International Finance Discussion Papers

Number 155

March 1980

STICKY NOMINAL WAGES AND THE OPTIMAL EMPLOYMENT RULE

by

Douglas G. Waldo

Sticky Nominal Wages and the Optimal Employment Rule

by

Douglas G. Waldo*

Summary

Conventional monetary theory implies increases in the money supply will increase aggregate demand, and, consequently, raise the general price level and real output. One approach to the division of increased aggregate demand between prices and real output is the Natural Rate-Rational Expectations (NRRE) hypothesis. Papers by Robert Lucas (1972), Thomas Sargent (1973), and Thomas Sargent and Neil Wallace (1975), show that this hypothesis implies a very pessimistic view of the ability of the monetary authority to dampen cyclical fluctuations in real output. Under the NRRE hypothesis real output is independent of the systematic component of monetary policy.1/ Some economists are critical of the auction market basis of the NRRE hypothesis. For instance, Stanley Fischer (1977a) hypothesizes a macroeconomic model where sticky nominal wage contracts prevent workers and entrepreneurs from offsetting the effects of systematic policy on real wages. These changes

*/ I am grateful to Richard Cothren, Richard Froyen, Thomas Kniesner, Robert MacKay, Michael Salemi, Roger Waud, and an anonymous referee for helpful comments. This paper is a part of my dissertation which was supported by the Brookings Institutions and the Social Science Research Council. The views I express are my own and should not be attributed to either the board or staff members of the Brookings Institution, the Federal Reserve System, or the Social Science Research Council. This paper will appear in the Journal of Monetary Economics.

1/ A systematic policy is any policy which is known with certainty one period in advance by agents forecasting rationally.

in the real wage are assumed to induce changes in employment. Thus, in Fischer's model sticky nominal wage contracts imply a role for systematic monetary policy and also provide a rationale for the positive correlation between prices and real output.

Robert Barro (1977) questions Fischer's assumption of a negative relation between real wages and employment.[2/] Though Barro does not use an explicit optimization framework, he bases his comments on previous work which derives optimal wage and employment schedules for labor contracts.[3/] Barro argues this optimal contracting approach implies an employment rule which fixes employment with respect to contemporaneously perceived changes in the general price level, even when the nominal wage rate is constant. Though the nominal wage is set before the general price level is observed, the resulting unanticipated changes in the real wage have no effect on employment as long as they are perceived.[4/]

2/ Barro's criticisms also apply to the related paper of Jo Anna Gray (1978).

3/ See Azariadas (1975), Baily (1974), and Gordon (1974). These papers focus on stochastic relative prices whereas this paper focuses on a stochastic general price level and holds relative prices constant.

4/ In this paper all changes in the general price level are assumed to be perceived contemporaneously but not (necessarily) anticipated in advance.

The main purpose of this paper is to analyze Barro's argument in an explicit optimization framework. This analysis consists of the specification of a contract model with sticky nominal wages, the derivation of the optimal employment rule, and the analysis of the properties of that rule under different assumptions about the utility functions of entrepreneurs and workers. Under some conditions Barro's argument is correct. If one assumes workers and entrepreneurs are indifferent toward fair bets in the consumption good, a type of risk neutrality, then the optimal employment rule calls for a level of employment which is constant for all states of the world. On the other hand, the assumption that the representative entrepreneur is risk neutral, the representative worker is risk averse, and that the representative worker's income effect dominates his substitution effect implies a negative relation between real wages and employment. Thus the main finding of this paper is that the theory of optimal contracts does not predict the sign of the relation between unanticipated changes in the real wage and employment.

This paper is divided into two sections and a conclusion. Section 1 analyzes a flexible nominal wage contract to provide a benchmark for Section 2's analysis of a sticky nominal wage contract.

1. Flexible nominal wage model

Fischer's model and Barro's criticisms deal with sticky nominal wage contracts. However, my first model will analyze a flexible nominal wage contract to provide a benchmark for the subsequent analysis of a sticky nominal wage contract.

Assume the aggregate price level, P_i, is a random variable with an associated probability of q_i where the subscript i indexes sample outcomes or states of nature. I assume there are two types of agents in the labor market, entrepreneurs and workers. Agents within each class are assumed to be identical so that the analysis can be conducted in terms of a representative worker and a representative entrepreneur. The entrepreneur's preferences are represented by an expected utility function, $E[Q(\hat{C}_i)]$, that has the entrepreneur's (random) consumption, $\hat{C}_i$, as an argument. The marginal utility of consumption is assumed to be positive and non-increasing.

$$Q'(\hat{C}_i) > 0, \; Q''(\hat{C}_i) \leq 0 \qquad (1)$$

The assumption of nonincreasing marginal utility guarantees the entrepreneur is risk neutral or risk averse, meaning that an even gamble between commodity bundles is never preferred to an arithmetic mean of those bundles. The assumption of nonincreasing marginal utility also guarantees the expected utility function is concave.5/

I assume the entrepreneur is the residual claimant6/ on the income stream generated by the firm, thus his consumption equals the real profits

5/ A linear combination of concave functions is concave.

6/ Azariadas (1975) and Baily (1974) argue that since entrepreneurs are self selected residual claimants on the income stream and since they are likely to have superior access to capital markets, one can treat entrepreneurs as risk neutral agents. That is $Q'' = 0$. Since at least one other contract model (Azariadas, 1978) allows entrepreneurs to exhibit risk aversion, I will generally not impose this restriction.

of the firm.

$$C_i = f(L_i) - \frac{W_i}{P_i} L_i = \pi_i \tag{2}$$

where L_i is the level of employment, W_i/P_i is the real wage, and $f(L_i)$ is a production function. The marginal product of labor is assumed to be positive and a decreasing function of labor.

The worker's preferences are represented by the expected utility function, $E[U(C_i, T - L_i)]$, whose arguments are consumption, C_i, and leisure. The worker's leisure is given by his fixed endowment of time, T, less the time at work, L_i. Both the marginal utility of consumption and the marginal utility of leisure are assumed to be positive and non-increasing.

$$U_1, U_2 > 0; U_{11}, U_{22} \leq 0 \tag{3}$$

Consumption and employment must satisfy the budget constaint

$$\frac{W_i}{P_i} L_i - C_i = 0 \tag{4}$$

The worker's utility function is assumed to be concave generating the additional restriction

$$U_{11}U_{22} - U_{12}^2 \geq 0 \tag{5}$$

An economic interpretation of a concave utility function is that the worker is allowed to be risk neutral or risk averse.[7/] Furthermore, I assume indifference curves are strictly convex to the origin in the goods - leisure space.

$$U_{11}\frac{U_2^2}{U_1^2} - 2U_{21}\frac{U_2}{U_1} + U_{22} < 0 \qquad (6)$$

The worker and the entrepreneur are assumed to make an (implicit or explicit) labor contract which sets the nominal wage and the level of employment conditional on the general price level. Thus the labor contract determines a wage and employment schedule. Optimizing agents have an incentive to exploit all mutually advantageous trades. This implies restrictions on the form of the labor contract. These restrictions are derived by maximizing the expected utility of the one agent subject to a floor on the expected utility of the other. Let θ be the floor on the expected utility of the worker and let λ be the Kuhn-Tucker multiplier. The optimization problem is written as

$$\max_{L_i, W_i} \sum_{i=1}^{I} q_i Q(\pi_i) + \lambda\{\sum_{i=1}^{I} q_i U(\frac{W_i}{P_i} L_i, T - L_i) - \theta\} \qquad (7)$$

Note that this maximization is with respect to a wage rate, W_i, and level of employment, L_i, which are contingent on the general price level. Thus there are no constraints imposed on the degree to which wages are indexed.

7/ This multivariate definition of risk aversion is a straightforward extension of the univariate definition and can be found in sources like Arrow (1964).

I assume the existence of a (possibly nonoptimal) wage and employment schedule such that the constraint is satisfied as a strict inequality. This assumption insures Slater's condition is satisfied, and allows me to use the Kuhn-Tucker theorem[8/] to derive necessary and sufficient conditions for a maximum. Assuming the constraint is binding, the necessary and sufficient conditions for a maximum imply

$$q_i Q'(f' - \frac{W_i}{P_i}) + \lambda q_i (U_1 \frac{W_i}{P_i} - U_2) = 0 \qquad i = 1,\ldots,I \tag{8}$$

and

$$-q_i Q' \frac{L_i}{P_i} + \lambda q_i U_1 \frac{L_i}{P_i} = 0 \qquad i = 1,\ldots,I \tag{9}$$

These conditions imply the resulting labor contract is Pareto optimal ex post and achieves a Pareto optimal distribution of risk.

Ex post Pareto optimality can be demonstrated by using (9) to substitute λ out of (8)

$$f = \frac{U_2}{U_1} \tag{10}$$

If this condition is not met, there are trades of labor and goods which increase the ex post utility of one agent without decreasing the ex post utility of the other. For instance, if the marginal product is greater than the marginal rate of substitution, increases in employment and

8/ See Takayama (1974, pp. 69 and 96-97).

the worker's consumption of goods increase the utility of both parties provided the ratio of the change in goods to the change in labor is less than the marginal product and greater than the marginal rate of substitution.

Note that the condition for ex post Pareto optimality (equation (10)) gives employment as an implicit function of the real wage. Before discussing the second implication of the optimal contract, a Pareto optimal distribution of risk, it is interesting to analyze the properties of this implicit function. The locus of real wages and levels of employment which satisfy (10) is drawn in Figure 1 and is labeled PO. Figure 1 also contains indifference curves and offer curves for both agents, thus it is essentially an Edgeworth-Bowley box diagram with the real wage instead of goods on the vertical axis.

Along any worker's indifference curve (IW_1, IW_2, or IW_3) the workers utility is constant. Thus the slope of a worker's indifference curve is given by[9/]

$$\left.\frac{\partial W_i/P_i}{\partial L_i}\right|_{U=U_0} = \frac{\frac{U_2}{U_1} - \frac{W_i}{P_i}}{L_i} \qquad (11)$$

9/ Throughout this paper the partial derivative operator will denote a small movement from variables associated with the ith state of nature toward variables associated with adjacent states of nature. The operator is not intended to denote changes in variables associated with the ith state of nature.

Given the level of employment, an increase in the real wage will increase the worker's utility. Consequently IW_1, IW_2, and IW_3 are associated with successively higher levels of utility for the worker. Given any real wage, the worker maximizes his utility by choosing a level of employment that equates the marginal rate of substitution and the real wage. Hence at points where the worker's indifference curve intersects his offer curve (L_s), the slope of the indifference curve is zero.[10/] Under the assumptions made earlier, the worker's indifference curves are positively sloped to the right of his offer curve and negatively sloped to the left of his offer curve.

Along any entrepreneur's indifference curve (IE_1, IE_2 or IE_3) the entrepreneur's utility is constant. Thus the slope of an entreprenuer's indifference curve is given by

$$\left.\frac{\partial W_i/P_i}{\partial L_i}\right|_{Q=Q_0} = \frac{f' - \dfrac{W_i}{P_i}}{L_i} \tag{12}$$

Given the level of employment, a decrease in the real wage will increase the entrepreneur's utility. Consequently IE_1, IE_2, and IE_3 are associated with successively higher levels of utility for the entrepreneur. Given

---

10/ Along a horizontal line the real wage is constant, and the highest level of utility is achieved where such a line is tangent to an indifference curve.

any real wage, the entreprenuer maximizes his utility by choosing a level of employment that equates the marginal product and the real wage. Hence at points where the entrepreneur's indifference curve intersects his offer curve (L_D), the slope of the entrepreneur's indifference curve is zero. Under the assumptions made earlier, the entrepreneur's indifference curves are positively sloped to the left of his offer curve and negatively sloped to the right of his offer curve.

Along the locus of points which are Pareto optimal ex post (PO), the marginal product equals the real wage by equation (10), and indifference curves are tangent by equations (10), (11), and (12). The PO curve does not pass through quadrant II because in that quadrant the marginal product is less than the real wage, and the real wage is less than the marginal rate of substitution. The PO curve does not pass through quadrant IV because in that quadrant the marginal product is greater than the real wage, and the real wage is greater than the marginal rate of substitution. It is interesting to note that the PO curve is essentially the same as the contract curve in an Edgeworth-Bowley box diagram. Whereas points along the contract curve are described as the outcome of the decision of welfare maximizing planner, points along the PO curve are the outcome of a mutually advantageous contract.

The slope of PO is found by differentiating (10)

$$\frac{\partial W_i/P_i}{\partial L_i} = \frac{U_1 f'' + \frac{W}{P}[U_{11}\frac{U_2}{U_1} - U_{21}] + [U_{22} - U_{12}\frac{U_2}{U_1}]}{-L[U_{11}\frac{U_2}{U_1} - U_{21}]} \qquad (13)$$

and the assumed restrictions on the utility functions of workers (convex indifference curves and positive nonincreasing marginal utilities) are not sufficient to sign this expression. However, some assumptions about the income elasticities of leisure and goods are sufficient to determine the slope of PO.

If both consumption and leisure are normal goods, the following conditions hold.

$$U_{11} \frac{U_2}{U_1} - U_{12} < 0 \tag{14}$$

and

$$U_{22} - U_{21} \frac{U_2}{U_1} < 0 \tag{15}$$

These conditions imply the expression in (13) is negative.

Certain types of utility functions imply both consumption and leisure are normal. For instance, the set of homothetic utility functions and its subset, the class of homogeneous utility functions, are sometimes used in the study of labor markets.[11/] As shown by Whitaker and McCallum (1971), among others, homothetic utility functions have linear expansion paths that lie along rays originating at the origin. If corner solutions are ruled

11/ A utility function is homothetic if for any two consumption bundles $(C_1, T-L_1)$ and $(C_2, T-L_2)$ and an arbitrary constant t, $U(C_1, T-L_1) = U(C_2, T-L_2)$ implies $U(tC_1, t(T-L_1)) = U(tC_2, t(T-L_2))$. See Barzel and McDonald (1973) for an example of the use of homothetic utility functions in the study of labor markets.

out, the income expansion path has a positive slope. Thus for any homothetic utility function, both leisure and consumption are normal goods, and the slope of PO is negative.

Second if leisure is perfectly income inelastic, PO is vertical. The income inelasticity of leisure implies goods are normal. Under these conditions the expression in (14) is zero and the expression in (15) is negative. Thus PO is vertical.[12/]

The second implication of the optimal contract is a Pareto optimal distribution of risk. This can be demonstrated by differentiating (8) and (9) with respect to the real wage, employment, and the price level.

$$\frac{\partial W_i/P_i}{\partial P_i} = \frac{\partial L_i}{\partial P_i} = 0 \tag{16}$$

Employment and the real wage are unaffected by changes in the general price level and the nominal wage is perfectly indexed. This condition implies a Pareto optimal distribution of risk, there is no redistribution of risk which can increase the expected utility of one agent without decreasing the expected utility of the other. In fact there is no redistribution of risk which increases the expected utility of either agent. Given any set of contracts with the same expected level of employment and real wage bill, neither the entrepreneur or the worker can find a contract which yields a higher level of utility than the contract with a fixed real wage and a fixed level of employment.

The condition for ex post Pareto optimality rules out all points that are not on the PO curve, and the condition for Pareto optimal risk sharing

12/ Under any other assumptions about income elasticities, (14) and (15) have opposite signs and the slope of PO is ambiguous.

implies that the contract consists of some single point on that curve. Optimality provides no basis for determining which point on the PO curve will be chosen, for that is a bargining problem. If the market for contracts was cleared by an auctioneer who called out the real wage term of the contract and received responses in the form of the employment term of the contract, then the optimal contract would duplicate the market clearing rule for an auction market. However, other results are also consistent with the optimality of the contract.

This section analyzed a contract model where the nominal wage was perfectly flexible and showed that the mutually advantageous contract implies a fixed real wage rate. Since fully indexed nominal wage rates are rarely observed, it is interesting to analyze the optimal contract when the nominal wage must be set before the price level is observed. The next section conducts this analysis.

2. Sticky nominal wage model

The purpose of this section is to analyze the issues raised by Robert Barro (1977) in an explicit optimization framework. Barro argues that even when the nominal wage rate is set before the price level is observed, agents exploiting all mutually advantageous trades will make a contract which sets the level of employment to equate the marginal product of labor and the marginal rate of substitution between consumption and leisure. Furthermore, neglecting the effects of changes in real wages on the marginal value of time, this employment rule implies employment is fixed with respect to unanticipated real wage movements and prices. Thus Barro argues that Fischer's model depends on an arbitrary rule to determine employment.

Fischer's response (1977b) to Barro rests on the observation that existing labor contracts specify a sticky nominal wage and leave the determination of the level of output to the entrepreneur. But this observation does not contradict the existence of a mutually advantageous employment rule. Assume the worker evaluates the labor contract on the basis of both explicit and implicit features and that one of the implicit feature of the contract is the employment rule used by the entrepreneur. If the worker's evaluation is affected by the employment rule, then the entrepreneur has an incentive to choose a rule which is mutually advantageous. The worker is willing to make concessions on the other terms of the contract to induce the entrepreneur to follow a mutually advantageous employment rule.

To analyze Barro's arguments I shall modify the model discussed in the previous section so that the nominal wage is constant ex post, derive the mutually advantageous employment rule, and analyze the properties of that rule under different assumptions about the utility function of entreprenurs and workers. The mutually advantageous employment rule is the solution to the following optimization problem.

$$\max_{L_i, W} \sum_{i=1}^{I} q_i Q(\pi_1) + \mu\{\sum_{i=1}^{I} q_i U(\frac{W}{P_i} L_i, T - L_i) - \theta\} \tag{17}$$

which differs from the previous problem in that one nominal wage rate applies no matter what the price level.[13/] I assume the existence of a possibly nonoptimal nominal wage rate and an employment schedule that satisfy the constraint as a strict inequality, so that Slater's condition is satisfied. Assuming the constraint is binding, the necessary and sufficient conditions for a maximum imply

$$q_i Q'(f' - \frac{W}{P_i}) + \mu q_i (U_1 \frac{W}{P_i} - U_2) = 0 \qquad i = 1,\ldots,I \tag{18}$$

and

$$-\Sigma\{q_i Q' \frac{L_i}{P_i}\} + \mu\Sigma\{q_i U_1 \frac{L_i}{P_i}\} = 0 \tag{19}$$

To interpret these conditions note that at the optimum combination of an employment schedule and a nominal wage rate, a small change in the expected utility of the worker, $d\theta$, changes the expected utility of the entrepreneur in the opposite direction by $\mu d\theta$. Thus μ is a multiplier that converts small changes in the worker's expected utility into their effects on the expected utility of the entrepreneur. To interpret equation (18), note that a change in the level of employment associated with P_i has a direct effect on the expected utility of the entrepreneur. This direct effect is composed of the direct effect on profits (marginal

13/ Fischer's macroeconomic model deals with a multiperiod labor contract while the model that I present here is a one period model. This one period model can be extended to a J period model where the resulting J labor schedules have the same properties as the labor schedule in the one period model.

product less real wage) which is weighted by the marginal utility of consumption and a probability. An indirect effect arises because the change in employment affects the expected utility of the worker by changing consumption and leisure. If, for example, the change in employment raises the expected utility of the worker above the floor, the entrepreneur can change other terms of the contract to increase his own expected utility and bring the expected utility of the worker back down to the floor. The second term in (18) captures this indirect effect. Equation (18) says that at the optimum no change in the level of employment associated with P_i will increase expected profits when both the direct and indirect effects are considered. Equation (19) makes the analogous statement for a change in the nominal wage rate.

Equation (18) is graphed as the locus MA in Figure 2. Any level of employment and the real wage which satisfy this condition must lie in quadrants I or III. An optimal contract will never specify employment in quadrant II since in that quadrant decreases in employment increase the utility of both agents (the real wage is greater than the marginal product and less than the marginal value of time). An optimal contract will never specify employment in quadrant IV since in that quadrant increases in employment increase the utility of both agents (the real wage is less than the marginal product and greater than the marginal value of time). Thus the contractual elimination of all mutually advantageous ex ante trades rules out all points where changes in employment increases the ex post utility of both parties. However the terms of the contract go further than this, in the process of eliminating all mutually advantageous trades the contract calls for some actions which increase the ex post utility of one agent at the expense of the other's ex post utility. For instance, consider the real wage outcome W/P_1. If the entrepreneur were free to choose the level of employment ex post (as in an auction market), he would not increase employment past L_1. However, in a labor contract the entrepreneur agrees that for some contingencies he will take actions which decrease his ex post utility, for instance the increase in employment from L_1 to L_2. In return for this agreement the worker promises a lower expected real wage bill or an amount of labor that is greater than the amount specified by his offer curve under contingencies like W/P_3. As long as there is a positive net gain in (ex ante) expected utility, the entrepreneur will agree to contingent actions which decrease his ex post utility. Similar statements hold for the worker.

The slope of MA is found by differentiating (18) with respect to real wages and employment to obtain

$$\frac{\partial W/P_i}{\partial L_i} = \frac{-\{Q'f'' + Q''(f' - \frac{W}{P_i})^2 + \mu(U_{11}(\frac{W}{P_i})^2 - 2U_{12}\frac{W}{P_i} + U_{22})\}}{\mu(U_{11}\frac{W}{P_i} - U_{12})L_i - Q''(f' - \frac{W}{P_i}) + (\mu U_1 - Q')} \tag{20}$$

To interpret (20) note that the marginal effect of changes in employment on the entrepreneur's utility decreases as employment increases. Thus the term in brackets in the numerator is unambiguously negative. The effect of changes in the real wage on the marginal expected utility of the entrepreneur is ambiguous. The first term in the denominator captures the income effect on the worker. As the worker's income rises the marginal utility of his consumption falls and this has a negative indirect effect on employment, however the effect on the marginal utility of leisure is ambiguous. The second term captures the entrepreneur's income effect. The sign of this term depends on the relation between the marginal product and the real wage. The third term captures the compensated substitution effect. Even if the worker and the entrepreneur are compensated for the changes in their income, a rise in the real wage induces the worker to want more employment and the entrepreneur to want less.

Barro (1977) argues the mutually advantageous employment rule will set the level of employment to equate the marginal product of labor and the marginal rate of substitution between consumption and leisure, thus it satisfies the condition for ex post Pareto optimality given in Section I. Furthermore, neglecting the effects of changes in the real wage on the marginal value of time, this employment rule implies employment is fixed with respect to unanticipated changes in the real wage. Inspection of (20) reveals the sign of the relation between unanticipated changes in the real wage and the level of employment is ambiguous. Also, since the nominal wage is fixed, the condition for Pareto optimal risk sharing is violated, and the theory of second best implies the remaining condition for Pareto optimality may not be satisfied. Thus the mutually advantageous employment rule need not satisfy the condition for ex post Pareto optimality.

One set of restrictions on utility functions does give an unambiguous sign to (20) and insures the condition for ex post Pareto optimality is satisfied. Consider the following set of assumptions; the worker's marginal utility of consumption is constant ($U_{11} = 0$), the worker's marginal utility function is addatively separable in consumption and leisure ($U_{12} = 0$), and the entrepreneur is risk neutral ($Q'' = 0$). This set of assumptions is necessary and sufficient for Barro's claim (ex post Pareto optimality and constant employment) to hold.14/

To show necessity, I shall derive an expression for the entrepreneur's marginal utility as a function of the worker's marginal utility of consumption, and then show that expression implies both marginal utilities are constant. Assume the optimal sticky wage contract, which satisfies (18) also satisfies the condition for ex post Pareto optimality, equation (10). These two equations imply

$$(U_1\mu - Q')\{\frac{W}{P_i} - \frac{U_2}{U_1}\} = (U_1\mu - Q')\{\frac{W}{P_i} - f'\} = 0 \text{ for all } P_i \tag{21}$$

From (21)

$$Q' = \mu U_1 \tag{22}$$

14/ Barro (1977, pp. 311, footnote 6) explicitly assumes the marginal value of time is unaffected by changes in the real wage. In my model this restriction guarantees the locus of ex post Pareto optimal points, (10), is vertical, but is not strong enough to guarantee that a sticky nominal wage contract satisfies the condition for ex post Pareto optimality.

at any real wage and level of employment which does not equate the marginal product and the marginal rate of substitution. Now assume constant employment and differentiate (22) with respect to the real wage to obtain

$$- Q''L = \mu U_{11}L. \tag{23}$$

Since the Kuhn-Tucker multiplier is positive, both Q" and U_{11} must be zero, and since the utility function of the worker is assumed to be concave, $U_{12} = 0$.

To see sufficiency, note that if $U_{11} = U_{12} = Q'' = 0$, then U_1 and Q' are constants, and (19) implies

$$\mu = Q'/U_1 \tag{24}$$

Next, substitute for μ in (18) to demonstrate the marginal product equals the marginal rate of substitution. Finally, by (20) the level of employment does not change in response to unanticipated changes in the real wage.[15/]

The economic interpretation of the restriction $Q'' = U_{11} = 0$ is that both agents are indifferent toward fair bets in the consumption good.[16/] This restriction is necessary and sufficient to prevent the introduction of

---

15/ Though a fixed level of employment does not imply $U_{11} = U_{12} = Q'' = 0$, I know of no other meaningful restrictions on the utility functions of workers and entrepreneurs which are sufficient for the expression in (20) to be zero.

16/ The additional assumption of addative separability implies the worker's utility function is concave, and rules out the possibility that the worker prefers fluctuating employment to fixed employment.

wage rigidities from violating the conditions for a Pareto optimal distribution of risk. Any agents whose utility functions satisfy this restriction are indifferent between all contracts with the same (fixed) level of employment and the same expected real wage bill (and visa versa). Since the condition for the Pareto optimal distribution of risk is not violated, the (fixed) level of employment satisfies the remaining condition for ex post Pareto optimality (10), thus PO and MA are the same. In addition, since the sticky nominal wage contract satisfies both conditions for Pareto optimality, it involves no welfare losses.

Finally consider the assumptions that the entrepreneur is risk neutral and that for any point that satisfies (18) and (19) the worker's offer curve passing through that point has a negative slope.[17/] This set of assumptions is sufficient (but not necessary) for a negative relation between employment and the real wage. The possibility of risk neutral entrepreneurs has been recognized in several articles on labor contracts. For instance, Azariadas (1975) and Baily (1974) argue that since entrepreneurs are self selected residual claimants on the income stream generated by the firm and since they have superior access to capital markets, they are essentially risk neutral. The assumption of risk neutrality implies the entrepreneur's income effect is zero. The possibility of a backward

---

17/ I assume that for any point on the worker's preference map there is an endowment such that an offer curve passes through the point.

bending supply curve of labor has long been recognized.[18/] This assumption implies the worker's income effect is negative and dominates his compensated substitution effect. Thus the expression in (20) is negative under these restrictions and employment and unanticipated changes in the real wage are negatively related. Mutually advantageous sticky nominal wage contracts can result in a negative relation between employment and real wages.

18/ See Barzel and McDonald for a discussion of the backward bending supply curve of labor. Using the properties of agents' offer curves to deduce the properties of the locus of mutually advantageous points may seem inconsistent. The theory of the labor market which I have discussed in this section deals with contractual exchange when agents make price forecast errors. Under this theory, exchange does not take place at the intersection of agents' offer curves but along the locus of mutually advantageous points. However offer curves are not superfluous. In the long run, changes in nonlabor income, technology, and other factors would dominate price forecast errors in their influence on the labor market. Thus agents' offer curves are the appropriate tool for long run analysis.

Conclusion

In the first section of this paper I analyzed a flexible nominal wage contract. When this contract eliminates all mutually advantageous ex ante trades, the resulting levels of employment and the real wage are Pareto optimal ex post, achieve a Pareto optimal distribution of risk, and are constant ex post.

Since a fully indexed nominal wage is rarely observed, I derived the optimal employment rule for a sticky nominal wage contract in the second section of this paper. One property of this rule is a special case of the theory of second best. The introduction of ex post wage rigidities prevents a contract from achieving a Pareto optimal distribution of risk, consequently, the mutually advantageous employment rule is not Pareto optimal ex post. A second property of this rule is that the relation between unanticipated changes in the real wage and employment is ambiguous in sign.

Robert Barro (1977) argues the employment rule for this second type of contract will fix the level of employment to satisfy the condition for ex post Pareto optimality. In my model this argument holds if both workers and entrepreneurs are indifferent toward fair bets in the consumption good. On the other hand, if one assumes that entrepreneurs are risk neutral, and that the workers' income effect is negative and strong relative to the substitution effect, then the level of employment and unanticipated changes in the real wage are negatively related. In the absence of restrictions on the utility functions of entrepreneurs and workers, the sign of the relation between the level of employment and unanticipated changes in the real wage is ambiguous.

Figure 1

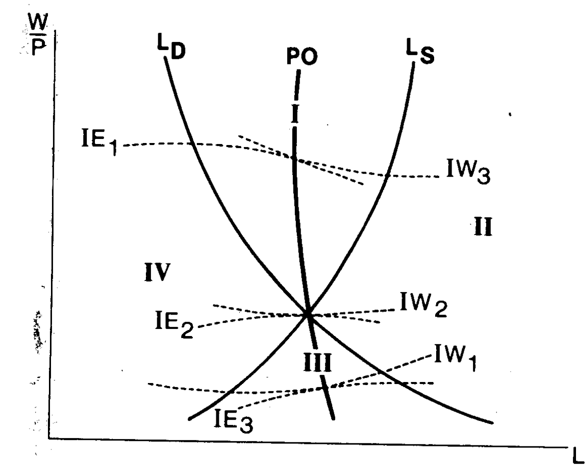
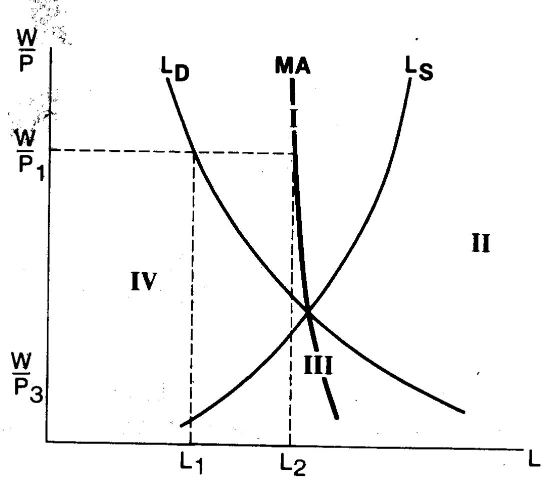


Figure 2



References

1. Arrow, K.J., 1964, The Role of Securities in the Optimal Allocation of Risk Bearing, The Review of Economic Studies 31, 91-96.

2. Azariadas, C., 1975, Implicit Contracts and Underemployment Equilibria, Journal of Political Economy 83, 1183-1202.

3. Azariadas, C., 1978, Escalator Clauses and the Allocation of Cyclical Risks, Journal of Economic Theory 18, 119-155.

4. Baily, M.N., 1974, Wages and Employment Under Uncertain Demand, Review of Economic Studies 41, 37-50.

5. Barro, R.J., 1977, Long-Term Contracts, Sticky Prices, and Monetary Policy, Journal of Monetary Economics 3, 305-316.

6. Barzel, Y. and R.J. McDonald, 1973, Assets, Subsistence, and the Supply Curve of Labor, American Economic Review 63, 621-633.

7. Fischer, S., 1977a, Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule, Journal of Political Economy 83, 241-254.

8. Fischer, S., 1977b, Long-Term Contracting, Sticky Prices, and Monetary Policy: A Comment, Journal of Monetary Economics 3, 317-323.

9. Gordon, D.F., 1974, A Neoclassical Theory of Keynesian Unemployment, Economic Inquiry 12, 431-459.

10. Gray, J.A., 1978, Indexation and Contract Length, Journal of Political Economy 86, 1-18.

11. Lucas, R., 1972, Expectations and the Neutrality of Money, Journal of Economic Theory 4, 103-124.

12. Sargent, T.J., 1973, Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment, Brookings Papers on Economic Activity 2, 429-479.

13. Sargent, T.J., and N. Wallace, 1975, Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule, Journal of Political Economy 83, 241-254.

14. Takayama, A., 1973, Mathematical Economics, (Dryden Press, Hinsdale, Illinois).

15. Whitaker, J.K. and B.T. McCallum, 1971, On the Homotheticity of Production Functions, Western Economic Journal 9, 57-63.