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COINTEGRATION, SEASONALITY, ENCOMPASSING, AND THE DEMAND FOR MONEY IN THE UNITED KINGDOM

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ABSTRACT

Virtually all previous narrow money demand studies for the United Kingdom have used seasonally adjusted data for money, prices, and expenditure. This paper develops a constant, data-coherent M_1 demand equation for the United Kingdom with seasonally *unadjusted* data. For that model, we address issues of cointegration, error correction, general-to-specific modeling, dynamic specification, model evaluation and testing, parameter constancy, and exogeneity. We also establish theoretical and empirical relationships between seasonally adjusted and unadjusted data, and so between models using those data. Finally, we derive and implement encompassing tests for comparing models using adjusted data with models using unadjusted data. Unlike the "standard" encompassing framework, variance dominance is *not* always a necessary condition for encompassing.

Key words and phrases: cointegration, conditional models, dynamic specification, encompassing, error correction models, exogeneity, general-to-specific modeling, model evaluation, money demand, parameter constancy, sequential reduction, testing, United Kingdom.

Cointegration, Seasonality, Encompassing, and the Demand for Money in the United Kingdom

Neil R. Ericsson, David F. Hendry, and Hong-Anh Tran[1]

1 Introduction

Wallis (1974) and Sims (1974) examine the effects of seasonal adjustment when estimating econometric relationships. Wallis considers the implications of estimation with seasonally adjusted data when the underlying economic relation involves the unadjusted data. Sims investigates the converse situation, in which the relation is in terms of the non-seasonal components of the economic variables, but the econometrician uses the unadjusted seasonal data. In each case, the empirical model is mis-specified and coefficient estimates generally are inconsistent.

Using two recently developed concepts, cointegration and encompassing, this paper sheds new light on the use of adjusted and unadjusted data in econometric modeling.[2] First, under mild assumptions about the seasonal adjustment procedure, the adjusted and unadjusted series of a given variable are cointegrated, with cointegrating vector $(+1 : -1)$. Second, the cointegrating vector for a set of variables is invariant to the choice of adjusted or unadjusted data. Even so, conducting inference may be problematic with adjusted data. In particular, dynamic adjustments and exogeneity status are usually altered. Third, parameter-encompassing statistics can be

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[2]A recent burgeoning literature discusses integration and cointegration at seasonal frequencies; cf. Dickey, Hasza, and Fuller (1984), Hylleberg, Engle, Granger, and Yoo (1990), Bell and Wilcox (1990), Ghysels, Lee, and Noh (1991), Hylleberg, Jørgensen, and Sørensen (1991), Lee (1992), and Beaulieu and Miron (1993) *inter alia*. Osborn (1988, 1991), Birchenhall, Bladen-Hovell, Chui, Osborn, and Smith (1989), and Franses and Kloek (1991) consider seasonally varying slope coefficients for cointegrated processes. Our paper focuses on cointegration at the zero frequency only, both for seasonally unadjusted data and for seasonally adjusted data.

constructed to compare a model using unadjusted data with one using adjusted data. These statistics aim to test empirically the dissimilar premises of Sims and Wallis. Contrasting with the classical encompassing framework, variance dominance is not always a necessary condition for parameter encompassing.

Section 2 constructs an analogue model of seasonal adjustment and obtains the cointegration results for the adjusted and unadjusted series. Section 3 modifies an existing parameter-encompassing statistic to compare "adjusted" with "unadjusted" models. Sections 4 and 5 illustrate the cointegration propositions and the encompassing test via a substantive empirical study of narrow money demand in the United Kingdom. Section 4 reports Johansen's system-based cointegration tests and estimates for adjusted and unadjusted series, variable by variable, and for sets of variables, whether adjusted or unadjusted. The theoretical and empirical results match closely. Section 5 develops a conditional money demand model with the unadjusted data, summarizes Hendry and Ericsson's (1991b) model with adjusted data, and applies the encompassing test of Section 3 to these two models. Surprisingly, the model with adjusted data does not encompass the model with unadjusted data, even though the former has a long and favorable track-record. Section 6 concludes. Appendix A documents data sources, Appendix B describes a sequential reduction for obtaining the conditional model of money demand using the unadjusted data, and Appendix C compares the results of cointegration analysis for several variants on the money demand system.

2 Relationships Between Seasonally Adjusted and Unadjusted Data

This section establishes several theoretical results on the cointegration relationships between adjusted and unadjusted data, where seasonal adjustment is approximated by a certain two-sided linear filter. As a preliminary, Section 2.1 discusses linear filters, defines an analogue model of X–11 seasonal adjustment, establishes a notation for cointegration, and defines mean equality between series. Section 2.2 considers conditions under which the original and filtered series are cointegrated and under which their mean difference is zero, and relates these conditions to the properties of seasonal adjustment filters, based on the analogue model. Section 2.3 extends the analysis to vector processes and matrix filters, specializing these for seasonal adjustment filters. If the unadjusted series are cointegrated, the adjusted series are also cointegrated, and with the same cointegrating vector(s) as for the unadjusted series. Section 2.4 comments on the approximations made in using the analogue model of seasonal adjustment. For comparison with these analytical results, Section 4 provides empirical evidence on the relationships between actual adjusted and unadjusted data. Below, the abbreviation NSA means "not seasonally adjusted", and SA means "seasonally adjusted" or "seasonal adjustment", depending upon the context.

2.1 Preliminaries

After briefly reviewing some properties of linear filters, this subsection formulates a linear analogue model of the X–11 SA procedure and derives implications of that analogue model for the NSA and SA data.

Linear filters. Consider a single variable x, and denote the filtered and unfiltered series of that variable as $\{x_t^f\}$ and $\{x_t\}$ respectively, where t is the time subscript. Throughout, x_t and x_t^f have the following relationship:

$$x_t^f = f(L)x_t , \tag{1}$$

where $f(L)$ is a finite-order, two-sided linear filter in the lag operator L:

$$f(L) = \sum_{i=-n}^{n} f_i L^i , \tag{2}$$

with half-length n and fixed, finite weights $\{f_i\}$. Some f_i could be zero, so $f(L)$ could be a one-sided filter in practice. Only *finite*-order filters are considered. Generalization to an infinite-order filter is feasible, but is of limited interest in the context of actual SA procedures.

When examining the properties of $f(L)$ in the context of SA filters, it will prove useful to re-express $f(L)$ as:

$$\begin{aligned} f(L) &= f(1) + f^*(L)\Delta \\ &= f(1) + f^*(1)\Delta + f^{**}(L)\Delta^2 , \end{aligned} \tag{3}$$

where Δ is the difference operator $1-L$; and $f^*(L)$ and $f^{**}(L)$ are themselves finite-order, fixed-weight, two-sided linear filters with polynomial coefficients denoted $\{f_i^*\}$ and $\{f_i^{**}\}$. Here and below, superscript asterisks * and ** denote polynomials obtained as in (3). While (3) finishes with a polynomial in Δ^2, the recursion can be repeated to any order. The sum of coefficients in each successive lag polynomial can be obtained recursively, noting that:

$$f^*(1) = -\left.\frac{\partial f(L)}{\partial L}\right|_{L=1} ,$$

$$f^{**}(1) = -\left.\frac{\partial f^*(L)}{\partial L}\right|_{L=1} , \tag{4}$$

etc. Consequently, the sums $f^*(1)$ and $f^{**}(1)$ are $-\sum_{i=-n}^{n} i \cdot f_i$ and $-\sum_{i=-n}^{n-1} i \cdot f_i^*$.

An analogue model of seasonal adjustment. In the analogue model of SA, x_t^f and x_t correspond to SA and NSA series respectively. As the analogue SA filter, $f(L)$ satisfies three assumptions:

1. the weights $\{f_i\}$ sum to unity,
2. $f(L)$ is symmetric in L, and
3. $f(L)$ eliminates deterministic seasonals.

While this analogue model only approximates the highly complex X–11 SA procedure, the approximation appears a good one for linear properties; cf. Nerlove (1964), Wallis (1974, 1983), Cleveland and Tiao (1976), and Bell (1992). See Wallis (1974, 1978) and Sims (1974) on the possible econometric consequences of using SA (or NSA) data. See Lovell (1963), Grether and Nerlove (1970), Granger (1978), Kenny and Durbin (1982), Wallis (1982, 1983), Burridge and Wallis (1984), Hylleberg (1986), and Maravall and Pierce (1987) on properties of existing and more "optimal" SA procedures.

The nature of the three assumptions for the analogue model is now discussed.

ASSUMPTION 1. *The sum $f(1)$ is unity.*

Under Assumption 1, the first equality in (3) is:

$$f(L) \ = \ 1 + f^*(L)\Delta \, . \tag{5}$$

By restricting the sum of coefficients in the scalar polynomial $f(L)$ to unity, Assumption 1 ensures that x_t^f and x_t are in the same units. Assumption 1 is with loss of generality by excluding unit roots in $f(L) = 0$, but otherwise represents a normalization of $f(L)$. The normalized polynomial is sufficiently general for current purposes since the focus is on seasonal adjustment. Further, if $f(1)$ were zero rather than unity, x_t^f would be a finite-weight, finite-order distributed lag of Δx_t, and so would be integrated of an order different from that of x_t. The restriction $f(1) = 0$ thus affects long-run properties of the data whereas seasonal adjustment is meant to leave those properties intact, so $f(1) \neq 0$ is a relatively innocuous assumption.

ASSUMPTION 2. *The polynomial $f(L)$ is symmetric in the lag operator.*

Assumption 2 means that $f(L) = f(L^{-1})$ or, equivalently, $f_i = f_{-i}$, $i = 1, \ldots, n$. From (3) and (4), Assumption 2 implies $f^*(1) = 0$, and so $f(L)$ can be written as:

$$f(L) \ = \ f(1) + f^{**}(L)\Delta^2 \, . \tag{6}$$

That $f^*(1)$ is zero can be seen by solving for the coefficients $\{f_i^*\}$ in terms of the $\{f_i\}$. In general, the polynomial $f^*(L)$ is:

$$\begin{aligned} f^*(L) \ &= \ \sum_{i=-n}^{n-1} f_i^* L^i \\ &= \ \sum_{i=1}^{n} f_{-i}^* L^{-i} + \sum_{i=1}^{n} f_{i-1}^* L^{i-1} \, . \end{aligned} \tag{7}$$

From (3), the coefficients $\{f_i^*\}$ are:

$$f_i^* \ = \ \begin{cases} \displaystyle\sum_{j=|i|}^{n} f_{-j} & \text{for} \quad -n \leq i < 0 \\[2ex] \displaystyle -\sum_{j=i+1}^{n} f_j & \text{for} \quad 0 \leq i < n \, . \end{cases} \tag{8}$$

Under symmetry, $f_i = f_{-i}$ $(0 < i \leq n)$, so $f^*_{-i} = -f^*_{i-1}$ $(0 < i \leq n)$. Substituting (8) into (7), $f^*(1)$ is zero. Symmetry is sufficient for $f^*(1) = 0$, but is not necessary. For example, a unit root in $f^*(L)$ ensures $f^*(1) = 0$, but does not imply symmetry. Extensions of this result appear in Osborn (1993) and Wallis (1993).

ASSUMPTION 3. *The polynomial $f(L)$ has a factor $v(L)$, where*

$$v(L) \;=\; s^{-1} \cdot \sum_{i=0}^{s-1} L^i \ ,$$

and s is the periodicity of seasonality.

Assumption 3 ensures that the SA filter eliminates any fixed seasonal pattern. Other assumptions about $f(L)$ will achieve the same result, but Assumption 3 is one of the simplest and most intuitive: $v(L)$ averages the data over the seasonal interval. Even so, Assumption 3 is not innocuous. An SA filter with a factor of $v(L)$ will eliminate seasonal unit roots in x_t if they are present. If they are not present, application of the SA filter will be similar to over-differencing; see also Maravall (1993).

A fixed seasonal pattern can be represented by $s(L)S_{1t}$, where $s(L)$ is an $(s-1)$th-order polynomial and S_{it} is a mean-adjusted dummy for the ith season. Hence $f(L)$ should annihilate $s(L)S_{1t}$:

$$f(L)s(L)S_{1t} \;=\; 0\ . \tag{9}$$

Under Assumption 3, $f(L) = f^o(L)v(L)$, where $f^o(L)$ is a fixed-weight, finite-order polynomial; and the superscript ^{o} indicates that $v(L)$ has been factored from the polynomial. Because S_{it} is mean-adjusted, it follows that:

$$\begin{aligned} f(L)s(L)S_{1t} &= f^o(L)v(L)s(L)S_{1t} \\ &= f^o(L)s(L)[v(L)S_{1t}] \\ &= 0\ . \end{aligned} \tag{10}$$

Thus, Assumption 3 ensures that $f(L)$ annihilates the seasonal dummies.

To summarize, under Assumptions 1–3, $f(L)$ may be written as:

$$\begin{aligned} f(L) &= 1 + f^{**}(L)\Delta^2 \\ &= f^o(L)v(L)\ . \end{aligned} \tag{11}$$

In discussing filtered series satisfying Assumptions 1–3, x^f_t will be referred to as x^a_t ("a" for analogue or adjusted), the weights $\{f_i\}$ as $\{a_i\}$, and so the filter $f(L)$ as the analogue SA filter $a(L)$.

Cointegration. Cointegration is discussed at length elsewhere; cf. Engle and Granger (1987), Hendry (1986), Johansen (1988, 1991), Johansen and Juselius (1990), Phillips (1991), Ericsson (1992a), and Banerjee, Dolado, Galbraith, and Hendry (1993). Here, the interest is in cointegration of the pair of integrated series $(x^f_t : x_t)'$. If x_t is I(d) and $x^f_t - x_t$ is (at most) I(k) for integers d and k such that $d > k \geq 0$, then x^f_t and x_t cointegrate from I(d) to I(k) with unit coefficients, denoted CI1(d, k). In the context of SA, filters for which x_t^f and x_t are $\mathrm{CI1}(d,0)$ are of particular interest, yet the order of integration d may be unknown in practice.

Mean equality. Even for a filter ensuring $\mathrm{CI1}(d,0)$, the expectation of $x_t^f - x_t$ need not be zero. (Throughout, this expectation is assumed to exist.) Yet, it may be desirable to have an SA procedure such that the adjusted and unadjusted series are equal "on average". When the SA procedure does so, the series satisfy "mean equality". Because integrated processes need not have finite means and because fixed seasonal patterns are non-ergodic, mean equality is defined as $\mathcal{E}(\sum_s[x_t^f - x_t]/s) = \mathcal{E}(v(L)[x_t^f - x_t]) = 0$, where the summation is over the seasonal interval, the expectation $\mathcal{E}(\cdot)$ is over possible realizations, and x_t^f and x_t are $\mathrm{CI1}(d,0)$.

Implications for the SA filter $a(L)$ follow directly. Let Δ_s $[= (1-L^s)]$ be the annual change operator, and note that $\Delta v(L) = s^{-1}\Delta_s$. Since $x_t^a = a(L)x_t$ and $a(L) = 1 + a^{**}(L)\Delta^2$ from (11), then:

$$\begin{aligned}\mathcal{E}\left(\sum\nolimits_s [x_t^a - x_t]/s\right) &= \mathcal{E}\left(v(L)[a^{**}(L)\Delta^2 x_t]\right)\\ &= \mathcal{E}\left(a^{**}(L)\Delta^2 v(L)x_t\right)\\ &= \mathcal{E}\left(a^{**}(L)\Delta\Delta_s x_t\right)/s\,. \end{aligned} \tag{12}$$

Thus, mean equality requires a lag polynomial $a^{**}(L)$ for which $a^{**}(L)\Delta\Delta_s x_t$ is zero on average. Suitable conditions are discussed in the next subsection.

2.2 Relationships Between Series

This subsection considers the relationships between x_t^f and x_t under Assumptions 1–3 about the polynomial $f(L)$, applying those assumptions only as necessary.

The following result establishes the relationship between unit cointegration and the properties of the linear filter $f(L)$. Without loss of generality, suppose:

$$f(L) = f(1) + h(L)\Delta^q\,, \tag{13}$$

where $h(L)$ is a finite-order, fixed-weight linear filter; $h(1)$ is finite; and q is positive. Under Assumption 1, x_t^f and x_t are $\mathrm{CI1}(d,0)$ if and only if a q exists such that $q \geq d$. From (1),

$$\begin{aligned} x_t^f - x_t &= [1 + h(L)\Delta^q]x_t \;-\; x_t\\ &= h(L)\Delta^q x_t\,, \end{aligned} \tag{14}$$

which is I(0) if and only if $q \geq d$. For autoregressive processes of x_t as in (19) below, mean equality requires an extra order of differencing to ensure that $\mathcal{E}[h(L)\Delta^q x_t]$ is not a function of (e.g., for $d = 1$) the variable's average growth rate.

Under Assumption 1 alone, x_t^f and x_t are $\mathrm{CI1}(d, d-1)$ from (5). Thus, if $d = 1$, the filtered and raw data are cointegrated in the usual sense. If the filter is symmetric as well (Assumption 2), $q = 2$ by construction from (6). Hence, for $d = 1$, x_t^f and x_t will also be mean equal. As a corollary, x_t and the series from the analogue SA process x_t^a are $\mathrm{CI1}(d, d-2)$, and are $\mathrm{CI1}(d, d-1)$ with mean equality (provided the expectation exists). From the general properties of cointegrated series, Granger causality must run in at least one direction between x_t^f and x_t.

2.3 Implications for Sets of Series

For a given type of data (filtered or unfiltered), several series may themselves be cointegrated. This subsection shows that such cointegration implies that the corresponding series of the "other" type are cointegrated with the same cointegrating vector(s).

Consider p variables and denote the filtered and unfiltered series of the jth variable as x_{jt}^f and x_{jt} respectively ($j = 1, \ldots, p$). The associated $p \times 1$ vectors of data are denoted x_t^f and x_t, with their context below clarifying that they are vectors (rather than scalars as in Sections 2.1 and 2.2). The vectors x_t^f and x_t have the relationship:

$$x_t^f = F(L)x_t \, , \tag{15}$$

where

$$\begin{aligned} F(L) &= \sum_{i=-n}^{n} F_i L^i \\ &= F(1) + \Delta \sum_{i=-n}^{n-1} F_i^* L^i \\ &= F(1) + F^*(L)\Delta \, , \end{aligned} \tag{16}$$

and the F_i are $p \times p$ matrices. For the time, assume $F(L)$ is the diagonal matrix of scalar polynomials, $\mathrm{diag}[f_1(L) \ldots f_p(L)]$, where $f_j(L)$ is the filter generating x_{jt}^f from x_{jt} ($j = 1, \ldots, p$). That is, each filtered series is a weighted moving average of the corresponding unweighted series, and that unweighted series alone. Filters for different series need not be the same. Under Assumption 1, $F(1) = I_p$.

Cointegration within x_t. If, under Assumption 1, β is a cointegrating vector for the integrated vector process x_t, then β is also a cointegrating vector for x_t^f. The converse also applies. Intuitively, seasonal adjustment should affect only the dynamics of a process, and not its long-run properties. A general proof for equivalent cointegrating vectors follows directly from the definition of $F(L)$ under Assumption 1. The special case where x_t can be represented as a finite-order Gaussian vector autoregression (VAR) illustrates this invariance to data type.

A Proof. Under Assumption 1, premultiplying (15) by β' yields:

$$\beta' x_t^f = \beta' x_t + \beta' F^*(L)\Delta x_t \, . \tag{17}$$

If x_t is I(1), $\beta' x_t$ and Δx_t are both I(0) in (17), and so $\beta' x_t^f$ must be I(0). That is, β is a cointegrating vector for x_t^f. More generally, if x_t is CI(d, k) with cointegrating vector β, x_t^f is CI$(d, d-1)$ with cointegrating vector β, and $\beta' x_t$ and $\beta' x_t^f$ differ by a term that is I$(d-1)$. For x_t and x_t^f interchanged, the proof is immediate. A corollary also follows directly: if x_t has r cointegrating vectors, then so does x_t^f, and vice versa.

Under Assumptions 1–3, equation (17) becomes:

$$\beta' x_t^a \;=\; \beta' x_t + \beta' A^{**}(L)\Delta^2 x_t \,, \tag{18}$$

in an obvious notation. The disequilibrium measures $\beta' x_t^a$ and $\beta' x_t$ differ by the term $\beta' A^{**}(L)\Delta^2 x_t$, which is two orders of integration less than the order of integration of x_t. If x_t is I(1), then $\beta' x_t^a$ and $\beta' x_t$ satisfy mean equality. Figure 8 in Section 4.3 below shows how close the empirically estimated $\beta' x_t^a$ and $\beta' x_t$ can be.

Importantly, the cointegration results both for pairs of series and for sets of series require assumptions about the seasonal filter only. *No* assumption is made about the actual seasonality (or lack thereof) in the data, other than implicitly (and rather weakly) through the assumed order of integration of the data.

If Assumption 1 is *not* satisfied, the cointegrating vector(s) for x_t and those for x_t^f need not be the same. Thus, unless implemented carefully, multiple-series SA procedures run the risk of affecting long-run as well as short-run relationships between the series; cf. Bartelsman and Cleveland (1993).

VARs. In the cointegration literature, Johansen (1988) and Johansen and Juselius (1990) have stimulated interest in finite-order Gaussian VARs. If x_t follows such a process, then inference in a VAR for the SA data x_t^a generally is affected, even though β and the number of cointegrating vectors r are invariant to the transformation from x_t to x_t^a. Also, as discussed in Hendry and Mizon (1978) and Davidson, Hendry, Srba, and Yeo (1978), series may be filtered, but not relationships. In practice, the relationship for the SA data is obtained by filtering each NSA series individually (potentially using different filters) and then combining the filtered (SA) data. Thus, the remainder of this subsection derives the VAR for x_t^a from a VAR for x_t by using the relationship between the NSA and SA data in (15) and discusses empirical implications for analyzing x_t^a rather than x_t.

Suppose x_t has the representation:

$$x_t \;=\; \mu + \sum_{i=1}^{\ell} \pi_i x_{t-i} + \Phi S_t + \varepsilon_t \qquad \varepsilon_t \sim IN(0, \Omega) \,, \tag{19}$$

where μ is a $p \times 1$ vector of constants, $\{\pi_i\}$ are $p \times p$ matrices of autoregressive coefficients, ℓ is the maximal lag length, S_t is a vector of seasonal dummies $(S_{1t} \ldots S_{st})'$, Φ is the corresponding matrix of coefficients, and ε_t is a mean zero Gaussian innovation with covariance matrix Ω. By adding and subtracting various lags of x_t, (19) may be rewritten as:

$$\Delta x_t \;=\; \mu + \pi x_{t-1} + \sum_{i=1}^{\ell-1} \Gamma_i \Delta x_{t-i} + \Phi S_t + \varepsilon_t \,, \tag{20}$$

where the $\{\Gamma_i\}$ and π are:

$$\Gamma_i \;=\; -(\pi_{i+1} + \cdots + \pi_\ell) \qquad i = 1, \ldots, \ell - 1 \,, \tag{21}$$

$$\pi \;=\; \left(\sum_{i=1}^{\ell} \pi_i\right) - I_p \;. \tag{22}$$

For convenience below, (20) can be rewritten as a polynomial in Δx_t:

$$\Gamma(L)\Delta x_t \;=\; \mu + \pi x_{t-1} + \Phi S_t + \varepsilon_t \;, \tag{23}$$

where $\Gamma(L) = I_p - \sum_{i=1}^{\ell-1} \Gamma_i L^i$. Also, for simplicity and ease of exposition, assume that x_t is I(1).

The matrix π contains the "impact" coefficients of the lagged level x_{t-1}. Defining r as rank(π), π can be expressed as the outer product of two (full column rank) $p \times r$ matrices α and β:

$$\pi \;=\; \alpha\beta' \;, \tag{24}$$

for $0 < r \leq p$. The matrix β' is the set of cointegrating vectors, α is the matrix of "weighting elements", and r is the number of cointegrating vectors. If $r = 0$, then $\pi = 0$, in which case Δx_t is solely a function of its lags, μ, S_t, and ε_t, and there is no cointegration. If $r > 0$, the representation in (24) is unique only up to nonsingular $r \times r$ linear transformations, since $\alpha\beta' = (\alpha Q)(Q^{-1}\beta') = \alpha^\dagger \beta^{\dagger\prime}$ (say) for a nonsingular $r \times r$ matrix Q. It is assumed that there are sufficient a priori restrictions on α and β to identify them uniquely.

To derive the VAR for x_t^a, two operations are useful: pre-multiplication of (23) by $F(L)$ and substitution of x_t by $x_t^a + (x_t - x_t^a)$. In combination with Assumptions 1 and 3, these operations obtain the VAR for x_t^a. From the first operation, (23) becomes:

$$F(L)\Gamma(L)\Delta x_t \;=\; F(L)\mu + F(L)\pi x_{t-1} + F(L)\Phi S_t + F(L)\varepsilon_t \;. \tag{25}$$

The terms in (25) are transformed as follows. Applying the second operation, the left-hand side of (25) is:

$$F(L)\Gamma(L)\Delta x_t \;=\; F(L)\Gamma(L)\Delta x_t^a + F(L)\Gamma(L)\Delta(x_t - x_t^a) \;. \tag{26}$$

Using Assumption 1, $F(L)\mu = \mu$. Applying the second operation and using Assumption 1, the second term on the right-hand side of (25) is:

$$\begin{aligned} F(L)\pi x_{t-1} \;&=\; F(L)\pi x_{t-1}^a + F(L)\pi(x_{t-1} - x_{t-1}^a) \\ &=\; \pi x_{t-1}^a + F^*(L)\pi\Delta x_{t-1}^a + F(L)\pi(x_{t-1} - x_{t-1}^a) \;. \end{aligned} \tag{27}$$

Under Assumption 3, the seasonal dummies vanish in (25):

$$\begin{aligned} F(L)\Phi S_t \;&=\; F^o(L)v(L)\Phi S_t \\ &=\; F^o(L)\Phi v(L) S_t \\ &=\; 0 \;, \end{aligned} \tag{28}$$

in an obvious notation. Thus, by substitution and re-arrangement of terms, (25) may be rewritten as:

$$G(L)\Delta x_t^a \;=\; \mu + \pi x_{t-1}^a + \eta_t \;, \tag{29}$$

where:

$$G(L) = F(L)\Gamma(L) - F^*(L)\pi L\,, \tag{30}$$

$$\eta_t = -F(L)\pi(L)(x_t - x_t^a) + F(L)\varepsilon_t\,, \tag{31}$$

and $\pi(L)$ is defined as $I_p - \sum_{i=1}^{\ell} \pi_i L^i$ (equivalently, $\pi(L) = \Gamma(L)\Delta - \pi L$).

While (29) is formally similar to (23), several important differences exist. First, (23) is a conditional model, whereas (29) generally is not: $G(L)$ is in general a two-sided polynomial. Second, no seasonal dummies appear in (29) because $F(L)$ annihilates them. Third, ε_t in (23) is an innovation whereas η_t in (29) is not. This can be problematic when conducting inference on (29); cf. Ghysels and Perron (1993) for the univariate case. Fourth, as noted by Wallis (1974, p. 21), use of SA data may induce residual autocorrelation at seasonal lags: note the presence of the seasonal factor $x_t - x_t^a$ in the error η_t.

From (17), the number of cointegrating vectors r and the cointegrating vectors β are invariant to the type of data (NSA or SA). From (23) and (29), the impact matrix π also appears invariant in a conditional VAR, but it is not. In (29), πx_{t-1}^a is $\alpha\beta' x_{t-1}^a$; $\beta' x_{t-1}^a$ is I(0); and in general $\beta' x_{t-1}^a$ is correlated with η_t. Consequently, the weighting matrix α is not invariant in conditional VAR models, so neither is the product $\alpha\beta'$. Thus, whether or not a set of variables is weakly exogenous may depend upon which type of data is used. Additionally, because $G(L)$ is in general a two-sided non-diagonal polynomial matrix, the conditional representation of (29) confounds dynamics from future Δx^a with $\beta' x_{t-1}^a$, and dynamics in η_t with lagged dynamics in Δx^a. Both affect α, and so weak exogeneity for β. Even with the invariance of cointegration itself, the empirical power of cointegration tests on SA and on NSA data may differ, as illustrated in Lee and Siklos (1991, 1993).

2.4 Comments

Wallis (1974) finds that the linear filter (2) approximates the actual X–11 procedure well. However, as Wallis (1974, p. 20) notes, the linear filter ignores several features of X–11: graduation of extreme values, constraints on calendar-year totals, corrections at the ends of series, and multiplicative models of SA. For integrated data, the first three of the ignored features affect only the short-run dynamics for a wide class of x_t processes, so the results above should still hold. However, see Granger and Hallman (1991) and Ermini and Granger (1993) on nonlinear transformations of integrated processes, as arise in multiplicative models.

3 Encompassing Tests in Theory

Consider two empirical models, one developed on NSA data and the other on SA data. The following description of parameter encompassing suggests how to compare and evaluate these two models. Suppose one model (Model 1) has an estimate $\hat{\theta}$ for its parameter θ, and the other model (Model 2) implies that that parameter should be $\tilde{\theta}$. Then test the closeness of $\tilde{\theta}$ to $\hat{\theta}$, accounting for the uncertainty from estimation. If $\tilde{\theta}$ is "statistically close" to $\hat{\theta}$, Model 2 parameter-encompasses Model 1. That is, Model 2 explains why Model 1 obtains the estimate that it does. See Mizon and Richard (1986) and Hendry and Richard (1989) for extensive discussions of encompassing.

Numerous forms of encompassing have been proposed, including those of: the error variance [Cox (1961, 1962), Mizon and Richard (1986)], parameters [Hendry (1983), Mizon and Richard (1986)], the reduced form [Ericsson (1983), Hendry and Mizon (1993)], exogeneity [Hendry (1988)], and forecasts [Chong and Hendry (1986), Ericsson (1992b)]. However, none of these tests are directly applicable because the dependent variables of the SA and NSA models are inherently different. This section modifies the parameter-encompassing test to address this problem, using the relation between the adjusted and unadjusted data from the SA filter. Because of a fundamental asymmetry in the two models, this section first considers whether or not the SA model encompasses the NSA model (Section 3.1), and then the converse (Section 3.2).

3.1 Does an SA Model Encompass an NSA Model?

For ease of exposition, this section switches to and modifies a notation common to the literature on encompassing and non-nested hypothesis tests, even while this new notation does conflict with that in Section 2. Denote the conditional (regression) models on the SA and NSA data as:

$$M_{SA}: \quad y_t^a = x_t^{a\prime}\beta + u_{0t} \qquad u_{0t} \sim IN(0, \sigma_0^2) \tag{32}$$

$$M_{NSA}: \quad y_t = z_t'\gamma + u_{1t} \qquad u_{1t} \sim IN(0, \sigma_1^2)\ , \tag{33}$$

where y_t^a and y_t are the SA and NSA observations on the dependent variable at time t, x_t^a and z_t are vectors of SA and NSA regressors for the models M_{SA} and M_{NSA}, and β and γ are the corresponding coefficients. The error u_{0t} (u_{1t}) is assumed to be independently and normally distributed with mean zero and variance σ_0^2 (σ_1^2) under the hypothesis that M_{SA} (M_{NSA}) is correctly specified. Independence and normality are chosen for expositional simplicity. The regressors in M_{NSA} are z_t rather than x_t so as to allow for different dynamic structures in the two models. Even so, x_t^a and z_t may well derive from the same set of basic variables.

In the standard framework, the F-statistic for the significance of z_t in M_{SA} would be used to test whether or not M_{SA} parameter-encompasses M_{NSA}. Under the hypothesis M_{SA}, this F-statistic is distributed as an F-ratio asymptotically, and possibly in finite samples as well. However, the dependent variables in (32) and (33) differ, so some modification of the statistic is necessary. One possibility is to use the regression:

$$y_t^a = x_t^{a\prime}\beta + z_t'c + (y_t - y_t^a)d_0 + e_{0t}\ , \tag{34}$$

with regression error e_{0t}, and test for the significance of z_t and the seasonal factor $(y_t - y_t^a)$. Under the hypothesis M_{SA}, $c = 0$ and $d_0 = 0$, and both z_t and $(y_t - y_t^a)$ are valid conditioning variables.[3] As usual, variance dominance ($\sigma_0^2 < \sigma_1^2$) is a necessary condition for encompassing. Under M_{NSA}, $\beta = 0$, $c = \gamma$, and $d_0 = -1$ although, as implied by Section 3.2 below, the precise power of the test may be difficult to derive.

The encompassing test proposed in (34) is easy to calculate. Several variants of the test are possible, depending upon which variables are added to (32): z_t and $(y_t - y_t^a)$ [as proposed], z_t only, x_t and $(y_t - y_t^a)$, and x_t.[4] While only the first implies a nesting model for M_{SA} and M_{NSA}, the others may prove useful as general diagnostic procedures.

3.2 Does an NSA Model Encompass an SA Model?

In light of (34), the obvious procedure for testing whether or not M_{NSA} encompasses M_{SA} is to estimate the regression:

$$y_t = z_t'\gamma + x_t^{a\prime}b + (y_t - y_t^a)d_1 + e_{1t}\ , \tag{35}$$

with regression error e_{1t}, and test for the significance of x_t^a and $(y_t - y_t^a)$, i.e., test $b = 0$ and $d_1 = 0$. Unfortunately, this procedure is invalid: in general, x_t^a and $(y_t - y_t^a)$ are not valid conditioning variables under M_{NSA} because they include future values of y and x. The problem is most obvious when x_t^a contains the SA lagged dependent variable y_{t-1}^a. From the two-sided nature of the SA filter, y_{t-1}^a includes y_t, y_{t+1}, and so on. Under M_{NSA}, these are not valid "explanatory" variables.

An encompassing test is feasible, but it requires analysis of y_t, z_t, and x_t jointly; and it is inherently more difficult to calculate. The procedure is as follows, where $x_t = z_t$ (assumed purely for ease of exposition). Suppose y_t and z_t are both modeled, as in a VAR or in a conditional/marginal factorization. From that model, the moments of $(y_t : z_t')'$ can be calculated. The matrix SA filter $F(L)$ transforms $(y_t : z_t')'$ to $(y_t^a : z_t^{a\prime})'$; and conditioning y_t^a on z_t^a generates the regression coefficient on z_t^a in terms of the moments of $(y_t^a : z_t^{a\prime})'$. Thus, from the model for $(y_t : z_t')'$, an implied coefficient from regressing y_t^a on z_t^a can be calculated via $F(L)$. The encompassing statistic compares that implied coefficient with the estimated coefficient.[5]

[3] Lovell (1963, p. 995) discusses the desirability of an SA procedure being orthogonal, i.e., where $\sum_t (y_t - y_t^a)y_t^a = 0$. Even so, some SA procedures are not orthogonal, in which case the resulting SA data contain a seasonal component. Unless seasonal adjustment is orthogonal, $(y_t - y_t^a)$ and the dependent variable in (34) will be correlated.

[4] Other possible inclusions are: $(x_t - x_t^a)$ and $(y_t - y_t^a)$; $(x_t - x_t^a)$; $(z_t - z_t^a)$ and $(y_t - y_t^a)$; $(z_t - z_t^a)$; z_t, $(z_t - z_t^a)$, and $(y_t - y_t^a)$; and z_t and $(z_t - z_t^a)$. These emphasize the seasonal discrepancies between the adjusted and unadjusted data. The first two are equivalent to two of the main variants.

[5] In light of Campos, Ericsson, and Hendry (1990), this procedure parallels what would be required to test whether or not Hendry and Ericsson's (1991a) U.K. money demand equation on annual data parameter-encompasses Friedman and Schwartz's (1982) money demand equation on phase-average data. The empirical *lack* of parameter encompassing in the reverse direction follows directly from those equations' standard errors, where variance dominance is necessary for parameter encompassing.

This procedure has several difficulties. First, analysis of y_t conditional on z_t may have been chosen precisely because modeling $(y_t : z_t')'$ is more difficult. E.g., in Sections 4–5, a congruent, constant-parameter, parsimonious, economically interpretable, conditional money demand model was (relatively) easy to obtain. Finding an economically and statistically acceptable model of interest rates, inflation, and total final expenditure is much more difficult. Second, the mapping by $F(L)$ only approximates X–11. Third, the actual numerical calculations are substantial, and nontrivial to program. Thus, Section 5 does not calculate this statistic, but reports statistics from regressions like (35), recognizing that those regressions may include invalid conditioning variables.

Equations (34) and (35) are algebraically identical, with $d_1 = 1 + d_0$, $\gamma = c$, and $b = \beta$, as follows from adding $(y_t - y_t^a)$ to both sides of (34). That is, the coefficients on y_t and y_t^a sum to unity. A parallel structure appears in Ericsson's (1992b) modification to Chong and Hendry's (1986) forecast-encompassing test statistic. The validity of (34) [or (35)] as a maintained hypothesis depends upon the null hypothesis, whether (32) or (33). Specifically, the validity of conditioning on the "additional" variables in (34) [or (35)] is at issue.

Surprisingly, variance dominance is *not* a necessary condition for M_{NSA} to encompass M_{SA}. Under M_{NSA}, the model M_{SA} (possibly) conditions inappropriately on x_t^a, giving M_{SA} an "artificially low" error variance. Parallel situations arise in comparing least squares and instrumental variables error variances in a simultaneous equations framework, and in testing conditional versus expectational models; cf. Hendry (1988) Favero and Hendry (1992) on the latter. Also, M_{SA} may have a smaller error variance than M_{NSA} even if conditioning is not an issue: by averaging the dependent variable, the SA filter may reduce the dependent variable's variability.

4 Empirical Results on Cointegration

This section analyzes U.K. data on money demand with the system-based cointegration procedures in Johansen (1988) and Johansen and Juselius (1990). Section 4.1 examines cointegration between the adjusted and unadjusted series, variable by variable. Sections 4.2 and 4.3 analyze the unadjusted data and adjusted data as separate sets. The remainder of this introduction summarizes the economic theory of money demand and describes the data. The notation is one common to the money demand literature, albeit conflicting occasionally with the notation above.

The standard theory of money demand posits:

$$m^d - p = \delta_0 \cdot y + \delta' R \,, \tag{36}$$

As an alternative encompassing approach, foreshadowed by Wallis (1974), premultiply both sides of (33) by $f_y(L)$ to obtain the same dependent variable as in (32). Then apply standard encompassing tests. This approach is complicated by the induced two-sided moving averages of the error and z_t in the modified (33).

where M^d is nominal money demanded, P is the price level, Y is a scale variable ("income"), R is a vector of interest rates, and variables in lower case are in logarithms. Equation (36) assumes log-linearity in money, prices, and incomes and linearity in interest rates, a common functional form. The income elasticity δ_0 is one half in Baumol's (1952) and Tobin's (1956) transactions demand theory and unity in Friedman's (1956) quantity theory. The elements in δ are semi-elasticities for interest rates, and a given element is negative (positive) if the associated asset is excluded from (included in) the selected monetary aggregate. See Laidler (1985) and Goldfeld and Sichel (1990) for general discussions of money demand, Goodhart (1984, 1989) on U.K. financial institutions, and Miller and Orr (1966), Milbourne (1983), and Smith (1986) for additional developments on the theory of money demand.

In the empirical analysis below, M, Y, and P are nominal M_1, real total final expenditure (TFE) at 1985 prices, and the TFE deflator. There are two interest rates, the three-month local authority interest rate ($R3$) and the M_1 retail sight-deposit interest rate (Rr). The first is the dominant short-term interest rate in the secondary market and measures the return on (some) assets outside M_1. The second is the interest rate on checkable interest-bearing accounts at commercial banks and is a return on an asset within M_1. Three derived variables are of interest: the inflation rate (Δp), the learning-adjusted retail sight-deposit interest rate (Rra; see Baba, Hendry, and Starr (1992), Hendry and Ericsson (1991b), and Appendix A for details), and the net interest rate or opportunity cost (defined as $R3 - Rra$ and denoted R^*). Money and expenditure are in £ millions, the deflator is unity for 1985, and interest rates are in fractions. The data are quarterly, 1963(1)–1989(2). Allowing for lags and transformations, estimation is over 1964(3)–1989(2), which is 100 observations ($T = 100$). For details on the data, see Appendix A.

Data description begins with six pairs of graphs, where the first of each pair (e.g., Figure 1a) plots the NSA and SA series for a given variable and the second (e.g., Figure 1b) plots the difference between them, denoted the seasonal component. Figures 1–6 show m, p, $m - p$, y, $y + p$ (nominal TFE), and $m - p - y$ (inverse velocity) over 1963(1)–1989(2). Visually, all NSA and SA series appear I(1) at least; the augmented Dickey-Fuller (1979, 1981) [ADF] test statistics in Table 1 support this.[6] For m and p in particular, system analysis in Johansen (1992c) suggests that they are I(2), but the evidence is not conclusive. The hypothesis of a unit root in Δm (and in Δp) can not be rejected with the ADF statistic at standard significance levels, but the estimated root for Δm is only +0.31 (+0.87 for Δp). Thus, agnosticism on the order of integration for m and p seems appropriate, so Appendix C considers the

[6]Here and below, a maximum of five lags is chosen, thereby allowing for possible stochastic seasonality in the quarterly data while not being too profligate in parameters. For instance, the fifth-order VAR for System I below entails 29 coefficients in each of five equations estimated on 100 observations.

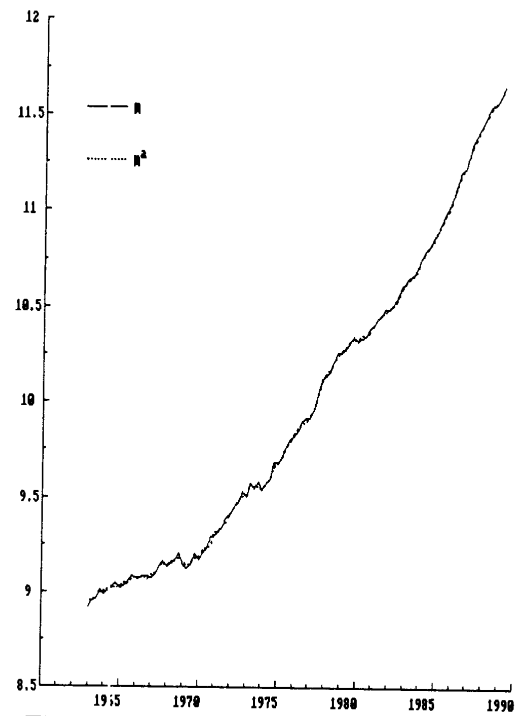

Figure 1a. The logs of NSA and SA nominal money stocks [m_t and m_t^a].

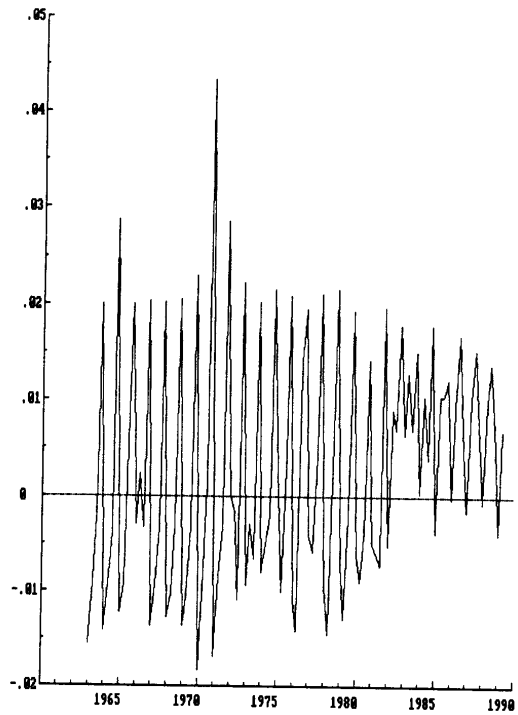

Figure 1b. The seasonal component of the nominal money stock [$m_t - m_t^a$].

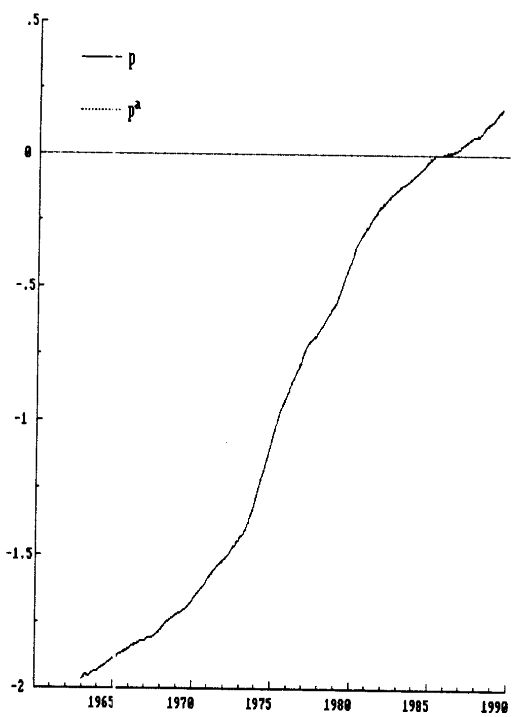

Figure 2a. The logs of NSA and SA prices [p_t and p_t^a].

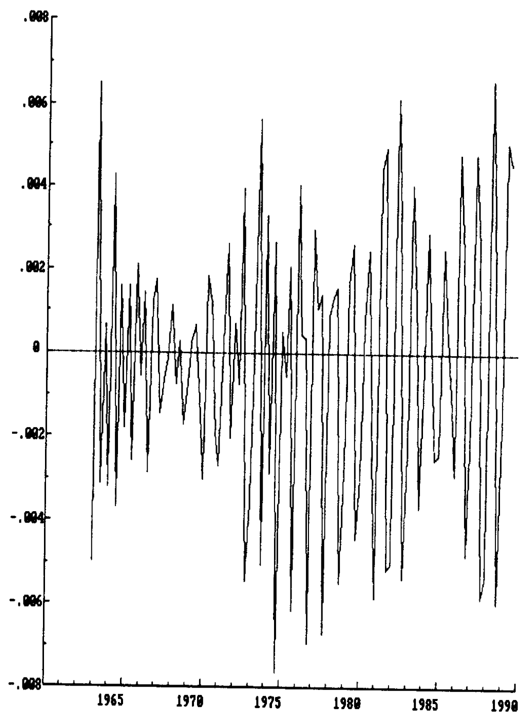

Figure 2b. The seasonal component of prices [$p_t - p_t^a$].

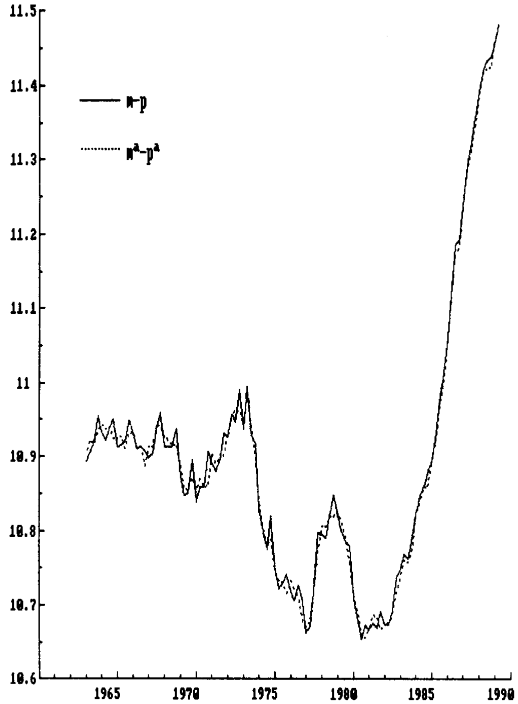

Figure 3a. The logs of NSA and SA real money stocks $[(m-p)_t$ and $(m^a-p^a)_t]$.

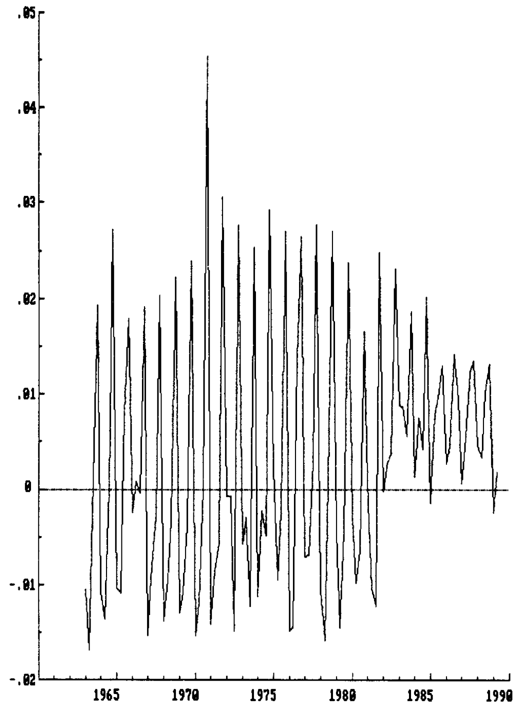
Figure 3b. The seasonal component of the real money stock $[(m-p)_t - (m^a-p^a)_t]$.

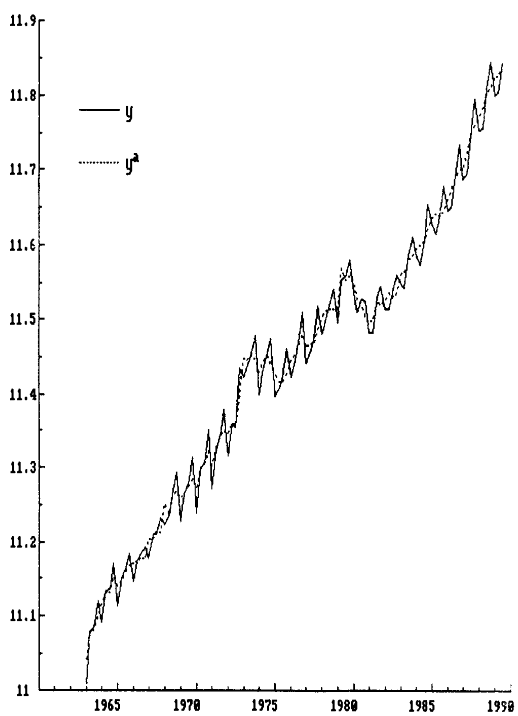

Figure 4a. The logs of NSA and SA real incomes $[y_t$ and $y_t^a]$.

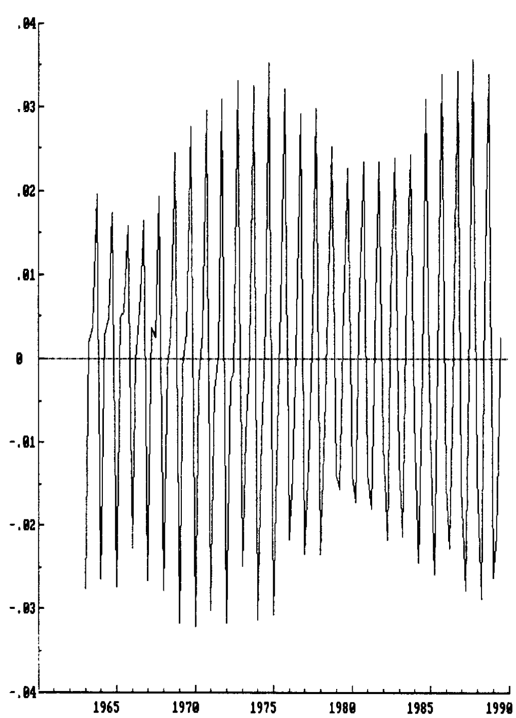
Figure 4b. The seasonal component of real income $[y_t - y_t^a]$.

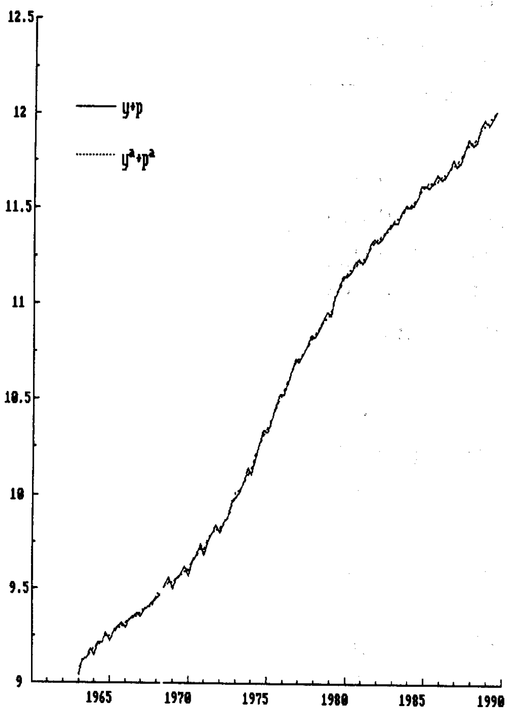

Figure 5a. The logs of NSA and SA nominal incomes [$(y+p)_t$ and $(y^a+p^a)_t$].

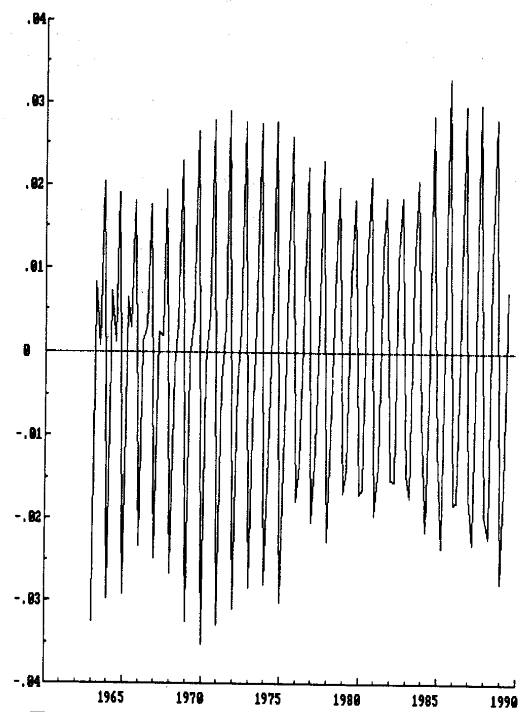

Figure 5b. The seasonal component of nominal income [$(y+p)_t - (y^a+p^a)_t$].

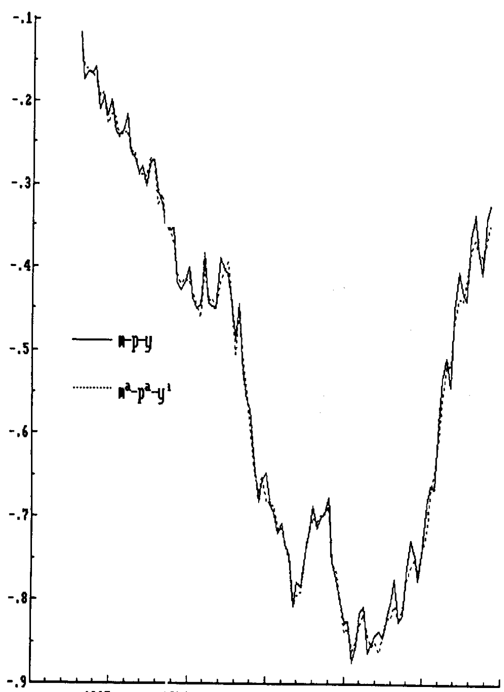

Figure 6a. The logs of NSA and SA inverse velocities [$(m-p-y)_t$ and $(m^a-p^a-y^a)_t$].

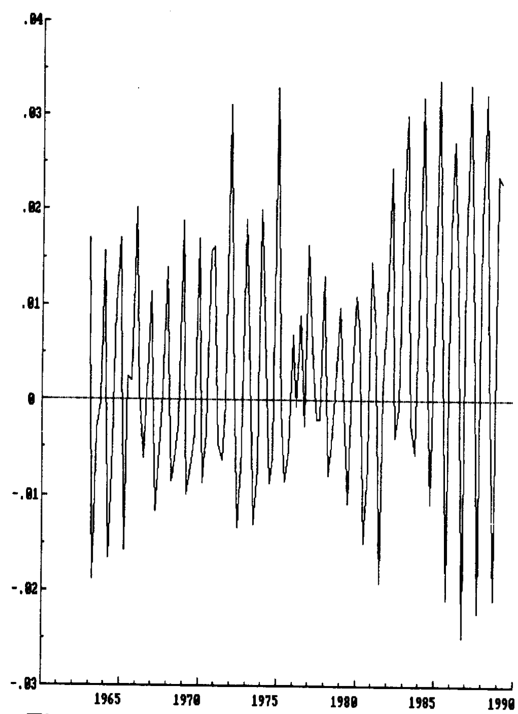

Figure 6b. The seasonal component of inverse velocity [$(m-p-y)_t - (m^a-p^a-y^a)_t$].

Table 1.
Augmented Dickey-Fuller Test Statistics

Variable	Type of data			
	NSA		SA	
m	–1.57	(–0.02)	–1.37	(–0.02)
p	–2.48	(–0.02)	–2.16	(–0.01)
$m-p$	–0.81	(–0.01)	–0.23	(–0.00)
y	–1.80	(–0.10)	–1.61	(–0.08)
$m-p-y$	–0.17	(–0.00)	+0.26	(+0.00)
Δm	–3.37	(–0.69)	–3.44	(–0.63)
Δp	–2.06	(–0.13)	–2.00	(–0.13)
$\Delta(m-p)$	–2.84	(–0.42)	–2.84	(–0.38)
Δy	–4.70	(–1.30)	–4.86	(–1.13)
$\Delta(m-p-y)$	–3.11	(–0.69)	–3.39	(–0.66)
$\Delta^2 m$	–5.77	(–2.84)	–6.63	(–2.90)
$\Delta^2 p$	–4.93	(–1.61)	–4.90	(–1.59)
$R3$	–3.45	(–0.22)		
Rra	–1.38	(–0.03)		
R^*	–2.07	(–0.08)		
$\Delta R3$	–5.19	(–1.15)		
ΔRra	–5.50	(–0.68)		
ΔR^*	–5.04	(–1.08)		

Notes: Each pair of entries includes the fourth-order ADF statistic with a constant term and trend and (in parentheses) the estimated coefficient associated with that ADF statistic. The ADF regressions for NSA data (except those for $R3$, Rra, and R^*) also include seasonal dummies. The sample is 1964(3)–1989(2) [$T = 100$], except for $\Delta^2 m$, $\Delta^2 p$, $\Delta^2 m^a$, and $\Delta^2 p^a$, which use 1964(4)–1989(2) [$T = 99$]. The 90%, 95%, and 99% critical values for $T = 100$ from MacKinnon (1991, Table 1) are –3.15, –3.45, and –4.05 respectively.

implications that m and p being I(2) or I(1) has for this analysis.[7]

All six NSA series display strong seasonality, as evidenced by the seasonal components (of the form $x - x^a$). In Figure 1b, $m - m^a$ is often 1–3% in absolute value, compared to an average quarterly growth rate for m of about 3%. The seasonal component for prices (Figure 2b) is nearly an order of magnitude smaller, often 0.2–0.6%, contrasting with an average growth rate for p of about 3% per quarter. Thus, the seasonal component for real money (Figure 3b) is virtually identical to that for nominal money, noting that $(m - p) - (m^a - p^a) = (m - m^a) - (p - p^a)$. The seasonality in real and nominal TFE (Figures 4b and 5b) is the most regular of those observed, with a strong cyclical pattern superimposed on the seasonality. Real TFE also has the largest seasonal component of the series examined: often 2–3%, compared to an average growth rate of under 1% per quarter.

The two dominant seasonal components are from nominal money and real income, and are of the same magnitude but contrast in pattern. The seasonality of inverse velocity (Figure 6b) is essentially the difference of those two components. Seasonal patterns in the levels of $m-p$, y, $y+p$, and $m-p-y$ are clearly detectable in Figures 3a, 4a, 5a, and 6a. Seasonal patterns for m and p are not apparent in Figures 1a and 2a because the range of the data is so great.

Figure 7 graphs the local-authority interest rate $R3$, the learning-adjusted M_1 retail sight-deposit interest rate (or "own rate") Rra, and the annual inflation rate $\Delta_4 p$. The two oil price increases are evident, and inflation declines to single-digit levels in the 1980s. Substantial differences between $\Delta_4 p$ and $R3$ persist over long episodes. Thus, inflation may play a role in the money demand function as the return on an illiquid, non-financial asset. The interest rate offered on M_1 (Rra) was introduced in 1984(3) and parallels $R3$ with a relatively constant spread. Importantly for money demand, the opportunity cost ($R3 - Rra$) declines from (often) double-digit figures in the 1970s and early 1980s to only a couple percent in the late 1980s.

4.1 Evidence on Pairs of Series

Table 2 presents Johansen's maximum likelihood cointegration test statistics, estimated cointegrating vectors (β'), and estimated weighting coefficients (α) for x and x^a, where x is any of m, p, y, $m-p$, $y+p$, or $m-p-y$. In each case, a fifth-order bivariate VAR is estimated for x and x^a with unconstrained seasonal dummies and a constant. The first four statistics in Table 2 are the maximal eigenvalue and trace eigenvalue statistics (λ_{max} and λ_{trace}) and those statistics adjusted for degrees of freedom lost in estimation (λ^a_{max} and λ^a_{trace}), with all four statistics being based on the largest eigenvalue.[8] Those statistics test the hypothesis that x and x^a are not cointe-

[7]The order of integratedness need not even be an inherent property of a time series. E.g., a series's order of integration could differ for different time periods.

[8]The hypothesis that there is at most one cointegrating vector (i.e., at least one zero eigenvalue) was not rejected for any of the series except m, and then only marginally so.

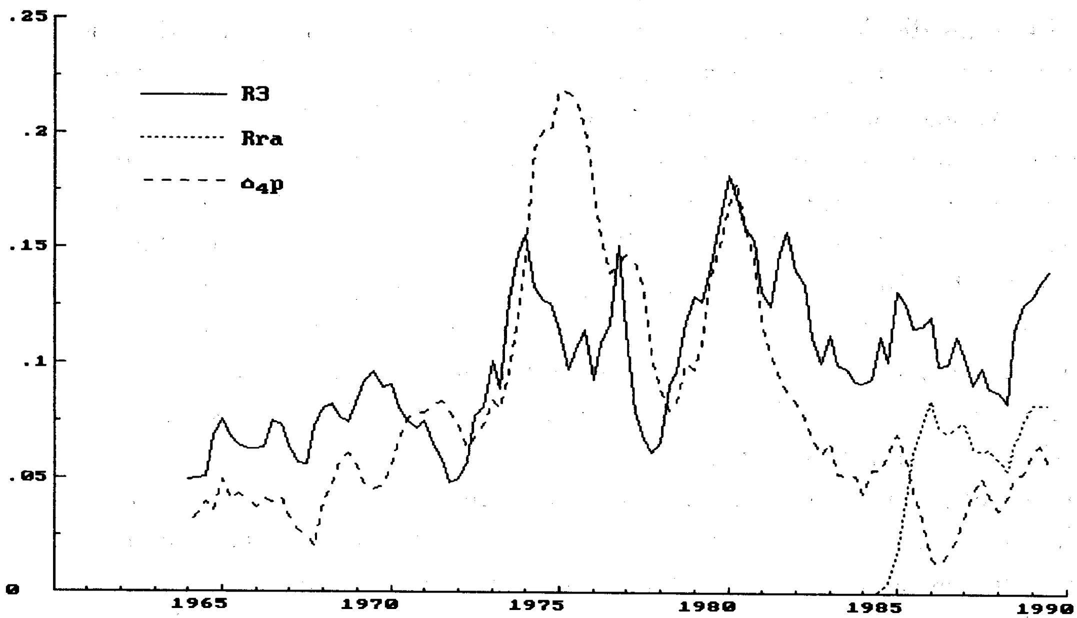

Figure 7. The three-month local authority interest rate ($R3_t$), the learning-adjusted M_1 retail sight-deposit interest rate (Rra_t), and the annual inflation rate ($\Delta_4 p_t$).

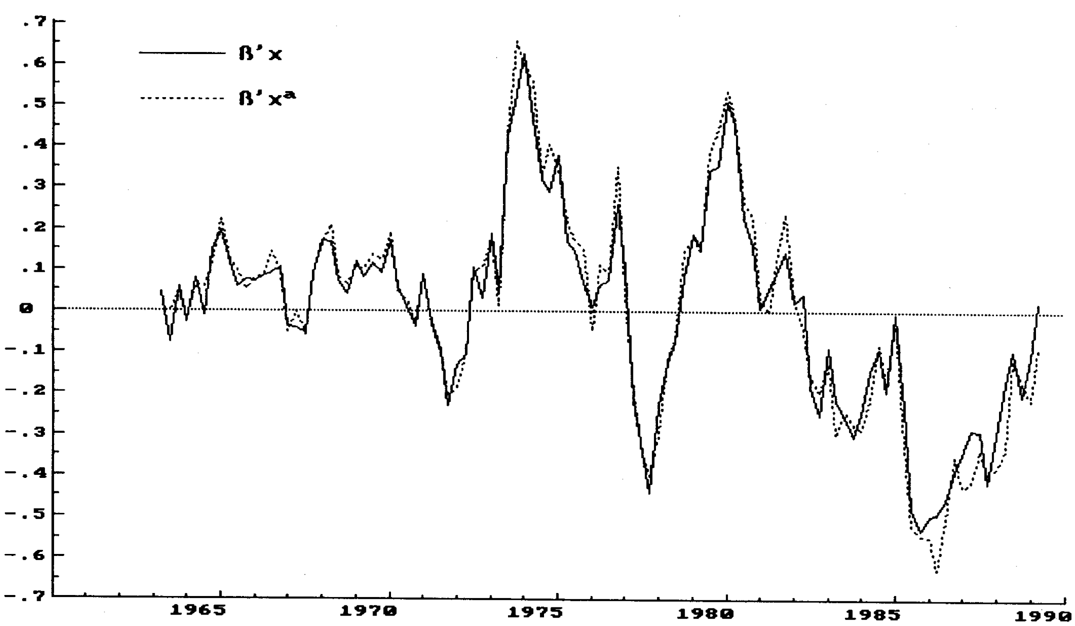

Figure 8. The NSA and SA disequilibrium measures $\beta' x_t$ and $\beta' x_t^a$.

Table 2.
Cointegration Analysis for Pairs of NSA and SA Series

Variable	Statistic					First row of:	
	λ_{max}	λ_{max}^{a}	λ_{trace}	λ_{trace}^{a}	$\chi^2(1)$	β'	α'
				Full sample: 1964(3)–1989(2)			
m	13.0	11.7	17.2	15.4	0.01	(1 –0.9998)	(+1.20 +1.66)
p	40.3	36.3	40.8	36.8	0.09	(1 –1.0000)	(–3.40 –0.68)
y	19.7	17.7	20.0	18.0	0.63	(1 –0.9994)	(–4.11 –2.07)
$m-p$	18.0	16.2	19.2	17.2	0.35	(1 –0.9974)	(+1.57 +2.18)
$y+p$	24.7	22.3	24.8	22.3	0.06	(1 –1.0000)	(–1.09 +1.31)
$m-p-y$	16.9	15.2	18.6	16.8	0.01	(1 –1.0000)	(+2.22 +2.77)
				Short sample: 1964(3)–1982(4)			
m	20.7	17.9	22.9	19.8	2.19	(1 –0.9985)	(+2.34 +3.44)
$m-p$	17.6	15.2	20.8	18.0	0.58	(1 –0.9957)	(+1.03 +2.23)
$m-p-y$	16.2	14.0	17.2	14.9	0.02	(1 –0.9997)	(+1.30 +2.58)
				Critical values			
90%	12.1	12.1	13.3	13.3	2.71		
95%	14.1	14.1	15.4	15.4	3.84		
99%	18.6	18.6	20.0	20.0	6.63		

grated against the alternative that there is at least one cointegrating vector. Critical values for the cointegration statistics are from Osterwald-Lenum (1992, Table 1). The fifth statistic in the table, denoted $\chi^2(1)$, tests the hypothesis $\beta' = (1 : -1)$ and is asymptotically distributed as $\chi^2(1)$ under that null hypothesis. For variables involving m and m^a, the statistics are also calculated over a "short sample", noting that the properties of the seasonal factor $m - m^a$ change after 1982; see Figure 1b.

Beginning with nominal money, the "max" and "trace" statistics reject at the 90% level; and the estimated cointegrating vector for $(m : m^a)'$ is $(1 : -0.9998)$, which is statistically and numerically insignificantly different from $(1 : -1)$. That is, $m - m^a$ appears I(0). Similar, even stronger evidence for cointegration appears for the other five variables and for nominal money over the short sample. In no case is the hypothesis $\beta' = (1 : -1)$ rejected.

For nominal money, the estimated weighting coefficients are +1.20 and +1.66, which are the coefficients on (approximately) $(m - m^a)_{t-1}$ in the equations for Δm_t and Δm^a_t respectively. Weighting coefficients in the other bivariate VARs are similar or larger in magnitude and of either sign. These coefficients for the bivariate VARs are numerically much larger than in typical empirical analyses of sets of variables; see Sections 4.2 and 4.3.

4.2 System Cointegration Analysis of the Unadjusted Data

This subsection tests for cointegration among the unadjusted series (m, p, y, $R3$, Rra). For both the NSA data set and the SA data set, inference could be affected by whether $R3$ and Rra enter separately or only via the opportunity cost R^*, and by whether m and p are I(1) or I(2). To assess the sensitivity of the cointegration tests to these factors, Appendix C examines four systems with the following variables:

System I. m, p, y, $R3$, Rra;

System II. m, p, y, R^*;

System III. $m - p$, Δp, y, $R3$, Rra; and

System IV. $m - p$, Δp, y, R^*.

The cointegrating vector and the weighting coefficients are little affected by the choice of system, so this subsection focuses on System IV. The system is a fifth-order vector autoregression with a constant term and seasonal dummies, but no trend. The estimation period is 1964(3)–1989(2).

Table 3 summarizes the cointegration results. It lists the eigenvalues related to $\hat{\pi}$ from largest to smallest, the max and trace statistics, the standardized estimated α and β', and statistics for testing restrictions on α and β'. The cointegration tests strongly reject the null of no cointegration ($r = 0$), but not the null of at most one cointegrating vector ($r \leq 1$), so there appears to be a single cointegrating vector for $(m - p, \Delta p, y, R^*)'$. The estimated cointegrating vector implies a long-run solution of:

$$m - p \;=\; 0.99y - 6.46\Delta p - 6.76R^* \;, \tag{37}$$

Table 3.
A Cointegration Analysis of NSA Data: $\{m-p,\ \Delta p,\ y,\ R^*\}$

Eigenvalues	0.345	0.121	0.049	0.015	
Hypotheses	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$	
λ_{max}	42.4	12.9	5.0	1.5	
λ^a_{max}	33.9	10.3	4.0	1.2	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	61.7	19.3	6.5	1.5	
λ^a_{trace}	49.4	15.5	5.2	1.2	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	$m-p$	Δp	y	R^*	
	1	6.46	−0.99	6.76	
	−0.05	1	−0.04	−0.46	
	−0.89	16.33	1	−4.95	
	−1.53	−4.82	−0.18	1	
	Standardized adjustment coefficients α				
$m-p$	−0.18	−0.08	−0.00	0.00	
Δp	0.03	−0.09	−0.00	−0.00	
y	−0.00	0.34	−0.01	−0.00	
R^*	0.03	0.17	0.00	−0.01	
	Test statistics for restrictions on β'				
Variable	$m-p$	Δp	y	R^*	Joint
$\chi^2(\cdot)$	—	—	0.0	—	—
p-value			[0.917]		
	Test statistics for zero restrictions on α				
Variable	$m-p$	Δp	y	R^*	Joint
$\chi^2(\cdot)$	28.6	5.0	0.0	1.3	5.6
p-value	[0.000]	[0.025]	[0.928]	[0.260]	[0.130]

with income, inflation, and interest rate elasticities in line with theory. A unit long-run homogeneity restriction on income can not be rejected.

The weighting coefficient on the cointegrating vector is -0.18 in the equation for money and is virtually zero in each of the other equations. While the coefficient in the inflation equation appears to be statistically significantly different from zero, the coefficients in the income, inflation, and the net interest rate equations jointly appear to be zero.[9] Those zeros are necessary for inflation, income, and the interest rate to be weakly exogenous for the parameters in the money equation; cf. Johansen (1992a). Section 5.1 assumes weak exogeneity and develops a model conditional on those variables.

4.3 System Cointegration Analysis of the Adjusted Data

Ericsson, Campos, and Tran (1990) test for cointegration in the SA data using Johansen's procedure. Table 4 replicates and adds to their System IV results, in which $m^a - p^a$, Δp^a, and y^a replace $m - p$, Δp, and y, and where a constant term (but no seasonal dummy) is included in the VAR. The eigenvalues, test statistics, and the estimated cointegrating vector in Table 4 are strikingly similar to those in Table 3, as implied by the theoretical analysis in Section 2.[10] Surprisingly, the estimated weighting matrices in Tables 3 and 4 are nearly identical. In general, they need not be, although they would be if, for all the SA filters, f_0 [in (2)] were approximately unity and the other f_i's were relatively small. Weights in the SA filters might have those values even for very seasonal data (such as m or y) if the seasonal component were well approximated by seasonal dummies and the SA filter included adjustment by dummies.

Figure 8 plots the estimated disequilibria $\beta' x_t$ and $\beta' x_t^a$, where β is numerically different for x_t and x_t^a. The choice of NSA or SA data matters little for the properties of the estimated disequilibrium. Shocks to the system and subsequent adjustment toward equilibrium are evident for higher inflation and interest rates (in 1973 and 1980), lower inflation and interest rates (in 1977 and 1982), and a lower opportunity cost (in 1985). Further, the disequilibria were substantial, sometimes exceeding $\pm 40\%$.

5 Single-equation Analysis

This section obtains a single-equation conditional model for money demand on NSA data (Section 5.1), summarizes Hendry and Ericsson's (1991b, equation (6))

[9] The "joint" test of zero restrictions on α does not include the zero restriction on α for the equation determining money. That latter restriction is resoundingly rejected, so money can not be assumed weakly exogenous in an equation determining inflation, income, or the interest rate.

[10] Hendry and Mizon (1993) analyze this SA data over a sample for which Rra is zero. They obtain a cointegrating vector for money demand that is virtually identical to the one for System IV.

Table 4.
A Cointegration Analysis of SA Data: $\{m^a - p^a,\ \Delta p^a,\ y^a,\ R^*\}$

Eigenvalues	0.386	0.128	0.050	0.009	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	
λ_{max}	48.8	13.7	5.1	0.9	
λ^a_{max}	39.0	11.0	4.1	0.8	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	68.6	19.8	6.1	0.9	
λ^a_{trace}	54.9	15.9	4.9	0.8	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	
	1	7.22	−1.08	7.16	
	−0.08	1	−0.04	−0.79	
	−1.26	16.03	1	−7.00	
	1.33	6.58	−0.12	1	
	Standardized adjustment coefficients α				
$m^a - p^a$	−0.18	−0.03	0.00	−0.00	
Δp^a	0.02	−0.05	−0.00	0.00	
y^a	−0.00	0.23	−0.01	−0.00	
R^*	0.03	0.14	0.00	0.01	
	Test statistics for restrictions on β'				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	Joint
$\chi^2(\cdot)$	—	—	0.8	—	—
p-value			[0.380]		
	Test statistics for zero restrictions on α				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	Joint
$\chi^2(\cdot)$	34.6	3.4	0.0	2.3	4.5
p-value	[0.000]	[0.065]	[0.996]	[0.133]	[0.215]

model on the SA data (Section 5.2), and tests each against the other with the encompassing tests designed in Section 3 above (Section 5.3).

5.1 Single-equation Analysis of the Unadjusted Data

As noted above, income, prices, and interest rates appear weakly exogenous, so single-equation modeling starts with an unrestricted autoregressive distributed lag (ADL) model for money. A parsimonious, economically interpretable, data-coherent simplification of that model is obtained via sequential reduction.

To match $\ell = 5$ in the VAR, the unrestricted single-equation model is a fifth-order ADL of m, conditional on p, y, $R3$, and Rra.[11] Table 5 lists coefficient estimates and their estimated standard errors (in parentheses). The long-run, static, non-stochastic solution to the model in Table 5 is:

$$\begin{aligned} m_t \;=\;& \underset{(0.09)}{0.98}\, p_t + \underset{(0.33)}{1.05}\, y_t - \underset{(1.6)}{6.4}\, R3_t + \underset{(0.9)}{7.0}\, Rra_t \\ & - \underset{(3.8)}{0.4} - \underset{(0.08)}{0.12}\, S_{1t} + \underset{(0.08)}{0.10}\, S_{2t} + \underset{(0.08)}{0.15}\, S_{3t}\,, \end{aligned} \tag{38}$$

where S_{it} is a seasonal dummy for the ith quarter, and estimated standard errors are in parentheses. Equation (38) corresponds to (36). The estimates in (38) closely match the system estimates of the first cointegrating vector when $\Delta p = 0$.

Table 5 (and also the regressions (39) and (41) below) includes diagnostic statistics for testing against various alternative hypotheses: residual autocorrelation (dw and AR), skewness and excess kurtosis ($Normality$), autoregressive conditional heteroscedasticity ($ARCH$), RESET ($RESET$), heteroscedasticity ($Hetero$), and heteroscedasticity quadratic in the regressors (alternatively, mis-specification of functional form) ($Form$).[12] The null distribution is designated by $\chi^2(\cdot)$ or $F(\cdot,\cdot)$, where the degrees of freedom fill the parentheses. For AR and $ARCH$, the first degree of freedom is the maximal lag. No statistic in Table 5 is significant at its 95% critical value.

The model in Table 5 has an equivalent error correction representation, which may be simplified to (39) below. Details of the simplification process appear in Appendix B.

[11] This specification is the least restrictive of the conditional ADLs associated with the four systems in that it does not impose restrictions on the long-run coefficients for prices and interest rates. Those restrictions *are* imposed early on in the sequential reduction; and their imposition from the start of the sequential reduction does not affect the final outcome.

[12] For references on the test statistics, see Durbin and Watson (1950, 1951), Box and Pierce (1970), Godfrey (1978), and Harvey (1981, p. 173); Jarque and Bera (1980); Engle (1982); Ramsey (1969); and White (1980, p. 825) and Nicholls and Pagan (1983) (the latter two on both *Hetero* and *Form*).

Table 5.
A General Autoregressive Distributed Lag for Nominal Money, Conditional on Prices, Incomes, and Interest Rates (NSA Data)

Variable	lag i (or summation over lags)						
	0	1	2	3	4	5	$\sum_{i=0}^{5}$
m_{t-i}	-1	0.588	0.127	-0.193	0.367	-0.042	-0.152
	(-)	(0.126)	(0.142)	(0.132)	(0.130)	(0.114)	(0.043)
p_{t-i}	0.430	0.024	-0.213	-0.031	-0.342	0.282	0.149
	(0.258)	(0.404)	(0.380)	(0.348)	(0.341)	(0.214)	(0.039)
y_{t-i}	0.022	0.219	0.044	-0.372	0.107	0.138	0.159
	(0.108)	(0.117)	(0.116)	(0.116)	(0.123)	(0.116)	(0.057)
$R3_{t-i}$	-0.433	-0.259	-0.342	0.081	0.187	-0.209	-0.974
	(0.137)	(0.204)	(0.208)	(0.204)	(0.201)	(0.150)	(0.211)
Rra_{t-i}	0.223	-0.024	2.250	-2.249	0.491	0.370	1.060
	(0.515)	(0.995)	(1.111)	(1.143)	(1.089)	(0.621)	(0.327)
constant	-0.055						
	(0.565)						
S_{it}		-0.018	0.016	0.023			
		(0.011)	(0.011)	(0.010)			

$T = 100$ [1964(3) – 1989(2)] $\quad R^2 = 0.9998 \quad \hat{\sigma} = 1.403\%$

$dw = 2.09 \quad AR: F(4,63) = 1.37 \quad Normality: \chi^2(2) = 2.48$

$ARCH: F(4,59) = 0.87 \quad Hetero: F(61,5) = 0.12 \quad RESET: F(1,66) = 1.70$

$$
\begin{aligned}
\Delta(\widehat{m-p})_t \;=\; & -\underset{[0.13]}{0.95}\,[\Delta_3(m-p)_{t-1}/3] \;-\; \underset{[0.12]}{1.07}\,[(\Delta p_t+\Delta p_{t-4})/2] \\
& +\underset{[0.05]}{0.16}\,\Delta^2 y_{t-2} \;-\; \underset{[0.080]}{1.189}\,[(R^*_t+R^*_{t-1}+R^*_{t-2})/3] \\
& -\underset{[0.011]}{0.174}\,(m-p-y)_{t-1} \\
& +\underset{[0.006]}{0.038} \;-\; \underset{[0.005]}{0.012}\,S_{1t} \;+\; \underset{[0.005]}{0.010}\,S_{2t} \;+\; \underset{[0.008]}{0.018}\,S_{3t}
\end{aligned}
\tag{39}
$$

$$
\begin{array}{lll}
T = 100\ [1964(3)-1989(2)] & R^2 = 0.84 & \hat{\sigma} = 1.348\% \\
AR: F(4,87) = 1.07 & dw = 1.95 & ARCH: F(4,83) = 0.44 \\
Normality: \chi^2(2) = 4.08 & RESET: F(1,90) = 1.64 & \\
Hetero: F(13,77) = 1.12 & Form: F(38,52) = 0.62\,. &
\end{array}
$$

Jack-knife, heteroscedasticity-consistent, estimated standard errors appear in square brackets under coefficient estimates; see White (1980), Nicholls and Pagan (1983), and MacKinnon and White (1985).

The first three terms on the right-hand side of (39) capture the effects of the lagged dependent variable, inflation, and income growth. Noting the near (negative) unit coefficient on $(\Delta p_t + \Delta p_{t-4})/2$, (39) can be transformed to have nominal money growth Δm_t as the dependent variable and $+0.5\Delta_4\Delta p_t$ on the right-hand side. That is, inflation per se has no immediate effect on nominal money demand; only its acceleration does. In the long run, both inflation and its acceleration affect money demand via the error correction. From the interest rate and error correction coefficients, the long-run solution of (39) is virtually (38), with unit income and price elasticities imposed as part of the sequential reduction.

Statistically, (39) appears satisfactory. None of the diagnostic tests reject, and the F-statistic for testing (39) against Table 5 is $F(24,67) = 0.70$ [83%], where the tail probability is in square brackets.

Constancy is an additional, crucial statistical property, particularly in the context of money demand equations; see Judd and Scadding (1982) and Goldfeld and Sichel (1990). Recursive least squares and the associated sequences of test statistics provide incisive tools for investigating constancy; cf. Brown, Durbin, and Evans (1975) and Dufour (1982). Graphs efficiently summarize the large volume of output. Figure 9a records the one-step residuals and the corresponding calculated equation standard errors for (39), i.e., $\{y_t - \hat{\beta}_t' x_t\}$ and $\{0.0 \pm 2\hat{\sigma}_t\}$ in a common notation. The equation standard error $\hat{\sigma}$ varies little. Figure 9b plots the "break-point" Chow (1960) statistics for the sequence {1969(3)–1989(2), 1969(4)–1989(2), 1970(1)–1989(2), ..., 1989(1)–1989(2), 1989(2)}, none of which are significant at even the 5% level. Figures 9c–9k show the numerical values of all the recursively estimated coefficients and plus-or-minus twice their recursively estimated standard errors, denoted $\hat{\beta}_t$ and $\hat{\beta}_t \pm 2ese(\hat{\beta}_t)$ respectively in the graphs. Coefficients on economic variables vary only slightly rela-

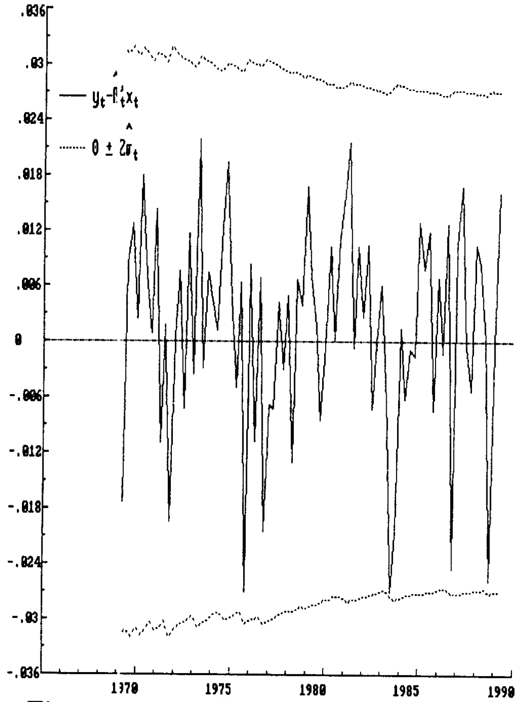

Figure 9a. One-step residuals and ±2 times the corresponding calculated equation standard errors.

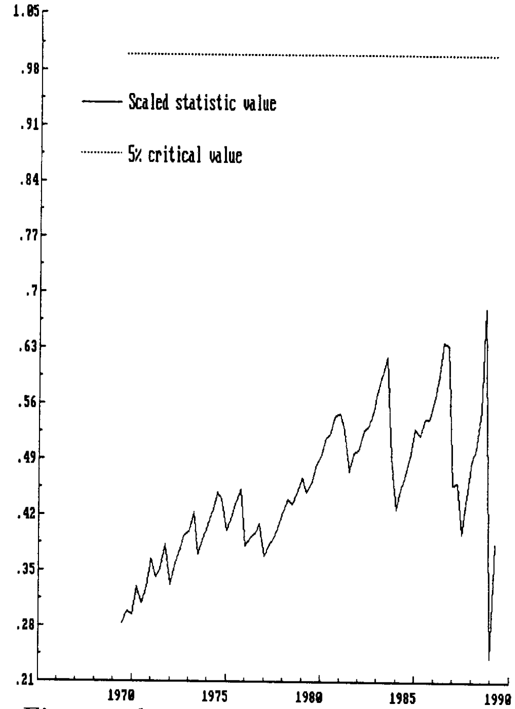

Figure 9b. The sequence of break-point Chow statistics, with the statistics scaled by their one-off 5% critical values.

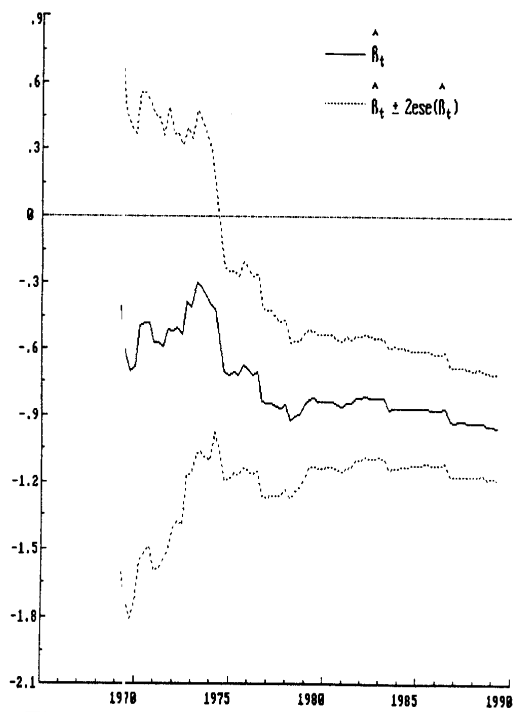

Figure 9c. Recursive estimates of the coefficient of $\Delta_3(m-p)_{t-1}/3$, with ±2 estimated standard errors.

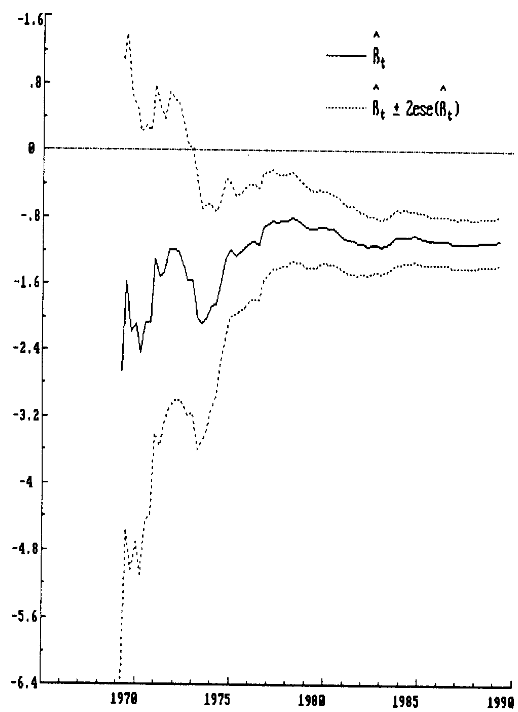

Figure 9d. Recursive estimates of the coefficient of $(\Delta p_t + \Delta p_{t-4})/2$, with ±2 estimated standard errors.

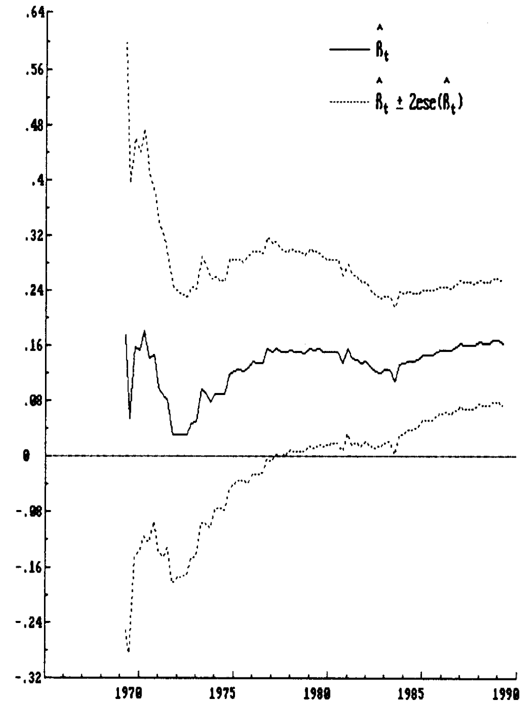

Figure 9e. Recursive estimates of the coefficient of $\Delta^2 y_{t-2}$, with ± 2 estimated standard errors.

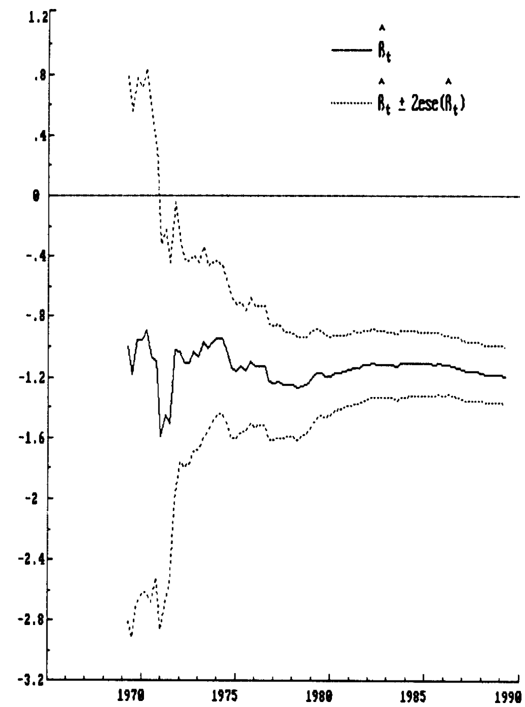

Figure 9f. Recursive estimates of the coefficient of $(R_t^* + R_{t-1}^* + R_{t-2}^*)/3$, with ± 2 estimated standard errors.

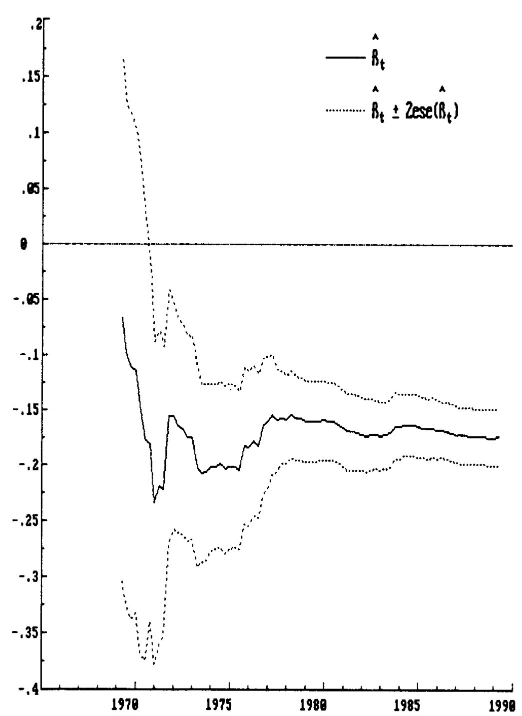

Figure 9g. Recursive estimates of the coefficient of $(m - p - y)_{t-1}$, with ± 2 estimated standard errors.

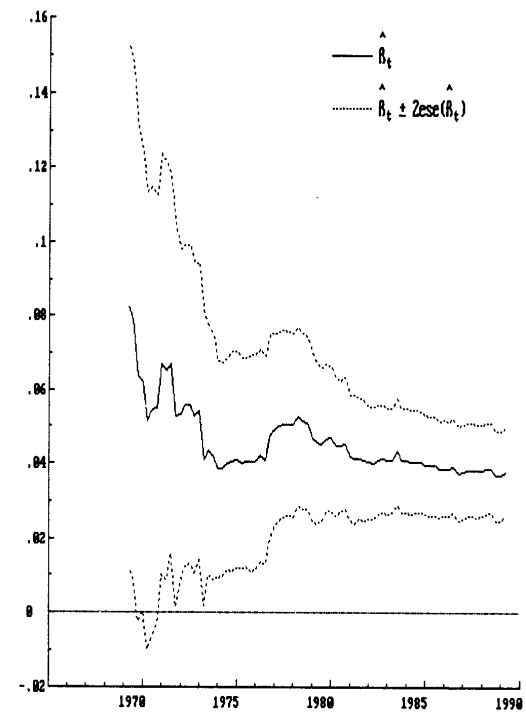

Figure 9h. Recursive estimates of the coefficient of the constant term, with ± 2 estimated standard errors.

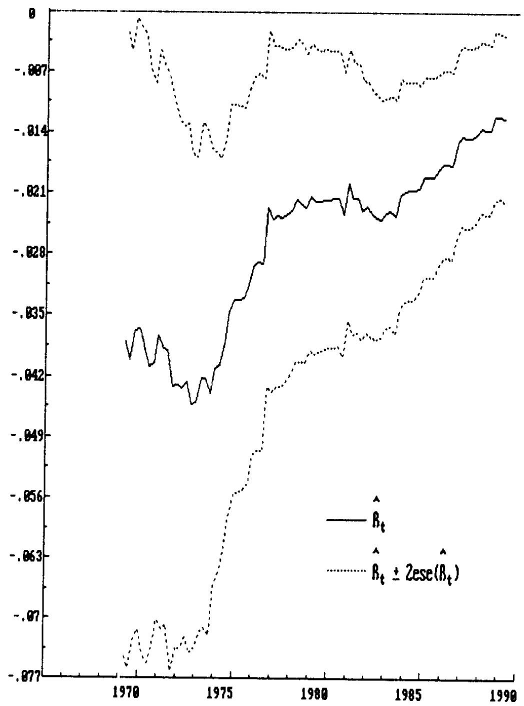

Figure 9i. Recursive estimates of the coefficient of S_{1t}, with ± 2 estimated standard errors.

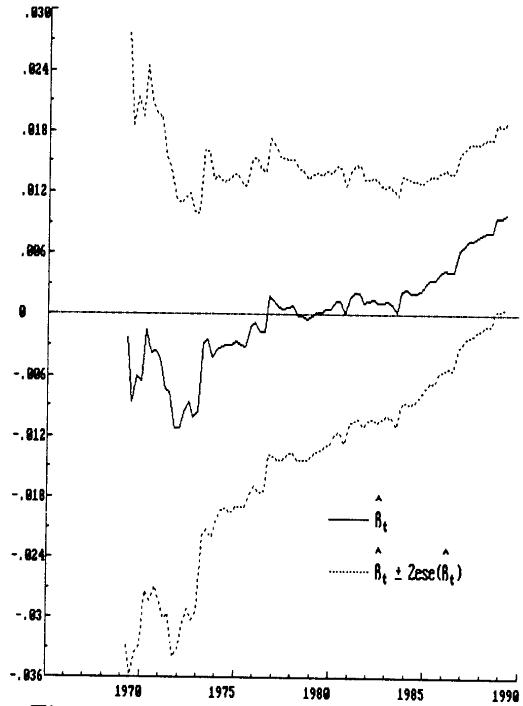

Figure 9j. Recursive estimates of the coefficient of S_{2t}, with ± 2 estimated standard errors.

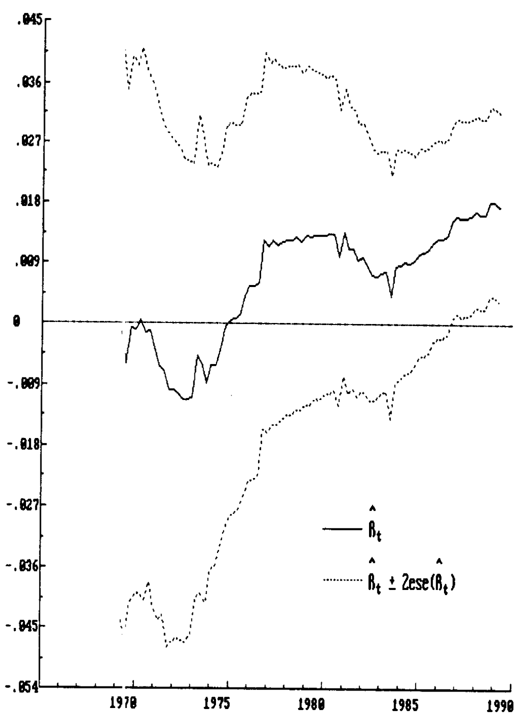

Figure 9k. Recursive estimates of the coefficient of S_{3t}, with ± 2 estimated standard errors.

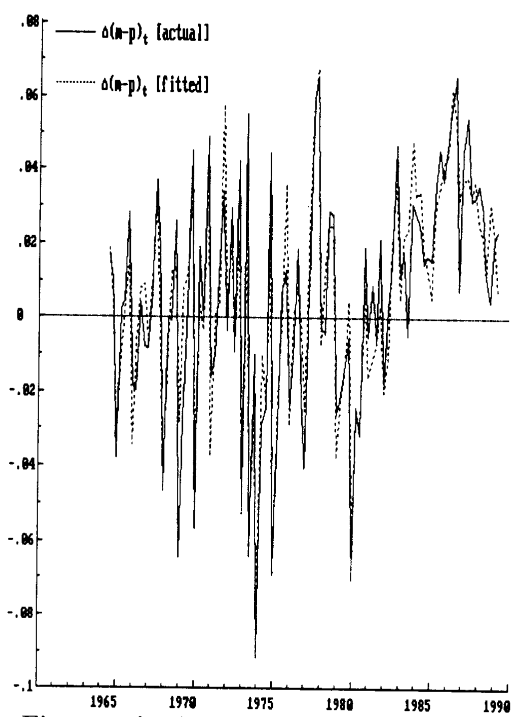

Figure 9ℓ. Actual and fitted values of $\Delta(m-p)_t$.

tive to their *ex ante* standard errors, and all those variables except $\Delta^2 y_{t-2}$ are highly significant by 1980. The quarterly dummies are statistically constant, but appear to drift numerically. Even with the full sample, their coefficients are only marginally significant. Figure 9ℓ plots the actual and fitted values for $\Delta(m-p)_t$ and shows how well (39) explains the data.

Hendry and Ericsson (1991b) document large changes in the data properties. Together with the constancy of (39), those changes imply the super exogeneity of expenditure, prices, and interest rates for the parameters in (39); cf. Engle, Hendry, and Richard (1983), Hendry (1988), and Engle and Hendry (1993). To summarize, (39) is a constant, economically interpretable, data-coherent model of NSA money demand in the United Kingdom.

5.2 Single-equation Analysis of the Adjusted Data

The SA data have been thoroughly studied in a sequence of papers, starting with Hacche (1974), Coghlan (1978) and Hendry (1979). The latter develops a constant, parsimonious error correction model over 1964(1)–1977(4). Subsequent models by Trundle (1982), Hendry (1985), Davidson (1987), Cuthbertson (1988), Hendry (1988), Ericsson, Campos, and Tran (1990), Hall, Henry, and Wilcox (1990), Hendry and Ericsson (1991b), and Hendry and Mizon (1993) are similar in form and numerical parameter values, with the main differences arising from using different sample periods, and data series with different base years.

From a fifth-order ADL on the current SA data set, Ericsson, Campos, and Tran (1990) obtain the following long-run, static, non-stochastic solution:

$$m_t^a = \underset{(0.08)}{0.96}\, p_t^a + \underset{(0.30)}{1.17}\, y_t^a - \underset{(1.5)}{6.7}\, R3_t + \underset{(0.8)}{7.0}\, Rra_t - \underset{(3.4)}{1.7} \tag{40}$$

$$T = 100\ [1964(3) - 1989(2)].$$

These estimates closely match those in (38) on NSA data, and the system estimates of the first cointegrating vector on both SA and NSA data.

Hendry and Ericsson (1991b, equation (6)) obtain the following error correction model (ECM).

$$\begin{aligned}\widehat{\Delta(m^a - p^a)_t} = &- \underset{[0.14]}{0.69}\, \Delta p_t^a - \underset{[0.06]}{0.17}\, \Delta(m^a - p^a - y^a)_{t-1} - \underset{[0.053]}{0.630}\, R_t^* \\ &- \underset{[0.008]}{0.093}\, (m^a - p^a - y^a)_{t-1} + \underset{[0.004]}{0.023}\end{aligned} \tag{41}$$

$T = 100\ [1964(3) - 1989(2)]$ $\quad R^2 = 0.76 \quad \hat{\sigma} = 1.313\%$
$AR: F(4,91) = 1.94$ $\quad dw = 2.18 \quad ARCH: F(4,87) = 0.74$
$Normality: \chi^2(2) = 1.53$ $\quad RESET: F(1,94) = 0.08$
$Hetero: F(8,86) = 1.36$ $\quad Form: F(14,80) = 1.05$.

Ericsson, Campos, and Tran (1990) find that (41) is a valid sequential reduction from the ADL, with an F-statistic of $F(25,70) = 0.97$ [51%]. From the coefficients on the interest rate and the error correction term, the long-run solution of (41) is little changed from (40). Extensive evaluation by Ericsson, Campos, and Tran (1990) and Hendry and Ericsson (1991b) shows that (41) is a constant, data-coherent, conditional model with sensible economic properties. Prices, expenditure, and interest rates appear super exogenous.

5.3 Encompassing Tests in Practice

From the studies to date, (41) appears a well-specified model of SA money demand. However, the encompassing tests from Section 3 will show that (41) does not encompass the NSA model (39). Other than mixing data sets, no obvious respecification of (41) results in an improved model of the SA data. Because of computational and modeling issues, the formally correct encompassing test in the reverse direction is not performed. However, evidence from the "invalid" encompassing test suggests that the NSA model does encompass the SA model.

To test whether or not the SA model encompasses the NSA model, $\Delta(m-p)_t - \Delta(m^a - p^a)_t$ and the regressors from the NSA model are added to the SA model, and the significance of the added variables is tested. The resulting estimated equation is the following, which implements (34).

$$
\begin{aligned}
\widehat{\Delta(m^a - p^a)_t} = & - \underset{[0.20]}{0.51}\, \Delta p^a_t - \underset{[0.07]}{0.09}\, \Delta(m^a - p^a - y^a)_{t-1} - \underset{[0.164]}{0.241}\, R^*_t \\
& - \underset{[0.116]}{0.048}\, (m^a - p^a - y^a)_{t-1} + \underset{[0.007]}{0.026} \\
& - \underset{[0.19]}{0.29}\, [\Delta(m-p)_t - \Delta(m^a - p^a)_t] - \underset{[0.16]}{0.53}\, [\Delta_3(m-p)_{t-1}/3] \\
& - \underset{[0.18]}{0.39}\, [(\Delta p_t + \Delta p_{t-4})/2] + \underset{[0.05]}{0.09}\, \Delta^2 y_{t-2} \\
& - \underset{[0.23]}{0.70}\, [(R^*_t + R^*_{t-1} + R^*_{t-2})/3] - \underset{[0.116]}{0.088}\, (m-p-y)_{t-1} \\
& + \underset{[0.008]}{0.008}\, S_{1t} + \underset{[0.005]}{0.012}\, S_{2t} + \underset{[0.007]}{0.015}\, S_{3t}
\end{aligned}
\tag{42}
$$

$T = 100$ [1964(3) – 1989(2)] $\quad R^2 = 0.81 \quad \hat{\sigma} = 1.225\%$

The F-statistic for the significance of the additional regressors is $F(9,86) = 2.57$ [1.1%]. If the seasonal component $\Delta(m-p)_t - \Delta(m^a - p^a)_t$ is not included, the F-statistic is still large: $F(8,87) = 2.50$ [1.7%].

As discussed in Section 3, reversing the roles of SA and NSA data is not valid because it generally entails invalid conditioning for the NSA model. Even so, the results from reversal may be of interest, to the extent that future values play a small role in the added regressors. Adding $\Delta(m-p)_t - \Delta(m^a - p^a)_t$ and the SA regressors in (41) to the NSA model (39) obtains (42), except that $\Delta(m-p)_t$ is the dependent variable and so the coefficient on $\Delta(m-p)_t - \Delta(m^a - p^a)_t$ is $+0.71$ (implied by Section 3.2). The F-statistic for the additional regressors is $F(5, 86) = 4.83$ [0.06%]. However, rejection appears entirely explained by the presence of the seasonal component $\Delta(m-p)_t - \Delta(m^a - p^a)_t$. If that seasonal component is not included in the regression, the F-statistic is $F(4, 87) = 1.54$ [19.9%].

In summary, the SA model does not encompass the NSA model, whereas the NSA model appears to encompass the SA model. The NSA model appears well specified otherwise and is sensible economically, so it represents the currently best available empirical model for U.K. narrow money demand. While the long-run solutions for the SA and NSA models are virtually identical empirically (as implied by the theoretical analysis), estimated short-run dynamics do differ. For instance, inflation and the net interest rate enter the SA model current-dated only, whereas they appear as time averages in the NSA model. Development of the NSA model also shows how new tests and new data play a central role in a progressive research program, whereby existing models are supplanted by new models that encompass the existing models and offer some "value added".

6 Concluding Remarks

Seasonality and seasonal adjustment procedures have stimulated a wealth of theoretical and empirical studies. With tools from the cointegration literature, this paper derives central relationships between pairs of SA and NSA data, and between sets of SA and NSA data. The encompassing framework provides a basis for comparing models developed on SA data with those developed on NSA data. This paper extends the analysis of Wallis and Sims to cointegrated series and develops methods for evaluating NSA and SA models directly against each other. Contrasting with results from Sims and Wallis in a stationary framework, there *are* invariants to seasonal adjustment of cointegrated data: the number of cointegrating vectors and the cointegrating vectors themselves. That said, inference may well be affected by the choice of data type.

A substantive model development of NSA narrow money demand in the United Kingdom illustrates the analytical results. Hendry and Ericsson's (1991b) SA model of narrow money demand appeared well-specified on existing tests, but was found deficient in the presence of the new NSA model. This result demonstrates the value of the new tests and of the NSA data. It also highlights the importance of statistical agencies providing NSA data even if they already provide SA data, since the latter need not be the appropriate data for empirical economic modeling.

Appendix A. The Data

Table A1.
Data Definitions and Sources

Variable	Definition	Source
GDP	Gross domestic product (expenditure-based) at market prices [£ million, current prices]	DJAF (NSA) DJBB (SA)
$GDP85$	Gross domestic product (expenditure-based) at market prices [£ million, 1985 prices]	DJCX (NSA) DJDI (SA)
IMP	Imports of goods and services at market prices [£ million, current prices]	DJAG (NSA) DJBC (SA)
$IMP85$	Imports of goods and services at market prices [£ million, 1985 prices]	DJCY (NSA) DJDJ (SA)
M	Monetary aggregate M_1: notes and coin in circulation with the public plus UK private sector sterling sight bank deposits, both non-interest-bearing and interest-bearing (financial year constrained when seasonally adjusted) [£ million, current prices]	AGAF (NSA) AGBA (SA)
P	Implicit deflator for total final expenditure [$= (GDP + IMP)/(GDP85 + IMP85)$] [1985 = 1.00]	—
$R3$	Interest rate on deposits with local authorities, for a minimum of three months and thereafter at seven days' notice (quarterly average of the rate on the last Friday of each month) [fraction]	AJOI
R^*	Learning-adjusted net interest rate ($= R3 - Rra$) [fraction]	—
Rr	Interest rate on (M_1) sterling retail sight deposits at banks [fraction]	Hendry and Ericsson (1991b)
Rra	Learning-adjusted interest rate on retail sight deposits at banks ($= w_t \cdot Rr_t$) [fraction]	—
w_t	Weighting function representing agents' learning about interest-bearing retail sight deposits [$= (1 + \exp[\kappa_0 - \kappa_1(t - t_0 + 1)])^{-1}$ for $t \geq t_0$, zero otherwise; $t_0 = 1984(3)$, $\kappa_0 = 5.0$, and $\kappa_1 = 1.2$]	Hendry and Ericsson (1991b)
Y	Total final expenditure at market prices ($= GDP85 + IMP85$) [£ million, 1985 prices]	—

Sources. The data sources are: *Bank of England Quarterly Bulletin*, various issues (*BEQB*); *Economic Trends Annual Supplement*, 1990 Edition, No. 15 (*ETAS*); *Financial Statistics*, various issues (*FS*); and *Monthly Digest of Statistics*, various issues (*MDS*). The first is a publication of the Bank of England, London; the other three are published by the Central Statistical Office (CSO), Her Majesty's Stationary Office, London. The four-character sequence is the CSO databank series number.

GDP, $GDP85$, IMP, and $IMP85$ are from $ETAS$ (Table 3), with minor changes for data revisions from MDS (Table 1.2). M is the M_1 series in FS (January 1989, Supplementary Table S32, Columns 6 [NSA] and 7 [SA]) and $BEQB$ (November 1989, Table 11.1, Columns 4 [NSA] and 14 [SA]). $R3$ is from various issues of the $BEQB$ (e.g., May 1989, Table 9.2) and FS (e.g., February 1990, Table 13.14). Rr is zero prior to 1984(3), and as listed in Hendry and Ericsson (1991b, Table A.2) thereafter. We are grateful to Stephen Hall at the Bank of England for providing Rr.

All data are quarterly and span 1963(1)–1989(2), unless otherwise noted.

Adjustments. Topping and Bishop (1989) document numerous breaks in the series for M_1. We account for the four primary breaks in M_1, proportionately rescaling data before the break to match the post-break value of M_1 for the quarter in which the break occurred. Adjusting the data for these breaks is critical, statistically as well as economically. The breaks range from −1.5% to +6.3%, but $\hat{\sigma}$ is only 1.3% in (39) and (41). See also Healey, Mann, Clews, and Hoggarth (1990).

Topping and Bishop's breaks are for NSA data, and are reported in Table A2 below. We use the same breaks for SA data, as suggested by Topping and Bishop (1989, p. 11).

Table A2.
The Four Primary Breaks in M_1

Date	Break $\pounds\times10^6$	Break per cent	M_1 after break		Explanation
1971(4)	+403	+3.8% +3.9%	11088 10765	(NSA) (SA)	A break occurs "... due to the incorporation of new information collected from the London clearing banks ... on the sector split of current and deposit accounts ...". (p. 25)
1975(2)	+618	+4.1% +4.0%	15791 15929	(NSA) (SA)	"New, more comprehensive, statistical returns introduced in May 1975 further reduced the estimation necessary to calculate M_1 ...". (p. 26)
1976(1)	-266	-1.5% -1.5%	17421 17588	(NSA) (SA)	"This is due to the incorporation of data on public corporations' holdings of notes and coin ...", i.e., which are *not* included in M_1. (pp. 26–27)
1981(4)	+2081	+6.1% +6.3%	35956 35257	(NSA) (SA)	"... the 'monetary sector' was introduced in place of the 'banking sector'; amongst others, this brought the [Trustee Savings Banks] into the monetary sector." (pp. 12, 28)

Source for quotes and breaks: Topping and Bishop (1989); see their Table 2(a) for breaks.
Units: £ million, unless otherwise noted.

Appendix B. Sequential Reduction Analysis

This appendix describes a sequential reduction from the general ADL model in Table 5 to the parsimonious ECM in (39). Many other "routes" for the reduction are possible. However, the values of the F-statistics for this sequence are small, implying that the F-statistics for other routes are unlikely to be statistically significant.

First, the model in Table 5 is rewritten as an equivalent ECM representation. Two types of transformations are used. Levels and lagged levels are written as differences and a current or lagged level; and levels on two variables are written as a differential between the two variables and the level of one of the variables. See Ericsson, Campos, and Tran (1990) for a motivation and further discussion. The specific transformations are:

1. nominal money m and prices p are transformed to real money $m-p$ and prices;
2. the interest rates $R3$ and Rra are transformed to the spread R^* and Rra;
3. each of the variables $m-p$, p, y, and Rra is transformed to a single log-level (or level) and a set of current and lagged differences, with the log-levels $m-p$, p, and y at the first lag and the level Rra current; and
4. the variables $(m-p)_{t-1}$ and y_{t-1} are transformed to $(m-p-y)_{t-1}$ and y_{t-1}, where $(m-p-y)_{t-1}$ is the potential error correction term.

The resulting coefficient estimates and estimated standard errors appear in Table B1, and provide the starting point for the sequential reduction.

To aid in the sequential reduction of the model in Table B1, we list several variables in Table B1 with highly statistically significant coefficients and which are economically reasonable to retain, as well as several variables whose coefficients appear numerically and statistically insignificant. The following are highly significant. The error correction term $(m-p-y)_{t-1}$ enters with a coefficient of -0.152, close to the first term in the α matrix for the system analysis (Table 3). The current net interest rate R_t^* and the current inflation rate Δp_t each enter with large negative coefficients, interpretable as reflecting costs to holding money when other assets (or goods) yield a return. The first and third lags of the dependent variable $\Delta(m-p)_t$ are statistically significant.

The following do not appear either numerically or statistically significant:

i. the variables p_{t-1} and y_{t-1};
ii. the variable Rra_t; and
iii. all current and lagged values of ΔRra.

Four additional reductions are considered:

iv. the coefficients on R_t^*, R_{t-1}^*, and R_{t-2}^* are equal, and those on R_{t-3}^*, R_{t-4}^*, and R_{t-5}^* are zero;
v. the coefficients on $\Delta(m-p)_{t-1}$, $\Delta(m-p)_{t-2}$, and $\Delta(m-p)_{t-3}$ are equal, and that on $\Delta(m-p)_{t-4}$ is zero;

Table B1.
The Unrestricted Error Correction Model for NSA Data

Variable	lag i					
	0	1	2	3	4	5
$\Delta(m-p)_{t-i}$	–1	–0.260	–0.133	–0.325	0.042	
	(—)	(0.111)	(0.119)	(0.113)	(0.114)	
Δp_{t-i}	–0.570	0.045	–0.041	–0.265	–0.240	
	(0.258)	(0.273)	(0.272)	(0.247)	(0.247)	
Δy_{t-i}	0.022	0.083	0.126	–0.246	–0.138	
	(0.108)	(0.137)	(0.125)	(0.131)	(0.116)	
R^*_{t-i}	–0.432	–0.259	–0.342	0.082	0.187	–0.209
	(0.137)	(0.204)	(0.208)	(0.204)	(0.201)	(0.150)
ΔRra_{t-i}	–0.297	–0.580	1.328	–0.840	–0.161	
	(0.485)	(0.619)	(0.642)	(0.642)	(0.617)	
$(m-p-y)_{t-i}$		–0.152				
		(0.043)				
p_{t-i}		–0.003				
		(0.014)				
y_{t-i}		0.007				
		(0.049)				
Rra_{t-i}	0.088					
	(0.276)					
constant	–0.055					
	(0.565)					
S_{it}		–0.018	0.016	0.023		
		(0.011)	(0.011)	(0.010)		

$T = 100$ [1964(3) – 1989(2)] $\quad R^2 = 0.87 \quad \hat{\sigma} = 1.403\%$
Note: All residual-based statistics are identical to those in Table 5.

vi. the coefficients on Δy_t, Δy_{t-1}, and Δy_{t-4} are zero, and those on Δy_{t-2} and Δy_{t-3} are equal and opposite; and

vii. the coefficients on Δp_t and Δp_{t-4} have equal coefficients, and those on Δp_{t-1}, Δp_{t-2}, and Δp_{t-3} are zero.

Eight models arise from treating these seven restrictions sequentially:

Model 1. The unrestricted ECM in Table B1 (equivalently, in Table 5);

Model 2. Model 1, excluding p_{t-1} and y_{t-1} (long-run unit homogeneity of prices and income imposed);

Model 3. Model 2, excluding Rra_t (long-run restriction of "opposite sign, equal magnitude" coefficients on $R3$ and Rra imposed);

Model 4. Model 3, excluding current and lagged values of ΔRra (short-run restriction of "opposite sign, equal magnitude" coefficients on $R3$ and Rra imposed);

Model 5. Model 4, excluding R^*_{t-1}, R^*_{t-2}, R^*_{t-3}, R^*_{t-4}, and R^*_{t-5} [once R^*_t, R^*_{t-1}, and R^*_{t-2} are transformed to $\sum_{i=0}^{2} R^*_{t-i}/3$, R^*_{t-1}, and R^*_{t-2}];

Model 6. Model 5, excluding $\Delta(m-p)_{t-2}$, $\Delta(m-p)_{t-3}$, and $\Delta(m-p)_{t-4}$ [once $\Delta(m-p)_{t-1}$, $\Delta(m-p)_{t-2}$, and $\Delta(m-p)_{t-3}$ are transformed to $\Delta_3(m-p)_{t-1}/3$, $\Delta(m-p)_{t-2}$, and $\Delta(m-p)_{t-3}$];

Model 7. Model 6, excluding Δy_t, Δy_{t-1}, Δy_{t-3}, and Δy_{t-4} [once Δy_{t-2} and Δy_{t-3} are transformed to $\Delta^2 y_{t-2}$ and Δy_{t-3}]; and

Model 8. Model 7, excluding Δp_{t-1}, Δp_{t-2}, Δp_{t-3}, and Δp_{t-4} [once Δp_t and Δp_{t-4} are transformed to $(\Delta p_t + \Delta p_{t-4})/2$ and Δp_{t-4}].

Table B2 reports $\hat{\sigma}$ and the Schwarz criterion (SC) for each model, the F-statistics for the reductions between all model pairs, and the associated tail probability values. Throughout the reduction sequence, $\hat{\sigma}$ remains relatively constant, the Schwarz criterion is always declining, and no reductions are statistically significant at the 5% level, whether considered individually or as sub-sequences. The complete reduction appears valid, with $F(24,67) = 0.70$ [0.83].

Table B2.
F- and Related Statistics for the Sequential Reduction
from the Fifth-order ADL Model in Table 5 (NSA Data)

Null Hypothesis				Maintained Hypothesis (Model Number)						
Model	k	$\hat{\sigma}$	SC	1	2	3	4	5	6	7
1	33	1.403%	-7.41	–						
				–						
↓ (i)				–						
2	31	1.383%	-7.50	0.02						
				[0.98]						
↓ (ii)				(2,67)						
3	30	1.378%	-7.54	0.15	0.43					
				[0.93]	[0.51]					
↓ (iii)				(3,67)	(1,69)					
4	25	1.392%	-7.69	0.85	1.16	1.31				
				[0.56]	[0.34]	[0.27]				
↓ (iv)				(8,67)	(6,69)	(5,70)				
5	20	1.388%	-7.86	0.86	1.04	1.12	0.90			
				[0.59]	[0.42]	[0.36]	[0.49]			
↓ (v)				(13,67)	(11,69)	(10,70)	(5,75)			
6	17	1.382%	-7.97	0.84	0.99	1.04	0.85	0.77		
				[0.64]	[0.48]	[0.43]	[0.56]	[0.51]		
↓ (vi)				(16,67)	(14,69)	(13,70)	(8,75)	(3,80)		
7	13	1.360%	-8.14	0.73	0.84	0.87	0.67	0.51	0.31	
				[0.78]	[0.65]	[0.61]	[0.78]	[0.83]	[0.87]	
↓ (vii)				(20,67)	(18,69)	(17,70)	(12,75)	(7,80)	(4,83)	
8	9	1.348%	-8.29	0.70	0.79	0.81	0.64	0.53	0.44	0.59
				[0.83]	[0.73]	[0.70]	[0.84]	[0.88]	[0.89]	[0.67]
				(24,67)	(22,69)	(21,70)	(16,75)	(11,80)	(8,83)	(4,87)

Notes: The first four columns report the model number (with reduction), and for that model: the number of unrestricted parameters k, the estimated equation standard error $\hat{\sigma}$, and the Schwarz criterion SC, defined as $\ln(RSS_T/T) + k \cdot (\ln T)/T$. The text of Appendix B defines the models. The three entries within a given block of numbers are: the F-statistic for testing the null hypothesis (indicated by the model number to the left of the entry) against the maintained hypothesis (indicated by the model number above the entry), the tail probability associated with that value of the F-statistic (in square brackets), and the degrees of freedom for the F-statistic (in parentheses).

Appendix C. Cointegration Analysis of the Four Systems

This appendix compares the results of cointegration analyses for four systems with the NSA data and four parallel systems with the SA data. The cointegrating vector and weighting coefficients are little affected by the choice of system or type of data.

C.1 System Cointegration Analysis of the Unadjusted Data

This subsection tests for cointegration among the unadjusted series (m, p, y, $R3$, Rra). For both the NSA data set and the SA data set, inference could be affected by whether $R3$ and Rra enter separately or only via the opportunity cost R^*, and by whether m and p are I(1) or I(2). If m and p are I(2), they may cointegrate with a cointegrating vector $(+1 : -1)$ to form an I(1) variable $m-p$. If they do, cointegration analysis could proceed with the I(1) variables $m-p$ and Δp rather than the I(2) variables m and p; see Johansen (1992c). Both statistically and computationally, analysis of I(2) variables is more cumbersome than that of I(1) variables, so valid transformation of the data to I(1) space is appealing.

To assess the sensitivity of the cointegration tests to these factors, four systems are examined:

System I. m, p, y, $R3$, Rra;

System II. m, p, y, R^*;

System III. $m-p$, Δp, y, $R3$, Rra; and

System IV. $m-p$, Δp, y, R^*.

The systems are fifth-order ($\ell = 5$) vector autoregressions of the corresponding variables: a constant term and seasonal dummies are included in all cases (but no trend); and the estimation period is 1964(3)–1989(2). Tables C1–C4 present the results.[13]

Tables C1–C4 list the eigenvalues related to $\hat{\pi}$ from largest to smallest, the max and trace statistics (λ_{max} and λ_{trace}), and those statistics adjusted for degrees of freedom lost in estimation (λ^a_{max} and λ^a_{trace}). The max and trace statistics are defined in Johansen (1988) and Johansen and Juselius (1990), and critical values are taken from Osterwald-Lenum (1992, Table 1). For a system having p eigenvalues total ($p = 5$ for models I and III, $p = 4$ for models II and IV), the hypothesis being tested is that there are at least g zero eigenvalues (unit roots) in the system, where $g = p - r$ and r is the number of nonzero eigenvalues (equally, the rank of π). If we reject that there are at least g unit roots, then we infer that there are at least $p - g + 1$ $(= r + 1)$ cointegrating vectors. Thus, it is convenient to pose the null hypotheses in terms of the number of nonzero eigenvalues r (as in the tables) rather than in terms of g.

From the rejections obtained on all systems, there is at least one cointegrating vector. In System I, there may be a second, although the critical values used may not be appropriate if m and p are I(2); see Johansen (1992b, 1992c).

[13] For this dataset, these results appear robust to the maximal lag length of the VAR.

Table C1.
A Cointegration Analysis of NSA Data: $\{m, p, y, R3, Rra\}$

Eigenvalues	0.379	0.239	0.160	0.093	0.005	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	
λ_{max}	47.7	27.4	17.4	9.8	0.5	
λ^a_{max}	35.7	20.5	13.0	7.4	0.4	
95% critical value	33.5	27.1	21.0	14.1	3.8	
λ_{trace}	102.7	55.0	27.7	10.3	0.5	
λ^a_{trace}	77.0	41.3	20.8	7.7	0.4	
95% critical value	68.5	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'					
Variable	m	p	y	$R3$	Rra	
	1	−1.09	−0.43	6.41	−8.12	
	−1.71	1	2.03	4.55	12.78	
	−0.08	−0.08	1	−1.15	−1.23	
	0.32	−0.21	−0.26	1	−2.17	
	−2.02	1.62	1.76	0.39	1	
	Standardized adjustment coefficients α					
m	−0.14	0.02	0.09	0.02	0.00	
p	0.05	0.01	0.05	0.01	−0.00	
y	0.02	−0.02	0.06	−0.11	0.00	
$R3$	0.04	−0.03	0.06	−0.00	−0.00	
Rra	0.00	−0.01	0.01	0.02	0.00	
	Test statistics for restrictions on β'					
Variable	m	p	y	$R3$	Rra	Joint
$\chi^2(\cdot)$	—	1.2	4.3	0.9	—	4.7
p-value		[0.268]	[0.037]	[0.338]		[0.197]
	Test statistics for zero restrictions on α					
Variable	m	p	y	$R3$	Rra	Joint
$\chi^2(\cdot)$	14.2	9.0	0.4	2.0	0.4	12.0
p-value	[0.000]	[0.003]	[0.542]	[0.160]	[0.519]	[0.018]

Table C2.
A Cointegration Analysis of NSA Data: $\{m, p, y, R^*\}$

Eigenvalues	0.355	0.171	0.102	0.002	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	
λ_{max}	43.8	18.8	10.8	0.2	
λ^a_{max}	35.0	15.0	8.6	0.2	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	73.6	29.8	11.0	0.2	
λ^a_{trace}	58.9	23.8	8.8	0.2	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	m	p	y	R^*	
	1	–1.16	–0.45	7.99	
	–5.19	1	22.33	–7.31	
	–0.58	0.31	1	–2.86	
	–3.37	2.71	2.94	1	
	Standardized adjustment coefficients α				
m	–0.10	0.01	–0.00	0.00	
p	0.04	0.00	0.00	–0.00	
y	0.01	–0.00	0.06	0.00	
R^*	0.02	–0.00	0.03	–0.00	
	Test statistics for restrictions on β'				
Variable	m	p	y	R^*	Joint
$\chi^2(\cdot)$	—	3.2	2.9	—	3.2
p-value		[0.075]	[0.087]		[0.204]
	Test statistics for zero restrictions on α				
Variable	m	p	y	R^*	Joint
$\chi^2(\cdot)$	12.3	9.6	0.1	0.7	9.8
p-value	[0.000]	[0.002]	[0.765]	[0.393]	[0.020]

Table C3.
A Cointegration Analysis of NSA Data: $\{m-p, \Delta p, y, R3, Rra\}$

Eigenvalues	0.384	0.197	0.107	0.050	0.022	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	
λ_{max}	48.4	21.9	11.4	5.1	2.2	
λ^a_{max}	36.3	16.4	8.5	3.9	1.6	
95% critical value	33.5	27.1	21.0	14.1	3.8	
λ_{trace}	89.0	40.6	18.7	7.3	2.2	
λ^a_{trace}	66.8	30.5	14.0	5.5	1.6	
95% critical value	68.5	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'					
Variable	$m-p$	Δp	y	$R3$	Rra	
	1	5.03	−0.64	5.14	−7.61	
	−0.21	1	−0.53	2.48	3.17	
	0.87	−12.81	1	4.03	−8.81	
	0.16	−3.00	−0.19	1	−0.65	
	0.36	2.22	−0.02	−0.19	1	
	Standardized adjustment coefficients α					
$m-p$	−0.22	−0.02	0.00	0.00	−0.01	
Δp	0.04	−0.00	0.01	0.02	0.01	
y	0.02	−0.09	−0.02	0.04	−0.01	
$R3$	0.07	−0.09	−0.00	−0.02	0.01	
Rra	0.01	−0.02	0.00	−0.01	−0.01	
	Test statistics for restrictions on β'					
Variable	$m-p$	Δp	y	$R3$	Rra	Joint
$\chi^2(\cdot)$	—	—	3.6	3.8	—	3.9
p-value			[0.059]	[0.051]		[0.145]
	Test statistics for zero restrictions on α					
Variable	$m-p$	Δp	y	$R3$	Rra	Joint
$\chi^2(\cdot)$	26.1	7.4	0.3	4.7	1.6	10.4
p-value	[0.000]	[0.006]	[0.591]	[0.030]	[0.203]	[0.034]

Table C4.
A Cointegration Analysis of NSA Data: $\{m-p,\ \Delta p,\ y,\ R^*\}$

Eigenvalues	0.345	0.121	0.049	0.015	
Hypotheses	$r=0$	$r\le 1$	$r\le 2$	$r\le 3$	
λ_{max}	42.4	12.9	5.0	1.5	
λ^a_{max}	33.9	10.3	4.0	1.2	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	61.7	19.3	6.5	1.5	
λ^a_{trace}	49.4	15.5	5.2	1.2	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	$m-p$	Δp	y	R^*	
	1	6.46	−0.99	6.76	
	−0.05	1	−0.04	−0.46	
	−0.89	16.33	1	−4.95	
	−1.53	−4.82	−0.18	1	
	Standardized adjustment coefficients α				
$m-p$	−0.18	−0.08	−0.00	0.00	
Δp	0.03	−0.09	−0.00	−0.00	
y	−0.00	0.34	−0.01	−0.00	
R^*	0.03	0.17	0.00	−0.01	
	Test statistics for restrictions on β'				
Variable	$m-p$	Δp	y	R^*	Joint
$\chi^2(\cdot)$	—	—	0.0	—	—
p-value			[0.917]		
	Test statistics for zero restrictions on α				
Variable	$m-p$	Δp	y	R^*	Joint
$\chi^2(\cdot)$	28.6	5.0	0.0	1.3	5.6
p-value	[0.000]	[0.025]	[0.928]	[0.260]	[0.130]

Tables C1–C4 also list the standardized estimated eigenvectors (β') and weighting matrices (α) for the four systems, where α and β' include vectors corresponding to (postulated) zero as well as nonzero eigenvalues. Coefficients of the (first) cointegrating vector are similar across systems, noting the different treatments of prices and interest rates. A unit long-run homogeneity restriction on prices appears satisfied when tested (Systems I and II), as does the restriction of "opposite sign, equal magnitude" on the interest rate coefficients (Systems I and III). A unit long-run homogeneity restriction on income appears less clear-cut in System I, although the test would not be interpretable as a homogeneity restriction if that system had two cointegrating vectors. In all systems, the joint test of unit long-run price and income homogeneities and/or the interest rate restriction (as appropriate) is always satisfied.

The weighting coefficients for the first cointegrating vector are similar across systems, being approximately -0.15 in the equation for money and virtually zero in the other equations.[14] Those zeros are necessary for prices, incomes, and interest rates to be weakly exogenous for the parameters in the money equation; cf. Johansen (1992a, 1992c). However, statistically, the test of those zero restrictions is rejected for Systems I–III, primarily due to a statistically significant (but numerically small) coefficient for the price equation. This slight lack of weak exogeneity does not appear to affect inferences in single-equation modeling (Section 5.1). Note that the "joint" tests of zero restrictions on α do *not* include the zero restriction on the equation determining money. That latter restriction is resoundingly rejected for all systems, so money (whether real or nominal) can not be assumed weakly exogenous in an equation determining prices, inflation, income, or either interest rate.

C.2 System Cointegration Analysis of the Adjusted Data

Tables C5–C8 reproduce and add to Ericsson, Campos, and Tran's (1990) SA results, which are for Systems I–IV with (m^a, p^a, y^a) replacing (m, p, y) and where a constant term (but no seasonal dummy) is included in the VARs. Thus, Tables C5–C8 (SA data) parallel Tables C1–C4 (NSA data). The eigenvalues and test statistics in Tables C5–C8 are strikingly similar to those in Tables C1–C4, as implied by the theoretical analysis in Section 2. Likewise, the estimated cointegrating vectors in Table C5–C8 are close to those in Table C1–C4. Surprisingly, the estimated weighting matrices in Tables C1–C4 and C5–C8 are nearly identical. In general, they need not be, although they would be if, for all the SA filters, f_0 [in (2)] were approximately unity and the other f_i's were relatively small.

Figures C1–C4 plot for the four systems the estimated disequilibria $\beta' x_t$ and $\beta' x_t^a$, where β is numerically different for x_t and x_t^a. The choice of system and of data type matter little for the properties of the estimated disequilibrium.

[14]Entries have been rounded relative to PcFiml's output, with the sign of the estimate retained even if the rounded value is zero.

Table C5.
A Cointegration Analysis of SA Data: $\{m^a,\ p^a,\ y^a,\ R3,\ Rra\}$

Eigenvalues	0.406	0.246	0.166	0.097	0.005	
Hypotheses	$r=0$	$r\leq 1$	$r\leq 2$	$r\leq 3$	$r\leq 4$	
λ_{max}	52.1	28.3	18.2	10.2	0.5	
λ^a_{max}	39.1	21.2	13.6	7.7	0.4	
95% critical value	33.5	27.1	21.0	14.1	3.8	
λ_{trace}	109.3	57.2	28.9	10.7	0.5	
λ^a_{trace}	81.9	42.9	21.7	8.0	0.4	
95% critical value	68.5	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'					
Variable	m^a	p^a	y^a	$R3$	Rra	
	1	−1.00	−0.77	5.80	−7.77	
	−1.81	1	1.08	11.31	15.50	
	−0.08	−0.12	1	0.03	−1.26	
	0.33	−0.21	−0.30	1	−2.18	
	−0.68	0.59	0.45	−0.41	1	
	Standardized adjustment coefficients α					
m^a	−0.19	0.00	0.07	0.02	0.00	
p^a	0.04	0.00	0.08	0.00	−0.00	
y^a	0.01	−0.01	0.02	−0.10	0.00	
$R3$	0.06	−0.02	0.04	−0.01	−0.01	
Rra	0.01	−0.01	0.01	0.02	0.00	
	Test statistics for restrictions on β'					
Variable	m^a	p^a	y^a	$R3$	Rra	Joint
$\chi^2(\cdot)$	—	0.0	1.0	1.6	—	2.4
p-value		[0.984]	[0.317]	[0.200]		[0.487]
	Test statistics for zero restrictions on α					
Variable	m^a	p^a	y^a	$R3$	Rra	Joint
$\chi^2(\cdot)$	23.7	5.4	0.2	4.1	1.5	8.0
p-value	[0.000]	[0.021]	[0.686]	[0.044]	[0.213]	[0.091]

Table C6.
A Cointegration Analysis of SA Data: $\{m^a,\, p^a,\, y^a,\, R^*\}$

Eigenvalues	0.375	0.166	0.115	0.002	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	
λ_{max}	47.0	18.2	12.2	0.2	
λ^a_{max}	37.6	14.5	9.8	0.1	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	77.6	30.5	12.4	0.2	
λ^a_{trace}	62.0	24.4	9.9	0.1	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	m^a	p^a	y^a	R^*	
	1	–1.04	–0.95	7.46	
	1.21	1	–9.80	–8.11	
	–0.61	0.35	1	–3.06	
	1.17	–1.05	–0.70	1	
	Standardized adjustment coefficients α				
m^a	–0.15	–0.01	–0.01	–0.00	
p^a	0.03	–0.01	0.01	0.00	
y^a	0.01	0.00	0.05	–0.00	
R^*	0.03	0.00	0.03	0.00	
	Test statistics for restrictions on β'				
Variable	m^a	p^a	y^a	R^*	Joint
$\chi^2(\cdot)$	—	0.3	0.0	—	1.0
p-value		[0.610]	[0.855]		[0.597]
	Test statistics for zero restrictions on α				
Variable	m^a	p^a	y^a	R^*	Joint
$\chi^2(\cdot)$	24.8	4.2	0.1	1.9	4.8
p-value	[0.000]	[0.040]	[0.737]	[0.169]	[0.184]

Table C7.
A Cointegration Analysis of SA Data: $\{m^a - p^a,\ \Delta p^a,\ y^a,\ R3,\ Rra\}$

Eigenvalues	0.417	0.226	0.112	0.050	0.022	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$	
λ_{max}	53.9	25.6	11.8	5.1	2.3	
λ^a_{max}	40.4	19.2	8.9	3.9	1.7	
95% critical value	33.5	27.1	21.0	14.1	3.8	
λ_{trace}	98.8	44.8	19.2	7.4	2.3	
λ^a_{trace}	74.1	33.6	14.4	5.5	1.7	
95% critical value	68.5	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'					
Variable	$m^a - p^a$	Δp^a	y^a	$R3$	Rra	
	1	5.67	−0.77	5.82	−7.72	
	−0.26	1	−0.47	1.98	3.40	
	1.19	−12.87	1	6.24	−11.76	
	0.19	−1.85	−0.16	1	−0.65	
	1.53	13.11	−0.04	−0.23	1	
	Standardized adjustment coefficients α					
$m^a - p^a$	−0.22	0.00	0.00	−0.01	−0.00	
Δp^a	0.04	−0.02	0.01	0.03	0.00	
y^a	0.00	−0.07	−0.02	0.04	−0.00	
$R3$	0.07	−0.11	−0.00	−0.03	0.00	
Rra	0.01	−0.02	0.00	−0.01	−0.00	
	Test statistics for restrictions on β'					
Variable	$m^a - p^a$	Δp^a	y^a	$R3$	Rra	Joint
$\chi^2(\cdot)$	—	—	1.4	2.1	—	2.3
p-value			[0.239]	[0.148]		[0.317]
	Test statistics for zero restrictions on α					
Variable	$m^a - p^a$	Δp^a	y^a	$R3$	Rra	Joint
$\chi^2(\cdot)$	28.3	5.8	0.0	4.9	1.7	7.9
p-value	[0.000]	[0.016]	[0.890]	[0.027]	[0.189]	[0.095]

Table C8.
A Cointegration Analysis of SA Data: $\{m^a - p^a,\ \Delta p^a,\ y^a,\ R^*\}$

Eigenvalues	0.386	0.128	0.050	0.009	
Hypotheses	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	
λ_{max}	48.8	13.7	5.1	0.9	
λ^a_{max}	39.0	11.0	4.1	0.8	
95% critical value	27.1	21.0	14.1	3.8	
λ_{trace}	68.6	19.8	6.1	0.9	
λ^a_{trace}	54.9	15.9	4.9	0.8	
95% critical value	47.2	29.7	15.4	3.8	
	Standardized eigenvectors β'				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	
	1	7.22	−1.08	7.16	
	−0.08	1	−0.04	−0.79	
	−1.26	16.03	1	−7.00	
	1.33	6.58	−0.12	1	
	Standardized adjustment coefficients α				
$m^a - p^a$	−0.18	−0.03	0.00	−0.00	
Δp^a	0.02	−0.05	−0.00	0.00	
y^a	−0.00	0.23	−0.01	−0.00	
R^*	0.03	0.14	0.00	0.01	
	Test statistics for restrictions on β'				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	Joint
$\chi^2(\cdot)$	—	—	0.8	—	—
p-value			[0.380]		
	Test statistics for zero restrictions on α				
Variable	$m^a - p^a$	Δp^a	y^a	R^*	Joint
$\chi^2(\cdot)$	34.6	3.4	0.0	2.3	4.5
p-value	[0.000]	[0.065]	[0.996]	[0.133]	[0.215]

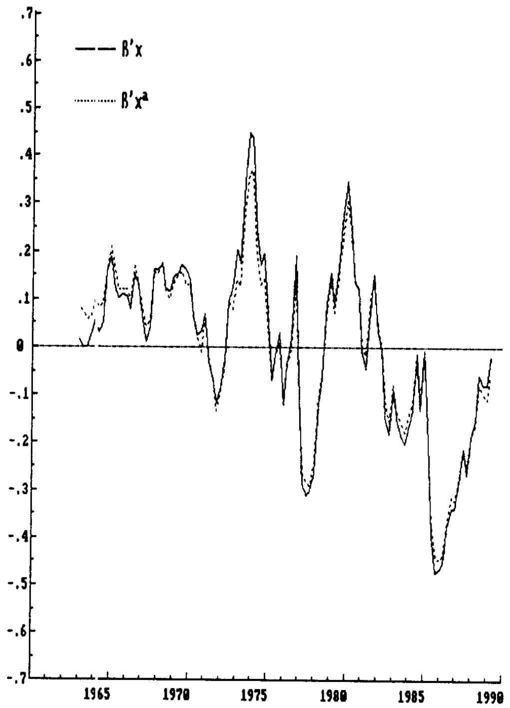

Figure C1. The NSA and SA disequilibrium measures $\beta' x_t$ and $\beta' x_t^a$ for System I.

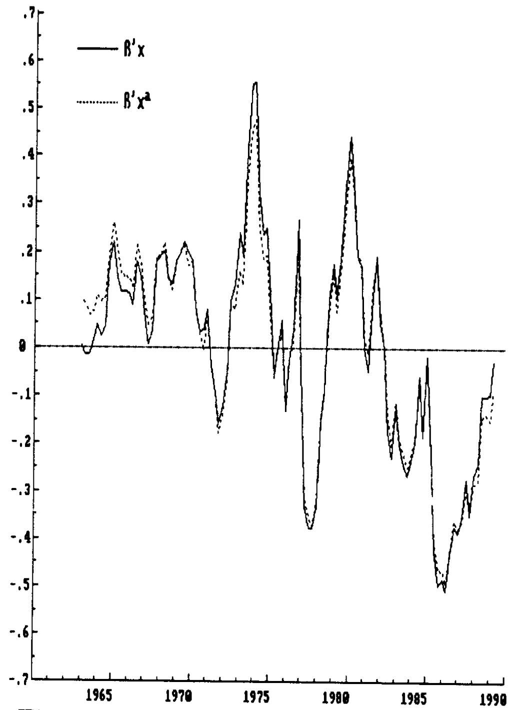

Figure C2. The NSA and SA disequilibrium measures $\beta' x_t$ and $\beta' x_t^a$ for System II.

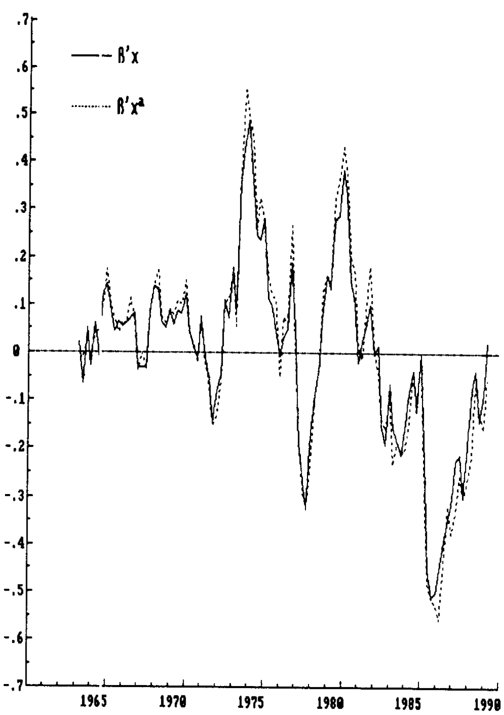

Figure C3. The NSA and SA disequilibrium measures $\beta' x_t$ and $\beta' x_t^a$ for System III.

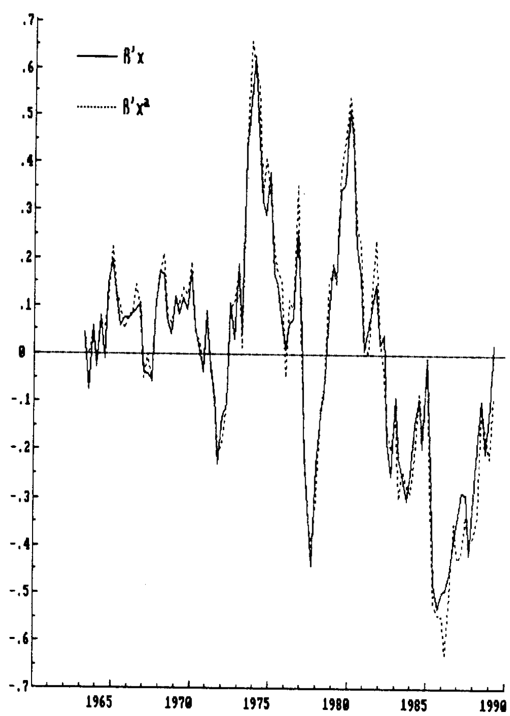

Figure C4. The NSA and SA disequilibrium measures $\beta' x_t$ and $\beta' x_t^a$ for System IV.

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