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MACROECONOMIC RISK AND ASSET PRICING: ESTIMATING THE APT WITH OBSERVABLE FACTORS

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ABSTRACT

This paper develops and applies a new maximum likelihood method for estimating the Arbitrage Pricing Theory (APT) model with observable risk factors. The approach involves simultaneous estimation of the factor loadings and risk premiums and can be appled to return panels with more securities than time series observations per security. Observable economic factors are found to account for 25 to 40 percent of the covariation in U.S. equity returns, and the APT pricing restrictions cannot be rejected for most sample periods. A significant "firm size anomaly" is measured, but it may be partly due to sample selection bias.

Macroeconomic Risk and Asset Pricing: Estimating the APT with Observable Factors

John Ammer[0]

1. Introduction

With models like CAPM and the APT, the finance profession has made substantial progress in determining how the means of asset returns should be related, taking their variances and covariances as given. We have been somewhat less successful at explaining the sources of risk at the root of those variances and covariances. One strand of the literature has attempted to relate asset return innovations to news about future variables.[1]

The approach taken in this paper has its earliest antecedents in Chen, Roll, and Ross (1986). These authors treat news about the economy as observable risk in the context of a factor model. Burmeister and McElroy (1988) go further in

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[0] The author is a staff economist in the International Finance Division of the Board of Governors of the Federal Reserve System. Opinions expressed herein do not necessarily concur with those of the Federal Reserve Board or any other employees of the Federal Reserve System. I would like to thank Jianping Mei and workshop participants at the Federal Reserve Board and Princeton University for helpful conversations, and Jianping Mei, Tina Sun, and Chris Turner for assistance in obtaining data. However, I made all of the errors.

[1] See Fama (1990), Campbell (1990), Campbell and Ammer (1991), and Campbell and Mei (1991).

this direction, devising a means for estimating the Arbitrage Pricing Theory (APT) model with observable risk factors. This paper makes further technical progress in a new estimation method for the APT with observable factors which can handle a large enough amount of data to do justice to Ross' (1976) concept of no *asymptotic* arbitrage opportunities. The next section of the paper briefly reviews the APT, and the third section presents a maximum likelihood estimation method for the model. The next section discusses construction of the factor space and preliminary results of an application to U.S. equity return data. The fifth section develops improvements in measuring the observable factors, by incorporating revisions in expectations of future variables. The following section presents estimates and the results of some hypothesis tests, and the subsequent one undertakes a brief investigation of the firm size anomaly. The eighth section concludes the paper.

2. The APT Model

Assume that excess returns (over the risk-free rate) on assets are generated by a linear factor model:

$$z_{i,t} = \mu_i + \sum_{j=1}^{K} b_{i,j} f_{j,t} + w_{i,t} \tag{1}$$

for i=1, n and t=1, T

where $E(f_j) = 0$, $E(w_i) = 0$, $E(f_j w_i) = 0$, and $E(w_j w_i) = 0$.

The absence of asymptotic arbitrage opportunities (Ross 1976) requires that for some vector λ:

$$\mu_i = \sum_{j=1}^{K} b_{i,j} \lambda_j \tag{2}$$

where the λ are factor prices (risk premiums). With this restriction, (1) can be rewritten:

$$z_{i,t} = \sum_{j=1}^{K} b_{i,j} \left(\lambda_j + f_{j,t} \right) + w_{i,t} \tag{3}$$

or in matrix form:

$$Z = \iota \Lambda' B + FB + W \tag{4}$$

where F is a matrix of (zero mean) random risk factors, B is a matrix of factor loadings,[2] Λ is a vector of factor prices, ι is a vector of ones, and the residuals W_t are independently and identically distributed with zero mean and diagonal covariance matrix Ω.

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[2] More general specifications of the APT allow the factor loadings to vary over time, for the factors and residuals to be heteroscedastic, and for the residuals to have non-zero covariances. However, the more restrictive version of the model we use here is common in the APT literature.

3. Estimation with Observable Factors

If one is to evaluate the importance of particular risk factors in the APT model, it is essential that the estimation method employed have three properties. First, it should be able to accommodate observable factors as inputs. In addition, one would like to be able to obtain consistent estimates and standard errors for the factor prices. A third property is also important if any asymptotic hypothesis testing will be done: the method should be capable of handling a large amount of data; in particular it should not require there to be more return observations per security than there are securities in the model.

All of the estimation methods in the published literature on the APT fail to simultaneously satisfy all three of the criteria listed above. The techniques of Ross and Roll (1980) and Mei (1990), treat the factors as unobserved and cannot extract them. The methods presented in Lehmann and Modest (1988), Connor and Korajczyk (1988), and Mei (1991) infer the risk factors (F) from the covariance structure of returns. The factor estimates are returns on particular portfolios (with the means subtracted), and the means of these returns are consistent estimates of the risk premiums (Λ). Unfortunately, with these methods, an additional step would be required to relate an observable risk factor, such as inflation risk, to the extracted factor space. Without such further analysis, there is little that one would be able to say about the nature of the risk that is priced.

Methods which use observable factors directly are potentially more appealing. Chen, Roll, and Ross (1986) apply the two step "cross-sectional regression" procedure of Fama and MacBeth (1973) to a panel of twenty size-sorted portfolios.[3] The first step is to obtain estimates of the factor loadings (B) by applying ordinary least squares to (1) with an unrestricted intercept (μ_i) for each security. Next, for each time period, the cross-section of security excess returns is regressed on the estimated factor loadings, to obtain estimates of the factor prices (Λ). Unfortunately, this technique suffers from an errors-in-variables problem in the second stage regression, which in general causes the precision of the estimates of the risk premiums (Λ) to be overstated.[4] In addition, there is no means for imposing the model restrictions when estimating the factor loadings with this method, so that it cannot truly be deemed a procedure for estimating the APT.

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[3] It is not clear why Chen, Roll, and Ross use so few assets, since their methodology is not constrained in this dimension. Statistical power is lost by bundling assets into portfolios instead of allowing them to enter the estimation individually.

[4] See Shanken (1992).

The APT estimation methods of Burmeister and McElroy (1988) and King, Sentana, and Wadhwani (1990) can handle both observed and unobserved factors, but both techniques require that there to be more return observations per security than there are securities in the model. Since the time series dimension of applications involving observable factors tends to be severely limited by the availability of macroeconomic data, this shortcoming can be quite constraining.

The estimation method we use is the first to satisfy all three of our criteria above. Note that if all risk factors are observed, and a parameterization is chosen for the distribution of the residuals (W), equation (4) can be estimated directly by numerical methods, choosing the values of B and Λ which maximize the likelihood function.[5] we take this approach, allowing observable excess returns on well diversified portfolios to proxy for the unobservable dimensions of the factor space,[6] and assuming that the residuals are drawn from a

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[5] One negative feature of both our method and that of Burmeister and McElroy (1988), is that it requires restrictions on the covariance matrix of the residuals. The Chen, Roll, and Ross (1986) paper is not subject to this criticism.

[6] Burmeister and McElroy (1988) also use returns as proxies for latent factors. It is important to account for any unobservable risk factors that might be present in returns, because it will be assumed that the residuals will be uncorrelated across assets.

multivariate normal distribution.[7] If the returns on these portfolios have no idiosyncratic risk,[8] they can be written:

$$Z_p = \left(\iota \, \Lambda_o' + F_o\right) B_{o,p} + \left(\iota \, \Lambda_u' + F_u\right) B_{u,p} \qquad (5)$$

where ι is a T-length vector of ones, the subscripts of F distinguish observed and unobserved factors, and Z_p and F_u are assumed to have the same dimensions. If $B_{u,p}$ is nonsingular, the observable factors and diversified portfolio excess returns will jointly span the underlying factor space:

$$\begin{pmatrix} F_o' + \Lambda_o \iota' \\ F_u' + \Lambda_u \iota' \end{pmatrix} = \begin{pmatrix} I & 0 \\ B_{o,p}' & B_{u,p}' \end{pmatrix}^{-1} \begin{pmatrix} F_o' + \Lambda_o \iota' \\ Z_p' \end{pmatrix} \qquad (6)$$

In order to insure that the first matrix on the right side of (6) is invertible, one should choose portfolios for Z_p with

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[7] This does not mean that the excess returns are themselves multivariate normal, as not distributional assumption is made about the risk factors.

[8] As discussed by Burmeister and McElroy (1988), non-zero residuals in the portfolio returns proxying for the latent factors can bias the estimates of the risk premiums to the extent that the sample mean of those residuals differs from zero. In one of their applications, they attempt to avoid this problem by substituting projections of these returns onto instruments which are contemporaneous returns on assets that are neither in the portfolios or in the data panel on the right side of (4). Wanting as many securities as possible to be included in Z, we chose not to take their approach. As long as residual risk in our portfolio returns is small compared to the factor risk, the effect on our estimates of Λ should be negligible.

some distinguishing features; if they have identical factor loadings, Z_p will not be able to span a multidimensional latent factor space.

Without loss of generality, we will assume that the unobserved factors are orthogonal to the observed factor space,[9] that they are mutually orthogonal, and that they have unit variances. Under those assumptions, equation (5) implies that

$$B_{o,p} = (Var(F_o))^{-1} \; Cov(F_o, Z_p) \qquad (7)$$

and

$$B_{u,p}'B_{u,p} = Var(Z_p) - Cov(Z_p, F_o) \; B_{o,p} \qquad (8)$$

and

$$B_{u,p} = Chol(B_{u,p}'B_{u,p}) \qquad (9)$$

where Chol(·) denotes a Cholesky decomposition of a symmetric matrix into an upper triangular matrix (and its transpose).

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[9] As, in some sense, they must be to truly be "unobserved".

Maximum likelihood estimation of (4) is facilitated by the following shortcut:

$$\max_{\Lambda, B} \mathcal{L}(\Lambda,\ B,\ F,\ Z) \quad = \quad \max_{\Lambda} \left[\max_{B} \mathcal{L}(B,\ F,\ Z \mid \Lambda)\right] \tag{10}$$

Conditional on Λ, maximum likelihood estimates of B can be computed by ordinary least squares. Thus only K_o (the number of observable factors) of the Kn+K model parameters need to be involved directly in the numerical maximization of the likelihood function. For a given Λ_o, $F+\Lambda$ is computed from (6). Then the OLS residuals from (3) can be used to compute the likelihood function. Asymptotic standard errors for the Λ_o estimates[10] can be computed from the second derivatives of

$$M(\Lambda \mid F,\ Z) \ \equiv \ \max_{B} \mathcal{L}(B,\ F,\ Z \mid \Lambda) \tag{11}$$

taken with respect to Λ.

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[10] The procedure does not provide standard errors for B, but most of the hypotheses one would want to test involve only Λ.

4. Simple VAR Residuals as Factors

If financial markets are informationally efficient, asset prices should react to news about relevant economic variables. Accordingly, for the results reported in Tables 2-6, the observable factors (F_o) are the residuals from a 5-lag vector autoregression estimated from 1952 to 1990,[11] scaled to unit variance:

$$X_t = \sum_{j=1}^{5} A_j X_{t-j} + F_{o,t} \tag{12}$$

Table 1 lists the macroeconomic variables (X) in the VAR specification.

The returns panel for a given sample period consists of all NYSE and AMEX firms for which there was a complete set of monthly returns on the 1990 CRSP tapes. The sample periods, which were chosen to correspond to Connor and Korajczyk (1988) and Mei (1991), were 1979-1983, 1984-1988, 1964-1968, 1969-1973, and 1974-1978. An additional 1979-1983 sample was drawn for firms which were listed at the beginning of the period and had at least 30 monthly return observations.[12] Note

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[11] The Akaike Information Criterion was applied to choose the lag length.

[12] Allowing missing values in the returns panel substantially

that it is necessary to have at least K return observations to identify the factor loadings for a given asset.

The portfolio returns[13] used to identify the latent factors were the first ten principal components of the individual excess returns panels.[14] These principal components are excess returns on portfolios formed from the securities in the panel. Similar results were obtained when capitalization-based New York Stock Exchange (NYSE) decile portfolios were used instead.

Estimates of the model[15] for the five sample periods appear in Tables 2-6. The second column of each table contains the estimated factor prices and standard errors from the negative inverse of the estimated Hessian matrix. The third column reports the average effect on the mean of an asset return from exposure to each risk factor, which is simply the product of

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increases the computation time for each evaluation of the likelihood function because the moment matrix of the factors is no longer the same for each return in the data panel.

13 Excess returns are over the one-month treasury bill return from Ibbotson (1991).

14 This is the Connor and Korajczyk factor extraction method. The portfolio returns are the eigenvectors associated with the largest eigenvalues of ZZ'.

15 We used the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm provided in the Gauss 2.01 MAXLIK module, with numerical derivatives. Although analytical derivatives of the likelihood function are tractable, they are computationally very complicated, and some initial experimentation suggested that their use would actually increase the computation time required to achieve convergence.

the factor price (plus the sample mean realization of the factor) and the mean factor loading. By choosing an order of orthogonalization, it is possible to decompose the R^2 of each return into contributions attributable to each of the factors; these are reported in the rightmost column of the tables.

To interpret the estimated elements of Λ associated with the observable macroeconomic risk factors, it is helpful to borrow some rough intuition from intertemporal asset pricing. Generally, assets that perform well when the endowment process (or economy) is doing poorly will have some hedging value, which will compensate for a lower mean return. Accordingly, assets with returns that covary positively with good news will be poor hedges, and should pay a premium return to compensate. Thus one would expect to estimate positive risk premiums for "good news" risk factors and negative prices for "bad news" factors.

A negative price was estimated for the PROD factor in all five sample periods, which was statistically significant in all but one case, implying that higher industrial production represents bad news for the economy, and is a contingency against which it is valuable to be able to hedge. This counter-intuitive result is the opposite of what was found by Chen, Roll, and Ross (1986) over a similar sample period.

Perhaps more disturbing, in the 1984-1988 sample, the average factor loading for PROD was negative, implying that good news about industrial production was bad news for the typical U.S. equity security in the sample.

5. Constructing Forward-Looking Factors

There are two obvious problems with using VAR residuals to represent relevant economic factors. First, for the pricing of long-term assets, current innovations in macroeconomic variables may not matter as much as news about their future values. Secondly, if there is information other than lags of these variables which helps to forecast them, some portion of the estimated residuals from the VAR system, which excludes this extra information, reflects old news which should have already been incorporated into asset prices.

In the results reported in Tables 8-18, these issues are addressed by measuring the observable factors as

$$F^{*}_{o,t} = (E_t - E_{t-1}) \frac{1}{13} \sum_{j=0}^{12} X_{t+j} \tag{13}$$

where expectations are formed in the context of a VAR(5) in

$$Y \equiv \begin{pmatrix} X & Z_m \end{pmatrix} , \tag{14}$$

where Z_m is the excess return on the NYSE value-weighted market portfolio. Thus measurement of the factors will reflect information about future realizations of X which have been incorporated into the general level of U.S. equity prices.[16] By choosing to aggregate news about the economic variables for a period zero through twelve months ahead, we have arbitrarily limited (to a one-year horizon) the extent to which investors can be forward-looking.[17] It would be easy to modify the assumptions, by either changing the horizon or using a discounted sum of forecast revisions.

Table 7 presents correlations of the excess return on the NYSE equal-weighted market portfolio with both the "residuals" factors F_o and the (forward-looking) "news" factors F_o^*. These correlations are generally stronger for the "news" factors, which is a preliminary vindication of the forward-looking factor measurement method. This is in part because of the ability of ZVWM to help predict next month's macroeconomic variables (in the vector autoregression). Apparently investors have a lot of information about very short-term economic prospects.

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[16] We implicitly assume that the variables in Y are observed upon realization. This is not strictly true for data on aggregate sales, production, and consumer prices. However, Huberman and Schwert (1985) found that investors in indexed Israeli government bonds were very good at forecasting the consumer price index data releases to which the bonds were indexed.

[17] A horizon of at least one year seemed appropriate because Fama (1990) has found that market portfolio returns predict production growth up to 12 months ahead.

6. Results and Hypothesis Tests

Tables 8-12 contain estimates of (4) for all five sample periods using the "news" factors. Significantly positive risk premiums were estimated for the PROD* factor in the 1979-1983 sample, and for the SALE* factor[18] in three of the sample periods. These results suggest that investors demand a higher mean return from assets with pro-cyclical returns, because they have poor hedging value.

For three of the samples, a negative risk premium was estimated for the oil price factor. Under the logic of intertemporal asset pricing, this would be consistent with high oil prices being bad news for the economy of the United States, a net importer. Using similar methodology, Ammer (1992, chapter 3) measured a positive oil factor price for equities traded in the UK, a net oil exporter.

For four of the periods, a negative risk premium was estimated for inflation news.[19] This result suggests that

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[18] Our postive risk premium estimates for the SALE* factor in the 1974-1978 and 1979-1983 periods contrast the negative factor price estimated for a final sales factor by Burmeister and McElroy (1988) using a sample of 70 securities from 1972 to 1982.

[19] Chen, Roll, and Ross (1986) also estimated a negative inflation factor price for the period 1958-1984.

securities which are good hedges against inflation are perceived to have extra value.

Negative risk premiums were estimated for both of the yield spread factors in most of the periods. For the corporate bond quality spread, this is consistent with default risk increasing during periods of bad economic news. The negative risk premium estimated for the maturity yield curve factor suggests that relatively high long term interest rates are bad for the economy.

The contribution of the observable factors to the mean R^2 ranges from 25 to 40 percent in these samples, compared to 20 to 30 percent when the "residual" factors were used. Nevertheless, unobservables are still driving the majority of the covariation in asset returns.

Likelihood ratio (LR) tests of the APT pricing restrictions can be performed by comparing the likelihood values reported in the tables to those obtained from unrestricted ordinary least squares estimation of equation (1). We fail to reject at the five percent level in four out of five cases.

In addition, variation in factor loadings typically accounts for between 35 and 40 percent of the cross-sectional variation in returns in a given time period. Variations in the sensitivity of returns to the six observable factors usually accounts for about 10 percent of this cross-sectional variation.

Table 13 presents estimates of Λ_o which have been restricted to be the same across all five samples.[20] A likelihood ratio test leads to strong rejection of the restriction, suggesting that factor prices may not be stable over time.

All of the results discussed so far are for panels of returns with no missing values. In other words, the samples have excluded companies for which listing on the NYSE or AMEX was discontinued sometime during the period. If such firms are fundamentally different from the survivors, the results may be biased. Table 14 presents estimates for 1979-1983 for all firms with at least thirty (not necessarily consecutive) return observations. The estimated factor prices are very close to the results reported in Table 8.

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[20] Note that it would not be appropriate to similarly restrict Λ_u, because the portfolios from which the unobservable factors are constructed vary across sample periods.

Tables 15 and 16 contain results for 1979-1983 in which one of the 16 factors has been dropped from the specification reported in Table 8. In Table 15, the tenth latent factor is omitted, which has little effect on the estimated risk premiums for the observable factors. Nevertheless, a likelihood ratio test leads to strong rejection of the null hypothesis that the sixteenth factor is redundant. Table 16 reports estimates for a 15-dimensional factor space which excludes ROIL*. Because this model is nested by the 16-factor model, it appears at first blush that an LR test statistics would have an asymptotic χ^2 distribution when the restricted model is the true model. This is *not* the case. When all of the (n) loadings for a particular factor are restricted to be zero, the price of that factor becomes unidentified.[21] However, the results of Monte Carlo simulations (see Appendix A) suggest that when n is large, the 95% quantile of the statistic is fairly close to that of of the $\chi^2(n+1)$ distribution. With a test statistic of about 2744, one can confidently reject the null hypothesis of exclusion of the ROIL* factor. Yet an application of the Akaike Information Criterion, which would be more oriented to parsimonious model choice than LR tests on large numbers of restrictions, would lead to dropping the oil factor.

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[21] Note that this is a feature of the APT model in general, *not* of this estimation method. Garcia and Perron (1990) document a similar problem for testing for the number of discrete states in a regime switching model.

7. The Firm Size Anomaly

This estimation framework can also be used to test the APT against specific alternatives which nest it. Some of the interesting alternatives are anomalies against which the older CAPM model was rejected, such as the "small firm effect", under which the equities of firms with lower ex ante market capitalizations have higher mean returns than larger firms, even after adjusting for market risk.[22] One could test for a security-specific effect by augmenting the APT equation (4) to

$$Z = \iota A S' + (\iota \Lambda' + F) B + W \tag{15}$$

where S is a vector of security characteristics and A is a scalar to be estimated.

Under this alternative model, the excess returns on well diversified portfolios would be

$$Z_p = \iota A S_p' + \left(\iota \Lambda_o' + F_o\right) B_{o,p} + \left(\iota \Lambda_u' + F_u\right) B_{u,p} \tag{16}$$

where $S_p = G'S$ and G is the matrix of portfolio weights.

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[22] This was first discovered by Banz (1981).

If G is known, estimation can proceed as above. However, when the portfolios are principal components of the return panel (Z), extracting the portfolio weights can be computationally expensive.[23] For the application presented here, we chose the alternative of estimating S_p (along with A and Λ_o) by allowing it to enter as parameters to the numerical likelihood maximization.

Table 17 presents estimates of (15) in which S is the difference between the firm's log capitalization at the beginning of the sample (January 1, 1979) and the mean log capitalization at that time for all firms in the sample. A significant size effect is measured here, but it could be due to sample selection bias. Smaller firms are much more likely to drop out of the sample (from ceasing to be listed on the exchange). If, as seems reasonable, the casualties tend to be securities that have "performed" poorly, the ex post mean returns for survivors will overstate their ex ante means.

The sample selection bias problem is reduced in the results reported in Table 18. Assets are included in the sample if they made it at least half of the way through. The estimated size effect coefficient is only slightly smaller, and

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[23] In particular, it involves computing eigenvectors of an n by n matrix.

translates to two percent being either added to or subtracted from the annual return on a typical security in the sample.

This result contrasts what was found by Chan, Chen, and Hsieh (1985), who applied the Fama and MacBeth (1973) cross-sectional regression methodology to the same data set used by Chen, Roll, and Ross (1986). In particular, they used the excess returns on twenty size-sorted portfolios as Z in (1). They found that high correlation between their portfolio of the smallest firms and a factor similar to RISK, for which they had estimated a large negative risk premium, explained the high mean return on this portfolio. However, because of the statistical power which is sacrificed by bundling securities into portfolios, it is hard to know whether the assets in this portfolio which cause its return to be correlated with the default risk spread are the same ones which cause the mean return to be high. For example, some of the firms in the sample may have equity returns which are negatively correlated with RISK simply because their debt securities are constituents of the measured BAA bond yield, while others have a high mean return for unrelated reasons.

8. Conclusions

This paper develops a new maximum likelihood method for estimating the Arbitrage Pricing Theory (APT) model with observable risk factors. The use of observable factors in the APT enables greater economic interpretation of the systematic risk which is (or is not) priced. The technique produces consistent standard errors for the factor prices, it allows one to impose (and test) the APT model pricing restrictions, and can be applied to panels of return data with more securities than time series observations per security.

The ability to handle large cross-sections appear to have allowed us to estimate the risk premiums more precisely. We estimate factor prices significantly different from zero (using a test size of five percent) more often than do Burmeister and McElroy (1988). In addition, note that the standard errors reported by Chen, Roll, and Ross (1986) overstate the precision of their estimates because they do not correct for the errors-in-variables problem that arises in their multi-stage estimation procedure. If the APT model is true, then our method, in which one can impose the APT restrictions should produce more precise estimates, as well as consistent standard errors.

Our technique is applied to several large panels of U.S. equity returns. Observable economic factors are found to account for 25 to 40 percent of the common variation in excess returns over the risk-free rate. A number of factor prices are found to be significantly different from zero, but estimates do not appear to be stable over different sample periods. The APT pricing restrictions cannot be rejected for most of the sample periods.

A significant "firm size anomaly" is measured, which appears not to be entirely due to sample selection bias.

Appendix A: Monte Carlo Distribution of LR Statistic

For each simulation, the APT was estimated with zero and one factors on 60 observations on 1797 returns, where the true model was white noise (zero factors). Likelihood ratio statistics were computed for 1000 simulations.

95% quantile of empirical distribution:	971.123
95% quantile of $\chi^2(1798)$ distribution:	948.738

χ^2 quantile of empirical critical value:	0.009
empirical quantile of χ^2 critical value:	0.186

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Table 1

Construction of Factor Space for APT Models

Observable Factors are based on the residuals from a VAR(5) :

PROD	growth in industrial production (seasonally adjusted)
ROIL	log relative (to CPI) wholesale price of oil (SA)
GCPI	Consumer Price Index (all items) inflation (SA)
SALE	growth in total real final retail sales (SA)
RISK	yield spread between BAA bonds and AAA corporate bonds
TERM	yield spread between 10-year and 3-month treasury securities

For the results reported in tables 8-18, the following variable is added to the VAR system to improve forecasting power:

ZVWM excess return (over the 1-month treasury bill) on the New York Stock Exchange Value-Weighted Market Index

In addition, the factors used there are *news* about the average values of the VAR variables for the current and next 12 months

$$F^{*}_{o,t} = (E_t - E_{t-1}) \frac{1}{13} \sum_{j=0}^{12} x_{t+j}$$

instead of the VAR residuals used for tables 2-6

$$F_{o,t} = (E_t - E_{t-1}) \; x_t \; .$$

The *Latent Factor* space is comprised by the components of excess returns (over the 1-month treasury bill) on well-diversified portfolios which are orthogonal to the observable factor space.

Table 2

Estimates of 16 Factor APT Model, 1/79-12/83

1797 returns

ln($\mathcal{L}$) = 137379.753 average R^2 = 0.551

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD	-0.052 (0.031)	-0.003	0.025
ROIL	-0.212 (0.024)	-0.005	0.015
GCPI	-0.285 (0.038)	0.010	0.023
SALE	0.026 (0.028)	0.007	0.022
RISK	0.029 (0.063)	-0.001	0.018
TERM	0.017 (0.070)	-0.002	0.014
		total:	0.116

with APT restrictions relaxed: ln($\mathcal{L}$) = 138230.339

χ^2 P-value for LR test: 0.911

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 3

Estimates of 16 Factor APT Model, 1/84-12/88

1529 returns

$\ln(\mathcal{L}) = 124137.187$ average $R^2 = 0.576$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD	-0.096 (0.024)	0.016	0.032
ROIL	-0.208 (0.054)	0.010	0.019
GCPI	0.054 (0.044)	-0.001	0.012
SALE	-0.339 (0.049)	0.026	0.020
RISK	-0.038 (0.037)	-0.004	0.013
TERM	0.051 (0.076)	0.001	0.012
		total:	0.106

with APT restrictions relaxed: $\ln(\mathcal{L}) = 124934.547$

χ^2 P-value for LR test: 0.118

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 4

Estimates of 16 Factor APT Model, 1/64-12/68

1529 returns

ln($\mathcal{L}$) = 122654.691 average R^2 = 0.506

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD	-0.064 (0.032)	0.000	0.013
ROIL	0.123 (0.006)	0.000	0.016
GCPI	0.103 (0.028)	0.000	0.019
SALE	0.434 (0.052)	-0.031	0.016
RISK	-0.079 (0.018)	0.051	0.026
TERM	-0.024 (0.009)	-0.002	0.014
		total:	0.104

with APT restrictions relaxed: ln($\mathcal{L}$) = 123324.675

χ^2 P-value for LR test: 0.999

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 5

Estimates of 16 Factor APT Model, 1/69-12/73

1800 returns

$\ln(\mathcal{L}) = 141107.095$ average $R^2 = 0.608$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD	-0.251 (0.028)	-0.045	0.037
ROIL	0.251 (0.009)	0.001	0.016
GCPI	-0.046 (0.037)	-0.009	0.024
SALE	-0.235 (0.032)	-0.000	0.013
RISK	-0.081 (0.016)	0.003	0.028
TERM	-0.019 (0.009)	-0.035	0.015
		total:	0.133

with APT restrictions relaxed: $\ln(\mathcal{L}) = 141975.141$

χ^2 P-value for LR test: 0.788

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 6

Estimates of 16 Factor APT Model, 1/74-12/78

1808 returns

$\ln(\mathcal{L}) = 142455.091$ average $R^2 = 0.651$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD	-0.121 (0.039)	-0.005	0.017
ROIL	-0.066 (0.018)	0.010	0.028
GCPI	-0.378 (0.037)	0.049	0.043
SALE	0.059 (0.038)	0.027	0.031
RISK	0.311 (0.029)	-0.033	0.016
TERM	-0.201 (0.012)	0.050	0.013
		total:	0.149

with APT restrictions relaxed: $\ln(\mathcal{L}) = 143473.602$

χ^2 P-value for LR test: 0.000

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 7

Correlation of Factors and Equal-Weighted NYSE, 1/64 - 12/88

factors which are contemporary residuals

PROD	ROIL	GCPI	SALE	RISK	TERM	ZEWM	
1.00	0.05	0.06	0.29	-0.13	-0.13	0.02	PROD
	1.00	0.16	0.05	-0.02	-0.06	0.05	ROIL
		1.00	-0.07	0.05	-0.05	-0.21	GCPI
			1.00	-0.18	-0.00	0.06	SALE
				1.00	-0.07	-0.14	RISK
					1.00	0.07	TERM
						1.00	ZEWM

factors which are news about mean of variable 0-12 months ahead

PROD*	ROIL*	GCPI*	SALE*	RISK*	TERM*	ZEWM	
1.00	-0.04	-0.19	0.76	-0.69	0.23	0.48	PROD*
	1.00	0.52	-0.21	0.26	-0.54	0.07	ROIL*
		1.00	-0.38	0.16	-0.36	-0.18	GCPI*
			1.00	-0.44	0.41	0.40	SALE*
				1.00	-0.38	-0.36	RISK*
					1.00	0.00	TERM*
						1.00	ZEWM

Table 8

Estimates of 16 Factor APT Model, 1/79-12/83

1797 returns

$\ln(\mathcal{L}) = 137282.520$ average $R^2 = 0.551$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.119 (0.037)	-0.022	0.041
ROIL*	-0.216 (0.030)	-0.001	0.016
GCPI*	-0.278 (0.037)	0.002	0.025
SALE*	0.204 (0.036)	0.036	0.023
RISK*	-0.118 (0.061)	0.006	0.020
TERM*	0.032 (0.068)	0.024	0.019
		total:	0.146

with APT restrictions relaxed: $\ln(\mathcal{L}) = 138129.939$

χ^2 P-value for LR test: 0.927

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 9

Estimates of 16 Factor APT Model, 1/84-12/88

1529 returns

$\ln(\mathcal{L}) = 124065.147$ average $R^2 = 0.576$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	-0.054 (0.029)	-0.028	0.070
ROIL*	-0.300 (0.057)	-0.014	0.016
GCPI*	-0.046 (0.047)	0.013	0.016
SALE*	-0.166 (0.043)	0.008	0.023
RISK*	-0.174 (0.037)	0.011	0.012
TERM*	0.206 (0.077)	-0.027	0.018
		total:	0.156

with APT restrictions relaxed: $\ln(\mathcal{L}) = 124860.991$

χ^2 P-value for LR test: 0.129

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 10

Estimates of 16 Factor APT Model, 1/64-12/68

1529 returns

$\ln(\mathcal{L}) = 122828.669$ average $R^2 = 0.509$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.008 (0.023)	0.002	0.039
ROIL*	0.124 (0.009)	-0.000	0.016
GCPI*	0.168 (0.028)	-0.012	0.029
SALE*	0.193 (0.032)	-0.016	0.016
RISK*	-0.022 (0.016)	0.097	0.041
TERM*	-0.067 (0.009)	0.024	0.023
		total:	0.164

with APT restrictions relaxed: $\ln(\mathcal{L}) = 123537.948$

χ^2 P-value for LR test: 0.959

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 11

Estimates of 16 Factor APT Model, 1/69-12/73

1800 returns

$\ln(\mathcal{L}) = 141102.873$ average $R^2 = 0.609$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	-0.174 (0.029)	0.014	0.146
ROIL*	0.175 (0.014)	0.004	0.016
GCPI*	-0.046 (0.035)	0.002	0.012
SALE*	-0.189 (0.034)	-0.037	0.014
RISK*	-0.016 (0.017)	-0.022	0.039
TERM*	-0.039 (0.010)	0.027	0.014
		total:	0.240

with APT restrictions relaxed: $\ln(\mathcal{L}) = 142003.209$

χ^2 P-value for LR test: 0.387

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 12

Estimates of 16 Factor APT Model, 1/74-12/78

1808 returns

$\ln(\mathcal{L}) = 142515.480$ average $R^2 = 0.652$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	-0.020 (0.034)	-0.005	0.128
ROIL*	-0.086 (0.021)	0.000	0.029
GCPI*	-0.281 (0.037)	0.020	0.026
SALE*	0.117 (0.031)	0.071	0.022
RISK*	0.178 (0.025)	-0.043	0.035
TERM*	-0.205 (0.011)	-0.020	0.021
		total:	0.261

with APT restrictions relaxed: $\ln(\mathcal{L}) = 143580.814$

χ^2 P-value for LR test: 0.000

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 13

Estimates of 16 Factor APT Model, 1/64-12/88

factor prices restricted to be the same for 5 sub-periods

$\ln(\mathcal{L}) = 667193.625$

name of factor	$\hat{\lambda}$
PROD*	-0.005 (0.015)
ROIL*	0.104 (0.007)
GCPI*	0.052 (0.019)
SALE*	0.064 (0.017)
RISK*	-0.005 (0.011)
TERM*	-0.068 (0.008)

with λ restrictions relaxed:	$\ln(\mathcal{L}) = 667794.640$
χ^2 P-value for LR test:	0.000

Notes: 10 latent factors were derived from the first 10 principal components of each excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance.

Table 14

Estimates of 16 Factor APT Model, 1/79-12/83

2123 returns

$\ln(\mathcal{L}) = 152587.326$ average $R^2 = 0.595$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.120 (0.035)	-0.023	0.042
ROIL*	-0.229 (0.031)	-0.002	0.017
GCPI*	-0.305 (0.038)	0.003	0.028
SALE*	0.215 (0.038)	0.042	0.026
RISK*	-0.122 (0.059)	0.006	0.021
TERM*	0.023 (0.072)	0.025	0.020
		total:	0.155

with APT restrictions relaxed: $\ln(\mathcal{L}) = 153720.179$

χ^2 P-value for LR test: 0.012

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance. Assets with missing returns were included if they had at least 30 observations in the 60 month sample period.

Table 15

Estimates of 15 Factor APT Model, 1/79-12/83

1797 returns

$\ln(\mathcal{L}) = 135324.253$ average $R^2 = 0.535$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.118 (0.038)	-0.021	0.041
ROIL*	-0.213 (0.030)	-0.000	0.016
GCPI*	-0.270 (0.036)	0.002	0.025
SALE*	0.202 (0.037)	0.035	0.023
RISK*	-0.120 (0.063)	0.006	0.020
TERM*	0.039 (0.069)	0.023	0.019
		total:	0.146

with APT restrictions relaxed: $\ln(\mathcal{L}) = 136142.601$

χ^2 P-value for LR test: 0.993

χ^2 P-value for adding 16th factor: 0.000

Notes: 9 latent factors were derived from the first 9 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 16

Estimates of 15 Factor APT Model, 1/79-12/83

1797 returns

$\ln(\mathcal{L}) = 135910.605$ average $R^2 = 0.539$

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.102 (0.036)	-0.017	0.041
GCPI*	-0.317 (0.038)	-0.004	0.021
SALE*	0.219 (0.037)	0.045	0.023
RISK*	-0.071 (0.060)	-0.002	0.014
TERM*	0.015 (0.067)	0.035	0.031
		total:	0.131

with APT restrictions relaxed: $\ln(\mathcal{L}) = 136745.636$

χ^2 P-value for LR test: 0.971

χ^2 P-value for adding 16th factor: 0.000

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 17

Estimates of 16 Factor APT Model, 1/79-12/83, allowing for firm size effect on mean return

1797 returns

$\ln(\mathcal{L}) = 137322.520$ average $R^2 = 0.551$

annualized (relative log) size effect: -0.014 (0.002)

mean absolute value of size effect on mean: 0.021

effect on smallest firm: 0.064
effect on largest firm: -0.089

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.153 (0.036)	0.002	0.041
ROIL*	-0.232 (0.029)	-0.000	0.016
GCPI*	-0.385 (0.039)	0.002	0.025
SALE*	0.296 (0.037)	-0.002	0.023
RISK*	-0.107 (0.059)	-0.000	0.020
TERM*	0.120 (0.067)	-0.001	0.020
		total:	0.145

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance.

Table 18

Estimates of 16 Factor APT Model, 1/79-12/83, allowing for firm size effect on mean return

2123 returns

$\ln(\mathcal{L}) = 152644.653$ average $R^2 = 0.595$

annualized (relative log) size effect: -0.013 (0.002)

mean absolute value of size effect on mean: 0.020

effect on smallest firm: 0.063
effect on largest firm: -0.084

name of factor	$\hat{\lambda}$	mean annual boost	average variance share
PROD*	0.135 (0.035)	0.001	0.042
ROIL*	-0.224 (0.031)	-0.000	0.017
GCPI*	-0.356 (0.038)	0.001	0.028
SALE*	0.284 (0.038)	-0.001	0.026
RISK*	-0.080 (0.056)	0.000	0.021
TERM*	0.094 (0.072)	-0.000	0.020
		total:	0.155

Notes: 10 latent factors were derived from the first 10 principal components of the excess return panel. Standard errors are in parentheses. Factors are scaled to unit variance. The contributions to R^2 are calculated by orthogonalizing the observable factors in their order of appearance. Assets with missing returns were included if they had at least 30 observations in the 60 month sample period.

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