The Lead of Output Over Inflation in Sticky Price Models

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Abstract

Output growth is negatively correlated with inflation, and detrended output is positively correlated with inflation, in the major North American and European economies. In addition, output growth and detrended output lead inflation. I explore the consistency of these correlations with three models of price adjustment: the partial adjustment model, a staggered price setting model, and the P-bar model. The ratio of the variance of supply to demand shocks necessary to match the pattern of output-inflation correlations can be ranked across the three models; the P-Bar model requires the lowest ratio, and the partial adjustment model requires the highest ratio. These results reveal that the recent burst of researchers using the partial adjustment model will find a larger role for supply shocks than alternative models of price rigidity.

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1. Introduction

This paper explores the consistency of sticky price and/or imperfect information models of price adjustment, under fairly simple specifications of the shocks hitting the economy, with the following stylized facts regarding inflation and output for the major North American and European economies (documented in section 2 below):

1. Inflation and output growth are negatively correlated;
2. Inflation and detrended output (HP-filtered) are positively correlated.
3. Output leads inflation, in the sense that the correlations of inflation with past output growth or detrended output are larger than the correlations of inflation with future output.

In contrast to previous research, several different sticky price models are considered, and the comparison of different models of nominal rigidity reveals that the type of price stickiness utilized can strongly alter, both quantitatively and qualitatively, the conclusions reached regarding the relative importance of supply and demand shocks for the comovement of inflation and output.

While much recent research has emphasized the contemporaneous correlations between inflation and various measures of detrended output, I also emphasize the strong lead of output over inflation experienced in the major North American and European economies (i.e., the *phase shift* between output and inflation). While the leading relationship of output to inflation is well documented, it has been underemphasized in the evaluation of models with slow price adjustment. Evaluation of several popular models reveals that the most heavily exploited model of nominal price rigidity in aggregate macro models, the partial adjustment model, is incapable of matching the lead of output over inflation and the sign of the contemporaneous inflation-output growth correlation when only demand shocks are considered. Perhaps even more importantly, the partial adjustment model requires a larger ratio of the variance of supply to the variance of demand shocks to match the sign and phase shift which characterizes the patterns of output-inflation correlations than is required by the other models of price adjustment considered, suggesting that the recent burst of research exploiting the partial adjustment model of price adjustment will find a larger role for supply shocks than would be found using other models of sticky prices.

The slow price adjustment models considered are the partial adjustment model of prices developed in Calvo (1983) and Rotemberg (1982,1987,1994), the Taylor staggered price setting model, and the P-bar model (McCallum (1994)). The staggered price setting model and the P-Bar model do generate the lead of output over inflation in response to demand shocks, and require a smaller ratio of the variance of supply to the variance of demand shocks to match the output-inflation relationship than the partial adjustment model. Interestingly, these models match the phase shift and place less emphasis on supply shocks because of the information imperfections regarding nominal shocks imbedded in the models, not the nominal price rigidity aspect per se. This emphasis on information problems as an important determinant of the output-inflation relationship follows a long line of thought on monetary economics, from Hume (1752) to Lucas (1973,1977,1987) and Brunner and Meltzer (Brunner, Cukierman, and Meltzer (1983) and Meltzer (1995)).

I place emphasis on the importance the models give to supply shocks in generating the lead of output over inflation because the phase shift is one of the central "stylized facts" of macroeconomics. For example, a high level of output today often enters discussions in the financial press as an indicator of future inflation because of the systematic lead of output over inflation. My demonstration analytically that the partial adjustment model of prices fails to generate the proper comovement of inflation and output in response to demand shocks, and requires a larger variance of supply shocks to match the output-inflation relationship, suggests that recent attempts to incorporate sticky prices into dynamic general equilibrium models through the partial adjustment model of prices, such as Hairault and Portier (1993), Rotemberg (1994), Yun (1994), Kimball (1995), King and Watson (1996), King and Wolman (1996), Ireland (1996), Kim (1996), and Woodford (1996), will require more emphasis on supply shocks than alternative models to match the comovement of nominal and real variables over the business cycle.

The next section presents evidence on the comovement between inflation and output. Section 3 presents the models. Section 4 examines the consistency of the models with the pattern of correlations between output growth and inflation, and section 5 examines the consistency of the correlations between detrended (HP-filtered) output and inflation.

2. Evidence on the Comovement of Prices, Inflation, and Output

Much recent research has focused on the cyclical behavior of the price level. This research has concluded that the price level is countercyclical, i.e., when output lies above trend, prices lie below trend (Cooley and Ohanian (1991), Chadha and Prasad (1994), Rotemberg (1994), Judd and Trehan (1995), and den Haan (1996)). In contrast, inflation appears to be procyclical (Chadha and Prasad (1994)). Since non-stationarity in the aggregate demand (or "money") process results in a non-stationary price level but stationary output in all of the models considered below (i.e., "long-run monetary neutrality"), I focus on the correlations between inflation and various measures of detrended output. This focus on the output-inflation relationship is consistent with the long-standing focus on "Phillips curve" type models.

Table 1 presents the correlation between the growth rate of output (GDP) and inflation in the major North American and European economies since 1960. While the statistical significance of the correlations varies across countries, two patterns emerge. In every case, the contemporaneous correlation is negative (or extremely close to zero). There is also a clear pattern to the lead-lag relationship between inflation and output growth; in every country, inflation is more strongly positively correlated with lagged output than with future output (i.e., $\rho(\Delta p_t, \Delta y_{t-i}) > \rho(\Delta p_t, \Delta y_{t+i})$).

Table 2 presents the correlation between detrended (HP-filtered) output at various lags and leads, and inflation. The HP-filtered output series is examined because it focuses on business cycle frequencies. Again, two patterns emerge. First, detrended output and inflation are positively correlated. Second, in every country, inflation is more strongly positively correlated with lagged output than with future output (i.e., $\rho(\Delta p_t, y_{t-i}) > \rho(\Delta p_t, y_{t+i})$).

Table 1: Correlation of Inflation and Output Growth

	U.S.	Canada	Great Britain	France	Germany
$\rho(\Delta p_t, \Delta y_{t-4})$	-0.08 (0.10)	0.17 (0.12)	-0.08 (0.10)	0.01 (0.12)	0.26* (0.11)
$\rho(\Delta p_t, \Delta y_{t-3})$	-0.10 (0.11)	0.08 (0.10)	-0.11 (0.08)	0.06 (0.13)	0.25* (0.13)
$\rho(\Delta p_t, \Delta y_{t-2})$	-0.21 (0.11)	0.11 (0.10)	-0.13 (0.08)	0.13 (0.12)	0.20 (0.11)
$\rho(\Delta p_t, \Delta y_{t-1})$	-0.19* (0.09)	0.05 (0.10)	-0.14 (0.12)	0.07 (0.14)	0.07 (0.12)
$\rho(\Delta p_t, \Delta y_t)$	-0.26* (0.12)	-0.19 (0.11)	-0.26** (0.09)	0.00 (0.14)	-0.05 (0.14)
$\rho(\Delta p_t, \Delta y_{t+1})$	-0.32** (0.09)	-0.03 (0.10)	-0.23** (0.08)	0.01 (0.14)	-0.12 (0.12)
$\rho(\Delta p_t, \Delta y_{t+2})$	-0.33** (0.09)	-0.08 (0.10)	-0.17 (0.09)	-0.13 (0.17)	-0.23 (0.14)
$\rho(\Delta p_t, \Delta y_{t+3})$	-0.30** (0.10)	-0.14 (0.11)	-0.17 (0.12)	-0.01 (0.13)	-0.11 (0.11)
$\rho(\Delta p_t, \Delta y_{t+4})$	-0.24 (0.13)	-0.18 (0.11)	-0.19* (0.08)	0.03 (0.12)	--0.12 (0.10)

Correlations reflect author's calculations based on OECD Main Economic Indicators-CDROM data.
Standard errors in parentheses are robust to heteroskedasticity and autocorrelation (Newey-West) with bandwidth parameter set at $N^{1/3}$.
Sample periods reflect maximum period available: U.S. (61:2-93:4), Canada (61:2-93:4), Great Britain (61:2-93:4), France (71:2-93:4), and Germany (69:2-93:4).
* significant at the 5% level.
** significant at the 1% level.

Table 2: Correlation of Inflation and HP-filtered Output

	U.S.	Canada	Great Britain	France	Germany
$\rho(\Delta p_t, y_{t-4})$	0.29* (0.14)	0.25* (0.11)	0.31* (0.13)	0.08 (0.20)	0.28* (0.11)
$\rho(\Delta p_t, y_{t-3})$	0.31* (0.13)	0.30** (0.10)	0.29* (0.13)	0.08 (0.20)	0.35** (0.09)
$\rho(\Delta p_t, y_{t-2})$	0.28* (0.13)	0.36** (0.10)	0.24* (0.12)	0.12 (0.20)	0.40** (0.07)
$\rho(\Delta p_t, y_{t-1})$	0.26* (0.13)	0.38** (0.10)	0.20 (0.16)	0.13 (0.19)	0.39** (0.08)
$\rho(\Delta p_t, y_t)$	0.20 (0.15)	0.26* (0.11)	0.08 (0.14)	0.10 (0.19)	0.33** (0.10)
$\rho(\Delta p_t, y_{t+1})$	0.10 (0.16)	0.24* (0.12)	-0.02 (0.15)	0.08 (0.19)	0.23 (0.13)
$\rho(\Delta p_t, y_{t+2})$	-0.01 (0.17)	0.19 (0.12)	-0.08 (0.15)	-0.03 (0.19)	0.09 (0.16)
$\rho(\Delta p_t, y_{t+3})$	-0.10 (0.17)	0.11 (0.13)	-0.12 (0.13)	-0.07 (0.19)	0.02 (0.18)
$\rho(\Delta p_t, y_{t+4})$	-0.16 (0.16)	-0.00 (0.14)	-0.19 (0.13)	-0.09 (0.19)	--0.06 (0.17)

Correlations reflect author's calculations based on OECD Main Economic Indicators-CDROM data.
Standard errors in parentheses are robust to heteroskedasticity and autocorrelation (Newey-West) with bandwidth parameter set at $N^{1/3}$.
The smoothing parameter for the HP-filter was set at 1600.
Sample periods reflect maximum period available: U.S. (61:1-93:4), Canada (61:1-93:4), Great Britain (61:1-93:4), France (71:1-93:4), and Germany (69:1-93:4).
* significant at the 5% level.
** significant at the 1% level.

For each of the major North American and European economies, there is a pronounced phase shift between output and inflation, in the sense that inflation is more strongly correlated with lagged output than with contemporaneous or future values of output. These results have been underemphasized in the evaluation of models of price adjustment (a notable exception is Taylor (1987)), which has focused exclusively on contemporaneous correlations. For example, Chadha and Prasad (1994) report a phase shift, but do not discuss the phase shift, instead emphasizing the procyclical inflation rate. In fact, of the papers mentioned in the section below on the partial adjustment model, only Yun (1994) reports the implications of his model for the phase shift between output and inflation. This *lead of output over inflation* is a robust stylized fact of business cycles, and I will demonstrate below that the popular partial adjustment model of slow price adjustment requires a larger ratio of the variance of supply shocks to the variance of demand shocks to match the phase shift between output and inflation than both other models of sticky prices considered. In addition, the partial adjustment model of sticky prices will be shown to imply a positive correlation between inflation and output growth in response to demand shocks, whereas the P-bar sticky price model will match the negative correlations found in Table 1, even when aggregate demand shocks are considered in isolation.

3. Three Sticky Price Models

Each of the models considered herein will employ very simple aggregate demand determination and driving processes for aggregate supply and demand shocks, in order to focus on the differences in the endogenous dynamics implied by different models of price rigidity. Aggregate demand is giving by

$$y_t = m_t - p_t \,, \tag{3.1}$$

where y is (log) real output, m is (log) money (nominal aggregate demand), and p is the (log) price level. This simple specification of aggregate demand (the quantity equation with constant velocity) is used in many models to explore the implications of alternative aggregate supply (i.e., price adjustment) specifications (such as Lucas (1973), Ball, Mankiw and Romer (1988), Blanchard and Fischer (1989), Rotemberg (1994), and Kimball (1995)). Aggregate demand fluctuations are a random walk

$$m_t = m_{t-1} + \epsilon_t, \tag{3.2}$$

where ϵ is an i.i.d. shock. Aggregate supply (x_t) is also a random walk,

$$x_t = x_{t-1} + \nu_t, \tag{3.3}$$

where ν is an i.i.d. shock. These simple driving processes insure that the focus is on the endogenous dynamics of the sticky price models, rather than the dynamics of the driving processes.[1]

The flexible price equilibrium of this kind of model consists of output at its natural, or aggregate supply determined, rate ($y_t = x_t$), and prices equal to the flexible price equilibrium value, $p_t^e = m_t - x_t$.

3.1 The Partial Adjustment Model

The first model of price rigidity examined is the partial adjustment model, and is

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[1] Allowing for serial correlation in money growth (as is typical in the monetary business cycle literature (for example, Rotemberg (1994), Yun (1994), Christiano and Eichenbaum (1995), Cooley and Hansen (1995), Kiley (1996), and King and Watson (1996)) does not change the results discussed in this paper, but does considerably complicate the analysis.

closely related to Rotemberg (1994). In the partial adjustment model of price adjustment, the (log) aggregate price level evolves according to

$$p_t = (1-\pi)p_{t-1} + \pi\, p_t^* , \tag{3.3}$$

where $0<\pi\leq 1$ is the speed of price adjustment, and p_t^* is the target price. The target price evolves according to

$$p_t^* = (1-(1-\pi)\rho)\sum_{j\geq 0} ((1-\pi)\rho)^j\, E_t\, p_{t+j}^e , \tag{3.4}$$

where the information set at t includes m_t and x_t, and ρ is the subjective discount rate of the price setter(s) under the Calvo (1983) interpretation discussed below.

The partial adjustment model can be given two interpretations. The standard interpretation involves the price setter(s) facing quadratic costs of price adjustment (Rotemberg (1982)). It is well known that quadratic costs of adjustment lead to partial adjustment as in (3.3) and (3.4), where the speed of price adjustment π is determined by the parameters of the adjustment cost function. Note that under this first interpretation, the price setters change price in each period, but only adjust their price part of the way towards the equilibrium price. The second interpretation assumes that firms face some constant probability π of adjusting price in any given period (Calvo (1983), Rotemberg (1987)); assuming a large number of firms with identical target prices and aggregating individual prices yields (3.3), which states that the price level in period t is determined by the fraction π of price setters who change price in period t to p_t^*, and the fraction $(1-\pi)$ of price setters whose prices remain unchanged from period t-1 to t. The target price (3.4) simply indicates that firms set current price with an eye on current and future market clearing prices ($p_t^e = m_t - x_t$) (due to the probability $(1-\pi)^s$ that nominal price has not been adjusted by time s under the constant hazard interpretation of the partial adjustment model). The interesting aspect of this interpretation is that individual firms do not adjust in every period, even partially, yet the aggregate price level evolves according to the partial adjustment mechanism (3.3). This unique aspect of the Calvo constant probability model of price adjustment makes the model extremely tractable (and hence popular in dynamic general equilibrium models such as Hairault and Portier (1993), Rotemberg (1994), Yun (1994), Kimball (1995), King and Watson (1996), King and Wolman (1996), Ireland (1996), Kim (1996), and Woodford (1996)), but also underlies the stark differences between the partial adjustment model and the staggered price setting model.

3.2 The P-bar Model

The model considered in this section is what McCallum (1994) calls the "P-bar" model of price adjustment. This type of slow price adjustment specification was introduced by Grossman (1974), and has been used by Barro and Grossman (1976), McCallum (1979,1994), Mussa (1981a,1981b,1982), Obstfeld and Rogoff (1984), and Flood and Hodrick (1986), and Chadha and Prasad (1993).[2] The P-bar model assumes that price adjustment depends upon movements in the equilibrium price level, p_t^e,

$$p_t - p_{t-1} = \pi(p_{t-1}^e - p_{t-1}) + E_{t-1}(p_t^e - p_{t-1}^e)\ , \tag{3.5}$$

where $0<\pi\leq 1$ is again a "price flexibility" parameter.[3] (3.5) indicates that in the P-bar

[2] As is clear from the reference list, the contributions of Mussa seem to have made this model of slow price adjustment more popular in the literature on exchange rates and slow price adjustment.

[3] The interpretation of the parameter π is specific to each model, but in all the models considered herein $(1-\pi)$ will partially determine the persistence of output fluctuations away from trend. Note also that Taylor (1993) has success matching output persistence with the speed of price adjustment in his econometric models, as does Kiley (1996) in cross-country comparison of the empirical behavior of output implied by the partial adjustment model.

model, the price level at t adjusts based on the degree of "disequilibrium" in t-1, and adjusts for any expected (as of t-1) movements in the equilibrium price. The idea behind this price adjustment equation is that the degree of disequilibrium indicates inflationary pressure in a "sticky" price world ($\pi<1$), and prices are set before trading in a period so price setters can only adjust at t to the movements in equilibrium prices expected at t-1. This latter aspect of the P-bar model is the sense in which it is an imperfect information model, and is necessary to generate output effects of monetary shocks (i.e., if the expectations operator in (3.5) is dated t, $p_t = p_t^e \; \forall t$).

3.3 Staggered Price Setting

This section presents a version of the Taylor (1980) model of staggered price setting. Taylor's original specification is motivated by wage contracts; herein I consider nominal prices fixed for two periods as in Blanchard and Fischer's (1989) model of staggered price setting. The interest in the Taylor type specification is twofold. First, it is a well known, tractable model of slow price adjustment. More importantly from my perspective is a comparison of the staggered price setting model's ability to match the pattern of correlation between output growth (detrended output) and inflation with the partial adjustment model's implications for the output-inflation relationship *because the popularity of the partial adjustment model is due, in part, to its close approximation to the staggered price setting model* (Calvo (1983), Blanchard and Fischer (1989)).

In the model, the economy consists of two (equally sized) types of price setters, type 1 and type 2. Both sets of price setters choose a fixed nominal price for two periods, but differ in the period in which they change price, with type 1 price setters changing price in odd periods and type 2 price setters changing price in even periods. Following Blanchard and Fischer (1989), a firm of type j has desired price in period t of

$$p_{jt}^{*} = cp_t + (1-c)p_t^e , \tag{3.6}$$

which indicates that a firm's desired price depends on the prices charged by all other firms (the price level, p) (with weight 0<c<1), and on the long run equilibrium price (p^e). Approximating the aggregate price level as $p_t = (½)(p_{1t} + p_{2t})$, and substituting into (3.6) yields type j's desired price in terms of type -j's chosen price,

$$p_{jt}^{*} = bp_{-jt} + (1-b)p_t^e , \ b=c/(2-c). \tag{3.7}$$

In period t, only half of price setters change their nominal price, which will hold for two periods. Assuming the price setters at t set their chosen nominal price, Z_t, equal to the average of their expected desired nominal price (given by (3.7)) over the two periods for which their nominal price is fixed yields the following equation for their chosen nominal price:

$$Z_t = (½)(bZ_{t-1} + (1-b)p_t^e + E_t\,[bZ_{t+1} + (1-b)p_{t+1}^e]) . \tag{3.8}$$

Solving (3.8) yields

$$Z_t = (1-\pi)Z_{t-1} + [(1-\pi)(1-b)/b] \textstyle\sum_{i\geq 0} (1-\pi)^I E_t\,[p_{t+i}^e + p_{t+1+i}^e] , \ (1-\pi) = [1-(1-b^2)^{½}]/b. \tag{3.9}$$

The price level is then given by $p_t = (½)(Z_t + Z_{t-1})$.

The staggered price setting model differs from the partial adjustment model in that it has an important imperfect information component; firms choose their price for two periods, and hence must choose a nominal price at time t which depends on their expectation of p_{t+1}^e, a feature which does not enter the partial adjustment model ((3.3) & (3.4)).

4. Output Growth and Inflation

The time series behaviors of output growth and inflation are easily determined for each model using (3.1)-(3.9), and are summarized in table 3 (where L is the lag operator).

Table 3: Output Growth and Inflation Processes

Partial Adjustment Model	
$\Delta p_t = \pi[1-(1-\pi)L]^{-1}[\epsilon_t - \nu_t]$	(4.1)
$\Delta y_t = (1-\pi)[1-(1-\pi)L]^{-1}[1-L]\epsilon_t - \pi[1-(1-\pi)L]^{-1}\nu_t$	(4.2)
P-Bar Model	
$\Delta p_t = \pi[1-(1-\pi)L]^{-1}[L][\epsilon_t - \nu_t]$	(4.3)
$\Delta y_t = [1-(1-\pi)L]^{-1}[1-L]\epsilon_t - \pi[1-(1-\pi)L]^{-1}L\nu_t$	(4.4)
Staggered Price Setting Model	
$\Delta p_t = (\pi/2)[1-(1-\pi)L]^{-1}[1+L\}[\epsilon_t - \nu_t]$	(4.5)
$\Delta y_t = ((2-\pi)/2)[1-(1-\pi)L]^{-1}[1-L]\epsilon_t - (\pi/2)[1-(1-\pi)L]^{-1}[1+L]\nu_t$	(4.6)

The output growth implications of the three models ((4.2), (4.4), and (4.6)) are very similar; each model of price adjustment implies that aggregate demand shocks cause an output expansion, followed by negative growth as output returns to its long run, supply determined rate. In this sense, all three models of price adjustment are very similar. Examination of the inflation processes reveal that the implications of the models for price dynamics are very different. In particular, both the P-bar model and the staggered price setting model ((4.4) and (4.6)) imply delayed responses of inflation to aggregate demand shocks (as can be seen by the presence of the lag operator L in the numerator of the lag polynomials), a result which stems from the imperfect information (lagged response to time t information about shocks) aspects of these models. While this point is a clear result of the timing assumptions embedded in the models regarding price changes by firms, and is quite simple, its implications are very important for the literature exploring the dynamic behavior of inflation and output over the business cycle, and have not as yet been acknowledged.

To see the importance of the information imperfections for the output-inflation relationship, the covariances between output growth and inflation implied by the models are reported in table 4 (where σ_ϵ and σ_v are the standard deviation of demand and supply shocks, respectively).

Table 4: Covariances of Inflation and Output Growth

	$cov(\Delta p_t, y_{t-1})$	$cov(\Delta p_t, y_t)$	$cov(\Delta p_t, y_{t+1})$
Partial Adjustment	$\pi^2[1-(1-\pi)^2]^{-1}(1-\pi)[(1-\pi)\sigma_\epsilon^2-\sigma_v^2]$	$-\pi^2[1-(1-\pi)^2]^{-1}[(\pi-1)\sigma_\epsilon^2+\sigma_v^2]$	$-\pi^2[1-(1-\pi)^2]^{-1}(1-\pi)[\sigma_\epsilon^2+\sigma_v^2]$
P-Bar	$\pi^2[1-(1-\pi)^2]^{-1}[\sigma_\epsilon^2-(1-\pi)\sigma_v^2]$	$-\pi^2[1-(1-\pi)^2]^{-1}[\sigma_\epsilon^2+\sigma_v^2]$	$-\pi^2[1-(1-\pi)^2]^{-1}(1-\pi)[\sigma_\epsilon^2+\sigma_v^2]$
Staggered Price Setting	$[1-(1-\pi)^2]^{-1}[(\pi(2-\pi)/4)(1-(1-\pi)^2)\sigma_\epsilon^2 -(\pi^2/4)(2(1-\pi)+1+(1-\pi)^2\sigma_v^2]$	$-\pi^2(1-\pi/2)[1-(1-\pi)^2]^{-1}\ \sigma_v^2$	$-[1-(1-\pi)^2]^{-1}[(\pi(2-\pi)/4)(1-(1-\pi)^2)\sigma_\epsilon^2 +(\pi^2/4)(2(1-\pi)+1+(1-\pi)^2\sigma_v^2]$

Comparing first the implications of each model for the contemporaneous covariance of output growth with inflation reveals that the P-bar model implies that demand shocks result in a negative covariance, the staggered price setting model implies no contribution of demand shocks to the contemporaneous covariance, and the partial adjustment model implies that demand shocks result in a positive covariance between output growth and inflation. These results are intuitive; in the P-bar model, output growth responds positively at first to an aggregate demand shock, but in the next period inflation rises as output growth falls. In the partial adjustment model, inflation and output growth both respond immediately (and positively) to an aggregate demand shock, resulting in a positive covariance. These results imply

Proposition 1: In order to match the negative correlations between output growth and inflation found in the major North American and European economies, the partial adjustment model requires a larger ratio of the variance of supply shocks to the variance of demand shocks than the staggered price setting model, which in turn requires a larger ratio than the P-bar model.

The covariances reported in table 4 also imply that all three models are capable of matching the phase shift between output growth and inflation reported in table 1, as $cov(\Delta p_t, \Delta y_{t-1}) > cov(\Delta p_t, \Delta y_{t+1})$ in all three models.

5. Detrended Output and Inflation

As in the previous section, the models presented in section 3 can be used to derive the implication of the different sticky price models for the dynamic behavior of the level of output. These implications are presented in table 5.

Table 5: Output Processes

Partial Adjustment Model	
$y_t = (1-\pi)[1-(1-\pi)L]^{-1}\epsilon_t - \pi[1-(1-\pi)L]^{-1}x_t$	(5.1)
P-Bar Model	
$y_t = [1-(1-\pi)L]^{-1}\epsilon_t - \pi[1-(1-\pi)L]^{-1}Lx_t$	(5.2)
Staggered Price Setting Model	
$y_t = ((2-\pi)/2)[1-(1-\pi)L]^{-1}\epsilon_t - (\pi/2)[1-(1-\pi)L]^{-1}[1+L]x_t$	(5.3)

Each model generates similar responses of the level of output to aggregate demand shocks, and passes the non-stationary behavior of aggregate supply (x) to output.

Of course, the behavior of inflation is still that given in table 3, and the information imperfections embedded in the staggered price setting model and the P-bar model imply different dynamics for inflation from those of the partial adjustment model. In the previous section, the discovery that the partial adjustment must rely more on supply shocks to match the contemporaneous covariance between output growth and inflation revealed that these differences in the inflation dynamics are important for one of the basic questions of macroeconomics - the relative importance of demand and supply shocks in the output-inflation relationship.

In order to determine whether the different dynamics imply similar results for the relationship between the deviation of output from a smooth trend and inflation, the effects of the HP-filter must be briefly discussed. Since output inherits a unit root from the unit root in potential output (aggregate supply), the output series must be rendered stationary before computing correlations with inflation. The HP-filter is one method of extracting a smooth trend from an integrated series. As shown in King and Rebelo (1993) and Baxter and King (1995), the HP-filter is equivalent, in large samples, to applying the following two sided moving average to a series;

$$HP = \lambda[1-L]^2[1-L^{-1}]^2[1+\lambda[1-L]^2[1-L^{-1}]^2]^{-1},$$

where λ is a smoothing parameter (which for quarterly data is traditionally set at 1600, so that the HP-filter emphasizes fluctuations with periodicity less than 8 years).

Two properties of the HP-filter are immediately obvious. First, the first differences in the numerator imply that the HP-filter renders stationary a unit root process. Secondly, inspection reveals that the HP-filter is a symmetric filter, i.e. that the coefficients on the kth order lag equals the coefficient on the -kth order lag. This implies that applying the HP-filter to one series does not alter the phase shift (or pattern of lead-lag correlations) between two stationary series (Sargent (1987), King and Rebelo (1993)).

This phase shift preserving property of the HP-filter can be used to examine the implications of the three sticky price models for the relative importance of demand shocks in generating the correlations between HP-filtered output and inflation reported in table 2. As is clear from the above formula for the HP-filter, computing the covariances between inflation and detrended output in terms of the parameters governing price adjustment and the driving processes is tedious, and relatively uninformative. Fortunately, the phase shift between inflation and output can be computed in the frequency domain, which is considerably simpler; (4.1), (4.3), (4.5) and (5.1)-(5.3) express inflation and output in terms of a sequence of innovations, or in the *time domain*. Alternatively, inflation and output at t can be expressed as a weighted sum of the periodic functions $\cos(\omega t)$ and $\sin(\omega t)$, where ω denotes a particular frequency (the *frequency domain*). The lead-lag relationship between output and inflation is summarized in the frequency domain by the phase statistic $P^{y,\Delta p}(\omega)$, which gives the lead ($P^{y,\Delta p}(\omega)>0$) or lag ($P^{y,\Delta p}(\omega)<0$) of output (y) over inflation (Δp) at frequency ω. Appendix I discusses the phase statistic, which essentially reveals asymmetry in the pattern of cross-correlations between output and inflation, so that the phase shift ($P^{y,\Delta p}(\omega)$) is positive when $\rho(\Delta p_t, y_{t-i}) > \rho(\Delta p_t, y_{t+i})$, as reported for the economies considered in table 2.

Therefore, focusing on the effects of demand shocks (i.e., when $\sigma_v = 0$), the phase statistics implied by the various models summarized in tables 3 and 5 are

Partial adjustment: $P^{y,\Delta p}(\omega) = 0$

P-Bar: $P^{y,\Delta p}(\omega) = \arctan(\sin(\omega)/\cos(\omega)) = \omega$

Staggered Price Setting: $P^{y,\Delta p}(\omega) = \arctan(\sin(\omega)/(1+\cos(\omega)))$.

We can transfer from frequencies ω to cycles of varying periodicity c by the equation $c=2\pi/\omega$. Defining the business cycle as cycles of 1 to 8 years, we see that $\omega \in [\pi/2,\pi/16]$ for quarterly data. The above formula for the phase shift between y and Δp then implies that output leads inflation for the P-Bar and staggered price setting model ($P^{y,\Delta p}(\omega)>0$), but not the partial adjustment model. These results lead to proposition 2:

Proposition 2: The partial adjustment model requires supply shocks to match (qualitatively), the leading behavior of output over inflation reported in table 2. The P-bar and staggered price setting model do not.

Of course, proposition 2 is not meant to cast doubt on the presence of supply shocks; proposition 2 simply implies, as does proposition 1, that the partial adjustment model relies on supply shocks more than the P-bar and staggered price setting models do when attempting to match the pattern of output-inflation correlations found in the economies considered in section 2.

6. Discussion and Conclusion

This paper has documented the following stylized facts for the major North American and European economies:

1. Inflation and output growth are negatively correlated;
2. Inflation and detrended output (HP-filtered) are positively correlated.

3. Output leads inflation, in the sense that the correlations of inflation with past output growth or detrended output are larger than the correlations of inflation with future output.

The consistency of several sticky price models with these stylized facts was than explored. The comparison of different models of nominal rigidity reveals that the type of price stickiness utilized can strongly alter, both quantitatively and qualitatively, the conclusions reached regarding the relative importance of supply and demand shocks for the comovement of inflation and output.

In particular, the partial adjustment model has become very popular in business cycle models attempting to match the correlations between prices and output over the business cycle, largely because of its simplicity. This paper has demonstrated that the partial adjustment model is more reliant on supply shocks in generating the negative correlation between output growth and inflation, and the lead of detrended output over inflation, than other popular sticky price models. The problem arises because the partial adjustment specification allows all firms to respond immediately to information about monetary shocks, whereas the data suggests the response of inflation is delayed. For example, a traditional explanation for the lead of output over inflation (i.e., an explanation appearing in an undergraduate text or the financial press), argues that output rises in the presence of excess nominal demand, revealing the excess nominal pressure and resulting in inflation. Without imperfect information, the rise in output does not reveal information about nominal pressure, and hence price stickiness alone does not result in output leading inflation. The P-bar model and staggered price setting both constrain the response of price setters to information about monetary shocks, and can therefore match the lead of output over inflation. The emphasis on imperfect information in explaining the relationship between prices and output has a long history, starting with Hume and continuing in the work of Lucas, and Brunner and Meltzer.

The differences between the partial adjustment model and the other models considered indicates that the recent literature using partial adjustment to model sticky prices in dynamic general equilibrium economies (Hairault and Portier (1993), Rotemberg (1994), Yun (1994), Kimball (1995), King and Watson (1996), King and Wolman (1996), Ireland (1996), Kim (1996), and Woodford (1996)) will tend to match the cyclical relationship between nominal and real variables by placing greater emphasis on aggregate supply disturbances than would be placed by alternative models. While the simplicity of the partial adjustment model is attractive, and previous research has viewed the partial adjustment model as a reasonable approximation to staggered price setting (Calvo (1983), Blanchard and Fischer (1989)), the implications of the partial adjustment model for the comovement between inflation and output are quite different from those of a staggered price setting model (or the P-bar model).

Appendix I:
The Phase Shift Between Output and Inflation

An introduction to frequency domain analysis with macroeconomic applications is Sargent (1987). All of the models considered in section 5 express output and inflation as polynomial functions in the lag operator of the shock to money or nominal aggregate demand. Denoting these functions as $c_y B_y(L)$ and $c_{\Delta P} B_{\Delta P}(L)$, the cross covariance generating of y and ΔP, $g_{y,\Delta P}(z)$, is

$$g_{y,\Delta P}(z) = c_y c_{\Delta P} \sigma_\epsilon B_y(z) B_{\Delta P}(z^{-1}).$$

This cross covariance generating function is a simple way to summarize the pattern of cross correlations between output and inflation.

A straightforward method for evaluating whether output leads inflation, i.e. whether inflation is more strongly correlated with lagged output than future output, is to look at the asymmetry in the cross covariance generating function. A measure of this asymmetry is the phase statistic in frequency domain analysis, which requires the cross spectrum.

The cross spectrum is found by evaluating the cross covariance generating function at $z=e^{-i\omega}$, where $i=(-1)^{1/2}$, and ω refers to the frequency, $\omega \in [-\pi . \pi]$. The phase statistic is found by taking the cross spectrum ($g_{y,\Delta P}(e^{-i\omega})$) and expressing it in terms of its real and imaginary parts,

$$g_{y,\Delta P}(e^{-i\omega}) = co(\omega) + iqu(\omega),$$

where the real part (co) is called the cospectrum and the imaginary part (qu) is called the quadrature spectrum. The phase statistic between y and ΔP is then

$$P^{y,\Delta P}(\omega) = \arctan(qu(\omega)/co(\omega)).$$

Application of this technique to the various models yields the phase statistics given in the text. Output leads inflation at frequency ω if $P^{y,\Delta P}(\omega)>0$, and output lags inflation at frequency ω if $P^{y,\Delta P}(\omega)<0$.

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