sum on money growth, the results that allow for lags closely agree with the results using M1 growth. The intercept dummy does not appear in the regressions because the per-unit term already adjusts for the slowdown in potential.

Results deteriorate when the sample period is 1959–2008. The post-1973 intercept dummy no longer seems to capture the growth slowdown well, and, while inclusion of lags of money growth raises the coefficient sum on money growth, the sum is still only 0.4. The results with per-unit money growth are poorer. The 1990s are not a decade where the nonneutral effects of monetary policy average out; adjusting money growth for output growth worsens money growth as an indicator of inflation under these circumstances.

Do the full-sample results refute the quantity theory, or at least show its lack of practical usefulness in understanding inflation behavior? We would argue not. The collapse of nominal interest rates during the 1990s in Japan led to a series of permanent increases in real money demand that distorted the money growth/inflation correlation, as they did in the United States in the 1980s. From the viewpoint of the quantity theory, the trend in the opportunity cost of holding money in Japan during the 1990s provides a legitimate reason why there can be surges in money growth that never have a counterpart in inflation—particularly for a very interest-elastic aggregate like M1. That trend has left an indelible impression on the Japanese data, one that is unlikely to go away even with the taking of long averages. Nevertheless, an interest-rate trend is not something that can be confidently extrapolated. Once the economy has completely adjusted to a permanent decline in interest rates, the quantity theory suggests that the \textit{ceteris paribus} unitary relation between money growth and inflation should become more evident.

Let us now consider the reduced-form relation between money growth and inflation in the United States. Table 4 presents regressions of inflation on money growth. Consider first the results with M1 as the measure of money. For the 1963–1979 sample, the sum of lags 0–3 on M1 growth is significant and very large. Indeed, it is well above unity. Allowing for the post-1973 growth slowdown with an intercept dummy brings the coefficient sum close to unity. But extending the sample period to 1989 destroys this result, making the sum negative.
The 1963–89 regression supports earlier evidence of breakdowns in bivariate M1/inflation relations when the 1980s are included (see, for example, Friedman and Kuttner, 1992). As noted previously, this deterioration reflected the protracted recovery of real M1 balances in response to permanent declines in nominal interest rates in the 1980s. Adding the years 1990 to 2008 to the sample seems to restore some significance to M1 growth, but the coefficient sum is far below unity, and the explanatory power of the regression is low.

Moving to M1 growth per unit of output produces a near-unit sum on money growth for 1963–79. But it makes the money growth/inflation relation contemporaneous for the reasons discussed previously. There is a deterioration in the relation in the 1980s (not as great as the deterioration using M1 growth, because rapid output growth in 1983 and 1984 makes inflation in those years easier to reconcile with M1 behavior) and a further fall in the coefficient sum as 1990–2008 data are included.

The use of money per unit of output in the preceding regressions implicitly entailed an assumption of a unitary income elasticity of money demand; otherwise, it would not be appropriate to impose a unit weight on output growth in constructing a “money growth relative to output” series. For Japan, a unitary long-run elasticity of real M1 demand appears to have empirical support (Rasche, 1990), and many econometric studies for the U.S. real M1 demand also support a unitary income elasticity (see Lucas, 1988; Hoffman and Rasche, 1991). There is some evidence, however, that the long-run income elasticity of M1 demand is better characterized empirically as 0.5 rather than 1.0 (see, for example, Ball, 2001). That being so, the “money growth relative to output” concept relevant for discussions of inflation should be measured as $\Delta \log(M1) – 0.5 \Delta \log Y$ rather than money growth per unit of output, $\Delta \log(M1) – \Delta \log Y$.

Results imposing the alternative income elasticity of 0.5 appear as the final three regressions of Table 4. The results agree closely with those that used a unit weight on output growth, with similar equation standard errors and comparable performances across different sample periods. In addition, as before, the expression of money growth in relative-to-output terms makes the coefficient on current money the dominant term in the sum of coefficients.

In Table 5 we present regressions of CPI inflation on M2 growth. The results help explain why many researchers (such as Benati, 2009) prefer to use that aggregate in empirical studies
Table 4. Regressions for U.S. CPI inflation using M1

<table>
<thead>
<tr>
<th>Monetary Variable</th>
<th>Sample Period</th>
<th>Coefficients</th>
<th>Lag</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>Sum</td>
<td>D74</td>
<td>R²</td>
<td>SEE</td>
</tr>
<tr>
<td>M1 growth 1963–1979</td>
<td>-0.034</td>
<td>0.624</td>
<td>0.995</td>
<td>0.214</td>
<td>1.748</td>
<td>---</td>
<td>0.831</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.336)</td>
<td>(0.348)</td>
<td>(0.270)</td>
<td>(0.262)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.103</td>
<td>0.507</td>
<td>0.877</td>
<td>1.244</td>
<td>0.026</td>
<td>0.005</td>
<td>0.947</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.309)</td>
<td>(0.198)</td>
<td>(0.204)</td>
<td>(0.166)</td>
<td>(0.184)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.044</td>
<td>0.003</td>
<td>0.143</td>
<td>0.384</td>
<td>---</td>
<td>0.156</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
<td>(0.354)</td>
<td>(0.355)</td>
<td>(0.435)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 growth 1963–1989</td>
<td>0.051</td>
<td>0.003</td>
<td>0.127</td>
<td>0.136</td>
<td>0.368</td>
<td>0.007</td>
<td>0.169</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.246)</td>
<td>(0.249)</td>
<td>(0.169)</td>
<td>(0.150)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 growth 1963–2008</td>
<td>0.024</td>
<td>0.001</td>
<td>0.127</td>
<td>0.136</td>
<td>0.368</td>
<td>0.007</td>
<td>0.169</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.234)</td>
<td>(0.249)</td>
<td>(0.169)</td>
<td>(0.150)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 growth relative to output:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y) 1963–1979</td>
<td>0.034</td>
<td>0.925</td>
<td>0.030</td>
<td>0.065</td>
<td>0.125</td>
<td>1.145</td>
<td>---</td>
<td>0.735</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.264)</td>
<td>(0.268)</td>
<td>(0.255)</td>
<td>(0.230)</td>
<td>(0.210)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y) 1963–1989</td>
<td>0.039</td>
<td>0.398</td>
<td>0.225</td>
<td>0.464</td>
<td>---</td>
<td>0.184</td>
<td>0.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.245)</td>
<td>(0.282)</td>
<td>(0.306)</td>
<td>(0.259)</td>
<td>(0.236)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y) 1963–2008</td>
<td>0.036</td>
<td>0.273</td>
<td>0.109</td>
<td>0.376</td>
<td>---</td>
<td>0.188</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y0.5) 1963–1979</td>
<td>-0.001</td>
<td>0.925</td>
<td>0.294</td>
<td>0.474</td>
<td>1.532</td>
<td>---</td>
<td>0.715</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.541)</td>
<td>(0.548)</td>
<td>(0.497)</td>
<td>(0.445)</td>
<td>(0.281)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y0.5) 1963–1989</td>
<td>0.031</td>
<td>0.242</td>
<td>0.312</td>
<td>0.468</td>
<td>---</td>
<td>0.108</td>
<td>0.031</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.353)</td>
<td>(0.438)</td>
<td>(0.516)</td>
<td>(0.417)</td>
<td>(0.290)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ log (M1/Y0.5) 1963–2008</td>
<td>0.029</td>
<td>0.234</td>
<td>0.155</td>
<td>0.399</td>
<td>---</td>
<td>0.167</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.175)</td>
<td>(0.268)</td>
<td>(0.271)</td>
<td>(0.174)</td>
<td>(0.141)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5. Regressions for U.S. CPI inflation using M2

<table>
<thead>
<tr>
<th>Monetary Variable</th>
<th>Sample Period</th>
<th>Coefficients</th>
<th>( \Delta \log )</th>
<th>( \Delta \log )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lag</td>
<td>( \Delta \log )</td>
<td>( \Delta \log )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constant</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sum</td>
<td>D74</td>
<td>( R^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SEE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2 growth</td>
<td>1963–1979</td>
<td>-0.072</td>
<td>0.119</td>
<td>0.193</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.030)</td>
<td>(0.235)</td>
<td>(0.259)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.550</td>
<td>0.680</td>
<td>1.543</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.269)</td>
<td>(0.229)</td>
<td>(0.363)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>---</td>
<td>0.714</td>
<td>0.018</td>
</tr>
<tr>
<td>M2 growth</td>
<td>1963–1979</td>
<td>-0.014</td>
<td>-0.050</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.053)</td>
<td>(0.263)</td>
<td>(0.281)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.395</td>
<td>0.333</td>
<td>0.705</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.287)</td>
<td>(0.346)</td>
<td>(0.731)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.026</td>
<td>0.263</td>
<td>0.753</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.020)</td>
<td></td>
<td>0.018</td>
</tr>
<tr>
<td>M2 growth</td>
<td>1963–2008</td>
<td>-0.003</td>
<td>0.006</td>
<td>-0.053</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td>(0.185)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.206</td>
<td>0.522</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.239)</td>
<td>(0.181)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>---</td>
<td>0.417</td>
<td>0.021</td>
</tr>
<tr>
<td>M2 growth</td>
<td>1963–2008</td>
<td>-0.014</td>
<td>0.060</td>
<td>0.240</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.013)</td>
<td>(0.185)</td>
<td>(0.237)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.237</td>
<td>0.474</td>
<td>0.723</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.181)</td>
<td>(0.152)</td>
<td>(0.008)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.012</td>
<td>0.449</td>
<td>0.021</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td>1963–1979</td>
<td>0.005</td>
<td>0.340</td>
<td>0.324</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>(0.012)</td>
<td>(0.229)</td>
<td>(0.240)</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>0.324</td>
<td>0.651</td>
<td>1.107</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>(0.249)</td>
<td>(0.235)</td>
<td>(0.263)</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>---</td>
<td>0.657</td>
<td>0.020</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td>1963–2008</td>
<td>0.018</td>
<td>0.364</td>
<td>0.120</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>(0.006)</td>
<td>(0.133)</td>
<td>(0.153)</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>0.120</td>
<td>0.309</td>
<td>0.718</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>(0.151)</td>
<td>(0.133)</td>
<td>(0.148)</td>
</tr>
<tr>
<td>( \Delta \log )</td>
<td></td>
<td>---</td>
<td>0.387</td>
<td>0.022</td>
</tr>
</tbody>
</table>

rather than M1. In the regressions with a post-1973 intercept dummy, the coefficient sum on M2 growth changes little as the sample is extended from 1979 to 2008, and has 1.0 within its confidence interval throughout. Not all is well with the M2/inflation relation; for example, the regression standard error rises as the sample is extended. But the greater resilience of the M2 results to the inclusion of more recent years’ data supports two points stressed earlier: that a filter is not required to establish a relation between money growth and inflation, and that, while measurement problems with money are undoubtedly significant in practice, many of the discrepancies that arose between M1 growth and inflation, especially those prior to 1994, are attributable to the substantial interest sensitivity of M1 balances, rather than to measurement problems with M1.

Money demand nominal homogeneity

---

21 The M2 series corresponds to the annual average of the M2 series plotted in Figure 1, but with an adjustment corresponding to the introduction of money market deposit accounts in 1983, using the adjustment magnitude in Friedman (1988) that in turn agrees with the estimate of the effect in Hall, Porter, and Small (1989).

22 Assenmacher-Wesche and Gerlach’s (2007) treatment of the U.S. data with a low-frequency filter does not deliver a point estimate on M2 growth closer to unity than we obtain in Table 5 using unfiltered annual data.
Our definition of the quantity theory does not associate the quantity theory closely with propositions about the money demand function. We have, however, insisted that zero degree homogeneity with respect to nominal variables is a property of money demand that is implied by the quantity theory—the demand is for real balances, in terms of real determinants. We consider U.S. M1 and M2 demand further in this light.

The nominal homogeneity restriction implies $g_1 = 0$ in the relation:

$$\Delta \log VM_t = g_0 + g_1 \pi_t + g_2 \Delta OPP_t + u_t.$$ 

where $VM_t$ is velocity, defined as nominal GDP divided by nominal money, and $OPP_t$ is the opportunity cost of the relevant aggregate. We measure $OPP_t$ for M1 by the federal funds rate (annual average) and $OPP_t$ for M2 by the spread between the federal funds rate and the M2 own-rate. A money demand relation can be cast as a velocity relation (with no separate real income term) if the money demand function has a unitary income elasticity, a property often found for M2 demand and, as noted above, also a common finding for M1. Note that this recasting of the relationship as a velocity relation means a change in sign when interpreting the coefficient on the interest rate: a negative money demand interest semielasticity implies a positive velocity interest semielasticity.

Given the definition of velocity, the natural price series to use in testing the nominal homogeneity restriction is the GDP deflator. For completeness, however, we also present results using CPI inflation. We express the relation in first differences rather than levels to allow for the likely presence of permanent money demand shocks, which produce nonstationarity in velocity and imply that levels of real money and real income are not cointegrated (see McCallum, 1993).

Because GDP deflator inflation and velocity growth have a definitional relation with one another, measurement errors in inflation may produce a correlation between inflation and velocity growth. These errors would tend to bias tests in the direction of rejecting nominal homogeneity. To protect against this bias, we estimate by instrumental variables, with two lags of each series (velocity growth, inflation, and first difference of opportunity cost) serving as instruments.

23 The nominal interest rate in this context measures the real opportunity cost of holding real money balances, as it reflects the difference between the real rates of return on money and interest-bearing assets.

24 The M2 own-rate is a standard variable in M2 demand studies published since the 1980s (for example, Hall, Porter, and Small, 1989). We use annual averages of the series available from the Federal Reserve Bank of St. Louis’ FRED site.
Estimates, using annual data, are presented in Table 6 for M1 and in Table 7 for M2. We consider the full sample (starting in 1962 for the M1 velocity estimation, a year later for M2), and results for samples starting in 1980. Because of the increased importance of sweeps for M1 behavior after 1993, we also present results for the 1962–1993 in the case of M1 velocity.

Nominal homogeneity of money demand is not rejected irrespective of the inflation series used, the definition of money chosen, or sample period considered. As the final rows in each table show, this continues to be the case if we relax the assumption of a unitary income elasticity of money demand. Thus, nominal homogeneity of money demand, a fundamental aspect of the quantity theory, appears to be consistent with the U.S. data.

7. Implications of a diminishing role for money

Benjamin Friedman (1999, 2000) has suggested that technological improvement in the financial sector raises the prospect of the near-obsolescence of central bank money. In terms of the subject matter of this paper, the scenario that Friedman envisages is consistent with continuing quantity-theory relations between inflation and money growth, provided that the latter refers to growth in deposit-inclusive measures of money. But both deposit creation and market interest rates would become disconnected, in this scenario, from central bank actions, with associated loss of central bank control over nominal spending. Friedman’s argument does not involve the complete disappearance of money, but instead a state of affairs in which the role of base money diminishes to the point where central banks’ ability to influence aggregate demand in a dependable fashion would be in jeopardy. Reactions to these conjectures include those of Goodhart (2000) and Woodford (2000, 2001). In the following paragraphs we attempt to outline the main contours, and evaluate the merits, of the debate.

Base money includes, of course, both currency and bank reserves. Goodhart (2000) argues convincingly that private sector demand for currency will persever for the foreseeable future, in part because of the anonymity conferred on currency transactions. In principle, the interest elasticity of currency demand gives central banks scope to manipulate interest rates without departing from its traditional policy of providing the amount of currency that the public demands at prevailing income and interest rates. But this would constitute a departure from

\[25\] A positive coefficient on real income growth in these estimates implies an income elasticity of money demand below unity.

\[26\] King (1999) advances similar arguments.
### Table 6: Tests of nominal homogeneity of M1 demand

Dependent variable: Log-difference in M1 velocity

<table>
<thead>
<tr>
<th>Sample period</th>
<th>Constant</th>
<th>GDP deflator</th>
<th>CPI</th>
<th>∆OPP</th>
<th>Δ log Yt</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962–2008</td>
<td>0.021</td>
<td>-0.010</td>
<td>---</td>
<td>0.505</td>
<td></td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.240)</td>
<td></td>
<td>(0.535)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1962–1993</td>
<td>0.002</td>
<td>0.301</td>
<td>---</td>
<td>0.520</td>
<td></td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.260)</td>
<td></td>
<td>(0.543)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980–2008</td>
<td>0.029</td>
<td>-0.344</td>
<td>---</td>
<td>1.427</td>
<td></td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.401)</td>
<td></td>
<td>(0.745)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1962–2008</td>
<td>0.020</td>
<td></td>
<td>0.016</td>
<td>0.595</td>
<td></td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td></td>
<td>(0.209)</td>
<td>(0.519)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1962–1993</td>
<td>0.005</td>
<td></td>
<td>0.199</td>
<td>0.574</td>
<td></td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td></td>
<td>(0.221)</td>
<td>(0.536)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980–2008</td>
<td>0.026</td>
<td></td>
<td>-0.224</td>
<td>1.339</td>
<td></td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
<td>(0.352)</td>
<td>(0.672)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1962–2008</td>
<td>0.037</td>
<td>-0.152</td>
<td>---</td>
<td>0.498</td>
<td>-0.328</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.329)</td>
<td></td>
<td>(0.556)</td>
<td>(0.475)</td>
<td></td>
</tr>
<tr>
<td>1962–1993</td>
<td>-0.018</td>
<td>0.497</td>
<td>---</td>
<td>0.306</td>
<td>0.318</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.376)</td>
<td></td>
<td>(0.672)</td>
<td>(0.462)</td>
<td></td>
</tr>
<tr>
<td>1962–2008</td>
<td>0.040</td>
<td></td>
<td>-0.162</td>
<td>0.612</td>
<td>-0.386</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td></td>
<td>(0.328)</td>
<td>(0.576)</td>
<td>(0.534)</td>
<td></td>
</tr>
<tr>
<td>1962–1993</td>
<td>-0.017</td>
<td></td>
<td>0.399</td>
<td>0.246</td>
<td>0.361</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td></td>
<td>(0.381)</td>
<td>(0.567)</td>
<td>(0.554)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 7: Tests of nominal homogeneity of M2 demand

Dependent variable: Log-difference in M2 velocity

<table>
<thead>
<tr>
<th>Sample period</th>
<th>Constant</th>
<th>GDP deflator</th>
<th>CPI</th>
<th>∆OPP</th>
<th>Δ log Yt</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963–2008</td>
<td>-0.002</td>
<td>0.130</td>
<td>---</td>
<td>0.634</td>
<td></td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.147)</td>
<td></td>
<td>(0.372)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980–2008</td>
<td>0.005</td>
<td>-0.006</td>
<td>---</td>
<td>1.076</td>
<td></td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.237)</td>
<td></td>
<td>(0.860)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1963–2008</td>
<td>-0.004</td>
<td></td>
<td>0.172</td>
<td>0.732</td>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td></td>
<td>(0.125)</td>
<td>(0.360)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980–2008</td>
<td>0.004</td>
<td></td>
<td>0.036</td>
<td>1.394</td>
<td></td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td></td>
<td>(0.179)</td>
<td>(0.653)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1963–2008</td>
<td>0.010</td>
<td>0.048</td>
<td>---</td>
<td>0.740</td>
<td>-0.290</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.193)</td>
<td></td>
<td>(0.421)</td>
<td>(0.290)</td>
<td></td>
</tr>
<tr>
<td>1963–2008</td>
<td>0.011</td>
<td></td>
<td>0.049</td>
<td>0.802</td>
<td>-0.299</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td></td>
<td>(0.185)</td>
<td>(0.437)</td>
<td>(0.320)</td>
<td></td>
</tr>
</tbody>
</table>
One part of Friedman’s (2000) argument is that technological progress makes it possible for buyers to make payments through accounts—bank or nonbank—that are not subject to reserve requirements. The existence of such arrangements is widely accepted by all participants in the debate. Woodford (2000) argues convincingly that the magnitude of required reserves is irrelevant. After all, several central banks do not rely on reserve requirements in their arrangements for setting interest rates. Overnight interest rates in these economies are typically controlled by means of “channel” arrangements, involving standing facilities that set both a floor and a ceiling on overnight rates. These rates apply to the operational reserve balances, useful for settlement purposes, which financial intermediaries hold with the central bank. An arrangement consistent with the channel system involves central bank payment of interest on reserves, including excess reserves; this possibility is discussed by Woodford (2000, 2001) and Goodfriend (2002). The arrangement offers the promise of securing a positive demand for central bank money in a technologically advanced financial system. If settlement reserves with the central bank are held by banks, along with overnight securities, then the interest rate on the latter will equal the sum of the interest rate paid on reserve balances plus the marginal service yield provided by these balances. By adjusting the interest paid on reserves, the central bank can exert a dominant influence on the overnight interest rate. The Federal Reserve introduced interest payments on reserves in October 2008.

One part of Woodford’s optimism about the prospects of monetary policy in an economy with a negligible medium of account appears debatable. That part is his statement that “the unit of account in a purely fiat system is defined in terms of the liabilities of the central bank” (Woodford, 2000, p. 257). Certainly the liabilities of the central bank would be a favored candidate for the role of unit of account in an economy with no medium of exchange, but there is no necessity that it be the one that prevails. Goods prices will, in a market economy, be quoted in terms of the medium that market participants find most convenient. Just as central bank currency can be supplanted by some other medium of exchange if its issue is managed too badly (e.g., under hyperinflation conditions), the central bank’s contender for the unit of account can conceivably lose out to another medium. And it is the unit of account actually prevailing in market transactions that is of macroeconomic importance; it is
stickiness in terms of prices used in actual transactions that is relevant for the definition of real rates of interest that influence aggregate demand.

8. **Interest rates vs. money in price level analysis**

The diminishing role for money provides a natural point of departure for a discussion of recent approaches to analyzing price level determination without money. The trend of professional work in recent years can be put in context by juxtaposing two observations from earlier decades: Patinkin’s (1972, p. 898) statement that “one of the primary tasks of monetary theory is indeed to explain the determination of the wage and price levels,” and Gowland’s (1991, p. 122) observation that “the term ‘monetary policy’ seems inappropriate in a model without money.” The recent literature can be thought of as embracing the first observation while rejecting the second. In particular, the “cashless” and “neo-Wicksellian” treatment in Woodford (2003) is a crystallization of a framework in which the central bank manipulates interest rates and there is no medium of exchange, with price level variations capable of being traced back to deviations of the real interest rate from the natural rate of interest.

The limiting case of no medium of exchange would, in our terms, indeed be a nonmonetary economy; there would be no monetary policy, literally defined. Nevertheless, as discussed above, there would be scope for different types of policy measures regarding price level behavior, with the price level being regarded as some general index of prices in terms of the unit of account.

Interest in a Wicksellian approach to price-level analysis showed some signs of reviving at a policy level even in the early 1990s (for example, Kohn, 1990), but it has exploded in recent years in light of Woodford’s (2003) emphasis on the role of the natural rate of interest in dynamic stochastic general equilibrium models. We have not contrasted Wicksellian and quantity-theory approaches in this paper because, with a medium of exchange present, the two are compatible, being in essence alternative ways of viewing the same process, as Woodford (2003, p. 53) acknowledges. Wicksell (1906/1935), for one, emphasized the money stock adjustments that were implied by the banking system’s variations in interest rates, although he also considered a “pure credit” economy. And in dynamic general equilibrium models, the money demand function that implies a connection between steady-state money growth and inflation comes from the same private sector optimization that delivers the IS and Phillips curves that Woodford uses.
As a practical matter, it may be more convenient to analyze price level adjustment by reference to interest rates than by reference to monetary growth. This could be the case if, for example, problems with measuring money in practice meant that researchers could be more confident in their estimates of the natural level of interest than their estimates of the non-inflationary rate of money growth. On the other hand, there are instances in the study of historical U.S. data where ignoring money growth appears to produce considerable loss of information. For example, during the credit controls episode of 1980, both monetary growth and interest rates collapsed. Looking solely at interest rates, Bordo, Erceg, Levin, and Michaels (2007) interpret this period as one of extreme monetary policy ease; likewise, the estimated monetary policy shock coming from Smets and Wouters’ (2007) dynamic general equilibrium model (estimated without money stock data) finds 1980 Q2 to have featured the most expansionary monetary policy shock in postwar U.S. history. By contrast, estimating a monetary policy shock series from a VAR that does include money, Blanchard and Watson (1986) find that 1980 Q2 featured one of the most contractionary monetary policy shocks in U.S. postwar history. The mid-1980 collapse in the U.S. economy suggests that the interpretations of monetary policy tightness that make use of monetary aggregates are the correct ones, and that evaluations based solely on interest rates were unreliable.

9. Conclusions

The present paper has considered what, if any, relationship there is between monetary aggregates and inflation, and whether there is any substantial reason for modifying the current mainstream mode of policy analysis, which frequently does not consider monetary aggregates at all. The quantity theory, as we have defined it, centers on the prediction that there will be a long-run reaction of prices to an exogenous increase in the nominal money stock. The fact that policymakers in practice do not set money growth rates exogenously does not rob the quantity theory of empirical content. Likewise, the observation that policymakers frequently are concerned with price behavior at horizons shorter than the very long run does not deprive the quantity theory of policy significance. On the contrary, the nominal homogeneity conditions that deliver the quantity-theory result are the same as those that deliver monetary neutrality, an important principle behind policy formulation. Furthermore, the quantity theory implies a ceteris paribus unitary relationship between

27 Note that if policymakers set their monetary instrument, be it an aggregate or an interest rate, actively in response to the state of the economy, then they would not be setting it exogenously.
inflation and money growth. After allowing for lags, this unitary relationship tends to emerge from examination of time series; it does not appear to be the case that replacing the time series with long averages of the data is a necessary or valuable step in recovering that relationship.

Our discussion has not disputed the position that financial innovation can obscure the relationship between monetary growth and inflation. What is needed, however, is a sense of proportion. We believe that too much of the reaction to problems in measuring money has taken the form of abandoning the analysis of monetary aggregates, and too little has taken the form of more careful efforts at improved measurement. The problems of measurement associated with monetary aggregates have parallels in the measurement and estimation problems that occur with policy analysis that excludes money. Frameworks that include interest rates as the sole monetary variable in the analysis must, for example, grapple with the fact that the natural rate of interest is unobserved. Any shift in the natural real rate of interest will modify the consequences for inflation of a specified interest-rate policy. Such a shift in the natural interest rate would call not for leaving interest rates out of the analysis, but for more intense efforts at estimating the natural rate. Moreover, since the connections of interest rates and monetary growth to inflation are clouded by the presence of an imperfectly observed series (especially the natural rate, in the case of interest rates; financial innovations, in the case of money), studies of inflation and monetary policy behavior can benefit from including both interest rates and money in the empirical analysis.
References


