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A SUBSTITUTE FOR THE CAPITAL STOCK VARIABLE IN INVESTMENT FUNCTIONS

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ABSTRACT

Capital stock variables appearing in investment and other equations are almost always constructed by the "perpetual inventory method." Successive values are related by the well-known equation:

$$K(t) = I(t) + (1-\delta)K(t-1),$$

where K(t) is the measure of the real capital stock at time t, I(t) is the real rate of investment, and δ the rate of depreciation. By successive backward substitutions for K(t-1), K(t) can be expressed equivalently as a weighted sum of past levels of investment plus the depreciated value of an initial real capital stock:

$$K(t) = \sum_{i=0}^{t-1} [I(t-i)(1-\delta)^{i}] + K(0)(1-\delta)^{t}.$$

The initial real capital stock, K(0), that is implicitly a component of every measure of the capital stock calculated by this method can rarely be measured, however, with any degree of accuracy. As demonstrated in this paper, the measurement error can frequently lead to severe bias in the estimated coefficients of investment functions.

This paper proposes a method to bypass this source of measurement error. In important cases it is then possible to estimate unbiased and consistent coefficients.

A SUBSTITUTE FOR THE CAPITAL STOCK VARIABLE IN INVESTMENT FUNCTIONS

Guy V.G. Stevens[1]

An empirical investment function typically contains a real capital stock variable as one of its regressors.[2] The coefficient of this variable is a function of the rate of depreciation and, sometimes, the degree of lagged adjustment. Measurement of the capital stock is, however, beset with difficulties and, as illustrated below, the associated errors-in-variables problem can lead to estimated coefficients that are seriously biased. This note proposes a method to eliminate the estimation bias caused by one of the key sources of measurement error: the error introduced into the capital stock series by the (usually) inaccurate estimate of the **initial** real capital stock.

Virtually all measures of the capital stock appearing in equations explaining fixed investment are constructed by the use of some form of the "perpetual inventory method." Successive values are related by the well-known equation:

$$K(t) = I(t) + (1-\delta)K(t-1), \tag{1}$$

where K(t) is the measure of the real capital stock at time t, I(t) is the real rate of investment at t, and δ is the (constant) rate of depreciation

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1. Senior Economist, International Finance Division, Board of Governors of the Federal Reserve System. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors or other members of its staff. I am grateful to Sean Doyle, Neil Ericsson, Anil Kashyap, P.A.V.B. Swamy, and Charles Thomas for helpful comments and suggestions, and to Jodi Garner for dedicated research assistance.
2. See, for example, Abel and Blanchard (1986), Clark (1979) and Hall (1987). Although I have not studied production functions extensively, the findings of this paper would presumably apply to the estimation of these functions as well.

of the capital stock; δ need not be a constant, but almost always is assumed to be. To construct a capital stock series, the analyst usually starts at some initial period zero with a measure of the initial capital stock, K(0), and then substitutes the depreciation rate and the elements of an investment series into equation (1) to calculate successive values of K(t).

By successive backward substitution for K(t-1) in equation (1), we can relate K(t) directly to the initial value for the capital stock, K(0). K(t) becomes a weighted sum of all past levels of investment and the depreciated value of the initial real capital stock:

$$K(t) = \sum_{i=0}^{t-1} [I(t-i)(1-\delta)^{i}] + K(0)(1-\delta)^{t}. \qquad (2)$$

Measurement error may be introduced into the capital stock series through any of the three components of equation (2): the I(t) series, δ, or K(0). This note has nothing to say about avoiding errors in the choice of the first two, but it does argue that the estimation bias introduced by a poor choice of an initial value for the real capital stock can, in many cases, be eliminated entirely.

Various methods have been used to estimate this initial capital stock, but virtually all researchers acknowledge that the starting values are subject to large errors.[3] As a result, where at all possible, they

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3. Where a long time series for real investment is available, researchers, for reasons explained below, have often implicitly set the initial value of the capital stock equal to zero -- by ignoring it. See, e.g., Bischoff (1971) and Hall and Jorgenson (1971). However, researchers using disaggregated or microeconomic data are usually forced to estimate the initial capital stock. Jorgenson and Siebert (1968), using firm-level (Footnote continues on next page)

have chosen starting dates for the capital stock calculation 10, 20 or more years before the beginning of the estimation period for any regression work -- relying on the implication of equation (2) that the impact of K(0) on subsequent values of the capital stock decays exponentially.[4] In simulations below, I show that for a simple model this strategy can, in the limit, solve the problem of estimation bias; however, for a reasonable annual rate of depreciation such as 13 percent, a break of 20 and sometimes more years may be necessary to minimize the bias.[5] Although researchers fitting investment functions on highly aggregated U.S. data may be able to afford the luxury of ignoring 80 or more quarters

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(Footnote continued from previous page)

data, deflated a given firm's nominal book value at time zero by an industry-level price deflator constructed by the National Industrial Conference Board. Kashyap (1988) started with the nominal book value of each firm's capital stock ten years prior to the start of their estimation period. The present author, in Stevens and Lipsey (1988), when using the traditional method, was forced to start the capital stock calculation with the undeflated nominal book value only one year prior to the beginning of the estimation period.

4. In the official estimates of fixed private capital in the United States presented in Gorman, Musgrave, Silverstein, and Comins (1985), no starting value for K(0) is assumed. In the case of an aggregate series such as fixed capital in total manufacturing, which has a starting date of 1925, it was explained to me by the authors that the relevant investment series went well back into the 19th century; under such circumstances it is likely that, according to equation (2), any effect of the initial capital stock on the capital stock in 1925 would be miniscule. It might be noted, however, that some types of capital such as buildings and railroads have useful lives of 50 years or more, necessitating an accurate real investment series for at least that long to eliminate all bias from ignoring the initial capital stock.

Bischoff (1971), explaining aggregate U.S. equipment expenditures and using an estimation period that began in 1951, started his capital stock calculation as early as 1909. A similar procedure was followed by Hall and Jorgenson (1971) for aggregate U.S. manufacturing equipment and structures.

5. This is not surprising when one notes that, at an exponential rate of depreciation of 13 percent, over 25 percent of the initial capital stock remains after 10 years, 6 percent after 20 years. As far as the realism of the 13 percent rate of depreciation is concerned, Hall and Jorgenson (1971), for example, use a depreciation rate of 14.7 percent for equipment and 6.3 percent for structures.

of data, studies using disaggregated industry data or microeconomic, firm-level data typically find it impossible to discard such a large number of observations.[6]

I. An Error-Free Substitute for the Capital Stock

As noted, equation (2) illustrates that for an error-free measure of the capital stock, three error-free elements are required. However, for the purpose of avoiding the estimation bias introduced by errors in measuring the capital stock, only two such elements are needed: accurate measures of the real investment series and the depreciation rate.

Suppose we are interested in estimating the coefficient of the capital stock variable, but that no adequate measure of K(0) is available. Equation (2) can be used to replace K(t) by the **two** independent variables, $(1-\delta)^t$ and the weighted sum of past investment rates -- thereby avoiding the errors-in-variables problem associated with the use of K(t). The estimated coefficients from the modified regression will share most, if not all, of the desirable properties of the coefficients estimated with a K(t) that is measured completely without error.

For concreteness, consider the simple linear investment function:

$$I(t) = \alpha + \beta Q(t) + \gamma K(t) + \epsilon, \tag{3}$$

where Q(t) is real output, I(t) and K(t) are investment and the capital stock as defined above, α, β, and γ are the unobserved constant coefficients, and ϵ is an independently distributed random error. If the

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6. See the relevant references cited in footnote 3.

regressors are measured without error, assuming the standard properties for ϵ, the least squares estimators of α, β, and γ will be unbiased and consistent. However, as discussed above, the measurement of the capital stock without error is unlikely in many situations.

Despite this potential errors-in-variables problem, equation (2) implies that, insofar as measurement errors in the capital stock are caused by inaccuracies in the estimates of the initial capital stock, K(0), one can still obtain unbiased estimates of α, β, and γ. Let us substitute the alternative definition of the capitial stock from equation (2) for K(t) in equation (3); denoting the weighted sum of past investment rates in (2) by WS(t), we get the first line of equation (4) below. This, in turn, can be rewritten in the second line of (4) as a linear regression of I(t) on the regressors WS(t), $(1-\delta)^t$, and Q(t), with constant coefficients α, β, γ, and γK(0).

$$\begin{aligned} I(t) &= \alpha + \beta Q(t) + \gamma[WS(t) + K(0)(1-\delta)^t] + \epsilon \\ &= \alpha + \beta Q(t) + \gamma WS(t) + \gamma K(0)(1-\delta)^t + \epsilon. \end{aligned} \tag{4}$$

It should be noted that the last coefficient in equation (4), γK(0), is a constant like the other three -- the product of γ and the constant, initial capital stock, K(0). In this transformed version of the investment function, there is no measurement error in any of the three regressors, WS(t), $(1-\delta)^t$, and Q(t). The other desirable properties possessed by the error term and regressors in equation (3) carry over to equation (4), so that least squares applied to (4) will lead to unbiased estimators for α, β, γ, and the product γK(0).

The large-sample property of consistency is a more complicated issue because the variable $(1-\delta)^t$ approaches zero as t approaches infinity. However, following the approach of Swamy and Rao (1972), one can prove that the least squares or maximum liklihood estimators for α, β, and γ will be consistent, although the estimator for $\gamma K(0)$ will not necessarily be.

II. An Example Using Alternative Measures of the Capital Stock

In this section I present comparisons of the effects of a number of (usually erroneous) measures of the real capital stock on the estimated coefficients of a simple investment model, one constructed such that the true coefficients are known. The alternative measures of the capital stock differ only by their initial values, $K_i(0)$; the correct value for $K(0)$ is postulated to be 100. Alternative, biased measures are indexed by their initial values; thus, in the text and Table 1, the measure with a starting value of 75, 25 percent below the true value, is denoted by $K_{75}(t)$.

Investment is related to expected output, Q(t), and the lagged capital stock by the simplest of stock-adjustment equations (whose adjustment coefficient is, by assumption, 0.5):

$$I(t) = 0.5[Q(t) - K(t-1)] + 0.13K(t-1), \tag{5}$$

where K(t-1) refers, of course, to the correct capital stock, whose initial value is 100, and where the last term represents replacement investment. Implicit in the above equation are the assumptions that the desired capital/output ratio is 1.0 and the depreciation rate is 0.13.

For a hint of realism, the primary output series used in the regressions equals annual U.S. GNP for the 69 year period, starting in 1919.[7] Given an output series and the initial value of the real capital stock (100), the corresponding investment series is manufactured to make the "true" investment function, equation (5), fit the data perfectly. Thus, in the table below, all deviations from an R^2 of 1.0, estimated coefficients of +0.5 for output, -0.37 for the capital stock, and 0.0 for the constant term are the result of the error in the measurement of the initial capital stock. The regression which contains the proposed substitute measures for the capital stock term should show a coefficient of -0.37 for the variable representing the weighted sum of past investment (WS) and -37.0 (i.e., -0.37•100) for the power of the depreciation rate. From these two coefficients one can extract an estimate of K(0), although it will not generally be unbiased.

Regression results are presented in Table 1 for three different sample periods, all of which are 20 years in length. They differ only in their starting dates: the first, starting in period 1 (1920 for the output series, one year after the start of the constructed capital stock series), uses capital stock measures with the largest degree of measurement error; the second sample period starts eleven years after the start of the capital stock series (years 11-39); and the third starts an additional ten years later (years 21-40). For each sample period a set of regression coefficients is presented for five alternative capital stock measures: K_0

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7. For GNP in constant dollars after 1947 see, **Economic Report of the President, January 1989** (1989), Table B-11, p. 321. For the data from 1919 to 1947, see U.S. Bureau of the Census (1960), Series **F 1-5**, p. 139.
For purposes of comparison, all regressions were also run with an alternative output series which consisted of a ten-year uniform cycle imposed on a trend rate of growth of 3 percent (the postwar average). The results were very similar to those reported in the table.

Table 1: Investment Regressions Using Alternative Measures of the Capital Stock Fitted Over Three Different Sample Periods

Variables	1 - 20					11 - 30					21 - 40				
	K_0	K_{75}	K_{105}	K_{500}	Alt	K_0	K_{75}	K_{105}	K_{500}	Alt	K_0	K_{75}	K_{105}	K_{500}	Alt
Q_t	.429	.490	.491	.348	.500	.545	.514	.497	.270	.500	.499	.500	.500	.492	.500
	(9.2)	(18.2)	(63.1)	(6.3)	(6.9E6)	(26.7)	(114)	(588)	(12.8)	(6.2E6)	(146)	(570)	(2824)	(30.9)	(1.1E6)
K_{t-1}	-.089	-.245	-.390	.017		-.374	-.374	-.369	-.204		-.358	-.367	-.371	-.413	
	(4.41)	(10.4)	(37.4)	(2.0)		(18.3)	(76.7)	(390)	(6.0)		(114)	(446)	(2208)	(24.1)	
WS_{t-1}					-.370					-.370					-.370
					(3.6E6)					(5.2E6)					(6.6E5)
$(1-\delta)^t$					-37.0					-37.0					-37.0
					(2.9E6)					(1.2E6)					(3.2E4)
C	-27.7	-15.8	4.04	-29.5	.000	-10.5	-2.56	.500	19.9	.000	-3.6	-.890	.180	16.3	.000
	(5.7)	(6.8)	(4.5)	(3.6)	(2.9)	(9.0)	(9.0)	(8.6)	(3.7)	(.39)	(12.4)	(12.8)	(12.9)	(12.8)	(2.7)
R^2	.839	.953	.996	.716	1.00	.986	.999	1.00	.909	1.000	.999	1.00	1.00	.987	1.000
SER	2.49	1.35	.401	3.31	.000	1.46	.357	.070	3.77	.000	.213	.055	.011	.998	.000

Sample Periods

(where the initial capital stock is assumed to be zero), K_{75}, K_{105}, K_{500} (where the initial capital stock is erroneously assumed to be five times the true value), and my suggested alternative (labeled ALT in the table).

The first result to notice is that the regression using the alternative measure of the capital stock (ALT), where no assumption is made about the starting value, fits the data perfectly for each choice of time period and output series.[8]

Moving from left to right across the table, one sees the effects of widening the gap between the starting date of the capital stock series and the beginning of the estimation period. As expected, for a given capital stock series (all of which have erroneous starting values), the results, in terms of R^2 and the degree of coefficient bias, improve monotonically from left to right as early observations, with their larger errors, are dropped from the sample.[9] However, even with a gap of 20 years between the start of the capital stock calculation and the estimation period, substantial errors can show up. Thus, as shown in the right-hand panel, for the greatly overestimated initial capital stock, K_{500}, the estimated capital stock coefficient (-.413) still shows a bias of 11 percent. For the regression in the same panel using K_0, the capital

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8. The perfect fit of the regression using the alternative measure is a by-product of the fact that the investment series was constructed such that a regression using the true values for the regressors fits the data perfectly (i.e. by construction, ϵ is zero). Thus, anything less than a perfect fit is due to measurement error. Since the alternative measure for the capital stock introduces no measurement error, in this particular case a perfect fit can be assured by selecting the true coefficients, α, β, and γ, as the estimated coefficients.

9. One apparent anomaly in the table is the increase in the standard error of the residual (SER) for the K_{500} regression as one moves from the left-hand panel to the middle panel. The multiple correlation coefficient rises considerably as expected, so this result may just be attributable to the higher variance of the dependent variable in this period.

stock series reflecting the typical practice of ignoring the initial capital stock, the bias in the same capital stock coefficient is still 3 percent -- small, but in view of the t ratio of 114, statistically significant.

Where there is no gap between the start of the capital stock calculation and the beginning of the estimation period, there are large biases in most of the estimated coefficients. Thus, in the left-hand panel, the **only** regression besides the suggested alternative that could be said to have small coefficient errors is that for K_{105}, where the initial error was only 5 percent. For K_{75} (showing a moderate initial error of 25 percent), besides the bias introduced into the constant term, the coefficient of the capital stock is underestimated by 51 percent (-.245 compared with -.370). Within the panel, the errors in the estimated coefficients increase monotonically as one moves away from the correct capital stock; for K_0 and K_{500} the estimated coefficients bear little resemblance to the true values.

Moving to the middle panel, where 10 years of data are dropped from the beginning of the previous estimation period, the results improve considerably for the regressions using the capital stock measures most in error. A small bias of 3 percent remains in the estimate of the capital stock coefficient for the K_{75} regression, along with a somewhat larger one of 9 percent in the K_0 regression. The coefficients of the K_{500} regression are still far from the mark. Thus it is clear that a ten-year gap is not sufficient to guarantee the elimination of the bias introduced by the error in the initial capital stock.

III. Conclusions

This note attempts to demonstrate the attractive theoretical and empirical properties of a substitute for traditionally-estimated capital stock variables used in investment and other regressions. In particular, a linear combination of a weighted sum of past investment rates and the variable $(1-\delta)^t$ is perfectly correlated with K(t) and requires no assumption about the initial value of the capital stock [K(0)]. The use of the recommended alternative avoids the estimation bias introduced by erroneous estimates of K(0) and allows the use of longer time series in estimation; it is no longer necessary to start the calculation of the capital stock series twenty or more years prior to the beginning of the estimation period.

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