The Auctions of Swiss Government Bonds: Should the Treasury Price Discriminate or Not?*

By Daniel Heller† and Yvan Lengwiler‡

January 14, 1998

Ever since Friedman's (1960) contribution, there has been an ongoing controversy about whether the Treasury should auction off its government debt with a discriminatory or with a uniform price format. Many industrialized countries, the United States or Germany, for instance, use discriminatory auctions, while Switzerland applies a uniform price rule. Using recent contributions to multi-unit auction theory, we analyze data on the bids submitted to Swiss Treasury bond auctions over the last three years. We then construct hypothetical bid functions that would occur under price discrimination. Based on these bid functions, we determine which auction format minimizes the government's costs of financing its debt.

JEL classification codes: D44, H63.

Keywords: government bonds, multi-unit auctions, price discrimination, uniform price.

1 Introduction

For decades, governments have been auctioning off fixed income securities to procure funds to finance their debt. Despite the quantitative importance of Treasury auctions, our theoretical knowledge on so called multi-unit auctions is still fairly limited. While auction theory has been a prolific field in economics, it has mostly focused on single-unit auctions — such as

*We thank Elmar Wolfstetter for stimulating our interest in the topic, and V. V. Chari, Ed Green, Michael Gordy, Peter Kugler, Dieter Nautz, Georg Nöldeke, and Walter Wasserfallen, as well workshop participants at the ESEM 1997 Annual Congress in Toulouse and at the annual meeting of the Swiss Society of Statistics and Economics in Luzern, 1997, and seminar participants at the Virginia Polytechnic Institute, at the Federal Reserve Bank of Minneapolis, and at the University of Basel for interesting comments. Parts of this paper were written while Lengwiler was on leave at the Board of Governors of the Federal Reserve System. The views expressed are not necessarily those of the Board of Governors of the Federal Reserve System or of the Swiss National Bank.

†Swiss National Bank.

‡Swiss National Bank and Board of Governors of the Federal Reserve System.

Please address correspondence to Yvan Lengwiler, Board of Governors of the Federal Reserve System, Division of Monetary Affairs, MAIL STOP 71, Washington, DC 20551, phone (202) 452 2607, e-mail ylengwiler@frb.gov

auctions for a painting, a bottle of wine, or drilling rights to an oil field. Multi-unit auctions are special in the sense that the good that stands for auction is divisible and that more than one unit of the same good is available. In an important contribution, Back and Zender (1993) show that the results from single-unit auctions do not necessarily carry over to multi-unit auctions. An early model that is especially relevant for Treasury auctions is Smith (1966). Nautz (1995b) and Nautz and Wolfstetter (1997) have developed explicit solutions for this model.

In practice, Treasuries apply two kinds of formats for auctions of fixed income securities, namely discriminatory and uniform price auctions. In a discriminatory auction a bidder pays the amount equal to his bid, if the bid is above the cut-off price set by the Treasury. In a uniform price auction, a bidder pays the cut-off price conditional on having submitted a bid that was equal or higher than the cut-off price. In most countries, bidders are allowed to submit multiple price-quantity pairs.

Considerable attention has been given to the question of which auction format yields greater revenues for the Treasury. Bikhchandani and Huang (1993) provide a survey over this controversy. At present, discriminatory auctions for bonds are more often used than uniform auctions, but many countries have been experimenting with the auction format. The United States, for instance, used to apply only discriminatory auctions, but has recently started to issue some bonds in uniform price auctions (Nyborg and Sundaresan, 1996). Germany went the opposite direction. The Bundesbank switched from uniform price to discriminatory auctions for their repos (Nautz, 1995a).

Switzerland is among the countries that apply the uniform price format. The aim of this paper is to investigate whether the Swiss Treasury could increase its revenues by switching to discriminatory auctions. We use Nautz' (1995b) model to derive optimal bid functions under the two auction regimes. Along the way, we also show that some small changes in the assumptions lead to a considerable simplification of Nautz' proofs. Then, we take observed bid functions from recent uniform price auctions and transform them into hypothetical bid functions under a discriminatory rule. With these bid functions, we can calculate which auction format would have in the past been better for the Swiss Treasury.

The paper is structured as follows. In the next section, we describe how Treasury bonds are currently auctioned off in Switzerland. In the third section we present the theoretical model for multi-unit auctions. Theoretical formulas for optimal bidding under the two auction regimes are derived. In the fourth section we show a method of how market participants can form expectations about the price of a just-to-be-issued bond. With the resulting density function a hypothetical bid function for a discriminatory auction is derived and the costs to the government of issuing bonds can be calculated. The last section draws policy conclusions.

2 The auction procedure for Swiss government bonds

As mentioned above, the Swiss Treasury uses uniform price (also called Dutch or competitive) auctions to issue fixed income securities. In this format, winning bidders pay the lowest accepted bid, the so called cut-off price. The following paragraphs describe in more detail the auction design that is applied by the Swiss Treasury for bonds. Money market debt register claims are auctioned in a slightly different way, and we do not discuss them in this paper.

The auctions for bonds take place on a bimonthly basis. The Treasury announces the characteristics of the bond that is to be issued, such as time to maturity, coupon, and callability. The Treasury also states the maximum number of bonds that will be issued. Usually, however, much less than this maximum is sold in the auction. The Treasury also reserve the right to cancel an auction if it does not consider the bids satisfactory.

Sometimes the Treasury chooses not to issue a newly designed bond, but simply to extend the volume of a previously issued series. In such a case, the new bonds have exactly the same characteristics (coupon, maturity, callability) as a previously issued series. For reasons explained later, we will use only the data on such auctions.

The bidders are invited to submit as many price-quantity bids as they want to. The bids are sealed and they have to be on a price grid of five cents. In addition to submitting price-quantity pairs, quantity bids without price can be placed. These unpriced bids have to be less than 100,000 Swiss francs. The importance of unpriced bids is small. They account for about 5% of the total amount sold in an average Treasury bonds auction.

After all bids are submitted, the Treasury decides on the cut-off price. All bids above this price are fully satisfied. Since the Treasury also sets an a priori limit to the number of bonds that are to be issued, rationing of bids submitted at the lowest winning price can occur. In recent years, however, rationing has become less frequent, since the maximum number of bonds that is issued in a single auction was substantially increased.

The circle of bidders is restricted to institutions and people holding accounts with the Swiss National Bank (SNB). Under the current regulation, institutional bidders admitted to the auctions are confined to banks with branches in Switzerland. All bidders are treated equally, i.e. there are no primary dealers. Also, no fee is charged to the participants.

3 The model: Comparing equilibria with or without price discrimination

3.1 Setup

In this section, we explain the construction and assumptions of a model of multi-unit auctions which has recently been proposed by Nautz (1995b), and which has been extended by Nautz and Wolfstetter (1997). In this model, the seller has a more or less passive role. Its only objective is to sell some given number of homogenous objects in such a way as to raise as much money as possible. Applied to the auction of government debt, we assume that the sole objective of the Treasury is to raise some given quantity of funds at minimum cost. The government does not act strategically, and as such the model is not a game. The bidders' problem is basically one of simple expected utility maximization, with no strategic interplay to consider.

We now turn to the details of the model. Let there be a finite *price grid* $P := \{p_1, \ldots, p_j, \ldots, p_k\}$. Prices are indexed such that a strictly increasing sequence results, $0 < p_1 < \cdots < p_j < \cdots < p_k$. There is a large number (ideally we would assume a continuum) of bidders i. Every bidder submits a *bid function* $b_i \colon P \to \mathbb{R}_+$, indicating the amount of debt he wishes to purchase at each price in the price grid. From these bids, the *aggregate bid function* is defined as $B(p) := \sum_i b_i(p)$.[1] It is important to note that the reported bids might deviate from the true willingness to pay, since bidders do not have to reveal their true preference. Let $d_i\colon \mathbb{R}_+ \to \mathbb{R}_+$ denote the true demand function. We assume that d_i is a strictly decreasing, invertible function. As before, $D(p) := \sum_i d_i(p)$.

After the auction, the seller announces a *cut-off price* π. All bids associated with prices weakly larger than π will be met, so $B(\pi)$ is the number of bonds sold to the bidders. The government sets π in order to meet some goal regarding the amount of funds raised. The revenue of the auction depends on its format, and will be discussed later. Let $R(\pi)$ denote this revenue. Then, the government sets π such that $R(\pi) = S$, when S is the amount of money that must be raised by the auction.

When deciding about their bid function, the bidders are unsure about the price that will result. This uncertainty is captured by some distribution function F from which π is drawn. The bidders take F as given because they are individually negligible, so that no one can individually have an influence on the seller's chosen cut-off price. Let $f(p_j) := F(p_j) - F(p_{j-1})$ for $j > 1$ and $f(p_1) := F(p_1)$.

Nautz (1995) characterizes optimal bidding in both auction formats, uniform and discriminating. We are able to provide simpler proofs to Nautz' results because we make somewhat different assumptions. First, we assume that all prices on the price grid have some probability of being chosen as the cut-off price, i.e. $f(p_j) > 0$ for all j. This is, in fact, not a strong assumption because we can always fix the price grid in such a way that it is contained in the support of the distribution of the price. Second, we do not restrict bidders to use decreasing bid functions. This allows us to use simple unconstrained maximization. This, too, is not a very strong assumption, because, as we will see in section 4.3, this constraint actually never binds empirically.

3.2 The uniform price auction

In a uniform price auction, all bidders pay the same price π for each unit of debt they purchase. Thus, the government's revenue is simply

$$R(\pi) := \pi B(\pi). \tag{1}$$

Notice that this is just like the revenue function of an ordinary monopolist. By assumption, the seller chooses π such that $\pi B(\pi) = S$. If B is not "too convex," then R is single peaked. As a consequence, there will usually be two πs that will yield the required amount of funds. In this case, the government chooses the higher price, because this is associated with fewer government bonds to be issued. Exceptionally, there might only be one such π, if S equals the maximum of $\pi B(\pi)$. If S is bigger than this maximum, then there is no π satifying the needs of the government.

[1] If there is a continuum of bidders (the unit interval, say), then $B(p) := \int_0^1 b_i(p)\,\mathrm{d}i$, assuming that b is integrable with respect to i.

Bidder i's expected payoff is

$$\sum_{j=1}^{k} f(p_j) \Bigg(\underbrace{\int_0^{b_i(p_j)} d_i^{-1}(q)\,\mathrm{d}q}_{\text{return}} - \underbrace{p_j b_i(p_j)}_{\text{cost}} \Bigg). \tag{2}$$

i chooses his bid function b_i such as to maximize (2). The following result, which is due to Nautz (1995b), says that in a uniform price auction, bidders reveal their true willingness to pay.

Proposition 1 (optimal bids in a uniform auction). *In a uniform price auction, each bidder i sets $b_i(p_j) = d_i(p_j)$ for all j.*

Proof. The bidder's problem is

$$\max_{b_i(p_1),\cdots,b_i(p_k)} \sum_{j=1}^{k} f(p_j) \left(\int_0^{b_i(p_j)} d_i^{-1}(q)\,\mathrm{d}q \ - \ p_j b_i(p_j) \right),$$

giving rise to k first order conditions,

$$f(p_j)(d_i^{-1}(b_i(p_j)) - p_j) = 0, \qquad j = 1, \ldots, k.$$

Since $f(p_i) > 0$ by assumption, it follows that $d_i^{-1}(b_i(p_j)) = p_j$, or equivalently, $b_i(p_j) = d_i(p_j)$, as claimed in the proposition. The second order condition is satisfied because d_i (and hence d_i^{-1}) is a decreasing function. QED

The intuition for this result is immediate: Consider the idea of bidding something else than actual demand at some price p_j. If π turns out different from p_j, this has no effect. Yet, if π turns out to equal p_j, then the bidder will not receive his utilitiy maximizing quantity, which is $d_i(p_j)$, but the quantity he bidded $b_i(p_j)$. So the bidder cannot gain by misrepresenting his demand. As a consequence, he will optimally reveal his true willingness to pay. This result is in contrast with Back and Zender (1993). In their model, bidders submit a steeper demand function than their true preference. Conditional on everyone behaving in the same way, the bidders know that submitting a steeper demand curve will yield a lower cut-off price while it increases the marginal costs on the other bidders. This result crucially hinges on the assumption that the Treasury will auction off a given quantity of bonds that is known in advance. In our model, such kind of strategic elements in bidding are not considered. Also, the number of bonds that will be issued is not known to the bidders, as it is the case in almost all Swiss bond auctions.

3.3 The price discriminating auction

To distinguish the equilibrium of both auction formats, a tilde will denote the endogenous variables in a price discriminating auction. Price discrimination can be used by the government in an attempt to extract the consumer surplus. Yet, bidders will try to evade this extraction and behave differently in a price discriminating auction than they do in a uniform price auction.

In a price discriminating auction, a bidder pays the price at which he submitted a bid, even if the cut-off price is lower. Accordingly, the government's revenue is

$$\tilde{R}(\tilde{\pi}) := \sum_{p_j \geq \tilde{\pi}} p_j(\tilde{B}(p_j) - \tilde{B}(p_{j+1})), \tag{3}$$

with $\tilde{B}(p_{k+1}) := 0$.

Bidder i's expected payoff is

$$\sum_{j=1}^{k} \tilde{f}(p_j)\Bigg(\underbrace{\int_0^{\tilde{b}_i(p_j)} d_i^{-1}(q)\,\mathrm{d}q}_{\text{return}} - \underbrace{\sum\nolimits_{j'=j}^{k} p_{j'}(\tilde{b}_i(p_{j'}) - \tilde{b}_i(p_{j'+1}))}_{\text{cost}}\Bigg), \tag{4}$$

again with $\tilde{b}_i(p_{k+1}) := 0$. The following result is also due to Nautz (1995b).

Proposition 2 (optimal bids in a discriminatory auction). *Bidder i's bid function is defined by*

$$\tilde{b}_i(p_j) := d_i\left(p_j + (p_j - p_{j-1})\frac{\tilde{F}(p_{j-1})}{\tilde{f}(p_j)}\right) < d_i(p_j) \tag{5}$$

for all $j > 1$, and $\tilde{b}_i(p_1) := d_i(p_1)$.

This result says that bidders underrepresent their true valuation for all prices except the lowest one.

Proof. Bidder i chooses his bids $\tilde{b}_i(p_1), \cdots, \tilde{b}_i(p_k)$ in such a way as to maximize (4). Consider first the first-order condition for $\tilde{b}_i(p_1)$,

$$\tilde{f}(p_1)(d_i^{-1}(\tilde{b}_i(p_1)) - p_1) = 0.$$

Since $\tilde{f}(p_1) > 0$ by assumption, this implies

$$\tilde{b}_i(p_1) = d_i(p_1),$$

as claimed in the proposition.

The first-order conditions for the other prices are more complicated,

$$\tilde{f}(p_j)d_i^{-1}(\tilde{b}_i(p_j)) - \Bigg(p_j \underbrace{\sum\nolimits_{j' \leq j} \tilde{f}(p_{j'})}_{=\tilde{F}(p_j) = \tilde{f}(p_j) + \tilde{F}(p_{j-1})} - p_{j-1} \underbrace{\sum\nolimits_{j' \leq j-1} \tilde{f}(p_{j'})}_{=\tilde{F}(p_{j-1})}\Bigg) = 0.$$

Rearranging yields

$$d_i^{-1}(\tilde{b}_i(p_j)) = p_j + (p_j - p_{j-1})\frac{\tilde{F}(p_{j-1})}{\tilde{f}(p_j)},$$

implying (5).

The inequality $\tilde{b}_i(p_j) < d_i(p_j)$ simply follows from the fact that $(p_j - p_{j-1})\tilde{F}(p_{j-1})/\tilde{f}(p_j)$ is strictly positive and d_i is a strictly decreasing function. QED

4 Constructing the outcome of hypothetical price discriminating auctions

4.1 The data and the strategy for interpreting it

The costs of paying back the debt is proportional to the number of bonds issued. Therefore it is optimal for the Treasury to choose the auction format that provides the same revenue with the lower number of bonds. In other words, the government's choice is $\min\{B(\pi), \tilde{B}(\tilde{\pi})\}$ subject to $R(\pi) \geq S$ and $\tilde{R}(\tilde{\pi}) \geq S$, with S being the amount of funds to be raised in the auction. Which format is better depends, of course, on the bid functions B and $\tilde{B}$. More specifically, it depends on the way agents shade their bids in the discriminating auction. Suppose bidders did report their true willingness to pay in a price discriminating auction, i.e. they did not shade, $B = \tilde{B}$. Then, the price discriminating auction would clearly be better for the government, because it would offer a means to collect the entire consumer surplus. Yet, as proposition 2 establishes, bidders do shade, and in fact, they shade precisely in order to save some of the consumer surplus. So, choosing between uniform price and price discrimination involves a trade-off for the government. In the uniform price auction, bidding is strong, but the government cannot collect the consumer surplus. In a price discriminating auction, the government collects at least some of the rent, but bidding is weak due to shading. In the rest of this paper, we answer the empirical question which auction procedure would have been better in the past for the Swiss Treasury.

Our data cover only uniform price auctions, because this is the only format used by the Swiss Treasury. Our strategy is to use these data to generate the distribution of cut-off price $\tilde{F}$ if price discrimination would have been used. Once we have $\tilde{F}$, we can apply propositions 1 and 2 to transform the observed bid function B into a hypothetical one $\tilde{B}$, describing how bidders would have behaved if the treasury was price discriminating. Finally, we compare the realized cost that the government has incurred to finance its debt with the cost that it would have had to bear had it used the price discriminating format.

In this whole process we disregard the unpriced bids (which are quantitatively not very important). We disregard them because our theory makes no prediction about why anyone would submit such bids. Therefore we have also no description of such bidders' behavior in a price discriminating auction. We also disregard the fact that, in the past, rationing of the lowest accepted class of bids has occured.

4.2 Estimating $\tilde{F}$ and constructing $\tilde{B}$

In the theoretical sections above, we have seen that the density function $\tilde{F}$ plays a crucial role in how agents shade their bids in a discriminatory auction. What do we know about $\tilde{F}$? Because different Treasury bonds are close substitutes, the price of a bond in a primary auction must in some way be related to the prices of the bonds that are traded in the secondary market. It seems reasonable to assume that every bidder follows closely the prices on the secondary market to extract a signal about the cut-off price that is about to be set by the Treasury. A bidder is not going to submit a bid at a price that he considers too high relative to what he would have to pay for a similar bond in the secondary market. In this way, the bid functions in the auctions will tend to shift according to market conditions. One way to capture this nexus is

$$\pi_z = \alpha + \beta\pi_z^s + \varepsilon_z, \tag{6}$$

with z indicating some auction.[2]

The cut-off price π is equal to the sum of three components. The constant α is a price wedge between primary and secondary market. The price π^s is the price at which a bond with the same characteristics (same time to maturity, same coupon, same callability) is traded on the secondary market just before the auction takes place. The final price component is an error term ε that stems from the Treasury's discretion that is unknown to the bidders. In addition to the secondary market, a bidder can also use the prices in the when-issued market to extract a signal about the cut-off price. Unfortunately, data of the when-issued prices are not available to us. Therefore, we are confined to data from the secondary market.

The characteristics of Treasury bonds that have been issued in Switzerland in the last few years vary widely: times to maturity range from roughly 3 to 20 years. Moreover, some bonds are not callable, others include one or several call options. In the most cases, at the time of the auction, there is no bond with exactly the same characteristics traded on the secondary market. One way to deal with this problem is to first estimate a zero-coupon yield curve (Deacon and Derry, 1994) and then to use this curve to determine the price that the to be auctioned bond would obtain on the secondary market. This would allow us to correct different maturities and coupons, but this route fails if some of the bonds are callable. In our case, there is an escape, though. We consider only those auctions in which the Treasury has not designed a new bond, but has chosen to sell more of an existing bond. As explained before, such bonds have exactly the same coupon, the same date of maturity, and the same call options as a previously issued bond that is still traded on the secondary market. π^s is then simply the secondary market price of such a previously issued bond at 11 o'clock a.m. of the day of the auction. This price should contain all the public information available about the value of the to-be-issued bond. Given that π^s is such a good signal, it is not surprising that α and β are statistically not different from zero and one, respectively.

¿From proposition 1 we know that the observed bid functions, B_z, are the true demand functions. If we knew the distribution of the cut-off price, we could apply proposition 2 and compute the shaded bid functions, $\tilde{B}_z$, of the price discrimination auctions. We can use the residuals of equation (6) to generate the distribution function of the price π_z of the uniform price auction, F_z. Yet, in order to apply proposition 2, we need the distribution of the cut-off price of the price discriminating auction, $\tilde{F}_z$. We proceed as follows: ¿From (6) we generate F_z, and, using proposition 2, we can compute some bid function $\tilde{B}_z^1$. This is the bid function that would have been realized in the price discriminating auction if the bidders expected $\tilde{\pi}_z$ to be distributed according to F_z. Then we use $\tilde{B}_z^1$ to generate the cut-off price $\tilde{\pi}_z^1$, which is the cut-off price that would just have been sufficient to raise an amount S_z of funds. Yet, this is not consistent, because we cannot expect that the price distribution is the same in both

[2]Because the variance of prices on financial markets is not constant over time, there is a potential heteroskedasticity problem. This problem is not easy to address here because the time between observations is not constant. Also, the small sample size prohibits the application of ARCH estimation procedures.

auction formats, so we need to iterate this process,

$$\begin{array}{rcccccc}
\pi_z = \alpha^0 + \beta^0 \pi_z^s + \varepsilon_z^0 & \Longrightarrow & F_z & \Longrightarrow & \tilde{B}_z^1 & \Longrightarrow & \tilde{\pi}_z^1 \\
\tilde{\pi}_z^1 = \alpha^1 + \beta^1 \pi_z^s + \varepsilon_z^1 & \Longrightarrow & \tilde{F}_z^1 & \Longrightarrow & \tilde{B}_z^2 & \Longrightarrow & \tilde{\pi}_z^2 \\
 & & \vdots & & & & \\
\tilde{\pi}_z^n = \alpha^n + \beta^n \pi_z^s + \varepsilon_z^n & \Longrightarrow & \tilde{F}_z^n & \Longrightarrow & \tilde{B}_z^{n+1} & \Longrightarrow & \tilde{\pi}_z^{n+1} \\
 & & \vdots & & & &
\end{array}$$

If this process converges, it leads to pairs of bid functions and distributions, $(\tilde{B}_z^n, \tilde{F}_z^n)$, that are more and more compatible with each other. In the limit, it gives us the outcome of hypothetical, price discriminating auctions. We stop the iteration when

$$\max_z \left| \tilde{B}_z^{n+1}(\tilde{\pi}_z^{n+1}) - \tilde{B}_z^n(\tilde{\pi}_z^n) \right| < \delta. \tag{7}$$

That means that we iterate as long as the computed number of bonds necessary to meet the target of raising an amount S_z of funds changes by more than δ in any single auction z. We set δ to be some reasonably small number (we choose 0.01, indicating an accuracy of the computation of bonds issued worth 10,000 Swiss francs). As an illustration, figure 1 shows the process of convergence for auction #7 of July 25, 1996.

---

figure 1 here

Harris and Raviv (1981) mention a set of empirical and experimental findings that correspond to our hypothetical bid functions.

> "1. Mean bid is larger under the competitive than under the discriminating auction.
>
> 2. The variance of bids is larger under the competitive auction.
>
> 3. The evidence regarding the comparison of seller's revenue under the two types of auctions is inconclusive." [Harris and Raviv (1981), page 1488].

Point 1 is a corollary of our proposition 2, and since our hypothetical bid functions satisfy this proposition by construction, they also satisfy Harris and Raviv's property 1. In fact, the hypothetical bid functions are strictly below the original bid functions for each of the twenty-nine auctions, which implies that the mean bid is lower in each of the hypothetical price discriminating auctions. Point 2 does also fit with our computed hypothetical bid functions, because these functions turn out to be always flatter than the original bid functions of the uniform price (or, "competitive") auction. Thus, the variance of the bids under price discrimination, as we have computed them, is smaller than under the uniform price auction. Point 3 is also in accordance with our computations. The next section discusses this aspect. The fact that the bid functions for the hypothetical price discriminating auctions fit those three empirical facts should provide some confidence that the computation procedure has generated hypothetical bid functions that do indeed make sense.

4.3 The results

Convergence was achieved after eight iterations. Table 1 contains all the results. It features the price signal of the secondary market π^s, the realized price of the uniform price auction π, the number of bonds issued $B(\pi)$, and the amount of funds raised $S(=\pi B(\pi))$.[3] Then follow the computed cut-off price of the hypothetical price discriminating auction $\tilde{\pi}$. Next is the number of bonds that would have to have been issued in this auction in order to raise S funds, $\tilde{B}(\tilde{\pi})$. The next two columns show by how much the price and the issued quantity differ in both formats. As one would expect, the discriminatory auction always has a lower cut-off price than the uniform price auction. The last column reports by what proportion the Treasury has fared better using the uniform compared to the discriminatory auction format. This is the percentage points by which $\tilde{B}(\tilde{\pi})$ exceeds $B(\pi)$.

---

table 1 here

The results demonstrate that in the past, price discrimination would not have done any good to the Treasury. In fact, in twenty-two out of twenty-nine auctions it would have required the Treasury to issue slightly more bonds in order to collect the same amount of funds. As a result, the price discriminating auction would have on average increased the government's debt finance cost by 0.2%. Over the whole period covering forty month, the Treasury would have had to issue additional bonds worth 20 million francs in order to raise the same revenue. This difference is small, and is in fact not significantly different from zero. What we find, then, is *empirical revenue equivalence* (or more precisely, cost equivalence) of both auction procedures.

We just mention that the generated $\tilde{B}$-functions are decreasing in all twenty-nine observations, so that we can say with some certainty that the constraint on the bidders to use downward sloping bid functions (which is imposed in Nautz, 1995b, and Nautz and Wolfstetter, 1997) is empirically irrelevant, at least for the Swiss Treasury bond auctions.

5 Conclusions and caveats

The question which auction procedure the Treasury should use is the subject of an ongoing controversy. In this paper, we have shown that theory alone cannot settle this question. We explore, subject to the validity of the Nautz model, a possibility of how to deal with the question, even when no experiments with parallel auctions using both auction formats are available. Our method allows us to retrieve hypothetical bid functions that would have occured under price discrimination, using data that have been observed under uniform price auctions only.

¿From this exercise, we conclude that the Swiss Treasury has wisely chosen to use the uniform price format. Moving to a price discriminating auction would not have made much

[3] $B(\pi)$ as in the table is simply the quantity of bids at the cut-off price. It can deviate from the number of bonds that were actually issued for two reasons. First, we disregard unpriced bids. Second, we also disregard rationing that has occured now and then. Both omissions tend to cancel each other to some extent.

difference in terms of the cost of financing the government debt. If anything, it would have made it a little more expensive for the Treasury. Moreover, the uniform price auction has another advantage. It is strategically much simpler than the price discriminating auction (an observation stressed by Chari and Weber, 1992, and by Nautz, 1995b). Since in a uniform price auction the bidders will optimally just reveal their true demand function, they do not have to use resources to figure out how the other bidders will behave and they need not engage in any strategic thinking. Thus, from a social welfare point of view, uniform price auctions are the better choice in the Nautz model.[4]

Let us nevertheless mention two caveats. First, we have assumed that the government simply wants to raise some given amount of funds *per auction*. Yet, the government's problem is dynamic: By issuing more bonds when bidding is strong, and issuing less when bidding is weak (hoping that the next auction will turn out better), it can smooth bidders' swings and reduce its cost. Thus, a more complete analysis would consider a sequence of auctions and a government that is optimizing intertemporally. Second, the assumptions needed for Nautz' results (our propositions 1 and 2) may not apply to the situation in Switzerland. Specifically, the market is currently dominated by three players — the three largest Swiss banks. They usually buy about two thirds to three quarters of the bonds. Despite this oligopolistic market structure, no episodes of market cornering or collusion are known. Yet it is quite clear that these big players do not take prices (i.e. the distribution function F or $\tilde{F}$, respectively) as given, thus invalidating one of the assumptions of the model. On the other hand, the scope for such oligopolistic behavior might be limited by the fact that the banking market is close to being perfectly contestable: If the banks were able to extract large rents by their market power, it would be worthwhile (and not very expensive in terms of setup costs) for a potential competitor to enter the banking market and out-bid the established banks in the Treasury bond auctions.

[4]This calls for some qualification. If we drop the assumption that bidders take the price distribution as given, full revelation of the true demand does not hold anymore, see e.g. Wilson (1979), Noussair (1995), Tenorio (1997).

References

Back, K. and Zender, J. F. (1993) "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *The Review of Financial Studies*, 6(4), 733–764.

Bikchandari, S. and Huang, C.-F. (1993) "The Economics of Treasury Security Markets," *Journal of Economic Perspectives* 7, 117–134.

Chari, V. V. and Weber, R. J. (1992) "How the U.S. Treasury Should Auction its Debt," *Federal Reserve Bank of Minneapolis Quarterly Review* 16(4), 3–12.

Deacon, M. and Derry, A. (1994) "Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds," Bank of England, Working Paper Series no. 23.

Friedman, M. (1960) *A Program for Monetary Stability*, New York: Fordham University Press.

Harris, M. and Raviv, A. (1981) "Allocation Mechanisms and the Design of Auctions," *Econometrica* 49(6), 1477–1499.

Nautz, D. (1995a) "Zur Feinsteuerung des Geldmarktes durch die Wertpapierpensionsgeschäfte der Bundesbank," *Zeitschrift für Wirtschafts- und Sozialwissenschaften* 115(4), 623–644.

Nautz, D. (1995b) "Optimal bidding in multi-unit auctions with many bidders," *Economics Letters* 48, 301–306.

Nautz, D. and Wolfstetter, E. (1997) "Bid Shading and Risk Aversion in Multi-Unit Auctions with Many Bidders," *Economics Letters* 56(2), 195–200.

Noussair, Ch. (1995) "Equilibria in a Multi-Object Uniform Price Sealed Bid Auction with Multi-Unit Demands," *Economic Theory* 5(2), 337–51.

Nyborg, K. G. and Sundaresan, S. (1996) "Discriminatory versus Uniform Treasury Auctions: Evidence from When-Issued Transactions," *Journal of Financial Economics* 42(1), 63–104.

Smith, V. L. (1966) "Bidding Theory and the Treasury Bill Auction: Does Price Discrimination Increase the Bill Price?" *Review of Economics and Statistics* 48, 141–146.

Tenorio, R. (1997) "On Strategic Quantity Bidding in Multiple Unit Auctions," *The Journal of Industrial Economics* 45(2), 207–17.

Wilson, R. (1979) "Auctions of Shares," *Quarterly Journal of Economics* 93(4), 675–689.

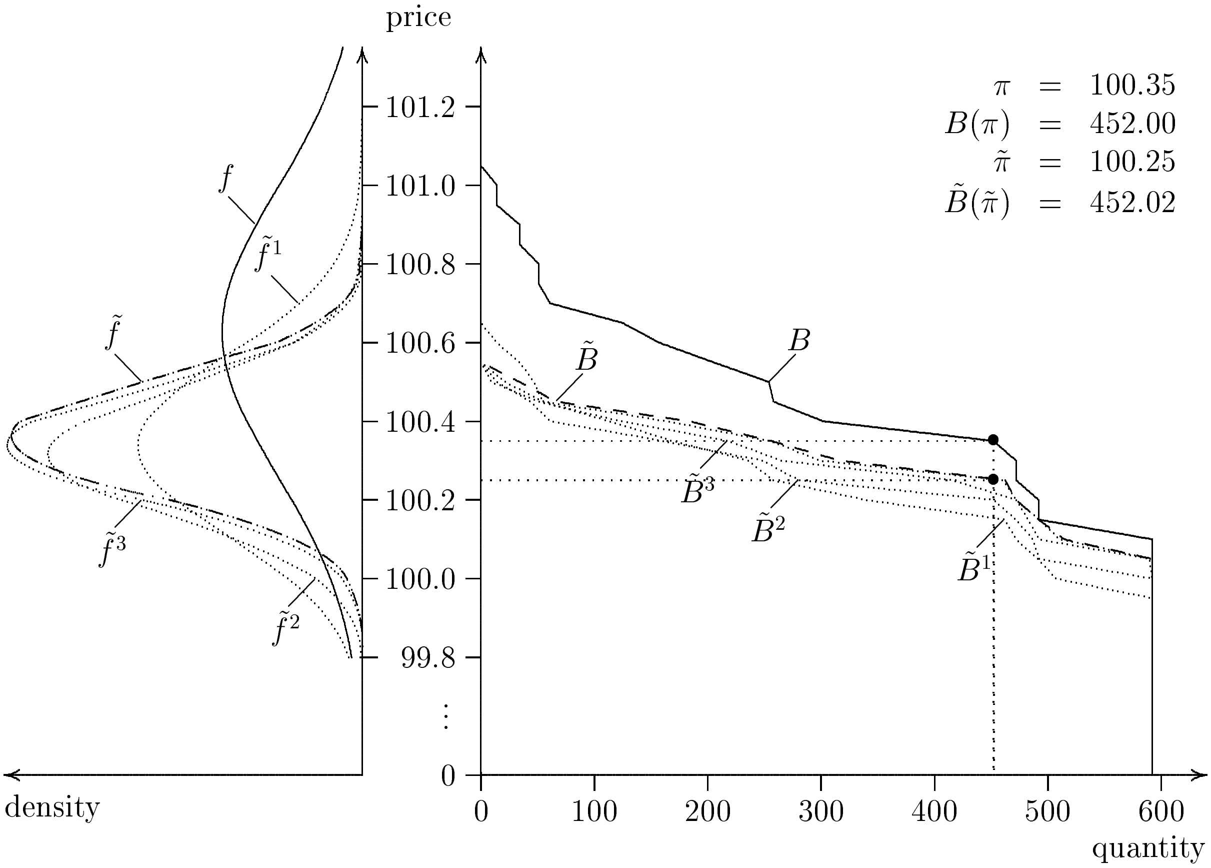

Figure 1. *Auction #7, bid functions (right) and density of expected cut-off price (left), original data and first five iterations.*

Table 1. Comparison of actual and simulated auctions.

obs	date	π^s	π	$B(\pi)$	S	$\tilde{\pi}$	$\tilde{B}(\tilde{\pi})$	$\tilde{\pi}-\pi$	$\tilde{B}(\tilde{\pi})-B(\pi)$	advantage uniform
1	27.03.1997	98.60	98.50	242.40	238.76	98.30	242.68	−0.20	+0.28	+0.12%
2	27.03.1997	106.87	106.90	305.20	326.26	106.70	305.70	−0.20	+0.50	+0.16%
3	23.01.1997	105.47	105.45	878.53	926.41	105.28	879.65	−0.17	+1.12	+0.13%
4	28.11.1996	103.78	103.45	726.63	751.70	103.38	726.39	−0.07	−0.25	−0.03%
5	26.09.1996	100.61	100.60	1079.50	1085.98	100.37	1081.36	−0.23	+1.86	+0.17%
6	26.09.1996	111.73	111.50	294.70	328.59	111.41	294.63	−0.09	−0.07	−0.02%
7	25.07.1996	100.65	100.35	452.00	453.58	100.25	452.02	−0.10	+0.02	+0.01%
8	25.07.1996	103.40	103.10	300.60	309.92	103.00	300.59	−0.10	−0.01	−0.00%
9	23.05.1996	104.53	104.50	219.00	228.86	104.31	219.34	−0.19	+0.34	+0.16%
10	25.01.1996	99.52	100.75	68.02	68.53	99.48	68.90	−1.27	+0.88	+1.29%
11	23.11.1995	105.29	105.40	302.55	318.89	105.15	303.25	−0.25	+0.70	+0.23%
12	28.09.1995	92.75	92.75	93.00	86.26	92.40	93.23	−0.35	+0.23	+0.24%
13	27.07.1995	97.80	97.90	585.20	572.91	97.60	586.99	−0.30	+1.79	+0.31%
14	24.05.1995	96.75	96.80	516.15	499.63	96.53	517.61	−0.27	+1.46	+0.28%
15	23.03.1995	100.72	100.90	605.12	610.56	100.57	607.20	−0.33	+2.08	+0.34%
16	23.02.1995	101.46	101.50	505.07	512.65	101.26	506.08	−0.24	+1.01	+0.20%
17	26.01.1995	101.55	101.50	429.54	435.98	101.31	430.13	−0.19	+0.59	+0.14%
18	22.09.1994	100.08	100.00	347.32	347.32	99.80	347.90	−0.20	+0.58	+0.17%
19	25.08.1994	93.00	92.40	194.50	179.72	92.35	194.21	−0.05	−0.29	−0.15%
20	25.08.1994	98.85	98.40	200.18	196.98	98.34	200.07	−0.06	−0.11	−0.05%
21	28.07.1994	100.20	100.30	299.14	300.03	100.00	299.90	−0.30	+0.77	+0.26%
22	28.07.1994	95.35	95.10	351.80	334.56	94.95	351.85	−0.15	+0.05	+0.01%
23	28.06.1994	99.90	99.80	257.50	256.99	99.61	257.82	−0.19	+0.32	+0.12%
24	28.04.1994	97.90	97.30	337.75	328.63	97.25	337.09	−0.05	−0.66	−0.20%
25	28.04.1994	97.96	98.40	197.42	194.26	97.83	198.53	−0.57	+1.11	+0.56%
26	24.03.1994	99.15	99.30	484.70	481.31	98.96	486.15	−0.34	+1.45	+0.30%
27	24.03.1994	98.80	99.20	303.60	301.17	98.68	305.23	−0.52	+1.63	+0.54%
28	24.02.1994	101.60	101.20	423.52	428.60	101.10	422.76	−0.10	−0.76	−0.18%
29	27.01.1994	102.65	103.00	895.67	922.54	102.55	899.52	−0.45	+3.85	+0.43%